On Using Pareto Distributions for Measuring Top-Income Gender Disparities*

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Abstract

Atkinson et al. (2018) propose a measure of the glass ceiling exploiting that top incomes are approximately Pareto distributed. We clarify how this glass-ceiling coefficient describes the increasing scarcity of women further up in the income distribution and show how it relates to the top-income gender gap. If interpreting top income gender differences as caused by a female-specific income tax, the gender gap and glass-ceiling coefficient measure its level and progressivity, respectively. Using Danish data on earnings, we show that the top gender gap and the glass-ceiling coefficient evolves differently across time, the life cycle, and educational groups.

Keywords: decomposition, gender gap, glass ceiling, summary statistics

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1 Introduction

There is wide agreement that women are underrepresented in the top of the earnings distribution, a phenomenon often referred to as the glass ceiling, but there is no general agreement on how to quantify the underrepresentation of women.[1] Atkinson et al. (2018) have suggested a measure of the glass ceiling exploiting that top incomes are well described by a Pareto distribution. Specifically, Atkinson et al. (2018) suggested using the ratio of the slope coefficients of separately estimated Pareto distributions for male and female incomes, respectively. In this paper, we investigate this measure of the glass ceiling, discuss its interpretation, and use it to analyze gender disparities in top earnings in Denmark 1980-2013.

First, we clarify how Atkinson et al. (2018)'s *glass-ceiling coefficient* relates to the *top-income gender gap*, which is a more commonly used measure of top-income gender disparities. Specifically, exploiting that both female and male top incomes are well described by Pareto distributions, we show that the entire difference between male and female top earnings distributions can be summarized with the top-income gender gap and the glass ceiling coefficient. The top-income gender gap is a level difference between the income distributions and captures the income difference between men and women at a given position in the income distribution. The glass-ceiling coefficient is a shape difference between the distributions and describes the rate at which women become increasingly underrepresented the further up in the income distribution we move.

Second, we show that the difference between the male and female top income distributions are observationally equivalent to an exponential tax schedule on women's income of the Feldstein (1969)-form. Hence, if we interpret top-earnings gender disparities as caused by a tax on women's income, this tax takes a particularly convenient functional form. In this tax schedule, the *top-income gender gap* corresponds to the level of the tax whereas the *glass-ceiling coefficient* corresponds to the progressivity of the tax.

The analysis suggests that the top-income gender gap and the glass-ceiling coefficient are not necessarily related. The final contribution of our paper is to show that the two statistics vary independently in the data. We use administrative earnings data from Denmark 1980-2013 to document that the top-earnings gender gap and the glass-ceiling coefficient display (i) different evolution in the aggregate, (ii) different evolution over the life cycle, and (iii) substantial variation in the level and evolutions across four high-earning

[1] For example, Albrecht et al. (2003) write that the gender gap "increases throughout the wage distribution and accelerates in the upper tail" and interpret it as a "strong glass ceiling effect", Bertrand (2018) writes that "women remain as of today underrepresented in the upper part of the earnings distribution" and that this phenomenon "is often referred to as the glass ceiling", while Blau and Kahn (2017) write that "there was a relatively large gender gap at the top of the distribution" and that "the wage gap fell more slowly (...) at the top than at other portions of the distribution. These two patterns suggest the notion of a 'glass ceiling.'"

education groups.

Specifically, we find that the aggregate top-earnings gender gap is steadily falling over the period 1980-2013 while the glass-ceiling coefficient is large and without trend over the period. Although the top earnings of women have approached that of men, the probability that a woman in the top 1% also belongs to the top 0.1% of the earnings distribution has been stable since 1980.

We follow cohorts through the life cycle and compute the cohort-specific top-earnings gender gap and glass-ceiling coefficient. The cohort top-earnings gender gap is comparatively small early in life, increasing until around age 40, and then decreasing. In contrast, the cohort glass-ceiling coefficient is steadily increasing throughout the life cycle.

Finally, we compute the top-earnings gender gap and the glass-ceiling coefficient separately for business majors, STEM majors, medical doctors and law majors. There is substantial variation in both the top-earnings gender gap and the glass-ceiling coefficient across these educational groups and within all of them, the evolution of the gap is different from the evolution of the glass ceiling. The most striking contrast is found for business majors, which display a rapidly increasing gap together with a decreasing glass-ceiling coefficient over the period.

