STAT588 Homework 1

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August 30, 2020

1 Question 1

Roll a pair of dice, one red and one green. Now, define the following events: A is the event that at least one of the dice will be an odd number; B is the event that the sum of the die will be equal to 7; C is the event that at least one of the dice is equal to 6.
(a) List all the outcomes for each of the events A, B and C explicitly.
Answer:
Let $S(I)$ be the sample space of event I.

$S(A) = \{$ (Red = 1, Green = 1), (Red = 1, Green = 2), (Red = 1, Green = 3), (Red = 1, Green = 4), (Red = 1, Green = 5), (Red = 1, Green = 6), (Red = 2, Green = 1), (Red = 2, Green = 3), (Red = 2, Green = 5), (Red = 3, Green = 1), (Red = 3, Green = 2), (Red = 3, Green = 3), (Red = 3, Green = 4), (Red = 3, Green = 5), (Red = 3, Green = 6), (Red = 4, Green = 1), (Red = 4, Green = 3), (Red = 4, Green = 5), (Red = 5, Green = 1), (Red = 5, Green = 2), (Red = 5, Green = 3), (Red = 5, Green = 4), (Red = 5, Green = 5), (Red = 5, Green = 6), (Red = 6, Green = 1), (Red = 6, Green = 3), (Red = 6, Green = 5) $\}$. The capacity of $S(A)$ is 27.

$S(B) = \{$(Red = 1, Green = 6), (Red = 2, Green = 5), (Red = 3, Green = 4), (Red = 4, Green = 3), (Red = 5, Green = 2), (Red = 6, Green = 1)$\}$. The capacity of $S(B)$ is 6.

$S(C) = \{$(Red = 6, Green = 1), (Red = 6, Green = 2), (Red = 6, Green = 3), (Red = 6, Green = 4), (Red = 6, Green = 5), (Red = 6, Green = 6), (Red = 1, Green = 6), (Red = 2, Green = 6), (Red = 3, Green = 6), (Red = 4, Green = 6), (Red = 5, Green = 6)$\}$. The capacity of $S(C)$ is 11.

(b) Find P(A), P(B), and P(C). Round all answers to 3 decimal places.
Answer:
There are 6 possible outcomes when rolling Red, and 6 possible outcomes when rolling Green. Therefore, there are $6 \times 6 = 36$ possible outcomes when rolling both of the dice. Hence,
$P(A) = \frac{27}{36} = 0.75$
$P(B) = \frac{6}{36} = 0.167$
$P(C) = \frac{11}{36} = 0.306$

(c) Are events A and C independent? Explain using appropriate probabilities, theorems, and/or definitions. Round all probabilities to 3 decimal places.
Answer:
The sample space of $A \cap C$ is $\{$(Red = 6, Green = 1), (Red = 6, Green = 3), (Red = 6, Green =

5), (Red = 1, Green = 6), (Red = 3, Green = 6), (Red = 5, Green = 6)}. The capacity of this set is 6. Then,
$P(A \cap C) = \frac{6}{36} = 0.167 \neq P(A) \times P(C) = \frac{3}{36} \times \frac{11}{36} = 0.025$
Hence, A and C are not independent.

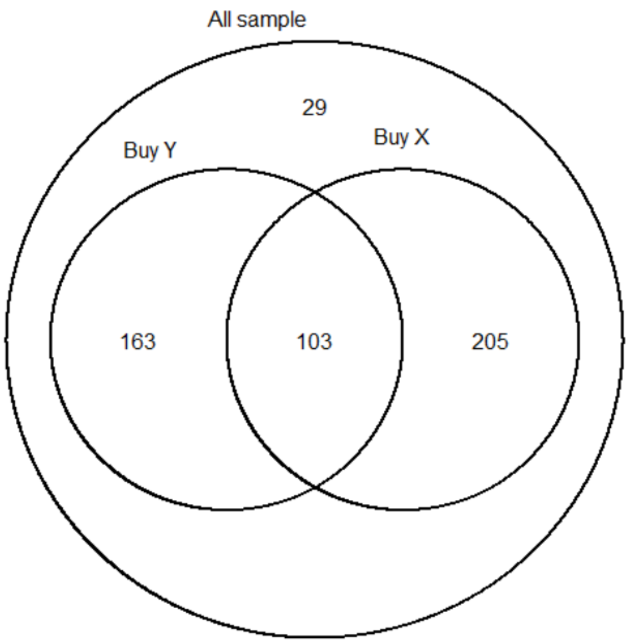
(d) Are events B and C disjoint (mutually exclusive)? Explain using appropriate probabilities,theorems, and/or definitions. Round all probabilities to 3 decimal places.
Answer:
$B \cap C = \{(\text{Red} = 1, \text{Green} = 6), (\text{Red} = 6, \text{Green} = 1)\} \neq \emptyset$. Hence the two events are not mutually exclusive.

2 Question 2

A city has 2 daily newspapers, the Chronicle and the Times. A survey of 100 residents of the city was conducted to determine the readership of the two newspapers. It was found that 80 read the Chronicle, 55 read the Times, and 15 read neither paper. Assuming that the survey reflects the actual readership of the city, what is the probability that a resident selected at random: (HINT: Draw a picture of the sample space and the events)
(a) Reads at least one of two papers?
Answer:
The number of people who read newspapers is $100 - 15 = 85$
The probability of selecting a person who reads at least one of the two papers is $\frac{85}{100} = 0.85$. (b) Reads both papers?
Answer:
The number of people who read both of the papers is $80 + 55 - 85 = 50$
The probability of selecting a person who reads both of the two papers is $\frac{50}{100} = 0.5$.

