HW - Markovian Drone

Note-1: I got the basic code from CS237B_HW1 It had three files: `q_learning.py, utils.py, and value_iteration.py`. I made `utils_other.py` to be separate from the main `utils.py` file and filled in the missing parts in `q_learning.py` and `value_iteration.py`.

Value Iteration

The first four questions are about value iteration, and the remaining ones are about Q-learning; therefore, I have used the headers accordingly.

Question-1

Given $n = 20$, $\sigma = 1$, $\gamma = 0.95$, $x_{eye} = (15, 7)$, and $x_{goal} = (19, 9)$, write code that uses value iteration to find the optimal value function for the drone to navigate the storm. Recall that value iteration repeats the Bellman update

$$\begin{cases} R(x,u) + \gamma \sum_{x' \in X} p(x' \mid x, u) V(x') & \text{if } x \text{ is not a terminal state} \\ R(x,u) & \text{otherwise} \end{cases}$$

until convergence, where $p(x' \mid x, u)$ is the probability distribution of the next state being x' after taking action u in state x, and R is the reward function.

Answer-1

To solve the question, I had to complete the missing parts of the `value_iteration` function in the `value_iteration.py` file. I have included all the necessary information as comments in the code.

```python
def value_iteration(problem, reward, terminal_mask, gam):
    Ts = problem["Ts"]  # Transition matrices for each action
    sdim, adim = Ts[0].shape[-1], len(Ts)  # State and action dimension
    # sdim = 400, adim = 4
    V = tf.zeros([sdim])  # Initial value function estimate

    assert terminal_mask.ndim == 1 and reward.ndim == 2

    # perform value iteration
    for _ in range(10000):
        ######### Your code starts here #########

        # perform the value iteration update
        # V has shape [sdim]; sdim = n * n is the total number of grid state
        V_prev = tf.identity(V)
        V_new = tf.zeros_like(V)

        # Ts is a 4 element python list of transition matrices for 4 actions
        for action in range(adim):
            # Transition probabilies -> p(x'|x,u)
            T = Ts[action]

            # R(x, u) + gamma * sum(p(x'|x,u) * V(x')))
            V_action = tf.reduce_sum(T * V_prev, axis=1)
            V_temp = reward[:, action] + gam * V_action

            # V changes all the time and takes the best action
            V_new = tf.maximum(V_new, V_temp)

        # Apply the terminal state mask: Set the value of terminal states to 0
        V_new = tf.where(terminal_mask, reward[:, 0], V_new)
        # compute err = tf.linalg.norm(V_new - V_prev) as a breaking condition
        err = tf.linalg.norm(V_new - V_prev)

        ######### Your code ends here ###########

        V = V_new

        if err < 1e-7:
            break

    return V
```