2 Describing top income gender disparities using Pareto distributions

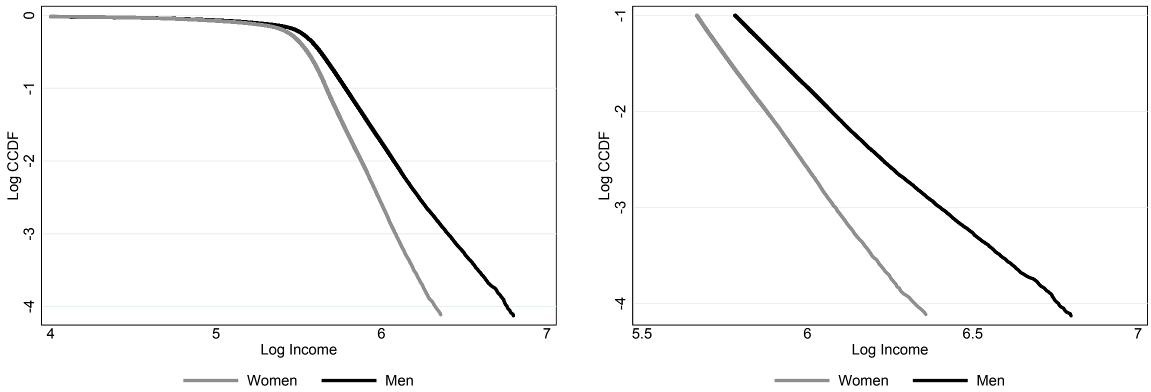
It is well-known that top incomes are described well by Pareto distributions (Gabaix, 2016).[2] Figure 1 shows base 10 log-log plots of the countercumulative distribution function (CCDF) of earnings for men and women in Denmark for the year 2009, for the full sample of all Danish workers and the top percent sample, respectively. For the top ten percent samples, the countercumulative distributions are virtually straight lines, indicating Pareto distributions $F_i(y) = 1 - L_i y^{-\alpha_i}$ where $i \in \{m, w\}$:

$$F_i(y) = 1 - L_i y^{-\alpha_i} \tag{1}$$

$$\Leftrightarrow \quad \log(1 - F_i(y)) = \log L_i - \alpha_i \log y. \tag{2}$$

Because top incomes are well described by Pareto distributions, we can summarize how the distributions of male and female top earnings differ with two parameters only. The male and female Pareto distributions in Figure 1b differ in their level and in their slopes.

[2] Throughout this paper, we will refer to top incomes as incomes higher than the 90th percentile of a given distribution. While this definition is line with much of the literature on top incomes, the exact cutoff is, of course, arbitrary.



(a) Full sample

(b) Top 10 percent sample

Figure 1: Log-log plot of the countercumulative distribution of earnings for both men and women, Denmark 2009. Earnings are measured in Danish kroner. The x-axis is log (base 10) earnings, so a value of 6 corresponds to 1,000,000 DKK ($\sim$ 150,000 USD)

The difference in the level, $\delta \equiv \log L_m - \log L_w$, measures the horizontal distance between the two lines in Figure 1 at the point where $\log(1 - F(y)) = -1$, i.e., it measures the earnings gap between the woman and the man at the 90th percentile of their repsective gender-specific distributions. In the figure, this distance is approximately 0.13 base 10 log points, meaning that a male in the 90th percentile earns approximately $10^{0.13} \approx 35$ percent more than a woman in the 90th percentile. Note that in terms of summarizing the difference between male and female top incomes, the choice to evaluate the gap at the 90th percentile is arbitrary, one could measure a top-earnings gender gap at the 95th percentile or the 99th percentile just as well.

As for the difference in the slopes, the steeper line for women means that the tail of the earnings distribution of women is thinner than the tail of the earnings distribution of men. Atkinson et al. (2018) note that the ratio of these slopes is a natural measure of the glass ceiling as it captures the increased scarcity of women as we move further up the income distribution. Following their reasoning, we define $\gamma \equiv \alpha_f/\alpha_m - 1$ and call it the *glass-ceiling coefficent*. Proposition 2.1 shows that the glass-ceiling coefficient describes how, for example, the share of women in the top 0.1% relates to the share of women in the top 1%. If the glass ceiling coefficient is 0, then the share of women in the top 0.1% is equal to the share of women in the top 1%, but if the glass-ceiling coefficient is positive, then the share is lower. Note that this way of measuring the glass ceiling does not depend on an arbitrary choice of a cutoff, like the definition of the top-earnings gender gap δ did.

