LECTURE NOTE

2018 FALL SEMESTER
9. 17

- Expectation Maximum
- Optimal Control

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Expectation Maximum

- The EM is an effective iterative method that is used to estimate the most probable model when certain information is hidden.

- Usually we use the Maximum Likelihood estimation method to estimate the parameters of the model that are appropriate for the observed data.

- $P(D,\theta)$ Is the likelihood for θ when there is observation data D

- Log likelihood function $L(\theta) = \log P(D,\theta)$

- What we want is to find the model θ that maximizes $L(\theta)$

Expectation Maximum

$D = \{ y_1 ,,, y_m \}$ Want to divide into 2 groups

Gaussian
- group 1 ~ $N(\mu_1 , \sigma_1^2)$
- group 2 ~ $N(\mu_2 , \sigma_2^2)$

$$P(group\text{'s } taken\ on) = \pi$$

Problem : We have to find parameter values which one don't know

Introduce hidden valuable

$$H = \{ H_1 , H_2 ,,, H_m \}$$

H_i = 1 when y_i is associated with g_1

H_i = 0 when y_i is associated with g_2

a. Guess an initial parameter value θ_0 : $t = 0$

b. Compute $P(H | \theta_t , D) \log P(D, H | \theta)$

c. $\theta_{t_1} = \arg\max_\theta E_p(H \mid \theta_i , D)$

REPEAT

Expectation Maximum

a. Guess an initial parameter value θ_0 : $t = 0$

b. Compute $P(H|\theta_t, D)\log P(D, H|\theta)$

$$P(H_i = 1 \mid \theta_t, D) = P(H_i = 1 \mid \theta_t, y_i)$$

Bayesian's rule

$$= \frac{P(y_i \mid H_i = 1, \theta) P(H_i = 1 \mid \theta)}{P(y_i|\theta)}$$

$$= \frac{\pi N(y_i; \mu_1, \sigma_1)}{\underbrace{\pi N(y_i; \mu_1, \sigma_1)}_{\text{group1}} + \underbrace{(1-\pi) N(y_i; \mu_2, \sigma_2)}_{\text{group2}}} = \sigma_i$$

$$P(H_i = 0 \mid \theta_t, D) = P(H_i = 0 \mid \theta_t, y_i) = 1 - \gamma_i$$

Soft assignment

Expectation Maximum

c. $\theta_{t_1} = \arg\max_\theta E_p(H \mid \theta_i , D)$

Define $\theta_t := \arg\max_\theta E_p(H \mid \theta_t , D) \log P(D, H \mid \theta)$

$$= \arg\max_\theta E_p(H \mid \theta_t , D)(\log P(H \mid \theta) + \log P(D \mid H, \theta))$$

$$= \arg\max_\theta E_p(H \mid \theta_t , D) \sum_{i=1}^{m} \log \pi^{H_1}(1-\pi)^{H_2}$$

$$+ \log N(y_i;\mu_1,\sigma_1)^{H_1} N(y_i;\mu_2,\sigma_2)^{1-H_1}$$

$$= \arg\max_\theta E_p(H \mid \theta_t , D) \sum_{i=1}^{m} [H_i \log \pi + (1-H_i)\log(1-\pi)$$

$$+ H_i\left(\frac{y_i-\mu_1}{\sigma_1}\right)^2 + H_i \log \ldots$$

$$+ (1-H_i)\left(\frac{y_i-\mu_2}{\sigma_2}\right)^2 + (1-H_i)\log \ldots]$$

Does not include π

Expectation Maximum

$$\pi = \text{argmax}_\pi E_p(H|\theta_t, D) H_i \log\pi + (1 - H_i)(1 - \pi)$$

$$= \text{argmax}_\pi \sum_{i=1}^{m} \underbrace{P(H_i = 1|\theta_t, y_i)}_{\gamma_i} \log\pi + \underbrace{P(H_i = 0|\theta_t, y_i)}_{1-\gamma_i} \log(1 - \pi)$$

$$\frac{d}{dt} = \sum_{i=1}^{m} \left(\frac{\gamma_i}{\pi} - \frac{(1-\gamma_i)}{1-\pi}\right) = 0 \quad \Rightarrow \quad \pi = \frac{1}{m}\sum_{i=1}^{m} \gamma_i$$

Soft assumption

Because it is a hidden variable, it can not be estimated accurately.

$$\mu_1 = \frac{\Sigma\gamma_i y_i}{\Sigma\gamma_i} \qquad \boxed{\sigma_1^2 = \frac{\Sigma\gamma_i (y_i - \mu_1)^2}{\Sigma\gamma_i}}$$

Fundamental concept of EM

Optimal Control

- Optimal Control addresses the problem of finding control laws for a particular system and achieves certain optimization criteria

$$X = f(x,t) \text{ or } f(x,u,t)$$

Object function : $\min_{u(t)} \int_{t_0}^{t_1} L(X,u,t)\,dt + \Phi(X(t_1), t_1)$

- Finding $u(t)$ that minimizes the object function is optimal control.

- A special case of the general nonlinear optimal control problem given in the previous section is the linear quadratic (LQ) optimal control problem.

$$\min_{u(t)} \frac{1}{2} \int_{t_0}^{t_1} \underbrace{X^T \overset{>0}{Q} X}_{\text{state}} + \underbrace{\overset{>0}{u^T u}}_{\text{input vector}} \, dt + \frac{1}{2} X^T(t_1) R X(t_1) \quad \text{s.t} \quad \boxed{X(t) = A(t)X(t) + B(t)u(t)}$$

Constraint

State : movement minimize **Input vector** : actuator power => energy minimize

State + Input vector => we can reach state with minimum energy

Optimal Control – Riccati Equation

- When the order of the system is large, the determination of the state-transition matrix becomes a tedious and time-consuming task

- An alternative method of finding the optimal feedback gain matrix utilizes the Ricatti equation

Riccati equation $\dot{K}(t) = -A^T(t)K(t) - K(t)A(t) + K(t)B(t)B(t)^TK(t) - Q(t)$

symmetric

$$\int_{t_0}^{t_1} \frac{d}{dt} X^T(t)K(t)X(t) = \int_{t_0}^{t_1} \dot{X}^T KX + X^T \dot{K} X + X^T K \dot{X} \, dt$$

Thank you

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