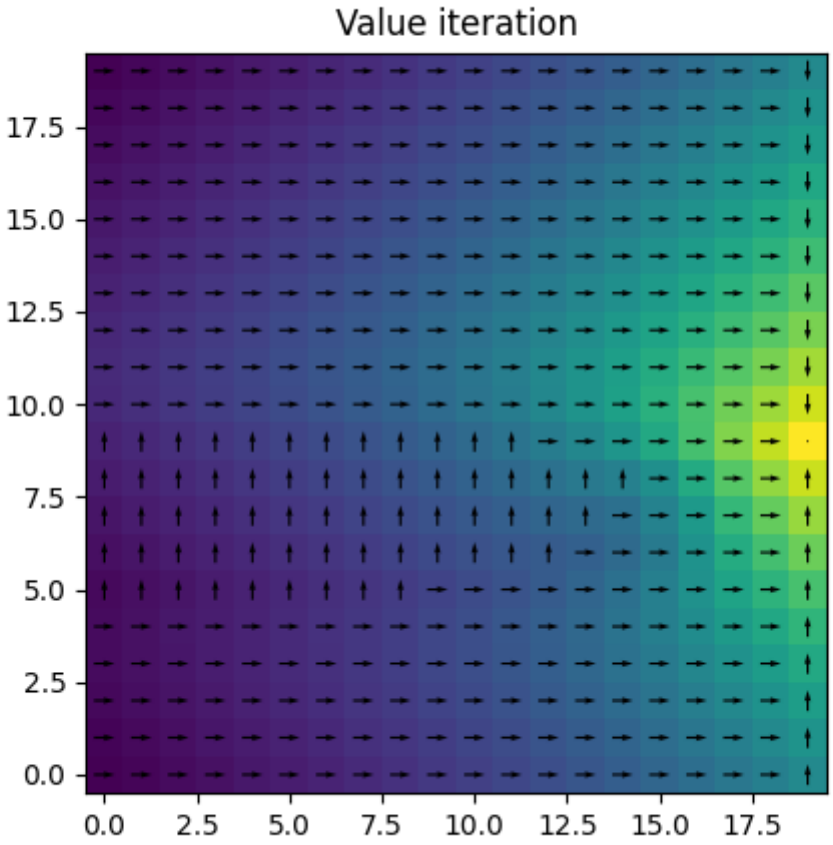
Question-2

Plot a heat map of the optimal value function obtained by value iteration over the grid X, with $x = (0, 0)$ in the bottom left corner, $x = (n - 1, n - 1)$ in the top right corner, the x_1-axis along the bottom edge, and the x_2-axis along the left edge. *Hint: We provide a function that plots the heat map: `visualize_value_function()

Answer-2

Using the Bellman equation and the code from the first answer, I was able to compute an optimal value function. For visualization, I utilized the

`visualize_value_function()` from the `utils.py` file as mentioned. The output is:



The output appeared fine and nearly perfect because the questions and the code indicated that there was wind at $(15, 7)$, and the arrows demonstrated that the wind should be avoided as much as possible to increase the chances of moving uninterruptedly.

Question-3

Recall that a policy π is a mapping $\pi : X \to U$ where $\pi(x)$ specifies the action to be taken should the drone find itself in state x. An optimal value function V^* induces an optimal policy π^* such that

$$\pi^*(x) \in \underset{u \in U}{\operatorname{argmax}} \left\{ R(x,u) + \gamma \sum_{x' \in X} p(x' \mid x, u) V^*(x') \right\}$$

Note that the optimal policy is only defined for non-terminal states. Use the value function you computed in part (a) to compute an optimal policy. Then, use this policy to simulate the MDP starting from $x = (0, 0)$ over $N = 100$ time steps.

Answer-3

To answer that question, I created two functions that reside in `utils_other.py`. In the first function, I utilized the formula provided by the question to determine the best action for each state and then used that information to establish the best policy. Below is the code, complete with comments that explain things as necessary:

```python
def derive_policy(V, problem, reward, gamma):
    Ts = problem["Ts"]  # Transition matrices for each action
    sdim, adim = Ts[0].shape[-1], len(Ts)

    policy = np.zeros(sdim, dtype=int)
    for s in range(sdim):
        # Initialize a list to store the values of each action
        action_values = []
        for a in range(adim):
            # Calculate the value for this action
            # R(x, u) + gamma * sum(p(x'|x,u) * V^*(x'))
            # Here V^*(x') is the optimal value function
            action_value = reward[s, a] + gamma * np.dot(Ts[a][s], V)
            action_values.append(action_value)

        # Select the action which gives the maximum value
        # Taking the argmax of the bellman equation with optimal value function
        best_action = np.argmax(action_values)
        policy[s] = best_action

    return policy
```

Then, the second function used this best policy and the probabilities of the value function to calculate the drone's route from the starting point to the ending (reward) point. Below is the code, complete with comments that

explain things as necessary:

```python
def simulate_mdp(policy, problem, start_state, N=100):
    n = problem["n"]
    Ts = problem["Ts"]  # Not each row sums to 1 because of floating point precision
    states = [start_state]
    current_state = start_state

    for _ in range(N):
        # Take the optimal action given current state
        action = policy[current_state]
        # Get the transition probabilities for the chosen action
        probabilities = Ts[action][current_state].numpy()
        # NOTE: I got error on the np.random.choice call below saying that probs did not sum to 1
        # I tried a lot of things to solve it and the error was because of floating point precision
        # So I just normalized the probabilities and it did not solve
        # But turning tf tensor to numpy array just solved it magically
        # Numpy handled floating point precision much better

        # Generate the next state based on the transition probabilities for the chosen action
        next_state = np.random.choice(np.arange(n * n), p=probabilities)
        states.append(next_state)
        current_state = next_state

    # Convert state indices back to (x, y) coordinates for visualization
    coords = [(state % n, state // n) for state in states]
    return coords
```

Important Note: There is a very important concept for these two functions. Since we calculated the best action for each state, we initially looped over states and then over actions. However, the transition matrices were created in the exact opposite order. This means that we have to change the ordering of rows and columns to achieve the finalized form of the policy and route. The process is straightforward because it involves just transposing, but this part might not be immediately obvious. I realized this when I attempted to visualize the results and saw the transposed version of what I was achieving.

