CS572 Intelligent Robotics: Lecture notes (2018/10/31)

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Linear system model: $X_{t+1} = A_t x_t + B_t u_t$

$$y_t = H_t x_t + v_t$$

$$E[v_t] = 0$$
$$E[w_t] = 0$$

$$C_v(t,s) = \begin{cases} 0, & \forall_t \neq s \\ R_t, & \text{t=s} \end{cases}$$

$$C_w(t,s) = \begin{cases} 0, & \forall_t \neq s \\ Q_t, & \text{t=s} \end{cases}$$

$$C_{wv}(t,s) = 0, \quad \forall_t, \forall_s$$

$$t^{-1} \rightarrow t$$
$$\widehat{x}_{t-1} \rightarrow \widehat{x}_{t|t-1} \rightarrow \widehat{x}_t$$

Using only model estimation in transition $\widehat{x}_{t-1} \rightarrow \widehat{x}_{t|t-1}$

$$\widehat{x}_{t|t-1} = A\widehat{x}_{t-1}$$
$$\widehat{y}_t = H\widehat{x}_{t|t-1}$$
$$\widehat{x}_t = \widehat{x}_{t|t-1} + K_t(y_t - H\widehat{x}_{t|t-1})$$

$$\overline{J} = E[(\widehat{x}_t - x_t)^T(\widehat{x}_t - x_t)]$$
$$e_t = \widehat{x}_t - x_t = \widehat{x}_{t|t-1} + K_t(y_t - H\widehat{x}_{t|t-1}) - x_t = (I - KH)\Sigma_t + K_t V_t$$

where $\Sigma_t = \widehat{x}_{t|t-1} - x_t$ A priori estimation error

(e_t Error from posteriori estimate)

$$e_t{}^T e_t = \Sigma^T\Sigma + \Sigma^T H^T K^T K H \Sigma - 2\Sigma^T K H \Sigma + 2\Sigma^T K V - 2V^T K^T K H \Sigma + V^T K^T K V$$

$$\frac{d}{dK}E[.] = 0 \rightarrow E\frac{d}{dK}$$

$$\textbf{i)}\quad f = \bar{a}^T K \bar{b}, \quad \frac{df}{dK} = \bar{a}\bar{b}^T$$
$$\textbf{ii)}\quad g = \bar{a}^T K^T K \bar{b}, \quad \frac{dg}{dK} = K\bar{b}\bar{a}^T + K\bar{a}\bar{b}^T$$

$$E\frac{d}{dK}e_t^T e_t = E[2KH\Sigma\Sigma^T H^T - 2(KH\Sigma V^T + KV\Sigma^T H^T) + 2KVV^T + 2(\Sigma V^T - \Sigma\Sigma^T H^T)] = 0$$

$$KHE[\Sigma\Sigma^T]H^T - KHE[\Sigma V^T] - KE[V\Sigma^T]H^T + KE[VV^T] + E[\Sigma V^T] - E[\Sigma\Sigma^T]H^T = 0$$

$$(\text{where} \quad VV^T \to R)$$

$$E[\Sigma V^T] = E[(\widehat{x}_{t|t-1} - x_t){V_t}^T]$$
$$= E[(A_{t-1}\widehat{x}_{t-1}{V_t}^T] - E[x_t {V_t}^T]$$

$E[\Sigma\Sigma^T] \triangleq P_{t|t-1} = E[(\widehat{x}_{t|t-1} - x_t)(\widehat{x}_{t|t-1} - x_t)^T]$ Error covariance of a priori estimation

$$KHP_{t|t-1}H^T + KR - P_{t|t-1}H^T = 0$$
$$K_t = P_{t|t-1}{H_t}^T[H_t P_{t|t-1}{H_t}^T + R_t]^{-1}$$ KALMAN GAIN

$P_t \triangleq E[(\widehat{x}_t - x_t)(\widehat{x}_t - x_t)^T] = E[e_t {e_t}^T]$ Error covariance of a posteriori estimation

