



# Intraday trading patterns in an intelligent autonomous agent-based stock market<sup>☆</sup>

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## ABSTRACT

Market microstructure studies of intraday trading patterns have established that there is a regular pattern of high volumes near both the open and close of the trading day. O'Hara (1995) points out the many difficulties in specifying all the necessary elements of a strategic model for determining and attaining an equilibrium describing intraday patterns. We develop an autonomous agent-based market microstructure simulation with both informed agents and uninformed liquidity-motivated agents. Both types of agents can learn when to trade, but are zero-intelligence on all other behavior. We do not impose an equilibrium concept but instead look for emergent behavior. Our results demonstrate that trading patterns can arise in such a model as a result of interactions between informed and uninformed agents even though the agents are non-strategic and not fully rational. As long as there is rudimentary social or individual learning, uninformed liquidity-motivated agents can coordinate to avoid trading with informed agents and suffering adverse selection losses. The extent and pattern of coordination between uninformed agents depends on the learning specification, the percentage of informed agents and the degree of cooperation/competition among the informed agents.

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## 1. Introduction

Market microstructure studies of intraday trading patterns have established that there is a regular pattern of high volumes near both the open and close of the trading day. This U-shaped pattern has been documented for stocks trading on the New York Stock Exchange by Jain and Joh (1988), Lockwood and Linn (1990) and others. Admati and Pfleiderer (1988) furnish a possible explanation for this pattern. They build a model where concentrated trading patterns can arise as a result of strategic interactions between traders with asymmetric information. Our paper uses a different approach to look at similar questions. We build an intelligent autonomous agent-based model and use computer simulations to analyze intraday trading patterns.

O'Hara (1995, pp. 150–151), discusses the robustness of strategic trading models in general, and the Admati and Pfleiderer model in particular, and states:

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While the outcome argued by Admati and Pfleiderer seems economically plausible, it need not be the equilibrium that actually occurs. . . . The problem is that to characterize the actual equilibrium one would need to know not only the equilibrium concept being applied, but also the specific game being played. . . . This problem highlights a major difficulty in applying strategic models to market microstructure issues. To formally model the underlying game in a market requires specifying the rules of the game, the players, their strategy sets, and their payoffs. . . . Exactly how the players strategies and beliefs are tied together is crucial for determining the resulting equilibrium. . . . Moreover, the ability to ever attain the proposed equilibrium in any actual market setting may also be a serious concern. Nonetheless, strategic models of trader behavior can provide substantial insight and intuition into the trading process. . . .

O'Hara's comments correspond exactly to the type of problem for which intelligent autonomous agent-based models were developed. Agent-based computational economics studies complex systems involving agents who act according to exogenously specified, relatively simple behavioral rules. The autonomous agents can adapt or learn through interactions with others and from their past experiences. The approach is to devise a simulation where autonomous agents interact and to look for emergent behavior. Further, experiments can be conducted to explain the properties of the simulated economy.

We develop a system to analyze how asymmetric information might induce intraday trading patterns. Instead of the strategic modeling approach typically used by market microstructure theorists, we construct an autonomous agent-based model. Our model corresponds to typical market microstructure models in that there are two types of agents, informed agents and uninformed, liquidity-motivated agents. However, our agents learn when to trade during the day according to two types of learning: a social learning using genetic algorithms and an individual learning using a modified Roth-Erev reinforcement learning algorithm. Further, our model differs from many typical market microstructure models because we do not impose an equilibrium concept. Instead, all trading occurs in the framework of a double auction.

Agent-based approaches to economic modeling usually relax the assumption of perfect rationality and can allow specification of a detailed trading institution. Behavioral finance theorists usually replace rationality with a behavioral assumption (e.g., overconfidence) so that agents are rational in all respects except for the behavior of interest. As in other agent-based asset markets, our approach is to replace rationality with simple behavioral rules. Our agents are non-strategic and are near zero-intelligence. Since we are interested in intraday patterns, our agents behave randomly in all respects except for the decision of when during the day to trade. Our agents learn when to trade either through social learning or through individual learning.

Even though our approach differs from Admati and Pfleiderer's strategic trading model, our main result is quite similar. Liquidity-motivated agents learn that they can minimize losses by trying to trade at the same time during the day *if* there is some degree of social or individual learning. The result is important because it suggests that concentrated trading patterns can occur in environments other than those featured in many theoretical market microstructure models. Even markets with near zero-intelligence agents possessing very rudimentary learning abilities can feature concentrated trading patterns.

Specification of a trading institution provides additional results as well as insights into the coordination process. In many theoretical models prices are determined by application of an equilibrium concept. Our model does not. We model trading as a process resulting from agents arriving and placing orders in a limit order book.<sup>3</sup> When concentration and coordination occurs in our simulations, it almost always occurs in the first period of the trading day. As time progresses, profitable limit orders (from the agent accepting the order's perspective) are likely to have already been taken off the book through orders accepted by informed agents. Liquidity-motivated agents soon discover that trading later in the day is more likely to be unprofitable. Our simulated economy is a dynamic model which can be used to study the process as well as the conditions under which both concentration and coordination occur.

The organization of our paper is as follows. Section 2 summarizes previous research. Section 3 describes our autonomous agent-based model, and presents the experimental design for characterizing the economy. Our results are presented in Section 4, and Section 5 concludes.

## 2. Previous research

We are unaware of any previous studies using agent-based computer models to study intraday trading patterns. However there are empirical and theoretical studies of intraday trading patterns, as well as a number of applications of agent-based models to interday trading patterns and other finance topics.

### 2.1. Intraday trading patterns

Market microstructure studies have established a U-shaped trading volume pattern on several stock exchanges. [Jain and Joh \(1988\)](#) and [Lockwood and Linn \(1990\)](#) report this pattern for NYSE stocks. Similar patterns have also been reported for the Toronto Stock Exchange ([McInish and Wood, 1990](#)), the London Stock Exchange ([Werner and Kleidon, 1996](#)), the Hong Kong Stock Exchange ([Ho and Cheung, 1991](#)) and the Taiwan Stock Exchange ([Lee et al., 2001](#)).

<sup>3</sup> Other agent-based models (designed to study interday prices) have featured trade through agents interacting with a limit order book. See [Chiarella et al. \(2009b\)](#) for an example, as well as a discussion of these models.

Since new information is likely to stimulate trade, researchers have studied information arrival, both public and private, as an explanation for the U-shaped pattern. [Berry and Howe \(1994\)](#) examine the extent to which public information releases can explain the empirical trading patterns. They use the rate of Reuters news releases as a proxy for public information arrival, and report a weakly positive relation between volume and public information arrival. They conclude that public information arrival can only partially explain the U-shaped trading pattern.

[Admati and Pfleiderer \(1988\)](#) demonstrate how private information can induce an intraday trading pattern. They develop a theoretical model with informed strategic traders, liquidity traders and discretionary liquidity traders. Informed traders possess superior information about the asset value, but their information is short-lived. These informed traders must trade in order to profit from their information, but wish to minimize the information transmitted by their trading activities. Liquidity traders submit orders for reasons exogenous to the model. Both the size and the timing of their orders are random. Discretionary liquidity traders also must trade, but can choose when during the day to trade. Admati and Pfleiderer show that the discretionary traders are better off when they concentrate. Trading at the same time minimizes their adverse selection losses. Their model predicts that trading will be concentrated at some time during the day, but it does not specify exactly when. Even so, the model demonstrates that a trading pattern can arise from a model of strategic trading.

## 2.2. Autonomous agent-based models in finance

There is an extensive and growing literature using autonomous agent-based models to study financial markets, but the majority of this research focuses on the efficient markets hypothesis and interday trading patterns. The Santa Fe Artificial Stock Market, developed by LeBaron, Arthur, Holland, Palmer and Taylor in the early 1990s is an example. More recent examples and variants include [Chen and Yeh \(2002\)](#) and [Iori \(2002\)](#). The Santa Fe model and its successors are discussed in reviews by [Hommes \(2006\)](#) and [LeBaron \(2006\)](#). Additional agent-based applications pertaining to behavioral finance and market microstructure as well as informational efficiency are discussed in relation to other types of theoretical and empirical studies in surveys by [Lux \(2009\)](#), [Hommes and Wagener \(2009\)](#), and [Chiarella et al. \(2009a\)](#).

The approach is to create different types of agents, each with the ability to learn to forecast stock prices using fundamental, technical or behavioral rules. The agents interact in the simulation producing dynamic interday stock market data, which often contain unexpected patterns or properties. These results can then be compared both to the well-defined efficient market benchmark and to empirical interday stock market data. In some instances, the artificial stock markets produce interday data that contains patterns remarkably similar to empirically observed departures from the predictions of the efficient markets hypothesis.<sup>4</sup> [Lux and Schornstein \(2005\)](#) have had similar success using an agent-based model to generate simulated results which contain patterns similar to those observed in actual foreign exchange price data.

Another application is to use agent-based model to study the effect of relaxing some of the assumptions underlying theoretical models. The Santa Fe models relax the assumption of agent rationality in favor of simpler behavioral rules. In another application, [Chen and Huang \(2008\)](#) construct an agent-based model to study theoretical predictions by [Blume and Easley \(2006\)](#) and [Sandroni \(2000\)](#). The theoretical models predict that agents who forecast accurately will survive. Chen and Huang show that without perfectly rational agents, risk-preferences are a more important factor for survivability. Our approach is broadly similar to that used by agent-based models such as Chen and Huang's, however, we will analyze intraday trading patterns instead of survivability. We also relax the rationality assumption, but our agents will not learn to forecast stock prices. Instead they will learn when during the day to trade.

## 2.3. Zero-intelligence agent models

Zero intelligence agent models are also relevant for our study. [Gode and Sunder \(1993\)](#) use the term zero intelligence traders to refer to agents who are programmed to behave randomly subject to some constraints. Gode and Sunder study a basic market for a single good, traded via double auction. Zero intelligence buyers are willing to buy at a random price below their given private value for the good. Zero intelligence sellers are willing to sell at a random price above their given private cost for the good. Their main result is a rapid convergence to a competitive equilibrium which is important because it highlights the role of the trading institution in explaining market outcomes. Competitive results are obtained without the assistance of profit-maximizing sellers, or surplus-maximizing buyers. [Duffy \(2006, p. 971\)](#) discusses Gode and Sunder's model and reviews subsequent developments and controversies concluding that "the ZI approach is a useful benchmark model for assessing the marginal contribution of institutional features and of human cognition in experimental settings."

We, as well, will use zero intelligence traders as a benchmark for studying intraday trading patterns. By designing agents who are zero intelligence in all behaviors except those of interest, we can focus on the role of the non-zero intelligence behaviors in explaining trading patterns such as the timing of trades. We construct agents who are zero-intelligence with regard to the values of their posted offers, but can learn when to trade.

