THE EMBEDDED STRUCTURE OF COLLATZ CONJECTURE

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1. The Embedded Structure

The well known Collatz conjecture is a function $f : \mathbb{N} \longrightarrow \mathbb{N}$ oscillating between its $n/2$ and $3n+1$ branches respectively, based on n parity.

Definition 1.1. Let C be any sequence computed using $f(n)$ function:

$$C = \{c_1, ..., c_{k-1}, c_k, ...c_n \mid c \in \mathbb{N}\} \tag{1.1}$$

Lemma 1.2. *Any Collatz sequence C can be unequivocally transformed using just eighteen pairs.*

Proof. Using $\mathbb{Z}/2\mathbb{Z}$ to select proper branch from $f(n)$ function and using $\mathbb{Z}/9\mathbb{Z}$ to reduce $f(n)$ results to $\{\overline{0}_9, \overline{1}_9, \overline{2}_9, \overline{3}_9, \overline{4}_9, \overline{5}_9, \overline{6}_9, \overline{7}_9, \overline{8}_9\}$ elements, following eighteen combinations, $\mid \mathbb{Z}/2\mathbb{Z} \mid \times \mid \mathbb{Z}/9\mathbb{Z} \mid$, can be determined for any n value:

$$n \, mod \, 9 = 0 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 0 \tag{1.2}$$
$$n \, mod \, 9 = 0 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 1 \tag{1.3}$$
$$n \, mod \, 9 = 1 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 5 \tag{1.4}$$
$$n \, mod \, 9 = 1 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 4 \tag{1.5}$$
$$n \, mod \, 9 = 2 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 1 \tag{1.6}$$
$$n \, mod \, 9 = 2 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 7 \tag{1.7}$$
$$n \, mod \, 9 = 3 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 6 \tag{1.8}$$
$$n \, mod \, 9 = 3 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 1 \tag{1.9}$$
$$n \, mod \, 9 = 4 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 2 \tag{1.10}$$
$$n \, mod \, 9 = 4 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 4 \tag{1.11}$$
$$n \, mod \, 9 = 5 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 7 \tag{1.12}$$
$$n \, mod \, 9 = 5 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 7 \tag{1.13}$$
$$n \, mod \, 9 = 6 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 3 \tag{1.14}$$
$$n \, mod \, 9 = 6 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 1 \tag{1.15}$$
$$n \, mod \, 9 = 7 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 8 \tag{1.16}$$
$$n \, mod \, 9 = 7 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 4 \tag{1.17}$$
$$n \, mod \, 9 = 8 \wedge n \, mod \, 2 = 0 \longrightarrow f(n) \, mod \, 9 = 4 \tag{1.18}$$
$$n \, mod \, 9 = 8 \wedge n \, mod \, 2 = 1 \longrightarrow f(n) \, mod \, 9 = 7 \tag{1.19}$$

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Since $\forall\ n \in \mathbb{N} \rightarrow f(n) \in \mathbb{N}$, it is possible to apply above equations on two consecutive elements $\{c_k, c_{k+1}\} \in C$:

