Education as a Key to Rectifying Misspecified Models and False Beliefs

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by

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Abstract

This paper examines the role of education in decision-making in the context of social learning and misspecified models. We develop a model inspired by the $2 \times 2 \times 2$ model [1], exploring how individuals' decisions are influenced by their educational background and their predecessors' action. In this paper, we develop of model that focuses of such influence under imperfect information and misinformation. The paper highlights the potential for overestimation of education's efficacy in enhancing decision-making accuracy. Through a focused analysis of the first two agents' choices, the research reveals insights into the dynamics of social learning, the valuation of education, and delves deeper into the impact of Agent 1's choice onto these of Agent 2.

Contents

1 Introduction

1.1 Background and Context

In the contemporary digital age, the phenomenon of observational learning has emerged as a cornerstone in the dissemination of information, molding opinions, and shaping behavioral patterns. Observational learning plays an important role in the transmission of information, opinions and behavior. For instance, individuals often look to movie box office rankings when deciding which films to watch. The choice of adopting new technology or software in a workplace can be heavily influenced by colleagues' preferences and decisions. Social learning also influences behavioral choices, such as dietary habits or commitment to physical fitness, as well as shaping views on societal or political matters, like environmental conservation or political affiliations. Given that most of our decisions are influenced by our prior belief and observations, and that we observe an increasing amount of disinformation, we have trouble capturing the complexity of others' decision-making process. This phenomenon is captured the model designed by [2] J. Aislinn Bohren in his paper on *Informational herding with model misspecification.* There he shows that information inferences cause social learning to fail in its converge to right information. Thus, it explains how misinformation leads to some misinterpretation. It highlights the importance to understand how people learn from the actions of their peers and act accordingly. This paper explores how a misspecified model of information may interfere with one's decision making process, it delves deeper into the effects of misperception of this information and disinformation.

We often encounter a complex task when trying to interpret information based on others' actions. These actions often reflect a mix of interrelated pieces of information, to fully grasp the underlying details of this model, one must understand the primary information leading one's actions. Hence, we have decided to distinguish two types of people, these who are considered "influential" – who will be referred to as educated agents – and the others – the uneducated agents–. In reality, we can question our ability to judge these situations: what if individuals misjudge their surroundings or overestimate their own knowledge? What if they give too much credit to their information and their perceptions? These questions lead to a model that permits individuals to over-trust their information. While influence can come from multiple reasons, in the case of trusting people with their choice and their information, we often give our trust to people who received an education. Indeed, through education, we are trained to sift through misinformation and to exercise critical-thinking, not taking every source at face value and understanding that others do not navigate with full information. This model incorporates the potential for such an educational process, allowing individuals to uncover the truth and perceive the right model.

To set this model, we follow the setup of the 2x2x2 model from the Economic Models of Social Learning (Chamley. Rational Herds) [1]. This model is characterized by its framework, which involves two time periods, two potential states of the world, and two possible actions. There, in the first time period, an individual receives a private signal about the state of the world, which is imperfect but informative, and chooses one of two actions based on this signal as they try to match the state of the world. In the second period, another individual observes the action of the first but not their private signal, receives their own signal, and then makes a decision. This setup highlights the process of social learning, where the latter individuals' decisions are influenced not only by their private information but also by the observable actions of their predecessors. The model captures the nuances of information cascades, where rational agents might choose to follow the majority action observed, leading to potential herd behavior, despite possessing private information that might suggest a different course of action.[3] It serves as a critical tool in understanding how decisions are made in environments of uncertainty and imperfect information.

Here we consider a model where individuals have common-value preferences that depend on an unknown state of the world. They each observe the public information based on their prior belief and the actions of the previous agents, and their individual cost of education. Then, they decide whether or not they should follow education, after this decision, they receive a signal. Lastly, they choose to act according to their signal – their private information – and the public information. While each agent first thinks that education will increase the precision of their private information, it makes them observe their true environment: each agent receives a signal of constant accuracy no matter his choice of education. These socially informed agents understand that prior actions reveal information about their predecessors' choice of action but do not increase their private information's accuracy. Indeed, all agents observe their predecessors' choice of education and their actions. Formally, uninformed agents believe that any uneducated individual received a signal of probability p_m of being right and that all educated individual receive a signal of probability p_h of being right ; with $p_m < p_h$. But informed agents know that every agent receives a signal of probability of being right of p_l, with $p_l < p_m$, and that uneducated agents are not aware of such dynamics. The idea behind this misspecification is that uneducated agents attribute too much credit to their private information and to the action of educated agents.

This paper tries to capture the dynamic to understand how model misspecification affects decision-making process. It builds the foundation for a more general approach by studying and interpreting the case of the two first agents. It differentiates the cases and finds conditions on the costs of education and prior belief of the agents to maximize his payoff. It also covers the possibility of inefficient education: with some settings, uninformed agents may have no incentive to choose education in the first place.

1.2 Research Objectives

Model misspecification has important policy implications for interventions aimed at acting against inefficient social choices. In the presence of information misperception and false information, the cost, utility and strength of education – such as university degree or public information campaigns – are major components of agents' decision-making process.