Proposition 2.1. *Let male income follow a Pareto distribution* $F_m(y) = 1 - L_m y^{-\alpha_m}$ *and let female income follow a Pareto distribution* $F_w(y) = 1 - L_w y^{-\alpha_w}$*. Denote the glass* *ceiling coefficient* $\gamma = \alpha_w/\alpha_m - 1$. *Then,*

$$\frac{\text{Share women in top } q_1}{\text{Share women in top } q_0} = \left(\frac{q_1}{q_0}\right)^{\gamma} \left(\frac{\text{Share men in top } q_1}{\text{Share men in top } q_0}\right)^{1+\gamma}.$$

Proof. Let the cutoff for top q_i be y_i. The share of women in the top q_i is given by $\frac{L_w y_i^{-\alpha_w}}{q_i} \times \frac{N_w}{N}$, where N_w and N is the number of women and the total population, respectively. The ratio of the two shares is $\frac{\text{Share women in top } q_1}{\text{Share women in top } q_0} = \frac{\frac{L_w y_1^{-\alpha_w}}{q_1}}{\frac{L_w y_0^{-\alpha_w}}{q_0}} = \frac{q_0}{q_1}\left(\frac{y_1}{y_0}\right)^{-\alpha_w}$. Similarly, we have $\frac{\text{Share men in top } q_1}{\text{Share men in top } q_0} = \frac{q_0}{q_1}\left(\frac{y_1}{y_0}\right)^{-\alpha_m}$. Put the two together, and we get

$$\frac{\text{Share women in top } q_1}{\text{Share women in top } q_0} = \left(\frac{q_1}{q_0}\right)^{\alpha_w/\alpha_m - 1} \left(\frac{\text{Share men in top } q_1}{\text{Share men in top } q_0}\right)^{\alpha_w/\alpha_m}.$$

□

In the Danish data, men are overrepresented in the top of the earnings distribution. In 2013, the share of men in top 10, 1 and 0.1% is 75, 88 and 93%, respectively. If men are vastly overrepresented in the top, then the ratio $\frac{\text{Share men in top } q_1}{\text{Share men in top } q_0}$ is approximately 1 and we get the approximation

$$\frac{\text{Share women in top } q_1}{\text{Share women in top } q_0} \approx \left(\frac{q_1}{q_0}\right)^{\gamma}.$$

For example, if $\gamma = 0.4$, as is the case for the lines in Figure 1b, then the share of women in the top 0.1% is about 40 percent of the share of women in the top 1%, since $\left(\frac{0.01}{0.1}\right)^{0.4} \approx 0.4$.[3]

The notion of a glass ceiling may be interpreted as a progressive tax on female incomes. The idea is that men and women have the same distribution of income potential but women face a female-specific tax which reduces their income, and that this tax increases as we move further up the income distribution. If the income potential is Pareto distributed, the top-income gender gap captures the level of the female-specific tax and the glass-ceiling coefficient captures the progressivity of the female-specific tax. Because the underlying

[3]In addition to describing the underrepresentation of women in some top share conditional on their representation in some broader top share, the glass-ceiling coefficient also describes the underrepresentation of women above some earnings cutoff given their representation in some lower earnings cutoff. More specifically, given that male and female earnings are Pareto distributed, one can show that $\frac{\#\text{ women with income} \geq y_1}{\#\text{ women with income} \geq y_0} = \left(\frac{\#\text{ men with income} \geq y_1}{\#\text{ men with income} \geq y_0}\right)^{1+\gamma}$.

income distribution is Pareto, the progressivity of this tax also has a convenient functional form:[4]

Proposition 2.2. *(Tax interpretation) Let both male and female income potential Z follow $F(z) = 1 - Lz^{-\alpha}$. Let male earnings equal the earning potential, $Y_m = Z$ and let female income be subject to a tax scheme $Y_w = (1-\tau_0)Z^{1-\tau_1}$. Then the glass-ceiling coefficient is $\gamma = \tau_1/(1-\tau_1)$. In particular, the glass-ceiling coefficient is independent of the overall taxation level τ_0 and only depends on the progressivity τ_1 of the tax scheme.*

Proof. The distribution of incomes for women is given by

$$F_w(y) = P(Y_w \leq y) = P\left((1-\tau_0)Z^{1-\tau_1} \leq y\right) = P\left(Z \leq \left(\frac{y}{1-\tau_0}\right)^{1/(1-\tau_1)}\right) =$$
$$= 1 - (1-\tau_0)^{\alpha/(1-\tau_1)} L y^{-\alpha/(1-\tau_1)}.$$

Therefore, the glass-ceiling coefficient is $1/(1-\tau_1) - 1 = \tau_1/(1-\tau_1)$. □

Proposition 2.2 highlights that the top-income gender gap and the glass-ceiling coefficient are conceptually independent. A priori, a societal change that leads to a decreasing top-income gender gap (a lower level of the female-specific tax rate) need not have an effect on the glass-ceiling coefficient (the progressivity of the tax rate) and vice versa.

