A few notes about decomposing the two way fixed effects estimator

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What is this?

Objective

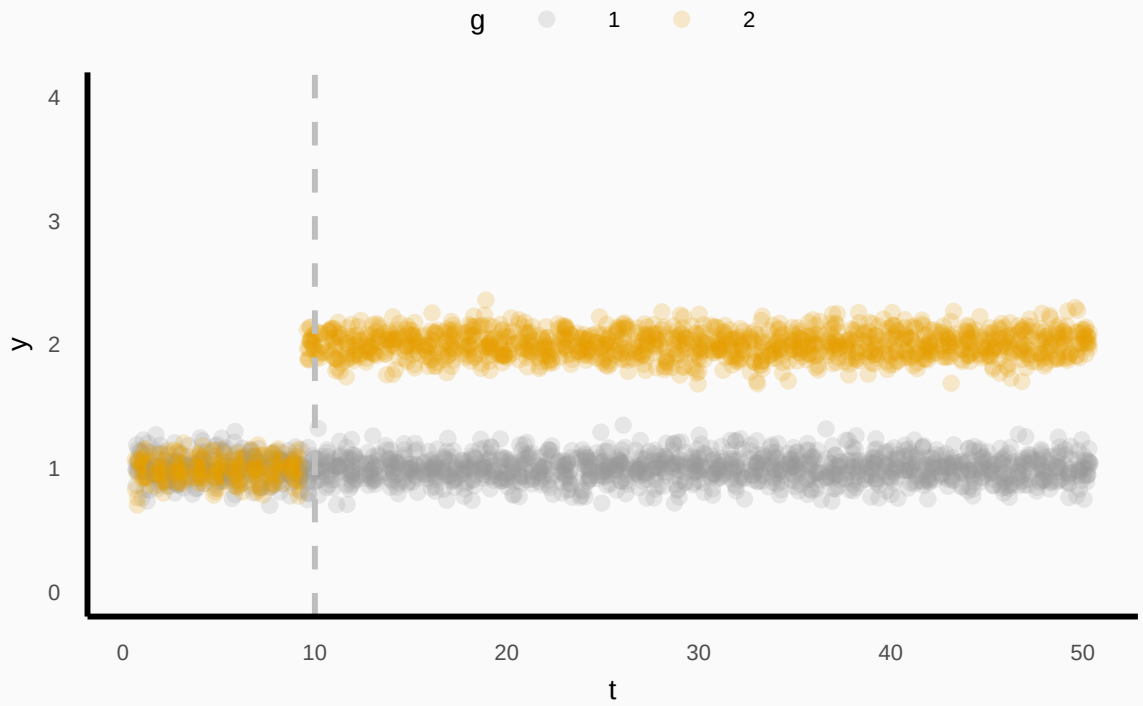
- Introduce the Chaisemartin & D'Haultfoeuille (forthcoming, AER; CD) & Goodman-Bacon (2019 WP; GB) decomposition of the two way fixed effects estimator and link them.

Disclaimer

- The slides represent my understanding. It might be wrong and there will be mistakesg. Please let me know.
- There is **a lot** more in CD's and GB's papers.
- I don't discuss assumptions (!) and extensions.
- Please see the original papers.

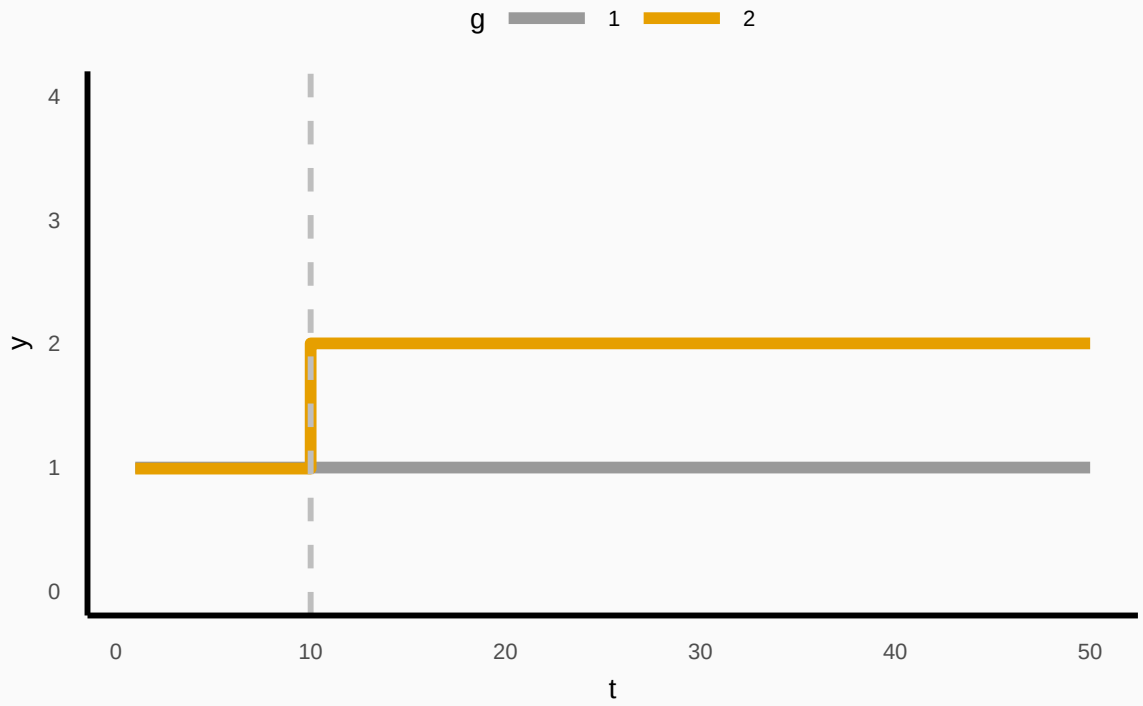
The traditional difference-in-differences (DD)

The running example



- t: period, g: group, y: outcome, d: treatment indicator, $g = 1$ is never treated, $g = 2$ is treated from $t = 10$.