In economical decision-making, education is often used as a marker of superior analytical capability and knowledge. The prevailing belief is that educated individuals, particularly those with advanced degrees (like master degrees), are better equipped to tackle complex issues due to their advanced knowledge. However, this assumption may overlook that while education broadens one's culture and range of knowledge, it doesn't necessarily endow an individual with comprehensive expertise across all domains. Although an education does not provide an expertise in all domains and does not give "magical tools", it does enhance critical thinking and develop a better perception of one"s environment.

This research focuses on comprehending how education impacts decision-making accuracy within misspecified models. The key research questions include exploring the extent to which formal education impacts the critical evaluation of information by individuals, the role of education in modifying the dynamics of information cascades, and whether the incorporation of education into economic models can enhance their descriptive and predictive accuracy. Through this inquiry, we aim to refine economic models for more accurate and effective policy implications.

2 The General Framework

We consider a discrete model of action in which each agent faces two choices. All agents are rational, indexed by $i \in \mathbb{N}$ and we considere the agents to be ordered (ie. Agent 1 acts first, Agent 2 acts second, Agent 3 acts third, etc.). Any agent will follow these three successive steps:

1. He observes the public information and his own education cost ;
2. He chooses to get an education or not ;
3. He chooses his own investment.

As previously mentioned, the model is based on a state of the world Ω. In this setting we consider:

$$\Omega \in \{0, 1\}$$

For any Agent i, for both education and investment we restrict or model to binary choices, and for simplicity we consider the following set up:

$$e_i \in \{0, 1\}$$

being Agent i's choice concerning education (ie. $e_i = 1$ if Agent i receives education and $e_i = 0$ if Agent i doesn't). This education occurs at an individual cost c_i, for multiple reasons this cost varies across agents and follows a probability distribution (that remain unknown and willingly not precised in this general framework): $c_i \sim F(.)$. We assume the cost to be independently identically distributed across all agents.
We consider the cost to be probabilistic for two main reasons. Firstly, it incorporates randomness and avoids having tedious indecision on agents' choices. Secondly, it better approaches real life cases: the cost of getting education widely differs from a person to another, while some might have their tuition fees and living expenses covered by relatives others might have to take loans to cover both types of expenses.

Furthermore, Agent i chooses his investment a_i, with:

$$a_i \in \{0, 1\}.$$

Here agents want their investment a_i to match the state of the world Ω, in others words Agent i wants to choose $a_i = \Omega$.
Indeed, we consider rational agents who want to maximize their own utility. The utility function is identical across agents and given by:

$$U_i = \tilde{U}(1- \mid a_i - \Omega \mid) - c_i \cdot e_i \tag{2.1}$$

with $\tilde{U}(.)$ a function defined and increasing on $[0,1]$ (with our binary model we could simply consider a step function such that $U(1) > U(0)$).

The public information – for Agent i –, as previously written, is the observable information given by Agent i predecessors' choice of action combined with a general prior belief μ_0. One can only observe one's predecessor's choice of education and investment: for all agents the private signal and the result of their investment remain private and unknowmn for future agents. We write this information in the following way:

$$h_i = \begin{cases} \phi \text{ if } i = 1 \\ \{(a_k, e_k) \mid k < i\} \end{cases} \tag{2.2}$$

By definition, $\forall k \in \mathbb{N}, h_{k-1} \subset h_k$.
And by induction, on can easily show that $\forall k \in \mathbb{N}, h_1 \subset h_k$.
This idea seems intuitive: it basically explains that any agent has more pieces of information than his predecessors. In particular, all agents have more pieces of information than the first agent.
The public belief is based on the public information and given by $F_i(\Omega \mid h_i)$ with $F(.)$ a probability distribution function. Here, as we work with binary models we observe $F(.)$ to be a step function.

We also assume that there is a common prior belief μ_0 on the state of the world. That is, all agents would believe that $\Omega = 1$ with probability μ_0 if they had no predecessors:

$$\mu_0 = \mathbb{P}[\Omega = 1 | h_1] \tag{2.3}$$

When μ_0 tends to 1, agents tend to be certain that Ω is 1. Whereas, when μ_0 tends to 0, it is the opposite: agents believe Ω is 0.
The closer μ_0 is to $\frac{1}{2}$, the more indecisive agents are before receiving any signal.

Let

$$\frac{1}{2} < p_l < p_m < p_h < 1 \tag{2.4}$$

be fixed and known by all agents.

After choosing whether or not to get an education, and before investing, each agent receives a private signal. Let s_i be Agent's i private signal, we also have $s_i \in \{0, 1\}$.
We know that $\mathbb{P}[s_i = \Omega] = q$ for some q depending on the situation ($q \in \{p_l; p_m; p_h\}$). In any case, the signal is symmetric meaning that

$$\mathbb{P}[s_i = \Omega | \, \Omega = 1] = \mathbb{P}[s_i = \Omega | \, \Omega = 0] = q$$

The private belief is based on the public information and the private signal the agent receives, it is given by $F_i\left(\Omega \mid h_i, s_i\right)$ with $F(.)$ the probability distribution function.

In this set up the misspecification from the model comes from the fact that – at first – any agent who did not receive education will consider the following:

∼ **the "uneducated" agents** receives a private signal of accuracy p_m ;

∼ **the "educated" agents** receives a private signal of accuracy p_h ;

The reality is the following every agents will receive a signal of accuracy p_l no matter their choice of education e_i.

