# Bottom up Parsing

Goal:

Given an input string *w* and a grammar G,
construct a parse tree by starting at the leaves
and working to the root.

The parser repeatedly matches the right-hand side(rhs) of a production against a substring in the current right-sentential form.

At each match, it applies a reduction to build the parse tree.

- Each reduction replaces the matched substring with the nonterminal on the left-hand side(lhs) of the production
- Each reduction adds an internal node to the current parse tree
- The result is another right-sentential form

The final result is a rightmost derivation, in reverse.

## Non-recursive predictive parsing

Consider the grammar

| | |
|---|---|
| 1 | <goal> ::= a <A> <B> e |
| 2 | <A> ::= <A> b c |
| 3 | \| b |
| 4 | <B> ::= d |

and the input string *abbcde*

| Production | Sentential Form | Handle |
|---|---|---|
| - | abbcde | 3,2 |
| 3 | a <A> bcde | 2,4 |
| 2 | a <A> de | 4,3 |
| 4 | a <A> <B> e | 1,4 |
| 1 | <goal> | - |

the notation *x,y* in handle field means $x^{th}$ production rule and the end position of the rule is *y* in the current sentential form.

The trick appears to be scanning the input and finding valid sentential forms.

## Handles

We trying to find a substring $\alpha$ of the current right-sentential form where:

- $\alpha$ matches some production A ::= $\alpha$
- reducing $\alpha$ to A is one step in the reverse of a rightmost derivation.

We call such a string a *handle*

Formally,

- a handle of a right-sentential form $\gamma$ is a production A ::= $\beta$ and a position in $\gamma$ where $\beta$ may be found.
- If (A ::= $\beta$, k) is a handle, then replacing the $\beta$ in $\gamma$ at position k with A produces the previous right-sentential form in a rightmost derivation of $\gamma$. Because $\gamma$ is a right-sentential form, the substring to the right of a handle contains only terminal symbols

## Handles

Provable fact:

If G is unambiguous, then every right-sentential form has a unique handle.

Proof: (by definition)

1. G is unambiguous => rightmost derivation is unique
2. => a unique production A ::= β applied to take $\gamma_{i-1}$ to $\gamma_i$
3. => a unique position k at which A ::= β is applied
4. => a handle (A ::= β, k) is unique

## Example

The left-recursive expression grammar
(original form, before left factoring)

| | |
|---|---|
| 1 | <goal> ::= <expr> |
| 2 | <expr> ::= <expr> + <term> |
| 3 | \| <expr> – <term> |
| 4 | \| <term> |
| 5 | <term> ::= <term> * <factor> |
| 6 | \| <term> / <factor> |
| 7 | \| <factor> |
| 8 | <factor> ::= number |
| 9 | \| id |

| Production | Sentential Form | Handle |
|---|---|---|
| - | <goal> | - |
| 1 | <expr> | 1,1 |
| 3 | <expr> – <term> | 3,3 |
| 5 | <expr> – <term> * <factor> | 5,5 |
| 9 | <expr> – <term> * id | 9,3 |
| 7 | <expr> – <factor> * id | 7,3 |
| 8 | <expr> – num * id | 8,1 |
| 4 | <term> – num * id | 4,1 |
| 7 | <factor> – num * id | 7,1 |
| 9 | id – num * id | 9,1 |

## Handle-pruning

The process we use to construct a bottom-up parsing tree is called handle-pruning.

To construct a rightmost derivation

$S = \gamma_0 => \gamma_1 => \gamma_2 => \ldots => \gamma_{n-1} => \gamma_n = w,$

We set i to n and apply the following simple algorithm

do i = n to 1 by – 1

(1) find the handle ($A_i = \beta_i$, $k_i$) in $\gamma_i$

(2) replace $\beta_i$ with $A_i$ to generate $\gamma_i - 1$

This takes 2n steps, where n is the length of the derivation

## Shift-reduce parsing

One scheme to implement a handle-pruning, bottom-up parser is called a shift-reduce parser.

Shift-reduce parsers use a stack and an input buffer

1. initialize stack with $
2. Repeat until the top of the stack is the goal symbol and the input token is "end of file"
   a) find the handle
      if we don't have a handle on top of the stack, shift an input symbol onto the stack
   b) prune the handle
      if we have a handle (A ::= $\beta$, k) on the stack, reduce
      i) pop | $\beta$ | symbols off the stack
      ii) push A onto the stack

## Back to “x – 2 * y”

| Stack | Input | Handle | Action |
|---|---|---|---|
| $ | id – num * id | none | shift |
| $ id | – num * id | 9,1 | reduce 9 |
| $ <factor> | – num * id | 7,1 | reduce 7 |
| $ <term> | – num * id | 4,1 | reduce 4 |
| $ <expr> | – num * id | none | shift |
| $ <expr> – | num * id | none | shift |
| $ <expr> – num | * id | 8,3 | reduce 8 |
| $ <expr> – <factor> | * id | 7,3 | reduce 7 |
| $ <expr> – <term> | * id | none | shift |
| $ <expr> – <term> * | id | none | shift |
| $ <expr> – <term> * id | | 9,5 | reduce 9 |
| $ <expr> – <term> * <factor> | | 5,5 | reduce 5 |
| $ <expr> – <term> | | 3,3 | reduce 3 |
| $ <expr> | | 1,1 | reduce 1 |
| $ <goal> | | none | accept |

1. shift until top of stack is the right end of a handle
2. find the left end of the handle and reduce

5 shifts + 9 reduces + 1 accept

## Shift-reduce parsing

Shift-reduce parsers

- are simple to understand
- have a simple, table-driven, shift-reduce skeleton
- encode grammatical knowledge in tables

A shift-reduce parser has just four canonical actions:

1. shift – next input symbol is shifted onto the top of the stack
2. reduce – right end of handle is on top of stack;
   locate left end of handle within the stack;
   pop handle off stack and push appropriate non-terminal *lhs*
3. accept – terminate parsing and signal success
4. error – call an error recovery routine

## LR(1) grammars

Informally, we say that a grammar G is LR(1) if,

given a rightmost derivation

$$S = \gamma_0 => \gamma_1 => \gamma_2 => \ldots => \gamma_{n-1} => \gamma_n = w,$$

we can, for each right-sentential form in the derivation,

1. isolate the handle of each right-sentential form, and
2. determine the production by which to reduce

by scanning $\gamma_i$ from left to right, going at most 1 symbol beyond the right end of the handle of $\gamma_i$

Formality will come later.

