

## APPENDIX

*A. Enumeration of  $n$ -hyperarc DHGs (Section III-A of the main paper)*

The systematic enumeration of all directed hypergraphlets (DHGs) for a fixed number  $n$  of hyperarcs is detailed in Algorithm 4. Note that it aims to enumerate hypergraphlets (i.e., all possible patterns), rather than their instances within a hypergraph. The procedure follows a three-stage approach: defining the structural search space, iterating through potential configurations, and filtering for validity and uniqueness.

**Defining the structural search space:** The search space is built upon the overlaps between hyperarcs. Each hyperarc consists of a head set and a tail set, which are disjoint under our assumption that hyperarcs are self-loop-free. When we overlay  $n$  hyperarcs, their head and tail sets intersect to form disjoint intersection regions. Each region represents a unique combination: for every hyperarc, the region is either inside its head, inside its tail, or outside both. This creates  $3^n$  possible regions in total. After removing the empty region (which lies outside all hyperarcs) we define the set  $R$  containing all valid overlap patterns ( $|R| = 3^n - 1$ ). Any specific DHG configuration can be uniquely represented as a subset  $S \subseteq R$ . For example, when  $n = 2$  (with hyperarcs  $e = (H, T)$  and  $e' = (H', T')$ ),  $R$  consists of the following 8 regions: (1)  $H \setminus H' \setminus T'$ , (2)  $H \cap H'$ , (3)  $H \cap T'$ , (4)  $H' \setminus H \setminus T$ , (5)  $T' \setminus H \setminus T$ , (6)  $H' \cap T$ , (7)  $T \cap T'$ , and (8)  $T \setminus H' \setminus T'$ .

**Configuration construction and validation:** The algorithm considers every subset  $S$  of  $R$ , where  $S$  represents a unique selection of occupied intersection regions. The function `ConstructHyperarcs` assembles the  $n$  hyperarcs by identifying which of these selected regions belong to which hyperarc. In the  $n = 2$  example, base hyperarcs are defined by gathering all regions associated with their respective heads ( $H$ ) and tails ( $T$ ). Hyperarc  $e$  is defined by  $H = \{1, 2, 3\}$  and  $T = \{6, 7, 8\}$ , while hyperarc  $e'$  is defined by  $H' = \{2, 4, 6\}$  and  $T' = \{3, 5, 7\}$ . As shown in Figure 13, when the algorithm selects the subset  $S = \{3, 6\}$ , these definitions are filtered to intersect with  $S$ , yielding the final hyperarcs  $e = (\{3\}, \{6\})$  and  $e' = (\{6\}, \{3\})$ .

To consider only valid and non-trivial structures, the algorithm discards  $S$  if it violates any of the three constraints:

- **No Duplicates:** All generated hyperarcs  $\{e_1, \dots, e_n\}$  must be unique. Two hyperarcs are identical only if they share the exact same tail and head sets, resulting in them consisting of the exact same intersection regions within the configuration.
- **Connectivity:** The union of all hyperarcs must form a connected structure, ensuring the configuration does not consist of isolated components.
- **Non-emptiness:** Every hyperarc must be well-defined; specifically, both its head set and tail set must be nonempty.

**Isomorphism and symmetry handling:** Isomorphic structures are configurations that are topologically identical but appear distinct due to the ordering of hyperarcs. As illustrated in Figure 14 for the  $n = 2$  case, the sequence  $(e, e')$  and the sequence  $(e', e)$  define the exact same topological structure despite their different permutations. To address this issue, the algorithm employs a `GetCanonicalIsomorphism`

**Algorithm 4:** Enumeration of  $n$ -hyperarc DHGs

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Input: Order  $n$  (number of hyperarcs)
Output: Set of unique DHGs  $\mathcal{D}$ 
1  $R \leftarrow \{H, T, \emptyset\}^n \setminus \{\emptyset\}^n$ 
2  $\mathcal{D} \leftarrow \emptyset$ 
3 foreach subset  $S \subseteq R$  do
4    $\{e_1, \dots, e_n\} \leftarrow \text{ConstructHyperarcs}(S)$ 
5   if  $\forall i, j, e_i \neq e_j$  and  $\bigcup e_i$  is connected and  $\forall i, e_i \neq \emptyset$ 
6     then
7        $C \leftarrow \text{GetCanonicalIsomorphism}(e_1, \dots, e_n)$ 
8       if  $C \notin \mathcal{D}$  then
9          $\mathcal{D} \leftarrow \mathcal{D} \cup \{C\}$ 
return  $\mathcal{D}$ 

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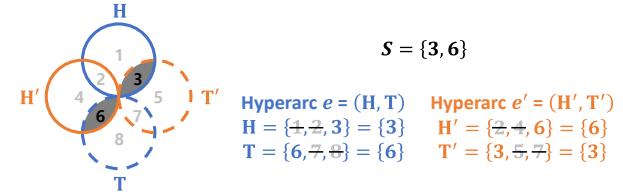


Fig. 13. Illustration of the configuration construction for  $S = \{3, 6\}$  with  $n = 2$ , showing the mapping of hyperarcs  $e$  and  $e'$  to their respective head ( $H$  or  $H'$ ) and tail ( $T$  or  $T'$ ) sets.

"lexicographically smallest"

$$S_{(e,e')} = \{3, 4, 6\} \quad \text{vs} \quad S_{(e',e)} = \{1, 3, 6\}$$

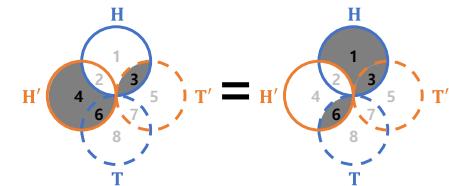


Fig. 14. Examples of isomorphic configurations for  $n = 2$ . The configuration  $S_{(e,e')} = \{3, 4, 6\}$  is filtered out in favor of the isomorphic, lexicographically smaller configuration  $S_{(e',e)} = \{1, 3, 6\}$ .

function to compute a canonical representation for each valid DHG. Specifically, we adapt a strategy (Appendix F of [17]) that evaluates all isomorphic permutations and selects the lexicographically smallest pattern as the canonical form (e.g.,  $\{1, 3, 6\}$  in Figure 14). By comparing this form against the set of previously discovered configurations  $\mathcal{D}$ , the algorithm ensures that only unique, non-isomorphic DHGs are included in the final output.

