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A Takagi–Sugeno fuzzy model combined with a support vector regression for stock trading forecasting

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ABSTRACT

The turning points prediction scheme for future time series analysis based on past and present information is widely employed in the field of financial applications. In this research, a novel approach to identify turning points of the trading signal using a fuzzy rule-based model is presented. The Takagi–Sugeno fuzzy rule-based model (the TS model) can accurately identify daily stock trading from sets of technical indicators according to the trading signals learned by a support vector regression (SVR) technique. In addition, when new trading points are created, the structure and parameters of the TS model are constantly inherited and updated. To verify the effectiveness of the proposed TS fuzzy rule-based modeling approach, we have acquired the stock trading data in the US stock market. The TS fuzzy approach with dynamic threshold control is compared with a conventional linear regression model and artificial neural networks. Our result indicates that the TS fuzzy model not only yields more profit than other approaches but also enables stable dynamic identification of the complexities of the stock forecasting system.

1. Introduction

The data collected from stock markets is one of the noisiest and most volatile signals. Stock markets are affected by many factors such as government policies, economic environments, interest rates, and inflation rates. In addition to the above-mentioned elements, the share prices of most listed companies also move up and down with other changing factors like market capitalization, earnings per share (EPS), price to earnings ratio, demand & supply, and market news. Despite the volatile nature of the stock markets, researchers still try to find certain correlations between these factors and stock prices.

The goal of investment activities is to make high or stable profits. How to provide investors an effective approaches or rules to get good profit in their financial objectives is very important, therefore, most researchers have tried to explain the relationship between financial markets and price variations to comprehend investment opportunity [1,3]. In stock markets, analysts and portfolio managers typically would apply a technical trading rule to conduct research and make buy or sell decisions. For a successful application of a trading rule, assigning values to parameters and exploring all possible combinations have been key issues to be resolved. However, the range of parameters often varies greatly, so it is difficult for users to search for the best parameter combination within a limited time frame. During the course of time, new approaches such as fuzzy systems, neural networks, and support vector machine have evolved in the areas of soft computing or evolutionary computation. Many researchers [3,4] used these approaches to predict stock prices but not to make trading decisions. However, the buy or sell timing (i.e. decision-making in trading) is crucial in achieving returns of investment. According to the best of our knowledge, up to now, only few research projects have explored the prediction of buy or sell timing [5,6]. For investors, deciding an appropriate timing to buy and sell presents a major challenge to the academic researchers and industrial practitioners.

Since the future movement of a stock price is unknown, predicting turning points is much more difficult than predicting stock price variations. Many researches in the past have focused on stock price prediction but not on trading decisions. However, to make profit in stock investment, investors need to determine the right time to trade instead of the stock price only. To resolve this problem, researchers have relied on knowledge learning and techniques from computational intelligence to reduce investment risks. A neural network methodology to provide trading signals is a core technique in their researches as background knowledge [6]. The support vector regression (SVR) is a tool on numerical prediction in support vector machine (SVM), and it is a powerful predication and learning approach [7,8]. The researchers proposed a dynamic threshold decision system combining SVR to determine the high-precision trading timing [9]. To further extend this concept, the

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objective of this paper is to further incorporate the Takagi–Sugeno (TS) fuzzy model [10] to identify the stock trading turning points to obtain better trade timing (trading decision). The proposed model can yield greater trading profits than a single model predication under time horizons from several days to several weeks. On the other hand, we have compared the TS fuzzy identification model with different learning algorithms to predict daily trading decisions for various stocks across different sectors in this study.

The rest of this paper is organized as follows. In Section 2, we describe different types of forecasting models used to make trading decisions in past researches. Then the TS fuzzy rule-based model is discussed in Section 3. Section 4 explains modeling for best historical trading using historical price and technical indicators data to make trading signals by the piecewise linear representation (PLR) method, and selecting highly correlated technical indicators by stepwise regression analysis (SRA), forecasting trading signals by SVR, and evaluating trading strategies. Section 5 explains how the TS fuzzy model identifies dynamic trading thresholds for stock trading decisions and compares the profits obtained from TS Fuzzy model with other three approaches. Finally, conclusions and directions for further research are discussed in Section 6.

2. Literature review

In recent years, stock market price prediction has been widely studied and investors are every interested in applying these forecasting models for making stock trading decisions. However, these models are based on the mathematic or statistic models such as regression models [11], generalized autoregressive conditional heteroskedasticity (GARCH) model [3,12]; autoregressive integrated moving average (ARIMA) model [13–15], and the probabilistic model [5]. These models have certain limits when applied in solving the stock trading problem. The stock trading decision recommendations are highly non-linear and non-stationary complex problem neither a mathematic formula nor a statistic model can predict precisely. The recent developments in machine learning and data mining techniques have provided a set of new tools for us to solve the financial stock trading decision problems. Especially, Computation Intelligence (CI) has been used in various areas of forecasting researches and its performance has often been shown better than traditional mathematic models [16,17].

The artificial neural network system (ANNs) has been widely used by different researchers for forecasting modeling due to its universal approximation property. In [18], a hybrid model by integrating K-mean cluster and fuzzy neural network (KFNN) was developed to forecast the future sales of a printed circuit board factory. In [19], the authors discussed the input variables, type of neural network models, performance comparisons for the prediction of foreign exchange rates, stock market index and economic growth. They concluded that most neural network inputs for exchange rate prediction are univariate, while those for stock market index prices and economic growth predictions are multivariate in most cases. A hybrid neuro-genetic system for stock trading forecasting was proposed in [20] and a recurrent neural network (NN) having one hidden layer is used for the prediction model. The neuro-genetic hybrid showed a significant improvement on the average over the buy-and-hold strategy. However, these two studies [19,20] showed that the proposed models have certain limits when the financial data is highly dynamic and volatile. In addition, a simple network structure is easier to apply but a high dimension and complex neural network may take significant time in training and learning the system.

The support vector regression is another approach that has been widely applied in solving the financial forecasting problems in recent studies. SVR constructs a hyper plane in a high-dimensional space and it can precise distinguish objects by a linear or nonlinear kernel function. The non-linear kernel function is suitable for solving the complex financial problem since the trading decisions involved in stock investment are highly non-linear and non-stationary. An interval-valued stock price index series over short and long horizons [21] was forecasted using multi-output support vector regression (MSVR) and their forecast performance compared with a simple trading strategy was very promising. As mentioned in [22], conventional modeling techniques such as the Box–Jenkins autoregressive integrated moving average (ARIMA) are not adequate for stock market price forecasting. They presented a SVR model with chaos-based firefly algorithm for stock market price of NASDAQ. They compared their approach with genetic algorithm-based SVR (SVR-GA), chaotic genetic algorithm-based SVR (SVR-CGA), firefly-based SVR (SVR-FA), ANNs and Adaptive Neuro-Fuzzy Inference Systems (ANFIS). Their proposed model performs the best than other traditional methods. However, they only predicted stock future prices using the optimized SVR and they still did not solve the trading decision problem.

A hybrid intelligent system for constructing mutual fund portfolios was presented which combines Genetic Algorithms (GA), Multi-Model Partitioning (MMP) theory and Extended Kalman Filters (EKF) [23]. They attempted to forecast the market status (inflating or deflating) for the next investment period. A hybrid dynamic neural network for stock market trend analysis was presented [25], they used a simple IIR filter based dynamic neural network (DNN) and an innovative optimized adaptive unscented Kalman filter for forecasting stock price indices of four different Indian stocks. Another intelligent hybrid was proposed in [26], and Adaptive Neural Fuzzy Inference Systems (ANFIS) based on n-period moving average model was applied for TAIEX stock forecasting. They mentioned that the more reasonable and understandable rules "if-then" rules can model the qualitative aspects of human knowledge.

Fuzzy systems are systems with variables that are based on a fuzzy logic algorithm and are one of the key intelligent approaches [10]. Specifically, the fuzzy logic inference system is based on mining techniques to extract crucial rules. The Wang & Mendel's (WM) method has been a famous algorithm as a benchmark method in the field and is among the first methods developed to design fuzzy inference systems from data [27]. In recent years, researchers have been improving the WM method to solve different problems, e.g. feature selection [28], electric load forecasting [29] and due-date assignment problem [30], in further, the concept of fuzzy system can be adopted to enhance the CI models, e.g. regularized least squares fuzzy support vector regression, and handle the financial time series forecasting [31].

