

## Homework 4

Concept:

$\mu$  = number of cells without final label. this is a well founded metric because...

$$\mu = \langle \sum a :: \neg \text{final label } a \rangle$$

show:

$$\mu = k > 0 \rightsquigarrow \mu < k$$

$$\mu = 0 \implies \text{post}$$

1.  $\text{init} \rightsquigarrow \text{post}$   
let's define  $\mu$ :

$$\mu = \langle \sum \forall x : \pi(x) :: L(x) \neq \lambda(x) \rangle$$

$$\mu = k \rightsquigarrow \mu < k$$

*because*

$$\pi(x) = (i, j) \wedge L(x) \neq \lambda(x) \wedge \pi(y) = (p, q) \wedge L(y) = \lambda(y)$$

$$L(i, j) := \min(L(i, j), L(p, q)) \text{ if } \text{SameRegion}((i, j), (p, q)) \text{ ensures } L(x) = \lambda(x)$$

$$\langle \forall x : \pi(x) = (i, j) :: \exists i, j : 0 \leq i, j \leq N :: L(x) = \lambda(x) \rangle$$

//need to show that for every region there exists at least one x such that

$$L(x) = \lambda(x)$$

//need to show that  $\mu \implies \text{post}$

## 2. **stable** post

$post \equiv \langle \forall x : \pi(x) :: L(x) = \lambda(x) \rangle$   
 $\wedge \langle \forall x, y : \pi(x) \wedge \pi(y) \wedge \neg(x \Gamma y) :: L(x) \neq L(y) \rangle$

*part1 :*

$post \equiv \langle \forall x : \pi(x) :: L(x) = \lambda(x) \rangle$   
*//this needs a proof*

*part2 :*

*invariant :*  $\langle \forall x, y : \pi(x) \wedge \neg(x \Gamma y) :: L(x) \neq L(y) \rangle$

*because :*

*initially*  $L(x) \neq L(y)$

*stable*  $L(x) \neq L(y)$