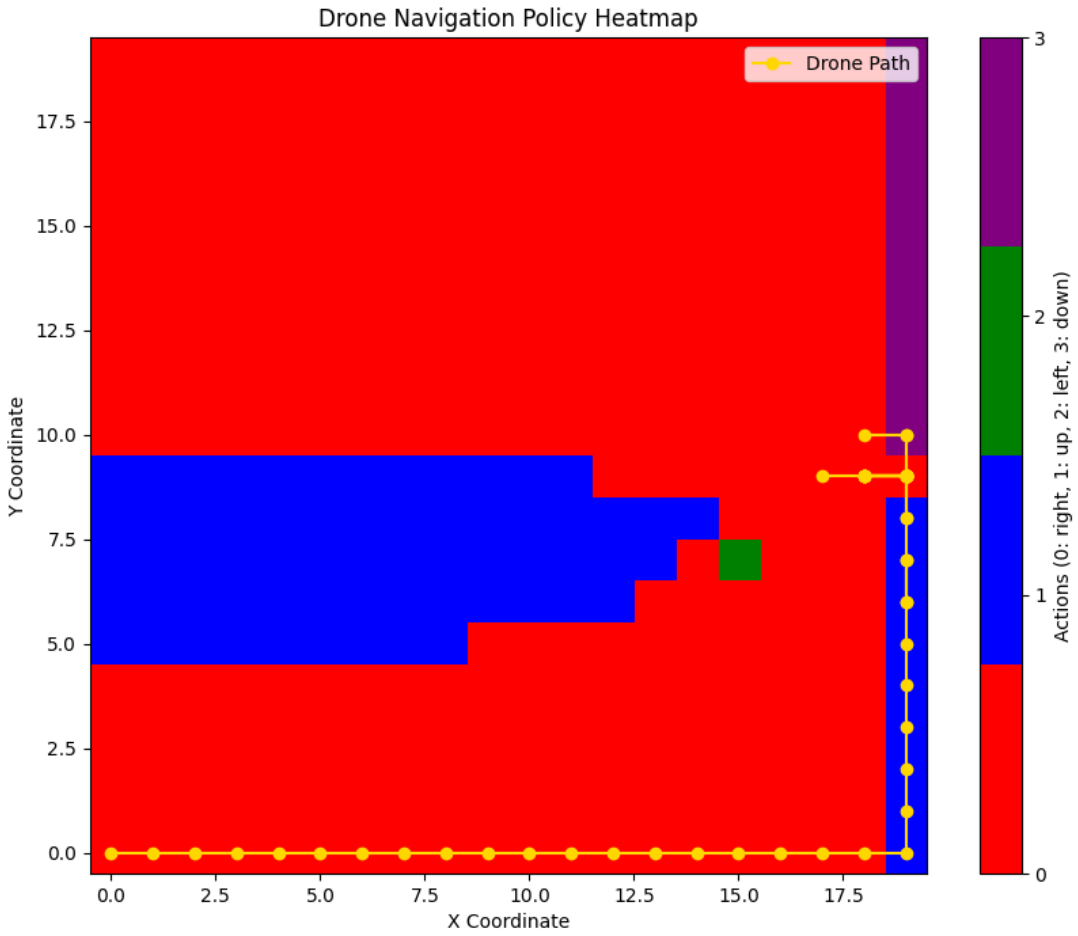
Question-4

Plot the policy as a heat map. Plot the simulated drone trajectory overlaid on the policy heat map, and briefly describe in words what the policy is doing. Hint: Feel free to modify `visualize_value_function()` or write it from scratch yourself.

Answer-4

Visualizing the value function wasn't the answer to this question, so I created my own visualization function, which can be found in

`utils_other.py`.
Here is the output:



This looked nearly perfect because the route adheres to the heatmap and follows the same rules. Normally, at the final part, it should terminate and not continue to move, but I couldn't figure out why it does not stop and continues to move instead. Even though it flew past the final point, the drone comes back because the best action is to return, which can also be observed on the heatmap.

Note: To reproduce the same results, run `value_iteration.py`. Although the output of the `value_function` will be the same, the last part may differ (most probably) because it utilizes the probabilities of the value function.

Q-Learning

Question-5

Given the same environment, sample 10^5 state transition triples

$$(x_i, u_i, x_i') \qquad \forall i \in [1, 10^5]$$

Make sure to generate transition samples with appropriate probability of getting blown off course by the storm. *-Important: We are representing the state as a two dimensional vector with the two grid coordinates and the action as a single element vector. You are free to use your own state and action representation, but another choice might require a significant amount of tuning of the resulting Q-network.