The Takagi–Sugeno fuzzy model has gained popularity in time series [32], control [33,34], modeling [35] and classification [36]. One of the reasons that the TS model has wide applications is that many linguistic terms like "Fast" and "High" are used to construct rules having close proximity to human recognition. Meanwhile, the language structure the model uses is arranged in a cause and effect format (an "IF . . . THEN" structure) which can facilitate human understanding and interpretation. The Takagi–Sugeno–Kang (TSK) model is one of the fuzzy models that are enhanced by the divide and conquer concept to solve complex problems where each fuzzy rule describes a part of model behavior and rules. Another reason for the popularity of the TSK model is due to its capability to understand each part of complex models [37]. Considering these merits above, the TS fuzzy model is employed in this research as a tool to solve the nonlinear and non-stationary financial stock market trading decision problems.

3. Takagi–Sugeno fuzzy rule-based model for stock trading decisions

The fuzzy model, proposed by the so-called Sugeno or Takagi–Sugeno–Kang [10,38], employs its input–output data relationships to build a fuzzy rule-based model. Generally, the identification task of the rule-based model is divided into two phases: (i) the structure learning and; (ii) the parameter learning. Structure learning is first of all determine the number of rules from cases, and then assign one or multiple fuzzy rules to input variables on each case according to the degree of membership and a membership function. Each rule selects the participating input variables and determines their membership functions. In the second phase (the parameter learning phase), the premise and the consequent parameters are calculated on the basis of a measure of an output error. The membership function of a fuzzy set A as $A(x)$, $x \in X$. All the fuzzy sets are associated with linear membership functions.

The TS fuzzy model consists of an if-then rule base.

$$\text{If}\, f(x_1\, \text{is}\, A_1, \ldots, x_k\, \text{is}\, A_k) \quad \text{then}\, y = g(x_1, \ldots, x_k) \tag{1}$$

where y denotes the inferred fuzzy value. $x_1,\ldots,x_k$ denotes the input variables in the part of the consequence. $A_1,\ldots,A_k$ denotes fuzzy sets with membership functions representing a fuzzy subspace. f denotes the logical function belonging to which parts of subspace. g denotes the function that implies the value of y when given $x_1,\ldots,x_k$.

Furthermore, the TSK fuzzy model is proposed given the background that the TS fuzzy model has gained an increasing interest in theoretical analysis and applications. The philosophy underlying this approach is to divide the input space into fuzzy sets and sub-models for predicting the system output from a set of simple linear sub-models. Since complex problems like stock trading decisions are solved by combining a set of simple linear sub-models to construct a non-linear model, a major advantage of the TSK fuzzy model is its capability to predict highly complicated problems using a simple small number of rules [39].

The rule antecedents partition a subset of the model variables into fuzzy sets. The consequent of each rule is a simple functional expression. Let us consider a stock trading system with k input variables as technical indicators $x=[x_1,\ldots,x_k]^{\mathrm{T}}$ and a single output as dynamic threshold y. The TSK fuzzy rules for structure learning are described as:

$$\begin{aligned} &\text{Rule}^{(r)}: \text{If}\, x_1\, is\, A_1^{(r)}\, \text{AND}\ldots\text{AND}\, x_k\, \text{is}\, A_k^{(r)} \\ &\text{THEN}\, y^{(r)} = g^{(r)}(x) = a_0^{(r)} + a_1^{(r)}x_1 + \cdots + a_k^{(r)}x_k \end{aligned} \tag{2}$$

where Rule(r) denotes the rth fuzzy sub-model, x_i denotes the ith model inputs as technical indicators, and $y^{(r)}$ denotes the local inferred output of the rth fuzzy sub-model (r). $A_i{}^{(r)}$ denotes the fuzzy antecedent linguistic variables and $a^{(r)}$ denotes the sub-model consequent parameters in the $Rule(r)$ fuzzy sub-model. For example, there are three variables and fuzzy sets so the number of sub-model is 27 (3^3) consisting of TS fuzzy structures. The technical indicators are fuzzed and mapped into multiple sub-models.

In the parameter learning that the value of a variable x_i belongs to a fuzzy set A_i with a truth value given by the membership functions $\mu_{A_i^{(r)}}(x_i) \in \Re \to (0, 1)$. On the other hand, since the global output is from each fuzzy sub-model combination, so the consequent parameters of each sub-model can be calculated using conventional least-squares method. For example, in this research if given an input vector as technical indicators $x=[x_1,\ldots,x_k]^{\mathrm{T}}$ to each function $g(x)$ then each fuzzy sub-model output inferred value have got. The inferred global output as dynamic threshold $\hat{y}$ of the TSK model is calculated by the fuzzy mean-weighted formula from each local inferred $g(x)$. The fuzzy weighted-mean computing is described as:

$$\hat{y} = \frac{\sum_{r=1}^{n} w^{(r)}(x) \times g^{(r)}(x)}{\sum_{r=1}^{n} w^{(r)}(x)} \tag{3}$$

where $w^{(r)}(x)$ denotes the degree of firing rth fuzzy rule, defined by

$$w^{(r)}(x) = \prod_{i=1}^{k} \mu_{A_j^{(r)}}(x_i) \tag{4}$$

where $\mu_{A_i^{(r)}}(x_i) \in \Re \to (0, 1)$ denotes the membership function of the antecedent fuzzy set $A_i^{(r)}$. The weighted-mean computing is very appealing for it facilitates the analysis of TSK fuzzy model in the framework of linear system [40].

A stock market is one of the most noisy and complex environments. In general, the stock price variations are uncertain and ambiguous. Since there is high uncertainty during some periods, it is difficult to make profitable trading decisions. However, a number of errors might occur in predicting the trading signal based on the SVR model. In this research, we want to use an identification approach to control the daily buy or sell timing (trading points), thus the TS fuzzy rule-based system is considered. This identification model can identify the trading point while the SVR model is used to predict the buy or sell trading point. Therefore, we use these buy and sell points as technical indicators (input space) and forecasting trading signals (output space) of the SVR model to build a TS fuzzy identification model on an assumption that these points can be embedded into a fuzzy rule-based structure.

3.1. The identification procedure of the TS fuzzy rule-based model for stock trading decision

In this research, we have used the TS fuzzy model to control the dynamic threshold for making timely decisions in daily stock trading. In Fig. 1, the automatic identification model implements a dynamic threshold technique to identify turning points as buy-sell trading points. The procedure includes the following stages.

- Stage 1: Build structures and parameters of the TS fuzzy model from training data set.
- Stage 2: Predict the dynamic threshold by the TS fuzzy model on a daily basis.
- Stage 3: Identify the turning points.

3.1.1. Stage 1: Build structures and parameters of the TS rule-based model from training data set

The TS fuzzy identification model could contain more rules by deriving an input-output relationship from training data. In Fig. 2, we calculate the actual trading signals (the blue line) and provide the most profitable trading points by stock price transition which has been explained in Section 4.1. The SVR model also provides more precise trading signals (the green line represents the forecasting output) from technical indicators. In other words, the dotted line represents the forecasting trading signals from SVR model whereas the continuous line means the actual trading signal from trading point transition. Therefore, the result indicates that there is a discrepancy between the actual trading signal (the orange dots) by PLR transition and the forecasting trading signal (the red dots) by SVR model output, where the trading point of the latter is not equal to the former one (i.e. the buy-sell trading signal is 0.5). Our task is to revise the trading signal to a new trading threshold for our proposed TS fuzzy model. The triangle dots are the forecasting trading points of SVR, which deviate from the actual trading pints (the square dots) by PLR.