*B. Randomization of Directed Hypergraphs (DHs) (Section III-B of the main paper)*

To randomize DHs, we extend the configuration model, which is widely-used for (hyper)graphs [63], [44], to DHs. Algorithm 5 outlines the process of obtaining a randomized DH  $G'$  from the input DH  $G$ . It ensures that  $G'$  and  $G$  have the same distributions of hyperarc sizes and node degrees. Hyperarcs are paired and the nodes in the head sets of each pair are shuffled to obtain a shuffled set  $E_{\text{temp}}$  of hyperarcs (Lines 3-13). The same process is applied to the tail sets on  $E_{\text{temp}}$  (Lines 14-24) to obtain the final set  $E'$  of hyperarcs.

*C. Characterization of Hypergraphs with Characteristic Profile using Z-Scores (Section V-B of the main paper)*

Instead of the suggested significance measure in Eq. (1), Z-scores can be used to compute the characteristic profiles (CPs)

**Algorithm 5:** Randomization of Directed Hypergraphs

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**Input:** a directed hypergraph:  $G = (V, E)$   
**Output:** a randomized directed hypergraph:  $G' = (V', E')$

- 1  $V' \leftarrow V, n_E \leftarrow |E|$
- 2  $E_{\text{temp}}, E' \leftarrow \emptyset, \emptyset$   
*// Shuffle the two head sets.*
- 3 **for**  $1 : \lfloor n_E/2 \rfloor$  **do**
- 4     Choose  $e_1, e_2 \in \binom{E}{2}$  uniformly at random  
 $E \leftarrow E \setminus \{e_1, e_2\}$   
 $H'_1, H'_2 \leftarrow \text{SHUFFLE}(H_1, H_2 | T_1, T_2)$   
 $E_{\text{temp}} \leftarrow E_{\text{temp}} \cup \{\langle T_1, H'_1 \rangle, \langle T_2, H'_2 \rangle\}$
- 5 **if**  $|E'| = 1$  **then**
- 6      $e_1 \in E$   
7     Choose  $e_2 \in E_{\text{temp}}$  uniformly at random  
8      $E_{\text{temp}} \leftarrow E_{\text{temp}} \setminus \{e_2\}$   
9      $H'_1, H'_2 \leftarrow \text{SHUFFLE}(H_1, H_2 | T_1, T_2)$   
10     $E_{\text{temp}} \leftarrow E_{\text{temp}} \cup \{\langle T_1, H'_1 \rangle, \langle T_2, H'_2 \rangle\}$   
11    *// Shuffle the two tail sets.*
- 12 **for**  $1 : \lfloor n_E/2 \rfloor$  **do**
- 13     Choose  $e_1, e_2 \in \binom{E_{\text{temp}}}{2}$  uniformly at random  
14      $E_{\text{temp}} \leftarrow E_{\text{temp}} \setminus \{e_1, e_2\}$   
15      $T'_1, T'_2 \leftarrow \text{SHUFFLE}(T_1, T_2 | H_1, H_2)$   
16      $E' \leftarrow E' \cup \{\langle T'_1, H_1 \rangle, \langle T'_2, H_2 \rangle\}$
- 17 **if**  $|E_{\text{temp}}| = 1$  **then**
- 18      $e_1 \in E_{\text{temp}}$   
19     Choose  $e_2 \in E'$  uniformly at random  
20      $E_{\text{temp}} \leftarrow E_{\text{temp}} \setminus \{e_2\}$   
21      $T'_1, T'_2 \leftarrow \text{SHUFFLE}(T_1, T_2 | H_1, H_2)$   
22      $E' \leftarrow E' \cup \{\langle T'_1, H_1 \rangle, \langle T'_2, H_2 \rangle\}$
- 23 **return**  $G' = (V', E')$   
*// Shuffle the two sets of nodes*
- 24 **Function**  $\text{SHUFFLE}(S_1, S_2 | F_1, F_2)$
- 25      $I \leftarrow S_1 \cap S_2$   
26      $R \leftarrow (S_1 \cup S_2) \setminus I$   
27      $R' \leftarrow R \setminus F_1 \setminus F_2$   
28      $S'_1 \leftarrow \text{Choose}(|S_1 \setminus I \setminus F_2|)$  elements in  $R'$  uniformly  
29       at random  
30      $S'_2 \leftarrow R' \setminus S'_1$   
31     **return**  $I \cup (S_1 \cap F_2) \cup S'_1, I \cup (S_2 \cap F_1) \cup S'_2$

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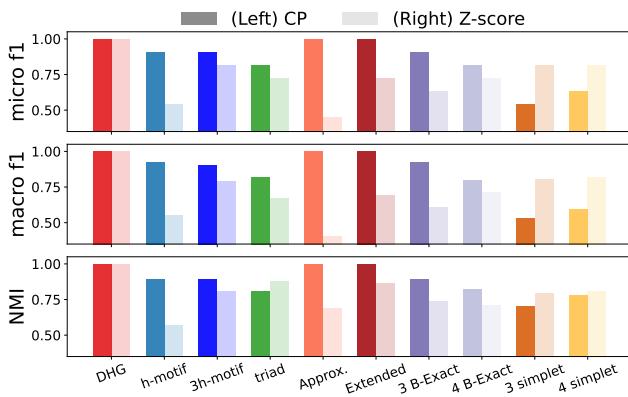


Fig. 15. In most cases, CPs based on the suggested significance measure demonstrate better clustering performance than the CPs based on Z-scores.

as follows:

$$\mu_i^G := \frac{|\Omega_i^G| - \overline{|\Omega_i^{G'}|}}{\text{std}(|\Omega_i^{G'}|)} + \epsilon, \quad (3)$$

where  $|\Omega_i^G|$  represents the count of the instances of DHG in  $G$ , and  $|\Omega_i^{G'}|$  and  $\text{std}(|\Omega_i^{G'}|)$  are the mean and standard deviation of the counts in ten randomized  $G'$ 's, respectively. We set  $\epsilon$  to 1, preventing from the case where  $\text{std}(|\Omega_i^{G'}|) = 0$ .

