Bartik Instruments: What, When, Why, and How

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Motivation: local labor market approaches

$$y_l = \beta_0 + \beta x_l + \epsilon_l$$

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- Autor, Dorn and Hanson (2013) setting:
  - l: location (commuting zone)
  - y_l: manufacturing employment *growth*
  - x_l: import exposure to China *growth*
  - β: effect of rise of China on manufacturing employment
  - an instrument for location-level exposure to trade with China

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Accounting identity #1:

$$x_l = \sum_{k=1}^{K} z_{lk} g_{lk}$$

- z_{lk}: location-industry shares (Z_l)
- g_{lk}: location-industry growth (in imports) rates (G_l)

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Accounting identity #2:

$$\underbrace{g_{lk}}_{\text{location-industry}} = \underbrace{g_k}_{\text{industry}} + \underbrace{\tilde{g}_{lk}}_{\substack{\text{idiosyncratic} \\ \text{location- industry}}}$$

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Infeasible Bartik:

$$B_l = \sum_{k=1}^{K} z_{lk} g_k$$

Other instruments have this structure

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China shock: e.g., Autor, Dorn and Hanson (2013)

- z_{lk}: location (l) industry (k) composition
- g_{lk}: location (l) industry (k) growth in imports from China
- g_k: industry k growth of imports (from China)

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Immigrant enclave: e.g., Altonji and Card (1991)

- z_{lk}: share of people from foreign k living in l (in a base period)
- g_{lk}: growth in number of people from k to l
- g_k: growth in people from k nationally

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Bank-lending relationships: e.g., Greenstone, Mas and Nguyen (2015)

- z_{lk}: location (l) share of loan origination from bank k
- g_{lk}: loan growth in location l by bank k
- g_k: part of loan growth due to bank supply shock

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Market size and demography: e.g., Acemoglu and Linn (2004)

- z_{lk}: spending share on drug l from age group k
- g_{lk}: growth in spending of group k on drug l
- g_k: growth in spending of group k (due to population aging)

What we'll show

Under plausible asymptotics:

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How to quantify which industries are "important":

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Various tests of the identifying assumption

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- Other asymptotics imply identification conditions in terms of the national shocks
  - We'll discuss why these are probably not good approximations to the DGP

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  - We'll discuss why these are probably not good approximations to the DGP
- We won't discuss spatial spillovers
- We don't discuss dynamics

Outline

- **Understanding the identifying assumption**
- Opening the black box
- Testing the plausibility of the identifying assumption

Autor, Dorn and Hanson (2013) empirical strategy: cross-CZ variation

OLS:

$$y_{lt} = \alpha + \beta x_{lt} + \epsilon_{lt}$$

- y: *change* in manufacturing employment share
- x: *change* in import exposure to China ($x_{lt} = \sum_k z_{lkt} g_{kt}^{\text{US}}$)
- g_{kt}^{US}: growth of imports from China to US

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IV first stage:

$$x_{lt} = \gamma_0 + \gamma B_{lt} + \eta_{lt}$$

- B_{lt}: Bartik instrument: $B_{lt} = \sum_k z_{lkt-1} g_{kt}^{\text{high-income}}$
- $g_{kt}^{\text{high-income}}$: growth of imports from China to 8 high income countries

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Two cross-sections: 1990-2000; 2000-2007 (▸ Details)

Benchmark estimates

	Second Stage (Bartik)
	Δ Mfg
China to US	-0.62
	(0.13)
Year and Census Division FE	Yes
Controls	Yes
Observations	1,444
R-squared	0.34

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IV estimate: $1/3$ of decline in manufacturing employment from 1990 to 2007 (▸ Details)

More general set-up

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x_{lt} &= \mathbf{D}_{lt}\tau + B_{lt}\gamma + \eta_{lt} \\
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Assumptions for IV in terms of B_{lt}:

- Exogeneity: $\mathbb{E}\left[B_{lt}\epsilon_{lt} | \mathbf{D}_{lt}\right] = 0$
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Question:

- What do these statements about B_{lt} imply about z_{lk0} and g_{kt}?

Three special cases

1. One time period, two industries
2. T time periods, two industries
3. One time period, K industries

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First-stage:

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The instrument is z_{l1}, while g_k affects relevance

‣ Why OLS is biased

Special case #2: T time periods, two industries

Panel Bartik:

$$B_{lt} = z_{l10}g_{1t} + z_{l20}g_{2t} = g_{2t} + \underbrace{\Delta_{gt}}_{g_{1t}-g_{2t}} z_{l10}$$

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- Industry shares times time period is the instrument
- (Updated industry shares: similar)

Special case #2: T time periods, two industries

- Analogy to continuous difference-in-differences
  - Δ_{gt} is size of policy
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- Analogy to continuous difference-in-differences
  - Δ_{gt} is size of policy
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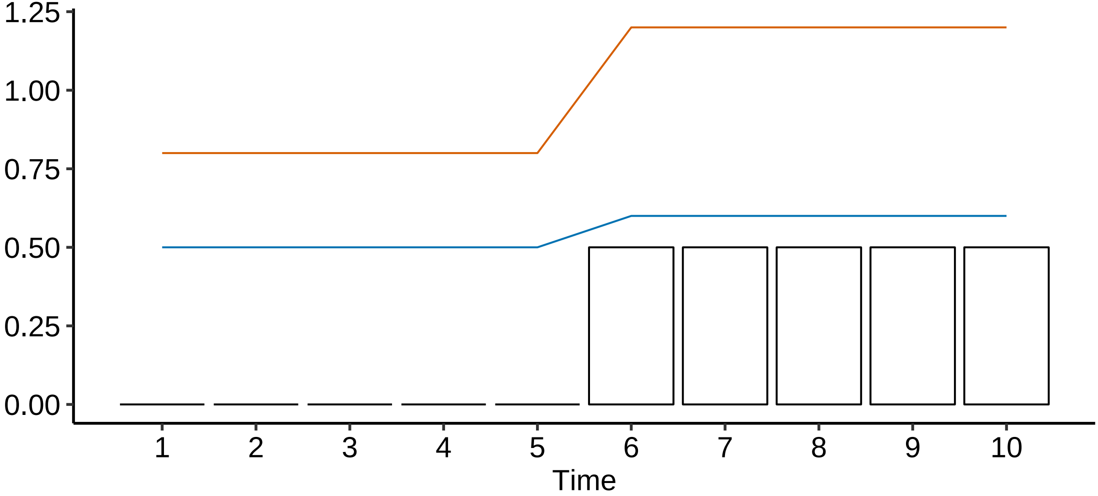
- Sometimes a "pre-period" before policy: test for parallel pre-trends
  - E.g., in ADH, what happens from 1970 to 1990?

