TELEROBOTICS AND CONTROL LABORATORY

Lecture Note
2018.10.22
Recursive Least-Squares Algorithm

Team 3
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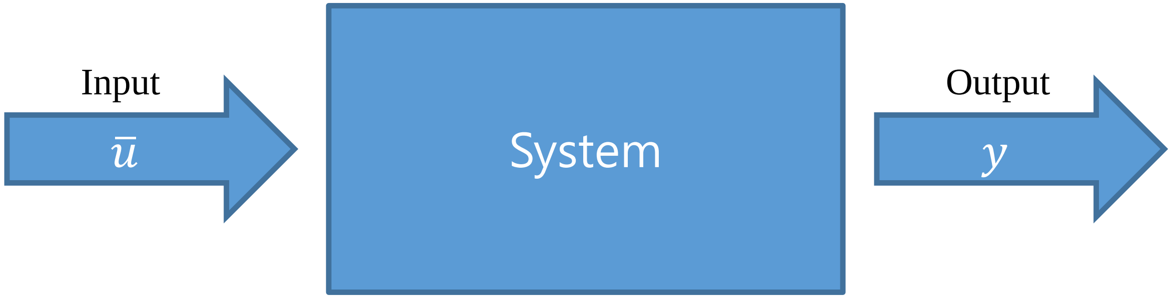
Simple system



$$\hat{y} = \bar{\theta}^T \bar{u}$$

$$V(\theta) = \frac{1}{N}\sum_{t=1}^{N}(\hat{y} - y)^2 \quad (cost - function)$$

$$\hat{\theta} = argminV_N(\theta), \hat{\theta} = PB$$

$$P = [\sum_{i=1}^{N} \bar{u}(i)\,\bar{u}^T(i)]^{-1} = [UU^T]^{-1}, B = \sum_{i=1}^{N}(y(i)\bar{u}(i))^2$$

Recursive Least-Squares Estimation (Split B_t, P_t)

$$1)\quad B_t = \sum_{i=1}^{t} y(i)\bar{u}(i) = \sum_{i=1}^{t-1} y(i)\bar{u}(i) + y(t)\bar{u}(t)$$

$$B_t = B_{t-1} + y(t)\bar{u}(t)$$

$$2)\quad P_t^{-1} = \sum_{i=1}^{t} \bar{u}(i)\,\bar{u}^T(i) = P_{t-1}^{-1} + \bar{u}(t)\bar{u}^T(i)$$

Multiply P_t on the left and P_{t-1} on the right

$$P_t P_t^{-1} P_{t-1} = P_t P_{t-1}^{-1} P_{t-1} + P_t \bar{u}(t)\bar{u}^T(i) P_{t-1}$$

$$P_{t-1} = P_t + P_t \bar{u}(t)\bar{u}^T(i) P_{t-1}$$

Multiply $\bar{u}(t)$ on the right

$$P_{t-1}\bar{u}(t) = P_t\bar{u}(t) + P_t\bar{u}(t)\bar{u}^T(i)P_{t-1}\bar{u}(t)$$

Recursive Least-Squares Estimation (Split B_t, P_t)

$$P_{t-1}\bar{u}(t) = P_t\bar{u}(t)(1 + \bar{u}^T(i)P_{t-1}\bar{u}(t))$$

$$P_t\bar{u}(t) = \frac{P_{t-1}\bar{u}(t)}{1 + \bar{u}^T(i)P_{t-1}\bar{u}(t)}$$

Multiply $\bar{u}^T(i)P_{t-1}$ on the right

$$P_t\bar{u}(t)\bar{u}^T(i)P_{t-1} = \frac{P_{t-1}\bar{u}(t)\bar{u}^T(i)P_{t-1}}{1+\bar{u}^T(i)P_{t-1}\bar{u}(t)} = P_{t-1} - P_t$$

$$P_t = P_{t-1} - \frac{P_{t-1}\bar{u}(t)\bar{u}^T(i)P_{t-1}}{1 + \bar{u}^T(i)P_{t-1}\bar{u}(t)}$$

can be obtained by using previous value P_{t-1} and the new data $\bar{u}(t)$

Design RLS (Recursive Least-Squares) Estimator

$$[A+BCD]^{-1} = A^{-1} + A^{-1}B[DA^{-1}B+C^{-1}]DA^{-1}$$

This is a inverse formula.

A and C is square matrix(Even though the right eqn looks more compulicated, it is easier to compute)

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K_t(y_t - \bar{u}^T(t)\hat{\theta}(t-1))$$

K_t is a type of gain for correcting prediction error.

$$\hat{\theta}(t) = P_t B_t, \qquad \hat{\theta}(t-1) = P_{t-1}B_{t-1}$$

Using the previously obtained P_t and B_t, we can summarize the eqn

$$\hat{\theta}(t) - \hat{\theta}(t-1) = P_t B_t - P_{t-1}B_{t-1} = \left(P_{t-1} - \frac{P_{t-1}\bar{u}(t)\bar{u}^T(i)P_{t-1}}{1+\bar{u}^T(i)P_{t-1}\bar{u}(t)}\right)\left(B_{t-1} + y(t)\bar{u}(t)\right) - P_{t-1}B_{t-1}$$

Finally, the K_t value can be obtained

$$\boxed{K_t = \frac{P_{t-1}\bar{u}(t)}{1+\bar{u}^T(i)P_{t-1}\bar{u}(t)}}$$

Design RLS (Recursive Least-Squares) Estimator

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P_{t-1}\bar{u}(t)}{1 + \bar{u}^T(i)P_{t-1}\bar{u}(t)}(y_t - \bar{u}^T(t)\hat{\theta}(t-1))$$

$The\ cost - function\ is\ as\ follows$

$$J_t(\bar{\theta}) = \frac{1}{2}\sum_{i=1t}(y(i) - \bar{u}^T(i)\bar{\theta})^2 + \frac{1}{2}(\theta - \overline{\theta_0})^T P_0^{-1}(\theta - \overline{\theta_0})$$

$This\ cost - function\ inclues\ squared\ estimation\ error\ term\ and\ weighted\ squared\ distance\ term$

$$\frac{dJ}{d\theta} = 0\ (in\ this\ condition)$$

1) $As\ t \rightarrow \infty, A\ gets\ lager$

2) $small\ , \theta\ is\ pulled\ towards\ \hat{\theta}(0).$

3) $If\ \lambda's\ of\ P_0^{-1}\ are\ small, \theta\ tends\ to\ change\ more\ quickly.$

4) $Initial\ P_0\ represents\ the\ level\ of\ confidence\ for\ the\ initial\ parameter\ \hat{\theta}(0).$