Answer-5

With the help of some research on the internet and the explanations in the code, I created transition triples. Here is the code for it:

```
######### Your code starts here #########
# x is the integer state index in the vectorized state shape: []
# u is the integer action shape: []
# compute xp, the integer next state shape: []

# make use of the transition matrices and tf.random.categorical
# tf.one_hot can be used to convert an integer state into a vector
# with 1 in the place of that index

# remember that transition matrices have a shape [sdim, sdim]
# remember that tf.random.categorical takes in the log of
# probabilities, not the probabilities themselves

# Convert the current state index into a one-hot encoded vector
x_vec = tf.one_hot(x, sdim)

# Multiply the transition matrix for the chosen action by the one-hot vector
# to get the probability distribution over the next states
prob_vec = tf.squeeze(tf.linalg.matvec(Ts[u], x_vec))

# Sample a new state index from the probability distribution
# using categorical sampling in the log-probability space
xp = tf.random.categorical(tf.math.log([prob_vec]), 1)[0, 0]
######### Your code ends here ###########
```

Question-6

In order to develop the Q-network loss function, we will start by writing down the Bellman equation of the optimal Q-function—our Q-network should aim to approximate that. Write down the expectation form of the optimal Q-function in terms of an equality

$$Q^*(x, u) = \dots$$

Hint: Your form should not contain the transition probabilities, since we do not know those.
Hint: Remember to account for the reward value and whether the state is terminal.

Answer-6

As I understand it, the loss function for Q-learning is similar to that of any regression problem; we can use the MSE (Mean Squared Error) loss function to calculate the loss. Since Q-learning utilizes Q-networks, the output of the network can be input into the MSE function along with the actual answer to calculate the loss. However, in addition to these elements, the prediction should also utilize the Bellman equation (to make it act like the real part?). Therefore, the final form of the error would look like this:

$$Loss = mean((R + \gamma \max_{u'} Q(x', u') - Q(x, u))^2)$$

Here, R is the immediate reward, γ is the discount factor, and $Q(s, a)$ are the values predicted by the network.
Here is the code for the given formula:

```
######### Your code starts here #########
# reward = tf.gather(reward_vec, tf.cast(X_, tf.int32))
reward = reward_fn(X_)
terminal = is_terminal_fn(X_)
not_terminal = 1 - tf.cast(terminal, tf.float32)
target_Q = reward + gam * not_terminal * next_Q
loss_ = tf.reduce_mean((target_Q - Q) ** 2)
######### Your code ends here ###########
```

Question-7

Create a feed forward neural network for representing the Q-network. Use 3 dense layers with a width of 64 (two hidden 64 neuron embeddings).

Answer-7

This is pretty straightforward, here is the code:

```
######### Your code starts here #########
# create the deep Q-network
# it needs to take in 2 state + 1 action input (3 inputs)
# it needs to output a single value (batch x 1 output) - the Q-value
# it should have 3 dense layers with a width of 64 (two hidden 64 neuron embeddings)
Q_network = Sequential(
    [
        InputLayer(shape=(3,)),
        Dense(64, activation="relu"),
        Dense(64, activation="relu"),
        Dense(1),  # output Q-value for each state-action pair
    ]
)

######### Your code ends here ##########
```

Question-8

Train the neural network filling in the provided Q_learning Python function. Use the Adam optimizer. Experiment with the following step sizes $10^{-3}, 10^{-2}, 10^{-1}$ and pick the one that works best.
We will now develop the Q-network loss as an L2 penalized equality (1) residual.

$$l = \frac{1}{n}\sum_{i=1}^{n} ||LHS_i - RHS_i||_2^2 \qquad \forall i \in S_{examples}$$

Hint: The expectation operator can be silently dropped since we are (1) using a quadratic residual penalty and (2) summing over all of the samples —which is equivalent to taking empirical expectation over data.

Answer-8

This is also pretty straight forward, since training loop is custom and can not be used by `tf.keras.model.Sequential().fit()` we have to create a

training loop using the `tf.GradientTape`. Here is the code:

```
######### Your code starts here #########
# create the Adam optimizer with tensorflow keras
# experiment with different learning rates [1e-4, 1e-3, 1e-2, 1e-1]
optimizer = tf.keras.optimizers.Adam(learning_rate=1e-3)

######### Your code ends here ###########

print("Training the Q-network")
for _ in tqdm(range(int(1e4))):
    ######### Your code starts here #########
    # apply a single step of gradient descent to the Q_network variables
    # take a look at the tf.keras.optimizers
    with tf.GradientTape() as tape:
        loss_val = loss()
    grads = tape.gradient(loss_val, Q_network.trainable_variables)
    optimizer.apply_gradients(zip(grads, Q_network.trainable_variables))

    ######### Your code ends here ###########
```

Question-10

Note: Question-9 is below, since it is not about the codes I answered it lastly.