This section sets the model for the rest of the paper but also for further investigation on the subject. While in most economics models this part is short and explains the basic assumption, we detail here where the model misspecifications come from. While there exist many types of misspecified models, the design of this one reflects a wish to model our tendency to overestimate the expertise given by education.

To illustrate this model's misspecification and its relevance, imagine a situation in which an investor has to choose to investment in a start up. He has to choose between Entrepreneur A and Entreupeneur B. The first one has a higher educational background than the second one. Often, the investor will favor Entreupreneur A over Entreupreneur B. However, this might be injust as Entrepreuner B might have a more precise expertise on this market.

The following sections will study the model under specific circumstances, for instance we will focus on the two first agents in the set of agents.

3 The Specific Model of Two Agents

We now only consider Agent 1 and Agent 2. These two agents have interesting properties as they determine the model following steps.
Furthermore, studying their behaviors will give insight for a broader model studying a countable number of agents.
Lastly, we will further detail in Section 4 but the choice of binary settings for this model makes it more incline to fall under educational cascades as this issue might arise after the second agent. Hence, focusing on the two first agents should give enough insights for this setup.

3.1 The Model Set Up

To simplify our analysis we reduce our analysis to more specific assumption. In particular, for all agent i, we will consider the following cost of education.

$$c_i \sim \cup[0,1], iid$$

Let us also consider the utility function to be given by:

$$u_i(a_i, c_i, e_i) = \lambda\left[1 - (a_i - \Omega)\right] - c_i \cdot e_i \quad \text{with } \lambda \in \mathbb{R}_+^*.$$

Without loss of generality we will set $\lambda = 1$.
We also consider the probability distribution function $F_i(.)$ to be the same across agents and such that all agents.
To keep generality, let us consider $\mu_0 = \mathbb{P}[\Omega = 1|h_1]$. It implies that $\mu_0 \in [0;1]$.
We also keep $\frac{1}{2} < p_l < p_m < p_h < 1$ unspecified.

This set up is practical to study the first two agents. We will first individually study their choice of education before studying their choice of investment.

3.2 The First Agent : analysis and cases differentiation

Let us now consider the first agent. Agent 1's prior belief is only μ_0 as $h_1 = \phi$.
At first the agent has to choose whether or not to get education. At that moment, the agent believes that the education, costing c_1, will increase his own private signal's precision.
Let E_e^1 be Agent 1's expected utility – at that time – of getting education.
Let E_{ne}^1 be Agent 1's expected utility – at that time – of not getting education.
Let us find the condition for which Agent 1 will undertake education:

$$E_e^1 > E_{ne}^1 \tag{3.1}$$

$$\Leftrightarrow \mathbb{E}[u_i(s_i, c_1, e_1 = 1)] > \mathbb{E}[u_i(s_i, c_1, e_1 = 0)]$$

$$\Leftrightarrow p_h \cdot 1 + (1 - ph) \cdot 0 - c_1 > p_m \cdot 1 + (1 - pm) \cdot 0$$

$$\Leftrightarrow p_h - p_m > c_1 \tag{3.2}$$

This condition defines a threshold cost for the education to be undertaken, when getting education is considered useful.
Indeed, most of the time, we assume to learn something significant while getting educated. Most of the time this new knowledge is valuable enough to shape our action. For instance, getting education in finance will likely shape our personal finances management. Receiving an education that has the possibility to shift your behaviors or beliefs is considered useful.
While this occurs most of time, it can happen that education is not considered useful. This occurs, if one's prior belief is too strong for education to have an effect on one's behaviour.
In our case, education is considered not useful when receiving a private signal (of probability p_h) wouldn't change agent's choice of investment when the signal contradicts the prior belief. This specific case will be studies later in this paper, for now we will consider condition 3.2 as the cost threshold.

The following figure represents the different impacts different signals have on Agent 1' choice of investment.

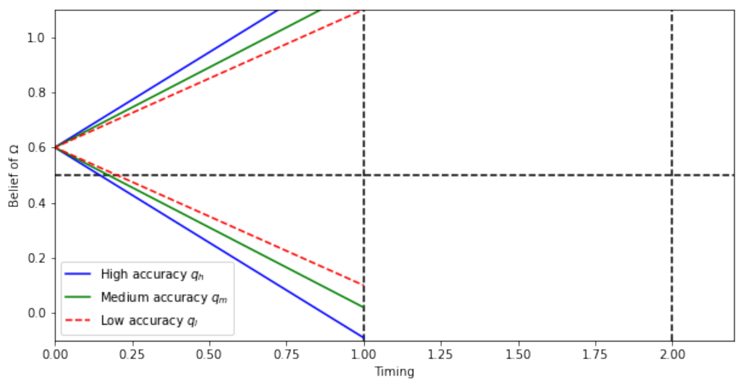

Figure 1: Agent 1's choices possibilities

Here the x-axis represents the time line, when $t = 1$ it is Agent 1's turn to act. The y-axis represents the public belief of the Ω. If it is above $\frac{1}{2}$, it is a public opinion that Ω is most likely to be 1 (the converse is true when it falls under $\frac{1}{2}$ with Ω thought to be 0).
Here the interception of all lines and the y-axis represent the prior belief μ_0.
The upward going lines represent the different effects on Agent 1's belief of Ω of receiving the signal $s_1 = 1$ in the three possible situations (prior belief on education, no education, true situation).