$$c_{k+1}\ mod\ 9 = 0 \rightarrow (c_k\ mod\ 9 = 0) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 0 \tag{1.20}$$
$$c_{k+1}\ mod\ 9 = 1 \rightarrow (c_k\ mod\ 9 = 6) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 6 \tag{1.21}$$
$$c_{k+1}\ mod\ 9 = 1 \rightarrow (c_k\ mod\ 9 = 0) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 0 \tag{1.22}$$
$$c_{k+1}\ mod\ 9 = 1 \rightarrow (c_k\ mod\ 9 = 3) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 3 \tag{1.23}$$
$$c_{k+1}\ mod\ 9 = 1 \rightarrow (c_k\ mod\ 9 = 2) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 2 \tag{1.24}$$
$$c_{k+1}\ mod\ 9 = 2 \rightarrow (c_k\ mod\ 9 = 4) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 4 \tag{1.25}$$
$$c_{k+1}\ mod\ 9 = 3 \rightarrow (c_k\ mod\ 9 = 6) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 6 \tag{1.26}$$
$$c_{k+1}\ mod\ 9 = 4 \rightarrow (c_k\ mod\ 9 = 8) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 8 \tag{1.27}$$
$$c_{k+1}\ mod\ 9 = 4 \rightarrow (c_k\ mod\ 9 = 7) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 7 \tag{1.28}$$
$$c_{k+1}\ mod\ 9 = 4 \rightarrow (c_k\ mod\ 9 = 4) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 4 \tag{1.29}$$
$$c_{k+1}\ mod\ 9 = 4 \rightarrow (c_k\ mod\ 9 = 1) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 1 \tag{1.30}$$
$$c_{k+1}\ mod\ 9 = 5 \rightarrow (c_k\ mod\ 9 = 1) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 1 \tag{1.31}$$
$$c_{k+1}\ mod\ 9 = 6 \rightarrow (c_k\ mod\ 9 = 3) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 3 \tag{1.32}$$
$$c_{k+1}\ mod\ 9 = 7 \rightarrow (c_k\ mod\ 9 = 8) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 8 \tag{1.33}$$
$$c_{k+1}\ mod\ 9 = 7 \rightarrow (c_k\ mod\ 9 = 5) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 5 \tag{1.34}$$
$$c_{k+1}\ mod\ 9 = 7 \rightarrow (c_k\ mod\ 9 = 5) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 5 \tag{1.35}$$
$$c_{k+1}\ mod\ 9 = 7 \rightarrow (c_k\ mod\ 9 = 2) \wedge (c_k\ mod\ 2 = 1) \implies c_k\ mod\ 9 = 2 \tag{1.36}$$
$$c_{k+1}\ mod\ 9 = 8 \rightarrow (c_k\ mod\ 9 = 7) \wedge (c_k\ mod\ 2 = 0) \implies c_k\ mod\ 9 = 7 \tag{1.37}$$

Derived from above equations, Generic Pairs represent any Collatz Pair $\{c_k, c_{k+1}\} \in C$ using m-multiplicities; for equation (1.38), $m = 1$ must be configured as base case instead of $m = 0$, since $\{0, 0\}$ is not a valid Collatz pair due to $f(n) \in \mathbb{N}$ definition. As follows:

$$\{(3\times3\times(1\times2\times m+0))+0,\ (1\times3\times3\times(1\times1\times m+0))+0\},\ m \geq 1 \tag{1.38}$$
$$\{(3\times3\times(1\times2\times m+1))+6,\ (1\times3\times3\times(3\times2\times m+5))+1\},\ m \geq 0 \tag{1.39}$$
$$\{(3\times3\times(1\times2\times m+1))+0,\ (3\times3\times3\times(1\times2\times m+1))+1\},\ m \geq 0 \tag{1.40}$$
$$\{(1\times3\times(3\times2\times m+0))+3,\ (1\times3\times3\times(3\times2\times m+1))+1\},\ m \geq 0 \tag{1.41}$$
$$\{(3\times3\times(1\times2\times m+0))+2,\ (1\times3\times3\times(1\times1\times m+0))+1\},\ m \geq 0 \tag{1.42}$$
$$\{(3\times3\times(1\times2\times m+0))+4,\ (1\times3\times3\times(1\times1\times m+0))+2\},\ m \geq 0 \tag{1.43}$$
$$\{(3\times3\times(1\times2\times m+0))+6,\ (1\times3\times3\times(1\times1\times m+0))+3\},\ m \geq 0 \tag{1.44}$$
$$\{(3\times3\times(1\times2\times m+0))+8,\ (1\times3\times3\times(1\times1\times m+0))+4\},\ m \geq 0 \tag{1.45}$$
$$\{(3\times3\times(1\times2\times m+0))+7,\ (1\times3\times3\times(3\times2\times m+2))+4\},\ m \geq 0 \tag{1.46}$$
$$\{(3\times3\times(1\times2\times m+1))+4,\ (1\times3\times3\times(3\times2\times m+4))+4\},\ m \geq 0 \tag{1.47}$$
$$\{(3\times3\times(1\times2\times m+0))+1,\ (1\times3\times3\times(3\times2\times m+0))+4\},\ m \geq 0 \tag{1.48}$$
$$\{(3\times3\times(1\times2\times m+1))+1,\ (1\times3\times3\times(1\times1\times m+0))+5\},\ m \geq 0 \tag{1.49}$$
$$\{(3\times3\times(1\times2\times m+1))+3,\ (1\times3\times3\times(1\times1\times m+0))+6\},\ m \geq 0 \tag{1.50}$$
$$\{(3\times3\times(1\times2\times m+1))+8,\ (1\times3\times3\times(3\times2\times m+5))+7\},\ m \geq 0 \tag{1.51}$$