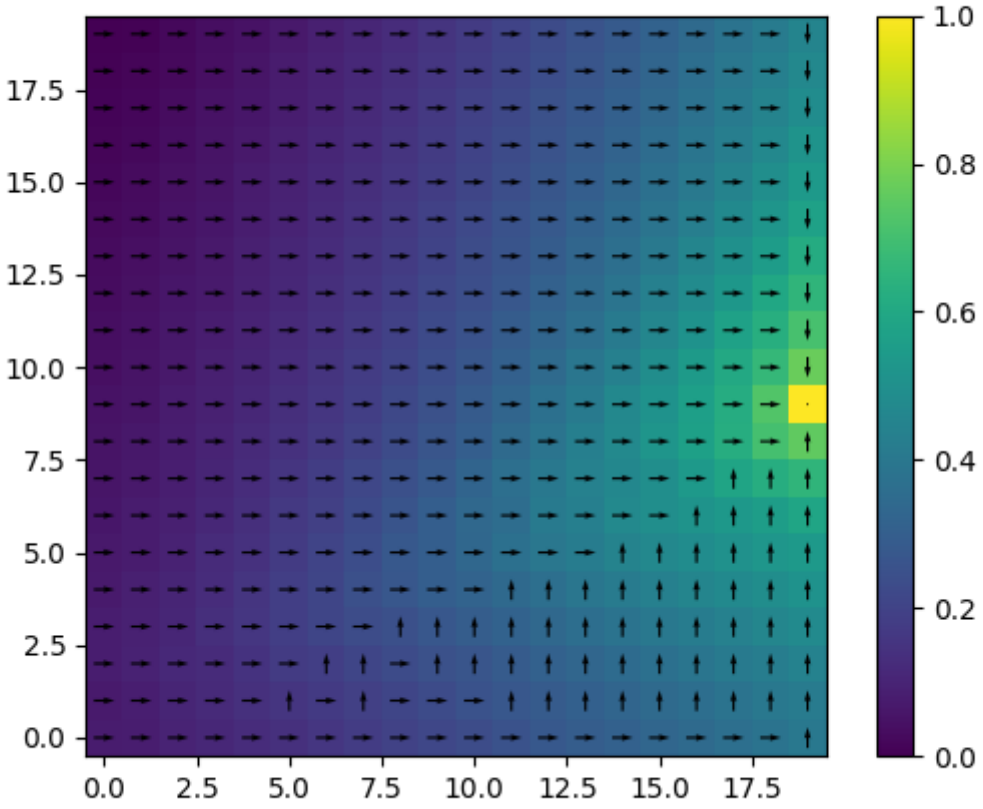
Question: Include a binary heat map plot that shows, for every state, if the approximate Q-network policy agrees or disagrees with the value iteration optimal policy.
Hint: Feel free to modify `visualize_value_function()` *or write it from scratch yourself.*

Answer-10

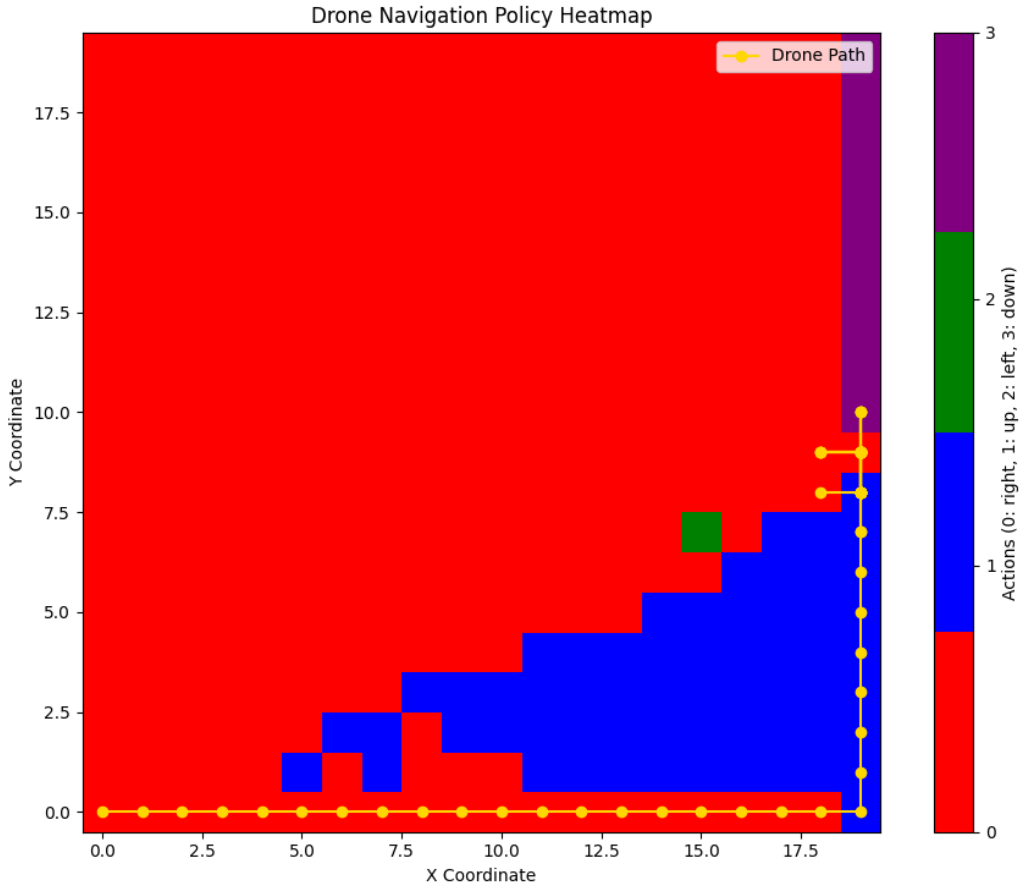
I made some experimenting in this part of the project because in oppose of value function the q-learning is a stocastic approach and to be able to get the best results we may need to adjust some hyperparameters, like networks size, step size and training loops etc.