The traditional DD

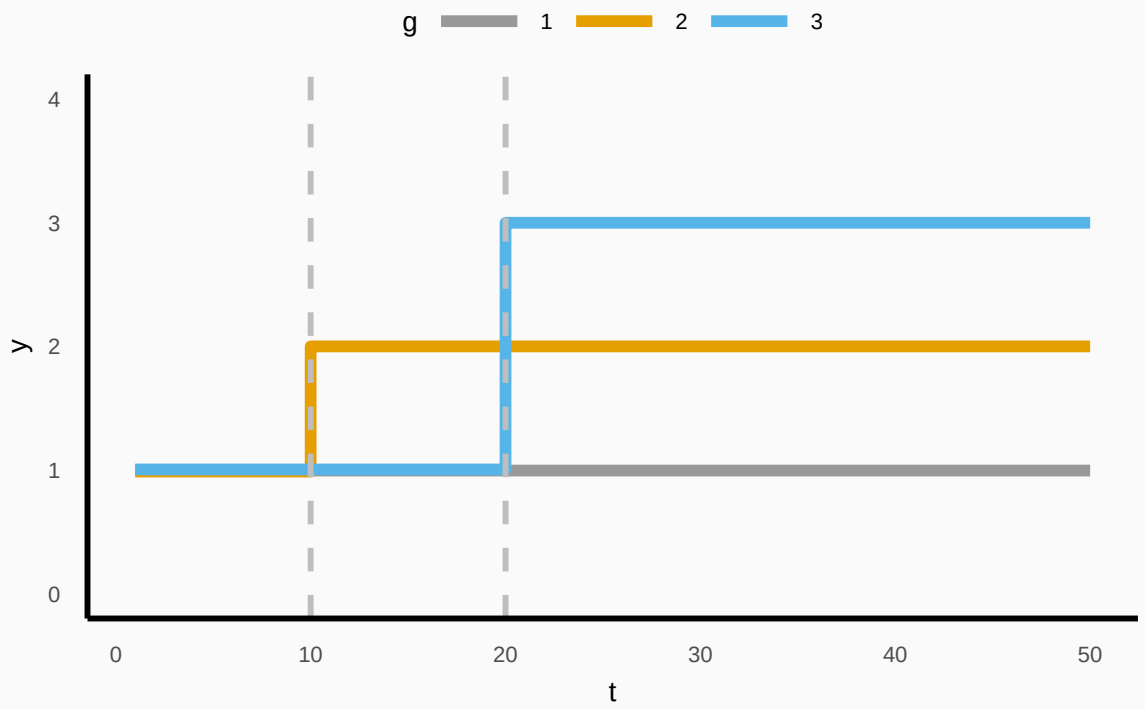


- t: period, g: group, y: outcome, d: treatment indicator, $g = 1$ is never treated, $g = 2$ is treated from $t = 10$.
- DD estimate:

$$\hat{\beta}^{DD} = (\bar{Y}_{g=2,t\geq 10} - \bar{Y}_{g=2,t<10}) - (\bar{Y}_{g=1,t\geq 10} - \bar{Y}_{g=1,t<10}) \qquad (1)$$

- As seen in for example *Card & Krueger (1994) "Minimum Wages and Employment: A Case Study of the Fast Food Industry in New Jersey and Pennsylvania", AER, 84, 772-784.*

DD with more groups



- Add another group, $g = 3$ that gets treated from period $t = 20$.
- DD estimate is then often estimated as $\hat{\beta}_{fe}$ from an OLS regression of:

$$y_{i,g,t} = \alpha + \gamma' G_g + \psi' T_t + \beta_{fe} D_{g,t} + e_{i,g,t} \tag{2}$$

- β_{fe} is also called the two-way FE estimator (we have both group and period fixed effects).

What β_{fe} capturing and how does it relate to $\hat{\beta}^{DD}$?

Bit question: What is β_{fe} capturing?

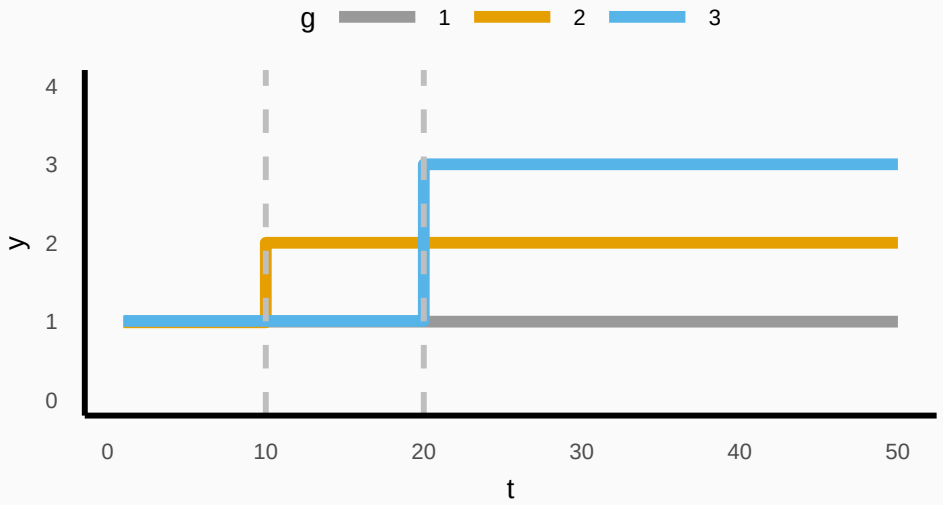
- *Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)*
  - **Decomposition of $\hat{\beta}_{fe}$ in weighted ATEs across (t, g) cells.**
  - Applicable to 2-way (e.g., group & time) fixed effects approaches.

- *Goodman-Bacon (GB): "Difference-in-Differences with Variation in Treatment Timing" (2019, WP)*
  - **Decomposition of $\hat{\beta}_{fe}$ in weighted $\hat{\beta}^{DD}$s**
  - Applicable to staggered adoptation designs.
- (There are other papers.)

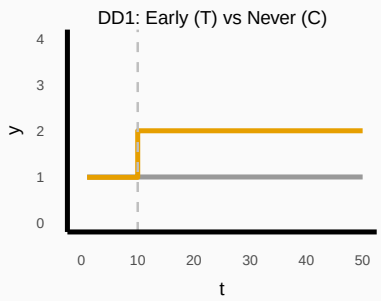
Let's start with the GB approach

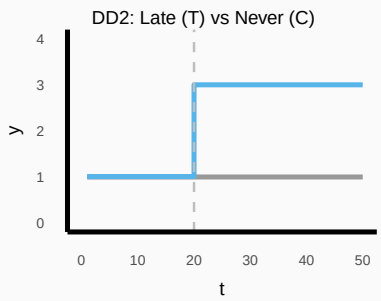
The GB decomposition

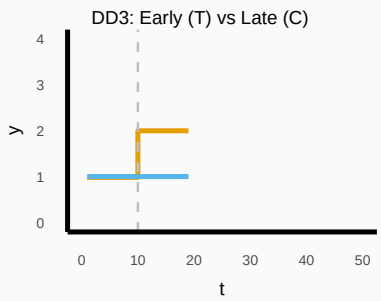
- We consider the example with 3 groups and a staggered introduction of a policy:

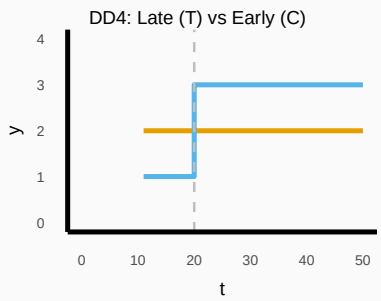


- Goodman-Bacon: we can decompose this in 4 separate $\hat{\beta}^{DD}$:









GB: β^{DD} is a weighted average of the separate DDs

- In our running example, β^{DD} is the weighted average across these 4 DDs:

$$\hat{\beta}_{fe} = w_1\hat{\beta}_1^{DD} + w_2\hat{\beta}_2^{DD} + w_3\hat{\beta}_3^{DD} + w_4\hat{\beta}_4^{DD} \tag{3}$$

- Key insight: $w \neq$ population shares, but also depends on when a group gets treated.
- See Theorem 1 in GB for the general expression of (3) (equation 10a in his paper) and the definition of the weights.
- GB then shows that:

$$\beta^{DD} = VWATT + VWCT - \Lambda ATT \tag{4}$$

where:

- $VWATT$: variance weighted ATE.
- $VWCT$: variance weighted common trends.
- ΔATT time varying treatment effect.