<sup>4</sup> Examples of some empirical anomalies include fat tails on the distribution of stock returns, volatility clustering where stock returns are characterized by successive periods of high and low volatility, and asset price bubbles (see [LeBaron, 2006](#)).

**Table 1**  
Model parameters.

		Base case
<i>Structural</i>		
$i = 0, \dots, N$	Number of agents	200
$d = 0, \dots, D$	Total number of days	2250
$t = 0, \dots, T$	Number of trading periods per day	8
$I$	Percentage informed agents	20
$A_d$	Asset value on day $d$	$U[20, 120]$
$E_{\min}, \dots, E_{\max}$	Range of expected asset value	$U[20, 120]$
$R_{\min}, \dots, R_{\max}$	Range around $E_{id}$ for which agent $i$ will place order	$U[5, 15]$
$t_o = 1, \dots, T$	Number of periods for which an order is valid	–
$\bar{t}_o = \{true, false\}$	Whether the number of periods an order is valid is random	true
$\delta_c$	Probability an order is randomly cancelled	0.1
$\bar{L}$	Number of days in a learning period	15
<i>Behavioral</i>		
Informed learn = $\{true, false\}$	Informed agents learn	true
Uninformed learn = $\{true, false\}$	Uninformed agents learn	true
Informed market = $\{true, false\}$	Informed agents compete	false
<i>Genetic algorithm learning</i>		
Group size	Number of uninformed agents in each learning group	40
Number crossover points	Number of crossover points used in the GA	1
Crossover rate	Probability crossover performed in the GA	0.7
Mutation rate	Mutation rate used in the GA	0.001
Elitism = $\{true, false\}$	Elitism used in the GA	false
<i>Modified Roth-Erev learning</i>		
$\beta$	Boltzman temperature cooling	30
$\epsilon$	Experimentation	.15
$q_j(0)$	Initial propensity	8000
$\theta$	Recency (forgetting)	0.10

### 3. Market microstructure

In this section, we describe the structure of our simulated market. The market requires specification of a number of parameters summarized in Table 1 and discussed below.

#### 3.1. Trading institution

The simulations consist of a number of trading days  $d = 0, \dots, D$ , each of which is further divided into a number of trading periods,  $t = 0, \dots, T$ . Before the first trading day, the simulation generates a specified number of agents,  $N$ , and randomly assigns each agent a timing chromosome  $C_i$ . The timing chromosome is a  $T$  bit binary number which is used to determine when agents enter the market for a potential trade. The timing chromosome for uninformed liquidity-motivated agents is coded and interpreted differently than the timing chromosome for informed agents, as shown in Fig. 1. Our model is designed so that uninformed agents go to the market once, and only once per day. Informed agents can go to the market during none, some or all of the trading periods in a trading day.

At the start of the simulation, timing chromosomes are randomly generated. However, during the simulation, agents' timing chromosomes will be altered as explained in Section 3.3. When the learning parameter is set to true agents can potentially learn when to trade, and possibly generate a non-random intraday pattern of trade. Uninformed agents can learn which period during the trading day to go to market. Informed agents can learn both when, and how often to go to market.

Before the first period of each trading day, the true asset liquidation value,  $A_d$ , is set to the realization of a random draw from a uniform distribution with bounds  $E_{\min}$  and  $E_{\max}$ , ( $U[20, 120]$  in our base case). Next, each agent is assigned an expected asset value  $E_{id}$  and a range  $R_{id}$ . If an agent goes to the market, these values, along with the state of the limit order book, will determine whether he or she places or accepts a bid or an ask. If the agent is uninformed, their  $E_{id}$  is a random draw from the uniform distribution  $U[E_{\min}, E_{\max}]$ . If the agent is informed, their  $E_{id}$  is set to the true asset liquidation value  $A_d$  at the beginning of the day. For both uninformed and informed agents the range,  $R_{id}$ , is a random draw from the uniform distribution  $U[R_{\min}, R_{\max}]$ .

After the agent's  $E_{id}$  and  $R_{id}$  are assigned for day  $d$ , the first period commences. Agents are randomly shuffled and then selected in succession. When selected, each agent's timing chromosome is checked to see whether he will go to the market. Going to the market consists of three steps.

First, any existing orders placed by the  $i$ th Agent are checked for cancellation. There is an independent random chance of cancellation for each of the agents bids and asks currently on the limit order book. The probability of an order being cancelled, that is removed from the limit order book, is equal to the parameter  $\delta_c$  (set to ten percent in our base case).

If there are 8 periods per day, uninformed agents' timing chromosomes are coded as a 3 bit binary number:

Timing Chromosome	Agent goes to Market in Period:
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Informed agents' timing chromosomes are coded as an 8 bit binary number where each bit corresponds to a single period. A "1" indicates the agent will go to market in the corresponding period. For example:

Timing Chromosome	Agent goes to Market in Period(s):
00000001	0
00000010	1
10000001	0,7
11000000	6,7
00001111	0,1,2,3
11111111	0,1,2,3,4,5,6,7

There are  $2^8$  or 256 possible values for informed agents' timing chromosomes.

**Fig. 1.** Timing chromosomes. With  $T$  periods per day, the informed agents timing chromosome will be a  $T$ -bit binary number. Uninformed timing chromosomes will be a  $S$ -bit binary number chosen so that  $2$  raised to the  $S$  equals  $T$ . For example, if  $T$  is eight, informed agents' timing chromosomes will be 8-bit and uninformed agents' will be 3-bit binary numbers.

Second, the agent forms an offer drawn randomly from  $U[E_{id} \pm R_{id}]$ .<sup>5</sup> If the draw is greater than the  $E_{id}$ , the agent places an ask at the value drawn. Conversely, a bid is placed at the value drawn when the draw is less than the agents  $E_{id}$ . The offer is compared to the current lowest ask and highest bid in the limit order book to determine whether a market order occurs. If a bid offer is greater than the lowest ask, the agent will buy a share, accepting the lowest ask. If the agents ask offer is less than the highest bid, the agent will sell a share, accepting the highest bid. If an agent's offer is between the limit order book's highest bid and lowest ask, the agent will place a limit order. This sequence of events is illustrated in Fig. 2.<sup>6</sup>

When a limit order is placed, the length of time until the order is automatically withdrawn is determined randomly. If the limit order is placed in period  $t$ , and there are  $T$  periods in the day ( $t = 0, \dots, T$ ), a draw from a uniform distribution  $U[1, T - t + 1]$  determines the number of periods,  $t_o$ , the order is valid. The order is withdrawn in the randomly chosen period, just before the simulation determines if the agent (who placed the limit order) will come to the market. Notice that an order may be cancelled with probability  $\delta_c$  prior to the time it is withdrawn, because agents also randomly check to see whether existing limit orders will be cancelled when they go to the market. Finally, all orders are cancelled and the limit order book is emptied before the next trading day commences.

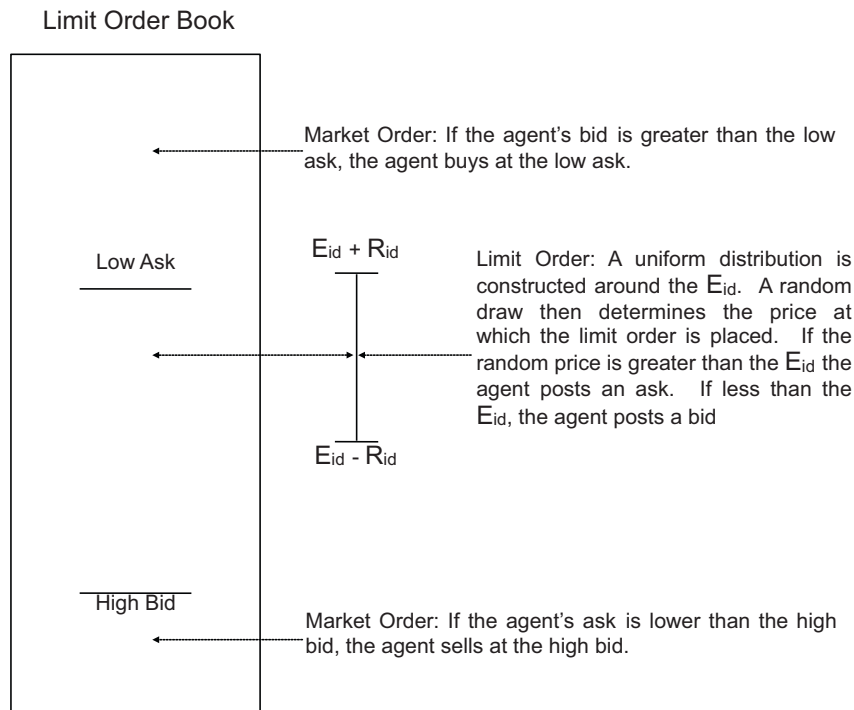
At the end of the day, each agent's profits are computed. For each purchase, the agent gains or loses the actual liquidation value,  $A_d$ , minus the purchase price. Similarly, for shares sold, the agent gains or loses the purchase price less  $A_d$ .

### 3.2. Informed agents

The next component of our artificial stock market is the addition of informed agents. Informed agents differ from the other zero-intelligence agents in that they know the value of the asset at the start of the trading day. Recall that the uninformed agents begin each day with their own  $E_{id}$  drawn from a uniform distribution. Although each uninformed agent's  $E_{id}$  is drawn from the same distribution as the asset liquidation value ( $A_d$ ),  $E_{id}$  differs across uninformed agents, and an uninformed agent's  $E_{id}$  will only equal  $A_d$  by chance. In the simulation the  $E_{id}$  and  $R_{id}$  is used (along with the best bid and ask posted in the order book) to determine whether the uninformed agent will place a buy or sell order, and whether the order will be a market or limit order. Instead of draws from a uniform distribution, the  $E_{id}$  for all informed agents is set equal to the asset liquidation value,  $A_d$ . The informed agent's  $R_{id}$ , is assigned in the same manner as for uninformed agents, as a random draw from the uniform distribution  $U[R_{\min}, R_{\max}]$ .

<sup>5</sup> The random interval the offer is drawn from is clipped by  $E_{\min}$  and  $E_{\max}$ . If the interval extends past the highest or lowest possible  $[E_{\min}, E_{\max}]$ , then the interval is will range from  $\max(E_{\min}, E_{id} - R_{id})$  to  $\min(E_{id} + R_{id}, E_{\max})$ .

<sup>6</sup> Note that an agent will never trade with themselves. Uninformed agents go to market only once per day, placing at most one limit or market order. Informed, cooperative agents go to market many periods of the day but only place market orders. Informed, competitive agents know the true value of the asset and would never place a market order for their own limit order since one of the orders would be unprofitable.