(1.52) $\{(3 \times 3 \times (1 \times 2 \times m + 1)) + 5,\ (1 \times 3 \times 3 \times (1 \times 1 \times m + 0)) + 7\},\ m \geq 0$

(1.53) $\{(3 \times 3 \times (1 \times 2 \times m + 0)) + 5,\ (1 \times 3 \times 3 \times (3 \times 2 \times m + 1)) + 7\},\ m \geq 0$

(1.54) $\{(3 \times 3 \times (1 \times 2 \times m + 1)) + 2,\ (1 \times 3 \times 3 \times (3 \times 2 \times m + 3)) + 7\},\ m \geq 0$

(1.55) $\{(3 \times 3 \times (1 \times 2 \times m + 1)) + 7,\ (1 \times 3 \times 3 \times (1 \times 1 \times m + 0)) + 8\},\ m \geq 0$

Determining Essential Pairs can be easily done by assigning base values for m multiplicity parameter on above Generic Pairs:

(1.56) $m = 1 \wedge (1.38) \rightarrow \{18, 9\}$

(1.57) $m = 0 \wedge (1.39) \rightarrow \{15, 46\}$

(1.58) $m = 0 \wedge (1.40) \rightarrow \{9, 28\}$

(1.59) $m = 0 \wedge (1.41) \rightarrow \{3, 10\}$

(1.60) $m = 0 \wedge (1.42) \rightarrow \{2, 1\}$

(1.61) $m = 0 \wedge (1.43) \rightarrow \{4, 2\}$

(1.62) $m = 0 \wedge (1.44) \rightarrow \{6, 3\}$

(1.63) $m = 0 \wedge (1.45) \rightarrow \{8, 4\}$

(1.64) $m = 0 \wedge (1.46) \rightarrow \{7, 22\}$

(1.65) $m = 0 \wedge (1.47) \rightarrow \{13, 40\}$

(1.66) $m = 0 \wedge (1.48) \rightarrow \{1, 4\}$

(1.67) $m = 0 \wedge (1.49) \rightarrow \{10, 5\}$

(1.68) $m = 0 \wedge (1.50) \rightarrow \{12, 6\}$

(1.69) $m = 0 \wedge (1.51) \rightarrow \{17, 52\}$

(1.70) $m = 0 \wedge (1.52) \rightarrow \{14, 7\}$

(1.71) $m = 0 \wedge (1.53) \rightarrow \{5, 16\}$

(1.72) $m = 0 \wedge (1.54) \rightarrow \{11, 34\}$

(1.73) $m = 0 \wedge (1.55) \rightarrow \{16, 8\}$

At this point, if presented Generic Pairs (or their base case representation Essential Pairs) would not be able express Collatz Pairs, it would necessarily mean that $n\ mod\ 2 \neq \{\overline{0}_2, \overline{1}_2\}$ or $n\ mod\ 9 \neq \{\overline{0}_9, \overline{1}_9, \overline{2}_9, \overline{3}_9, \overline{4}_9, \overline{5}_9, \overline{6}_9, \overline{7}_9, \overline{8}_9\}$, but any $c_k \in \mathbb{N}$ due to $f(n)$ definition. Additionally, note that any Essential Pair is compliant with $f(n)$ branches.

