Stat 500 Solutions to Homework #4 Fall 2020

Common mistakes:

- For one-sided tests, please do not state the conclusion as "conclude that A and B are different". This is so obvious a mistake.
- In SAS output, the results for Folded F-test is a separate table, not the one in the ANOVA table.That F statistic is the F test for model parameters, which you will learn later.

1. 5 points

   With $Y \sim Poisson(\mu)$, we have $Var(Y) = \mu$. For $X = g(Y) = \sqrt{Y}$, $g(Y)' = \frac{1}{2\sqrt{Y}}$, apply delta method, the variance of X is

   $$Var(X) = [g(\mu)']^2 Var(Y) = \frac{1}{4\mu}\mu = \frac{1}{4},$$

   that is, the variance of x is a constant after transformation. (Page 214 - page 219 of lecture notes)

2. 25 points

   (a) i. The pooled two-sample t-test produces a value of a t-statistic equal to -3.14 with 120 degrees of freedom, and the corresponding p-value is 0.0011. Because the p-value is small (less than 0.01), there is significant evidence that infection with tubercle Bacilli tending to decrease survival times.

   ii. The permutation p-value is 0.0011 (with seed = 1234567) for t-tests constructed from random permutations of the observations into the two treatment groups. Because the p-value is quite small, the data provide significant evidence that infection with tubercle Bacilli decreases the mean survival time relative to the controls. And this result closely agrees with the t-test based on a normal model with homogeneous variances that was considered in part a(i).

   (b)

   i.

Treatment	Rank
Bacilli	3189.5
Control	4313.5

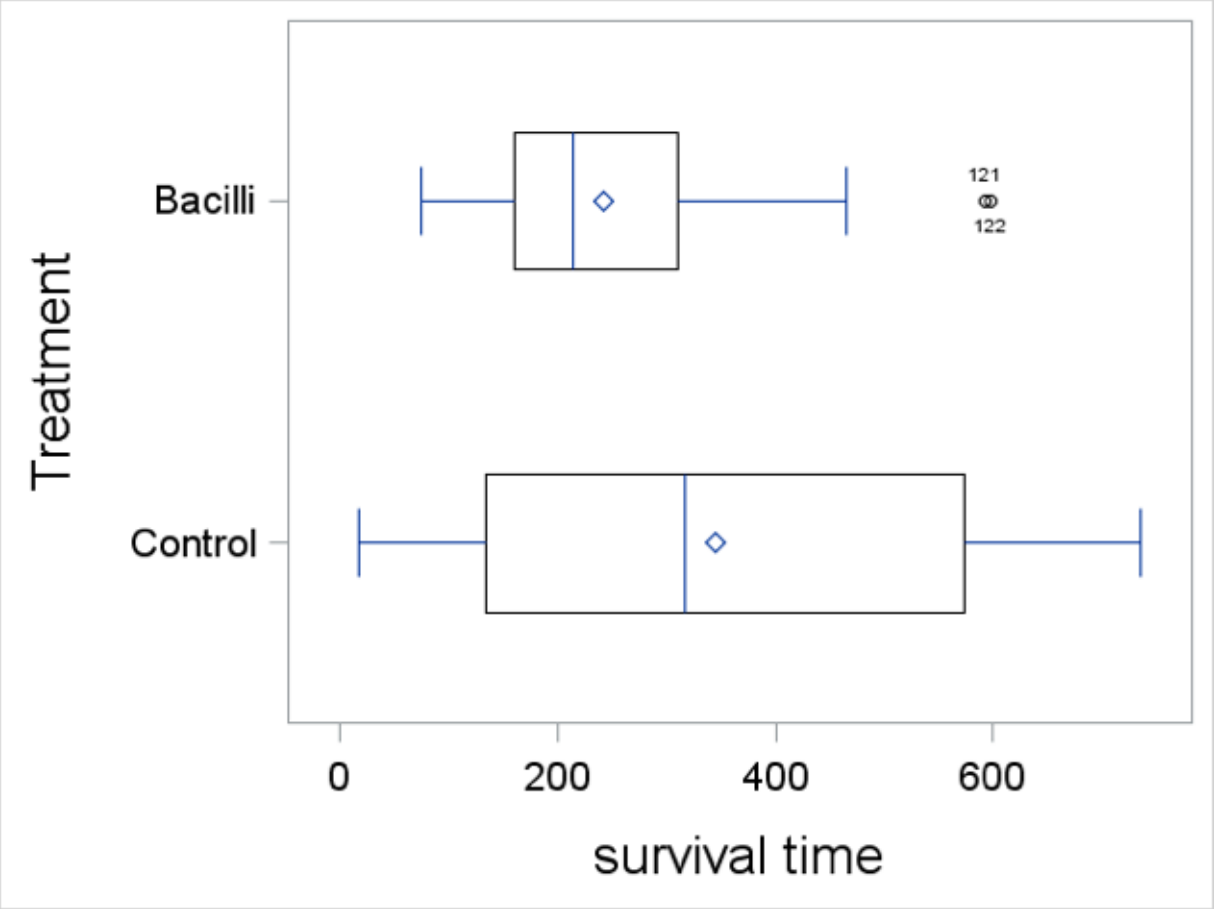
   ii. The p-value for the one-sided test is 0.0266.