**Fig. 2.** Sequence of events occurring when an agent goes to market. When going to the market, an agent forms an offer drawn randomly from a uniform distribution,  $U[E_{id} \pm R_{id}]$ . If the draw is greater than the  $E_{id}$ , the agent posts an ask at the value drawn. Conversely, a bid is placed at the value drawn when the draw is less than the agents  $E_{id}$ . The offer is compared to the current lowest ask and highest bid in the limit order book to determine whether a market order occurs. If a bid offer is greater than the lowest ask, the agent will buy a share, accepting the lowest ask. If the agent's ask offer is less than the highest bid, the agent will sell a share, accepting the highest bid. If an agent's offer is between the best quotes in the order book, the agent will place a limit order.

The random procedure for arrival is also unchanged. The uninformed and informed agents are shuffled together to randomly determine their order of arrival. Upon arrival the ZI-informed agent tries to place an order based on their timing chromosome. If an order is to be placed, then the informed agent's  $E_{id} = A_d$  is compared to the best posted bid ( $b$ ) and ask ( $a$ ) prices. If a profitable trade is possible because the  $A_d$  is greater than the lowest ask, the informed agent will buy a share, earning  $A_d - a$ . If  $A_d$  is lower than the highest posted bid, the informed agent will sell a share, earning  $b - A_d$ .

We now define two treatments corresponding to the behavior of the informed agents when no profitable trade is possible. Under the competitive informed agent treatment, if  $A_d$  is between the best posted bid and ask, the informed agent will post a limit order to the order book. The procedure is the same as for the uninformed agents. Since informed agents know the  $A_d$ , they will only place asks that are above the  $A_d$  or bids below the  $A_d$ .<sup>7</sup> Informed agents compete with each other to post the best bid and ask prices to the order book.

Under the cooperative informed agent treatment, the informed agents do not compete. If no profitable trade is possible, they do nothing. No limit order is posted to the market. Under the cooperative treatment the order book can only contain limit orders originated by uninformed agents. The cooperative and competitive treatments represent limiting behavior for the informed agents. In the cooperative treatment informed agents can take maximum advantage of the uninformed agents, but in the competitive treatment the uninformed agents are somewhat protected by competition between informed agents.

### 3.3. Learning

We consider two types of learning treatments. In the social learning treatment agents' timing chromosomes evolve according to a genetic algorithm. In the individual learning chromosomes evolve according to a modified Roth-Erev algorithm.<sup>8</sup> In all of our simulations, both informed and uninformed agents are in effect zero-intelligence agents with regard to prices. Agents do not behave strategically as their choice of market or limit order depends solely on their  $E_{id}$ , which is not modified by the learning algorithms. Therefore, our uninformed agents can never learn the true asset value,  $A_d$ . Instead, our agents can learn when to trade. The learning algorithms only modify agents' timing chromosomes as explained below.

<sup>7</sup> The random interval the offer is drawn from is clipped by the best posted bid ( $b$ ) and ( $a$ ) prices. If the interval extends past the highest bid or lowest ask, then the interval is  $\max(b, A_d - R_{id})$  to  $\min(A_d + R_{id}, a)$ .

<sup>8</sup> Individual learning using a classifier system (Holland, 1986), a type of genetic algorithm, is not well suited for our market because the payoff from any given timing chromosome can only be known by interaction with the other agents.



Because of adverse selection, the uninformed agents will minimize losses if they learn never to go to the market.<sup>9</sup> Many theoretical microstructure models avoid this no-trade equilibrium by making their uninformed agents' trading decisions random. One justification for doing so is that these agents are motivated by liquidity reasons which are exogenous to the models.

We too model uninformed agents so as to preclude the no-trade equilibrium. In social learning, uninformed liquidity motivated agents are modeled so that an agent's chromosome encodes the behavior of going to the market only once per trading day. Never going to the market during the day, or going to the market in more than one period is not possible because there are no permissible chromosomes for those actions in our model. Similarly, with individual learning, going to market more than once, or not going at all are not included as possible actions.

Notice that our implementation is not exactly the same as liquidity-motivated agents in typical microstructure models. We do not require agents to trade. Instead we require agents to go to the market. In our model it is possible for an uninformed agent to go to the market, and place a limit order which never results in an actual trade because no other agent subsequently accepts the offer.

Although liquidity-motivated agents are programmed so as to go to market exactly once per trading day, there are no such restrictions applied to our informed agents. Informed agents' choice set includes never going to the market, going to the market multiple times per day, and even going to the market every period during the trading day.

### 3.3.1. Genetic algorithms

Genetic algorithms (GAs) are based on the principles of genetic evolution in biological organisms whereby populations evolve over many generations through some variant of these processes: selection (reproduction), crossover (mating), and mutation.<sup>10</sup> First rigorously defined by Holland (1975) as an adaptive method to solve search and optimization problems, the use of GAs in many other problem domains, including learning, expanded during the 1980s and 1990s.<sup>11</sup> In search and optimization problems, each chromosome has an invariant fitness value which describes how well the chromosome "solves" the problem. In the learning context of our model, the principles of a GA are used to evolve the timing chromosomes of each group of agents. Uninformed agents are assigned to groups and the informed agents are assigned to a separate group. Our use of the GA differs from the typical use in search and optimization problems. In a search or optimization problem the fitness of a given chromosome is invariant as the population is evolved. In our model the fitness of a given chromosome varies by learning period based on the interactions of the agents in the market in our model.

Genetic algorithms as a learning mechanism for autonomous agent-based economics and finance models have been widely discussed and documented elsewhere (Reichmann, 1999). Many of the authors in the *Handbook of Computational Economics: Agent-based Computational Economics* (Tesfatsion and Judd, 2006) discuss the use of GAs as a learning process in computational models.<sup>12</sup> Chen (2002) contains numerous studies in finance that use genetic algorithms (and the closely related genetic programming), including optimal portfolio diversification, capital asset pricing models, and agent-based stock markets. A genetic algorithm was chosen as the learning representation for groups of agents because of the natural interpretation of the binary encoded chromosome as a timing strategy. Each bit of the chromosome represents an independent course of action the agent can take (whether or not to go to the market in period  $t$ ).

At the end of each learning period (15 days in the base case), each group of agents are separately put through a genetic algorithm to evolve their chromosomes. Each agent's chromosome is replaced by a new chromosome based on three processes: selection, crossover, and mutation. The chromosomes in a group of agents learning together constitute the possible pool of replacement chromosomes. In selection phase, each agent receives one of the original chromosomes in their pool determined by proportional random selection.<sup>13</sup> After selection, pairs of agents are randomly selected with a given probability to undergo crossover. When selected for crossover, the chromosomes of each of two agents is split at a random point and the parts are swapped.<sup>14</sup> Finally, during mutation, a mutation operator is applied to each bit of each chromosome. The mutation operator decides with a very low probability whether to randomly flip the bit.<sup>15</sup>

### 3.3.2. Modified Roth-Erev learning

The idea behind reinforcement learning is that an agent should be more likely to undertake an action for which he has experienced a higher reward and less likely to undertake an action which has yielded a lower reward. Again, many of the

<sup>9</sup> Although not reported in the results below, we find that if uninformed agents are allowed to have a timing chromosome that includes never going to the market they will rapidly learn to not go to the market under either social or individual learning.

<sup>10</sup> We do not use elitism, a process whereby a given percentage of the best chromosomes from one generation are automatically passed to the next generation without crossover and mutation.

<sup>11</sup> The Santa Fe Artificial Stock Market uses a classifier system (a variant of a GA) where agents individually evolve populations of technical and fundamental strategies for predicting asset prices on which to base their net demand.

<sup>12</sup> The *Handbook* provides an exhaustive review of the literature as of 2006. Genetic algorithms as a learning process are contrasted extensively with many alternative approaches.

<sup>13</sup> A roulette wheel mechanism is employed whereby the probability that a given chromosome is randomly chosen is proportional to the chromosome's fitness. Thus chromosomes that performed well in the learning period are more likely to be selected for the next learning period.

<sup>14</sup> For example, if chromosome  $C_{34}[11001011]$  is paired for crossover with chromosome  $C_{21}[01100010]$  between positions 3 and 4, the new chromosomes would be  $C'_{34}[11000010]$  and  $C'_{21}[01101011]$ .

<sup>15</sup> The probability that a given bit mutates is typically set very low, 0.1–1 percent.

authors in the *Handbook of Computational Economics: Agent-based Computational Economics* discuss the variants of reinforcement learning in computational models. Reinforcement learning provides a very sharp contrast in learning assumptions when compared to the genetic algorithm. Our implementation of the genetic algorithm implicitly assumes a very high level of information sharing through communication and mimicry among the agents learning together. Each agent, is in effect, aware of what strategies others have played, who has done well in their group, and who they want to mimic. Over time only the fittest chromosomes survive and are passed among the agents. In reinforcement learning agents only know the consequences of their own actions. They are not directly aware of the consequences of actions taken by other agents. Agents are not even really aware that they are participating in a market where strategic considerations might be important and therefore can be considered only minimally more rational than the zero-intelligence traders of Gode and Sunder (Duffy, 2006, p. 972).

Nicolaisen et al. (2001) modified the Roth-Erev learning algorithm which is one of many variants of reinforcement learning. With modified Roth-Erev learning each agent is given a complete set of all possible chromosomes. The set of timing chromosomes provides the possible actions an agent can take on a given day and initially are equally likely to be chosen by the agent. At the beginning of a learning period (15 days in the base case), each agent randomly selects an initial timing chromosome (action) to use. At the end of the learning period the agent updates the propensity to take that action again. Propensity is updated based on three parameters: initial propensity ( $q_j(0)$ ), recency (also known as forgetting,  $\theta$ ), and experimentation ( $\epsilon$ ).<sup>16</sup> Given updated propensities, the probabilities of choosing any chromosome (action) are updated using a Gibbs–Boltzman function.<sup>17</sup> The cycle of randomly choosing a timing chromosome (action), receiving the reward for using that timing, updating propensities based on the reward and then updating the probability distribution based on the updated propensities continues for each learning period of the simulation.

### 3.4. Experimental design

Our primary research question is to characterize the pattern of trade that results from interactions between informed and uninformed agents under different learning regimes. To study the impact of learning regime on trade patterns, we compare simulation results with social learning in different group sizes using genetic algorithms with individual learning using modified Roth-Erev learning. Fig. 3 contains a flowchart illustration of the simulation.

As shown in Fig. 3, and also in Table 1, the simulation has many parameters that must be specified.<sup>18</sup> Our approach is to first define a base case, and use the base case to compare the simulation results under alternative learning and parameter specifications. The parameter values for our base case are listed in Table 1. The second part of our design studies the impact of group size in social learning and the effects of varying the fraction of informed agents, the number of days, the frequency of learning, and the length of time between learning episodes. The degree of social learning is studied by dividing the uninformed agents into groups, and implementing the genetic algorithm separately for each group. The final part of the design replaces the genetic algorithm with modified Roth-Erev learning to examine the limiting case where there is no social learning.