The dotted line models the true situation: the effect on Agent 1's belief of receiving a certain signal if the model was not misspecified. It can also be seen as the signal effect on Agent 1 belief of Ω after receiving education.
The plain lines represent the effect a signal is thought to have.
The blue line represents the effect Agent 1 thinks the signal would have on his belief of Ω after receiving education.
The green line represents the impact of Agent 1's signal if he receives no education on his belief of Ω.

Lastly, one could note that the maximum cost of education – for it to be undertaken – can be determine by this graph, more specifically by the gap between the blue and the green line at $t = 1$.
This particular feature is cause by the specific utility function. Indeed, the utility function, in this section, is linear with a leading coefficient equal to 1. Hence, the difference in expected utility is equal to the difference in the signal accuracy – as seen at (3.3) – ; as the maximum price of undertaking education is the difference between the expected utility in both cases, we can observe it on Figure 1. If the leading coefficient is different from 1 (but we still consider a linear utility function), one could determine the maximum cost of education by multiplying this difference by the leading coefficient.

Now let us study more rigorously Agent 1's choice of investment. For this study we will distinguish four cases depending on the side of the prior belief (ie. if μ_0 is under or over $\frac{1}{2}$) and the signal received (if $s_1 = 0$ or $s_1 = 1$).

Note that, for rational agents, two of these four cases are very intuitive and similar. Indeed, when Agent 1 has a prior belief $\mu_0 < \frac{1}{2}$ and receives signal (whatever its accuracy) $s_1 = 0$, then Agent 1's signal only confirms his intuition. It seems intuitive to assume that Agent 1 will choose $a_1 = 0$.
Similarly, if the prior belief $\mu_0 > \frac{1}{2}$ and $s_1 = 1$, Agent 1 will choose $a_1 = 1$.
This decision is very intuitive: if we have to have to choose between two restaurants and we, a priori, believe the first restaurant to be better. Then reading any comment from a relative we trust advising us to go to the first restaurant instead of the second one, would only reinforce our determination to go there.

To start our analysis, let us assume Agent did not receive education and therefore believes he receives a signal s_1 of probability of being right p_m.

Case 1: $\mu_0 < \frac{1}{2}$ and $s_1 = 0$:
For Agent 1 to follow his signal and choose $a_1 = 0$, we need the following condition to be met:

$$\mathbb{P}[\Omega = 0 | s_1 = 0] > \frac{1}{2}$$

Using Bayes' rule as $\mathbb{P}[s_1 = 0] \neq 0$ we have:

$$\Leftrightarrow \frac{\mathbb{P}[s=0 \mid \Omega=0] \cdot \mathbb{P}[\Omega=0]}{\mathbb{P}[s_1=0|\Omega=0] \cdot \mathbb{P}[\Omega=0]+\mathbb{P}[s_1=0 \mid \Omega=1].\mathbb{P}[\Omega=1]}>\frac{1}{2}$$

$$\Leftrightarrow \frac{p_m \cdot (1-\mu_0)}{p_m \cdot (1-\mu_0)+(1-p_m)\, \mu_0}>\frac{1}{2}$$

$$\Leftrightarrow p_m > \mu_0 \tag{3.3}$$

Case 2: $\mu_0 > \frac{1}{2}$ and $s_1 = 1$:
For Agent 1 to follow his signal and choose $a_1 = 1$, we need the following condition to be met:

$$\mathbb{P}[\Omega=1|s_1=1]>\frac{1}{2}$$

$$\Leftrightarrow \frac{\mathbb{P}[s_1=1 \mid \Omega=1] \cdot \mathbb{P}[\Omega=1]}{\mathbb{P}[s_1=1|\Omega=0] \cdot \mathbb{P}[\Omega=0]+\mathbb{P}[s_1=1 \mid \Omega=1].\mathbb{P}[\Omega=1]}>\frac{1}{2}$$

$$\Leftrightarrow \frac{p_m \cdot \mu_0}{p_m \cdot \mu_0+(1-p_m) \cdot (1-\mu_0)}>\frac{1}{2}$$

$$\Leftrightarrow p_m > 1-\mu_0 \tag{3.4}$$

In both cases 1 and 2, we are sure that the condition studied above will realize. Indeed both conditions 3.3 and 3.4 are going to occur for any probability p_m under the case's specific conditions.
In Case 1, $\mu_0 < \frac{1}{2}$ and by definition $p_m > \frac{1}{2}$. Hence, 3.3 occurs at anytime – with these conditions –, and Agent 1 chooses $a_1 = 0$.
In Case 2, $\mu_0 > \frac{1}{2} \Leftrightarrow 1-\mu_0 < \frac{1}{2}$ and by definition $p_m > \frac{1}{2}$. Hence, 3.4 occurs at anytime – with these conditions –, and Agent 1 chooses $a_1 = 1$.