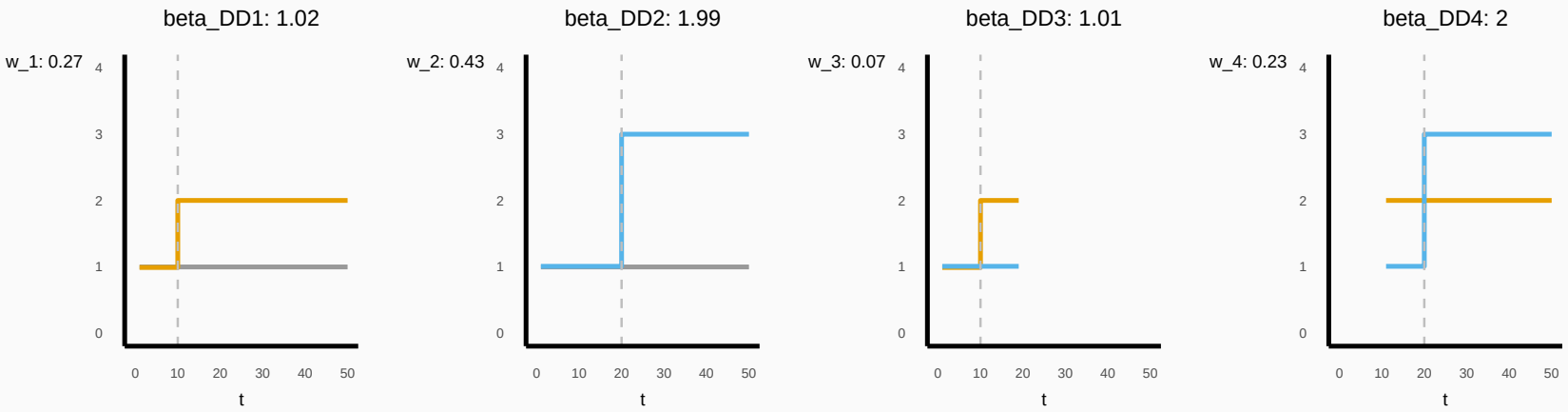
(see page 12 in Goodman-Bacon, 2019)

Is it a problem? Let's consider an example.

DGP

- treatment effects: group2=1 (yellow), group3=2 (blue)
- equal group sizes
- **population weighted average treatment effects=1.5**

Estimated effects

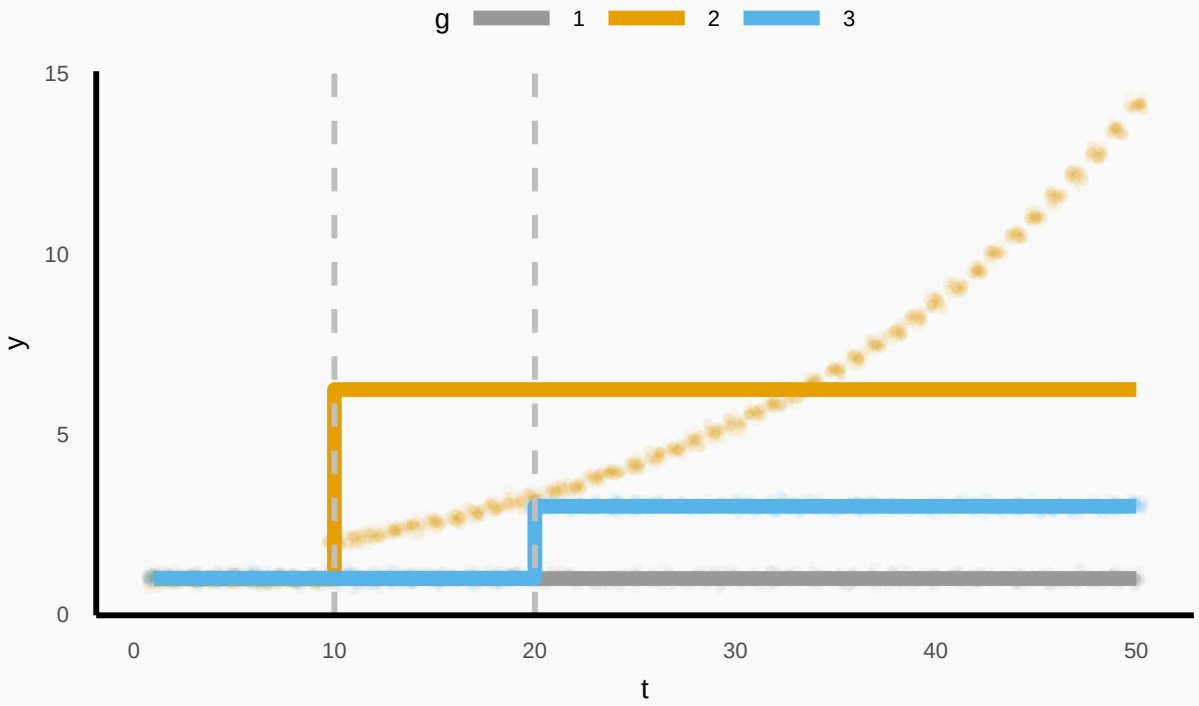


- $\hat{\beta}_{fe} = 0.27 \times 1.02 + 0.43 \times 1.99 + 0.07 \times 1.01 + 0.23 \times 2 = 1.66$
- Group 3 (blue) gets a higher weight because it is treated more in the middle.
- Not a problem if you know that it is not population weighted.

What about the last term (ΛATT)? Treatments effects varying over time.

DGP

- We now let the treatmetn effect grow with a rate of 3 percent per period in group 2
- **population weighted average treatment effect=2.42**



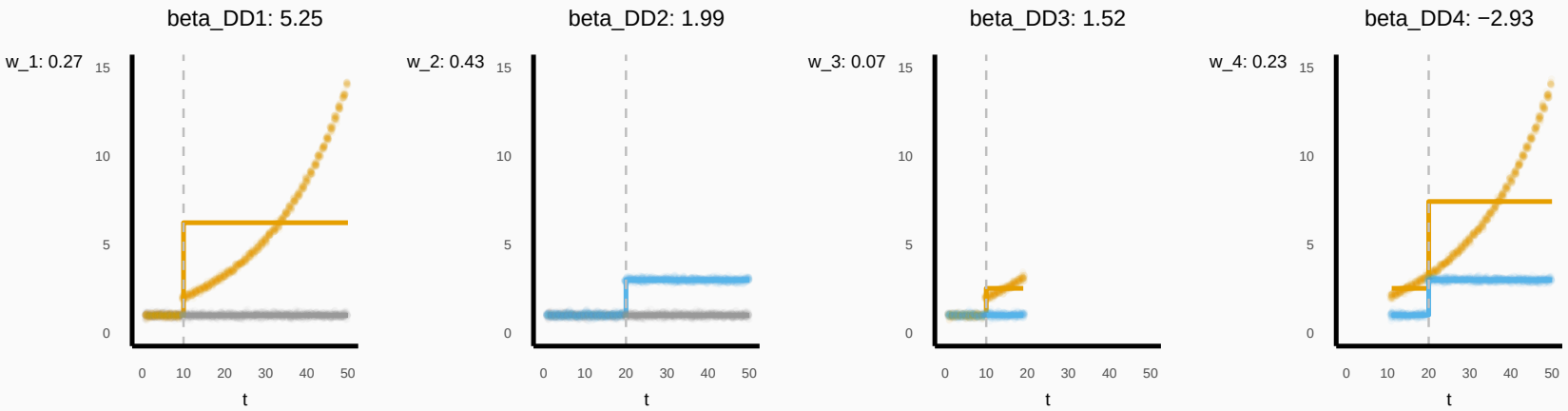
- **BUT** $\hat{\beta}_{fe} = 1.73$
- That issmaller than the average treatment effect in any of the groups!
- Why?