Example 1.3. Let us first obtain the Collatz sequence values for $f(17)$:

(1.74) $$f(17) = \{17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1\}$$

Let us group them as Collatz Pairs:

(1.75) $$f(17) = \{17, 52\}, \{52, 26\}, \{26, 13\}, \{13, 40\}, \{40, 20\}, \{20, 10\}, \{10, 5\}, \{5, 16\}, \{16, 8\}, \{8, 4\}, \{4, 2\}, \{2, 1\}$$

At this point, let us represent these Collatz Pairs with Generic Pairs and m-multiplicities:

(1.76) $\{(3 \times 3 \times (1 \times 2 \times 0 + 1)) + 8,\ (1 \times 3 \times 3 \times (3 \times 2 \times 0 + 5)) + 7\}$

(1.77) $\{(3 \times 3 \times (1 \times 2 \times 2 + 1)) + 7,\ (1 \times 3 \times 3 \times (1 \times 1 \times 2 + 0)) + 8\}$

(1.78) $\{(3 \times 3 \times (1 \times 2 \times 1 + 0)) + 8,\ (1 \times 3 \times 3 \times (1 \times 1 \times 1 + 0)) + 4\}$

(1.79) $\{(3 \times 3 \times (1 \times 2 \times 0 + 1)) + 4,\ (1 \times 3 \times 3 \times (3 \times 2 \times 0 + 4)) + 4\}$

(1.80) $\{(3 \times 3 \times (1 \times 2 \times 2 + 0)) + 4,\ (1 \times 3 \times 3 \times (1 \times 1 \times 2 + 0)) + 2\}$

(1.81) $\{(3 \times 3 \times (1 \times 2 \times 1 + 0)) + 2,\ (1 \times 3 \times 3 \times (1 \times 1 \times 1 + 0)) + 1\}$

(1.82) $\{(3 \times 3 \times (1 \times 2 \times 0 + 1)) + 1,\ (1 \times 3 \times 3 \times (1 \times 1 \times 0 + 0)) + 5\}$

(1.83) $\{(3 \times 3 \times (1 \times 2 \times 0 + 0)) + 5,\ (1 \times 3 \times 3 \times (3 \times 2 \times 0 + 1)) + 7\}$

(1.84) $\{(3 \times 3 \times (1 \times 2 \times 0 + 1)) + 7,\ (1 \times 3 \times 3 \times (1 \times 1 \times 0 + 0)) + 8\}$

(1.85) $\{(3 \times 3 \times (1 \times 2 \times 0 + 0)) + 8,\ (1 \times 3 \times 3 \times (1 \times 1 \times 0 + 0)) + 4\}$

(1.86) $\{(3 \times 3 \times (1 \times 2 \times 0 + 0)) + 4,\ (1 \times 3 \times 3 \times (1 \times 1 \times 0 + 0)) + 2\}$

$$\{(3 \times 3 \times (1 \times 2 \times 0 + 0)) + 2,\ (1 \times 3 \times 3 \times (1 \times 1 \times 0 + 0)) + 1\} \tag{1.87}$$

Finally, Generic Pairs are represented using Essential Pairs and m-multiplicities:

$$\begin{aligned} f(17) = \{&\{17,52,0\},\{16,8,2\},\{8,4,1\},\{13,40,0\},\{4,2,2\},\{2,1,1\},\\ &\{10,5,0\},\{5,16,0\},\{16,8,0\},\{8,4,0\},\{4,2,0\},\{2,1,0\}\} \end{aligned} \tag{1.88}$$

This is the multiplicities sequence for $f(17)$:

$$m = \{0,2,1,0,2,1,0,0,0,0,0,0\} \tag{1.89}$$

At this moment, it is still possible to revert the original Collatz sequence using Generic Pairs equations and the multiplicities as entry point.

□

2. The Essential Pairs Details

The capability to represent any C using a limited subset of pairs, eases pattern recognition. Let us analyze exposed Essential Pairs.

