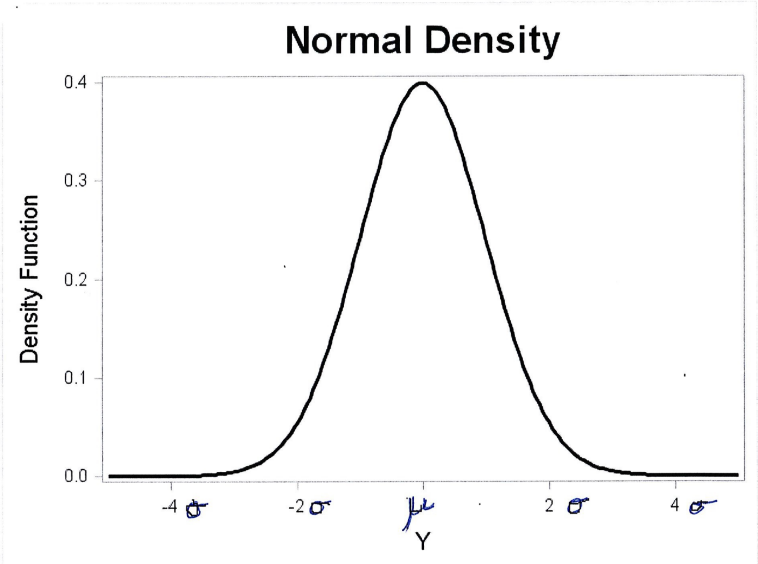
# STAT 500

## Normal Distribution Information

- Definition: A continuous random variable Y has a normal distribution if the density function of the random variable is of the form:

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{y-\mu}{\sigma}\right)^2} \qquad -\infty < y < \infty$$

- The parameters of the normal distribution are the mean $\mu$ and the variance $\sigma^2$. By definition, $E(Y) = \mu$ and $V(Y) = E[(Y-\mu)^2] = \sigma^2$
- Normal distributions are usually denoted as $Y \sim N(\mu, \sigma^2)$. Note: some resources use $\sigma$ instead of $\sigma^2$.
- Here is a picture of the density function of a normal distribution. The mean parameter $\mu$ controls the center of the distribution and the variance parameter $\sigma^2$ controls the variability of the distribution.




- A standard normal distribution has a mean $\mu = 0$ and variance $\sigma^2 = 1$. This is denoted as $Z \sim N(0,1)$
- Results for standard normal distributions.
  - If $Z \sim N(0,1)$ then $Y = (\mu + Z * \sigma) \sim N(\mu, \sigma^2)$
  - If $Y \sim N(\mu, \sigma^2)$ then $Z = \frac{Y-\mu}{\sigma} \sim N(0,1)$

- Here is a picture of the density function of a standard normal distribution.