What about the last term (ΛATT)? Treatments effects varying over time.

DGP

- We now let the treatmetn effect grow with a rate of 3 percent per period in group 2
- **population weighted average treatment effect=2.42**

Decomposing the estimated effect using GBs approach



- $\hat{\beta}_{fe} = 0.27 \times 5.25 + 0.43 \times 1.99 + 0.07 \times 1.52 + 0.23 \times -2.93 = 1.73$

What if we left out the last DD?

- $\hat{\beta}_{fe} = 0.27 \times 5.25 + 0.43 \times 1.99 + 0.07 \times 1.52 = 2.39$

GB Summary

Goodman-Bacon (GB): "Difference-in-Differences with Variation in Treatment Timing" (2019, WP)

Take aways

- $\hat{\beta}_{fe} =$ is a weighted average of all the DDs possible combinations
- Weights $\neq$ popualion shares, but depend on treatment timing. Groups that are treated towards the middle of the period receive more weight.
- If treatment effects vary over time, some DDs can be negative.
- Also relevant for the common trend assumption: Violations can offset each other.
- GB derives test for violation of weighted common trend assumption.

What we should do

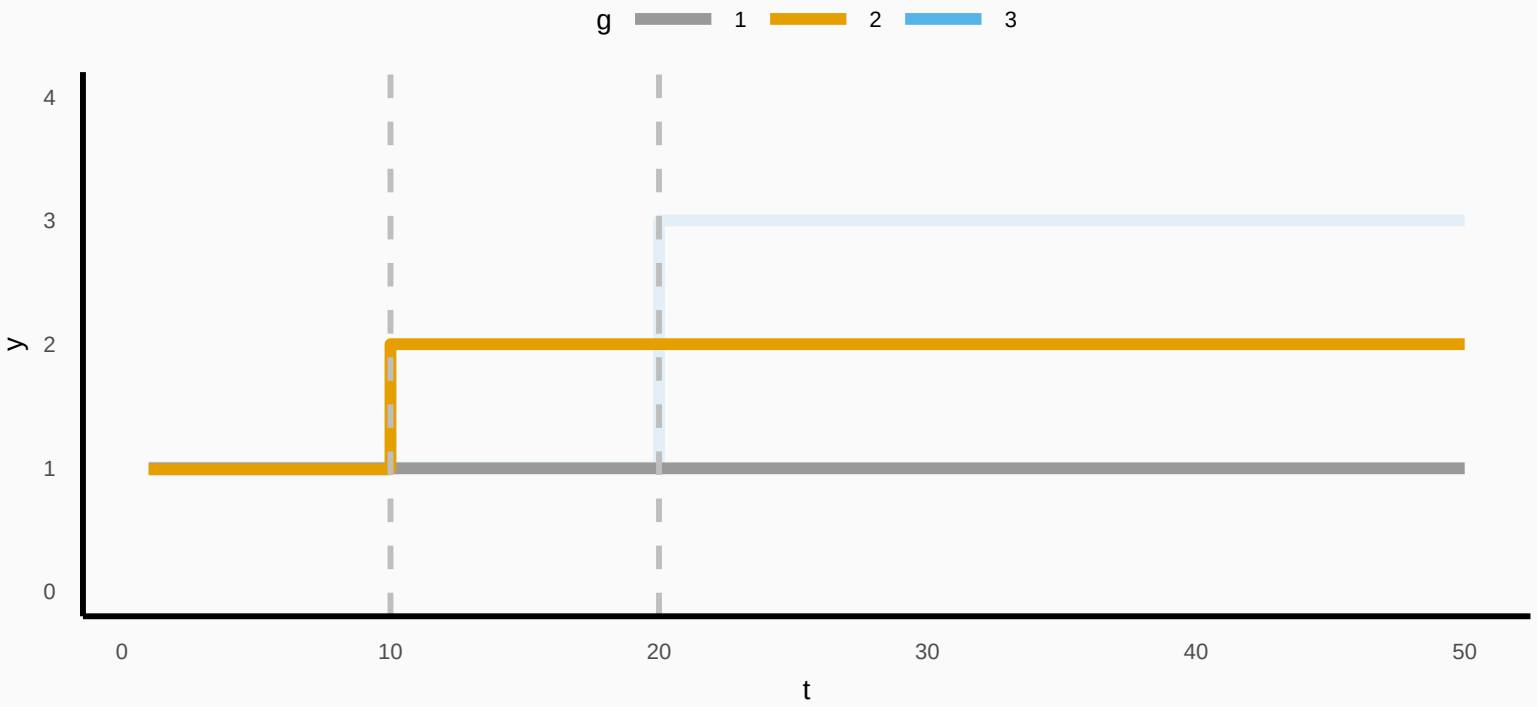
- Compute weights and DDs and check whether any of them are negative. ("bacondecomp" function in R and Stata).
- Test for violation of common trend assumption.
- Read the paper (there is much more than ocvered here)

Now to the CD decomposition

Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)

Decompose β_{fe} in weighted TEs across $(g, t) : D_{g,t} = 1$ cells

- (back to the original example with constant treatment effects over time)

