HOMEWORK 8 STAT 588.

(1) a) For $\Lambda = \lambda$, we have

$$f(x \mid \lambda) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \text{for } x = 0, 1, 2, \ldots$$

$$h(\lambda) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} \lambda^{\alpha-1} e^{-\lambda/\beta} & \text{for } \lambda > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Hence,

$$f(\lambda, x) = \frac{\lambda^x e^{-\lambda}}{x!} \frac{1}{\beta^\alpha \Gamma(\alpha)} \lambda^{\alpha-1} e^{-\lambda/\beta}$$

$$= \frac{1}{x! \, \beta^\alpha \Gamma(\alpha)} \lambda^{(x+\alpha)-1} e^{-\lambda(1+\frac{1}{\beta})}$$

for $x = 0, 1, 2, \ldots$ and $\lambda > 0$, and $f(\lambda, x) = 0$ elsewhere

$$g(x) = \int_0^\infty \frac{1}{x! \, \beta^\alpha \Gamma(\alpha)} \lambda^{(x+\alpha)-1} e^{-\lambda(1+1/\beta)} \, d\lambda.$$

$$= \frac{1}{x! \, \beta^\alpha \Gamma(\alpha)} \cdot \Gamma(x+\alpha) \left(1+\frac{1}{\beta}\right)^{-(x+\alpha)}$$

$$\int_0^\infty \frac{1}{\left(1+\frac{1}{\beta}\right)^{-(x+\alpha)} \Gamma(x+\alpha)} \lambda^{(x+\alpha)-1} e^{-\lambda(1+\frac{1}{\beta})} \, d\lambda.$$

$$= \frac{\Gamma(\alpha+x) \left(1+\frac{1}{\beta}\right)^{-(x+\alpha)}}{x! \, \beta^\alpha \Gamma(\alpha)}$$

for $x = 0, 1, 2, \ldots$

Thus, $\varphi(\lambda \mid x) = \dfrac{f(\lambda, x)}{g(x)}$

$$= \frac{1}{\left(1+\frac{1}{\beta}\right)^{-(x+\alpha)} \Gamma(x+\alpha)} \lambda^{(x+\lambda)-1} e^{-\lambda(1+\frac{1}{\beta})}$$

for $\lambda > 0$, and $\varphi(\lambda \mid x) = 0$ elsewhere.

As can be seen by inspection, this is a gamma distribution with the parameters $x+\alpha$ and $1+\frac{1}{\beta}$

b) The mean of the posterior distribution of Δ is $(x+\alpha)\left(1+\frac{1}{\beta}\right)$.

Hence, $\mathbb{E}[\Delta \mid x] = (x+\alpha)\left(1+\frac{1}{\beta}\right)$

(2)

$N = 7, \quad n = 2, \quad x = 0, 1, 2$

$H_0 : M = 2 \quad$ vs. $\quad H_a : M = 4$

a) $\alpha = \mathbb{P}(\text{Reject } H_0 \mid H_0 \text{ true})$

$= \mathbb{P}(x = 2 \mid M = 2)$

$= \dfrac{\binom{2}{2}\binom{5}{0}}{\binom{7}{2}} = 0.0476$

b) $\beta = \mathbb{P}(\text{Accept } H_0 \mid H_0 \text{ false})$

$= \mathbb{P}(x \neq 2 \mid M = 4)$

$= \mathbb{P}(x = 0 \mid M = 4) + \mathbb{P}(x = 1 \mid M = 4)$

$= \dfrac{\binom{4}{0}\binom{3}{2}}{\binom{7}{2}} + \dfrac{\binom{4}{1}\binom{3}{1}}{\binom{7}{2}} = 0.7143.$

(3) a) Committing type I error = Rejecting the null that 60 percent of its passengers object to using a cell phone inside the plane during flights while the null is true

b) Committing type II error = Fail to reject the null while the null is false, i.e., not 60% of passengers object to using cell phones

(4) Let pvalue be p. We reject the null hypothesis at a significance level of 0.05, in which case $p \leq 0.05$.

a) As $p \leq 0.05$, it does not necessarily mean $p \leq 0.01$, so the null will not always be rejected at a significance level of 0.01

b) When $p \leq 0.05$, $p < 0.1$, so the null is rejected at a significance level of 0.1

(5) a) At a significance level of 0.01, we fail to reject H_0

b) At a significance level of 0.05, we reject H_0

c) At a significance level of 0.1, we reject H_0

(6) $n = 12$, $\bar{x} = 33.6$, $s = 2.3$

$H_0 : \mu = 35$

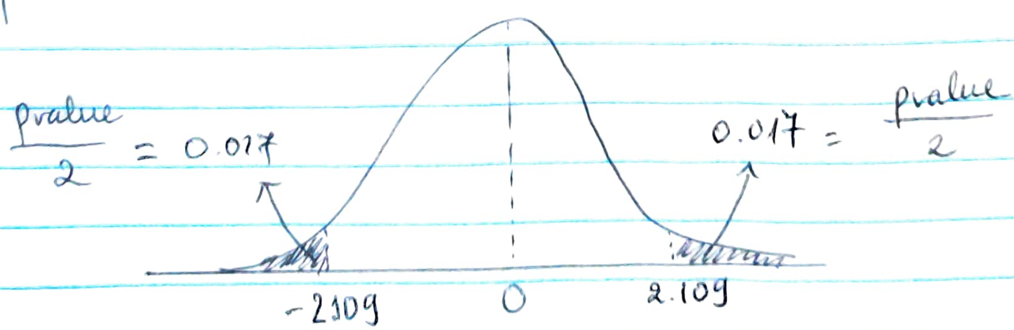
$H_a : \mu \neq 35$

$\alpha = 0.05$

a) t distribution with $n-1 = 11$ dfs

b) $$T_1 = \frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{n}} = \frac{33.6 - 35}{2.3/\sqrt{12}} = -2.109$$

$$\text{p-value} = 2 \times \mathbb{P}(t \leq T_1) = 2 \times 0.017 = 0.034$$



(7)

$n = 24$, $s = 238$

$H_0: \sigma = 250$

$H_a: \sigma < 250$.

$\alpha = 0.01$

a) Chi square distribution with $n-1 = 23$ dfs.

b) Test statistic $= 20.845$

p value $= 0.409$.

(8)

$n = 18$, $\bar{x} = 63.84$, $s = 2.75$

a) $H_0: \mu = 65$

$H_a: \mu \neq 65$.

b) $$T = \frac{\bar{x} - \mu}{\hat{\sigma}/\sqrt{n}} = \frac{63.84 - 65}{2.75/\sqrt{18}} = -1.7896$$

c) A 99% confidence interval for the average sale is (61.96, 65.72), which does contain our null hypothesis value of 65. Therefore, at a significance level of 0.01, we fail to reject the null.

d) We conclude that the population average sales is not significantly different from 65.