3 Question 3

A market research organization claims that, among 500 shoppers interviewed, 308 regularly buy Product X, 266 regularly buy Product Y, 103 regularly buy both, and 59 buy neither on a regular basis. Use a Venn diagram to check the claims made by this study. Are the claims valid or invalid?
Answer:
If 59 people do not buy any of the products on a regular basis, the number of people who buy either products regularly is $500 - 59 = 441$.
The claim is invalid as $308 + 266 - 103 = 471 \neq 441$.



4 Question 4

A company currently has two active projects, one in Asia and the other in Europe. Let A be the event that the Asian project is successful and B be the event that the European project is successful. Suppose that A and B are independent events with $P(A) = 0.4$ and $P(B) = 0.7$. If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning using "common sense." Round all results to 4 decimal places.

Answer:

$P(A' \cap B') = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(A \cap B) = 1 - P(A) - P(B) + P(A)P(B) = P(A^c)P(B^c)$

Because the two events are independent, if the Asia project fails, it has nothing to do with the result in Europe.

$P(B) = 0.7$, so the probability that the project in Europe fails is $P(B^c) = 0.3$

5 Question 5

One division of an Amazon fulfillment center employs 3 people, U, V, and W, who pull items from shelves and assemble them for subsequent verification and packaging. U makes a mistake in an order (gets a wrong item or the wrong quantity) one time in a hundred, V makes a mistake in an order five times in a hundred, and W makes a mistake in an order three times in a hundred. If U, V, and W fill, respectively, 30, 40, and 30 percent of all orders, what is the probability that if a mistake is made in an order, the order was filled by U? Round the answer to 5 decimal places.

Answer:

P(U makes a mistake) $= \dfrac{1}{100}$

P(V makes a mistake) $= \dfrac{5}{100}$

P(W makes a mistake) $= \dfrac{3}{100}$

P(U makes an order) $= \frac{30}{100}$
P(V makes an order) $= \frac{40}{100}$
P(W makes an order) $= \frac{30}{100}$
Then, P(A mistake is made) $= \frac{1}{100} \times \frac{30}{100} + \frac{5}{100} \times \frac{40}{100} + \frac{3}{100} \times \frac{30}{100} = 0.032$
P(U makes a mistake | A mistake is made) $= \frac{\frac{1}{100} \times \frac{30}{100}}{0.032} = 0.09375$

6 Question 6

In how many ways can 6 people form a circle for a team-building exercise?
Answer:
Assume all standing positions are the same, so the difference between the ways of arranging 6 people is the order of people. Then if we arbitrarily consider the position of one of the six people as fixed, we can arrange the other five people in $5! = 120$ different ways.

7 Question 7

A multiple-choice test consists of 20 questions, each permitting a choice of 4 alternatives. In how many ways may a student fill in the answers if they answer each question?
Answer:
The total number of ways is 4^{20}.

8 Question 8

A television director is scheduling a certain sponsor's commercials for an upcoming broadcast. There are eight slots available for commercials. In how many ways may the director schedule the commercials
(a) If the sponsor has eight different commercials, each to be shown once?
Answer:
There are 8 ways to choose one commercial for the first slot, 7 ways to choose a commercial for the second slot,... Hence, there are $8! = 40320$ ways to show eight different commercials in eight slots available.

(b) If the sponsor has four different commercials, each to be shown twice?
Answer:
There are $\frac{8!}{2! \times 2! \times 2! \times 2!} = 2520$ ways to show four different commercials, each twice, in four slots.
(c) If the sponsor has three different commercials, the first of which is to be shown five times, the second two times, and the third once?
Answer:
There are $\frac{8!}{5! \times 2! \times 1!} = 168$ ways.

9 Question 9

Suppose 100 people entered the door prize giveaway at a convention. The first place prize is a \$100 gift card, the second place prize is a \$50 gift card, and the third place prize is a \$25 gift card. If a person can only win 1 prize, how many ways can the door prizes be awarded?

Answer:

There are 100 choices for the person who receives the \$100 prize, 99 remaining choices for the \$50 prize and 98 choices for the \$25 prize, so there are $100 \times 99 \times 98 = 970200$ ways to award the prize.

10 Question 10

Suppose that a cancer patient decides to undergo a transplant operation in the hopes that it will increase the probability of survival. Round all answers to 4 decimal places.

(a) The probability of surviving this transplant operation is 0.55. If a patient survives the operation, the probability that his or her body will reject the transplant within a month is 0.20. What is the probability of being successful in both of these critical stages?

Answer:

If a patient survives the operation, the probability of success in the second stage is 0.8. We can write this probability as P(success in the second stage | success in the first stage) = 0.8.

Moreover, P(success in the first stage) = 0.55.

Hence, P(success in both stages) = P(success in the first stage $\cap$ success in the second stage) = P(success in the second stage | success in the first stage) $\times$ P(success in the first stage) = $0.8 \times 0.55 = 0.44$

(b) If the patient is successful with both stages of the transplant, then the probability that the patient will remain in remission (the cancer does not reoccur) for the next two years is 0.48. What is the probability of being successful in all three of these stages?

Answer:

Let A be the event of success at the first two stages. According to (a), P(A) = 0.44.

We have, P(success at stage 3 | A) = 0.48. Hence, P(success at three stages) = $0.48 \times 0.44 = 0.2112$