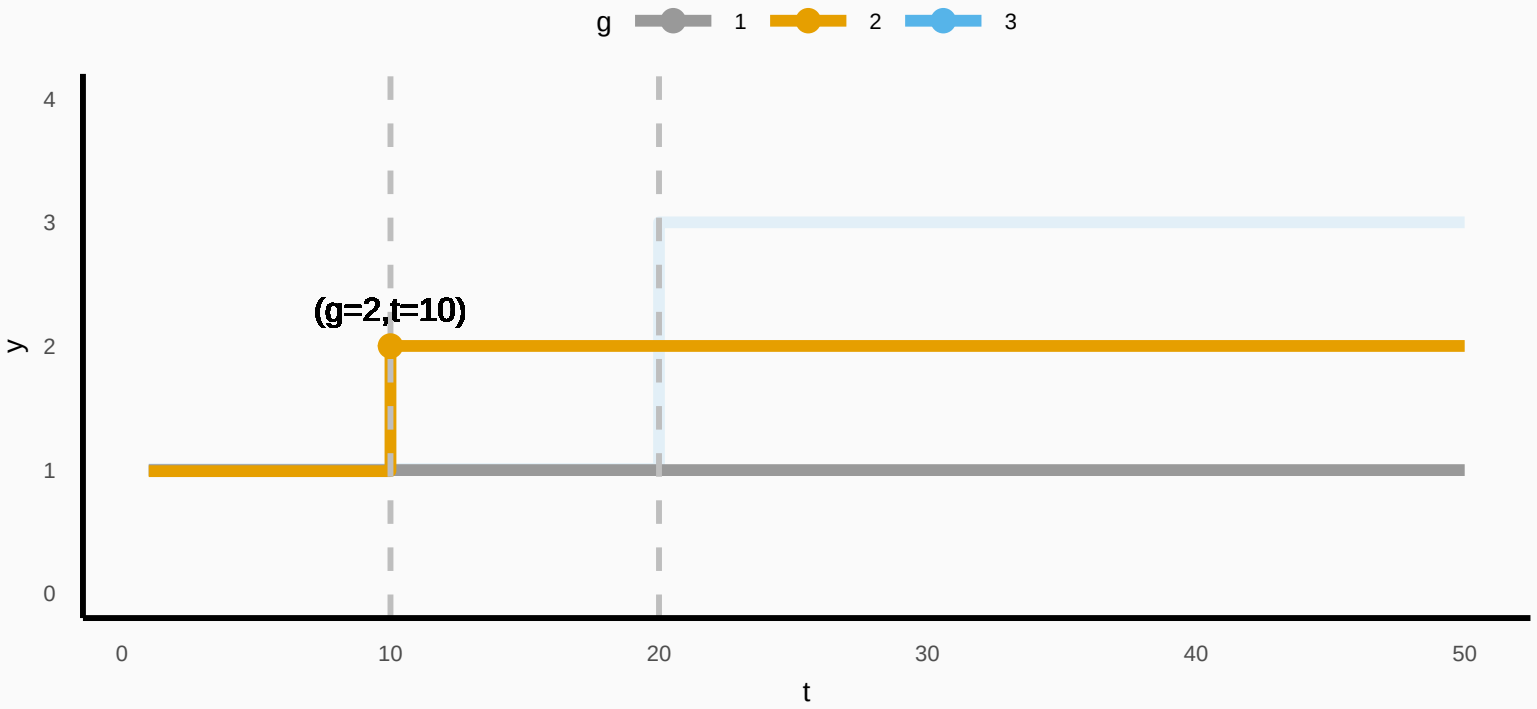


The CD decomposition

Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)

Decompose β_{fe} in weighted TEs across (g, t) : $D_{g,t} = 1$ cells

- i.e. for the cell $g = 2, t = 10$, weight number: 1

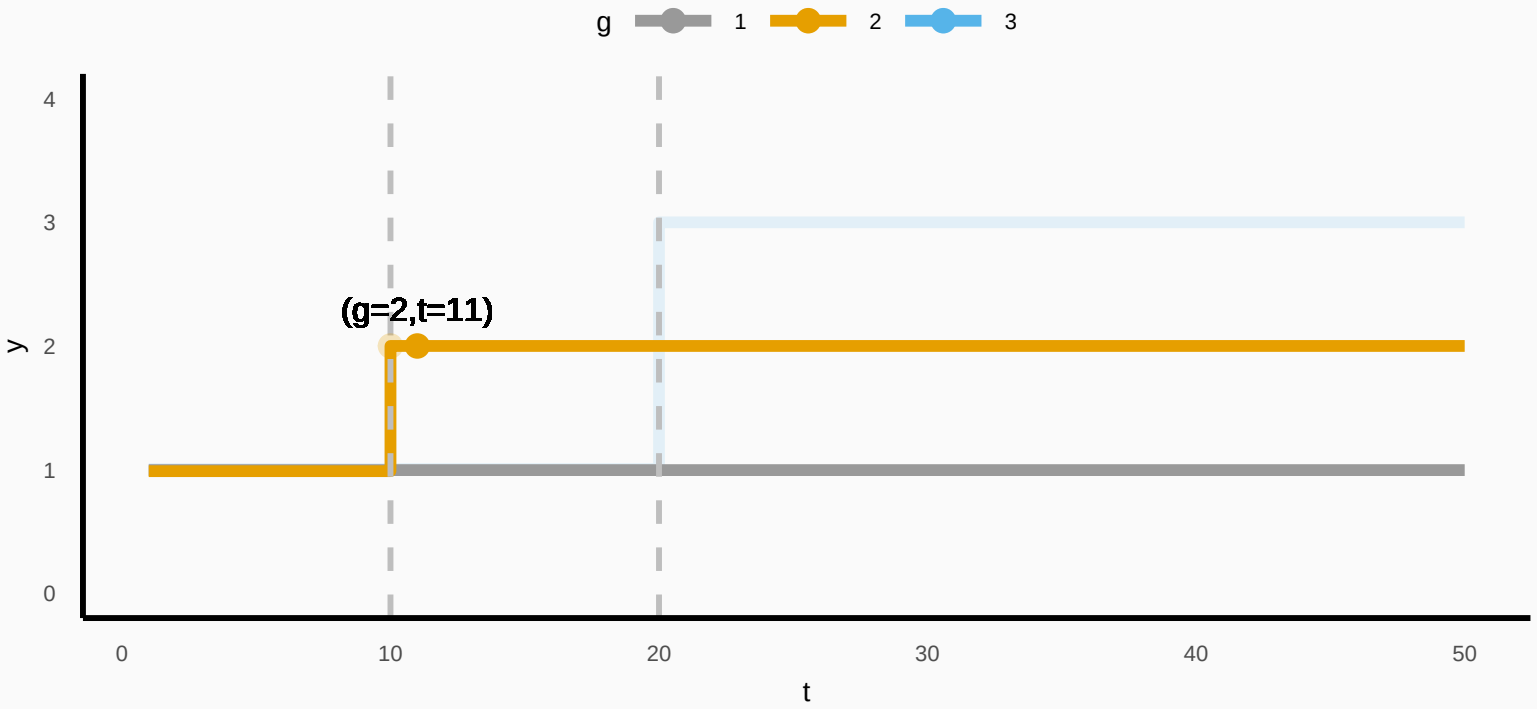


The CD decomposition

Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)

