

Homework 5: Formal properties

Program Description

Consider a square matrix B of size N. Let A represent the initial configuration of the matrix B.

1. Write a UNITY program that transposes the rows and columns of matrix B and preserves the following invariant:

$$\begin{aligned} \text{inv. } & p \leq q \wedge \\ & \langle \forall i, j : (1 \leq i < p \vee 1 \leq j < p \vee q < i \leq N \vee q < j \leq N) \wedge 1 \leq i \leq N \wedge 1 \leq j \leq N :: \\ & B[i, j] = A[j, i] \rangle \end{aligned}$$

Program Transpose

declare

$A : \text{array } [1..N, 1..N] \text{ of integer}$
 $p, q : \text{integer}$

initially

$$\begin{aligned} A &= B \\ p &= 1 \\ q &= N \end{aligned}$$

always

assign

$$\begin{aligned} & \langle \parallel i \in \{p+1, q-1\} \wedge (q-1) < j < (p+1) :: A[i, j] := B[j, i] \rangle \\ & \parallel \\ & \langle \parallel j \in \{p+1, q-1\} \wedge (q-1) < i < (p+1) :: A[i, j] := B[j, i] \rangle \\ & \parallel \\ & p, q := p+1, q-1 \text{ if } p \leq q \end{aligned}$$

2. Write a formal specification of the correctness of the program you designed. Such a specification often assumes the following general form:

$$init \rightsquigarrow Post$$

$$\mathbf{stable} \ Post$$

$$init \equiv A = B$$

$$post \equiv \forall i, j : 1 \leq i, j \leq N :: A[i, j] = B[j, i]$$

3. Explain in narrative form (no formal proof) the steps involved in proving these two properties.