After computing the significances using Z-scores, we compute the similarity matrix and perform clustering by following the procedures in Section V-B. Note that, regardless of significance measures, using DHGs results in better domain-based differentiation compared to the others, with a 27% clustering performance gain in terms of macro f1-score, as shown in Figure 15. Also note that, in most cases, using the suggested significance measure (i.e., Eq. (1)) leads to better clustering performance than using Z-scores.

#### D. Details on Data Structure Implementation (Section IV-A of the main paper)

As stated in the theoretical analysis, we assume the directed hypergraph (DH)  $G = (V, E)$  is stored using an adjacency list format implemented with hash tables, where the size of each entry is explicitly maintained. In this section, we elaborate on the specific requirements of this structure and analyze the increase in time complexity if this assumption fails.

**Requirements:** To ensure the algorithm runs within the theoretical bounds, the data structure must support average  $O(1)$  time complexity for membership queries (checking if an element exists) and for retrieving the size of entries.

- **For  $f(e, e')$  (in CODA-E, CODA-A):** As stated in Lemma 1, the sizes of  $H, T, H'$ , and  $T'$  must be obtainable in  $O(1)$ ; and intersections (e.g.,  $H \cap H'$ ,  $H \cap T'$ ) must be obtainable in  $O(|\bar{e}|)$ , for example, by iterating over the nodes of one set (e.g.,  $H$ ) and checking the existence of each in the corresponding set (e.g.,  $H'$ ) in average  $O(1)$  time.
- **For CODA-A:** Line 2 specifically requires retrieving the degree of node  $v$  in  $O(1)$  time.

**Data structure implications:** To satisfy these requirements efficiently, the data must be stored in structures that support average  $O(1)$  lookup and size retrieval. This ensures the algorithm runs within its theoretical bounds. Note that our hash table-based adjacency-list storage is one example that meets this condition, and any other storage format that does so can be used instead, such as bitset representations, perfect hashing, or hybrid array–hash structures.

**Impact of alternative data structures:** If the data structure fails to meet the discussed requirements, the overall time complexity will increase. For example, if the DH is stored using standard arrays or linked lists, checking for a node's existence within a hyperarc requires a linear scan ( $O(|e|)$ ) or a binary search on sorted arrays ( $O(\log |e|)$ ). This increases the intersection computation complexity from linear to quadratic relative to the hyperedge size, specifically  $O(|\bar{e}|^2)$  or  $O(|\bar{e}| \log |\bar{e}|)$ .

While standard sparse matrix formats are memory-efficient, they do not naturally support constant-time membership checks, thereby increasing time complexity:

- **CSR/CSC:** Checking if an element exists requires a binary search over the indices, taking  $O(\log |e|)$ . This increases the intersection complexity from  $O(|\bar{e}|)$  to  $O(|\bar{e}| \log |\bar{e}|)$ .
- **COO:** Checking for existence requires a linear scan  $O(|E|)$  (or  $O(\log |E|)$  if sorted), which is significantly slower.

**Preprocessing:** If the input data is not in the required format, we can preprocess the dataset to convert it into the hash-based

adjacency list format. The cost of this conversion is linear with respect to the DH size,  $O(\sum_{e \in E} |\bar{e}|)$ , for most input formats.

#### E. Implementation Details (Section V-A of the main paper)

For DHGs, we implement all counting algorithms in C++. For counting h/3h-motifs, we use the official C++ implementations [17], [43]. For counting configurations on simplicial complexes, we use the official Julia implementations [1]. For counting triads, we implement a widely-used subquadratic algorithm [52] in C++.

#### F. DHGs with Self-Loops (Section IV-A of the main paper)

As discussed in Section III-A, the original 91 DHGs exclude self-loops to maintain a manageable configuration space. Relaxing this constraint to allow unrestricted self-loops, where the head and tail sets of a hyperarc may intersect, would expand the configuration space to 16,381 valid states. Such a massive increase would compromise both the interpretability and simplicity of the DHG framework.

To address this issue, we instead consider a controlled expansion that incorporates self-loops in a simplified and restricted manner. For any pair of hyperarcs, we define four possible self-loop configurations based on the binary state of each hyperarc:

- No self-loops (the original configuration).
- Self-loop on hyperarc  $e$  only.
- Self-loop on hyperarc  $e'$  only.
- Self-loops on both hyperarcs  $e$  and  $e'$ .

By mapping these four configurations onto the baseline 91 DHGs and pruning redundant patterns resulting from symmetry, the set expands to a total of 341 unique extended DHGs. Refer to Section V-B of the main paper for an empirical comparison of the characterization power of these expansions and the original DHGs.

#### G. Count Distributions (Section V-B of the main paper)

We analyze the occurrence distributions of DHGs in real-world and randomized directed hypergraphs (DHs). To ensure statistical significance, we generate ten randomized DHs and report the average counts. As shown in Figure 16, the counts of DHGs in real-world directed hypergraphs are distinct from those in randomized directed hypergraphs.

#### H. Experimental Settings for Hyperarc Prediction (Section V-C of the main paper)

In this section, we list the hyperparameter settings of the feature vectors and classifiers used for the hyperarc prediction and report the detailed experimental setups.