Case 3: $\mu_0 < \frac{1}{2}$ and $s_1 = 1$:
For Agent 1 to follow his signal and choose $a_1 = 1$, we need the following condition to be met:

$$\mathbb{P}[\Omega=1|s_1=1]>\frac{1}{2}$$

$$\Leftrightarrow \frac{\mathbb{P}\left[s_1=1 \mid \Omega=1\right] \cdot \mathbb{P}[\Omega=1]}{\mathbb{P}\left[s_1=1 \mid \Omega=1\right] \cdot \mathbb{P}[0=1]+\mathbb{P}\left[s_1=1 \mid \Omega=0\right].\mathbb{P}[\Omega=0]}>\frac{1}{2}$$

$$\Leftrightarrow \frac{p_m \cdot \mu_0}{p_m \cdot \mu_0+(1-p_m) \cdot (1-\mu_0)}>\frac{1}{2}$$

We recall that as a sum and product of strictly positive terms, $p_m \cdot \mu_0 + (1-p_m) \cdot (1-\mu_0)$ is also strictly positive. Hence, we have:

$$\Leftrightarrow 2 \cdot p_m \mu_0 > p_m \cdot \mu_0 + (1-p_m)(1-\mu_0)$$

$$\Leftrightarrow p_m > 1 - \mu_0 \tag{3.5}$$

While the condition was always realised in Case 1 and in Case 2, it is not the case here. Indeed we observe that $\mu_0 > \frac{1}{2} \Leftrightarrow 1 - \mu_0 > \frac{1}{2}$. Hence, one can set p_m and μ_0 such that $\frac{1}{2} < p_m < 1 - \mu_0$; the prior belief of Ω being 0 is too strong for the signal – of accuracy p_m – to overthrow it ; in that case $a_1 = 0$.

Case 4: $\mu_0 > \frac{1}{2}$ and $s_1 = 0$:
For Agent 1 to follow his signal and choose $a_1 = 0$, we need the following condition to be met:

$$\mathbb{P}[\Omega = 0 \mid s_1 = 0] > \frac{1}{2}$$

$$\mathbb{P}[\Omega = 0 \mid s_1 = 0] = \frac{\mathbb{P}[s_1 = 0 \mid \Omega = 0] \cdot \mathbb{P}[\Omega = 0]}{\mathbb{P}[s_1 = 0]} > \frac{1}{2}$$

$$\Leftrightarrow \frac{p_m \cdot (1-\mu_0)}{p_m \cdot (1-\mu_0) + (1-p_m) \cdot \mu_0} > \frac{1}{2}$$

$$\Leftrightarrow 2 \cdot p_m \cdot (1-\mu_0) > p_m \cdot (1-\mu_0) + (1-p_m) \cdot \mu_0$$

$$\Leftrightarrow p_m - \mu_0 \cdot p_m > \mu_0 - p_m \cdot \mu_0$$

$$\Leftrightarrow p_m > \mu_0 \tag{3.6}$$

For similar reasons as the ones recalled in Case 3, one cane easily pick μ_0 and p_m such that Agent 1 follows his own prior belief whatever the value of s_1.
If 3.6 holds, then Agent 1 chooses $a_1 = 0$, otherwise we have $a_1 = 1$.

While these thresholds are found for $e_1 = 0$, meaning that Agent 1 did not receive an education and still have to undertake his choice of investment.

Let us now study these conditions in the case of $e_1 = 1$, meaning Agent 1 received an education.
In that case, Agent 1 expected a signal of higher with a probability of indicating the right state (p_h) but learns that he receives a signal with a lower probability (p_l).

Let us now deduce the following conditions for Agent 1 to follow the signal he receives once he received education.

Case 1: $\mu_0 < \frac{1}{2}$ and $s_1 = 0$:
The condition for Agent 1 to choose $a_1 = 0$ and follow his signal is:

$$p_l > \mu_0 \tag{3.7}$$

Case 2: $\mu_0 > \frac{1}{2}$ and $s_1 = 1$:
The condition for Agent 1 to choose $a_1 = 1$ and follow his signal is:

$$p_l > 1 - \mu_0 \tag{3.8}$$

Case 3: $\mu_0 < \frac{1}{2}$ and $s_1 = 1$:
The condition for Agent 1 to choose $a_1 = 1$ and follow his signal is:

$$p_l > 1 - \mu_0 \tag{3.9}$$

Case 4: $\mu_0 > \frac{1}{2}$ and $s_1 = 0$:
The condition for Agent 1 to choose $a_1 = 0$ and follow his signal is:

$$p_l > \mu_0 \tag{3.10}$$

These conditions are similar to the previous ones: only the probability of the signal being right differs.
Hence we find the similar intuitive results for case 1 and case 2. In other words if case 1 or 2 occurs, Agent 1, as a rational agent, will follow his signal (and prior belief).

In Case 3, the condition becomes narrower than when Agent 1 did not receive education (as $p_l < p_m$ and we assume $1 - \mu_0$ to be the same across both cases).
In this Case, if $p_l < 1 - \mu_0$, Agent 1 chooses $a_0 = 1$ despite is signal contradicting it.
We can observe that the situation in which $p_l < 1 - \mu_0$ but condition (3.5) is met represents a situation of great deception. It can be observed when $\frac{1}{2} < p_l < 1 - \mu_0 < p_m$. Indeed, the agent paid to receive education but education causes the agent to realize that signal did not have the possibility to overthrow prior belief.
This 'great deception' can be illustrated by a situation in which a friend decides to buy a travel guide to choose between traveling to destination A or B. She slightly prefers destination A to destination B. Before, buying this guide she thinks it would enable her to make a wiser decision for her vacation. She buys it and realizes that the guide advise destination B over destination A. While this information could be useful, she realizes that not only the guide provides less pieces of advice than she thought but it is not precise enough to help her make her decision.