The red dots are called “trading threshold” (the rules is only based on buy and sell knowledge) in building a fuzzy identification

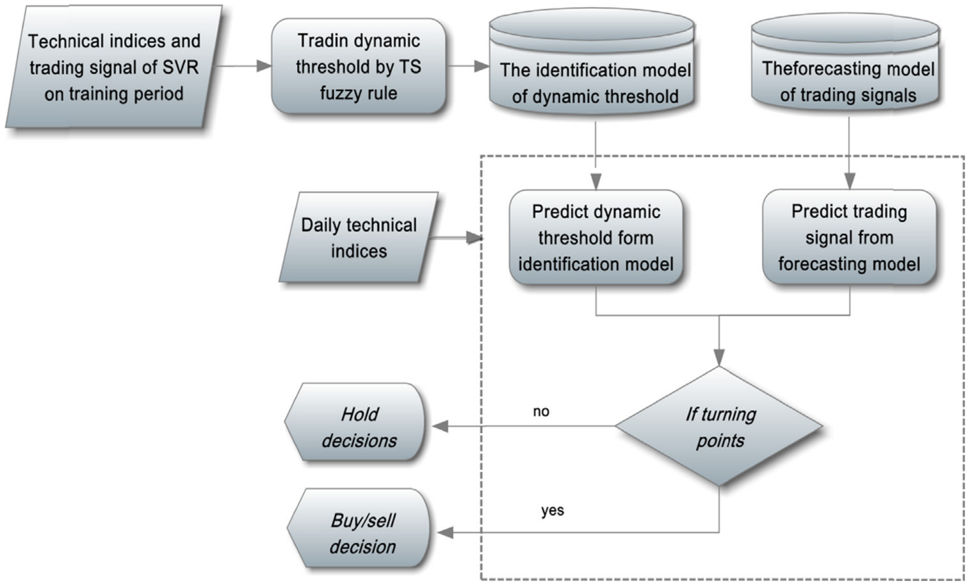

Fig. 1. The framework of trading point identification model based on the TS fuzzy rules.

model for generating trading thresholds. The structures and parameters of the TS fuzzy rule are calculated as follows:

- Step 1: Build the fuzzy structure of TS sub-models

We divide this step into two parts: (i) input data into the fuzzy set and membership degree, and (ii) build multiple TS fuzzy sub-models based on the TS fuzzy structure.

In the part (i) sub-model input space consists from technical indicators. Each input variable was divided into several fuzzy sets. For example, suppose we use the triangular membership function to calculate membership degree and there are three input variables and three fuzzy sets. The input variables $x = \{v1, v2, v3\}$ that each variable is divided into three fuzzy sets respectively (i.e. $v1$ has $A1$, $A2$, and $A3$; $v2$ has $B1$, $B2$ and $B3$; $v3$ has $C1$, $C2$ and $C3$). Therefore each input variable corresponds to three fuzzy sets with three degrees of membership respectively (i.e. md_1, md_2 and md_3). For example, if the value 26.814 of input variable $v1$ falls within the range $\{26.814, 33.62\}$, then this value belongs to three fuzzy sets A and it's fuzzy degree of membership is $\{1, 0, 0\}$ (Fig. 3).An example of three fuzzy sets with triangle membership function for three variables.

The membership degree of ith variable $\mu_{A_i^{(r)}}$ is calculated for Eq. (4) as follows:

$$md_{ij} = \begin{cases} \dfrac{\max(FS_j) - x_i}{\max(FS_j) - \min(FS_j)} & \text{if } j = \text{first of fuzzy set and} \\ & x_i > \min(FS_i), \\ \dfrac{\max(FS_j) - x_i}{(\max(FS_j) - \min(FS_j))/2} & \text{if } j \neq \text{first end of fuzzy set and} \\ & x_i > \min(FS_i) + \dfrac{\max(FS_j) = \min(FS_j)}{2}, \\ \dfrac{x_i - \max(FS_j)}{(\max(FS_j) - \min(FS_j))/2} & \text{if } j \neq \text{first/end of fuzzy set and} \\ & x_i < \min(FS_i) + \dfrac{\max(FS_j) = \min(FS_j)}{2}, \\ \dfrac{\max(FS_j) - x_i}{\max(FS_j) - \min(FS_j)} & \text{if } j = \text{end of fuzzy set and} \\ & x_i > \min(FS_j) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where md_{ij} denotes a value of the fuzzy degree of membership in ith variable and jth fuzzy set. $\max(FS_j)$ and $\min(FS_j)$ denotes a range of minimal and maximal values in jth fuzzy set. S denotes the number of the fuzzy set. x denotes an original value in ith variable. Therefore each value of variable is divided into three fuzzy sets and the degree

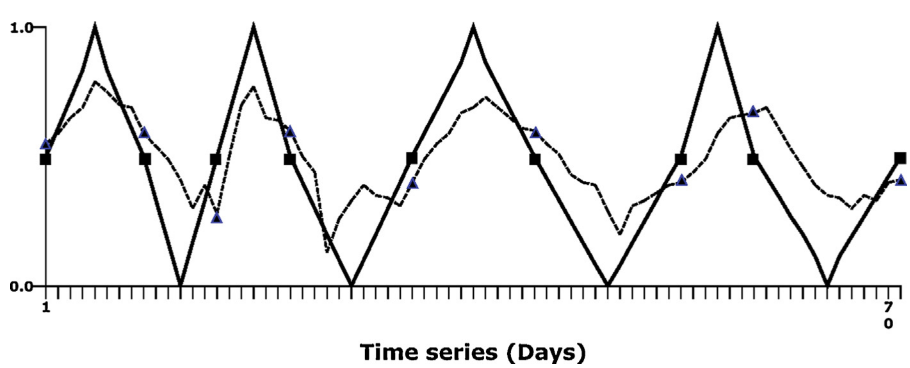

Fig. 2. The difference between the actual trading signals from PLR transition (the black continuous line) and the forecasted trading signals from SVR model (the black dotted line).

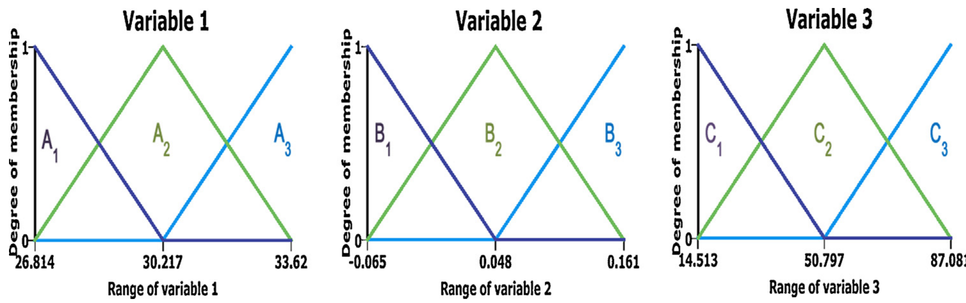

Fig. 3. An example of three fuzzy sets with triangle membership function for three variables.

of membership is represented by three values but in this case of membership function only has two values greater than zero.

Each technical index is transformed into a linguistic fuzzy set by previous part. In the part (ii) the total number of the sub-model of the TS fuzzy structure is n^m where n denotes the number of the fuzzy set, m denotes the number of variables. As shown in Fig. 4, the three variables represent three technical indicators with the values $TI = \{27.5, 0, 80\}$. Each variable corresponds to the fuzzy sets, which are $FS = \{(A_1, A_2, A_3), (B_1, B_2, B_3), (C_1, C_2, C_3)\}$. The degree of membership for the three fuzzy sets under each variable by Eq. (9) is $MD = \{(0.5, 0.5, 0), (0.2, 0.8, 0), (0, 0.75, 0.25)\}$. Consequently, the zero value of the degree of membership is excluded (i.e. in the fuzzy set A_3, B_3, C_1). Moreover, the number of sub-models only corresponds to 8 sub-linear models of the TS fuzzy structure, including the sub-model 2, 3, 5, 6, 11, 12, 14 and 15.