Experiment-1

3 Layered Network, stepsize 10^{-3}, training step 10000:



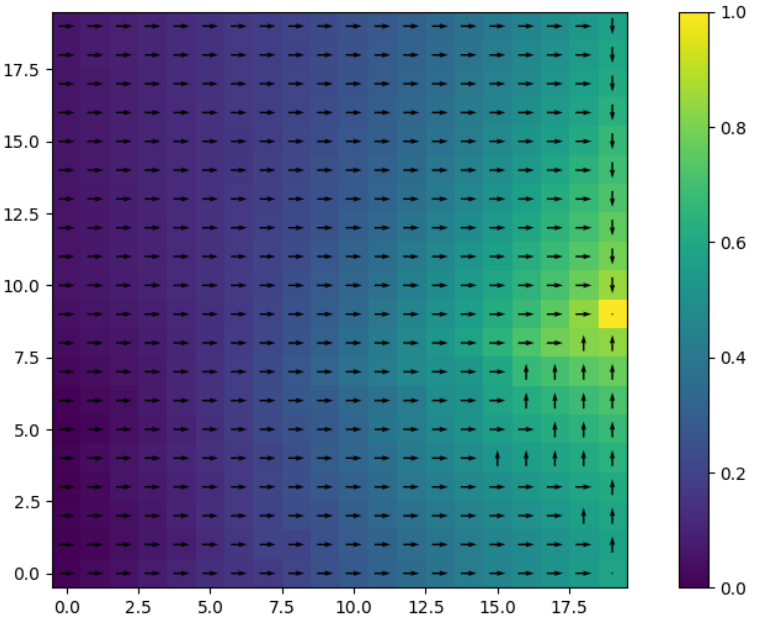
The value function does not look that well