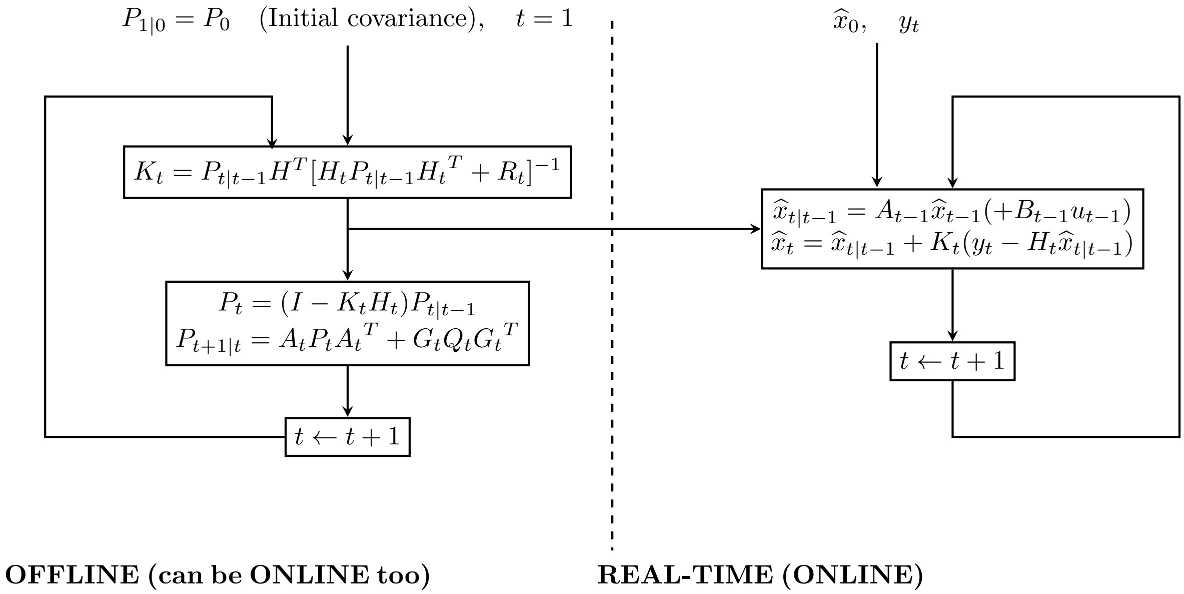
$$P_t = E[((I - KH)\Sigma + KV)((I - KH)\Sigma + KV)^T]$$
$$= (I - KH)E[\Sigma\Sigma^T](I - KH)^T + KE[VV^T]K^T$$
$$= (I - KH)P_{t|t-1}$$

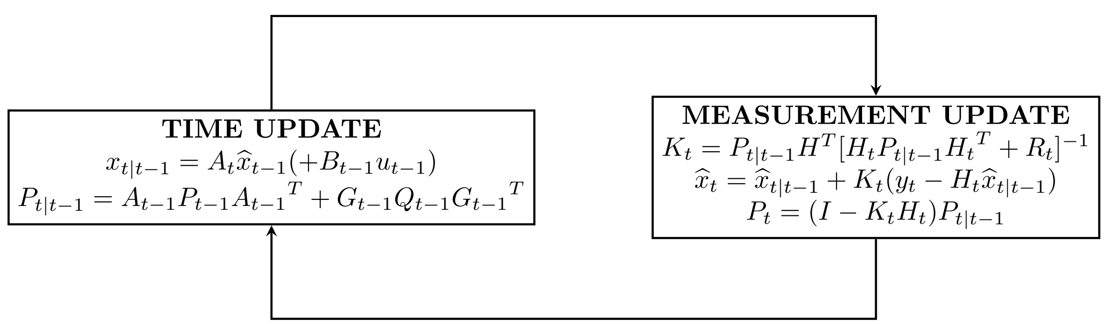
$$P_t = (I - KH)P_{t|t-1}(I - KH)^T + KRK^T$$
$$= (I - KH)P_{t|t-1}$$

$$\Sigma_{t+1} = \widehat{x}_{t+1|t} - x_{t+1} = A_t\widehat{x}_t - (A_t x_t + G_t w_t)$$
$$= Ae_t - Gw_t$$

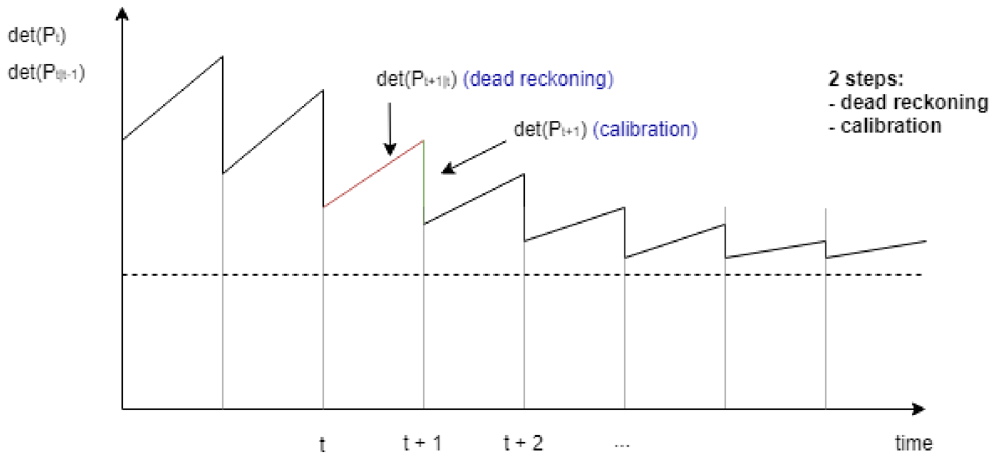
$$P_{t+1|t} = E[\Sigma_{t+1}{\Sigma_{t+1}}^T] = AE[ee^T]A^T - GE[we^T]A^T - AE[ew^T]G^T + GE[ww^T]G^T$$

