

Homework 9

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Problem 2

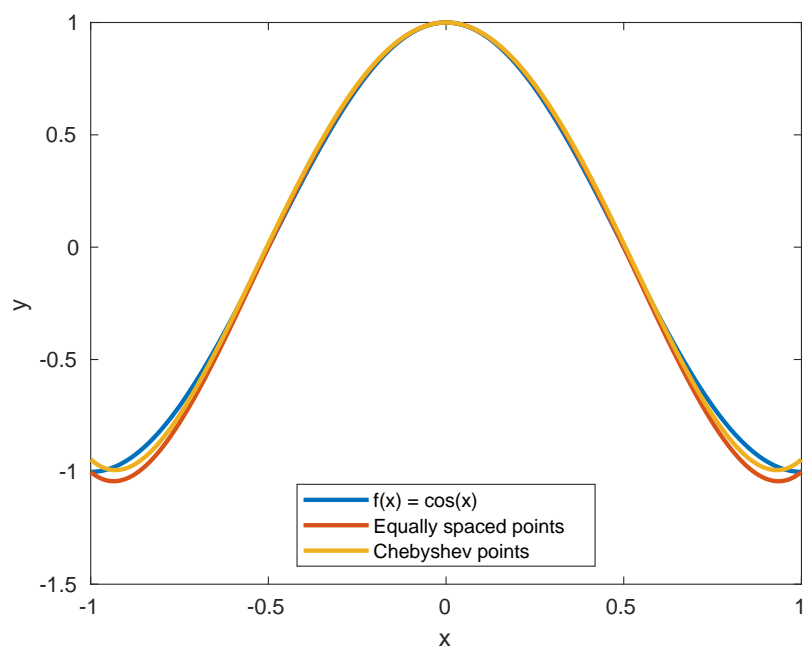


Figure 1: Approximation of $f(x) = \cos(\pi x)$ by interpolating polynomials with different choice of interpolation points.

In this plot, we see both f and two approximations of f overlaid. All three functions are very similar, so both approximation methods appear good in this case. However, we can consider the following plot of the absolute error to see more detail.

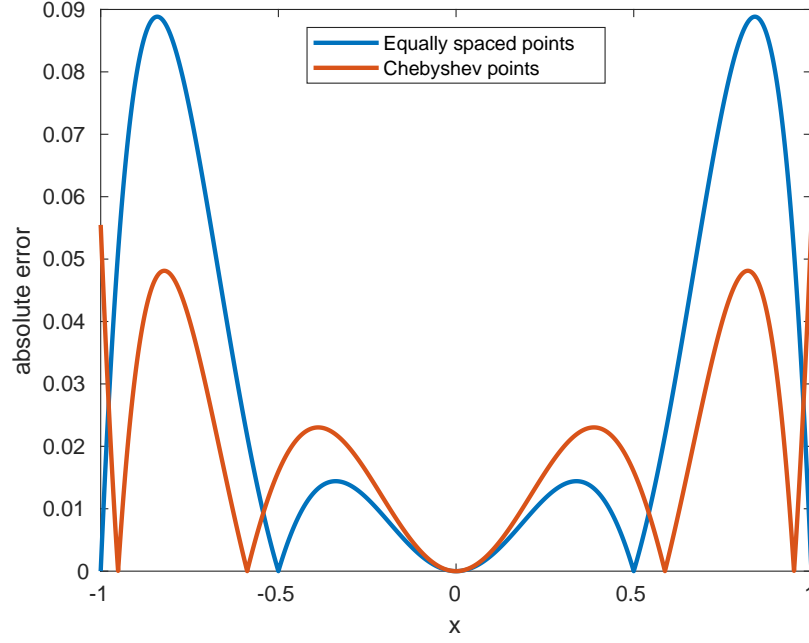


Figure 2: Absolute error between $f(x)$ and approximations.

From the plot of the error, we can visually confirm that the maximum error is larger for the equally spaced points than it is for the Chebyshev points. We can also verify that the error is 0 at the respective interpolation points for each polynomial, as we expect. We find the following values for the norm of the error:

$$\|p_{\text{equal}} - f\|_{\infty} = 0.0888$$

$$\|p_{\text{equal}} - f\|_2 = 0.0604$$

$$\|p_{\text{Chebyshev}} - f\|_{\infty} = 0.0555$$

$$\|p_{\text{Chebyshev}} - f\|_2 = 0.0341$$

In this case, choosing the Chebyshev points to interpolate over decreases the error of the approximation. While this is not always true, it is a typical case which is why we often prefer to interpolate over the Chebyshev points given the option.