3 Computing the top earnings gender gap and glass ceiling coefficient for Denmark 1980-2013

In the empirical analysis, we study gender disparities in top labor earnings in Denmark. Specifically, we track the evolution of the top-earnings gender gap and the glass-ceiling coefficient in Denmark over the period 1980-2013, both in the aggregate and also across the life cycle and educational groups. This analysis complements Atkinson et al. (2018), who study the evolution of the aggregate glass-ceiling coefficient for total income (including capital income and self-employment income) in several countries, without comparing with the top-income gender gap.

We compute the top-earnings gender gap as the difference between the male and female log earnings at the 90th percentiles. To ease interpretation, we report this difference in natural log points. We compute the glass-ceiling coefficient by first fitting a Pareto distribution, using maximum likelihood, to the top 10% of the earnings distribution for

[4] The same functional form is commonly used to approximate the progressivity of the US tax system (Feldstein, 1969; Heathcote et al., 2017).

men and women separately, and then computing the glass-ceiling coefficient from the shape parameters of the two fitted Pareto distributions, $\gamma = \alpha_f/\alpha_m - 1$.

3.1 Data

Data sources In our analysis we combine three Danish datasets on the population, income, and education level. The point of departure for our analysis is the Danish Civil Registration System[5] for the full population in Denmark 1980 to 2013. The register includes all individuals with residence in Denmark on December 31st of a given year. From this registry we obtain information about date of birth and gender. We link these individuals to the income and education registries using the unique personal identification number. From the income registry we obtain information on earnings, based on tax records. From the education registry we obtain data on the highest completed educational degree. Information on educations completed before 1970 is based on a census compiled in November 1970 and third-party reports. Information on educations completed after 1970 are based on information reported directly from the institutions (for educations completed in Denmark) and surveys (for educations completed outside Denmark).

Earnings variables From the income registry we create an earnings variable that includes taxable and nontaxable income from employment, the value of fringe benefits, the value of stock options, and severance pay. Income from self-employment is not included.

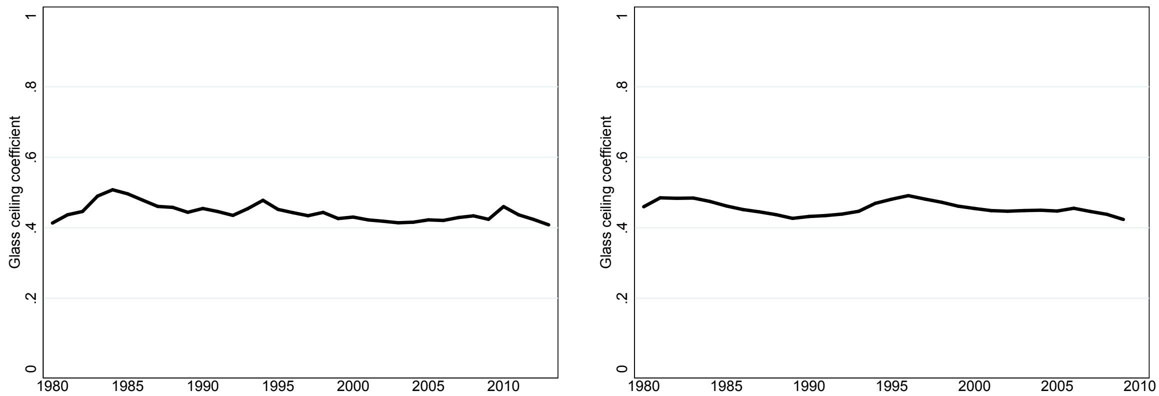
We present our results using two different earnings concepts. Specifically, we use a one-year and a five-year forward-looking moving average earnings concept. The former has the advantage of allowing us to include more years and have a less restricted population. The latter allows us to ignore yearly fluctuations, but requires restricting the sample to individuals observed in at least five consecutive years.

Educational classifications We classify individuals according to the highest completed education on October 1 in the given year implying that the highest educational degree is varying over time. The educational degrees are classified by field and level according to the International Standard Classification of Education (ISCED) (UNESCO, 2014). The highest degree is identified based on the ranking within the ISCED.

Sample selection We restrict the sample to individuals with positive earnings who are between 18 to 64 years old at the beginning of the year. We impose no further sample restrictions.

[5]In Danish: "Det Centrale Personregister" or "CPR-Registret".

(a) The top-earnings gender gap, 1-year earnings. **(b)** The top-earnings gender gap, 5-year earnings.



(c) The glass-ceiling coefficient, 1-year earnings. **(d)** The glass-ceiling coefficient, 5-year earnings.