## 4. Results

Our main focus concerns trading patterns arising from interactions between informed and uninformed agents. Empirical studies typically analyze behavior indirectly through trading volume, volatility and spreads. Agent-based simulations permit more direct additional data analysis. Since our model specifies the fundamental asset value for each day, we can study daily profits and losses. The timing chromosome describes which period(s) an agent will go to market. Therefore the distribution of chromosomes across agents directly measures agents' timing behavior. Because trade timing behavior is fully encoded in agents' chromosomes, we focus mainly on these instead of statistics based on either limit orders or transactions.

### 4.1. Concentration and coordination measures

In order to succinctly summarize the distribution of timing chromosomes, we now introduce two indices, a coordination index and a concentration index. The intuition for the coordination index comes from the Herfindahl index, which is widely

<sup>16</sup> Propensities are updated according to  $q_j(t+1) = [1 - \theta]q_j(t) + E_j(\epsilon, N, k, t)$  where  $q_j(t)$  is the propensity for choosing timing chromosome  $j$  in period  $t$ ,  $k$  is the last chromosome (action) chosen,  $r_k(t)$  is the profits earned by using  $k$  th chromosome,  $t$  is the current learning step, and  $N$  is the number of chromosomes (actions). Further

$$E_j(\epsilon, N, k, t) = \begin{cases} r_k(t)[1 - \epsilon] & j = k \\ q_j(t) \frac{\epsilon}{N-1} & j \neq k \end{cases}$$

allocates the reward between the chromosome chosen and all other chromosomes based on  $\epsilon$ .

<sup>17</sup> The Gibbs–Boltzman probability allows the updating to handle negative propensities which may arise for uninformed agents. Specifically, the probability of action  $j$  in learning period  $t$  is  $Prob_t(j) = e^{q_j(t)/\beta} / \sum_{n=1}^N e^{q_n(t)/\beta}$  where  $\beta$  is the temperature ("cooling") parameter.

<sup>18</sup> The simulation was implemented using the MASON multiagent simulation toolkit (Luke et al., 2005).



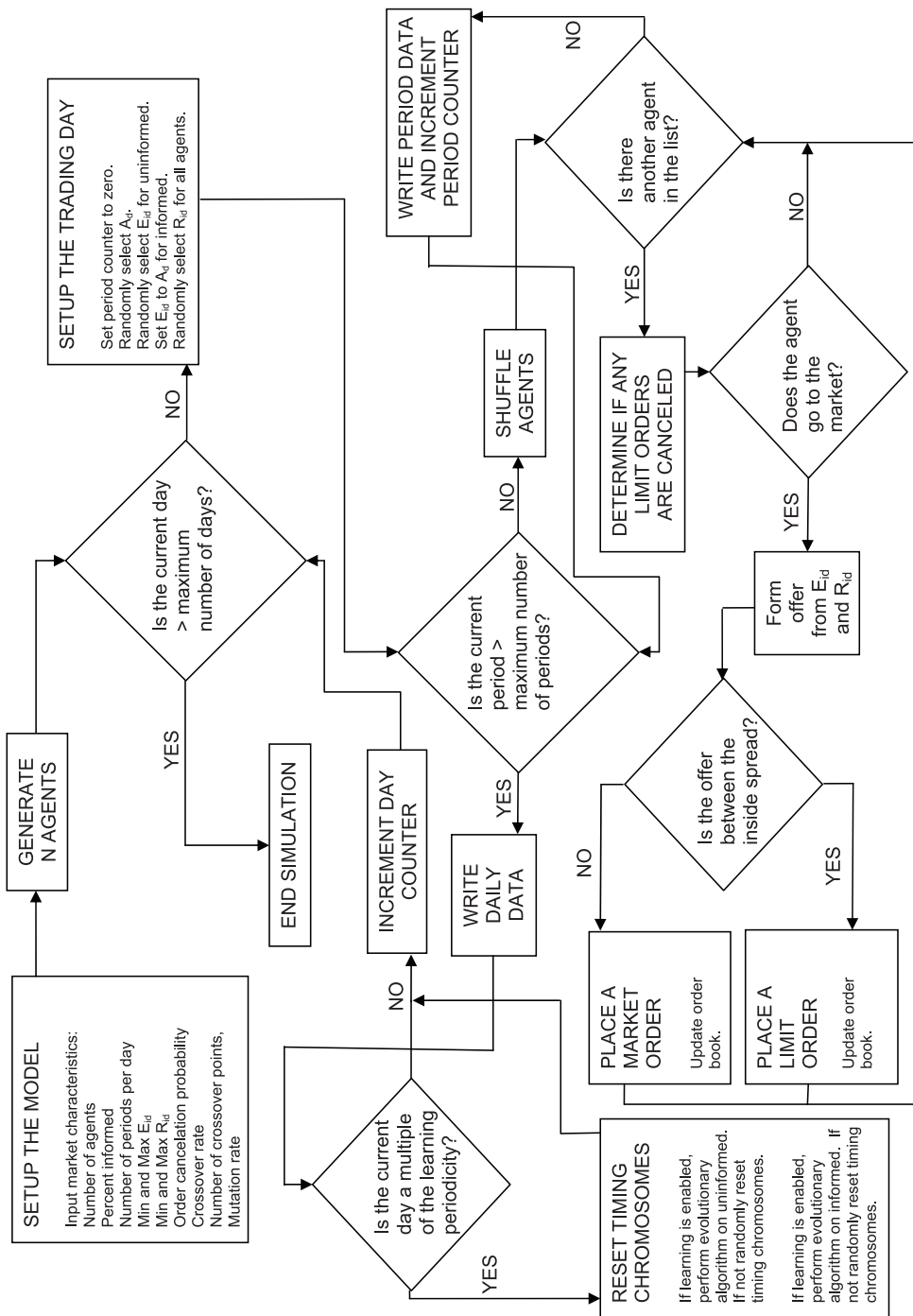


Fig. 3. Flow of the simulation.

used in Industrial Organization studies. Define the coordination index as  $CRD = \sum_{j=1}^T s_j^2$  where  $s_j$  is the fraction of agents possessing chromosome  $j$ .<sup>19</sup> For uninformed agents,  $T=8$ , as there are eight possible chromosomes. Therefore uninformed  $CRD$  values can range between  $1/8$  and  $1$ . For informed agents  $T=256$ , but since there are only 40 informed agents, the minimum possible value of  $CRD$  is  $1/40$ . For informed agents, the maximum  $CRD$  is one.

With individual learning, this procedure cannot be used to calculate the  $CRD$  because agents possess all timing chromosomes instead of a single chromosome. Each agents timing decisions are based on his or her distribution of probabilities associated with the set of chromosomes. We describe a modified calculation of the  $CRD$  measure for individual learning simulations below in Section 4.4.

We next define a concentration index based on the average number of times agents go to market. Define the concentration index as  $CON = N / \sum_{i=1}^N x_i$  where  $x_i$  is the number of times agent  $i$  goes to market. Therefore  $CON$  equals one divided by the average number of times agents go to market. Since uninformed agents are allowed to go to market only once per day, their  $CON$  is equal to 1 by construction.<sup>20</sup> However for informed agents, the  $CON$  can range between  $1/8$  and  $\infty$ , since informed can go to market anywhere from every period during the day to not at all during the day.

Although the  $CRD$  will be calculated separately for informed and uninformed agents, the  $CON$  will be reported only for the informed agents. As will be seen below, the  $CON$  will tend to be lower for informed agents as they tend to go to market more often. Informed agents gain when trading with the uninformed, so more trading activity results in higher profits. Conversely uninformed agents will tend to coordinate reflecting the greater safety from adverse selection loss when they are concentrated and coordinated.

#### 4.2. Base case

The base case in the experimental design provides a benchmark for examining how changes in the learning regime impacts trading patterns. The base case market contains 200 agents with 20 percent informed operating in 8-period days for 2250 trading days. The informed agents cooperate by only placing market orders. Learning occurs every 15-day period (150 generations of learning) and is social using a genetic algorithm. The 40 informed agents learn as a group and the uninformed agents are randomly divided into four learning groups of size 40. The group size of 40 was selected to provide an intermediate case between individual learning and the case with all uninformed agents belonging to a single social group. Each learning group is programmed as a separate population when implementing the genetic algorithm. The base case parameters are listed in Table 1.

All simulations including the base case were simulated 30 times using random starting seeds for the random number generators.<sup>21</sup> Unless otherwise specified all results for all experimental specifications are the average over the 30 trials.