In a similar way, in case 4, the condition is narrower when Agent 1 receives an education (as $p_l < p_m$). If we have $\frac{1}{2} < p_l < \mu_0 < p_m$, then the agent also faces a situation of great deception as described in case 3. Indeed, in that case, the agents pays for his education, believing that the signal would be beneficial for his behaviour (ie. choice of investment) but it is the opposite: he realises education has no effect on his choices.

Now let us study the case mentioned at the beginning of the section 3.2. We want to determine the minimal signal precision p_m for the education to be considered useful (we recall that we consider education to be useful when receiving a signal opposed to the prior belief can shift the agent's choice of investment). We want to find such condition given the agent prior belief of benefits of education.
Let us find p_m such that education is considered not useful:

- if $\mu_0 < \frac{1}{2}$, $s_1 = 1$, and:

$$\mathbb{P}[\Omega = 1 | s_1 = 1] < \frac{1}{2}$$

$$\Leftrightarrow p_h < 1 - \mu_0 \tag{3.11}$$

- if $\mu_0 > \frac{1}{2}$, $s_1 = 0$, and:

$$\mathbb{P}[\Omega = 0 \mid s_1 = 0] < \frac{1}{2}$$

$$\Leftrightarrow p_h < \mu_0 \tag{3.12}$$

We note that when $\mu_0 < \frac{1}{2}$, we have $1 - \mu_0 > \mu_0$. Similarly, when $\mu_0 > \frac{1}{2}$, we have $1 - \mu_0 < \mu_0$.
Hence, we find that education is considered not useful when:

$$p_h < \max(\mu_0; 1 - \mu_0). \tag{3.13}$$

In that situation, Agent 1 would not undertake education no matter the cost he faces (note that Agent 1 would strictly prefer not undertaking education if $c_1 > 0$ but would be indifferent if $c_1 = 0$).

In this Section, we have studied Agent 1's behavior and the different conditions under which he undertakes education, then follow his signal. While we determined some conditions under which education is considered be not useful, we will focus on the other situations which bring more insights for the rest of the model.
Now that Agent 1's turn is over, we can now study Agent 2. For readability, we will distinguish two situations. The case in which Agent 1 receives an education is discussed in section 3.3, while the case in which Agent 1 did not receive an education is discussed in section 3.4.

3.3 The Second Agent: analysis when Agent 1 received education

Let us now assume that Agent 1 received an education, meaning that Agent 2 believes that Agent 1 received a signal with probability of being right p_h.

We first note that the threshold cost c_2 above which education becomes too expensive is equal to Agent 1's. In other words, the condition for Agent 2 to undertake education is: $E_e^2 > E_{ne}^2 \Leftrightarrow p_h - p_m > c_1$.
It is actually the case that given this model, the cost threshold is the same across all agents. We will discuss this property in section 4.

The idea behind our analysis is to update the prior belief, as in many other experiments in social learning [4]. Thus we need to determine μ_1 taking into account both the prior belief (μ_0) and the public information h_2 (in this section $h_2 = (a_1, e_1)$). In that case, Agent 2's choice would be the same as Agent 1's with the updated prior belief.

In this Section and Section 3.4, we consider two different cases depending on Agent 1's choice of education. This choice is made for the readability and comprehension of mu_1

Let us now study Agent 2's investment decision with $e_1 = 1$.
For this, we compute the updated prior belief μ_1.

$$\mu_1 = \mathbb{P}\left[\Omega = 1 | H = h_2\right]$$

$$\Leftrightarrow \mu_1 = \frac{\mathbb{P}\left[H = h_2 \mid \Omega = 1\right] \cdot \mathbb{P}[\Omega = 1]}{\mathbb{P}\left[H = h_2\right]}$$

$$\Leftrightarrow \mu_1 = \frac{\mathbb{P}\left[H = h_2 \mid \Omega = 1\right] \cdot \mathbb{P}[\Omega = 1]}{\mathbb{P}\left[H = h_2 \mid \Omega = 1\right] \cdot \mathbb{P}[\Omega = 1] + \mathbb{P}\left[H = h_2 \mid \Omega = 0\right] \cdot \mathbb{P}[\Omega = 0]} \tag{3.14}$$

Let us first assume that Agent 2 does not receive an education, meaning $e_2 = 0$. We can now distinguish two cases:

- If Agent 1 chose $a_1 = 1$, without receiving an education, Agent 2 considers $\mathbb{P}[H = h_2 | \Omega_1] = p_h$. Therefore, we determine:

$$\mu_1 = \frac{p_h \cdot \mu_0}{p_h \cdot \mu_0 + (1 - p_h) \cdot (1 - \mu_0)}$$

- If Agent 1 chose $a_1 = 0$, before receiving an education, Agent2 considers $\mathbb{P}[H = h_1 | \Omega_1] = 1 - p_h$. Therefore, we determine :

$$\mu_1 = \frac{(1 - p_h) \cdot \mu_0}{(1 - p_h) \cdot \mu_0 + p_h \cdot (1 - \mu_0)}$$

By combining the two equations above we determine the general formula for μ_1 when Agent 1 received education and Agent 2 does not.

$$\mu_1 = \frac{\mu_0 \cdot [1 - p_h + a_1 \cdot (2 \cdot p_h - 1)]}{\mu_0 \cdot [1 - p_h + a_1 \cdot (2 \cdot p_h - 1)] + (1 - \mu_0) \cdot [p_h + a_1 \cdot (1 - 2 \cdot p_h)]} \tag{3.15}$$