- Step 2: Calculate parameters of each TS fuzzy sub-model

Based on the structure of the TS fuzzy sub-model collected from the previous step, we need to compute parameters of each sub-linear model respectively. Therefore, the ordinary least squares (OLS) approach is adopted to measure each fuzzy TS sub-model regression parameter because the TS fuzzy model has been evolved from a set of simple regression models. The object function of OLS estimator is obtained when the sum of squared residuals (SSR) is minimized as follows:

$$\begin{aligned}\text{Minimize } \left(R^2\right)^{(r)} &= \left(\sum \varepsilon_i^2\right)\\ &= \sum (y_i^{(r)} - (\alpha_{i0}^{(R)} + \beta_{i1}^{(R)} x_{i1} + \cdots + \beta_{ik}^{(r)} x_{ik}))\end{aligned} \quad (6)$$

where $(R^2)^{(r)}$ denotes the sum of squared residuals in the rth linear fuzzy sub-model. R^2 is a measure of the model fit. $y_i^{(r)}$ denotes the actual value as the trading threshold in ith case of rth linear fuzzy sub-model. $\beta_{ik}^{(r)}$ denotes the measure parameters in kth variable of rth linear fuzzy sub-model of ith case. The parameters of each variable of a sub-model are computed for $g^{(r)}(x)$ in Eq. (7).

$$\hat{\beta}_j = \frac{\sum x_{ij} - (1/n)\sum x_{ij} \sum y_i}{\sum x_{ij}^2 - (1/n)\left(\sum x_{ij}\right)}, \hat{\alpha}_0 = \bar{y} - \hat{\beta}\bar{x} \quad (7)$$

3.1.2. Stage 2: Predict the dynamic threshold by the TS fuzzy Model on a daily basis

In this research, we use the dynamic thresholds from the TS fuzzy model to identify turning points as daily trading signals. Eqs. (3) and (4) are employed to configure global thresholds of the TS fuzzy rule-based model. The values of dynamic thresholds are calculated as follows:

- Step 1: Use the membership function of the fuzzy set to analyze the daily technical indicators corresponding to which TS fuzzy sub-model. i.e. $TI = \{27.5, 0, 80\}$, $FS = \{(A1, A2), (B1, B2), (C2, C3)\}$, sub-model $\{2, 3, 5, 6, 11, 12, 14, 15\}$.
- Step 2: Use regression coefficients of the fuzzy TS sub-model to predict local inferred values i.e. from 8 sub-model output.
- Step 3: From Eqs. (3) and (4), we can calculate global values of the dynamic thresholds i.e. 8 local inferred values.

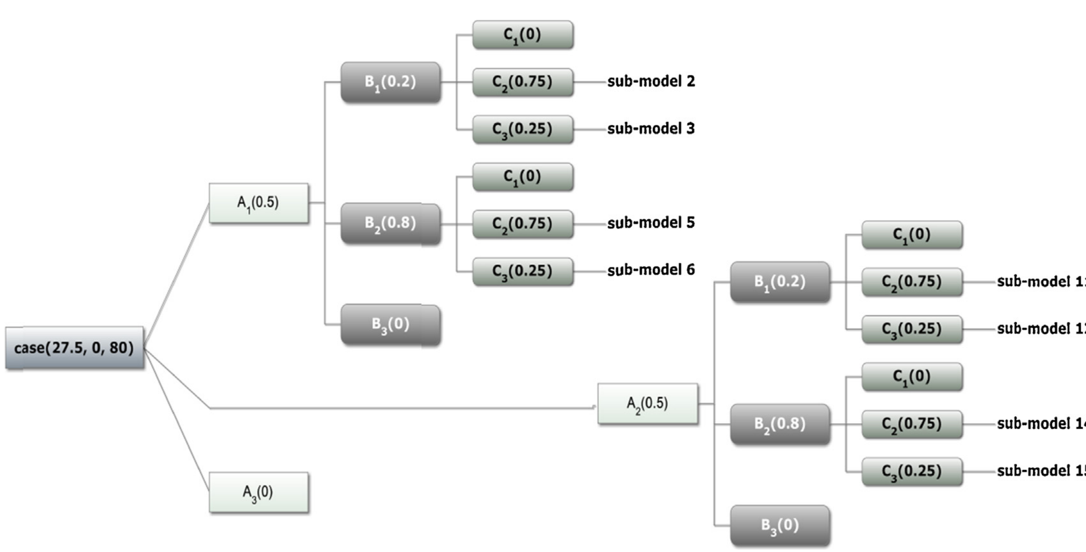

Fig. 4. An example that a case corresponding to 8 sub-models of the TS fuzzy structure.

Follow the 3 steps; each daily technical index can produce daily dynamic thresholds by the TS fuzzy rule-based model to identify turning points.

3.1.3. Stage 3: Identify turning points

In the identification step, the SVR model is used to predict daily trading signals while the TS fuzzy rule-based model is employed to calculate daily dynamic thresholds for identifying trading points. The principles for making trading decisions include:

- If the time series prediction of trading signals based on the SVR model goes up with dynamic thresholds of the TS fuzzy model when the trading signal of the TS fuzzy model intersects with a "buy" trading decision.
- If the time series prediction of trading signals based on the SVR model goes down with dynamic thresholds of the model when the trading signal of the TS fuzzy rule-based model intersects with a "sell" trading decision.
- A "hold" trading decision is made (or do not make any trading decision) when the forecasting trading signal does not intersect with dynamic thresholds.

4. The SVR model for stock trading signals forecasting

How to represent the time series is one of the most important challenges when conducting time series researches. A suitable identification model will greatly increase the easiness and efficiency of time series predication. In this paper, we use fuzzy linguistic variables to represent technical indicators data for identification and knowledge discovery because the degree of complex computing is greatly reduced so that investors can comprehend linguistic variables more easily [41].

First we need to model the trading signal by using the non-linear learning ability of the SVR approach to identify the trading point for making profits. A piecewise linear representation method is integrated with a SVR method to predict the stock trading signal. Then we use a PLR technique to decompose the historical data into various segments. The endpoints of these segments are the buy-sell points. These segments are converted into a continuous stream of trading signals for model learning. This model can provide crucial variables for better identification, therefore a SRA approach has been proposed to select high effect technical indicators.

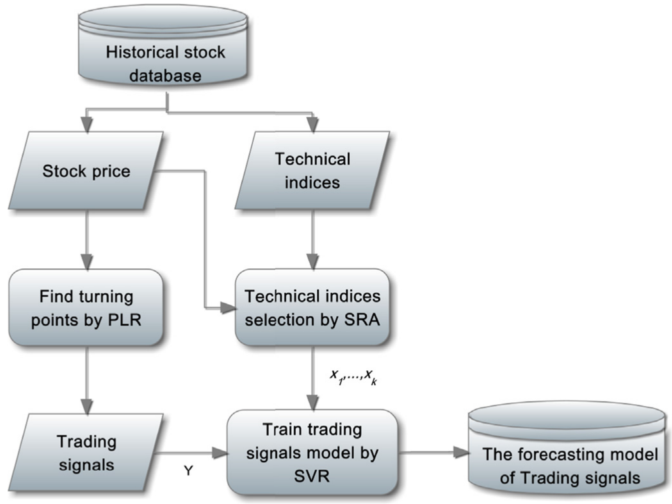

Fig. 5. The framework of the stock trading forecasting system using TS fuzzy model.

The framework of modeling the trading system is illustrated in Fig. 5. Technical indicators of stock will be selected using a SRA technique. The PLR approach is applied to decompose the historical stock data into different segments for building a lot of buy-sell trading points. Each segment is converted to a trading signal where the input will be the selected technical index and the output is the trading signal generated from PLR approach. The proposed system will be trained using these trading signals to build up a trading signal and a dynamic forecasting model with threshold properties. A detailed procedure of the modeling processes is explained in the following sections.

4.1. Modeling the trading by PLR

The output of SVR is the trading signal. The value of the trading signal is calculated from the segments obtained from the PLR approach which is described as follows:

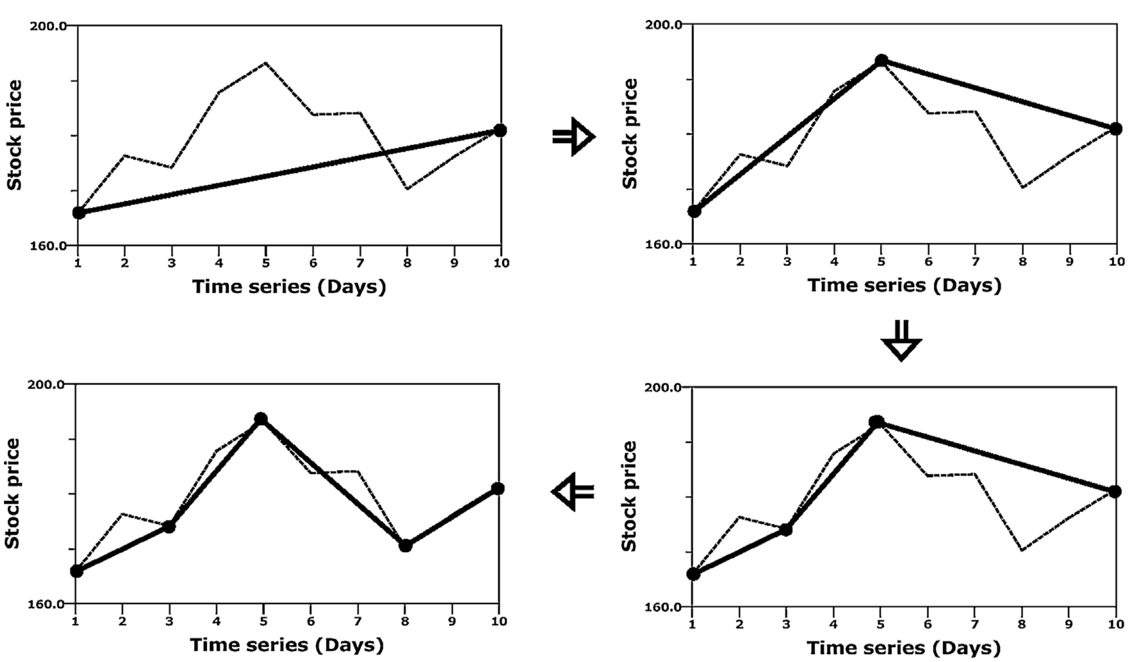

Fig. 6. Buy-sell trading segments (black line) determiner by PLR approach in stock price (dashed line).

