Statistics 500 - Homework #6, Fall 2020
Due: by noon Friday, 10/02/2020

Reading Assignment: Statistical Sleuth, Chapters 5 and 6. Review the conceptual exercises at the end of Chapter 5 and 6. Solutions to the conceptual exercises are given at the end of Chapter 5. Begin reading Chapter 23 and Section 24.4 of The Statistical Sleuth.

1. Consider the study on light speed in problem 4 of homework 5. In 1879, A. A. Michelson made 100 determinations of the velocity of light in air. The data used in this analysis were reported by Stigler (1977, Annals of Statistics, 5:4, 1075). The numbers are in km/sec, and have had 299,000 subtracted from them. The data for this exercise were modified by Stigler to correct for overall bias in Michelson's measurement for the speed of light in air. The measurements are grouped into five trials with 20 determinations for each trial. Since each determination was an attempt to measure the same "true" value of the speed of light in air, one might expect that the population means of possible measurements should be the same for all five trials. However, adjustments to the equipment or method for measuring the speed of light in air may have been made between trials, and these may cause the mean values to different across trials. (Note that although one can argue that the trails are random effects, for this problem, let's treat trails as fixed effects.) SAS code that also contains the data is posted in the file **lightspeed.sas**. (You have worked on this dataset for an overall F test in last homework.)

a. There is a question of which sets of trials have different means for the speed of light in air measurements. Use the Tukey-HSD method to compare means for measurements of speed of light in air for each pair of trials.

   (i) What is the minimum distance between two sample means required to reject the hypothesis that two trials have the same means using an experiment-wise type I error level of $\alpha = .05$?

   (ii) Indicate which means are significantly different at the 5% level by underlining sets of means that are not significantly different.

First Trial	Second Trial	Third Trial	Fifth Trial	Fourth Trial
909	856	845	831.5	820.5

b. If we are interested in using Bonferonni's method to perform hypothesis to test for differences between all pairs of means, what is the type I error rate that we need to control for each comparison so that the experiment-wise type I error rate is 0.05?

c. Consider the following set of orthogonal contrasts that examine potential chronological changes in the population means across the five trials:

   Contrast 1: (Mean for the first trial)-(mean of the other four trials)
   Contrast 2: (Mean for the second trial)-(mean of the last three trials)
   Contrast 3: (Mean for the third trial)-(mean of the last two trials)
   Contrast 4: (Mean for the fourth trial)-(mean of the last trials)

   For each contrast, report the contrast coefficients, the estimate of the contrast, the standard error of the estimated contrast, and the p-value for the t-test of the null hypothesis that the expected value of the contrast is zero. Using a Type I error level of 0.05 for each test, summarize your conclusions. (These are one-at-a-time t-tests and there is no control for an experiment-wise type I error level.)

d. Compute the sum of squares and corresponding F-statistic for each of the four contrasts listed in part b. Show that the sums of squares for the set of orthogonal contrasts sums to the between trials sums of squares.

e. Use the residuals to check the assumption of normality for the measurements of the speed of light in air. Construct a normal q-q plot and report the p-value for the Shapiro-Wilk test. State your conclusion.

f. Perform the Brown-Forsythe test the null hypothesis that population variances are homogeneous for the five trials. Report the p-value for the test and state your conclusion.

g. Apply the Kruskal-Wallis test to test the null hypothesis that each of the five sets of speed of light in air measurements, corresponding to the five trails run by Michelson, were samples from the same distribution of possible speed of light measurements. Report the value of the test statists, the p-value, and state your conclusion.

2. Consider the nesting cavity data from exercise 19 at the end of chapter 5 in the Statistical Sleuth. These data were examined in problem 2 of homework 5. Summary statistics shown below with samples means and sample standard deviations computed from the natural logarithms of the observed areas of cavity entrances.

