Hans Weeks
Software Specifications
September 24, 2017

Homework 4

Concept:
$\mu =$ number of cells without final label. this is a well founded metric because...
$\mu = \langle \sum a :: \neg \textit{final label } a \rangle$
show:
$\mu = k > 0 \leadsto \mu < k$
$\mu = 0 \implies post$

1. init $\leadsto$ post
   let's define μ:
   $\mu = \langle\ \Sigma\ \forall\ x : \pi(x) :: L(x) \neq \lambda(x)\ \rangle$

   $\mu = k \leadsto \mu < k$
   because
   $\pi(x) = (i,j) \wedge L(x) \neq \lambda(x) \wedge \pi(y) = (p,q) \wedge L(y) = \lambda(y)$
   $L(i,j) := min(L(i,j), L(p,q))\ if\ SameRegion((i,j),(p,q))\ ensures\ L(x) = \lambda(x)$
   $\langle \forall x\ : \pi(x) = (i,j)\ :: \exists\ i,j : 0 \leq i,j \leq N :: L(x) = \lambda(x)\ \rangle$

   //need to show that for every region there exists at least one x such that L(x) = λ(x)
   //need to show that $\mu \implies$ post

   $$\mu = 0 \implies post$$

   if the number of elements that don't have there final label is equal to zero, then all elements must have there final label
   ✓ $\mu = 0 \implies \langle \forall x : \pi(x) :: L(x) = \lambda(x) \rangle$

   $$L(x) = L(y) \wedge \neg(x \Gamma y) \implies L(x) \neq \lambda(x) \vee L(y) \neq \lambda(y)$$

   which implies that $\mu(x) > 0$
   therefore
   ✓$\mu(x) = 0 \implies \langle \forall x, y\ : \pi(x)\ \wedge \pi(x) \wedge \neg(x \Gamma y) ::\ L(x) \neq L(x)\ \rangle$

2. **stable** post

$$
\begin{aligned}
&post \equiv \langle \forall x \;:\; \pi(x) \;::\; L(x) = \lambda(x) \;\rangle \\
&\wedge \;\langle \forall x, y \;: \pi(x) \;\wedge \pi(x) \wedge \neg(x \Gamma y) ::\; L(x) \neq L(x) \;\rangle \\
\\
&part1 : \\
&post \equiv \langle \forall x \;:\; \pi(x) \;::\; L(x) = \lambda(x) \;\rangle \\
&// this\ needs\ a\ proof \\
\\
&part2 : \\
&invariant : \langle \forall x, y : \pi(x) \;\wedge\; \neg(x \Gamma y) :: L(x) \neq L(y) \rangle \\
&because : \\
&initially\ L(x) \neq L(y) \\
&stable\ L(x) \neq L(y)
\end{aligned}
$$