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Software Specifications
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Homework 4

Concept:
$\mu =$ number of cells without final label. this is a well founded metric because...
$\mu = \langle \sum a :: \neg final\ label\ a \rangle$
show:
$\mu = k > 0 \rightsquigarrow \mu < k$
$\mu = 0 \implies post$

1. init $\rightsquigarrow$ post
   let's define μ:
   $\mu = \langle\ \Sigma\ \forall\ x : \pi(x) :: L(x) \neq \lambda(x)\ \rangle$

   With this μ in mind, we would like to show 2 things:
   1. μ only decreases and 2. when $\mu = 0$, then we have reached post.

   Part 1:

   $$\mu = k \rightsquigarrow \mu < k$$

   μ increases or stays the same. If k is 0, then it never decreases, just remains 0. If k is greater than 0, then there exists a statement S_{ijpq} where (i, j) is an element that does not have its final label and (p, q) does have its final label. Then

   $$S_{ijpq}\ ensures\ \mu < k$$

   By invariant 1 we know that such an element pq always exists.

   Part2:

   $$\mu = 0 \implies post$$

   First, if the number of elements that don't have there final label is equal to zero, then all elements must have there final label

✓ $\mu = 0 \implies \langle \forall x : \pi(x) :: L(x) = \lambda(x) \rangle$

Second, if we have 2 elements that are not in the same region but have the same label, that implies that one of those elements does not have its final label:

$$L(x) = L(y) \wedge \neg(x\Gamma y) \implies L(x) \neq \lambda(x) \vee L(y) \neq \lambda(y)$$

which implies that $\mu(x) > 0$
therefore
✓ $\mu(x) = 0 \implies \langle \forall x, y \ : \pi(x) \ \wedge \pi(x) \wedge \neg(x\Gamma y) :: \ L(x) \neq L(x) \ \rangle$

2. **stable** post

$$post \equiv \langle \forall x \;:\; \pi(x) \;::\; L(x) = \lambda(x) \;\rangle$$
$$\wedge \;\; \langle \forall x, y \;\; : \pi(x) \;\; \wedge \pi(x) \wedge \neg(x \Gamma y) :: \;\; L(x) \neq L(x) \;\rangle$$

$part1:$
$post \equiv \langle \forall x \;:\; \pi(x) \;::\; L(x) = \lambda(x) \;\rangle$
$//this\ needs\ a\ proof$

$part2:$
$invariant: \langle \forall x, y : \pi(x) \;\; \wedge \;\; \neg(x \Gamma y) :: L(x) \neq L(y) \rangle$
$because:$
$initially\ L(x) \neq L(y)$
$stable\ L(x) \neq L(y)$