The PLR approach to determine appropriate trading points: Typically, investors make trading decisions by observing the variations in technical indicators. However, it is not easy for every investor to comprehend the meaning of variations in technical index. In this paper, we use the PLR approach to find high and low price points in the piecewise fitting as shown in Fig. 6. Below are the steps of PLR:

- Step 1: Collect the closing prices of a stock from training data. Suppose the time series of closing prices are denoted by P_1, P_2,. . .,P_n. The first point (P_1) connected to the last point (P_n) represent a segment for next step.
- Step 2: Set a price threshold to guarantee a return. Then find the point (P_i) with the greatest distance from this line (P_1, P_n). The (P_1, P_n) segment is divided into two parts where the segment 1 is P_1 to P_i and segment 2 is P_i to P_n. The criterion is satisfied only when the distance of pi with line (P_1, P_n) is greater than the threshold.
- Step 3: Repeat step 2 until the (P_1, P_n) segment cannot reach the price threshold.

The PLR approach provides a lot of segments with low and high price points. If the segment of stock price is set as an output for the SVR model, the turning points will be the highest and lowest stock price in a specific segment except first and last segments in training data. Therefore many segments belonging to an uptrend or downtrend have been generated. During dynamic usage of this approach for SVR model, these segments cannot be used directly because our target is turning points instead of stock prices. Therefore these values have to be converted into trading signals according to trend segment.

Trading signals transformation from trading points: The segments of stock price are a combination of many uptrend and downtrend segments. A mathematical formula is used to convert these segments of stock price into trading signals. This transformation can provide continuous values which are more suitable for the data learning in the SVR model. The transformation formulas are defined as follows:

For the uptrend segment:

$$T_i = \begin{cases} 0.5 - (i-1)/L & \text{if } i <= L/2 \\ i/L - 0.5 & \text{if } i > L/2 \end{cases} \tag{8}$$

For the downtrend segment:

$$T_i = \begin{cases} 0.5 + (i-1)/L & \text{if } i <= L/2 \\ 1.5 - i/L & \text{if } i > L/2 \end{cases} \tag{9}$$

where L denotes the length of the uptrend or downtrend series and T_i denotes the trading signal of ith day. The range of trading signal is 0.0–1.0. The turning points are the points at which the trading signal intersects the threshold. The threshold is equal to 0.5. If the down slope of the trading signal is lower than the intersection point, the intersection point is a sell point. On the contrary, if the down slope of the trading signal is over the intersection point, the intersection point is a buy point.

4.2. Modeling the features space using technical indicators as inputs

In this paper, we have considered 28 variables (technical indicators) as listed in Table 1. These variables are correlated with variations in stock prices to some degree. The quantity of correlation varies for different variables. Rather than using all the 28 variables, we select the variables with a greater correlation than a user-defined threshold. The variable selection is done by stepwise regression analysis. We apply the SRA approach to determine the technical indicators affecting the stock price. This is accomplished by selecting the variables repeatedly. The steps of the SRA approach are described as follows:

- Step 1: Find the correlation coefficient r for each technical index $v_1, v_2, \ldots, v_n$ with the stock price y in a stock. These correlation coefficients are stored in a matrix called correlation matrix.
- Step 2: The technical index with largest R^2 value is selected from the correlation matrix. Let the technical index be v_i. Derive a regression model between the stock price and technical index i.e. $\hat{y} = f(v_i)$.
- Step 3: Calculate the partial F-value of other technical indicators. Compare the R^2 value of the remaining technical indicators and select the technical index with highest correlation coefficient. Let the technical index be v_j. Derive another regression model i.e. $\hat{y} = f(v_i, v_j)$.
- Step 4: Calculate the partial F-value of the original data for the technical index v_j. If the F-value is smaller than the user defined threshold, v_j is removed from the regression model since it does not affect the stock price significantly.

Table 1
Technical indicators used as input variables.

Technical index	Variation of n days	Explanation
Moving Average (MA)	5MA,6MA,10MA, 20MA	Moving averages are used to emphasize the direction of a trend and smooth out price and volume fluctuations that can confuse interpretation
Bias (BIAS)	5BIAS, 10BIAS	The difference between the closing value and moving average line, which uses the stock price nature of returning back to average price to analyze the stock market
Relative Strength Index (RSI)	6RSI, 12RSI	RSI compares the magnitude of recent gains to recent losses in an attempt to determine overbought and oversold conditions of an asset
Nine Days Stochastic Line (K, D)	9K, 9D, 9KD	The stochastic line K and line D are used to determine the signals of over-purchasing, over-selling, or deviation
Moving Average Convergence and Divergence (MACD)	9 MACD	MACD shows the difference between a fast and slow exponential moving average (EMA) of closing prices. Fast means a short-period average, and slow means a long period one
Williams %R (pronounced "percent R")	12W%R	Williams %R is usually plotted using negative values. For the purpose of analysis and discussion, simply ignore the negative symbols. it is best to wait for the security's price to change direction before placing your trades
Transaction Volume (TV)	TV, 5TV, 10TV, 15TV	Transaction volume is a basic yet very important element of market timing strategy. Volume provides clues as to the intensity of a given price move
Differences of Technical Index (Δ)	Δ5MA, Δ6MA, Δ10MA, Δ5BIAS, Δ10BIAS, Δ6RSI, Δ12RSI, Δ12W%R, Δ9K, Δ9D, Δ9 MACD	Differences of technical index between the day and next day

- Step 5: Repeat step 3 to step 4. If the *F*-value of variable is more than the user-defined threshold; the variable should be added to the model otherwise it should be removed.

In this paper, the range of the input variables of TS and SVR model should be between 0 and 1. Hence, the selected technical indicators are normalized as follows:

$$\mathrm{Normal}\,(x_{ij}) = \frac{x_{ij} - \mathrm{Min}(x_i)}{\mathrm{Max}(x_i) - \mathrm{Min}(x_i)} \quad i = 1, \ldots, n; \quad j = 1, \ldots, m;\ n \text{ and } m \in \Re \tag{10}$$

where Normal(x_{ij}) denotes the normalized value of jth data point of ith technical index. Max(x_i) denotes the maximum value of ith technical index. Min(x_i)denotes the minimum value of ith technical index. x_{ij} denotes original value of jth data point of ith technical index. n and m denotes the total number of technical indicators and data points respectively.

4.3. Forecasting trading signals generated by SVR model

Support vector regression will be applied as a machine learning model to extract the hidden knowledge in the historic stock database. The single output is the trading signal and the multiple input features are technical indicators for SVR model. SVR learning model transforms multiple features into high multidimensional feature space and the transformed feature space can be mapped into a hyper-plane space to determine correct signals based on those support vector points. On the kernel function selection, we try to use RBF, polynomial, and sigmoid functions to generate better performance for the SVR model because the stock market is a very complicated non-linear environment. In our study, the threshold of PLR leads different result of SVR training, therefore, there are various PLR threshold value been testing in experimental results. For each PLR, the common combination for RBF and polynomial kernel includes cost, C, epsilon, ε and γ is selected by the grid search with exponentially growing sequences. C ranges from 10^{-3} to 10^{3}. ε starts from 10^{-4} to 10^{-1} and γ is fixed as 0. In "polynomial", the degree of polynomial function starts from 2^{-9} to 2^{-4}. The same range is also used as gamma in RBF kernel. This study chooses these famous kernel functions and the respective parameter setting of SVR with minimal root mean square error (RMSE) between actual and trained output.