Figure 2: The evolution of the top-earnings gender gap and glass-ceiling coefficient for Denmark 1980-2013. Five-year earnings are computed as a forward-looking moving average, see Section 3.1.

3.2 The top-earnings gender gap and glass-ceiling coefficient

In Figure 2, we show the evolution of the top-earnings gender gap and glass-ceiling coefficient from 1980 to 2013, both for one-year earnings and five-year earnings. While the top-earnings gender gap has decreased, the glass-ceiling coefficient has remained stable.

The top-earnings gender gap As seen from the upper row (Figures 2a and 2b), the top-earnings gender gap has steadily decreased from around 0.4 log points in 1980 to 0.3 log points in 2013. This is true for both one-year and five-year earnings. In 1980, a man at the 90th percentile of the men's earnings distribution earned $e^{0.4} = 49$ percent more than a woman at the 90th percentile of the women's earning distribution. By 2013, this gap had decreased to $e^{0.30} = 35$ percent.

The glass-ceiling coefficient As seen from the bottom row (Figures 2c and 2d), the glass-ceiling coefficient has fluctuated around 0.4 without trend over the sample period.

This is true for both one-year and five-year earnings.

What does a glass-ceiling coefficient of 0.4 mean? Recall the approximation from Section 2,

$$\frac{\text{Share women in top 1\%}}{\text{Share women in top 10\%}} \approx \frac{\text{Share women in top 0.1\%}}{\text{Share women in top 1\%}} \approx 0.1^{\gamma} \approx 0.40.$$

Thus, a glass ceiling coefficient of 0.4 means that the share of women in the top 0.1% is around 0.4 of the share of women in the top 1%, which in turn is around 0.4 of the share of women in the top 10%. The stability of the glass ceiling coefficient across time means that these ratios in 2013 were the same as in 1980.

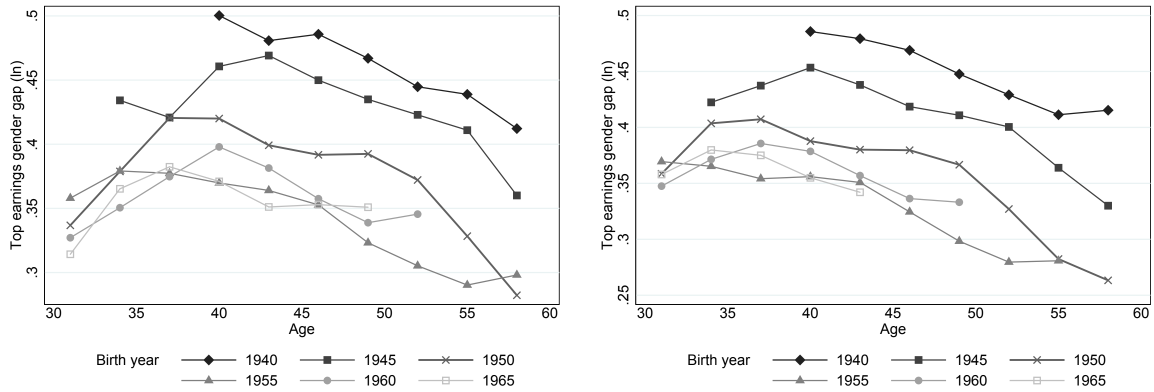
How should we interpret the combination of a falling gender gap and a stable glass ceiling coefficient? This joint observation tells us that although earnings difference between men and women at the 90th percentile has decreased, the representation of women in the very top of the earnings distribution conditional on being in the top of the earnings distribution (e.g, $\frac{\text{Share women in top 0.1\%}}{\text{Share women in top 1\%}}$) has remained stable.

3.3 Results over the life cycle

We now turn to the life-cycle dimension of top-earnings gender inequality. From a previous literature, we know that the gap in earnings between men and women is the largest around age 40, see, e.g., Manning and Swaffield (2008), Albrecht et al. (2015), and Albrecht et al. (2018). We also know that the Pareto tail parameter α of the total earnings distribution increases with age (Badel et al., 2018). Less is known about the evolution of the glass ceiling over the life cycle.