Diff in Diff Example



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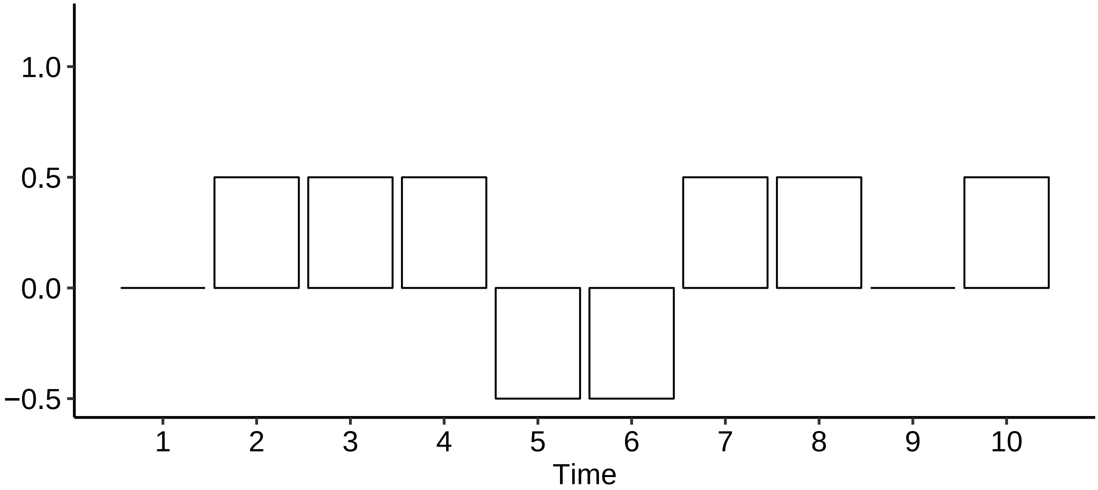

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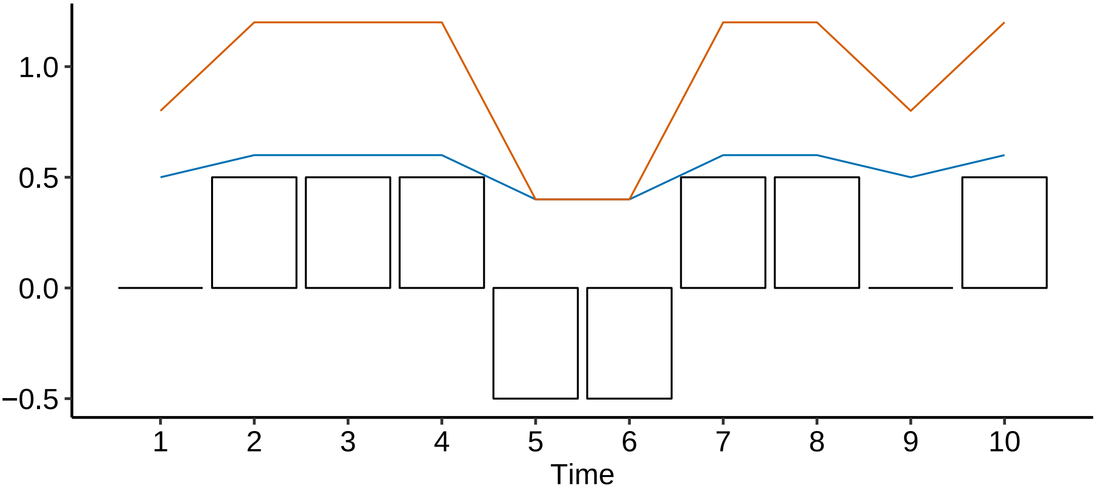

Special case #3: One time period, K industries

- G: $K \times 1$ vector of g_k
- Z: $L \times K$, matrix of Z_l
- $Y^{\perp}$, $X^{\perp}$, $B = (ZG)$: $L \times 1$, vectors of $y_l^{\perp}$, $x_l^{\perp}$ and B_l
- Ω: $K \times K$

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$$\hat{\beta}_{Bartik} = \frac{B'Y^{\perp}}{B'X^{\perp}}$$
$$\hat{\beta}_{GMM} = \frac{(X^{\perp\prime}Z)\Omega(Z'Y^{\perp})}{(X^{\perp\prime}Z)\Omega(Z'X^{\perp})}$$

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If $\Omega = (GG')$, then $\hat{\beta}_{Bartik} = \hat{\beta}_{GMM}$

▸ Proof

Summary

Two estimators are numerically identical:

- TSLS with Bartik instrument
- GMM with industry shares $\times$ time period as instruments and a particular weight matrix

When is the estimator consistent for the estimand of interest?

What is the identification condition?

$$\hat{\beta}_{Bartik} = \frac{\sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} z_{lkt} g_{kt} y_{lt}^{\perp}}{\sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} z_{lkt} g_{kt} x_{lt}^{\perp}}$$

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Two cases:

1. Fix K, fix T and let $L \to \infty$
2. Let $K \to \infty$, fix $T (= 1)$, and let $L \to \infty$
3. Fix K, let $T \to \infty$, and fix L (skip today)

Case #1: Fix K and T, $L \to \infty$

GMM identification condition:

$$\mathbb{E}\left[z_{lk0}\mathbb{1}(s=t)\epsilon_{lt}\middle|\mathbf{D}_l\right]=0, \forall k, s, t$$

Case #1: Fix K and T, $L \to \infty$

GMM identification condition:

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Bartik identification condition:

$$\sum_k \sum_s g_{ks}\mathbb{E}\left[z_{lk0}\mathbb{1}(s = t)\epsilon_{lt}|\mathbf{D}_l\right] = 0, \forall t$$

Case #2: Let $K \to \infty$, fix $T = 1$, and let $L \to \infty$

- Follow Kolesar, Chetty, Friedman, Glaeser and Imbens (2015), **many invalid instruments** (also, Borusyak, Hull and Jaravel (2018) and Adao, Kolesar and Morales (2018)):
  - Structural error: $\epsilon_l = \sum_k z_{lk}\lambda_k + \tilde{\epsilon}_l$, $\tilde{\epsilon}_l \perp\!\!\!\perp$ "everything"

