STAT 588 Homework 11

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1 Question 1

(a)

$H_0 : \mu_A = \mu_B$

$H_a : \mu_A \neq \mu_B$

$\alpha = 0.1$

(b) Assuming the true variances of the two populations are equal.
$S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \dfrac{12 * 10^2 + 16 * 16^2}{13 + 17 - 2} = 189.1429$
The test statistic $t = \dfrac{(\bar{X_A} - \bar{X_B}) - (\mu_A - \mu_B)}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}} = \dfrac{45 - 64}{\sqrt{189.1429 * (\dfrac{1}{13} + \dfrac{1}{17})}} = -3.749679$
(c) We have $t_{\alpha/2, n_1+n_2-2} = t_{0.05,28} = 1.701$. Then $|t| > t_{0.05,28}$. Therefore, at a significance level of 0.1, we reject the null. We conclude that the true mean additional sales are different between the two algorithms.

(d) P-value $= 2 * P(T \geq |t|) = 0.00082$, which is smaller than 0.1; hence, at a significance level of 0.1, we reject the null, agreeing with part (c).

2 Question 2

(a)

$H_0 : \sigma_A^2 = \sigma_B^2$

$H_a : \sigma_A^2 \neq \sigma_B^2$

$\alpha = 0.1$

(b) Since $s_A^2 < s_B^2$, when we reject the null if $\dfrac{s_B^2}{s_A^2} \geq f_{\alpha, n_2-1, n_1-1}$. $f_{\alpha, n_2-1, n_1-1} = f_{0.1,16,12} = 2.09381$
Test statistic $f = \dfrac{s_B^2}{s_A^2} = \dfrac{16^2}{10^2} = 2.56$
(c) Since $2.56 > 2.09381$, we reject the null at a significance level of 0.1. We conclude that the variances of the two populations are not equal.

(d) P-value $= P(F > f) = 0.05263$, which is smaller than 0.1. At a significance level of 0.1, we reject the null.

3 Question 3

H_0 : The data follows Poisson distribution.

H_a : The data does not follow Poisson distribution.

$\alpha = 0.1$

First, we estimate the mean of Poisson distribution using the mean of the observed distribution:
$\hat{\lambda} = \frac{0*8+1*11+2*13+3*8+4*3+5*3+6*3+7*1}{8+11+13+8+3+3+3+1} = 2.26$, which approximately is 2.3.
Using the Poisson statistical table, we get the following table:

Count	No. of Obs	Poisson prob $\lambda = 2.3$	Expected frequency
0	8	.1003	5.015
1	11	.2306	11.53
2	13	.2652	13.26
3	8	.2033	10.165
4	3	.1169	5.845
5	3	.0538	2.69
6	3	.0206	1.03
7	1	.0068	0.34

Test statistic:
$$\chi^2 = \frac{(8-5.015)^2}{5.015} + \frac{(11-11.53)^2}{11.53} + \frac{(13-13.26)^2}{13.26} + \frac{(8-10.165)^2}{10.156} + \frac{(3-5.845)^2}{5.845} + \frac{(3-2.69)^2}{2.69} + \frac{(3-1.03)^2}{1.03} + \frac{(1-0.34)^2}{0.34} = 8.737241$$
$\chi^2_{\alpha,m-t-1} = \chi^2_{0.1,5} = 9.23636$

Since $\chi^2 = 8.737241$ is less than 9.23636, we fail to reject the null at a significance level of 0.1.

4 Question 4

$H_0 : \theta_1 = \theta_2 = \theta_3$

$H_a : \theta_1, \theta_2, \theta_3$ are not all equal.

$\alpha = 0.1$

We reject the null hypothesis when $\chi^2 \geq \chi^2_{\alpha,k-1}$, where $\chi^2_{\alpha,k-1} = \chi^2_{0.1,2} = 4.60517$ and $\chi^2 = \sum_{i=1}^{2}\sum_{j=1}^{3} \frac{(f_{ij}-e_{ij})^2}{e_{ij}}$.

The pooled estimate of θ is $\hat{\theta} = \frac{2119}{4223} = 0.502$.

The expected frequencies are:

$e_{11} = 1458 * 0.502 = 731.916; e_{12} = 1382 * 0.502 = 693.764; e_{13} = 1383 * 0.502 = 694.266$

$e_{21} = 1458 * 0.498 = 726.084; e_{22} = 1382 * 0.498 = 688.236; e_{23} = 1383 * 0.498 = 688.734$
$$\chi^2 = \frac{(842-731.916)^2}{731.916} + \frac{(736-693.764)^2}{693.764} + \frac{(541-694.266)^2}{694.266} + \frac{(616-726.084)^2}{726.084} + \frac{(646-688.236)^2}{688.236} + \frac{(842-688.734)^2}{688.734}$$

$\chi^2 = 106.3524$

Since $\chi^2 \geq \chi^2_{\alpha,k-1}$, we reject the null hypothesis at a significance level of 0.1.

5 Question 5

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> summary(fit)

Call:
lm(formula = house.change ~ unemployment, data = data)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.142090 -0.079419  0.004148  0.070623  0.159417 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -0.06594    0.05386  -1.224    0.232
unemployment -0.98760    0.86061  -1.148    0.262

Residual standard error: 0.08994 on 26 degrees of freedom
Multiple R-squared:  0.04821,	Adjusted R-squared:  0.0116 
F-statistic: 1.317 on 1 and 26 DF,  p-value: 0.2616
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The estimated intercept is -0.06594 and the estimated slope is -0.9876.

When testing $H_0 : \beta = 0$ vs $H_a : \beta \neq 0$, we get the test statistic $t = -1.148$ with p-value $= 0.262$. At a significance level of 0.05, we fail to reject the null.