**Hyperparameter settings of feature vectors:** The embedding dimensions of node2vec, hyper2vec, and deep hyperedges are all fixed to 91. Other hyperparameters of these methods are fixed to their default settings at the following links:

- **node2vec (n2v):** <https://github.com/aditya-grover/node2vec>
- **hyper2vec (h2v):** <https://github.com/jeffjh/NHNE>
- **deep hyperedges (deep-h):** <https://github.com/Oxpayne/deep-hyperedges>

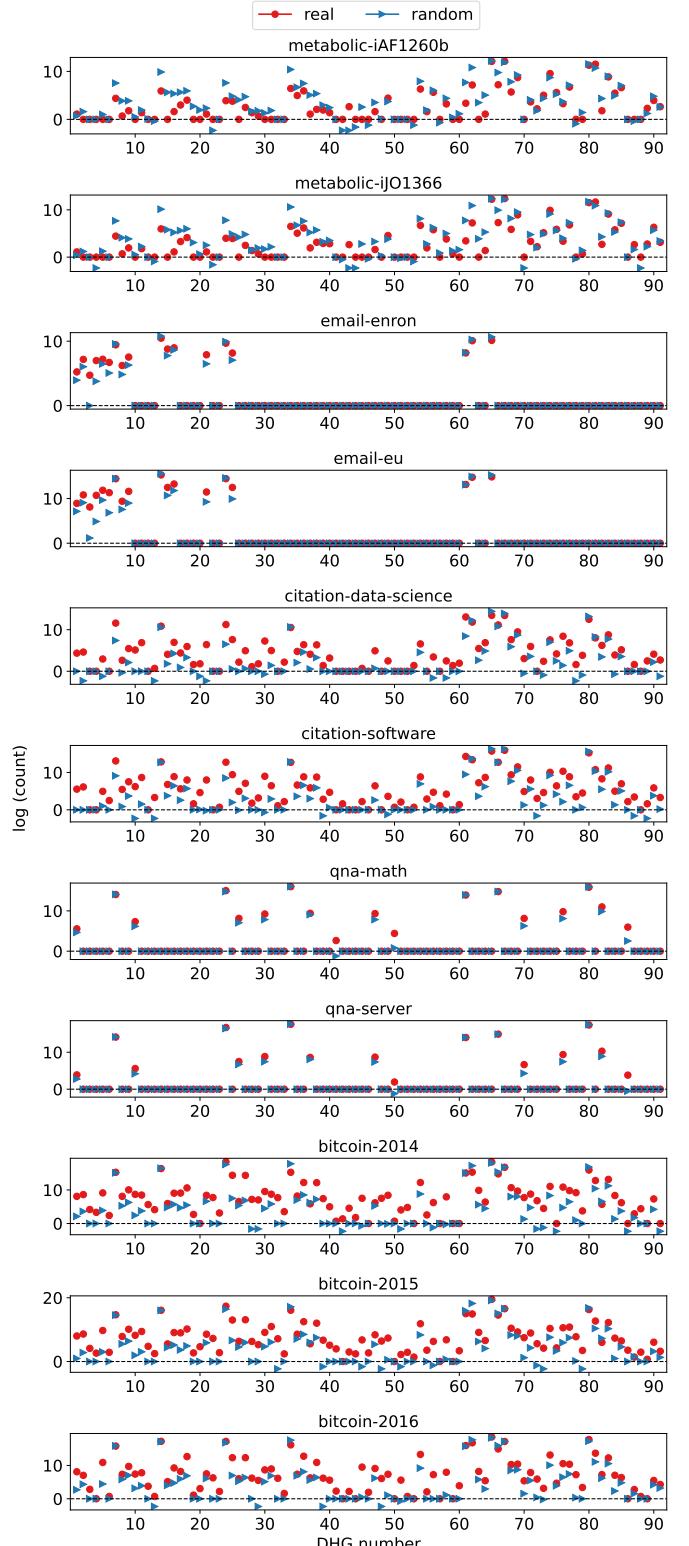


Fig. 16. Log counts of DHGs in real-world and randomized directed hypergraphs (DHs). The counts of DHGs are clearly distinguished in real-world and randomized DHs.

Note that h-motif and triad do not have any hyperparameters.

**Details of classifiers:** The hyperparameters of the tree-based classifiers (Decision Tree, Random Forest, XGBoost, and LightGBM), Logistic Regressor, KNN, and MLP are fixed to their default settings at the following links:

**Algorithm 6:** A2A

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**Input:** (1) a directed hypergraph:  $G = (V, E)$   
(2) # of samples  $n = q \cdot |E|$  for a given ratio  $q$

