Homework 9

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Problem 1
(d)

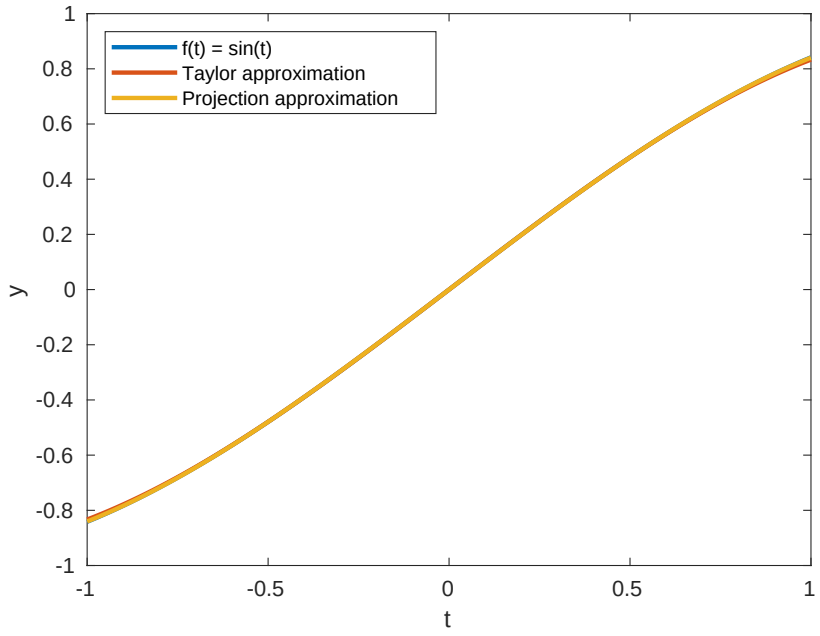

Figure 1: Approximations of $f(t) = \sin(t)$ by different methods.

We can see from the overlaid plot of $f(t)$ and its approximations in Figure 1 that both methods yield very similar results. In order to make the relationships more clear, the absolute error is plotted in Figure 2.

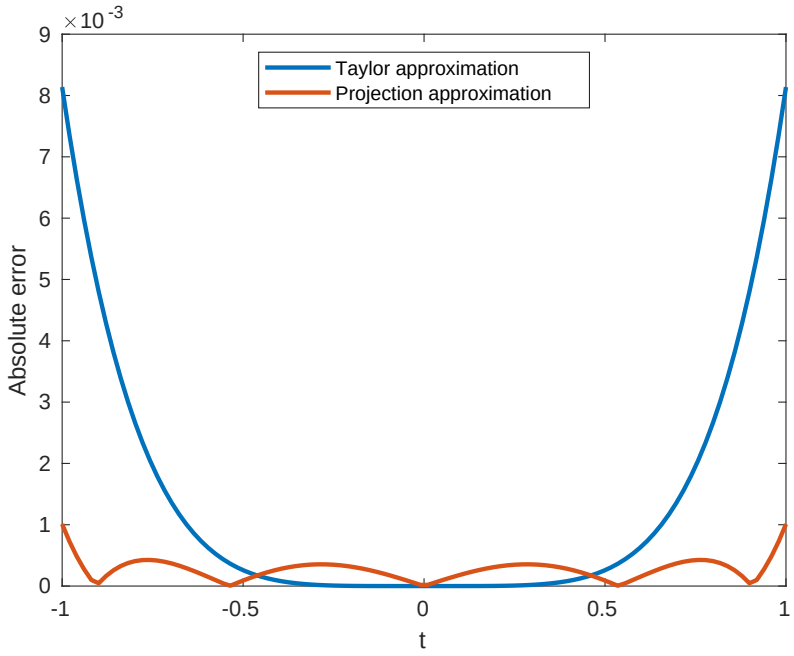

Figure 2: Absolute error of different approximation methods.

In Figure 2, it becomes very clear that the projection method approximates the sine function over this interval much better than the Taylor approximation does. This isn't true uniformly, since in the middle of the interval from about x=-0.5 to x=0.5, the error in the projection method is actually greater than in the Taylor approximation. However, the difference is most clear towards the ends of the interval, where the error of both methods attains its maximum. If $T(x)$ represents the Taylor polynomial and $p(x)$ represents the polynomial produced from the projection map, we can calculate that

$$\|\sin - T\|_\infty = 0.0081, \text{ and } \|\sin - p\|_\infty = 0.0010.$$

Thus, the projection method works better, which makes sense since it is optimized for this interval, whereas we will find the same Taylor polynomial centered at 0 for any interval.