$$P_{t+1|t} = A_t P_t {A_t}^T + G_t Q_t {G_t}^T$$

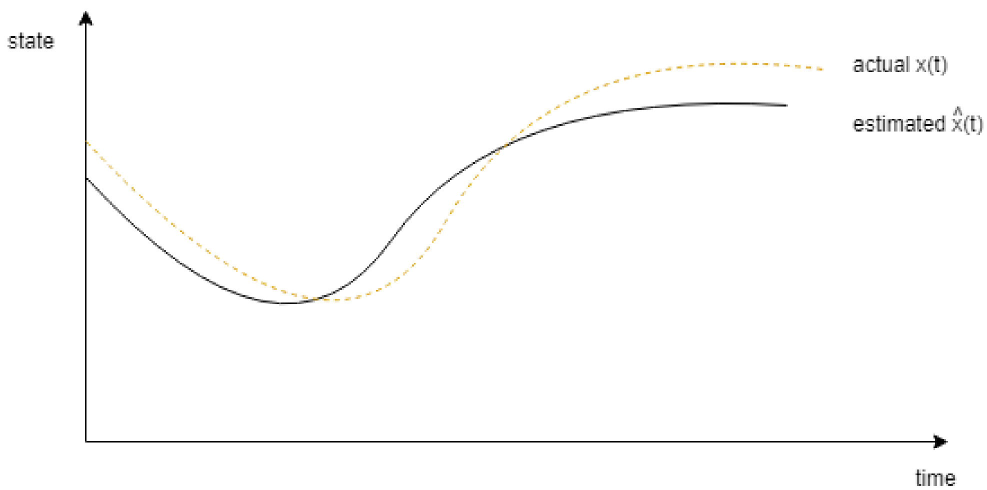

$P_{1|0} = P_0$ (Initial covariance), $t = 1$
$K_t = P_{t|t-1} H^T [H_t P_{t|t-1} {H_t}^T + R_t]^{-1}$
$P_t = (I - K_t H_t) P_{t|t-1}$
$P_{t+1|t} = A_t P_t {A_t}^T + G_t Q_t {G_t}^T$
$t \leftarrow t + 1$
$\widehat{x}_0, \quad y_t$
$\widehat{x}_{t|t-1} = A_{t-1} \widehat{x}_{t-1} (+B_{t-1} u_{t-1})$
$\widehat{x}_t = \widehat{x}_{t|t-1} + K_t (y_t - H_t \widehat{x}_{t|t-1})$
$t \leftarrow t + 1$
OFFLINE (can be ONLINE too)
REAL-TIME (ONLINE)


TIME UPDATE
$x_{t|t-1} = A_t \widehat{x}_{t-1} (+B_{t-1} u_{t-1})$
$P_{t|t-1} = A_{t-1} P_{t-1} {A_{t-1}}^T + G_{t-1} Q_{t-1} {G_{t-1}}^T$
MEASUREMENT UPDATE
$K_t = P_{t|t-1} H^T [H_t P_{t|t-1} {H_t}^T + R_t]^{-1}$
$\widehat{x}_t = \widehat{x}_{t|t-1} + K_t (y_t - H_t \widehat{x}_{t|t-1})$
$P_t = (I - K_t H_t) P_{t|t-1}$

Dead Reckoning - the process of estimating current position based on a previously determined position



EXTENDED KALMAN FILTER



$$x_{t+1} = f(x_t, t) + w_t$$

$$x_{t+1} - f(x_t, t) = w_t$$

$$x_{t+1} - f(x_t, t) \sim P w_t$$

$$y_t = h(x_t, t) + v_t$$

$$y_t - h(x_t, t) = v_t$$

$$y_t - h(x_t, t) \sim P v_t$$

$$\left.\frac{\partial f}{\partial x}\right|_{x=\widehat{x}_t} = \widehat{A}(\widehat{x}_t, t) = \widehat{A}_t$$

$$\left.\frac{\partial h}{\partial x}\right|_{x=\widehat{x}_t} = \widehat{H}(\widehat{x}_t, t) = \widehat{H}_t$$