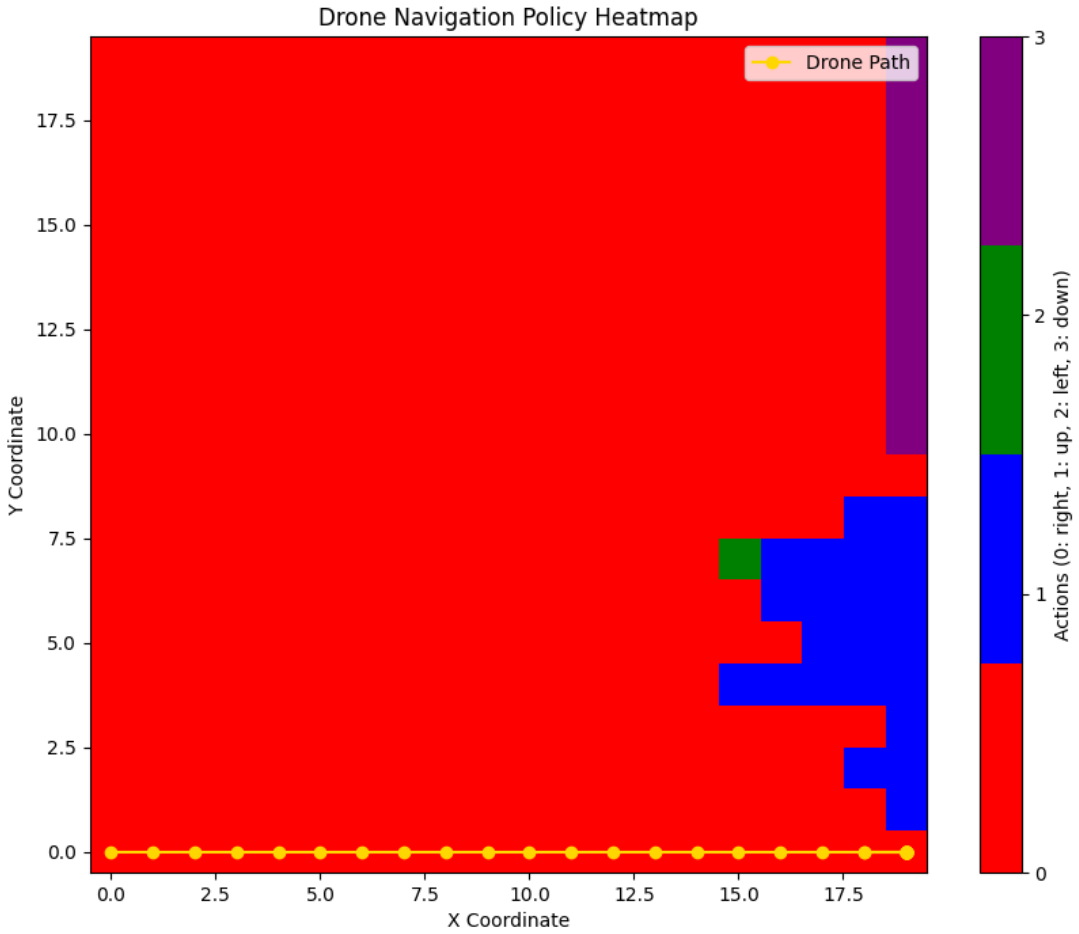


Same goes for heatmap as well, it nearly does not take storm (it directs to storm $(15, 7)$ while it should not) in to account and just worked like it is not there.