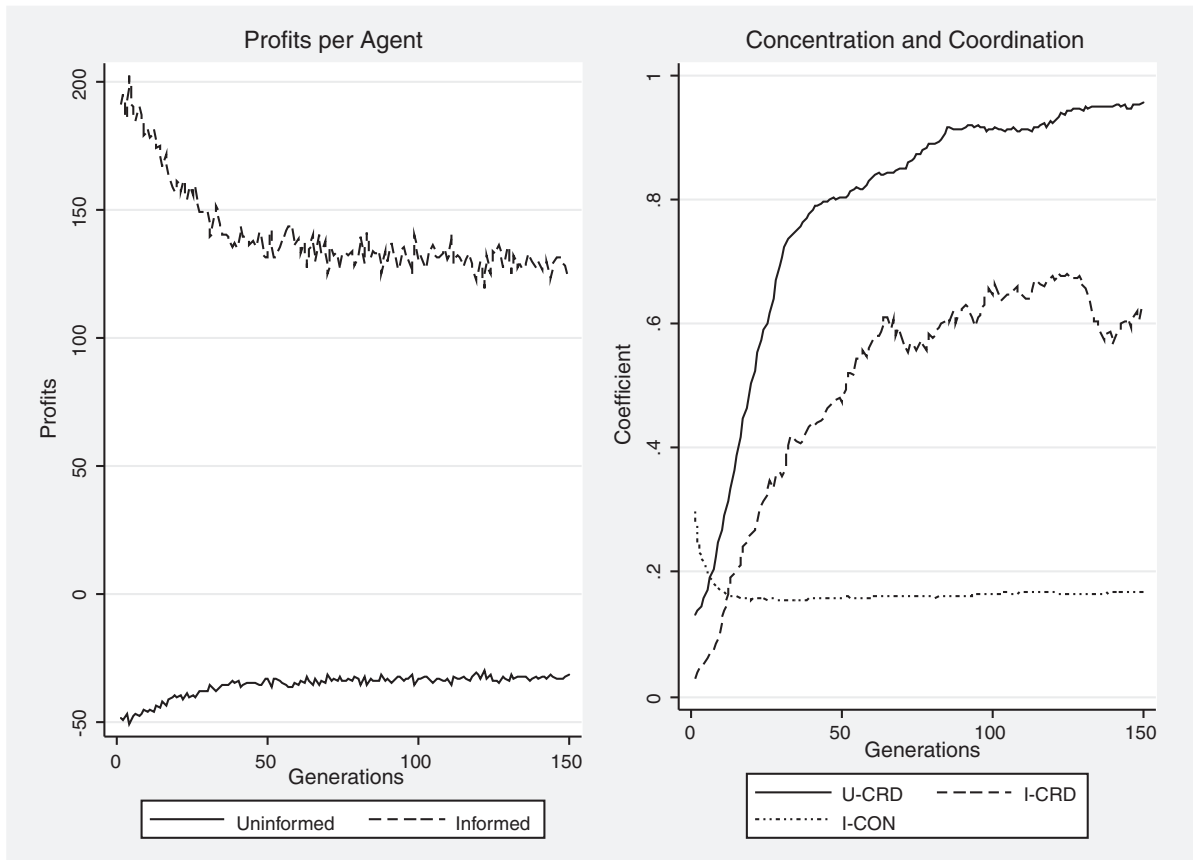
In the base case, uninformed agents are able to learn on average to coordinate their actions. Fig. 4 contrasts the average profit per informed and uninformed agent by learning generation with the concentration and coordination indices by generation. As the uninformed agents begin to coordinate in the first 30–40 generations, profits for the uninformed start to rise (but remain negative) while profits for the informed agents fall. The changes in profits per uninformed agents coincide with the rapid rise in  $CRD$  for the uninformed. Profits for informed agents also initially rise as these agents also learn to coordinate, but then decline and eventually level as the uninformed agents coordinate. Note that the average  $CRD$  for the uninformed for the base case is 0.956 which indicates a much higher degree of coordination than just within-group learning can account for. If all agents in a given group were to coordinate to a particular period, but each group coordinated on a different period, the  $CRD$  would equal 0.25. If all members of 2 of the 4 groups coordinate to the same period and the other groups coordinate to separate periods,  $CRD$  would rise to 0.375. If all members of three of the groups coordinated to the same period with all members of the fourth group choosing a different period,  $CRD$  would be 0.625. The base case average  $CRD$  is close to all members of all groups coordinating on one period ( $CRD = 1$ ). Informed agents also learn to coordinate, but not to the same degree as uninformed. The base case average  $CRD$  is only 0.628. However, the informed agents go to the market in period zero 99.8 percent of the time. Informed agents also learn to go to market often. The base case average  $CON$  for informed agents is 0.168, which corresponds to the average informed agent going to market 6.2 times per day.

Average coordination and concentration by generation hides the richness of the outcomes observed in the simulations. Fig. 5 provides histograms of the  $CRD$  for the uninformed and both the  $CRD$  and  $CON$  for the informed agents at the beginning and again at the end of each simulation run. At the beginning of the simulation, the distribution of both  $CON$  and  $CRD$  across runs is consistent with the initial random construction of chromosomes. By the end of the runs, the informed agents have both increased  $CRD$  and decreased  $CON$ . Informed agents are increasing the number of times they go to the market, as they

<sup>19</sup> When calculating the coordination index, uninformed timing chromosomes are recoded so as to be consistent with informed agents' chromosomes. The three-bit binary number is restated as an 8-bit number. Each possible three-bit number recodes to an 8-bit number with a single 1 and seven 0's. The location of the 1 corresponds to the period the uninformed agent goes to market.

<sup>20</sup>  $CON$  cannot be undefined for uninformed traders since these agents must go to market exactly once per day. However, when uninformed traders are not forced to go to the market at least once, the no-trade equilibrium can occur and the  $CON$  measure can become undefined as uninformed agents learn never to go to market.

<sup>21</sup> Starting seeds for every simulation run were retrieved from random.org to reduce any systematic bias in seeding the simulation's random number generators.



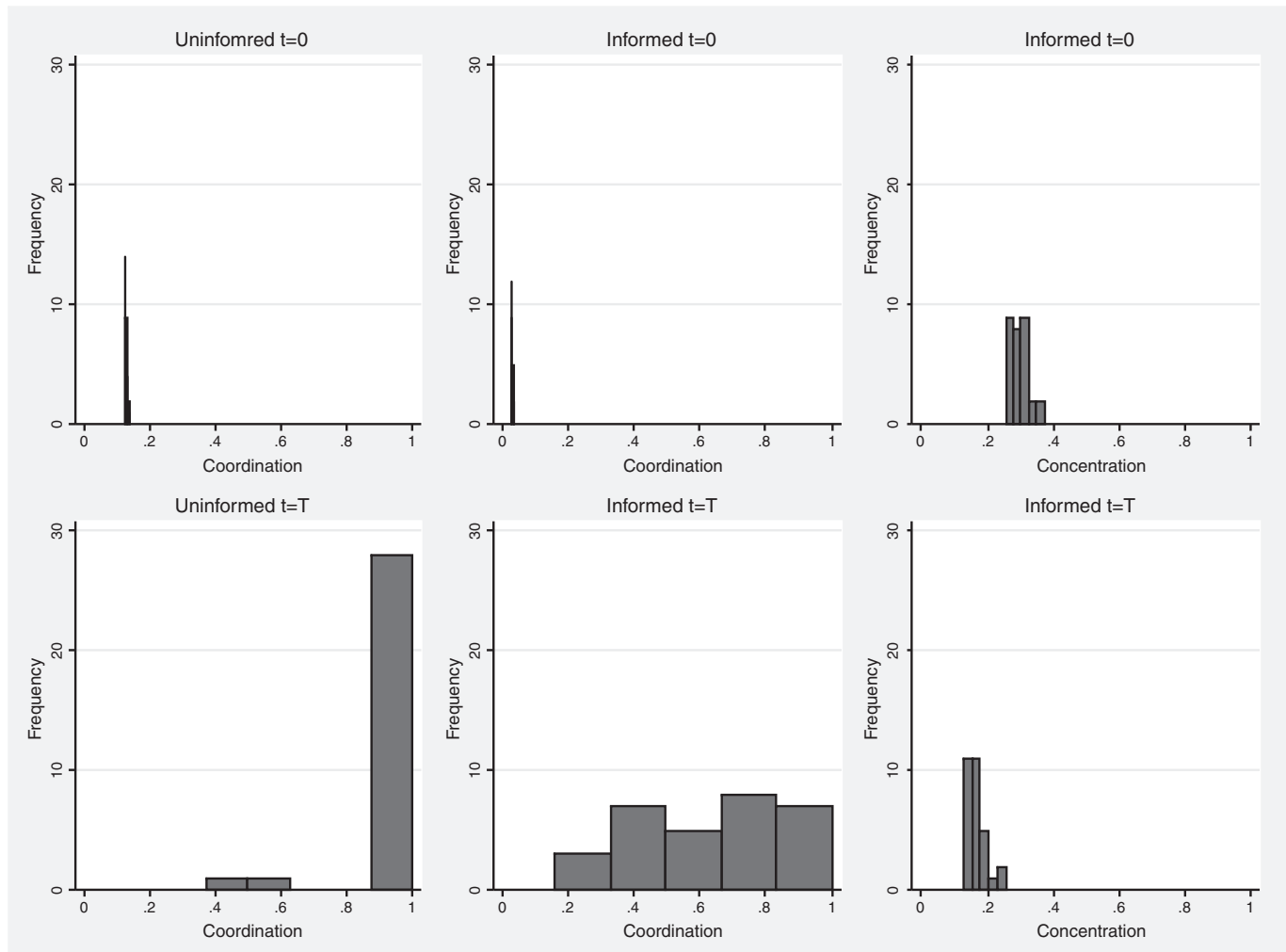
**Fig. 4.** These graphs contrast the average profit per informed and uninformed agent to the average *CON* and *CRD* indices for each learning generation. Results are averaged over 30 simulations of the base case.

earn additional profits with each trade. The informed *CRD* measure increases, but as the histogram shows, the magnitude of the increase varies by the run. The uninformed agents are also learning to coordinate. By the end of the simulation, in 28 of 30 runs, uninformed *CRD* s are consistent with all uninformed agents of every group coordinating to a single period during the day. The other two runs are consistent with two or three groups coordinating to one period and the remaining group(s) coordinating to a different period. Uninformed agents exhibited a high degree of coordination in every run of our base case.

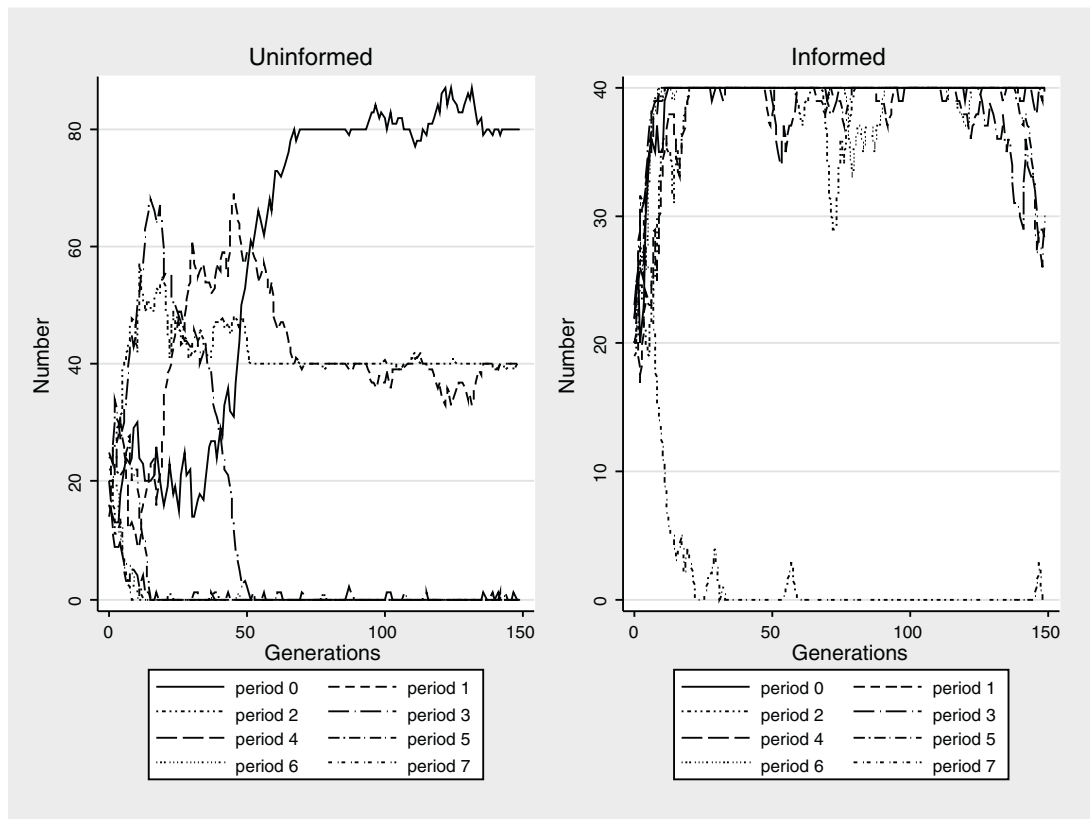
To visualize the process of coordination, Figs. 6–8 illustrate the number of agents going to market for each period by generation for particular base case runs. The three figures are of the minimum, median, and maximum uninformed *CRD* observed in the 30 runs. Coordination via the social learning of the base case occurs well within 50–75 generations of learning and generally persists out to 150 generations.<sup>22</sup> The impact of the mutation operator of the genetic algorithm can be observed in all three cases for the informed as their coordination for going to market every period waxes and wanes in later generations as the mutation operator induces some experimentation. Experimentation is also apparent for the uninformed in later generations as a few uninformed agents try to coordinate on a second period but quickly are punished for going to market alone. The minimum case (Fig. 6, *CRD* = 0.375) is consistent with two of the groups of 40 going to market in period zero, one group going to market in period one and the other group in period two.