Decompose β_{fe} in weighted TEs across (g, t) : $D_{g,t} = 1$ cells

- i.e. for the cell $g = 2, t = 11$, weight number: 2

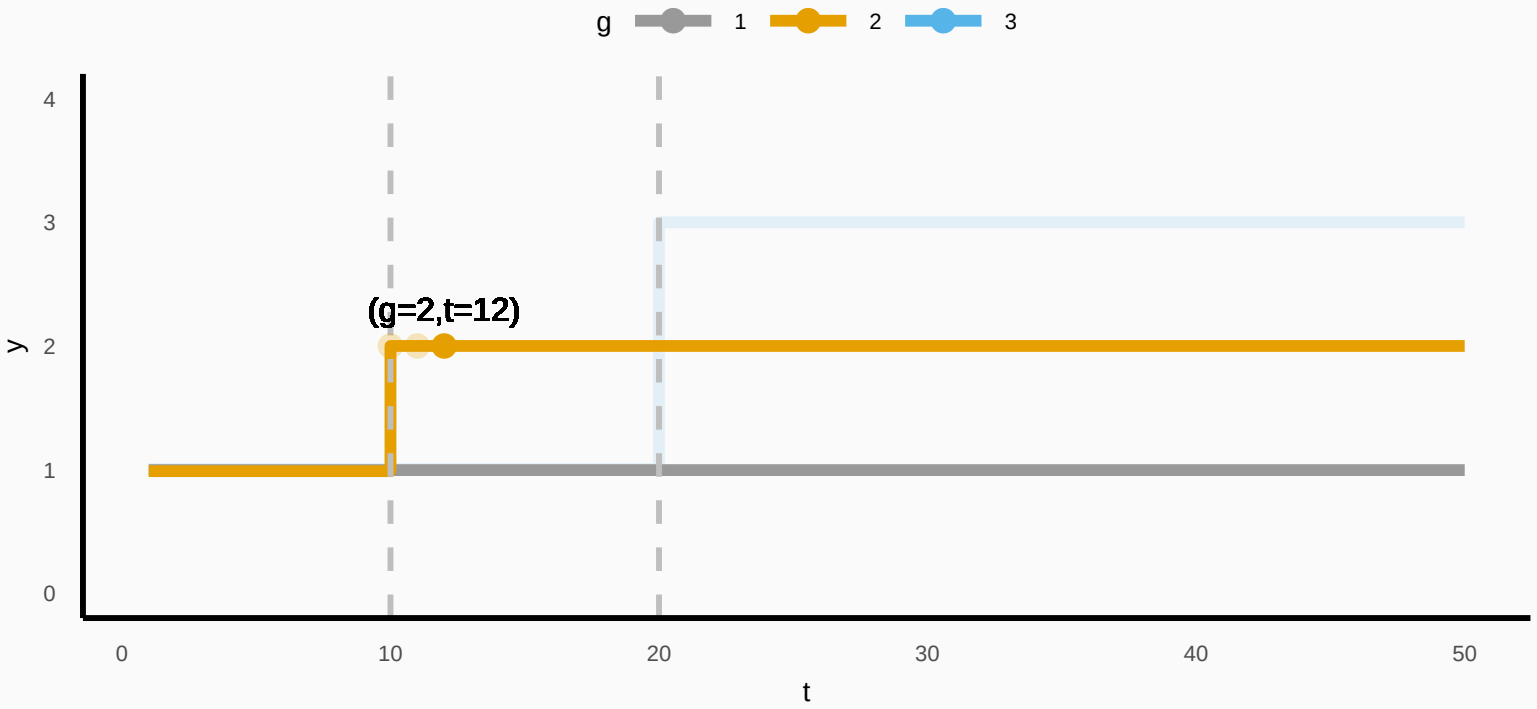


The CD decomposition

Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)

Decompose β_{fe} in weighted TEs across (g, t) : $D_{g,t} = 1$ cells

- i.e. for the cell $g = 2, t = 12$, weight number: 3

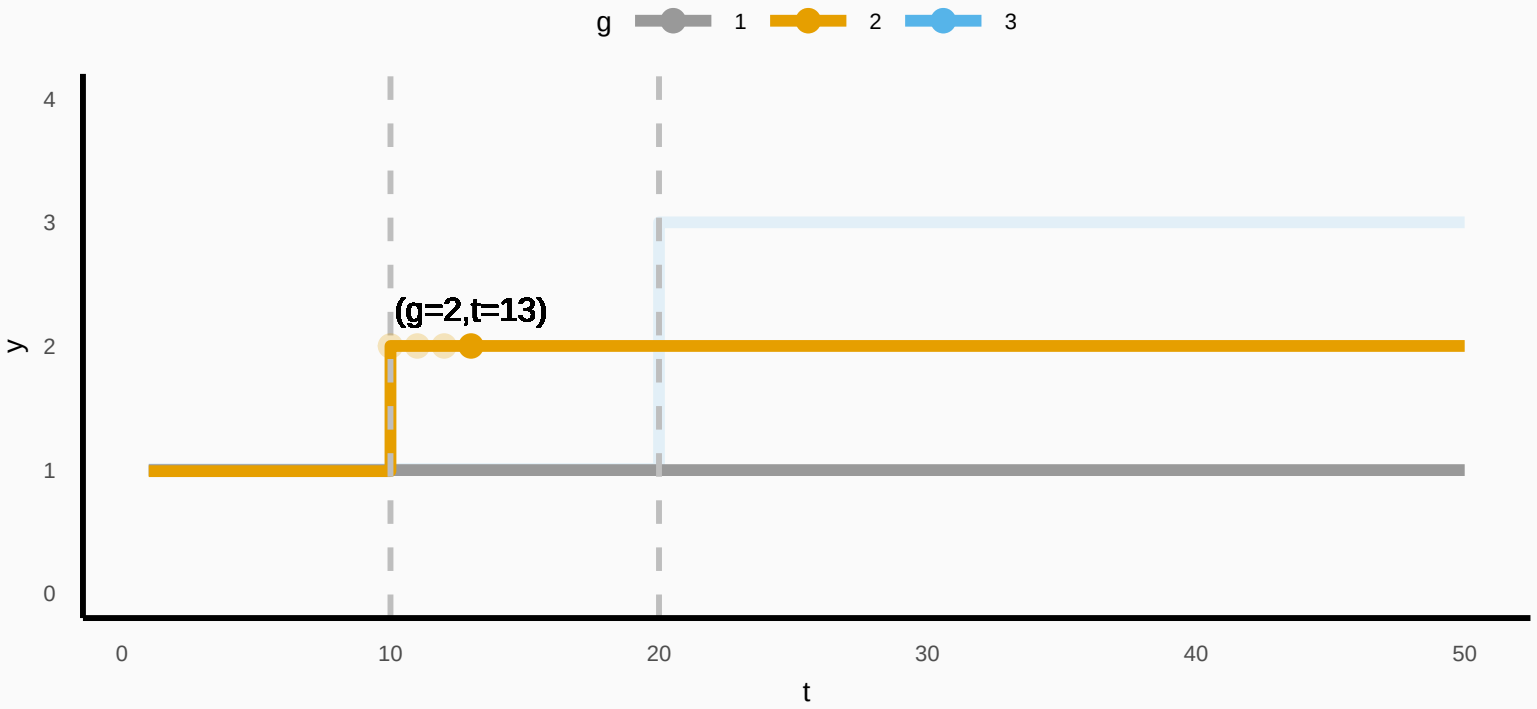


The CD decomposition

Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)