Experiment-2

2 Layered Network, stepsize 10^{-2}, training step 10000:

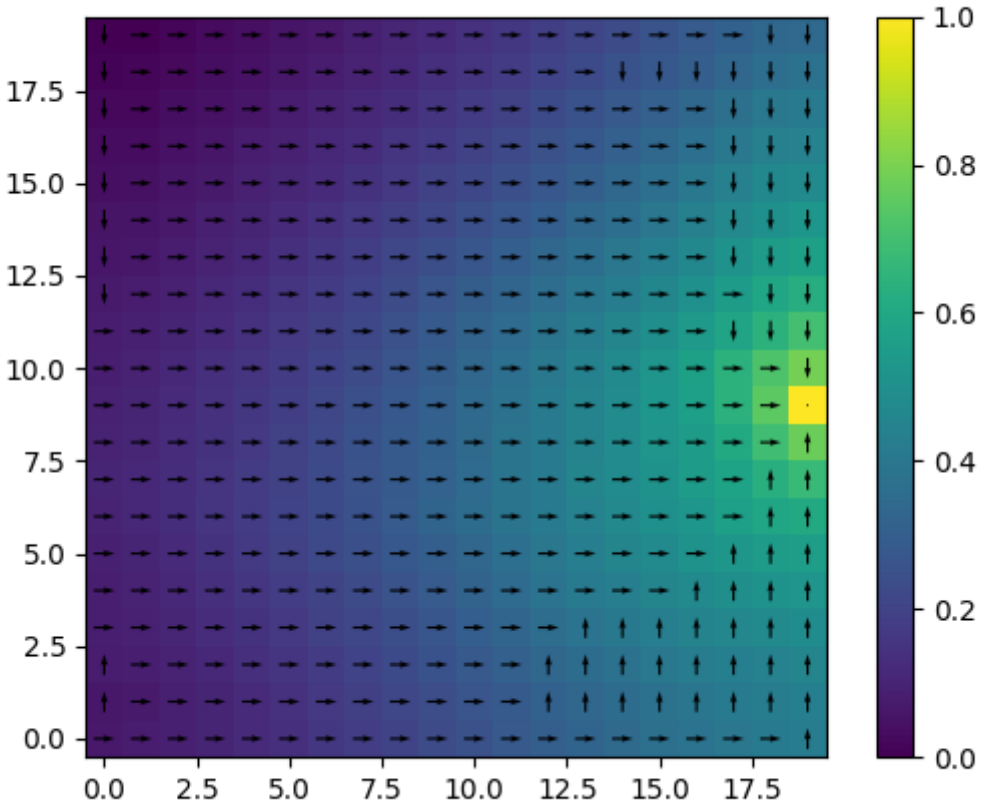


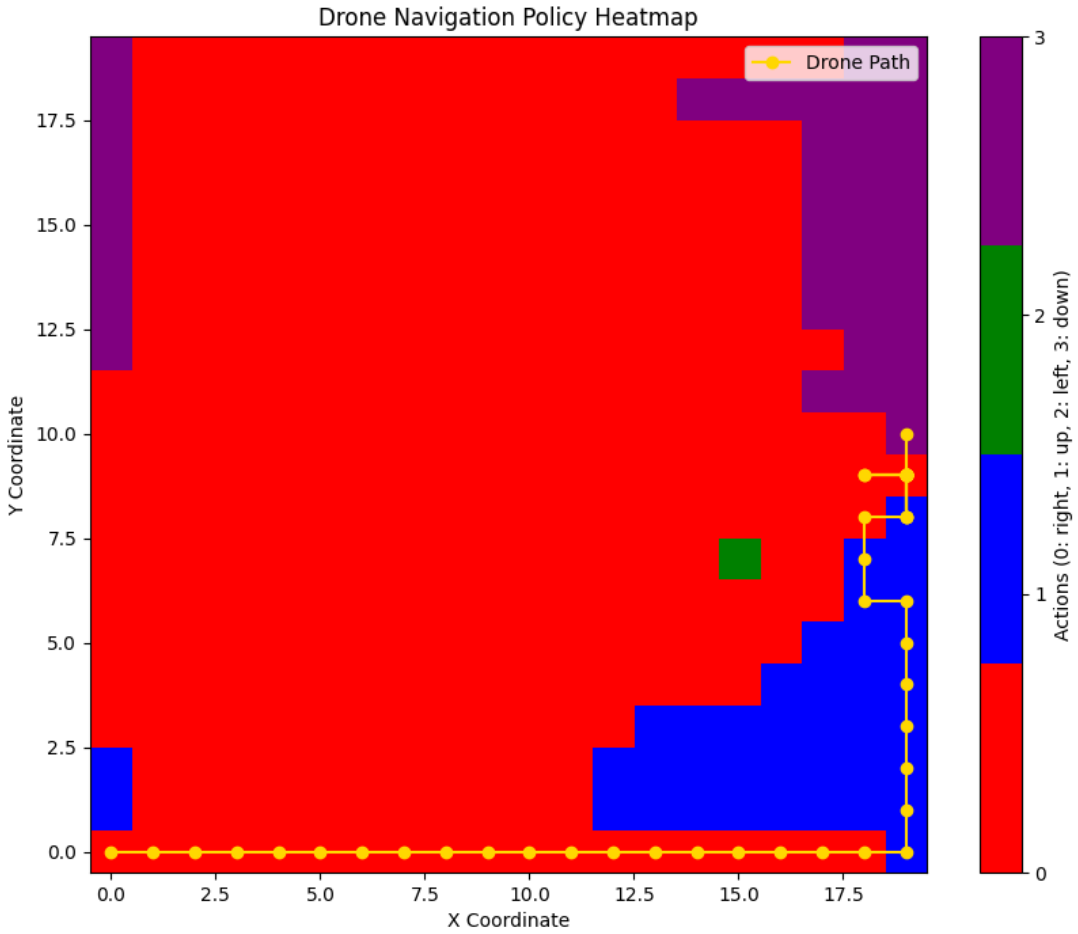


Same conclusion.

Experiment-3

3 Layered Network, stepsize 10^{-3}, training step 50000 (This took a lot longer):





Training it more and growing the network did not do well at the opposite it made worse.

Conclusion: My guess as to why Q-learning performed poorly compared to value iteration is that the problem might be too simple to require a network; basic methods might suffice for this straightforward issue.

Question-9

Describe a dynamical system or a dataset situation in which using Q-learning could be easier than using value iteration.
Hint: You don't need to limit the example to the field of robotics.

Answer-9

Since Q-learning employs neural networks to optimally solve problems, it might be more effective in large and stochastic environments compared to value iteration, which uses fixed methods without any dynamic elements like those found in deep learning models. One example could be autonomous driving. In such environments, there are numerous factors to consider, and value iteration might be very slow and computationally cumbersome. Although there are many variables, a neural network can learn patterns that a traditional mathematical function might not capture. Therefore, for very complex problems, Q-learning could be a preferable choice over value iteration. However, for a simpler problem like ours, value iteration might be more suitable.