   iii. H_0: Distributions of survival times are the same for controls and Bacilli infected guinea pigs
   H_1 : Survival times tend to be shorter for guinea pigs infected with the Bacilli

   Because the p-value from the Wilcoxon rank-sum test (0.0266) is smaller than 0.05, we can reject null hypothesis of that the survival time distributions are the same for the two treatment groups at the 0.05 level of significance. The data support the alternative that infection with tubercle Bacilli tends to decrease survival times.

iv. The results from Wilcoxon test are in accordance with the results from the two sample t-test in part (a), but the p-value is much smaller for the t-test in this situation.

(c) Yes, the Wilcoxon test in part (a) implies two distinct distributions, for which the Bacilli infected group has a smaller mean and a smaller median survival time. This coincides with the pattern in the box plots for which the box covering the middle half of the survival times for the Bacilli infected group is shifted below the box representing the middle half of the survival times for the controls.



(d) The hypotheses are:

$$\begin{cases} H_0 : \sigma^2_{Bacilli} = \sigma^2_{Control} \\ H_a : \sigma^2_{Bacilli} \neq \sigma^2_{Control} \end{cases}$$

And we obtained

i. Ratio of sample standard deviations

$$\frac{\max\{S_1, S_2\}}{\min\{S_1, S_2\}} = \frac{222.197}{117.931} \approx 1.89,$$

which indicates that there is little impact of variances of two groups on the two sample t tests.

ii.

iii.

Test	F Statistic	p-value
Folded F-test	3.55	<.0001
Brown-Forsythe Test	42.79	<.0001

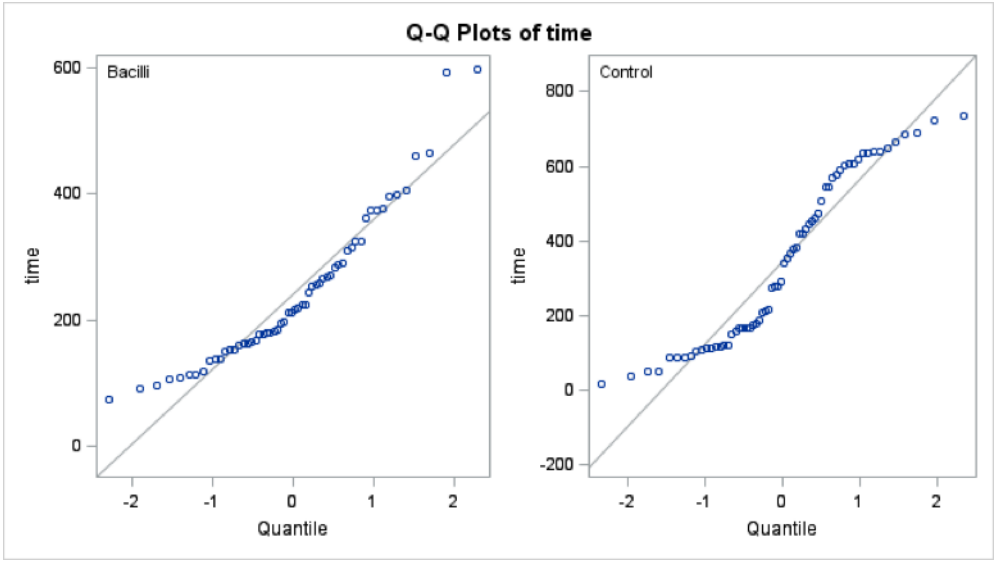
Both test yields significant p-values, indicating that we can reject null hypothesis and conclude that the variances of these two groups are heterogeneous. In this situation, the controls exhibit more variation in survival times.