Fig. 9 illustrates what happens to trading volumes in the base case. The first panel graphs the trading volume per period of the day by period averaged across all days of all runs. The second panel graphs the trading volume per day by generation averaged across all runs. Because informed agents do not place limit orders in the base case, they can only trade with uninformed agents. However uninformed agents can trade with either informed agents (I–U volume) or with other uninformed agents (U–U volume). Fig. 9 shows that U–U volume initially declines and then levels off. As the informed agents learn to go to market frequently, the volume of U–U trades declines, and the volume of I–U trades increase. By around 50 generations, the informed have learned to go to market very frequently, and the uninformed have learned to go only in the first period (see Figs. 6–8), so subsequent trading volume stabilizes.

<sup>22</sup> Allowing social learning to continue for longer periods of time, up to 450 learning periods does not induce movement away from the outcomes described herein.



**Fig. 5.** Agent timing strategies histograms indicate the distribution of CON and CRD indices for uninformed vs. informed at the beginning and end of each of 30 simulations of the base case.



**Fig. 6.** Agents timing strategies when uninformed  $CRD = 0.375$ . These graphs indicate the number of uninformed and informed agents going to market each period of the day by learning generation. Results are the minimum uninformed  $CRD$  achieved in 30 simulations of the base case.

In the base case we have observed a rich set of behavioral outcomes in the timing strategies of agents. The informed agents learn to go to market almost every period, maximizing their chances of trading with an uninformed trader. The uninformed traders coordinate their activities every time, either fully or nearly so depending on the particular run. The intuition is similar to Admati and Pfleiderer's; uninformed agents must concentrate and coordinate to reduce the severity of the adverse selection problem.

#### 4.3. Social and individual learning

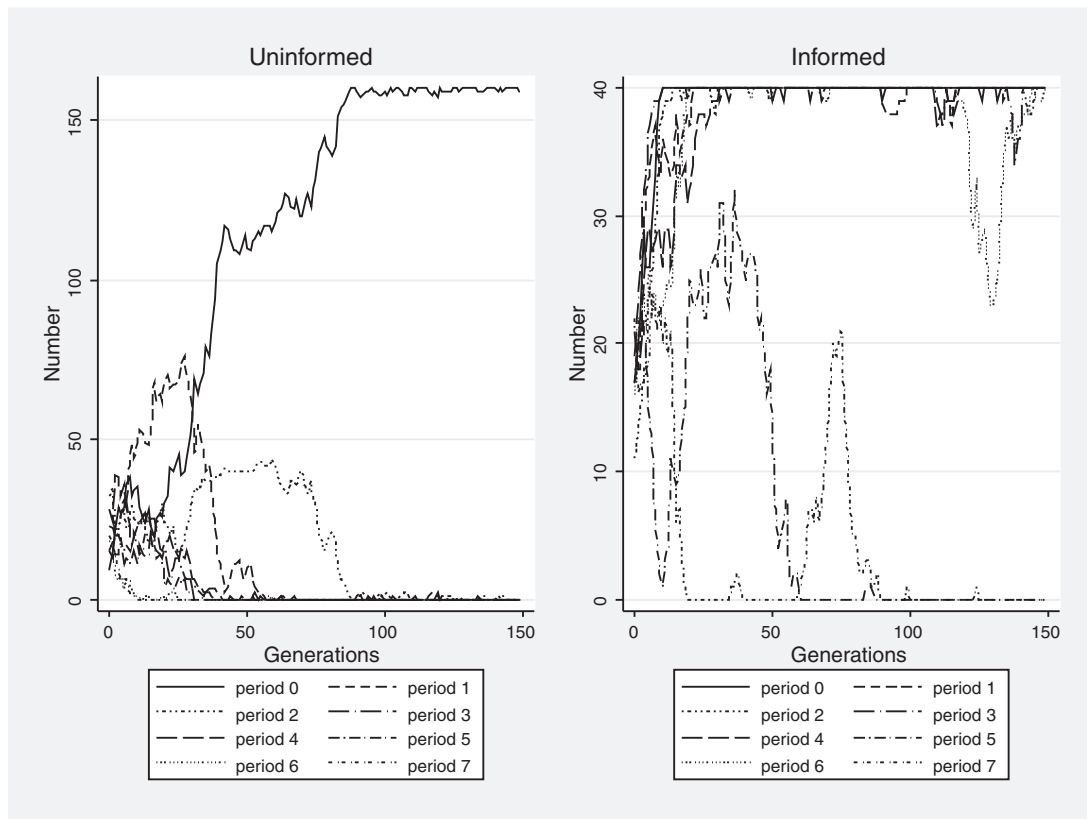
Table 2 reports the results of an experimental design varying both the size of the uninformed groups and the number of learning generations. As the size of the social learning group size gets smaller (and thus there are more groups), the degree of coordination declines. But even with smaller group sizes, there is still some coordination. The "Random Across Groups" row illustrates this result by calculating the expected uninformed  $CRD$  assuming that all members within each group are perfectly coordinated, but each group selects their timing chromosome randomly.<sup>23</sup> Even with 16 groups (a group size of ten agents) the uninformed agents'  $CRD$  is approximately double the Random AG value.

By increasing the number of generations while holding the learning periodicity constant, uninformed agents have more time to learn. Table 2 also reveals that lengthening the time to learn only marginally increases the uninformed coordination index. Additional learning time increases the uninformed  $CRD$  the most when it is lower to begin with.

Informed agents' concentration behavior is fairly invariant to changes in either learning periodicity or uninformed group size. These agents are going to market frequently, every period or almost every period of the day. If all of the informed went every period, their  $CON$  would be 0.125. Table 2 shows that the informed agents'  $CON$  is typically near 0.15. Informed agents go to market almost every period, hence they are coordinated, but not concentrated.

So far we have considered social learning using a GA. Lowering the group size and increasing the number of groups reduces the scope for social learning. We now consider cases where social learning is not possible. Table 3 contains the results from simulations where the GA is replaced with individual reinforcement learning via the modified Roth-Erev algorithm.

<sup>23</sup> This is a sensible benchmark because genetic algorithms using social mimicry often tends to ultimately adopt the same chromosome. Since each of our learning groups separately implement the GA, coordination across groups is not occurring simply because we are using the GA.



**Fig. 7.** Agents timing strategies when uninformed  $CRD = 0.988$ . These graphs indicate the number of uninformed and informed agents going to market each period of the day by learning generation. Results are the median uninformed  $CRD$  achieved in 30 simulations of the base case.

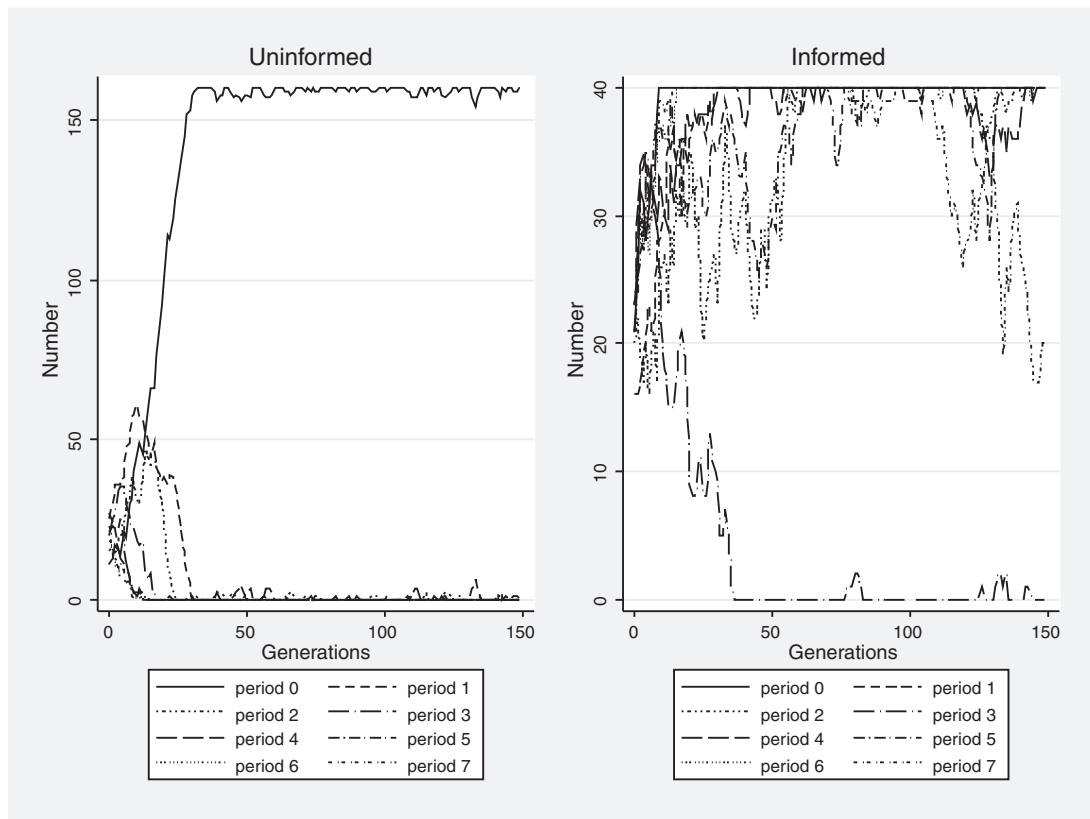
**Table 2**  
GA learning concentration and coordination.