**Output:**  $C[i]$  for every  $i \in [m]$

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1  $C[i] \leftarrow 0, \forall i \in [m]$ 
2 for  $1 : n$  do
3   Choose  $e \in E_{\geq 1}$  uniformly at random
4    $N_e \leftarrow \{e' \in E \setminus \{e\} : e \cap e' \neq \emptyset\}$ 
5    $C[f(e, e')] \leftarrow C[f(e, e')] + \frac{|E_{\geq 1}|}{2 \cdot n} \cdot HM(|N_e|, |N_{e'}|)$ 
6 return  $C$ 
```

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- **Decision Tree (DT):** <https://scikit-learn.org/stable/modules/generated/sklearn.tree.DecisionTreeClassifier>
- **Random Forest (RF):** <https://scikit-learn.org/stable/modules/generated/sklearn.ensemble.RandomForestClassifier>
- **XGBoost (XGB):** <https://xgboost.readthedocs.io/en/stable/>
- **LightGBM (LGBM):** <https://lightgbm.readthedocs.io/en/latest/pythonapi/lightgbm.LGBMClassifier>
- **Logistic Regressor (LR):** [https://scikit-learn.org/stable/modules/generated/sklearn.linear\\_model.LogisticRegression](https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression)
- **KNN:** <https://scikit-learn.org/stable/modules/generated/sklearn.neighbors.KNeighborsClassifier>
- **MLP:** [https://scikit-learn.org/stable/modules/generated/sklearn.neural\\_network.MLPClassifier](https://scikit-learn.org/stable/modules/generated/sklearn.neural_network.MLPClassifier)

The hyperparameters of the HNN-based classifiers (HGNN, FastHyperGCN, and UniGCNII) are set as follows: the number of layers and hidden dimension are all fixed to 2 and 128, respectively. We train HGNN and UniGCNII for 500 epochs using Adam with a learning rate of 0.001 and a weight decay of  $10^{-6}$ , and FastHyperGCN for 200 epochs using Adam with a learning rate of 0.01, a weight decay of  $5 \times 10^{-4}$ , and a dropout rate of 0.5.

For these HNN-based classifiers, we employ early stopping, and to this end, we divide the fake hyperarcs into the train, validation, and test sets using a 6:2:2 ratio. In each set, we uniformly sample the same number of real hyperarcs as the number of fake hyperarcs. For every 50 epochs, we measure the validation accuracy and save the model parameters. Then, we use the checkpoint (i.e., saved model parameters) with the highest validation accuracy to measure test performance.

*I. A2A: Baseline Algorithm (Section V-D of the main paper)*

In this section, we provide a detailed description of the baseline algorithm, A2A, including its unbiasedness, variance, and complexity. Its pseudocode is presented in Algorithm 6. Here,  $p(e, e') = \left( \frac{1}{|N_e|} + \frac{1}{|N_{e'}|} \right) \cdot \frac{1}{|E_{\geq 1}|}$  where  $E_{\geq 1} = \{e : N_e \geq 1\}$ . Assume  $E_{\geq 1}$  is given at first. Also, for space efficiency, we assume  $N_e$  is maintained (Line 4).  $HM(A, B)$  on Line 5 denotes the harmonic mean of  $A$  and  $B$ .

**Proposition 10** (Unbiasedness of A2A). *Algorithm 6 is unbiased, i.e.,  $\mathbb{E}[C[i]] = |\Omega_i|, \forall i \in [m]$ .*

*Proof.* Follow the flow of the proof of Proposition 2.  $\square$

**Proposition 11** (Variance of A2A). *For each  $i \in [m]$ , the*

**Algorithm 7: Generation of Directed Hypergraphs**


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**Input:** (1) the number  $n$  of nodes,  
(2) a ratio  $r$  of hyperedges to nodes,  
(3) the maximum size  $k$  of a hyperedge

**Output:** a directed hypergraph:  $G = (V, E)$