Lemma 2.1. *Any Essential Pair can be expressed using in turn Essential Pairs, being only {2,1} pair common to all these representations.*

Proof. List below contains all the Essential Pairs, enumerating required ones to represent themselves. As depicted, only {2,1} pair is present on any representation. Additionally, this pair has an unique property: it is the only self-contained one; it is named Root Pair.

$$\begin{aligned}\mathbf{\{18,\ 9\}} \rightarrow &\{18,9\},\{17,52\},\{16,8\},\{14,7\},\{13,40\},\{11,34\},\\ &\{10,5\},\{9,28\},\{8,4\},\{7,22\},\{4,2\},\{2,1\}\end{aligned} \tag{2.1}$$

$$\mathbf{\{15,\ 46\}} \rightarrow \{17,52\},\{16,8\},\{15,46\},\{10,5\},\{8,4\},\{4,2\},\{2,1\} \tag{2.2}$$

$$\begin{aligned}\mathbf{\{9,\ 28\}} \rightarrow &\{17,52\},\{16,8\},\{14,7\},\{13,40\},\{11,34\},\{9,28\},\\ &\{8,4\},\{7,22\},\{5,16\},\{4,2\},\{2,1\}\end{aligned} \tag{2.3}$$

$$\mathbf{\{3,\ 10\}} \rightarrow \{16,8\},\{10,5\},\{8,4\},\{5,16\},\{4,2\},\{3,10\},\{2,1\} \tag{2.4}$$

$$\mathbf{\{2,\ 1\}} \rightarrow \{2,1\} \tag{2.5}$$

$$\mathbf{\{4,\ 2\}} \rightarrow \{4,2\},\{2,1\} \tag{2.6}$$

$$\mathbf{\{6,\ 3\}} \rightarrow \{16,8\},\{10,5\},\{8,4\},\{6,3\}\{5,16\},\{4,2\},\{3,10\},\{2,1\} \tag{2.7}$$

$$\mathbf{\{8,\ 4\}} \rightarrow \{8,4\},\{4,2\},\{2,1\} \tag{2.8}$$

$$\begin{aligned}\mathbf{\{7,\ 22\}} \rightarrow &\{17,52\},\{16,8\},\{13,40\},\{11,34\},\{10,5\},\{8,4\},\{7,22\},\\ &\{5,16\},\{4,2\},\{2,1\}\end{aligned} \tag{2.9}$$

$$\mathbf{\{13,\ 40\}} \rightarrow \{16,8\},\{13,40\},\{10,5\},\{8,4\},\{5,16\},\{4,2\},\{2,1\} \tag{2.10}$$

$$\mathbf{\{1,\ 4\}} \rightarrow \{1,4\},\{4,2\},\{2,1\} \tag{2.11}$$

$$\mathbf{\{10,\ 5\}} \rightarrow \{16,8\},\{10,5\},\{8,4\},\{5,16\},\{4,2\},\{2,1\} \tag{2.12}$$

$$\begin{aligned}\mathbf{\{12,\ 6\}} \rightarrow &\{16,8\},\{12,6\},\{10,5\},\{8,4\},\{6,3\},\{5,16\},\{4,2\},\\ &\{3,10\},\{2,1\}\end{aligned} \tag{2.13}$$

$$\begin{aligned}\mathbf{\{17,\ 52\}} \rightarrow &\{17,52\},\{16,8\},\{13,40\},\{10,5\},\{8,4\},\{5,16\},\{4,2\},\\ &\{2,1\}\end{aligned} \tag{2.14}$$

$$\textbf{\{14, 7\}} \rightarrow \{17,52\},\{16,8\},\{14,7\},\{13,40\},\{11,34\},\{10,5\},\{8,4\},\{7,22\},\{5,16\},\{4,2\},\{2,1\} \tag{2.15}$$

$$\textbf{\{5, 16\}} \rightarrow \{16,8\},\{8,4\},\{5,16\},\{4,2\},\{2,1\} \tag{2.16}$$

$$\textbf{\{11, 34\}} \rightarrow \{17,52\},\{16,8\},\{13,40\},\{11,34\},\{10,5\},\{8,4\},\{5,16\},\{4,2\},\{2,1\} \tag{2.17}$$

$$\textbf{\{16, 8\}} \rightarrow \{16,8\},\{8,4\},\{4,2\},\{2,1\} \tag{2.18}$$

□

3. The Essential Pairs Relationships

Lemma 3.1. *Any Collatz sequence, represented by its Essential Pairs and m-multiplicities, is accepted by a finite-state machine.*