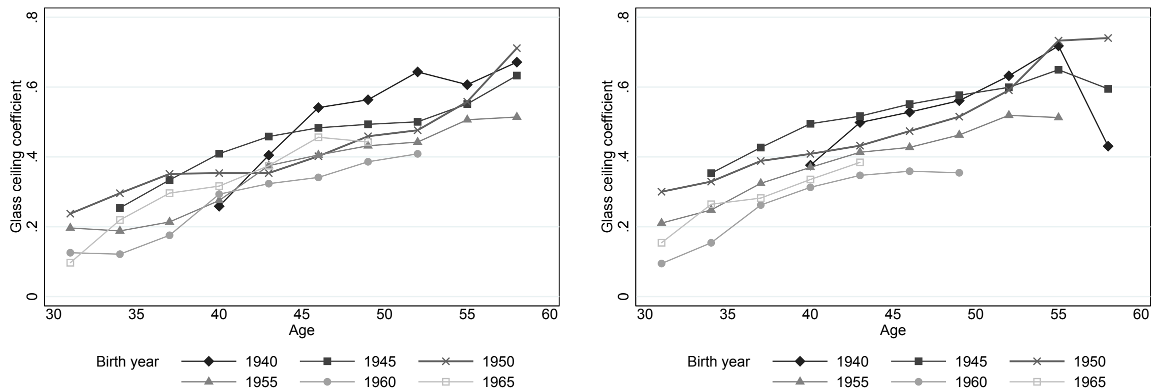
In Figure 3, we plot the evolution of the top-earnings gender gap and glass-ceiling coefficient for six cohorts over their working life. To gain precision, we compute the top-earnings gender gap and glass-ceiling coefficient based on data for three years around each age level. For example, the top-earnings gender gap at age 40 is computed from the pooled sample of all earnings at ages 39, 40 and 41 of any given cohort.

The cohort top-earnings gender gap The evolution of the top-earnings gender gaps over the life-cycle for the six different cohorts, for one-year and five-year earnings respectively, are displayed in Figures 3a and 3b. The top-earnings gender gap is positive and humped shaped over the life cycle. This is consistent with Manning and Swaffield (2008), Albrecht et al. (2015), and Albrecht et al. (2018). We can follow the cohort born in 1950 throughout their entire working life. At age 30-32, the top-earnings gender gap for the cohort is 0.34 log points (40 percent). The cohort top-earnings gender gap is increasing until age 36-38, peaking at 0.42 log points (52 percent). After age 40, the top-earnings



(a) Top earnings gender gap, 1-year earnings

(b) Top earnings gender gap, 5-year earnings



(c) Glass-ceiling coefficient, 1-year earnings

(d) Glass-ceiling coefficient, 5-year earnings

Figure 3: The evolution of the top-earnings gender gap and the glass-ceiling coefficient over the life cycle in Denmark 1980-2013. Five-year earnings is computed as a forward looking moving average, see Section 3.1.

gender gap decreases and before retirement the top-earnings gender gap is around 0.28 log points (32 percent).

We cannot follow the other cohorts throughout their entire working life, but the qualitative life-cycle pattern is similar for the ages we do observe. The level of the top-earnings gender gap is higher for the early cohorts, with a peak of 0.5 log points for the 1940 cohort compared to the 1965 cohort with a peak below 0.4 log points. This is consistent with the gradual decline of the aggregate top-earnings gender gap in Figures 2a and 2b.

The cohort glass-ceiling coefficient The evolution of the glass-ceiling coefficient over the life-cycle for the six different cohorts, for one-year and five-year earnings respectively, are displayed in Figures 3c and 3d. In contrast to the evolution of the top-earnings gender gap, the glass-ceiling coefficient is steadily increasing over the life cycle and stable across cohorts. For five-year earnings, the glass ceiling levels off or drops at the age 58-60. This, however, likely reflects the effect of retirement, as five-year earnings are computed as a five-year forward moving average.

For one-year earnings, the 1950 cohort has a glass-ceiling coefficient of 0.24 at age 30-32, which increases throughout their working life to a level of 0.64 at age 63-65. The life-cycle evolution of the glass-ceiling coefficient is similar for the other cohorts. Quantifying this, a glass-ceiling coefficient of 0.24 means that women are underrepresented in the top 0.1% by a factor $0.1^{0.24} = 0.58$ given their representation in the top 1%. Specifically, this means that the share of women in the top 0.1% was around 0.58 of the share of women in the top 1%, which in turn is around 0.58 of the share of women in the top 10%. A glass-ceiling coefficient of 0.64 means that the relative representation of women when moving up the same top shares falls off at the rate of $0.1^{0.64} = 0.23$.

The contrasting evolution of top-earnings gender gap and the glass-ceiling coefficient over the life cycle reinforces that they reflects independent dimensions of gender inequality: what is true about the top-earnings gender gap need not to be true about the glass ceiling.

