STAT588 HOMEWORK 8

① $n = 120$, $\bar{x} = 141.8$, $\sigma = 10.5$

a) $\alpha = 0.02$

b) Standard normal

c) $\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$\Rightarrow 141.8 - 2.33 \times \frac{10.5}{\sqrt{120}} < \mu < 141.8 + 2.33 \times \frac{10.5}{\sqrt{120}}$

$\Rightarrow 139.5667 < \mu < 144.0333$

② $n = 18$, $\bar{x} = 63.84$, $s = 2.75$

a) $\alpha = 0.01$

b) t distribution with $n-1 = 17$ dfs

c) $\bar{x} - t_{\alpha/2, n-1}\cdot\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2, n-1}\frac{s}{\sqrt{n}}$

$\Rightarrow 63.84 - 2.898 \times \frac{2.75}{\sqrt{18}} < \mu < 63.84 + 2.989 \times \frac{2.75}{\sqrt{18}}$

$\Rightarrow 61.9616 < \mu < 65.7184$.

③ $n_1 = 12$, $\bar{x}_1 = 13.8$; $n_2 = 15$, $\bar{x}_2 = 12.9$; $\sigma_1 = 1.2$; $\sigma_2 = 1.5$

a) $\alpha = 0.05$

b) Standard normal

c) $(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$\Rightarrow (13.8 - 12.9) - 1.96\sqrt{\frac{1.2^2}{12} + \frac{1.5^2}{15}} < \mu_1 - \mu_2 <$

$(13.8 - 12.9) + 1.96\sqrt{\frac{1.2^2}{12} + \frac{1.5^2}{15}}$

$\Rightarrow -0.1184 < \mu_1 - \mu_2 < 1.9184$

④ $n_1 = 12$; $\bar{x}_1 = 13.8$, $s_1 = 1.2$; $n_2 = 15$; $\bar{x}_2 = 12.9$, $s_2 = 1.5$

a) $\alpha = 0.05$

b) t distribution with $n_1 + n_2 - 2 = 25$ dfs

c) $s_p = \sqrt{\frac{11(1.2)^2 + 14(1.5)^2}{25}} = 1.3761$

$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2,25} \times s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2,25}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\Rightarrow (13.8 - 12.9) - 2.06 \times 1.3761 \sqrt{\frac{1}{12} + \frac{1}{15}} < \mu_1 - \mu_2 < (13.8 - 12.9) + 2.06 \times 1.3761 \sqrt{\frac{1}{12} + \frac{1}{15}}$$

$$\Rightarrow -0.1979 < \mu_1 - \mu_2 < 1.9979$$

⑤ $n = 100$, $x = 18$, $\hat{\theta} = \frac{18}{100} = 0.18$.

a) $\alpha = 0.01$

b) Standard normal

c) $$\hat{\theta} - z_{\alpha/2} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}} < \theta < \hat{\theta} + z_{\alpha/2} \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$$

$$\Rightarrow 0.18 - 2.575 \sqrt{\frac{0.18(1-0.18)}{100}} < \theta < 0.18 + 2.575 \sqrt{\frac{0.18(1-0.18)}{100}}$$

$$\Rightarrow 0.081 < \theta < 0.2789$$

(6) $\hat{\theta}_1 = \frac{48}{500} = 0.096$; $\hat{\theta}_2 = \frac{68}{500} = 0.136$.

a) $\alpha = 0.1$

b) Standard normal

c) $(\hat{\theta}_1 - \hat{\theta}_2) - z_{\alpha/2}\sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}} < \theta_1 - \theta_2$

$< (\hat{\theta}_1 - \hat{\theta}_2) + z_{\alpha/2}\sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}}$

$\Rightarrow \quad -0.0733 < \theta_1 - \theta_2 < -0.00675$

(7) $n = 12, \quad s = 0.625$

a) $\alpha = 0.06$

b) Chi square distribution with $n-1 = 11$ dfs

c) $\frac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}$

$\Rightarrow \quad \frac{11 \times 0.625^2}{3.99} > \sigma^2 > \frac{11 \times 0.625^2}{21.34}$

$\Rightarrow \quad 0.2014 < \sigma^2 < 1.0769$

(8) $n_1 = n_2 = 61, \quad \bar{x}_1 = 80.7, \quad s_1 = 19.4, \quad \bar{x}_2 = 88.1, \quad s_2 = 18.8$

a) $\alpha = 0.04$

b) F distribution with $n_1 - 1 = 60$ and $n_2 - 1 = 60$ dfs

c) $\frac{s_1^2}{s_2^2} \times \frac{1}{f_{\alpha/2, n_1-1, n_2-1}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} \times f_{\alpha/2, n_2-1, n_1-1}$

$\Rightarrow \quad \frac{19.4^2}{18.8^2} \times \frac{1}{1.71} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{19.4^2}{18.8^2} \times 1.71$

$\Rightarrow \quad 0.6227 < \frac{\sigma_1^2}{\sigma_2^2} < 1.8209$