2. Firstly, because the distribution of force is symmetric with respect to the axis x=L/2 (the differential equation as well) , so the solution should be symmetric with respect to the axis x=L/2. Hence, we can reduce our region into [0,L/2]. Also we get a boundary condition u'(L/2)=0.

In the region where $|x - L/2| > x_0$ (or $[0, x_d]$ where $x_d = \frac{L}{2} - x_0$), the equation becomes

$$\frac{d^2u}{dx^2} = -\frac{\rho g}{YI}$$

Which has a general solution

$$u = -\frac{1}{2}\frac{\rho g}{YI}x^2 + \beta x + \gamma$$

since u(0)=0, here $\gamma = 0$, rewrite u as $u = \alpha x^2 + \beta x$ (where β is unknown)

then, in the region $[x_d, \frac{L}{2}]$, we use finite difference method to discretize the equation (let's say, N sub-intervals) (the length of each sub-intervals is h)

$$\frac{u_{n+1} + u_{n-1} - 2u_n}{h^2} = -\frac{f_0}{YI}e^{-(\frac{x-\frac{L}{2}}{x_0})^2} + \frac{f_0 e^{-1} - \rho g}{YI} = g(x_n)$$

$$n = 2,3, \ldots N$$

at $x = x_d$, we let both the function value and first order derivative be continuous, i.e.

$$\alpha {x_d}^2 + \beta x_d = u_1$$

$$2\alpha x_d + \beta = \frac{1}{2h}(-3u_1 + 4u_2 - u_3)$$

At $x = L/2$, we have

$$\frac{3u_{N+1} - 4u_N + u_{N-1}}{2h} = 0$$

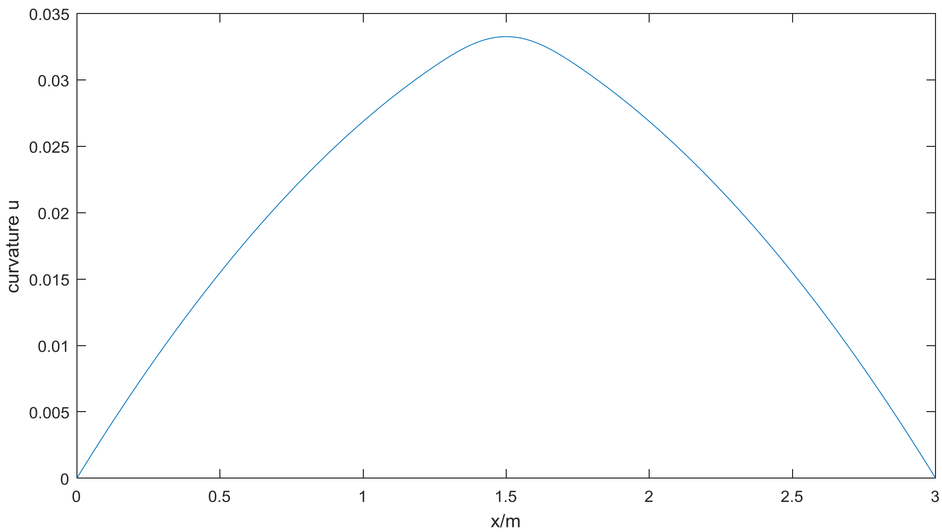
Then the totally N-1 equations can be expressed as

$$
\begin{pmatrix}
\frac{4}{3+\frac{2h}{x_d}}-2 & 1-\frac{1}{3+\frac{2h}{x_d}} & 0 & \cdots & 0 & \\
1 & -2 & 1 & & & \\
0 & 1 & -2 & & & \\
\vdots & & & \ddots & \vdots & \\
 & & & -2 & 1 & 0 \\
0 & & \cdots & 1 & -2 & 1 \\
 & & & 0 & \frac{2}{3} & -\frac{2}{3}
\end{pmatrix}
\begin{pmatrix} u_2 \\ u_3 \\ \vdots \\ u_{N-1} \\ u_N \end{pmatrix}
=
\begin{pmatrix} h^2 g(x_2)+\frac{2x_d\alpha h}{3+\frac{2h}{x_d}} \\ h^2 g(x_3) \\ \vdots \\ h^2 g(x_{N-1}) \\ h^2 g(x_N) \end{pmatrix}
$$

Which could be solved by Thomas method. After that we could calculate u_1, u_{N+1}, when β.

Result:

We plot the function at whole region of [0,L] (N=50)



Script Code:

```
f0=200; x0=0.25; L=3; w=0.2; t=0.03; ro=3; Y=1000000000; g=9.8;
I=t*t*t*w/3;
alpha=-ro*g/Y/I/2;
g=@(x) -f0/Y/I*exp(-(x-L/2).*(x-L/2)/x0/x0)+f0/exp(1)/Y/I-ro*g/Y/I;
N=30;
x=linspace(L/2-x0,L/2,N+1);
h=x(2)-x(1);  %preliminary condition
A=zeros(N-1,N-1);
for i=2:N-2
   A(i,i-1)=1;
   A(i,i+1)=1;
   A(i,i)=-2;
end
A(1,1)=4/(3+2*h/(L/2-x0))-2;
A(1,2)=1-1/(3+2*h/(L/2-x0));
A(N-1,N-2)=2/3;
A(N-1,N-1)=-2/3;
```

```
b=g(x(2:N))*h*h;
b(1)=b(1)+2*alpha*h*(L/2-x0)/(3+2*h/(L/2-x0));  %construct the matrix
U=thomas_tridiagonal(A,b);
u1=(4*U(1)-U(2)-(L/2-x0)*2*alpha*h)/(3+2*h/(L/2-x0));
un1=(4*U(N-1)-U(N-2))/3;
beita=(u1-alpha*(L/2-x0)*(L/2-x0))/(L/2-x0);
xx=linspace(0,L/2-x0,1000);
yy=xx.*xx*alpha+xx*beita;
UU=[yy,U,[un1],U(end:-1:1),yy(end:-1:1)];
XX=[xx,x(2:end),x(2:end)+x0,xx(2:end)+L/2+x0];  %extend into whole region
plot(XX,UU);
```