Proof. It is possible to describe a finite-state machine ready to digest any Collatz sequence C expressed using Essential Pairs and m-multiplicities. Note that multiplicities are flattened by $mmod2$, covering $f(n)$ branches. The FSM accepts the sequence on 210 suffix. As follows:

$$\begin{aligned}
M &= (Q, \Sigma, \delta, q_0, F) \\
Q &= \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}, q_{17}, q_{18}\} \\
\Sigma &= \{140, 141, 210, 211, 3100, 3101, 420, 421, 5160, 5161, 630, 631, 7220, 7221, \\
&\quad 840, 841, 9280, 9281, 1050, 1051, 11340, 11341, 1260, 1261, 13400, 13401, \\
&\quad 1470, 1471, 15460, 15461, 1680, 1681, 17520, 17521, 1890, 1891\} \\
q_0 &= q_0 \\
F &= q_1 \\
\delta &= see\ transitions\ below
\end{aligned}$$

$$\delta(q_0, 140) = q_4 \tag{3.1}$$
$$\delta(q_0, 141) = q_4 \tag{3.2}$$
$$\delta(q_0, 210) = q_1 \tag{3.3}$$
$$\delta(q_0, 211) = q_{10} \tag{3.4}$$
$$\delta(q_0, 3100) = q_{10} \tag{3.5}$$
$$\delta(q_0, 3101) = q_{10} \tag{3.6}$$
$$\delta(q_0, 420) = q_2 \tag{3.7}$$
$$\delta(q_0, 421) = q_{11} \tag{3.8}$$
$$\delta(q_0, 5160) = q_{16} \tag{3.9}$$
$$\delta(q_0, 5161) = q_{16} \tag{3.10}$$
$$\delta(q_0, 630) = q_3 \tag{3.11}$$
$$\delta(q_0, 631) = q_{12} \tag{3.12}$$
$$\delta(q_0, 7220) = q_4 \tag{3.13}$$
$$\delta(q_0, 7221) = q_4 \tag{3.14}$$
$$\delta(q_0, 840) = q_4 \tag{3.15}$$
$$\delta(q_0, 841) = q_{13} \tag{3.16}$$
$$\delta(q_0, 9280) = q_{10} \tag{3.17}$$
$$\delta(q_0, 9281) = q_{10} \tag{3.18}$$
$$\delta(q_0, 1050) = q_5 \tag{3.19}$$
$$\delta(q_0, 1051) = q_{14} \tag{3.20}$$
$$\delta(q_0, 11340) = q_{16} \tag{3.21}$$
$$\delta(q_0, 11341) = q_{16} \tag{3.22}$$
$$\delta(q_0, 1260) = q_6 \tag{3.23}$$
$$\delta(q_0, 1261) = q_{15} \tag{3.24}$$
$$\delta(q_0, 13400) = q_4 \tag{3.25}$$
$$\delta(q_0, 13401) = q_4 \tag{3.26}$$
$$\delta(q_0, 1470) = q_7 \tag{3.27}$$
$$\delta(q_0, 1471) = q_{16} \tag{3.28}$$
$$\delta(q_0, 15460) = q_{10} \tag{3.29}$$
$$\delta(q_0, 15461) = q_{10} \tag{3.30}$$