(e) If guinea pig survival time has a normal distribution for both control and Bacilli group, then the points in Q-Q plot would be roughly in a line. And from the QQ plots below, it is obvious that a normal distribution does not provide an accurate description of the distribution of survival timed for either group.

For the group exposed to Tubercle Bacilli, the points on the right side of the q-q plot are higher than the line, indicating a heavy right tail with more long survival times than would be expected from a normal distribution. Especially in the right tail, there are two points much higher above the line, implying a possibility for outliers. And the points in the middle part are below the line, indicating fewer observations in the middle of the distribution than one would expect from a normal distribution with the observed mean and standard deviation. The points are also above the line on the left side of the q-q plot indicating the lower survival times tend to be closer to the mean survival time than one would expect if the observations had a normal distribution. This is an indication of a right skewed distribution.

As for control group, the points on the q-q plot exhibit an "s" pattern. Points on the extreme right of the plot are below the line indicating that they tend to be closer to the mean survival time than one would expect if the survival times had been sampled from a normal distribution. Points on the extreme left of the q-q plot are above the line which suggests that the smallest survival times are closer to the mean survival time than would be expected from a sample from a normal distribution with the same mean and variance. This is an indication that the survival times were sampled from a distribution with shorter right and left tails than a normal distribution (kurtosis less than 3).

The distributions of survival times appear to have different shapes for the Bacilli infected and control guinea pigs, and neither distribution is a normal distribution.



(f)

Group	Shapiro-Wilk Statistic	p-value
Bacilli	0.9156	0.0006
Control	0.9101	0.0002

For each group, the p-value for the Shapiro-Wilk test is quite small, indicating that the null hypothesis of normality is incorrect. Therefore, we can conclude that survival times for neither group conform to a normal distribution.

(g) Brown-Forsythe test would be more appropriate for these data because it is less sensitive to departures from normal distributions, in the sense that the actual type I error level for the test will not be pushed too much above the nominal type I error level when the data are not sampled from normal distributions. It was evident from the Shapiro-Wilk tests in part (f), that survival times do not conform to a normal distribution for either the Bacilli infected group or the control group. In this particular case, however, both tests led to the same conclusion.

3. 20 points

(a) This is an experimental study.

(b) $(17.04 - 14.84) \pm t_{10+10-2,0.975} * S_p * \sqrt{1/10 + 1/10} = (-1.988213, 6.388213)$
(Or use Satterwhite t test, which gives $(-2.17, 6.57)$)
We are 95% confident that $(-1.988213, 6.388213)$ contains the true difference between those 2 groups.

(c) No. 0 is contained in the 95% CI

(d)

$$n_0 = 8(\frac{Z_{0.975}S_p}{1})^2 = 610.652$$

$$n_1 = 8(\frac{t_{n_0,0.975}S_p}{1})^2 \approx n_0 = 610.652 = 611$$

(Asks for normal approximation so we don't need to iterate. The actual minimum sample size is 612 as given by SAS, the difference due to the approximation)

(e)
```
proc power;
    twosamplemeans
    alpha = 0.05
    meandiff = 1
    stddev= 4.4576
    npergroup = .
    power = 0.975;
run;
```

(f)
- Independence: met by experimental design
- Normality: met by QQ-plot
- Equal variance: not met by Folded F test

(g) $H_0 : \mu_1 = \mu_2$ v.s. $H_a : \mu_1 \neq \mu_2$

$df = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{1}{n_1-1}(s_1^2/n_1)^2 + \frac{1}{n_2-1}(s_2^2/n_2)^2} = 11.2594$

Test Statistic: 1.10358

p-value: 0.2928

Conclusion: Under 95% confidence level, fail to reject the null hypothesis and conclude that mean of the 2 groups are the same.

(h) $H_0 : \mu_1 = \mu_2$ v.s. $H_a : \mu_1 \neq \mu_2$

Test Statistic: 1.3985

p-value (Normal approximation): 0.1620

Conclusion: Under 95% confidence level, fail to reject the null hypothesis and conclude that mean of the 2 groups are the same.