```

1  $V \leftarrow [n], E \leftarrow \emptyset$ 
2 for  $1 : r \cdot n$  do
3   Choose the hyperarc size  $d \in \{2, \dots, k\}$  uniformly at random
4   Choose  $U \subseteq V$  such that  $|U| = d$  uniformly at random
5    $T, H \leftarrow$  Split  $U$  into two groups of sizes  $\lfloor \frac{d}{2} \rfloor$  and  $\lceil \frac{d}{2} \rceil$ 
6    $E \leftarrow E \cup \{\langle T, H \rangle\}$ 
7 return  $G = (V, E)$ 
```

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variance of  $C[i]$  obtained by Algorithm 6 is

$$\begin{aligned} Var[C[i]] &= \sum_{(e, e') \in \Omega_i} \frac{1}{n} \left( \frac{1}{p(e, e')} - 1 \right) \\ &= \sum_{(e, e') \in \Omega_i} \frac{1}{n} \left( \frac{|E_{\geq 1}|}{2} \cdot HM(|N_e|, |N_{e'}|) - 1 \right). \end{aligned}$$

*Proof.* Follow the flow of the proof of Proposition 3.  $\square$

**Proposition 12** (Time and space complexity of A2A). *The time complexity of Algorithm 6 is  $O(n \cdot (\max_{e \in E} |\bar{e}| \cdot \max_{e \in E} |N_e|))$ . Its space complexity is  $O(\sum_{e \in E} |\bar{e}|)$ .*

*Proof.* The information of a given directed graph is stored in  $O(\sum_{e \in E} |\bar{e}|)$  space at first. For time complexity,  $O(\max_{e \in E} |\bar{e}| \cdot \max_{e \in E} |N_e|)$  time is required assuming  $O(p \cdot q)$  time is taken for set union when there are  $p$  sets, of which size bounded by  $q$ . For space complexity,  $O(|N_e|) \in O(\sum_{e \in E} |\bar{e}|)$  space is needed. Checking  $f(e, e')$  requires  $O(\max_{(e, e') \in \Omega} \min(|\bar{e}|, |\bar{e}'|))$ -time, which is bounded by  $O(\max_{e \in E} |\bar{e}| \cdot \max_{e \in E} |N_e|)$ .  $\square$

*J. Generation of Directed Hypergraphs (DHs) (Section V-D of the main paper)*

Algorithm 7 describes the process of generating a random DH of a desired size. The size of each hyperedge is determined uniformly at random (Line 3), and it consists of randomly selected nodes (Line 4), which are then randomly divided into head and tail sets of almost equal size (Line 5).

*K. Snapshot Construction for Time Analysis (Section V-E of the main paper)*

The citation-data-science dataset consists of 41 timestamps, and citation-software dataset includes 49 timestamps, with each publication year assigned as a timestamp. Regarding the email, qna, and bitcoin datasets, for each time-evolving DH  $G = (V, E)$  with timestamp  $\tau_e$  for each  $e \in E$ , i.e.,  $e = \langle H, T, \tau_e \rangle$ , we consider 10 timestamps  $\{t_1, t_2, \dots, t_{10}\}$  of the same interval, where  $t_1 < \dots < t_{10} = \max_{e \in E} \tau_e$  and  $t_2 - t_1 = t_1 - \min_{e \in E} \tau_e$ . For each timestamp  $t_i$  above, we create a snapshot (i.e., sub-DH) where the edge set is  $E_i = \{e : \tau_e \leq t_i\}$  and the node set  $V_i = \bigcup_{e \in E_i} \bar{e}$ . Then, we compute the occurrence ratio of each DHG in each sub-DH.

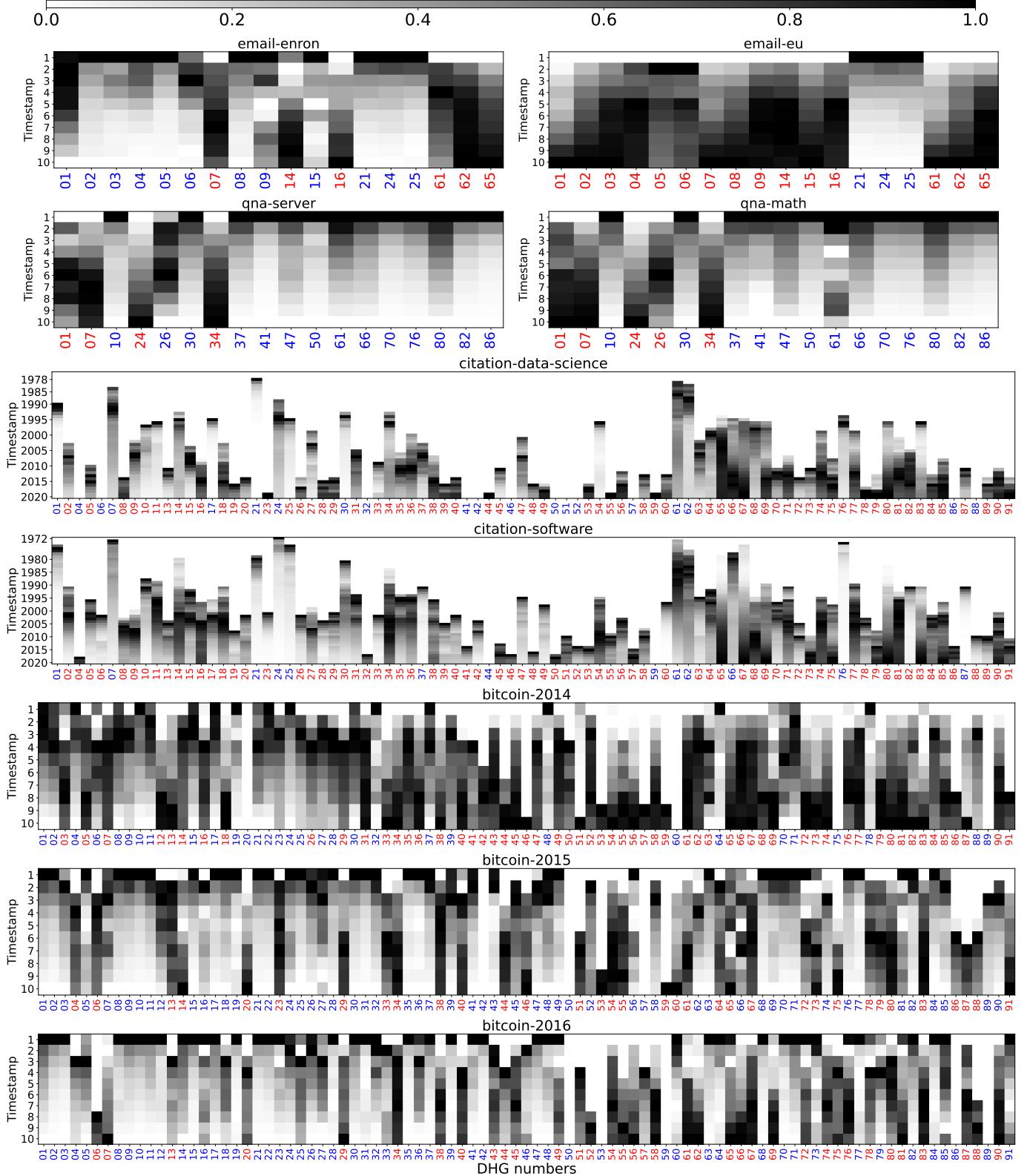


Fig. 17. Normalized DHG occurrence ratios over time, where rows represent timestamps and columns denote DHG numbers. DHG numbers are color-coded to indicate two temporal trend types: **increasing** and **decreasing**.

#### L. The Time-evolving Patterns (Section V-E of the main paper)

We extend the time-evolving pattern analysis to encompass all observed DHGs. To effectively compare the temporal trends independently of absolute frequencies, we normalize the DHG occurrence ratios using Min-Max scaling. Specifically, for

each DHG, we rescale the occurrence ratios across timestamps between  $[0, 1]$ , where 0 and 1 correspond to the minimum and maximum occurrence ratios of that DHG, respectively. Figure 17 illustrates the evolution of normalized DHG occurrence ratios, which reveals two distinct trends: **increasing** and