Decompose β_{fe} in weighted TEs across (g, t) : $D_{g,t} = 1$ cells

- i.e. for the cell $g = 2, t = 13$, weight number: 4

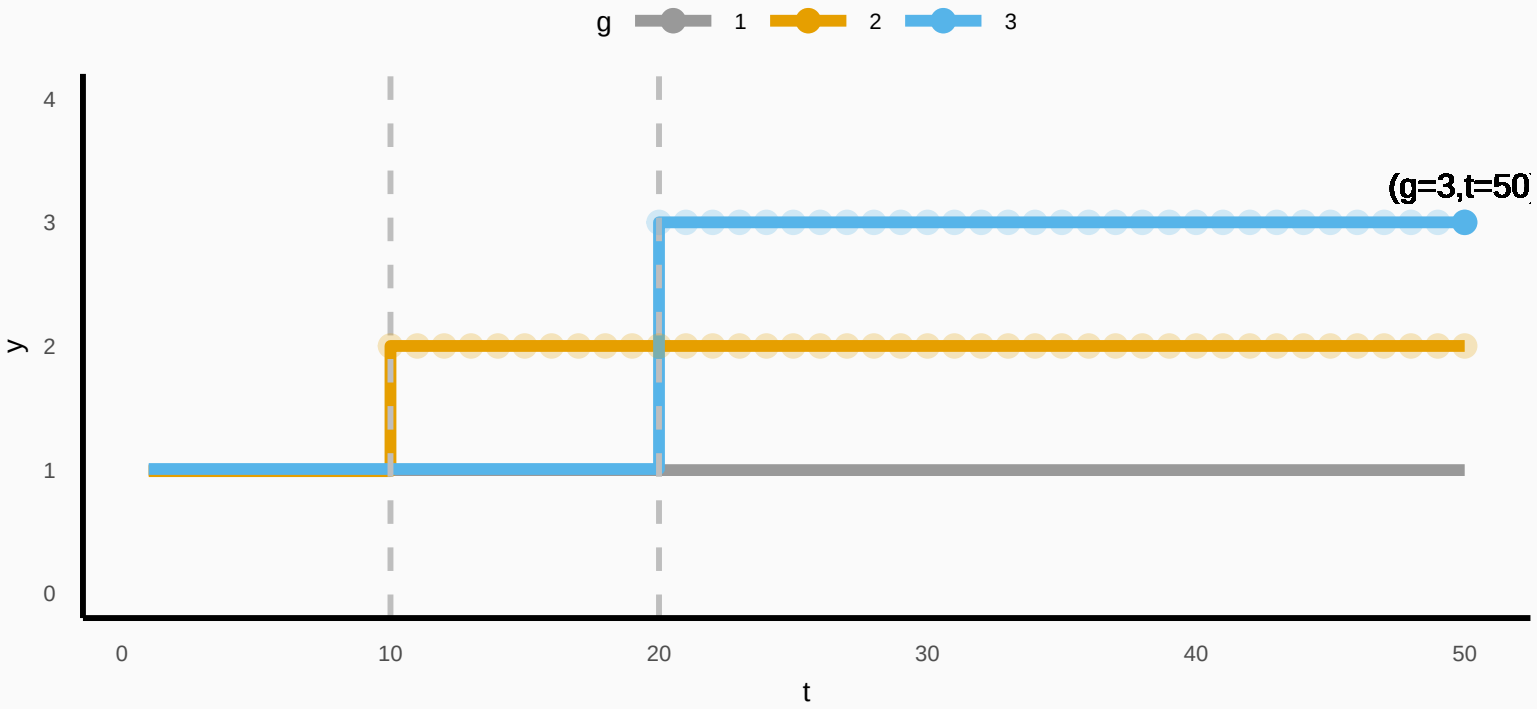


The CD decomposition

Chaisemartin & D'Haultfoeuille (CD): "Two-way fixed effects estimators with heterogeneous treatment effects" (forthcoming, AER)

Decompose β_{fe} in weighted TEs across (g, t) : $D_{g,t} = 1$ cells

- i.e. for the cell $g = 3, t = 50$, weight number: 72



The CD decomposition

Decompose β_{fe} in weighted TEs across $(g, t) : D_{g,t} = 1$ cells

- β_{fe} is then given by

$$\beta_{fe} = E\left[\sum_{(g,t):D_{g,t}=1} \frac{N_{g,t}}{N_{D=1}} w_{g,t}\Lambda_{g,t}\right] \tag{5}$$

where

- $w_{g,t}$ is the weight on group g in period t
- $\Lambda_{g,t}$ is the TE in group g in period t
- $N_{D=1}$ is the number of treated units.
- $N_{g,t}$ is the number of observations in cell g and t.

See Theorem 1 in CD

- How are the weights defined?

CD: Calculating the weights

Calculating the CD weights

1. Run a regression of $D_{g,t}$ on a constant, g and t fixed effects.
2. Save the residual, $e_{fe,g,t}$
3. Create the weight of that cell as

$$w_{fe,g_t} = \frac{e_{fe,g,t}}{\sum_{(g,t):D_{g,t}=1} \frac{N_{g,t}}{N_{D=1}} e_{fe,g,t}} \tag{6}$$

Intuition

- Consider cell (g, t) from a group that is treated for almost the entire period and at a time where almost all cells are treated.
- We would predict that this cell is treated (because it is from a group that is mostly treated at a time that is mostly treated).
- $\Rightarrow$ small or even negative residual
- $\Rightarrow$ small or even negative weight.

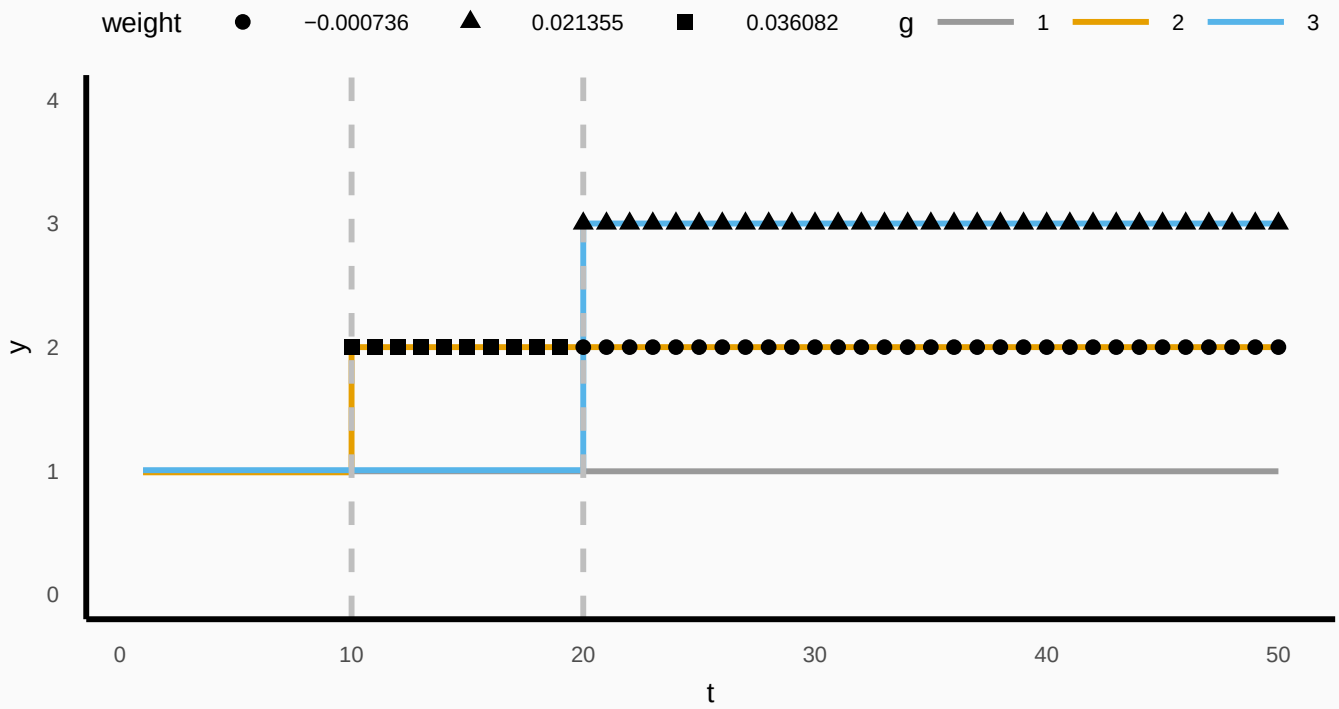
An example from CD

- 2 equally sized groups, 3 periods
  - group 1 is only treated in period 3
  - group 2 is treated in period 2 and 3

- The weights are then:
  - $e_{fe,1,3} = 1/6$
  - $e_{fe,2,2} = 2/6$
  - $e_{fe,2,3} = -1/6$

- So that $\beta_{fe} = 1/2 \times E(\Lambda_{1,3}) + 1 \times E(\Lambda_{2,2}) - 1/2 \times E(\Lambda_{2,3})$
- If for example $E(\Lambda_{2,3}) = 4$ & $E(\Lambda_{1,3}) = E(\Lambda_{2,2}) = 1$, then $\beta_{fe} = -1/2$
- Estimate is negative, although all cell's treatment effects are positive!
- This is only an issue with heterogenous treatment effects.