Summary Statistics for Natural Logarithm of Areas of Nesting Cavity Entrances

Species	*Sample Size (n)*	*Sample Mean Log(1000 mm²)*	*Sample Std. Dev. Log(1000 mm2)*
Mouse	127	7.347	0.4979
Pinyon mouse	44	7.369	0.4235
Bewick’s wren	24	7.428	0.3955
Mountain bluebird	41	7.487	0.3181
Ash-throated flycatcher	18	7.563	0.3111
Plain titmouse	16	7.568	0.4649
Northern flicker	11	8.214	0.2963
Western Screech-owl	7	8.272	0.3242
American kestrel	6	8.297	0.5842

a. After observing the data, the researchers decided to test the null hypothesis that the average of the logarithm of the area of cavity openings for the first six species (mouse, pinyon mouse, Bewick’s wren, mountain bluebird, ash-throated flycatcher and plain titmouse) is equal to the average of the logarithm of the cavity opening for the other three species (northern flicker, western screech owl, and American kestrel). Because they decided to do this test after examining the summary statistics and selecting what appeared to be large difference, they will need to use the Scheffe’ method to do this test. Report the test statistic and the critical value, using the alternative that the null hypothesis is not true. State your conclusion.

b. Use the Scheffe’ method to construct a confidence interval for the contrast

$$\frac{\mu_1+\mu_2+\mu_3+\mu_4+\mu_5+\mu_6}{6}-\frac{\mu_7+\mu_8+\mu_9}{3}$$

so that the confidence level is at least 95 percent.

c. Evaluate a confidence interval for the contrast in part (b) formula based on an ordinary t-test without any multiple comparison adjustment. (Refer to the formula of standard error for contrasts to construct this t-confidence interval). Report the upper and lower bounds of this interval. How does it compare to the result from the Scheffe’ method?

d. One researcher tried to argue that using the confidence interval from (c) in situations such as this is okay because only one confidence interval was constructed. How would you respond to this assertion?

3. For each of the following experiments, (i) determine if it is a completely randomized experiment, a randomized complete block experiment, or something else. (ii) If it is an experiment, determine the experimental units (or runs), the treatments, and the blocks, if any blocks are used. (iii) Outline an ANOVA table listing sources of variation and degrees of freedom.

a. An agronomist is interested in the effects of plant density on the yield of a certain crop. There are six densities of interest (7, 8, 9, 10, 11, and 12 plants/m^2) that are interesting. The agronomist has five fields and each field is divided into 6 plots of equal size. She suspects that there is non-homogeneity of soil fertility across fields. She chooses a design such that each density is randomly assigned to one plot in each of the five fields. A separate random assignment is done within each field. ???

b. A router is used to cut notches in a printed circuit board. The vibration of the board as it is cut is considered a major source of variation in the diameter of the notches. Three cutting speeds, 50, 70, and 90 rpm, were used in an experiment to assess the impact of cutting speed on vibration. For each cutting speed, notches were cut in five different circuit boards. Once a notch is cut into a board it cannot be used again, so 15 different boards were used. The assignment of boards to cutting speed was done at random.

c. Industrial psychologists wish to investigate the effect of music in the factory of the productivity of workers. Four distinct music programs and no music make up the five treatments. The experiment is run in 8 plants. Each music program is used for one week. Within each plant music programs are randomly assigned to weeks (1,2,3,4,5) so that all five music programs are used in each plant. Production at each plant is recorded for each of the five weeks.

4. Ten patients who suffered from a certain sleep disorder were examined in a study of the effectiveness of a particular drug as an aid to sleep. The average number of hours of sleep per night was recorded for three nights for each patient. During this time each patient was given a placebo, a pill that did not contain any active ingredient. Then each patient was given a pill containing the drug for three consecutive nights and average hours of sleep was recorded for those three nights for each patient. (Note that the design can be improved if the order of placebo and drug are randomly assigned.) The recorded data appear below.