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- Intuitive special case (Kolesar et al (2015, section 2)):
  - z_{lk} is binary
  - $\lim_{L \to \infty} \frac{K}{L} = constant$

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$$g_k \perp\!\!\!\perp \lambda_k$$

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- Each instrument correlated with the error term, but averages out
  - Example: industry-level supply and demand shocks independent
- "Large" K is not good evidence that $K \to \infty$ is a good approximation
  - Law of large numbers in industries, so it matters if some instruments are "important"
  - Next section develops machinery to quantify "importance"

Outline

- Understanding the identifying assumption
  - *If fixed K and T, and $L \to \infty$, then in terms of industry composition*
- **Opening the black box**
- Testing the plausibility of the identifying assumption

Decomposing Bartik

(Special case of Rotemberg (1983), proposition 1)

$$\hat{\beta}_{Bartik} = \sum_k \hat{\alpha}_k \hat{\beta}_k, \quad \sum_k \hat{\alpha}_k = 1$$

IV estimate using only the k^{th} instrument:

$$\hat{\beta}_k = (Z_k' X)^{-1} Z_k' Y$$

"Rotemberg" weight:

$$\hat{\alpha}_k = \frac{g_k Z_k' X}{\sum_{k=1}^K g_k Z_k' X}$$

▸ Proof

Interpretation: sensitivity to misspecification elasticity

Conley, Hansen and Rossi (2012); Andrews, Gentzkow and Shapiro (2017)

Local misspecification: $\epsilon_{lt} = L^{-1/2} V_{lt} + \tilde{\epsilon}_{lt}, \;\; Cov(V_{lt}, Z_{lt}) \neq 0,$

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Suppose $\beta_0 \neq 0$. Percentage bias:

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Industry with high α_k:

- an industry where it matters whether it is misspecified (endogenous)

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- $\sqrt{L}\,(\hat{\beta}_k - \beta_0) \xrightarrow{d} \tilde{\beta}_k, \qquad \mathbb{E}\left[\tilde{\beta}_k\right] =$ bias (misspecification) of kth instrument

Suppose $\beta_0 \neq 0$. Percentage bias:

$$\frac{\mathbb{E}[\tilde{\beta}]}{\beta_0} = \sum_k \alpha_k \frac{\mathbb{E}[\tilde{\beta}_k]}{\beta_0}$$

Industry with high α_k:

- an industry where it matters whether it is misspecified (endogenous)
  - because it is “important” in the estimate

Top five industries (out of 397)

	$\hat{\alpha}_k$	$g_k^{\text{high-income}}$	$\hat{\beta}_k$

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Electronic Computers	0.182	85.017	-0.620
Household Audio and Video	0.130	118.879	0.287
Computer Equipment	0.076	28.110	-0.315
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The **main source of variation in exposure is** *within-manufacturing specialization in industries subject to different degrees of import competition...there is differentiation according to* **local labor market reliance on labor-intensive industries**...*By 2007, China accounted for over 40 percent of US imports in four four-digit SIC industries (***luggage, rubber and plastic footwear, games and toys, and die-cut paperboard***) and over 30 percent in 28 other industries, including* **apparel, textiles, furniture, leather goods, electrical appliances, and jewelry***.*

— Autor, Dorn and Hanson (2013) , pg. 2123

‣ Correlation matrix

Outline

- Understanding the identifying assumption
  - *If fixed K and T, and $L \to \infty$, then in terms of industry composition*
- Opening the black box
  - *Which industries are "important" in estimates*
- **Testing the plausibility of the identifying assumption**

Three tests of the identifying condition

(And one test of the plausibility of alternative identifying conditions)

1. Confounds (or correlates)
2. Pre-trends
3. Alternative estimators and overidentification
4. Plausibility of many invalid instrument asymptotics

Test #1: Correlates of initial industry composition

- How are initial characteristics (D_{l0} less F.E.) related to Z_{l0}?
- Look at high-Rotemberg weight industries (and aggregate)

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Not definitive:

- Shows source of variation
- Can address by controlling for observables ($D_{l0} \times$ time)

Test #1: Correlates

	Games and toys	Electronic computers	Household audio and video	Computer equipment	Telephone apparatus	China to other
Share Empl in Manufacturing	0.01	0.21	0.08	0.21	-0.07	0.57
	(0.03)	(0.18)	(0.08)	(0.15)	(0.06)	(0.07)
Share College Educated	-0.08	0.20	0.01	0.22	-0.07	0.30
	(0.03)	(0.11)	(0.04)	(0.10)	(0.06)	(0.06)
Share Foreign Born	0.01	-0.01	-0.02	-0.01	-0.08	0.15
	(0.01)	(0.04)	(0.01)	(0.04)	(0.03)	(0.03)
Share Empl of Women	0.05	-0.04	-0.08	-0.02	-0.02	0.10
	(0.03)	(0.12)	(0.05)	(0.12)	(0.07)	(0.06)
Share Empl in Routine	0.04	-0.37	0.06	-0.36	-0.01	-0.08
	(0.03)	(0.14)	(0.05)	(0.12)	(0.07)	(0.13)
Avg Offshorability	0.02	0.33	0.00	0.29	0.23	-0.24
	(0.02)	(0.10)	(0.05)	(0.08)	(0.04)	(0.09)
1980 Population Weighted	Yes	Yes	Yes	Yes	Yes	Yes
N	1,444	1,444	1,444	1,444	1,444	1,444
R^2	0.02	0.08	0.01	0.08	0.05	0.22

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Test #2: Pre-trends

$$\Delta\text{Manufacturing Emp}_{lt} = \alpha + \sum_s \mathbb{1}(s = t)\gamma_{k,s} z_{lk,1980} + \epsilon_{lt}$$

- Four time periods: 1970-1980, 1980-1990, 1990-2000, 2000-2007
- Convert $\hat{\gamma}_{k,t}$ to levels ($1970 = 100$)