Let's calculate the weights in our example



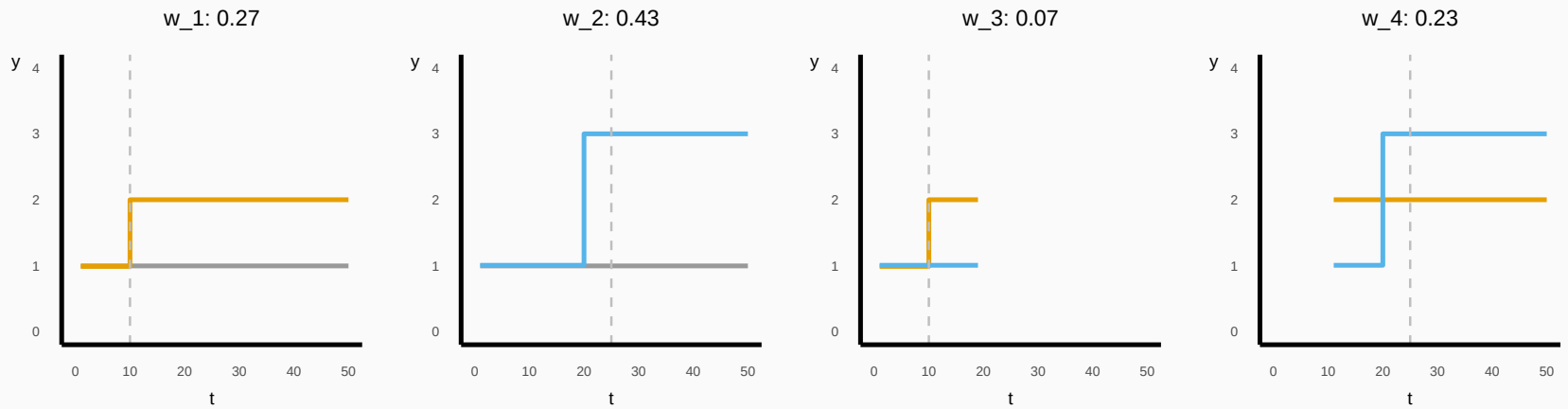
- Largest weight at the beginning: few groups are treated.
- Group 2 cells even get negative weight after $t = 20$ because they are from a group that is mostly treated and at a time that that is mostly treated.

And using equation (5) we get:

- $\hat{\beta}_{fe}$=10 $\times$ 0.03608 $\times$ 1+ 31 $\times$ 0.02135 $\times$ 2- 31 $\times$ 0.0007361 $\times$ 1=1.66
- Note: the cells' TEs are normally unobserved (we know the DGP).

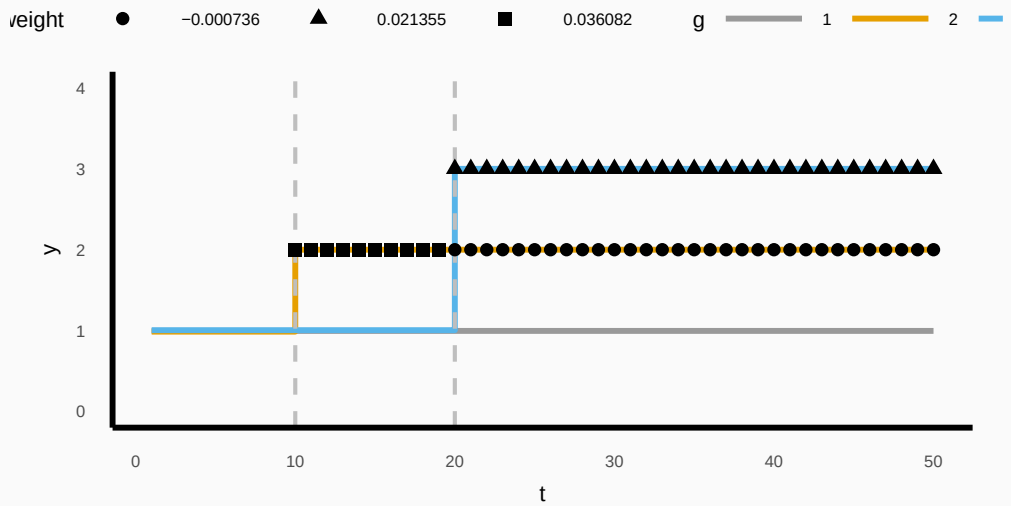
Can we link the CD and GB weights?

The GB weights



- Sum of weights where G=2 is treated: 0.27+0.07=0.34
- Sum of weights where G=3 is treated: 0.43+0.23= 0.66

The CD weights



- Sum of weights where G=2 is treated: 10 $\times$ 0.03608 -31 $\times$ 0.0007361=0.34
- Sum of weights where G=3 is treated: 31 $\times$ 0.02135=0.66.

Back to CD

So what should we do?

1. Calculate weights of the cells!
2. If you have negative weights: calculate $\underline{\sigma}_{fe}$ (See Corollary 1 in the paper)

- tells us the amount of treatment heterogeneity that is required to for the pop weighted average treatment effect to have the opposite sign of $\hat{\beta}_{fe}$.
- So if $\underline{\sigma}_{fe}$ is very small you should be worried!
- Are treatment effects likely to be correlated with weights? Check if weights are correlated with covariates.

3. If things still look bad: use their new estimator: DID_M

- Identies average effect across switching cells:

$$DID_M = \sigma_{t=2}^{T} \left(\overbrace{\frac{N_{1,0,t}}{N_S} DID_{+,t}}^{\text{"joiners"}} + \overbrace{\frac{N_{0,1,t}}{N_S} DID_{-,t}}^{\text{"leavers"}} \right) \tag{7}$$

4. In Stata

- `twowayfeweights` calculates weights and $\underline{\sigma}_{fe}$
- `did_multiplegt` estimates DID_M
- R is on the way

Read!

Read both papers. There are many more insights in the papers.

- Check assumptions!
- Adding controls
- First differences
- Placebo tests
- Correlation between weights and covariates

. . . and much more

Summary

Question: What is $\hat{\beta}_{fe}$ from OLS regression of:

$$y_{i,g,t} = \alpha + \gamma' G_g + \psi' T_t + \beta_{fe} D_{g,t} + e_{i,g,t}$$

capturing?

- Goodman-Bacon (2019, WP) shows that in a staggered adoptation design we can think of this as a weighted average of all possible difference-in-differences.
  - Weights are not equal to population weights, but depend on when units become treated!
  - Has implications for common trend assumption.
  - DDs might enter with opposite sign if treatment effect varies over time.

- Chaisemartin & D'Haultfoeuille (forthcoming, AER) show that $\hat{\beta}_{fe}$ is a weighted average across all treated (g, t) cells.
  - The weights can be negative and bias estimates when TEs are heterogeneous!
  - Provides a quantification of the degree of TE heterogeneity required in order to flip the sign.
  - Provides a new estimator.
  - And much more.

What I will do now

1. Calculate Chaisemartin & D'Haultfoeuille weights (with `twowayfeweights` in Stata or manually in R).
2. If I have negative weights, I'll check how much treatment heterogeneity is required to flip the sign (with `did_multiplegt` in Stata).
3. If 2. is small and it is meaningful, I'll apply their new estimator.

- Note it only identifies from switching cells!

4. If I have a staggered adoption design: check for negative DDs using GB approach (using `bacondecomp` in Stata).
5. Create event study charts if possible.
6. Use new toolbox to check placebo effects/ weighted common trend assumptions.

Thanks

Thanks to Clément, Xavier & Andrew for excellent papers!