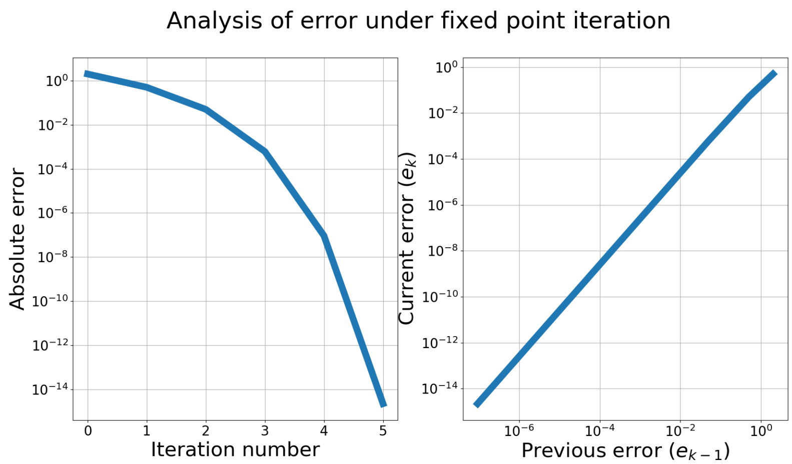
HOMEWORK 3

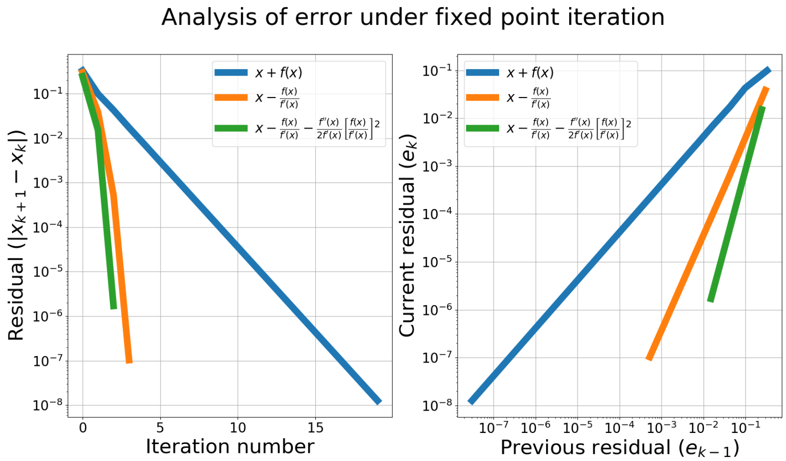
HUNTER SCHWARTZ

2.



(a) We have analyzed this problem of finding the fixed point of $g(x) = \frac{x}{2} + \frac{2}{x}$ before, concluding that the order of convergence should be equal to 2 since at the fixed point $z = 2$, we have $g'(z) = 0$. In the plots above, we began the fixed point iteration with an initial condition of $x_0 = 4$. We see based on the left plot that we have indeed converged. Furthermore, we see that the log-log plot comparing the error of consecutive iterations is a straight line, which is useful to us for determining the order of converge. This is because if we have $e_{k+1} = Ce_k^q$, then $\log(e_{k+1}) = q\log(e_k) + \log(C)$. Thus, if the order of convergence is a constant q, then it is represented by the slope of the straight line comparing these consecutive errors. For instance, from the plot, we have roughly the points $(10^{-6}, 10^{-12})$ and $(10^{-4}, 10^{-8})$, so we can estimate the slope to be $q = \frac{\log(10^{-12})-\log(10^{-8})}{\log(10^{-6})-\log(10^{-4})} = \frac{-12+8}{-6+4} = 2$. Thus, the data supports our claim that the order of convergence is 2.

Date: September 23, 2020.



(b) In the plots above, we started the iterations from the point $x_0 = 4$. We can see from the plot on the left that as we add more terms to our fixed point function, we are converging faster/in fewer iterations. We should expect this, since we have seen that with a typical fixed point we can expect linear convergence, but with Newton's method we can get up to quadratic convergence, and with more specific terms even higher orders of convergence. This is clearly demonstrated in the plot on the right comparing the consecutive residuals. As above, the slope of these lines represent the order of convergence of the iteration. We can estimate the slope of the blue line as close to 1, the orange line as close to 2, and the green line as close to 3, which agrees with our theory about the expected behavior of these functions.