When Agent 2 also receives an education (ie. $e_2 = 1$), we find :

$$\mu_1 = \frac{\mu_0 \cdot [1 - p_l + a_1 \cdot (2 \cdot p_l - 1)]}{\mu_0 \cdot [1 - p_l + a_1 \cdot (2 \cdot p_l - 1)] + (1 - \mu_0) \cdot [p_l + a_1 \cdot (1 - 2 \cdot p_l)]} \tag{3.16}$$

Hence, by combining (3.15) and (3.16), we find the general formula for μ_1 when Agent 1 received education:

$$\mu_1^e = \frac{\mu_0 \cdot I_1^e}{\mu_0 \cdot I_1^e + (1 - \mu_0) \cdot I_2^e} \tag{3.17}$$

With the following :

- $I_1^e = [1 - p_h + e_2 \cdot (p_h - p_l) + a_1 \cdot (2 \cdot p_h - 2 \cdot e_2 \cdot (p_h - p_l) - 1)]$
- $I_2^e = [p_h - e_2 \cdot (p_h - p_l) + a_1 \cdot (1 - 2 \cdot p_h + 2 \cdot e_2 \cdot (p_h - p_l))]$

Now that we have determined μ_1, let us study Agent 2 's choice of investment in the two last situations (case 3 and case 4). We do not study the two first ones as they constitute trivial cases.

Case 3: $\mu_1 < \frac{1}{2}$ and $s_1 = 1$:
The condition for Agent 1 to choose $a_1 = 1$ and follow his signal is similar to conditions (3.5) and (3.9). We determine:

$$p_m - e_2 \cdot (p_m - p_l) > 1 - \mu_1 \tag{3.18}$$

Case 4: $\mu_1 > \frac{1}{2}$ and $s_1 = 0$:
The condition for Agent 1 to choose $a_1 = 0$ and follow his signal is similar to conditions (3.6) (3.10). We find:

$$p_m - e_2 \cdot (p_m - p_l) > \mu_1 \tag{3.19}$$

We have now determine the conditions under which Agent 2 follows his signal when his predecessor received an education and no matter his choice of education.
The following section focuses on determining these conditions when Agent 1 did not receive an education.

3.4 The Second Agent: analysis when Agent 1 did not received education

We will now consider that Agent 1 did not undertake education (from (3.2 we have $p_h - p_m < c_1$). Hence, Agent 1 chose a_1 believing his signal would have a probability p_m of indicating the right value of Ω. If Agent 2 does not receive an education, he believes this too.
In section 3.3, Agent 1 chose a_1 knowing his signal had a probability p_l of indicating the right value of Ω. However, without receiving an education, Agent 2 considers this probability to be p_h.
Lastly, if Agent 2 receives an education, he knows that this probability is p_l, and that Agent 1 did not know it and acted believing his signal accuracy was p_m. By making the same computations as in the section 3.3 we determine the new updated belief μ_1 when Agent 1 did not receive education.

$$\mu_1^{ne} = \frac{\mu_0 \cdot I_1^{ne}}{\mu_0 \cdot I_1^{ne} + (1 - \mu_0) \cdot I_2^{ne}} \tag{3.20}$$

With:

- $I_1^{ne} = [1 - p_m + e_2 \cdot (p_m - p_l) + a_1 \cdot (2 \cdot p_m - 2 \cdot e_2 \cdot (p_m - p_l) - 1)]$
- $I_2^{ne} = [p_m - e_2 \cdot (p_m - p_l) + a_1 \cdot (1 - 2 \cdot p_m + 2 \cdot e_2 \cdot (p_m - p_l))]$

For Agent 2 to follow his own signal, we find similar conditions. In Case 1 and 2, the signal and the prior public belief both align and a_2 is trivial.

In Case 3 ($\mu_1 < \frac{1}{2}$ and $s_1 = 1$), the condition for Agent 2 to follow his private signal and choose $a_2 = 1$ is still given by (3.18) with μ_1 given as in (3.20).

In Case 4 ($\mu_1 > \frac{1}{2}$ and $s_1 = 0$), the condition for Agent 2 to follow his private signal and choose $a_1 = 0$ is still given by (3.19) with μ_1 given as in (3.20).

We have now determine the conditions under which Agent 2 follows his private signal, no matter his choice of education, when his predecessor did not receive education.
While this model easily model the dynamic behind the effect of education on misspecified models and false beliefs, it presents some limitations. Let us discuss these limitations in the section 4.

Let us now study the impact of Agent 1's investment on Agent 2's prior belief.
For this we study $\frac{\mu_1^{ne}}{\mu_1^e}$. One can easily determine:

$$\frac{\mu_1^{ne}}{\mu_1^e} = \frac{I_1^{ne}}{I_1^e} \cdot \frac{I_1^e + \frac{1-\mu_0}{\mu_0} \cdot I_2^e}{I_1^{ne} + \frac{1-\mu_0}{\mu_0} \cdot I_2^{ne}} \tag{3.21}$$

Now, we study the ratio given in (3.21) to study this impact.