	Uninformed group size				
	10 16 <sup>a</sup>	20 8 <sup>a</sup>	40 4 <sup>a</sup>	80 2 <sup>a</sup>	160 1 <sup>a</sup>
<i>Uninformed coordination</i>					
100 gens	0.322	0.663	0.851	0.945	0.984
150 gens	0.316	0.647	0.956	0.982	0.983
200 gens	0.385	0.672	0.935	0.989	0.986
250 gens	0.446	0.830	0.937	0.973	0.982
Random AG	0.180	0.235	0.343	0.563	1.000
<i>Informed coordination</i>					
100 gens	0.696	0.698	0.668	0.572	0.666
150 gens	0.780	0.738	0.628	0.612	0.640
200 gens	0.802	0.691	0.698	0.700	0.702
250 gens	0.810	0.661	0.678	0.683	0.733
<i>Informed concentration</i>					
100 gens	0.146	0.145	0.160	0.166	0.146
150 gens	0.143	0.150	0.168	0.164	0.157
200 gens	0.141	0.162	0.164	0.159	0.164
250 gens	0.136	0.161	0.172	0.158	0.161

Table indicates the *CON* and *CRD* indices by number of learning generations and uninformed group size. Informed group size is held constant at 40 in all cases. The Random AG (across groups) row provides some benchmark values. Random AG presents the expected *CRD* if all members within groups are perfectly coordinated but each group selects a timing chromosome randomly. By construction, the *CON* index equals 1 for all uninformed runs. Results are averaged over 30 simulations for each parameter combination. All other parameters held constant at the base case values listed in Table 1.

<sup>a</sup> Number of groups





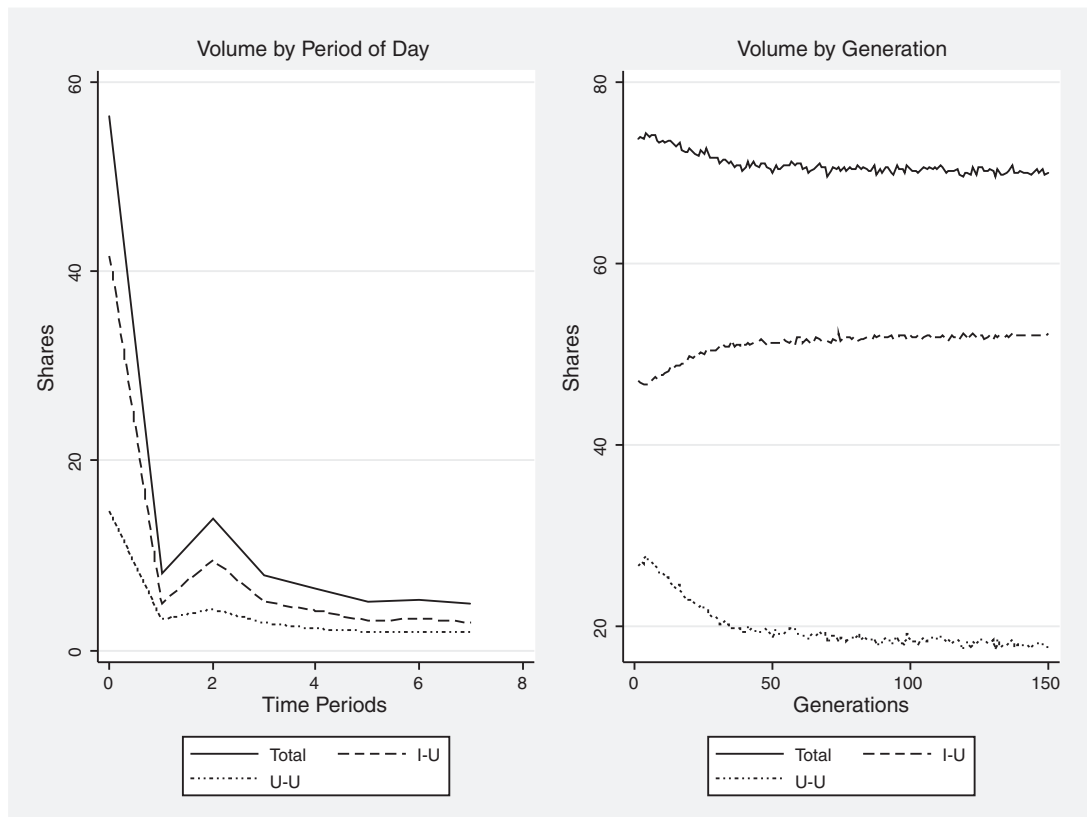
**Fig. 8.** Agents timing strategies when uninformed  $CRD = 1.000$ . These graphs indicate the number of uninformed and informed agents going to market each period of the day by learning generation. Results are the maximum uninformed  $CRD$  achieved in 30 simulations of the base case.

Recall that with individual learning, each agent possesses all possible timing chromosomes and a vector of corresponding probabilities. The calculation of the  $CRD$  must be modified to account for this. The  $CRD$  index used above is based on the fraction of agents possessing each chromosome,  $s_j$ . The generalization is to determine the fraction of agents playing similar probability distributions over the available chromosomes. To determine the groupings of uninformed agents (or informed agents, separately) with similar probability distributions, k-means clustering analysis is used (Hastie et al., 2008). In k-means clustering analysis, a number of groups must be specified. Since the number of groups naturally arising in the set of probability distributions should be endogenous, a k-means clustering analysis is conducted for each group size  $\geq 2$ . The

**Table 3**  
Reinforcement learning coordination.

Initial propensity 8,000				Recency			
		0.05	0.10	0.15	0.20	0.25	0.30
Experimentation	0.05	0.974	0.821	0.545	0.338	0.347	0.271
	0.10	0.968	0.813	0.527	0.333	0.280	0.270
	0.15	0.875	0.814	0.594	0.348	0.309	0.332
	0.20	0.959	0.815	0.548	0.344	0.313	0.356
	0.25	0.949	0.805	0.612	0.417	0.410	0.421
	0.30	0.934	0.807	0.601	0.424	0.472	0.470
Initial propensity 16,000				Recency			
		0.05	0.10	0.15	0.20	0.25	0.30
Experimentation	0.05	0.913	0.821	0.591	0.342	0.294	0.326
	0.10	0.967	0.817	0.579	0.301	0.287	0.298
	0.15	0.968	0.811	0.542	0.363	0.318	0.301
	0.20	0.955	0.813	0.545	0.352	0.356	0.400
	0.25	0.951	0.815	0.546	0.378	0.428	0.376
	0.30	0.939	0.812	0.575	0.433	0.380	0.484

Table indicates the  $CRD$  index for the uninformed agents as recency (columns) and experimentation (rows) varies for the base case as defined in Table 1 using modified Roth-Erev learning. Results averaged over 30 simulations for each parameter combination.



**Fig. 9.** Trade volume graphs indicate the average volume per agent per period of the day and average volume per agent per learning generation for all trades and for trades between uninformed–uninformed agents (U–U volume) and informed–uninformed (I–U volume) agents. Results are averaged over 30 simulations.

number of groups to use is then determined by a “stopping” rule (Caliński and Harabasz, 1974).<sup>24</sup> Once the groupings have been determined,  $s_j$  is calculated as the fraction of agents in each group and the CRD is calculated as described before.

Table 3 reports the results of replacing social learning with individual reinforcement learning. The parameters for reinforcement learning (see Table 1) include a recency and an experimentation parameter. The recency parameter adjusts the importance placed on recent profits or losses in updating the probabilities. Low recency values imply longer memory of past outcomes. The experimentation adjusts the frequency that agents will attempt actions to test whether different strategies will become more profitable. Table 3 shows that the uninformed coordination is quite sensitive to the recency value but relatively insensitive to experimentation. When recency is low, coordination is nearly complete. At 0.05 recency, the CRD is close to the maximum possible level of one. But when recency is very high coordination is much lower. With 0.30 recency, the CRD ranges only between 0.27 and 0.48.

#### 4.4. The coordination process

With both social and individual learning uninformed coordination can be near complete with at least some combinations of parameter values. But how does the process work? Some insight into the process comes from analyzing the intraday pattern. When there is coordination, it is almost always in the first period (period zero) of the trading day. In our social learning base case, in every case coordination occurs in the first period of the trading day. Similarly agents nearly always coordinate in the first period in the individual learning base case. The average probability weighting for the first period over all uninformed agents across the thirty base case runs was 93.6 percent.

Coordination in period zero is a result of the interaction between informed and uninformed within the microstructure of the simulation. Analyzing the profitability of both market and limit orders provides insight into the process. Table 4 is

<sup>24</sup> The Caliński–Harabasz rule has been identified as one of the better stopping rules to use (Milligan and Cooper, 1985). In the Caliński–Harabasz rule, a pseudo- $F$  statistic is calculated for each number of groups. Larger pseudo- $F$  values indicate more distinct groupings. The number of groups used are then determined by ordering the pseudo- $F$  values by number of groups (starting at 2), stopping when the pseudo- $F$  value decreases as the number of groups increase.

**Table 4**

Uninformed average profits by type of order and period placed/traded.

Period order placed	Period order traded							
	0	1	2	3	4	5	6	7
<i>Limit orders</i>								
0	2.73	4.60	25.74	27.51	29.60	31.87	34.32	38.55
1		−0.83	16.54	25.40	26.37	28.77	32.61	33.34
2			−4.50	15.37	23.04	25.81	26.33	28.91
3				−5.48	14.50	23.90	24.99	29.60
4					−8.24	14.38	24.07	25.78
5						−10.53	13.65	24.25
6							−11.26	14.29
7								−14.19
<i>Market orders</i>								
Period order placed and traded								
	−8.81	−16.43	−18.68	−19.52	−20.88	−22.22	−22.44	−23.18

Table indicates the average profits per period of the day for uninformed traders by type of order and period placed/traded. The data is drawn from a sample run of the base simulation with reinforcement learning.

generated from a representative single run of the base case of our individual learning treatment. All 15,000 trading days are used. Each trade is broken down according to when during the trading day the order was put on the limit order book and when the order was filled. The average profitability of the trade, from the point of view of the agent placing the limit order, broken down by both the time the order was placed and was accepted is shown in the upper portion of Table 4. In this run, all limit orders are placed by uninformed agents (in the cooperative informed treatment, informed agents are only allowed to place market orders). The lower portion of the table summarizes the average profits per market order by period. Here, averages are computed using only market orders placed by uninformed agents.

Table 4 reveals that uninformed market orders are very unprofitable, and become more unprofitable as the trading day progresses. The limit order book is empty at the start of the day, and gets filled up over the day as the uninformed submit bids and asks. Informed agents (in our base case) do not place limit orders. They only accept existing limit orders, if profitable. That is, an informed agent will buy only if there is an uninformed ask price less than the true asset value, or sell only if there is an uninformed bid greater than the true asset value. Therefore limit orders removed from the order book through trade are those that realize the greatest profits for the informed agent. So after enough informed trading, the true asset value will be within the inside spread. All market orders will be then become unprofitable, and informed agents will no longer trade as long as the true asset value remains within the inside spread. And once the true asset value lies within the spread, a wider spread will tend to make market orders more unprofitable.