	Hours of Sleep per Night	
Patient	Drug Given	Placebo Given
1	1.3	0.6
2	1.1	1.1
3	6.2	2.5
4	3.6	2.8
5	4.9	2.9
6	1.4	3.0
7	6.6	3.2
8	4.5	4.7
9	4.3	5.5
10	6.1	6.2

a. Identify the blocks and treatments for this experiment:
b. Compute the value of a *t*-statistic for testing the null hypothesis that mean hours of sleep are the same for the drug and the placebo, against a one-sided alternative that mean hours of sleep are greater when the drug is used. Report your test statistic, degrees of freedom, and your p-value.
c. Can the null hypothesis be rejected at the $\alpha = .05$ level by the *t*-test in part (b)?
d. Apply the Shapiro-Wilk test to test the normality of the differences in hours of sleep per night when the drug is given then when the placebo is given. Report a p-value and state your conclusion.

e. Apply the Wilcoxon signed rank test to these data. Report your test statistic, p-value, and your conclusion.
f. Apply the sign test to these data. Give your expression to calculate p-value, report the *p*-value and state your conclusion.
g. Using the data shown above, compute how large (the number of patients) a study of this type would need to be so that the width of a 95% confidence interval for the difference in the mean responses to the drug and the placebo would be about 0.75 hours.

5. Chemotherapy for cancer often produces undesirable side effects such as nausea and vomiting. The effectiveness of THC (the active ingredient in marijuana) in preventing these side effects was compared with the commonly used drug compazine. Of 46 chemotherapy patients who tried both drugs (but were not told which was which), 21 patients expressed no preference, 20 experimented less side effects with THC and the other 5 preferred compazine. Since the patients only indicated a preference, neither a *t*-test nor a Wilcoxon signed rank test can be applied. Use the sign test to test the null hypothesis that the two drugs are equally effective in preventing side effects against the two-sided alternative that the drugs are not equally effective. Report the *p*-value and state your conclusion.

6. A researcher studied the effects of three experimental diets with varying fat contents on total lipid (fat) levels in plasma. Total lipid level is a widely used predictor of risk of coronary heart disease. Fifteen male subjects who were within 20 percent of their ideal body weight were grouped into five blocks according to age. Within each block, the three experimental diets were randomly assigned to the three subjects. Measurements of reduction in total lipid level (in grams per liter) were made after the subjects were on the diet for a specific period of time. The data are displayed below. SAS code for the data input is posted as plasma_lipids_data.sas. For this question, you may treat the blocks as fixed.

	Fat Content of Diet		
Block	Extremely Low	Fairly Low	Moderately Low
Ages 15-24	0.73	0.67	0.15
Ages 25-34	0.86	0.75	0.21
Ages 35-44	0.94	0.81	0.26
Ages 45-54	1.40	1.32	0.75
Ages 55-64	1.62	1.41	0.78

a. Construct an ANOVA table for a model with additive block and diet effects.
b. Are there significant differences in the mean lipid reduction for the three diets? Apply an F-test and state your conclusion.
c. Apply the Tukey's HSD method for all pairwise comparison with an experiment-wise Type I error level of $\alpha = .05$. State your conclusions.
d. Writing the additive model for block and diet effects as $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$ for i=1,2,3 diets and j=1,2,3,4,5 blocks (age groups), one contrast among the diet means is $\gamma_1 = (-1)\alpha_1 + (0)\alpha_2 + (1)\alpha_3$. Note that this correspond to examining the linear trend of plasma lipid over fat content of diet. Obtain a second orthogonal contrast for examining differences among the mean plasma lipid levels for the three diets. For each contrast, compute the sum of squares, the value of F-statistic, and p-value for testing the null hypotheses that the contrast mean is zero. State your conclusions.
e. Examine the normal probability plot of the residuals. What does this plot suggest? What do the results of tests for normality suggest?
f. Estimate and interpret the relative efficiency of blocking for this experiment relative to a completely randomized experiment.