- In the cases in which Agent 2 decides to undertake education, no matter Agent 1's decision on education μ_1 will stay the same. Indeed, we find $\frac{\mu_1^{ne}(e_2=1,a_1=0)}{\mu_1^{e}(e_2=1,a_1=0)} = \frac{\mu_1^{ne}(e_2=1,a_1=1)}{\mu_1^{e}(e_2=1,a_1=1)} = 1$.
  This property highlights the fact that education enables the agent to perceive his environment as it is and detach himself from his predecessors' actions.

- In the case in which Agent 2 decides not to undertake education, and Agent 1 chose $a_1 = 1$, we find: $\frac{\mu_1^{ne}(e_2=0,a_1=1)}{\mu_1^{e}(e_2=0,a_1=1)} < 1 \Leftrightarrow \mu_1^{ne}(e_2 = 0, a_1 = 1) < \mu_1^{e}(e_2 = 0, a_1 = 1)$.
  This result is rather intuitive: Agent 2's belief tends toward $\Omega = 1$ when Agent 1 chose $a_1 = 1$. Agent 2 is more impacted by Agent 1's choice of investment if he observes Agent 1 received an education – he believes that Agent 1 received a signal of accuracy p_h rather than p_m–.

- In the case in which neither Agent 2, nor Agent 1 undertook education, we find: $\frac{\mu_1^{ne}(e_2=0,a_1=0)}{\mu_1^{e}(e_2=0,a_1=0)} > 1 \Leftrightarrow \mu_1^{ne}(e_2 = 0, a_1 = 0) > \mu_1^{e}(e_2 = 0, a_1 = 0)$.
  This result is similar to the previous one and the same explenation could be given.

4 Discussion

The $2 \times 2 \times 2$ model framework employed in this model initiates a cascading effect based on the interactions between two agents while simplifying the complex nature of decision-making. This simplified model provides a foundational understanding for real-life decisions and their consequences. To obtain more nuanced and realistic results, one could introduce a continuous variable, $\Omega \in [0, 1]$ with $a_i \subset [0, 1]$, to represent the spectrum of choices more accurately. However, this would necessitate advanced mathematical approaches, such as measure theory, to manage the increased complexity. Despite this, the current model offers valuable insights into the underlying dynamics, which is crucial before delving into more sophisticated models.

Furthermore, the assumption that probabilities p_l, p_m, and p_h remain constant across agents simplifies the impacts of education on individual decision-making. In reality, individuals benefit differently from education, which should ideally be reflected by having an individual signal precision to choose their investment in their subsequent choices (a_i for Agent i). The model's simplification in this regard aims to preserve clarity and interpretability of the results, under the premise that variations in educational outcomes among individuals are often minor.

Additionally, the model's binary approach to educational decisions does not fully encapsulate the complexities of real-world educational systems, which include various levels from secondary education to doctoral studies. This simplification, though it limits the model's depth, makes the dynamics easier to understand and serves as a foundation for more complex future models that may include a detailed representation of educational levels.

In conclusion, the discussion highlights the trade-off between model simplicity and the representation of real-world complexities. It emphasizes the model's capacity to elucidate fundamental dynamics and identifies areas for refinement and expansion in subsequent research.

5 Conclusion and Future Research

In this report, we embarked on an exploratory journey to discern the extent to which individuals depend on the actions of their peers in determining their own educational paths and rectifying false beliefs. The crux of our investigation was a theoretical model that strived to encapsulate the intricate dynamics arising from misspecified models and the complex interplay between individual decision-making and societal influences. While certain aspects of our findings, such as those detailed in cases (3.11) and (3.12), aligned with intuitive expectations, our model also offered a more nuanced and rigorous elucidation of the influence of prior beliefs and the accuracy of others' actions on one's educational choices. For instance, the updated prior belief μ_1 (3.17) and (3.20) allows to discern the effect of the different probabilities p_l, p_m, p_h, the common prior belief μ_0, and previous agent decisions (e_1, and a_1) on the new agent's decision.

The implications of our research are significant, laying a foundational stone for further inquiry with an increased number of agents. This model represents a novel contribution to the field of social learning in economics, particularly by introducing a distinctive form of model misspecification that highlights the potential dangers of overreliance on peer actions and underscores the corrective potential of education.

However, it is important to acknowledge the limitations of our model. The assumptions we adopted, while facilitating analytical clarity, oversimplify the complex nature of real-world interactions. Inspired by established economic models of social learning, such as those detailed in "Rational Herds" by Chamley, our model is prone to informational cascades, particularly when two consecutive agents mimic the actions of their predecessors. This feature, though insightful, poses challenges in extending the model's applicability beyond a few agents.

Looking ahead, there are several promising directions for future research. One approach could involve extending our model inductively, applying the established formulas with updated prior beliefs μ_{i-1} for each subsequent agent, with the aim of deriving a generalized expression for μ_i. Additionally, moving beyond the binary constraints of the $2 \times 2 \times 2$ model setup, as mentioned in section 4, to a continuum of values for Ω and agents' actions could mitigate the occurrence of informational cascades and broaden the model's relevance.

In conclusion, this thesis not only sheds light on the dynamics of social learning and education within economic models but also paves the way for future interdisciplinary research that bridges economics, psychology, and educational theory. The theoretical insights gleaned from our model hold the potential to inform real-world applications, particularly in the realm of public policy and educational reform, underscoring the enduring importance of education in shaping economic behaviors and societal outcomes. [5],

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