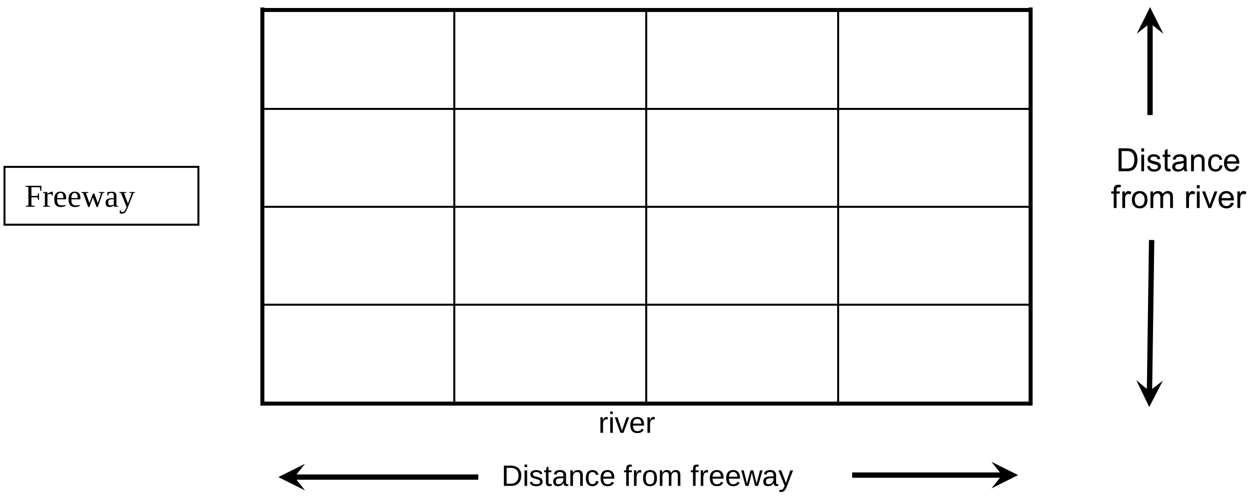
Stat 500 **Latin Square Example**

This is a study of four management treatments for increasing rare grass in a meadow. Two blocking factors are considered in the design. One is the distance from a stream (river) and another is the distance from the freeway. The yield of rare grass is the response variable.



The ANOVA table for the Latin square

Treatment mean estimates

Treatment	Mean	SE
Harvest	62.5	0.78
Windrow	54.5	0.78
In situ	51.2	0.78
Uncut	30.2	0.78

The efficiency: compared with an RCBD, did the Latin square (LS) help?

1. Did including rows (distances to the freeway) as blocks help?

   Compare with

2. Did including columns (distances to the stream) as blocks help?

   Compare with

Notes

1. We have not talked about testing rows and columns here, because they are only a way to

2. In a Latin-square design, the experimental unit (EU) is

Multiple Latin squares

If you have **multiple Latin squares**, the analysis depends on

Example. A 3×6 Latin rectangle with 3 treatments ($t = 3$) in $s = 2$ squares

First square Second square

1						
2						
3						

The ANOVA table for t rows, st columns, and t treatments (s squares)

The SE of the mean response for treatment k and the SE of the difference in mean responses for treatments k and l

Example. Multiple squares

<table>
<tr><th></th><th colspan="3">First square</th><th colspan="3">Second square</th></tr>
<tr><td>1</td><td></td><td></td><td></td><td colspan="3" rowspan="3"></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
<tr><td>4</td><td colspan="3" rowspan="3"></td><td></td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td></tr>
</table>

We need to lay out the table, not just t rows and t columns. The analysis is trickier.

The ANOVA table for the above example (with $s = 2$ squares)

ANOVA table for brome data from a LS design:

Source	df	SS	MS	F	p-value
Row		108.96	36.32		
Column		3.02	1.005		
Treatment		2275.77	758.6	312	<0.0001
Error		14.60	2.43		

Efficiency: did Latin Square help? (compare to RCBD)

1) Did including columns as blocks help? Compare with RCBD with only the row blocks.

   $MSE_{LS} = 2.43$
   $MSE_{RCBD} = (SS_{col} + SS_{Error}) / (df_{col} + df_{Error}) = (3.02 + 14.60) / (6 + 3) = 1.96 < 2.43$
   (RCBD with only the row blocks is more precise!)

Relative Efficiency = $MSE_{RCBD} / MSE_{LS} = 1.96 / 2.43 = 0.81$
   RCBD requires 0.81 as many observations.

2) Did including rows as blocks help? Compare with RCBD with only the column blocks.

   $MSE_{LS} = 2.43$
   $MSE_{RCBD} = (SS_{Row} + SS_{Error}) / (df_{Row} + df_{Error}) = (108.96 + 14.60) / (6 + 3) = 13.73$
   (LS is more precise than the RCBD with only column blocks!)

   Relative Efficiency = $MSE_{RCBD} / MSE_{LS} = 13.73 / 2.43 = 5.65$
   RCBD requires 5.65 times as many observations to get the same precision as the Latin square design.

Note that we haven't talked about testing rows and columns here. Row blocks and column blocks are a way to reduce the variability.

In a Latin Square design, the experimental unit is the intersection of a row and column in a field (one tire of an auto), that is what is randomly assigned to a treatment. Neither rows nor columns were assigned to any treatment factor.

What do you give up with Latin Square designs?

1) Cannot look at the look at the interactions, e.g., row and treatment interaction
2) When t is small, very few df in error → multiple squares

If you have multiple Latin squares, the analysis depends on how the squares are arranged.

Example 1, Latin rectangle. 3 treatments (t = 3). 3 x 6 rectangle, in s=2 squares.

	First square			Second Square		
1						
2						

3						

ANOVA table for t rows, st columns and t treatments (s squares)

Source	df	df (example)
Row	t-1	2
Column	***st-1***	5
Treatment	t-1	2
Error	(t-1)(***st-2***)	8
Total	***st^2-1***	17

Example 2, multiple squares

	First square			Second Square		
1						
2						
3						
4						
5						
6						

Need to lay out the table, not just t rows and t columns. The analysis is trickier.

ANOVA table for the example 2 (with s=2 squares)

Source	df	df (example, s= 2, t = 3)
Square	s-1	1
Row (square)	s(t-1)	4
Column (square)	s(t-1)	4
Treatment	t-1	2
Error	(st-s-1)(t-1)	6
Total	***st^2-1***	17

Example: brome2.sas (separate squares)