3.4 Results across educational groups

In this section, we investigate the evolution of the top-earnings gender gap and glass ceiling coefficient across four high-earning university-degree education groups: Business, Medicine, STEM, and Law majors. Due to their overrepresentation among top earners, several studies have documented and investigated the causes of gender disparities in these groups. For example, see Bertrand et al. (2010) on the gender gap for US MBA graduates, Azmat and Ferrer (2017) for lawyers, Jena et al. (2016) for medical doctors and Beede et al. (2011) for STEM graduates. An earlier literature has also documented the heterogeneity of gender gaps across university-level education groups, see, e.g. Black et al. (2008). To the best of our knowledge, no study has documented the heterogeneity of a measure of the glass ceiling across education groups.

We identify a person as having a university degree if the person holds a bachelor degree or higher (ISCED level degree of 6 or higher). The education groups are then identified using 4-digit level ISCED field codes.[6] In Table 1, we show the education group shares in the top 1% and top 10% of the earnings distribution in 2013 by gender. Together, these groups constitute 36.4(28.7)% of the population of top 1(10)% earners. Compared to women, male top earners are more likely to be STEM and business majors, whereas female top earners are more likely to be medicine and law majors.

[6]More specifically, in our classification, a business major has a degree in a field within "accounting and taxation", "finance, banking and insurance", "management and administration" or "marketing and advertising". A STEM major has a degree in a field within "natural sciences, mathematics and statistics", "information technologies and communication technologies" or "engineering and engineering trades". Law and Medicine have their own separate ISCED codes.

Table 1: Share of education groups among top earners, Denmark 2013.

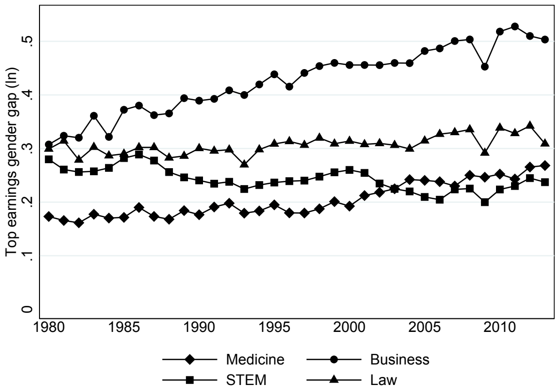
	Total	Women	Men
		Top 1%	
Full sample	100%	12.3%	87.7%
of which has a university degree in			
Business	11.9%	9.8%	12.2%
Medicine	7.6 %	13.3%	6.8%
STEM	12.3 %	9.8%	12.7%
Law	4.6 %	10.2%	3.9%
		Top 10%	
Full sample	100%	24.6%	75.4%
of which has a university degree in			
Business	6.7%	6.8%	6.7%
Medicine	3.3%	9.0%	3.7%
STEM	12.9%	8.8%	14.2%
Law	5.8%	6.4%	2.3%

Notes: Education specified according to the ISCED standard. Top 1 and 10 % are calculated from the one-year earnings distribution.

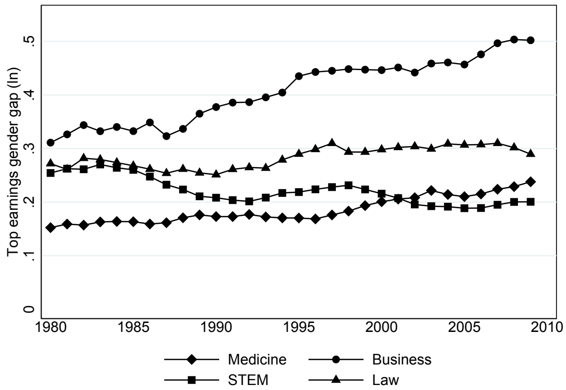
In Figure 4, we display the evolution of the top-earnings gender gap and the glass ceiling coefficient for the four education groups.

The top-earnings gender gap As seen from the top row of Figure 4, there is considerable variation in the top-earnings gender gap across educational groups. Focusing on one-year earnings in 1980, the lowest gaps are found within medicine. Here, the gap is below 0.2 log points (22 percent). In the other three groups, the gap was around 0.3 log points (35 percent). In contrast to the stable decline in the aggregate top-earnings gender gap (Figure 2), the gap has not declined within any of these groups over the period. For STEM and law majors, it has remained largely constant. For medicine, it has increased and converged towards the levels found in Law and STEM. For business majors, it has increased dramatically. In 2013, the top-earnings gender gap was 0.5 log points (65 percent), about twice as high as the level found in medicine and STEM. These patterns are largely the same for five-year earnings.