Since the SVR approach possesses high learning capability and accuracy in predicting continuous signals for building hidden knowledge among trading signals and technical indicators, it is a widely used tool for predicating the trading signals.

4.4. Stock trading practices and strategies

In stock markets, investors profit from the movement of stock prices. Investors buy stock to hold long time and profit from price increases. Investors have to pay transaction fees for the trading processes each time they buy or sell stock. Given different fees might apply in different countries; our trading strategy uses the proposed model to make trading decisions. Therefore, the identification mechanism of the TS fuzzy approach sets a highly stable control threshold to identify the trading signals from the SVR model.

4.5. Evaluation of trading decisions

In this research, the trading point (buy and sell timing) is identified by the TS fuzzy model based on the forecasting trading signal of SVR. In the experimental section, we also use various predication frameworks to the generated profiting trading points and compare their performances. The profit in each different framework is calculated as follows:

$$\mathrm{profits} = \sum_{i=1}^{k} \left\{ \frac{(1-a-b) \times p_{S_i} - (1+a) \times p_{B_i}}{(1+a) \times p_{B_i}} \right\} \times 100\% \tag{11}$$

where C is the total amount of money to be invested at the beginning as well as the capital of money, a refers to the tax rate of ith transaction, b refers to the handling charge of ith transaction, k is the total number of transaction, p_{Si} is the selling price of the ith transaction and p_{Bi} is the buying price of ith transaction.

5. Experimental results in the US stock market

In this research, we have selected six stocks and one index from the US stock market to compare the profit achieved by various trading models, the stocks including Apple Inc. (APPLE), The Boeing Company (BA), Caterpillar Inc. (CAT), Johnson & Johnson (JNJ), Exxon Mobil Corporation (XOM), Verizon Communications Inc. (VZ) and the index including S&P 500 (S&P500). Among all the stocks and index, 253 data points were collected for the training period from 1/2/2008 (mm/dd/yy) to 12/31/2008 while 124 data points were used for the testing period from 1/2/2009 to 6/30/2009. The number of tested fuzzy sets for the input variables varied from 2 to 5. Also, the PLR threshold values ranged from 0.5σ to 1σ by multiply the coefficient (from 0.5 to 1) to the standard deviation (σ) of historical stock price of each stock. Finally, the combination of the number of fuzzy sets and PLR threshold generating most profit was selected.

Generally, the lower the values of PLR thresholds, the more peaks and troughs there are. Due to the tax rates and handling charges, it is not sufficient to make profits by selling at peak and buying at trough. Thus, profit making for peaks and troughs were selected and converted into the fuzzy rules. As the number of fuzzy rules increases, more troughs and peaks are available to generate thresholds resulting in a higher profit for most of the cases. The PLR threshold ranges from 0.5σ to 1σ based on stock price. The best PLR thresholds and number of fuzzy rules in training data are shown in Table 2.

Taking APPLE stock as an example to describe the trading rules used in TS fuzzy approach. Fig. 7 shows that while threshold of PLR is set as 0.5σ, 12 trading points can be found during 1/2/2008 to 12/31/2008 (with two additional trading points, starting and ending day, the total trading points will be 14). The black line, black dot line and square points denote the trading signal by PLR, stock price and trading points respectively. The main idea in this paper is to find the fuzzy rule calculated by PLR. As through the PLR which has are 12 fuzzy rules for train TS fuzzy model.

The threshold of the PLR testing shows that both a stock with a large standard deviation/a low threshold and a stock with a low standard deviation/a high threshold can makes high profits. However, if a high value of threshold greater than 0.7 is selected, the failure rate of the identification model will be lower in most

Table 2
The best parameters in various stocks corresponding to different PLR threshold values.

Parameter	APPLE	BA	CAT	JNJ	XOM	VZ	S&P500
Threshold	0.5σ	0.7σ	0.5σ	0.7σ	0.9σ	0.8σ	0.7σ
Fuzzy rule	14	4	15	16	19	7	3

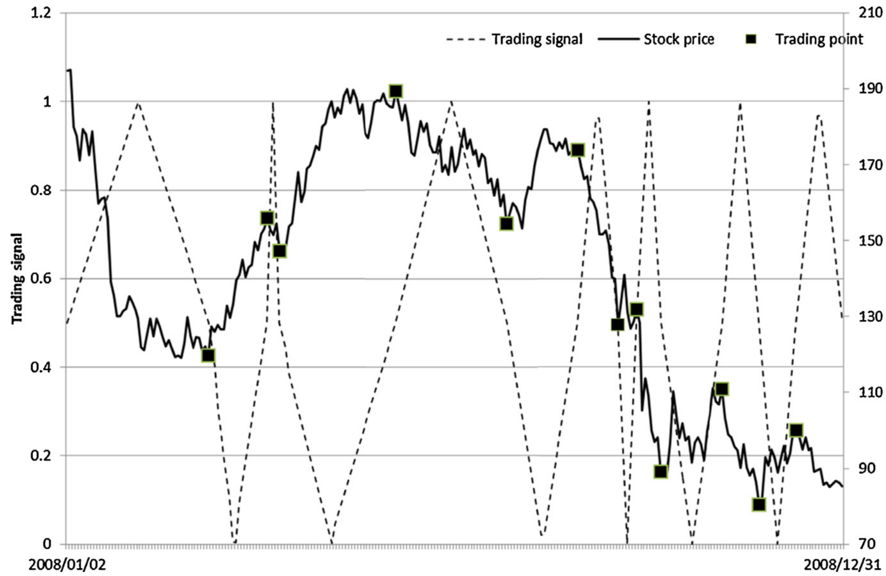

Fig. 7. The trading strategy and buy/sell decision in APPLE stock.

Table 3
Technical indicators selection by SRA.

Stock	Selected technical indicators	R^2	F-value
APPLE	5BIAS, 6MA and 12W%R	0.9990	83,031*
BA	5BIAS, 6MA, Δ5MA, Δ6MA and Δ9D	0.9993	74,678*
CAT	5BIAS, 6MA, 6RSI, Δ5MA and Δ6MA	0.9993	73,977*
JNJ	5BIAS, 6MA, 6RSI, Δ9K, Δ10MA and 10TV	0.9999	28,889*
XOM	5BIAS, 6MA, 6RSI, Δ6MA, Δ9D, 5TV and 15TV	0.9993	50,534*
VZ	5BIAS, 6MA, 9K, 12RSI, 9MACD, Δ5MA, Δ6MA, Δ6RSI, Δ9K, Δ9D, Δ9MACD, 9KD, Δ10MA and 5TV	0.9997	57,522*
S&P500	5BIAS, 6MA, Δ5MA, Δ6MA, Δ5BIAS, Δ9K, Δ9D, Δ12%R, 10MA, TV and 5TV	0.9997	81,453*

* Notes the p-value <0.025.

stock cases. Table 3 shows that how many technical indicators are selected by SRA. The 5BIAS and 6 MA are selected on all stocks.

In this research, we have compared our identification model of TS fuzzy rule-based approach with three other identification models developed in the past. The PLR-SVR model used a fixed threshold to determine trading points and the threshold is decided according to the average value plus/reduce one standard deviation of forecasted trading signal from the historical training data. The PLR-BPN model used neural networks in combination with PLR and exponential smoothing to determine the trading points. The statistical parameters such as moving average, rate of change and trading volumes to determine the buy-sell points and generated profit. Each identification rule provides trading points for each stock so the best profits of the three identification rules are shown in Table 4. The results turn out that our proposed TS fuzzy model generates the greatest returns for the six stocks and one index (i.e., #1, 2, 3, 5, 6 and 7) and outperforms other three models. The average profit rate of these seven stocks is 18.08% using the TS Fuzzy model whereas the average profit rate generated by other models like PLR-SVR, PLR-BPN and statistical model are −0.041%, 12.32% and 9.65% respectively. The buy and sell points obtained from the TS fuzzy identification model in each stock are shown in Fig. 8. The black square represents the buy point and the black triangle represents the sell point using a trading strategy to determine turning points as explained in experimental.

The case analysis in this paper shows that the TS fuzzy rule-based model can be used to identify the turning points of stocks trading signals in different stocks at the highest degree of profit. Our identification model based on the TS approach is an automatic trading point recognition technique from TS fuzzy sub-models inferred. Therefore it is essential to obtain a clear data set by linguistic variables, which can represent all possible stock market fluctuations. In an uptrend market, our identification model can generate the highest profit most of the time. Therefore, the combination of SVR and

Table 4
Comparison of profit obtained by various models.