[decreasing](#). Notably, we observe that datasets from the same domain exhibit similar time-evolving patterns for each DHG. These results confirm that DHGs effectively capture diverse temporal dynamics across different datasets.

#### *M. Application Results (Section V-C of the main paper)*

In this section, we report the full results of the hyperarc prediction problem. Table IV reports the accuracy and AUROC and results (average over 100 trials). The best performances are highlighted in bold, and the second-best performances are underlined. Notably, in terms of average ranking, using DHG vectors, including the dimension reduced versions, performs best in most settings, achieving up to 33% higher AUROC on the bitcoin-2016 dataset and a 47% higher accuracy on the bitcoin-2014 dataset than the second best features.

TABLE IV

HYPERCARC PREDICTION PERFORMANCE. WE COMPARE NINE HYPERARC FEATURE VECTORS USING TEN CLASSIFIERS. THE BEST PERFORMANCES ARE HIGHLIGHTED IN BOLD, AND THE SECOND-BEST PERFORMANCES ARE UNDERLINED. NOTABLY, USING DHG VECTORS LEADS TO THE BEST PERFORMANCE (UP TO 47% AND 33% BETTER IN TERMS OF ACCURACY AND AUROC, RESPECTIVELY) IN MOST SETTINGS, INDICATING THAT DHGs EXTRACT HIGHLY INFORMATIVE HYPERARC FEATURES.

(a) metabolic-iAF1260b

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.664±0.050	0.549±0.061	0.664±0.047	0.649±0.049	0.656±0.057	0.504±0.050	0.511±0.044	0.701±0.055	<b>0.705±0.044</b>
	RF	<b>0.734±0.046</b>	0.650±0.067	<u>0.733±0.048</u>	0.681±0.059	0.690±0.063	0.531±0.054	0.518±0.047	0.714±0.050	0.717±0.060
	DT	<b>0.695±0.055</b>	0.597±0.063	<u>0.683±0.051</u>	0.632±0.065	0.651±0.053	0.512±0.057	0.506±0.056	0.583±0.055	0.615±0.065
	KNN	0.702±0.047	0.625±0.067	0.703±0.044	0.696±0.058	0.696±0.049	0.537±0.050	0.534±0.056	0.704±0.046	<b>0.720±0.050</b>
	MLP	0.709±0.049	0.603±0.062	<b>0.716±0.053</b>	0.646±0.054	0.654±0.063	0.533±0.051	0.537±0.047	0.705±0.054	0.667±0.054
	XGB	0.729±0.048	0.625±0.066	<b>0.741±0.050</b>	0.666±0.059	0.708±0.062	0.514±0.048	0.519±0.045	0.695±0.054	0.717±0.055
	LGBM	0.728±0.047	0.626±0.063	<b>0.740±0.047</b>	0.658±0.053	0.697±0.062	0.527±0.054	0.516±0.052	0.698±0.053	0.713±0.060
	HGNN	0.538±0.047	0.538±0.063	0.546±0.053	0.543±0.040	<b>0.549±0.055</b>	0.483±0.045	0.498±0.067	0.526±0.057	0.540±0.055
	FHGCN	0.562±0.067	0.555±0.055	0.588±0.064	0.535±0.051	<b>0.666±0.067</b>	0.507±0.045	0.491±0.059	0.538±0.050	0.619±0.070
	UGCNII	<b>0.643±0.051</b>	0.596±0.069	0.623±0.057	0.625±0.038	0.618±0.061	0.491±0.050	0.494±0.054	0.597±0.070	0.619±0.066
Max		0.734±0.046	0.650±0.067	<b>0.741±0.050</b>	0.696±0.058	0.708±0.062	0.537±0.050	0.537±0.047	0.714±0.050	0.720±0.050
Avg.		0.670±0.066	0.596±0.036	<b>0.674±0.064</b>	0.633±0.051	0.659±0.045	0.514±0.017	0.512±0.015	0.646±0.072	0.663±0.059
Rank Avg.		2.500±1.360	6.600±0.663	<b>2.200±0.980</b>	5.200±1.536	3.900±1.578	8.500±0.500	8.500±0.500	4.600±1.800	3.000±1.265
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.735±0.059	0.580±0.067	0.727±0.052	0.724±0.049	0.728±0.056	0.506±0.067	0.509±0.052	0.785±0.051	<b>0.791±0.046</b>
	RF	0.815±0.045	0.703±0.087	0.817±0.046	0.784±0.058	0.826±0.046	0.539±0.066	0.533±0.068	0.793±0.051	<b>0.835±0.048</b>
	DT	<b>0.695±0.056</b>	0.599±0.062	<u>0.683±0.051</u>	0.632±0.065	0.651±0.053	0.512±0.057	0.506±0.056	0.583±0.055	0.615±0.065
	KNN	0.770±0.045	0.673±0.076	<u>0.774±0.048</u>	0.755±0.056	0.762±0.051	0.550±0.058	0.552±0.070	0.760±0.047	<b>0.788±0.051</b>
	MLP	0.759±0.053	0.622±0.071	<u>0.778±0.053</u>	0.694±0.074	0.696±0.068	0.543±0.065	0.559±0.059	<b>0.800±0.048</b>	0.710±0.063
	XGB	0.805±0.045	0.684±0.076	<u>0.816±0.046</u>	0.752±0.068	0.812±0.049	0.518±0.063	0.526±0.068	0.775±0.047	<b>0.824±0.047</b>
	LGBM	0.807±0.046	0.680±0.072	<u>0.817±0.046</u>	0.736±0.062	0.799±0.057	0.540±0.059	0.529±0.062	0.785±0.045	<b>0.823±0.049</b>
	HGNN	0.559±0.057	0.541±0.067	0.576±0.048	0.554±0.059	<b>0.594±0.060</b>	0.479±0.058	0.506±0.086	0.522±0.060	0.580±0.055