$$\delta(q_0, 1680) = q_8 \tag{3.31}$$
$$\delta(q_0, 1681) = q_{17} \tag{3.32}$$
$$\delta(q_0, 17520) = q_{16} \tag{3.33}$$
$$\delta(q_0, 17521) = q_{16} \tag{3.34}$$
$$\delta(q_0, 1890) = q_{18} \tag{3.35}$$
$$\delta(q_0, 1891) = q_9 \tag{3.36}$$
$$\delta(q_1, 140) = q_4 \tag{3.37}$$
$$\delta(q_1, 141) = q_4 \tag{3.38}$$
$$\delta(q_2, 210) = q_1 \tag{3.39}$$
$$\delta(q_2, 211) = q_{10} \tag{3.40}$$
$$\delta(q_3, 3100) = q_{10} \tag{3.41}$$
$$\delta(q_3, 3101) = q_{10} \tag{3.42}$$
$$\delta(q_4, 420) = q_2 \tag{3.43}$$
$$\delta(q_4, 421) = q_{11} \tag{3.44}$$
$$\delta(q_5, 5160) = q_{16} \tag{3.45}$$
$$\delta(q_5, 5161) = q_{16} \tag{3.46}$$
$$\delta(q_6, 630) = q_3 \tag{3.47}$$
$$\delta(q_6, 631) = q_{12} \tag{3.48}$$
$$\delta(q_7, 7220) = q_4 \tag{3.49}$$
$$\delta(q_7, 7221) = q_4 \tag{3.50}$$
$$\delta(q_8, 840) = q_4 \tag{3.51}$$
$$\delta(q_8, 841) = q_{13} \tag{3.52}$$
$$\delta(q_9, 9280) = q_{10} \tag{3.53}$$
$$\delta(q_9, 9281) = q_{10} \tag{3.54}$$
$$\delta(q_{10}, 1050) = q_5 \tag{3.55}$$
$$\delta(q_{10}, 1051) = q_{14} \tag{3.56}$$
$$\delta(q_{11}, 11340) = q_{16} \tag{3.57}$$
$$\delta(q_{11}, 11341) = q_{16} \tag{3.58}$$
$$\delta(q_{12}, 1260) = q_6 \tag{3.59}$$
$$\delta(q_{12}, 1261) = q_{15} \tag{3.60}$$
$$\delta(q_{13}, 13400) = q_4 \tag{3.61}$$
$$\delta(q_{13}, 13401) = q_4 \tag{3.62}$$
$$\delta(q_{14}, 1470) = q_7 \tag{3.63}$$
$$\delta(q_{14}, 1471) = q_{16} \tag{3.64}$$
$$\delta(q_{15}, 15460) = q_{10} \tag{3.65}$$
$$\delta(q_{15}, 15461) = q_{10} \tag{3.66}$$
$$\delta(q_{16}, 1680) = q_8 \tag{3.67}$$
$$\delta(q_{16}, 1681) = q_{17} \tag{3.68}$$
$$\delta(q_{17}, 17520) = q_{16} \tag{3.69}$$
$$\delta(q_{17}, 17521) = q_{16} \tag{3.70}$$
$$\delta(q_{18}, 1890) = q_{18} \tag{3.71}$$
$$\delta(q_{18}, 1891) = q_9 \tag{3.72}$$

The ability to represent any Collatz series using a limited subset of pairs eases the pattern recognition. As per 1.2, any Collatz Pair can be expressed as an Essential Pair and a m-multiplicity, being the multiplicities size (converging to zero) proportional to the Collatz Pair represented. Additionally, has been shown in 2.1 that any representation always uses the Root Pair and $m = 0$ as a final pair.

Example 3.2. Based on (1.88), $f(17)$ sequence transformation accepted for M is:

$$f(17) = \{17520, 1680, 841, 13400, 420, 211, 1050, 5160, 1680, 840, 420, 210\} \tag{3.73}$$

Acceptance path, in this case, is as follows:

$$q_0 \to q_{16} \to q_8 \to q_{13} \to q_4 \to q_2 \to q_{10} \to q_5 \to q_{16} \to q_8 \to q_4 \to q_2 \to q_1$$

□

Since any Collatz sequence C can be grouped in pairs, any of these pairs can be reworked as depicted during this paper and that final transformation is always be digested by the M acceptor on the suffix $\{210\}$, it is possible to state that Collatz conjecture is correct. Future job can be done validating this procedure on other sequences, studying relationship between Collatz sequences with same Essential Pairs or m-multiplicities, for instance.

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Abstract. Do Collatz sequences share a common structure? If so, does it prove that, from any initial value n, Collatz sequences always reach value 1? This paper tries to answer these two questions disclosing the embedded pattern present in any Collatz sequence.

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