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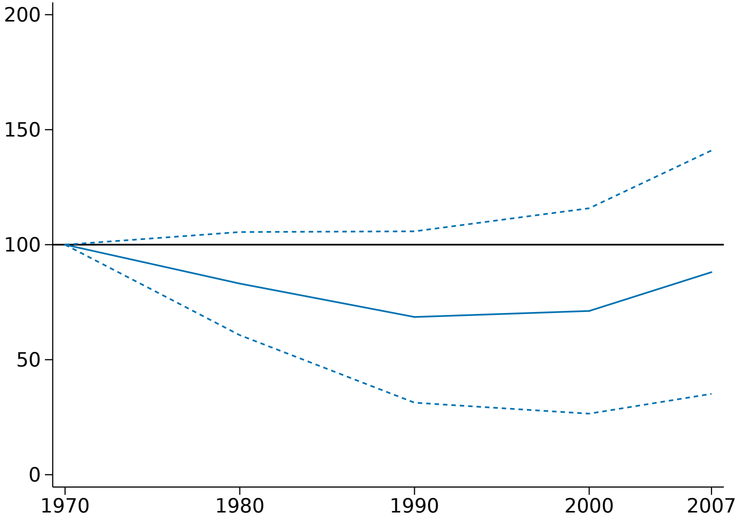
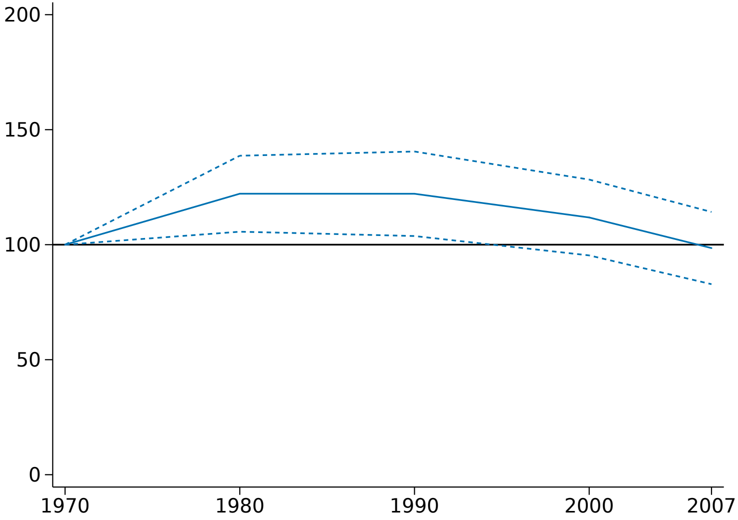
- Four time periods: 1970-1980, 1980-1990, 1990-2000, 2000-2007
- Convert $\hat{\gamma}_{k,t}$ to levels ($1970 = 100$)
- k is top five Rotemberg weight industries in 1980, and "aggregate"
  - Aggregate: 1980 shares, aggregated using $g_{k,1990-2000}^{\text{high-income}}$

"Pre-period" prior to 1990

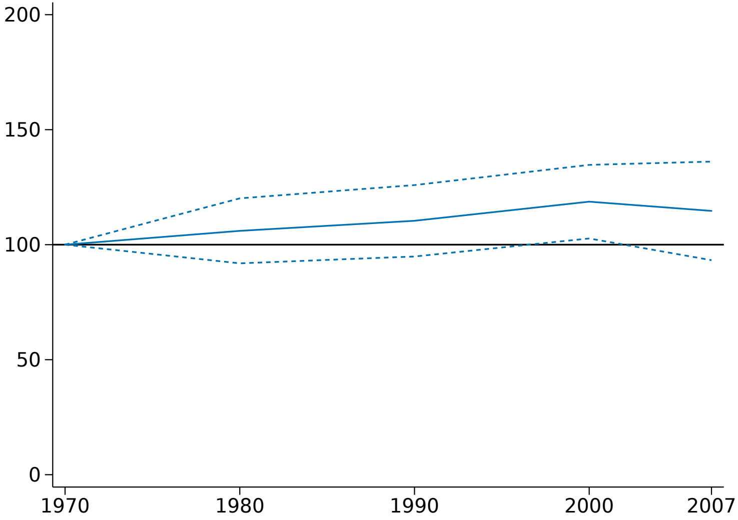
Games and toys, fixed 1980 industry shares, Rotemberg weight 0.182