Because later market orders are more costly, agents learn to arrive early. Recall that the agents do not choose whether to place a market or limit order. That choice depends on the state of the order book and an individual agents' randomly generated asset values,  $E_{id}$ . In our simulation, uninformed agents are not allowed to try to minimize losses by avoiding market orders. So even though Table 4 seems to suggest that limit orders are more profitable than market orders, an uninformed agent cannot exploit this. Our agents are near zero-intelligence and are by construction unable to learn anything about asset value from the state of the limit order book.

Table 4 also shows that limit orders that survive on the order book for longer are more profitable. Any limit order that is unprofitable to the agent placing the bid or ask will be profitable to the agent accepting the offer. Odds are that an informed trader will accept such unprofitable bids or asks. Therefore only profitable limit orders will survive.

When uninformed agents arrive later in the trading day, there is a greater chance of an unprofitable market order. The process of coordination therefore occurs as uninformed agents learn to arrive early. So when coordination occurs, agents learn to arrive in period zero, the initial period of the trading day.

#### 4.5. Variations

Table 5 reports two experimental designs to test the sensitivity of results from the base case to the percentage informed and to the learning periodicity. A greater percentage of informed agents may either increase or decrease the uninformed coordination. If there are more informed, the adverse selection problem is worse and there is a greater incentive for coordination. But more informed agents also means there are less uninformed, so it will become harder to coordinate. It will be more difficult for the uninformed to find each other.

This ambiguity is reflected in Table 5. The GA algorithm is implemented with the percent of informed varied from 5 percent to 40 percent. At the same time, the number of uninformed and the social learning group size also changes. In all cases there is some uninformed coordination, but perhaps because the number and composition of the social learning groups cannot be held constant, the average uninformed CRD does not respond smoothly to changes in fraction of informed.

**Table 5**

GA learning—sensitivity analysis.

CRD Index								
Learning groups	0.05	0.10	0.15	Percent informed				
	5	5	5	0.20	0.25	0.30	0.35	0.40
Uninformed	0.620	0.766	0.830	0.912	0.610	0.666	0.768	0.884
Informed	0.901	0.847	0.665	0.688	0.688	0.707	0.654	0.582
Learning groups	Learning periodicity							
	5	10	15	20	25			
Uninformed	0.767	0.858	0.943	0.914	0.930			
Informed	0.744	0.717	0.659	0.725	0.649			
CON Index								
Learning groups	0.05	0.10	0.15	Percent informed				
	5	5	5	0.20	0.25	0.30	0.35	0.40
Informed	0.151	0.133	0.142	0.170	0.133	0.132	0.137	0.159
Learning groups	Learning periodicity							
	5	10	15	20	25			
Informed	0.149	0.150	0.154	0.157	0.166			

Table indicates the *CON* and *CRD* indices for the GA Learning base case as defined in Table 1 but varying the percentage informed and the number of days in a learning period. When percent informed is varied, uninformed agents are placed into social learning groups of 40 as in the base case. However, if the number of uninformed is not evenly divisible by 40, the remaining uninformed are placed in their own group. When the learning periodicity is varied, the number of days in the simulation is also varied to maintain 150 learning generations. The *CON* index of the uninformed agents equals 1 by construction in all runs. Results averaged over 30 simulations for each parameter combination.

As the percent informed is varied, the informed *CON* tends to be low throughout. The informed agents are going to market frequently. However, the informed *CRD* declines as the percentage informed increases. Informed agents potential profits are lower if there are less uninformed agents, hence coordination is less profitable.

Results from a similar exercise using individual learning are presented in Table 6. Difficulty in coordination arises because there are fewer informed appears to outweigh any increased incentive to coordinate from lowering uninformed agents' expected profits. The impact on the *CRD* from increasing the percent informed is dependent on the recency parameter. With lower recency, coordination is strong and changing the fraction of informed has little impact. With higher recency, coordination is weak. However changes in the fraction informed has little impact. But for intermediate recency values, increasing the percent informed greatly reduced the degree of uninformed coordination.

The number of days in a learning period have a small, but not dramatic, impact on coordination by the uninformed. With social learning, as learning periodicity is increased from 5 days to 15 days, the average the *CRD* increases from 0.767 to 0.943 (see Table 5). A longer learning periodicity makes the uninformed agents' average daily profits over the learning period less volatile. A less noisy profit signal means learning and therefore coordinating is easier. Increasing the number of days in a learning period beyond 15 has little effect. A learning period of fifteen days provides a sufficiently clear profit signal to the

**Table 6**

Modified Roth-Erev learning—sensitivity analysis.

Uninformed							
Percent Informed	0.05	0.10	0.15	0.20	0.25	0.30	
	0.05	0.993	0.838	0.699	0.386	0.293	0.293
	0.10	0.953	0.837	0.664	0.376	0.339	0.294
	0.15	0.980	0.819	0.636	0.406	0.330	0.342
	0.20	0.966	0.811	0.578	0.342	0.338	0.312
	0.25	0.945	0.800	0.539	0.310	0.307	0.316
	0.30	0.914	0.773	0.434	0.327	0.307	0.314
	0.35	0.903	0.744	0.387	0.287	0.319	0.348
	0.40	0.845	0.675	0.329	0.266	0.317	0.289
Learning periodicity							
Learning groups	5	10	15	20	25		
	0.149	0.793	0.817	0.753	0.749		
Informed	0.030	0.029	0.030	0.029	0.029		

Table indicates the *CRD* index for the modified Roth-Erev learning base case as defined in Table 1 but varying the percentage informed (rows) versus recency (columns) and the number of days in a learning period. When the learning periodicity is varied, the number of days in the simulation is also varied to maintain 1000 learning generations. Results averaged over 30 simulations for each parameter combination.

**Table 7**

Cooperation versus competition among informed.

	Cooperative informed	Competitive informed
<i>CRD index</i>		
Uninformed		
100 gens	0.851	0.357
150 gens	0.956	0.359
200 gens	0.935	0.349
250 gens	0.937	0.336
Informed		
100 gens	0.668	0.959
150 gens	0.628	0.962
200 gens	0.698	0.940
250 gens	0.678	0.938
<i>CON index</i>		
Informed		
100 gens	0.160	0.172
150 gens	0.168	0.159
200 gens	0.164	0.167
250 gens	0.172	0.155

Table indicates the *CON* and *CRD* indices by number of learning generations. Cooperative informed is the base case where informed agents only place market orders. If there is no profitable market order possible, they do nothing. Competitive informed modifies the base case so that the informed traders place limit orders narrowing the bid-ask spread if there is not an opportunity for a profitable market order. *CON* for the uninformed equals 1 by construction. Results are averaged over 30 simulations for each parameter combination. All other parameters held constant at the base case values listed in Table 1.

uninformed agents given the parameters of the model.<sup>25</sup> For the informed agents in our simulations, both *CON* and *CRD* are relatively invariant to changes in the number of days in a learning period.

Table 7 considers one other variation of the base case whereby the informed agents are allowed to place limit orders when no profitable market order exists. Allowing informed to place limit orders narrows the bid-ask spread because when there is no profitable trade available, an informed limit order will improve the best bid or ask in the order book. As competition between the informed tends to narrow the spread and center it around the actual asset value, the order book becomes a “less dangerous” place for uninformed agents. This process mitigates the need for the uninformed to coordinate. The narrowing of the bid-ask spread “protects” uninformed agents who place limit orders farther away from the true asset value

## 5. Concluding remarks

Our results demonstrate that trading patterns can arise in an autonomous agent-based model as a result of interactions between informed and uninformed agents. Trading strategies can be measured directly through observation of agents’ timing chromosomes. In a wide range, but not for all model specifications, uninformed agents can successfully coordinate to avoid trading with informed agents and suffering adverse selection losses. Trading volumes and profits generated during the simulations are consistent with these results.

The economic intuition underlying coordination in our agent-based model is the same in spirit as Admati and Pfleiderer’s theoretical insight. Attempting to trade at the same time allows uninformed agents to avoid adverse selection losses. However, our agents are not rational, strategic optimizers. They have nearly zero intelligence. Our model does not use an equilibrium concept. Still, we observe coordination with uninformed agents going to market at the start of the trading day. Further, our agent-based model also details a dynamic process leading to the coordinated pattern of trade.

In our model trade is concentrated at the start of the trading day. If an uninformed agent arrives later in the trading day, informed agents will have had more time to accept favorable limit orders. Arriving later in the trading day therefore results in a greater likelihood that all remaining bids and asks will be unprofitable (from the viewpoint of the arriving agent) to accept. So uninformed agents learn to arrive early.

Of course, varying parameters can affect the likelihood of coordination. Social learning is especially important. We find partial coordination with smaller learning groups and more complete coordination with larger learning groups. With individual learning coordination depends critically on the recency parameter. If uninformed agents overweight recent days’ profits and underweight older experiences, then coordination fails. But with lower recency values, coordination is nearly complete.

In addition to providing evidence that very rudimentary learning can result in concentrated patterns of trade, agent based models can be used to study additional questions. There is no shortage of ideas for extensions of the model. For example, in our model all information arrives just prior to the start of the trading day. Our model could be extended to allow for information to arrive at different times or even randomly during the day. Another possible extension could involve generalizing the

<sup>25</sup> Table 6 also shows that lengthening the learning periodicity in the individual learning model eventually leads to the *CRD* declining. This result is most likely due to an increasing learning periodicity reducing the variation in profitability, thus slowing down how quickly the propensities and probability distributions are being changed over time.

competitive and cooperative treatments for the informed agents. A generalized treatment might allow informed traders to learn whether or not to place limit orders, making the competitive or cooperative feature endogenous. Another, more major refinement might be to program agents to be a little more intelligent, allowing them to learn about asset value and trade timing simultaneously. With this feature, it would also be interesting to construct an economy to study both interday and intraday trading patterns. Interday patterns in our simulations are not meaningful because the true asset value is reset at the start of every trading day. While this design choice greatly simplifies our model, we may well be eliminating some important features. For example, our simulations do obtain the concentration of trading at the start of the day, but do not obtain the full U-shaped pattern observed in many markets. It seems plausible that the end-of-day concentration of trade and the U-shaped pattern might be produced if agents are made to consider their positions at the start of the following day. Perhaps these and other model modifications will prove to be both feasible and instructive.

Market microstructure models (e.g., [Admati and Pfleiderer, 1988](#)) predict concentrated trading. These strategic trader models characterize equilibria, but are silent as to how such equilibria might be attained in actual markets. In our model coordination takes place through trading in a double auction environment. Further, coordination may occur even when agents are zero intelligence with regard to prices, and can only learn about when to trade. Our simulation results also show that partial coordination can occur. The degree of coordination depends upon learning parameters such as recency or group size, and can vary even within sets of thirty runs with the same parameter specifications.

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