2.

(a)according to the formula of up-limit of error

$$|\Delta f(x)| \leq \frac{\gamma}{4(n+1)} h^{n+1}$$

$$\gamma = \max|f^{n+1}(x)|$$
$$h = \max\,(x_{i+1} - x_i)$$

here n=1, $\gamma = \max\,(\sin(x)) \leq \sin\,(1)$

we calculate **h=0.00218**

since it's a sufficient condition, larger h may also allow $|\Delta f(x)|$ to smaller than the requirement, we decide to search for h in a more accurate way:

since in each segment we have $|\Delta f(x)| \leq \frac{\gamma}{8} h^2$, where the second derivative of sin(x) is –sin(x), it's reasonable to **assume that largest error always occur at segment [1-h,1]**, so we firstly set h, and then calculate 1000 points in segment [1-h,1] both for sin(x) and Lagrange interpolation and find the largest error among them. If the largest error is still smaller than the requirement, we increase h further, until some h result in a out-of-requirement error(here we just increase h by hand for simplicity, without any iteration method).

Code 1: for Lagrange interpolation

```
function yi = LagrangeInterp(x,y,xi)
n = length(x);
L = zeros(1,n);
for i = 1:n,
L(i) = 1;
for j = 1:n,
if j ~= i,
L(i) = L(i)* (xi - x(j))/(x(i) - x(j));
end
end
end
yi = sum(y.* L);
```

Code 2 : search for a more accurate h

```
%Lagrange Interpolation of sin(x) in [0,1] to 1th order
h=0.002181;  %set h
X=[1-h,1];
Y=[sin(1-h),sin(1)];
Err=zeros(1,1000);
for i=1:1000
    Err(i)=sin(1-h+i/1000*h)-LagrangeInterp(X,Y,1-h+i/1000*h);
end
Errmax=max(abs(Err))
```

And we find h is in **somewhere between 0.002181 and 0.002182** (the Errmax of 0.002181 is 4.9998e-07, and of 0.002182 is 5.0044e-07). So it seems that the estimation from the formula is already good in this case.

(b)in this case n=2, $\gamma = \max(\cos(x)) \leq 1$, we calculate **h = 0.018171**

Again we apply the same method to search for a more accurate h(only with difference that we assume the largest error always occur at [0,2h])

```
%Lagrange Interpolation of sin(x) in [0,1] to 2th order
h=0.0198;
X=[0,h,2*h];
Y=[0,sin(h),sin(2*h)];
Err=zeros(1,1000);
for i=1:1000
    Err(i)=sin(i/1000*2*h)-LagrangeInterp(X,Y,i/1000*2*h);
end
Errmax=max(abs(Err))
```

And we find h is in **somewhere between 0.0198 and 0.0199**

(the Errmax of 0.0198 is 4.9787e-07, and of 0.0199 is 5.0546e-07).

In this case the accurate h is more larger than estimation.

(c) in this case n=3, $\gamma = \max(\sin(x)) \leq \sin(1)$, we calculate **h = 0.05528**

Again we apply the same method to search for a more accurate h(only with difference that we assume the largest error always occur at [1-3h,1])

```
%Lagrange Interpolation of sin(x) in [0,1] to 3th order
h=0.062;
X=[1-3*h,1-2*h,1-h,1];
Y=[sin(1-3*h),sin(1-2*h),sin(1-h),sin(1)];
Err=zeros(1,1000);
for i=1:1000

Err(i)=sin(1-3*h+i/1000*3*h)-LagrangeInterp(X,Y,1-3*h+i/1000*3*h);
end
Errmax=max(abs(Err))
```

And we find h is in **somewhere between 0.062 and 0.063**

(the Errmax of 0.062 is 4.8997e-07, and of 0.063 is 5.2184e-07).

In this case the accurate h is more larger than estimation.

3. we write a general code applying Newton divided difference method

```
%Newton divided difference n order
%X, Y should be row
function [P,D]=Newtondivdif(X,Y,x)
n=length(X)-1;
D=zeros(n+1,n+1);
D(:,1)=Y';
for i=1:n
    for k=i:n
        D(k+1,i+1)=(D(k,i)-D(k+1,i))/(X(k-i+1)-X(k+1));
    end
end
XX=ones(1,n);
for p=1:n
    for q=p:n
        XX(1,q)=XX(1,q)*(x-X(p));
    end
end

P=D(1,1);
for j=1:n
    P=P+D(j+1,j+1)*XX(j);
end
```

divided difference table:

0.6900	0	0	0	0
1.1000	0.4100	0	0	0
1.3900	0.2900	-0.0600	0	0
1.6100	0.2200	-0.0350	0.0083	0
1.9500	0.1700	-0.0167	0.0046	-0.0008

$P_4(2.4)=1.2272$

$P_4(4.2)=1.6487$