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Software Specifications
October 1, 2017

Homework 5: Formal properties

Program Description
Consider a square matrix B of size N. Let A represent the initial configuration of the matrix B.

1. Write a UNITY program that transposes the rows and columns of matrix B and preserves the following invariant:

   $inv.\ p \leq\ q \wedge$

   $\langle\ \forall i,j\ :\ (1 \leq i < p \vee 1 \leq j < p \vee q < i \leq N \vee q < j \leq N) \wedge\ 1 \leq i \leq N \wedge 1 \leq j \leq N\ ::$
   $B[i,j] = A[j,i]\rangle$

   Program Transpose

   declare

   $$A : array\ [1..N,\ 1..N]\ of\ integer$$
   $$p, q : integer$$

   initially

   $$A = B$$
   $$p = 1$$
   $$q = N$$

   always
   assign

$$p := p + 1 \ if \ p \leq q$$
$$\sim q$$
$$\| \ q := q - 1 \ if \ p \leq q$$
$$\|$$
$$\langle \ \| \ j \in \{p, q\} \wedge q < i < p \ :: \ B[i, j] := B[j, i] \rangle$$
$$\|$$
$$\langle \ \| \ i \in \{p, q\} \wedge q < j < p \ :: \ B[i, j] := B[j, i] \rangle$$

2. Write a formal specification of the correctness of the program you designed. Such a specification often assumes the following general form:

$$init \ \leadsto \ Post$$
stable $Post$
$$init \equiv A = B$$
$$post \equiv \forall i, j : 1 \leq i, j \leq N :: A[i, j] = B[j, i]$$
inv 1
inv $\forall i, j : 1 \leq i, j \leq N :: B[i, j] = A[i, j] \vee A[j, i]$

3. Explain in narrative form (no formal proof) the steps involved in proving these two properties.