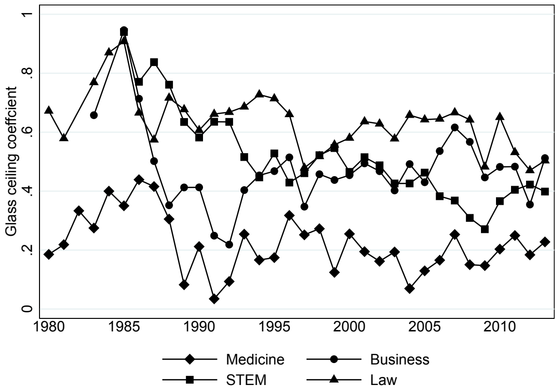
The glass-ceiling coefficient In the bottom row of Figure 4, we display the evolution of the glass-ceiling coefficient. Similarly to education-specific top-earnings gender gaps, there is also considerable variation here. In contrast, however, there is no positive trend within any education group over time. Given that sample sizes are considerably smaller here than in the previous analyses, the graphs are substantially smoother for five-year



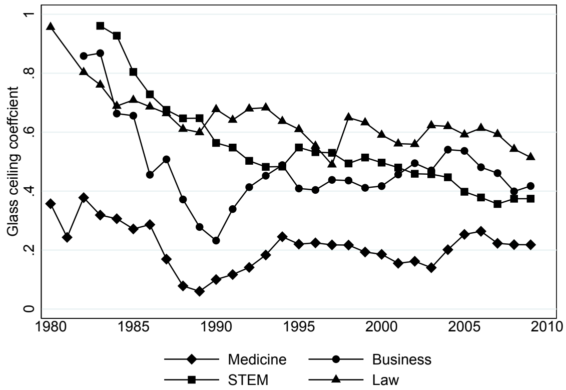
(a) Top earnings gender gap, 1-year earnings



(b) Top earnings gender gap, 5-year earnings



(c) Glass ceiling coefficient, 1-year earnings



(d) Glass ceiling coefficient, 5-year earnings

Figure 4: The evolution of the top-earnings gender gap and the glass-ceiling coefficient for Denmark 1980-2013, by education. Five-year earnings is computed as a forward looking moving average, see Section 3.1

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than for one-year earnings.[7] Focusing on five-year earnings, the lowest level of the glass-ceiling coefficient has been within medicine throughout the period, around 0.2 since 1995. This level means that the share of female medicine majors in the top 0.1% is around $0.1^{0.22} = 0.63$ of the share of female medicine majors in the top 1%, which in turn is around 0.63 of the share of female medicine majors in the top 10%. For the other three groups, the glass-ceiling coefficients have come down from very high levels in the early 1980's until today. In 2013, they lie all within the range $[0.35 - 0.55]$.

The contrasting evolution of the top-earnings gender gap and the glass-ceiling coefficient across these education groups again reinforces that they should be treated as separate phenomena. The large top-earnings gender gap among business majors suggests that their labor market is relatively less rewarding for women compared to men. How-

[7] For instance, law constitutes only about 5 percent of the sample of top 10 percent earners, as displayed in Table 1.

ever, business does not display a significantly higher glass-ceiling coefficient than law and STEM majors, meaning that a female business major is as likely a female STEM major to be in the very top of their respective earnings distribution conditional on being in the top of their respective earnings distribution. From this perspective, conditional on making it to the top 10 percent of the earnings distribution, business does not appear to be a less rewarding education for women compared to men.

4 Conclusion

Building on the insight of Atkinson et al. (2018), we have shown that top-earnings gender disparities can be summarized by two parameters: a top-earnings gender gap and a glass-ceiling coefficient. These parameters are informative and easily computed. We have also shown that the difference between the male and female top income distributions are observationally equivalent to a progressive tax schedule on women's income, on the Feldstein (1969)-form, with the progressivity of the tax schedule determined by the glass-ceiling coefficient. Finally, using Danish administrative data 1980-2013, we have documented how these two measures vary across time, the life cycle, and educational groups. The overarching conclusion is that the top-earnings gender gap and the glass-ceiling coefficient display different time-series patterns, suggesting that they should be treated as separate phenomena.

In Denmark, the top-earnings gender gap has been narrowing from a gap of 0.4 (49%) in 1980 to a gap of 0.3 in 2013 (35%). In contrast, the glass-ceiling coefficient is without trend, with a value of approximately 0.4 throughout the period. Over the life cycle, the cohort top-earnings gender gap is hump shaped while the cohort glass-ceiling coefficient is steadily increasing. Across education groups, there is substantial variation in both the top-earnings gender gap and the glass-ceiling coefficient. For all these groups, the evolution of the glass-ceiling coefficient is different from that of the top-earnings gender gap.

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