Electronic computers, fixed 1980 industry shares, Rotemberg weight 0.182

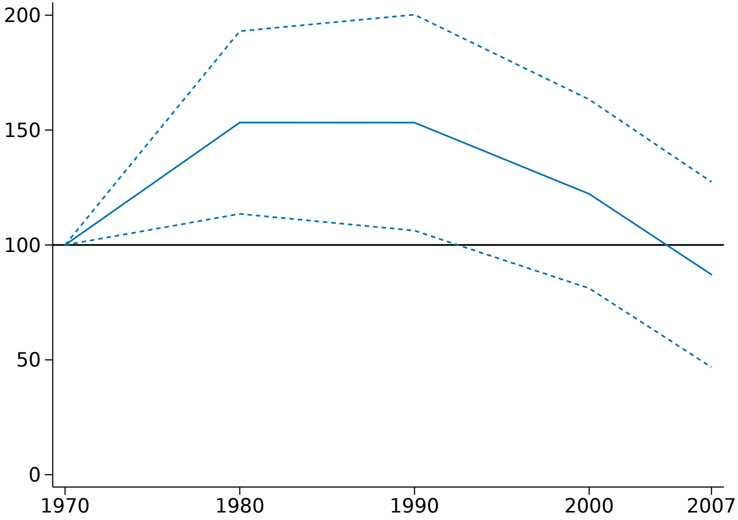


Household audio and video, fixed 1980 industry shares, Rotemberg weight 0.130



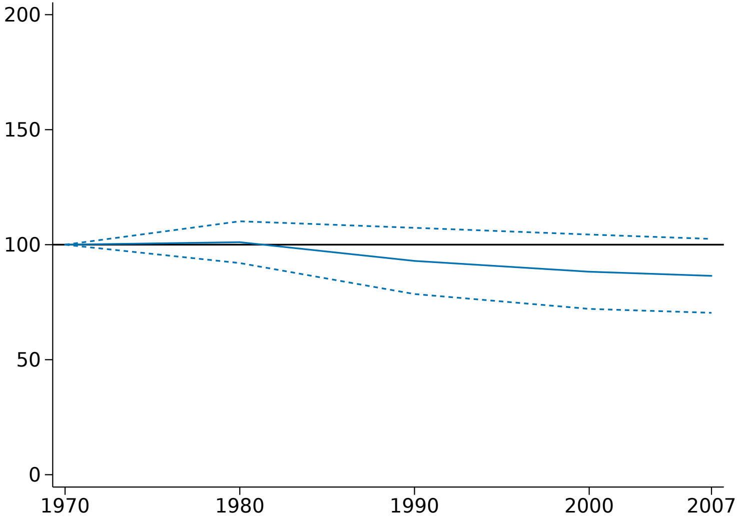
▸ 1990 Shares ▸ Updated shares

Computer equipment, fixed 1980 industry shares, Rotemberg weight 0.076



▸ 1990 Shares ▸ Updated shares

Telephone apparatus, fixed 1980 industry shares, Rotemberg weight 0.058



▸ 1990 Shares ▸ Updated shares

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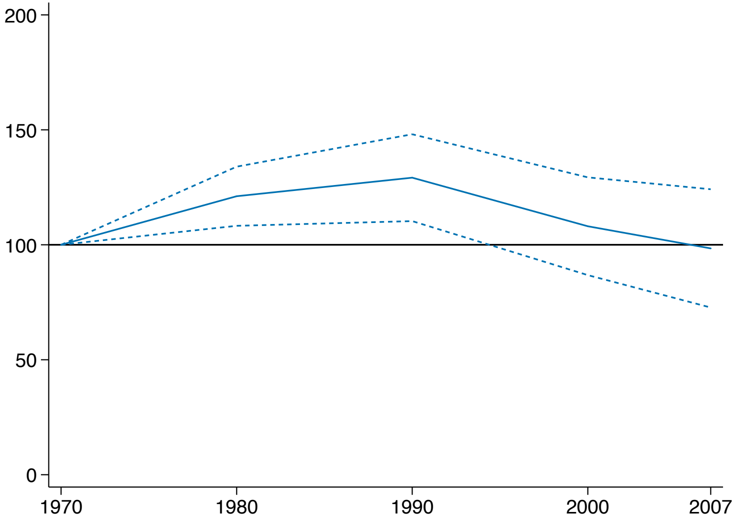

▸ 1990 Shares

Test #3: Alternative estimators and overidentification tests

Basic insight: many instruments

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- Estimators (maximum likelihood): LIML, Hausman, Newey, Woutersen, Chao and Swanson (2012) HFUL (heteroskedasticity-Fuller (1977))
- Estimators (two-step): TSLS (problematic), Bartik TSLS, MBTSLS (Anatolyev (2013), and Kolesar et al (2015))

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- Gap between maximum likelihood and two-step estimators is evidence of misspecification

Also, overidentification tests, which provides evidence of misspecification

Test #3: Alternative estimators and overidentification

	Δ Emp	Over ID Test
OLS	-0.17	
	(0.04)	
TSLS (Bartik)	-0.62	
	(0.11)	
TSLS	-0.22	872.69
	(0.06)	[0.00]
MBTSLS	-0.33	
	(0.05)	
LIML	-2.07	1348.50
	(3.52)	[0.00]
HFUL	-1.13	1141.08
	(0.04)	[0.00]
Year and Census Division FE	Yes	
Controls	Yes	
Observations	1,444	

Test #4: Plausibility of many invalid instrument asymptotics

- A few large Rotemberg weights suggests that $K \to \infty$ is a less plausible approximation
  - Recall, in ADH top five industries (out of 397) were 46% of overall positive weight

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- A few large Rotemberg weights suggests that $K \to \infty$ is a less plausible approximation
  - Recall, in ADH top five industries (out of 397) were 46% of overall positive weight
- $K \to \infty$ (in binary special case) implies that all Rotemberg weights go to zero as $L \to \infty$ (Proof)

$\beta = 2,\ L = 800,\ K = 228$					
	OLS	Infeasible Bartik		Top 5 α_k share	
	$\hat{\mathbb{E}}\left[\hat{\beta}\right]$	$\hat{\mathbb{E}}\left[\hat{\beta}\right]$	Med$[\hat{\beta}]$	$\hat{\mathbb{E}}$	Med
	(1)	(2)	(3)	(4)	(5)
(1) Standard					
(2) $\sigma^2_{\lambda_k} = 0.2\sigma^2_{g_k}$					
(3) $\sigma^2_{\lambda_k} = 1.0\sigma^2_{g_k}$					
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(5) $\sigma^2_{\lambda_k} = 1.0\sigma^2_{g_k}$, $\lambda_k = g_k$ (smallest 5 α_k)					
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Summary

- Understanding the identifying assumption
  - *If fixed K and T, and $L \rightarrow \infty$, then in terms of industry composition*
- Opening the black box
  - *Which industries are "important" in estimates*
- Testing the plausibility of the identifying assumption
  - *Correlates, pre-trends, overidentification/alternative estimators, distribution of weights*

Why OLS can be biased if Bartik is valid

$$\mathbb{E}[x_l \epsilon_l] \neq 0 \Rightarrow \mathbb{E}[\{g_l + z_{l1}\tilde{g}_{l1} + z_{l2}\tilde{g}_{l2} + B_l\}\epsilon_l] \neq 0$$

and

$$\mathbb{E}[B_l \epsilon_l] = 0$$

Implies:

$$\mathbb{E}[\{g_l + z_{l1}\tilde{g}_{l1} + z_{l2}\tilde{g}_{l2}\}\epsilon_l] \neq 0$$

▸ Back to two industry case

Data and details

Two cross-sections:

- 1990-2000
- 2000-2007

Other details:

- Controls: dummy for 2000-2007, % of manufacturing employment in -1, % college educated, % foreign born, % of employment among women, % of employment in routine occupations, % average offshorability index, Census division dummies
- Weighted by start of period CZ share of national population

(▸ Back to empirical strategy)

What numbers mean

$$\hat{\beta} = -0.6$$

Interpretation: \$1,000 per worker increase in import exposure over a decade reduces manufacturing employment per working age population by 0.6 percentage points

- From 1990-2000 Chinese import exposure rose \$1,140 per worker ($1.140 \times -0.6 = -0.70$)
- From 2000-2007 Chinese import exposure rose \$1,839 per worker $1.839 \times -0.6 \times 0.7 = -0.77$)

Context: Fall in manufacturing employment:

- 2.07 pp from 1990 to 2000
- 2.00 pp from 2000 to 2007

▸ Back to ADH results

GMM proof

Proof.
If $\Omega = (G - \bar{G})(G - \bar{G})'$, then

$$\begin{aligned}
\hat{\beta}_{L,GMM} &= \frac{X_L'(Z_L - \bar{Z}_L)(G - \bar{G})(G - \bar{G})'(Z_L - \bar{Z}_L)'Y_L}{X_L'(Z_L - \bar{Z}_L)(G - \bar{G})(G - \bar{G})'(Z_L - \bar{Z}_L)'X_L} \\
&= \frac{X_L'(B_L - \bar{B}_L)(B_L - \bar{B}_L)'Y_L}{X_L'(B_L - \bar{B}_L)(B_L - \bar{B}_L)'X_L} \\
&= \hat{\beta}_{L,Bartik}
\end{aligned}$$

(Intermediate step: $(Z_L - \bar{Z}_L)G = Z_L G - \bar{Z}_L G = B_L - \bar{B}_L$ and $(Z_L - \bar{Z}_L)\bar{G} = \mathbf{0}$ $(L \times 1)$.) □

▸ Back to K industries

Rotemberg proof

$$\alpha_k(C)\beta_k = \frac{c_k Z_k X}{\sum_k c_k Z_k X}(Z_k' X)^{-1} Z_k Y = \frac{c_k Z_k Y}{\sum_k c_k Z_k X}$$

$$\sum_{k=1}^{K} \alpha_k(C)\beta_k = \frac{\sum_k c_k Z_k Y}{\sum_k c_k Z_k X} = \frac{C' Z Y}{C' Z X}.$$

▸ Back to statement

Assumptions

(i) the data $\{\{x_{lt}, Z_{lt}, D_{lt}, V_{lt}, \epsilon_{lt}\}_{t=1}^{T}\}_{l=1}^{L}$ are independent and identically distributed with K and T fixed, and L going to infinity;

(ii) $\mathbb{E}[\epsilon_{lt}] = 0$, $\mathbb{E}[V_{lt}] = 0$ and $Var(\tilde{\epsilon}) < \infty$;

(iii) $\mathbb{E}[z_{lkt}\epsilon_{lt}] = 0$ for all values of k; $\mathbb{E}[z_{lt}V_{lt}] = \Sigma_{ZV}$, where Σ_{ZV} is a $1 \times K$ covariance vector with at least one non-zero entry; and $\mathbb{E}[Z_{lt}x_{lt}^{\perp}] = \Sigma_{ZX^{\perp}}$ is a $1 \times K$ covariance vector with all non-zero entries, and $\Sigma_{ZX^{\perp},k}$ is the k^{th} entry; and

(iv) and $Var(z_{lkt}\epsilon_{lt}) < \infty$, $Var(z_{lkt}V_{lt}) < \infty$ and $Var(z_{lkt}x_{lt}^{\perp}) < \infty$ for all values of k.

▸ Back to interpretation

Connection to first-stage coefficient

Exact decomposition:

$$x_l = B_l + \underbrace{g_l + Z_l' \tilde{g}_{lk}}_{\eta_l}$$

Regression version:

$$x_l = \pi B_l + \eta_l$$

- If $\mathbb{E}[G_l | Z_l] = \mathbb{E}[G_l]$, then $Cov(B_l, \eta_l) = 0$
- If $Cov(B_l, \eta_l) = 0$, then $lim_{L \to \infty} \hat{\pi} = 1$

Bartik dimension-reduction

Weights and growth rates are not the same thing

	α_k	G	β_k	Var(z_k)
α_k	1			
G	0.581	1		
β_k	-0.005	-0.041	1	
Var(z_k)	0.154	-0.038	0.054	1

▸ Back to Top 5

- Assume g_k is mean zero
- Assume $\tilde{g}_{lk}$ is mean zero
- Assume $\frac{K}{L} \rightarrow$ *constant*
- Let $\bar{z}_{L,k} = \sum_l z_{lk}$ for a sample of size L, be bounded
- Recall that z_{lk} are $0/1$, and each location only gets 1 (and we omit one)
- Hence, X is mean zero

Then:

$$\alpha_k = \frac{g_k Z_k' X}{\sum_k g_k Z_k' X}$$

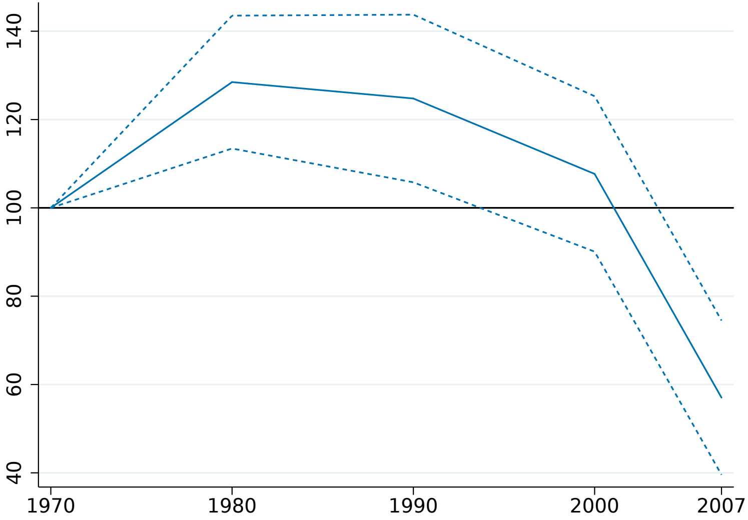
$$\alpha_k = \frac{g_k \bar{z}_{L,k} g_k + g_k \bar{z}_{L,k} \sum_{l,st.z_{lk}=1} \tilde{g}_{lk}}{\sum_k \left(g_k \bar{z}_{L,k} g_k + g_k \bar{z}_{L,k} \sum_{l,st.z_{lk}=1} \tilde{g}_{lk} \right)}$$

$$\alpha_k = \frac{g_k^2 \bar{z}_{L,k} + g_k \bar{z}_{L,k} \sum_{l,st.z_{lk}=1} \tilde{g}_{lk}}{\sum_k \left(g_k^2 \bar{z}_{L,k} + g_k \bar{z}_{L,k} \sum_{l,st.z_{lk}=1} \tilde{g}_{lk} \right)}$$

Note that as $L \rightarrow \infty$, the numerator remains bounded for any k, but the denominator explodes. Hence, $\alpha_k \rightarrow 0$.