## Why study LR(1) grammars?

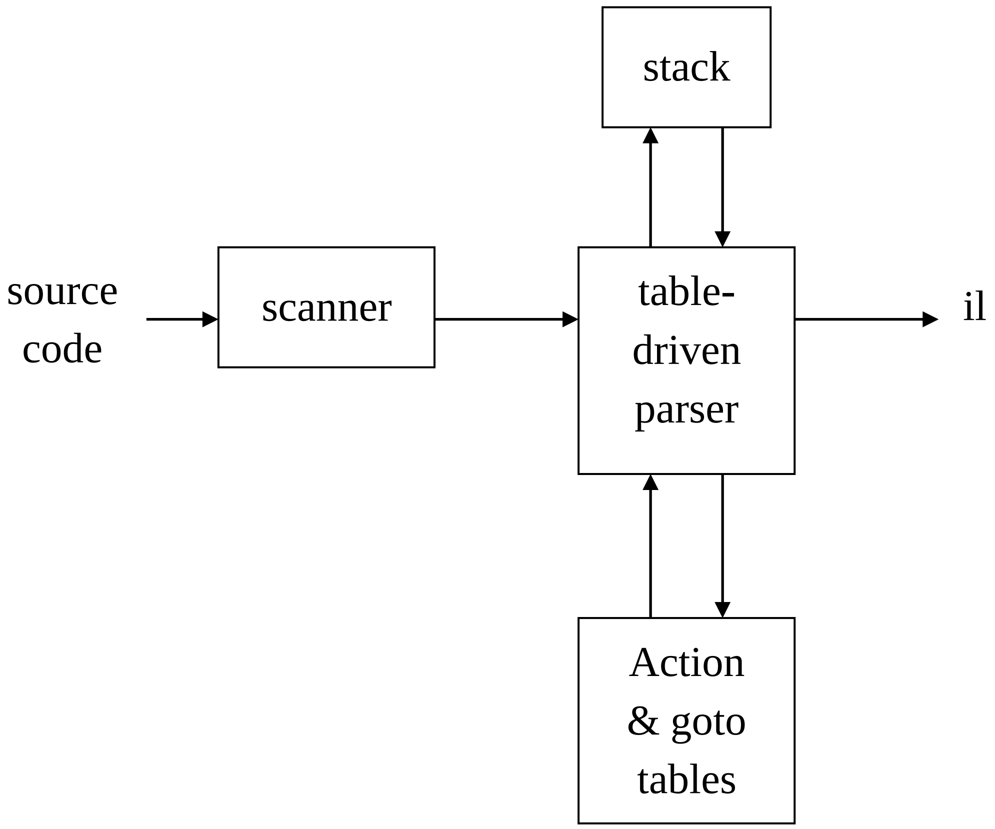
LR(1) grammars are often used to construct shift-reduce parsers.

We call these parsers LR(1) parsers.

- everyone’s favorate parser(EFP)
- virtually all context-free programming language constructs can be expressed in an LR(1) form
- LR grammars are the most general grammars that can be parsed by a non-backtracking, shift-reduce parser
- efficient shift-reduce parsers can be implemented for LR(1) grammars
- LR parsers detect an error as soon as possible in a left-to-right scan of the input
- LR grammars describe a proper superset of the languages recognized by LL(predictive) parsers

## Table-driven LR(1) parsing

A table-driven LR(1) parser looks like




Stack two items per state: state and symbol

Table building tools are readily available(yacc)

We’ll learn how to build these tables by hand!

## LR(1) parsing

The skeleton parser:

```
token = next_token()
repeat forever
    s = top of stack
    if action[s, token] = “shift s_i” then
        push token
        push s_i
        token = next_token()
    else if action[s, token] = “reduce A ::= β” then
        pop 2 * | β | symbols
        s = top of stack
        push A
        push goto[s, A]
    else if action[s, token] = “accept” then
        return
    else error()
```

This takes k shifts, l reduces, and 1 accept, where k is the length of the input string and l is the length of the reverse rightmost derivation.

## Example tables

| | Action | | | | GOTO | | |
|---|---|---|---|---|---|---|---|
| | Id | + | * | $ | <expr> | <term> | <factor> |
| $S_0$ | s4 | – | – | – | 1 | 2 | 3 |
| $S_1$ | – | – | – | acc | – | – | – |
| $S_2$ | – | S5 | – | r3 | – | – | – |
| $S_3$ | – | r5 | s6 | r5 | – | – | – |
| $S_4$ | – | r6 | s6 | r5 | – | – | – |
| $S_5$ | s4 | – | – | – | 7 | 2 | 3 |
| $S_6$ | s4 | – | – | – | – | 8 | 3 |
| $S_7$ | – | – | – | r2 | – | – | – |
| $S_8$ | – | r4 | – | r4 | – | – | – |

## The Grammar

1 <goal> ::= <expr>
2 <expr> ::= <expr> + <term>
3 | <term>
4 <term> ::= <term> * <factor>
5 | <factor>
6 <factor> ::= id