No.	Stock name	TS Fuzzy model	PLR-SVR model	PLR-BPN model	Statistical model
1	APPLE	45.16%	−21.73%	12.97%	20.50%
2	BA	32.66%	8.52%	17.50%	20.03%
3	CAT	17.01%	0.00%	9.36%	24.83%
4	JNJ	6.31%	−1.22%	16.88%	0.00%
5	XOM	14.78%	0.00%	−1.99%	−7.65%
6	VZ	−1.24%	−2.59%	27.72%	0.00%
7	S&P500	11.9%	−11.58%	3.77%	9.81%
Average		18.08%	−0.041%	12.32%	9.65%

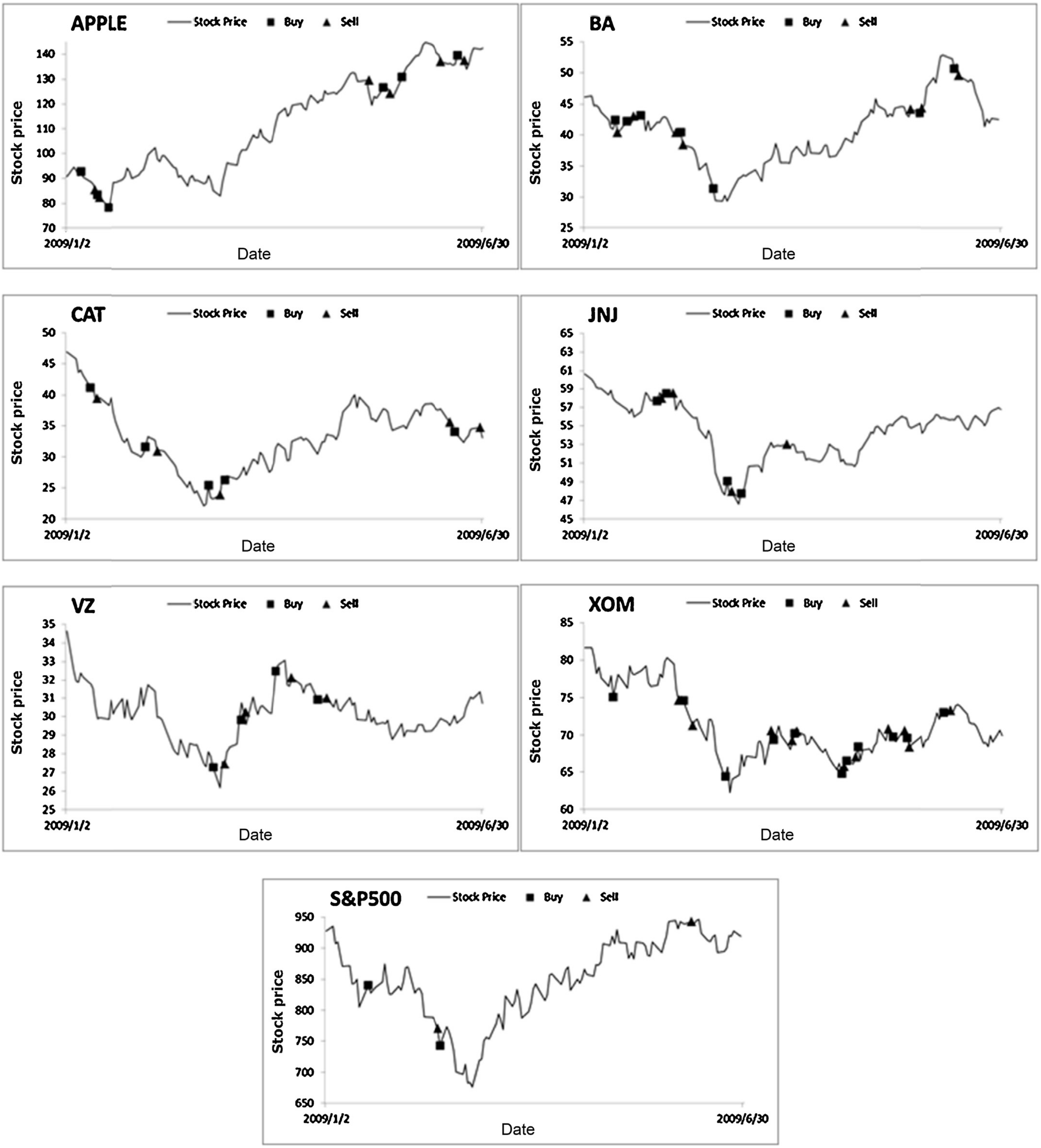

Fig. 8. Buy and sell points of 7 US stocks from our proposed TS fuzzy identification model (black square point: buy point; black triangle point: sell point).

TS fuzzy model is an excellent forecasting tool to predict trading signals and identify trading thresholds. In addition, implementing more sophisticated fuzzy models should further enhance the predictive capacity of the proposed fuzzy model.

Beside the Sharpe ratio (*SR*) and modified Sharpe ratio (m*SR*) estimators, we use annual log profit in excess of the risk-free rate (Treasury bill rate is very close to 4% in year 2009, U.S.). Table 5 provides some summary statistics on SR and m*SR*. In addition, the

Table 5
Summary statistics for monthly log profits in excess of the risk-free rate: mean (μ), standard deviation (θ) Sharpe ratio (*SR*), and modified Sharpe ratio (m*SR*).

Stock	μ	θ	*SR*	m*SR*
TS Fuzzy model	0.1408	0.1585	0.8883	2.4718
PLR-SVR model	−0.0809	0.0975	−0.8294	−0.2846
PLR-BPN model	0.0832	0.0976	0.8518	1.1401
Statistical model	0.0565	0.1253	0.4507	0.3912

Table 6
χ^2 test on the trading signals and forecasting trading signals.

	APPLE	BA	CAT	JNJ	XOM	VZ	S&P500
e	0.2126	0.0207	0.0096	0.1264	0.0932	0.0685	0.0648
R^2	0.4614	0.9949	0.9989	0.823	0.8988	0.9446	0.9284
χ^2	22.2610*	0.5688*	0.1226*	11.2331*	5.0011*	3.7738*	1.6713*

* p-value >0.05, accept the null hypothesis.

Table 7
The results on TS fuzzy model by winning and losing trade performance.

Stock	APPLE	BA	CAT	JNJ	XOM	VZ	S&P500	Total number
Winning trade	2	4	2	2	8	3	1	22
Losing trade	4	3	3	2	3	1	1	17

m*SR* estimator provides the corresponding 98% confidence interval with variance stabilizing transformation. According to SR and m*SR* estimators, all of three models are positive ratio that it mean all model can capture positive profit better than risk-free rate. Our proposed model as TS fuzzy has highest SR and m*SR* compare to other three models.

In addition, we perform some statistical tests including root mean squared error (e), R-square (R^2), Chi-square tests (χ^2), winning trade and losing trade. Table 6 provides some relevant summary statistics to check the over fitting of trading signals of our proposed model in training data. The TS fuzzy model is very effective since all stocks and index are significant. The degree of freedom for all stocks is 253.

Table 7 shows the number of winning (positive return) and losing (negative return) to check trading performance in testing data. In our proposed model, the total number of trading is 22 wins and 17 losses so that the TS fuzzy model can capture 56.41% winning trade and total average profit is 18.08%.

6. Conclusions

This paper has presented a novel approach to identify stock trading threshold by the TS fuzzy model based on SVR's forecasting trading signals. Also, PLR approach is applied to decompose the historical stock data into different segments for providing a set of best buy-sell segments. The identification of the TS fuzzy model is implemented by combining multiple fuzzy sub-models with the SVR method in training data. The experimental results demonstrate its high efficiency to be able to conduct time series analysis in complex stock markets. The buy or sell trading decisions can provide significant trading profits for investors as shown from the experimental results. The dynamic threshold of trading point determination by TS fuzzy model is better than using fixed threshold by PLR-SVR. The fixed threshold rapid to determine trading points but it not has sensitive detection for a complex stock price fluctuation.