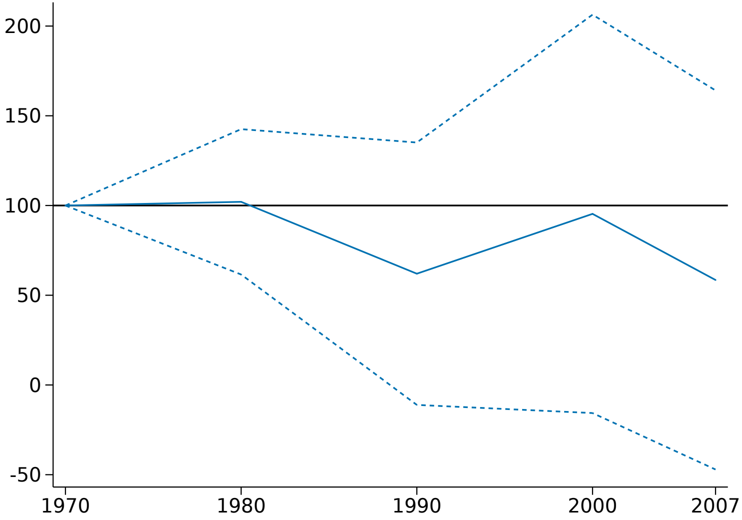
‣ Back to Case 1 vs. Case 2

Aggregate, fixed 1990 industry shares



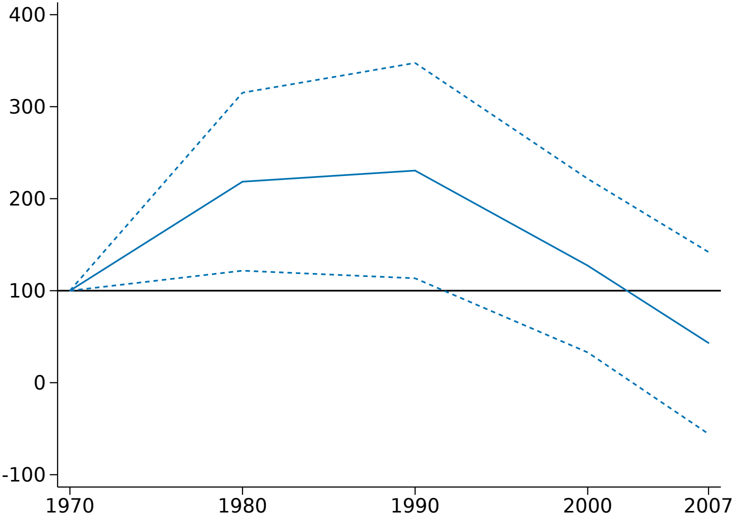
▸ 1980 Shares

Games and toys, fixed 1990 industry shares, Rotemberg weight 0.182



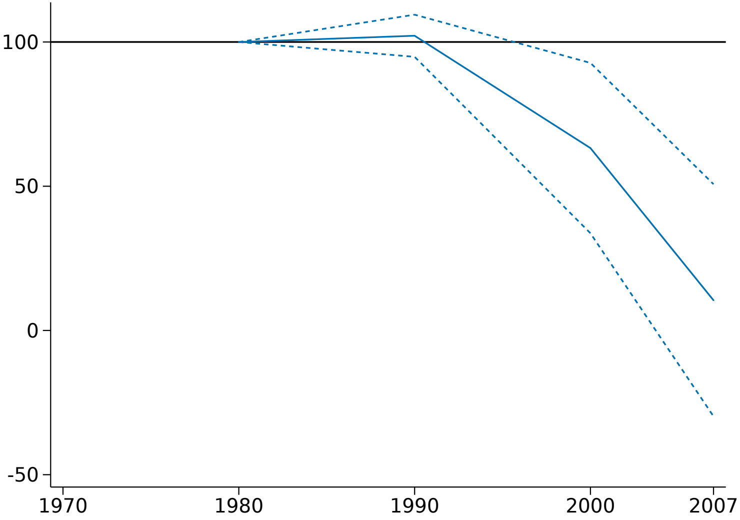
Games and toys, time-varying industry shares, Rotemberg weight 0.182

