Hans Weeks
Software Specifications
September 24, 2017

Homework 4

Concept:
$\mu =$ number of cells without final label. this is a well founded metric because...
$\mu = \langle \sum a :: \neg \mathit{final\ label\ a} \rangle$
show:
$\mu = k > 0 \rightsquigarrow \mu < k$
$\mu = 0 \implies post$

1. init $\rightsquigarrow$ post
   let's define μ:
   $\mu = \langle\ \Sigma\ \forall\ x : \pi(x) :: L(x) \neq \lambda(x)\ \rangle$

   $\mu = k \rightsquigarrow \mu < k$
   Let's say that:
   $\pi(x) = (i, j) \wedge L(x) \neq \lambda(x) \wedge \pi(y) = (p, q) \wedge L(y) = \lambda(y)$
   then
   $L(i, j) := min(L(i, j), L(p, q))\ if\ SameRegion((i, j), (p, q))\ ensures\ L(x) = \lambda(x)$
   and
   $\langle \forall x\ : \pi(x) = (i, j)\ :: \exists\ i, j : 0 \leq i, j \leq N :: L(x) = \lambda(x)\ \rangle$

   meaning that if x does not have its final label and y does have its final label, then we can select the assignment statement that takes x and y, and this statement ensures that μ decreases.
   Additionally, there always exists such a y that $L(y) = \lambda(y)$, because at least one element is set to the minimum label in its region.

   $$\mu = 0 \implies post$$

   if the number of elements that don't have there final label is equal to zero, then all elements must have there final label
   ✓ $\mu = 0 \implies \langle \forall x : \pi(x) :: L(x) = \lambda(x) \rangle$

   $$L(x) = L(y) \wedge \neg(x \Gamma y) \implies L(x) \neq \lambda(x) \vee L(y) \neq \lambda(y)$$

which implies that $\mu(x) > 0$
therefore
$\checkmark\, \mu(x) = 0 \implies \langle \forall x, y \ : \pi(x) \ \wedge \pi(x) \wedge \neg(x \Gamma y) :: \ L(x) \neq L(x) \ \rangle$

2. **stable** post

$$post \equiv \langle \forall x \ : \ \pi(x) \ :: \ L(x) = \lambda(x) \ \rangle$$
$$\wedge \ \langle \forall x, y \ : \pi(x) \ \wedge \pi(x) \wedge \neg(x \Gamma y) :: \ L(x) \neq L(x) \ \rangle$$

$part1:$
$post \equiv \langle \forall x \ : \ \pi(x) \ :: \ L(x) = \lambda(x) \ \rangle$
$//this\ needs\ a\ proof$

$part2:$
$invariant : \langle \forall x, y : \pi(x) \ \wedge \ \neg(x \Gamma y) :: L(x) \neq L(y) \rangle$
$because:$
$initially\ L(x) \neq L(y)$
$stable\ L(x) \neq L(y)$