The proposed identification model of the TS fuzzy system has the significant advantage of not requiring any human process in the decision process. Intensive experimental results using data from the U.S. stock and S&P500 index have shown that the TS fuzzy method based on trading signals provides better results than the conventional linear regression approach, artificial neural network and piecewise linear representation with support vector regression in terms of trading point predications. The future research will focus on improving the knowledge learning process in order to provide a more effective model. That will include (1) a sliding window for training our model, (2) different feature selection techniques for capturing more significant data and (3) a different knowledge learning model for improving the forecasting performance.

References

[1] P.C. Chang, C.H. Liu, A TSK type fuzzy rule based system for stock price prediction, Expert Syst. Appl. 34 (2008) 135–144.
[3] F.X. Diebold, R.S. Mariano, Comparing predictive accuracy, J. Bus. Econ. Stat. 20 (1) (2002) 134–144.
[4] W. Leigh, R. Purvis, J.M. Ragusa, Forecasting the NYSE composite index with technical analysis, pattern recognizer, neural network, and genetic algorithm: a case study in romantic decision support, Decis. Support Syst. 32 (4) (2002) 361–377.
[5] D. Bao, Z. Yang, Intelligent stock trading system by turning point confirming and probabilistic reasoning, Expert Syst. Appl. 34 (1) (2008) 620–627.
[6] P.C. Chang, C.Y. Fan, C.H. Liu, Integrating a piecewise linear representation method and a neural network model for stock trading points prediction, IEEE Trans. Syst. Man Cybern. Part C: Appl. Rev. 39 (2009) 80–92.
[7] V. Vapnik, The Nature of Statistical Learning Theory, Springer–Verlag, New York, 1995.
[8] B.J. Chen, M.W. Chang, C.J. Lin, A study on EUNITE competition 2001, IEEE Trans. Power Syst. 19 (2004) 1821–1830.
[9] P.C. Chang, T.W. Liao, L.L. Lin, C.Y. Fan, A dynamic threshold decision system for stock trading signal detection, Appl. Soft Comput. 11 (5) (2011) 2998–4010.
[10] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE Trans. Syst. Man Cybern. 15 (1) (1985) 116–132.
[11] S. Busuttil, Y. Kalnishkan, Weighted kernel regression for predicting changing dependencies, Lect. Notes Comput. Sci. 4701 (2007) 535–542.
[12] R. Gencay, Linear, non-linear and essential foreign exchange rate prediction with simple technical trading rules, J. Int. Econ. 47 (1) (1999) 91–107.
[13] G. Box, G. Jenkins, G. Reinsel, Time Series Analysis Forecasting and Control, second ed., John Wiley & Sons, New Jersey, 2008.
[14] A. Timmermann, C.W.J. Granger, Efficient market hypothesis and forecasting, Int. J. Forecast. 20 (1) (2004) 15–27.
[15] O. Valenzuela, I. Rojas, F. Rojas, H. Pomares, L.J. Herrera, A. Guillen, L. Marquez, M. Pasadas, Hybridization of intelligent techniques and ARIMA models for time series prediction, Fuzzy Sets Syst. 159 (7) (2008) 821–845.
[16] P.F. Pai, System reliability forecasting by support vector machines with genetic algorithms, Math. Comput. Model. 43 (3/4) (2006) 262–274.
[17] Z. Zhu, H. He, J.A. Starzyk, C. Tseng, Self-organizing learning array and its application to economic and financial problems, Inf. Sci. 177 (5) (2007) 1180–1192.
[18] P.C. Chang, C.H. Liu, C.Y. Fan, Data clustering and fuzzy neural network for sales forecasting: a case study in printed circuit board industry, Knowl.-Based Syst. 22 (5) (2009) 344–355.
[19] W. Huang, K.K. Lai, Y. Nakamori, S. Wang, L. Yu, Neural networks in finance and economics forecasting, Int. J. Inf. Technol. Decis. Mak. 6 (1) (2007) 113–140.
[20] Y.K. Kwon, B.R. Moon, A hybrid neurogenetic approach for stock forecasting, IEEE Trans. Neural Netw. 18 (3) (2007) 851–864.
[21] T. Xiong, Y. Bao, Z. Hu, Multiple-output support vector regression with a firefly algorithm for interval-valued stock price index forecasting, Knowl.-Based Syst. 55 (2014) 87–100.
[22] A. Kazem, E. Sharifi, F.K. Hussain, M. Saberi, O.K. Hussain, Support vector regression with chaos-based firefly algorithm for stock market price forecasting, Appl. Soft Comput. 13 (2) (2013) 947–958.
[23] N. Spanoudakis, K. Pendaraki, G. Beligiannis, Portfolio construction using argumentation and hybrid evolutionary forecasting algorithms, Int. J. Hybrid Intell. Syst. (IJHIS) 6 (4) (2009) 231–243.
[25] R. Bisoi, P.K. Dash, A hybrid evolutionary dynamic neural network for stock market trend analysis and prediction using unscented Kalman filter, Appl. Soft Comput. 19 (2014) 41–56.
[26] L.Y. Wei, C.H. Cheng, H.H. Wu, A hybrid ANFIS based on n-period moving average model to forecast TAIEX stock, Appl. Soft Comput. 19 (2014) 86–92.
[27] L.X. Wang, J.M. Mendel, Generating fuzzy rules by learning from examples, IEEE Trans. Syst. Man Cybern. 22 (6) (1992) 1414–1427.
[28] M.E. Cintra, C.H. Arruda, M.C. Monard, Fuzzy feature subset selection using the Wang & Mendel method, in: Proceedings of the 8th International Conf. on Hybrid Intelligent Systems (HIS 2008), Barcelona, 2008, pp. 590–595.
[29] X. Yang, J. Yuan, J. Yuan, H. Mao, An improved WM method based on PSO for electric load forecasting, Expert Syst. Appl. 37 (12) (2010) 8036–8041.
[30] P.C. Chang, J.C. Hieh, T.W. Liao, Evolving fuzzy rules for due-date assignment problem in semiconductor manufacturing factory, J. Intell. Manuf. 16 (4/5) (2005) 549–557.
[31] R. Khemchandani, S. Jayadeva, Chandra, Regularized least squares fuzzy support vector regression for financial time series forecasting, Expert Syst. Appl. 36 (1) (2009) 132–138.
[32] X. Xie, L. Lin, S. Zhong, Process Takagi–Sugeno model: a novel approach for handling continuous input and output functions and its application to time series prediction, Knowl.-Based Syst. 63 (2014) 46–58.
[33] P. Nikdel, M. Hosseinpour, M.A. Badamchizadeh, M.A. Akbari, Improved Takagi–Sugeno fuzzy model-based control of flexible joint robot via Hybrid-Taguchi genetic algorithm, Eng. Appl. Artif. Intell. 33 (2014) 12–20.
[34] W.H. Ho, S.H. Chen, J.H. Chou, Optimal control of Takagi–Sugeno fuzzy-model-based systems representing dynamic ship positioning systems, Appl. Soft Comput. 13 (7) (2013) 3197–3210.
[35] K. Guelton, T. Bouarar, N. Manamanni, Robust dynamic output feedback fuzzy Lyapunov stabilization of Takagi–Sugeno systems – a descriptor redundancy approach, Fuzzy Sets Syst. 160 (19) (2009) 2796–2811.

[36] Z. Zhang, H. Zhou, S. Liu, P.D.B. Harrington, An application of Takagi–Sugeno fuzzy system to the classification of cancer patients based on elemental contents in serum samples, Chemometr. Intell. Lab. Syst. 82 (1/2) (2006) 294–299.
[37] X. Guo, D. Wang, W. Liu, A Takagi–Sugeno–Kang (TSK) power model using compressed-sensing sampling, Chin. J. Chem. Eng. 20 (6) (2012) 1161–1166.
[38] M. Sugeno, G. Kang, Structure identification of fuzzy model, Fuzzy Sets Syst. 28 (1986) 329–346.
[39] R. Babuska, H.B. Verbruggen, Constructing fuzzy models by product space clustering, in: H. Hellendoorn, D. Driankov (Eds.), Fuzzy Model Identification, Selected Approaches, Springer-Verlag, New York, 1997, pp. 53–90.
[40] R.K. Lai, C.Y. Fan, W.H. Huang, P.C. Chang, Evolving and clustering fuzzy decision tree for financial time series data forecasting, Expert Syst. Appl. 36 (2) (2009) 3761–3773.
[41] K.Y. Kwon, R.J. Kish, Technical trading strategies and return predictability: NYSE, Appl. Financ. Econ. 12 (9) (2002) 639–653.