Electronic computers, fixed 1990 industry shares, Rotemberg weight 0.182



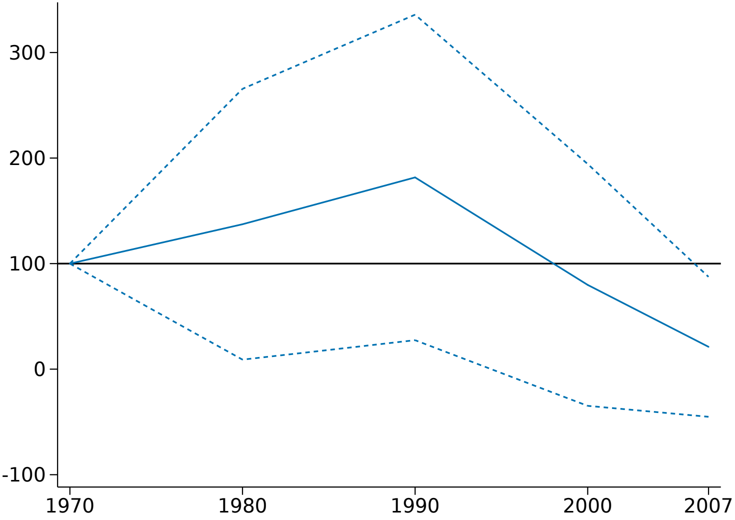
▸ 1980 Shares ▸ Updated shares

Electronic computers, time-varying industry shares, Rotemberg weight 0.182



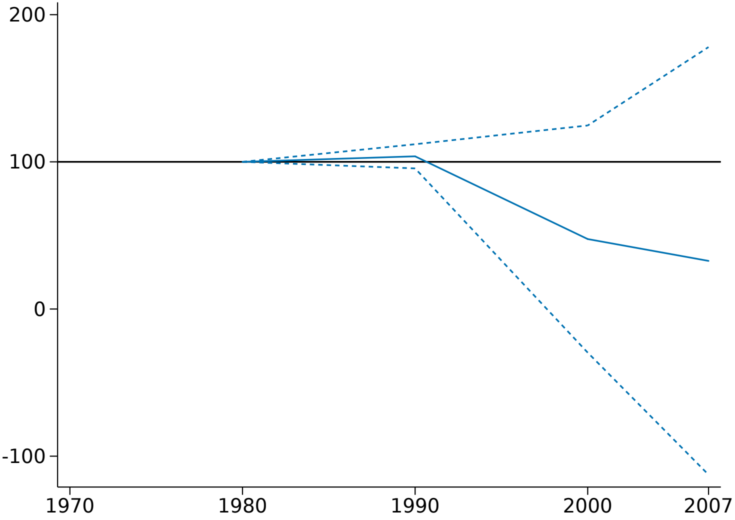
1980 Shares 1990 Shares

Household audio and video, fixed 1990 industry shares, Rotemberg weight 0.130

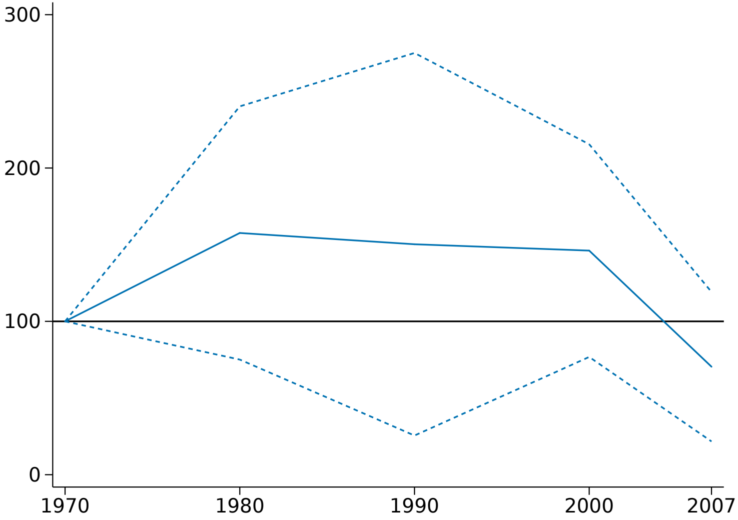


▸ 1980 Shares ▸ Updated shares

Household audio and video, time-varying industry shares, Rotemberg weight 0.130

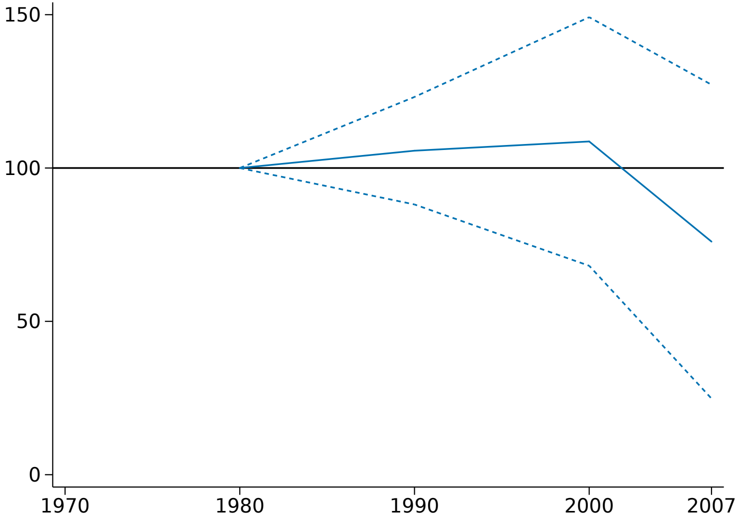


Computer equipment, fixed 1990 industry shares, Rotemberg weight 0.076



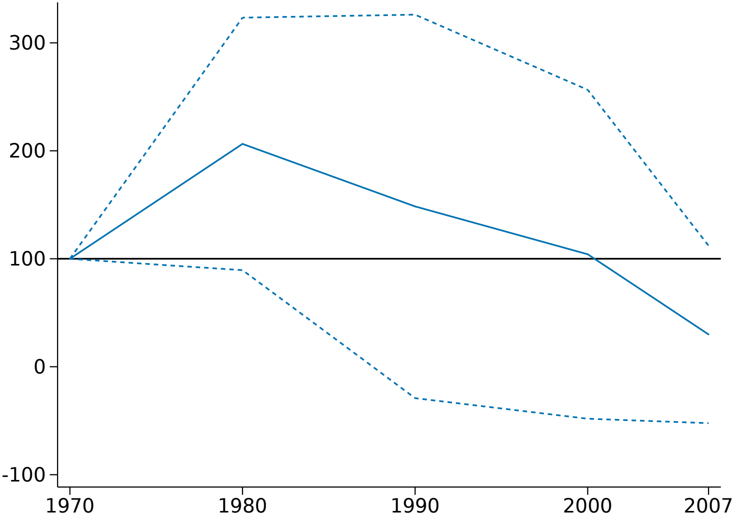
1980 Shares   Updated shares

Computer equipment, time-varying industry shares, Rotemberg weight 0.076

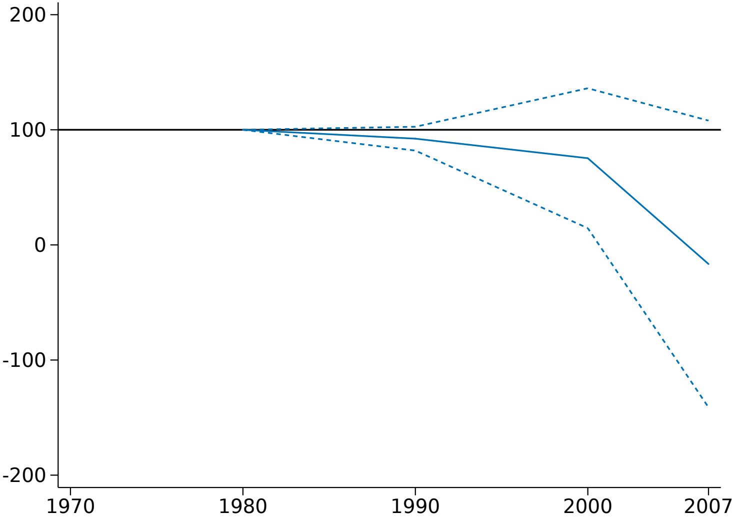


1980 Shares   1990 Shares

Telephone apparatus, fixed 1990 industry shares, Rotemberg weight 0.058



Telephone apparatus, time-varying industry shares, Rotemberg weight 0.058



▸ 1980 Shares ▸ 1990 Shares