	FHGCN	0.603±0.080	0.612±0.056	0.637±0.077	0.567±0.072	<b>0.729±0.065</b>	0.506±0.059	0.490±0.067	0.561±0.060	0.693±0.069
	UGCNII	<b>0.702±0.045</b>	0.641±0.097	<u>0.681±0.055</u>	0.674±0.048	0.680±0.060	0.488±0.049	0.489±0.068	0.607±0.092	0.679±0.085
Max		0.815±0.045	0.703±0.087	0.817±0.046	0.784±0.058	<b>0.826±0.046</b>	0.550±0.058	0.559±0.059	0.800±0.048	<b>0.835±0.048</b>
Avg.		0.725±0.083	0.634±0.050	<u>0.731±0.080</u>	0.687±0.076	0.728±0.071	0.518±0.023	0.520±0.023	0.697±0.108	<b>0.734±0.087</b>
Rank Avg.		3.100±1.221	6.400±0.917	<u>2.600±0.917</u>	5.600±0.663	3.000±1.265	8.600±0.490	8.400±0.490	5.100±2.022	<b>2.200±1.470</b>

(b) metabolic-iJO1366

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.662±0.051	0.584±0.055	0.658±0.050	0.648±0.051	0.656±0.048	0.506±0.052	0.505±0.042	<b>0.713±0.057</b>	0.699±0.051
	RF	<b>0.742±0.050</b>	0.666±0.065	<u>0.730±0.045</u>	0.704±0.059	0.698±0.068	0.532±0.045	0.527±0.045	0.723±0.048	0.710±0.066
	DT	<b>0.694±0.051</b>	0.612±0.064	<u>0.680±0.053</u>	0.641±0.057	0.656±0.071	0.509±0.053	0.517±0.049	0.582±0.061	0.612±0.055
	KNN	0.705±0.049	0.628±0.063	0.689±0.045	0.691±0.055	0.682±0.051	0.520±0.053	0.539±0.049	<b>0.725±0.054</b>	0.715±0.061
	MLP	<b>0.718±0.049</b>	0.618±0.064	0.705±0.042	0.653±0.055	0.656±0.059	0.537±0.048	0.539±0.052	0.710±0.055	0.644±0.051
	XGB	<b>0.741±0.050</b>	0.657±0.062	<u>0.732±0.045</u>	0.681±0.054	0.709±0.053	0.519±0.051	0.523±0.048	0.703±0.050	0.719±0.059
	LGBM	<b>0.740±0.052</b>	0.632±0.058	<u>0.734±0.051</u>	0.663±0.056	0.720±0.057	0.509±0.050	0.530±0.047	0.702±0.050	0.713±0.063
	HGNN	0.542±0.052	0.550±0.043	<u>0.564±0.046</u>	0.553±0.036	<b>0.569±0.055</b>	0.499±0.056	0.522±0.037	0.545±0.036	0.558±0.050
	FHGCN	0.591±0.052	0.566±0.059	0.599±0.054	0.532±0.053	<b>0.653±0.071</b>	0.504±0.045	0.498±0.048	0.548±0.060	0.619±0.067
	UGCNII	<b>0.660±0.059</b>	0.610±0.050	0.646±0.055	0.616±0.048	0.621±0.051	0.518±0.047	0.514±0.034	0.633±0.035	0.647±0.052
Max		<b>0.742±0.050</b>	0.666±0.065	<u>0.734±0.051</u>	0.704±0.059	0.720±0.057	0.537±0.048	0.539±0.049	0.725±0.054	0.719±0.059
Avg.		<b>0.680±0.064</b>	0.612±0.035	<u>0.674±0.055</u>	0.638±0.054	0.662±0.042	0.515±0.011	0.521±0.013	0.658±0.070	0.664±0.053
Rank Avg.		<b>2.300±1.900</b>	6.400±0.917	<u>2.800±0.980</u>	5.300±1.005	3.800±1.720	8.600±0.490	8.400±0.490	4.000±2.049	3.400±1.497
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.725±0.064	0.622±0.057	0.724±0.053	0.727±0.058	0.721±0.053	0.501±0.063	0.504±0.058	<b>0.795±0.048</b>	0.782±0.053
	RF	0.824±0.048	0.730±0.076	0.817±0.045	0.794±0.057	<b>0.834±0.048</b>	0.541±0.055	0.546±0.066	0.803±0.043	0.820±0.054
	DT	<b>0.694±0.051</b>	0.611±0.064	<u>0.680±0.053</u>	0.641±0.057	0.656±0.071	0.509±0.053	0.517±0.049	0.582±0.061	0.612±0.055
	KNN	0.774±0.051	0.673±0.077	0.762±0.046	0.752±0.055	0.746±0.054	0.535±0.065	0.560±0.059	<u>0.778±0.051</u>	<b>0.779±0.055</b>
	MLP	0.782±0.050	0.646±0.072	0.761±0.049	0.705±0.075	0.687±0.061	0.548±0.063	0.559±0.069	<b>0.804±0.048</b>	0.682±0.059
	XGB	0.814±0.049	0.710±0.070	0.813±0.047	0.765±0.054	<b>0.819±0.045</b>	0.530±0.063	0.537±0.061	0.792±0.047	0.818±0.049
	LGBM	0.818±0.049	0.687±0.063	0.813±0.049	0.747±0.055	<b>0.822±0.047</b>	0.515±0.057	0.541±0.064	0.794±0.046	0.812±0.047
	HGNN	0.571±0.060	0.552±0.052	0.587±0.048	0.551±0.049	<b>0.601±0.062</b>	0.509±0.051	0.526±0.027	0.534±0.043	0.596±0.053
	FHGCN	0.640±0.062	0.615±0.065	0.647±0.062	0.553±0.070	<b>0.720±0.072</b>	0.506±0.059	0.484±0.059	0.566±0.065	0.692±0.070
	UGCNII	<b>0.713±0.070</b>	0.647±0.056	0.692±0.059	0.663±0.044	0.663±0.058	0.502±0.045	0.503±0.035	0.642±0.043	0.704±0.069
Max		0.824±0.048	0.730±0.076	0.817±0.045	0.794±0.057	<b>0.834±0.048</b>	0.548±0.063	0.560±0.059	0.804±0.048	0.820±0.054
Avg.		<b>0.736±0.079</b>	0.649±0.050	<u>0.730±0.074</u>	0.690±0.082	0.727±0.075	0.520±0.016	0.528±0.024	0.709±0.108	0.730±0.080
Rank Avg.		<b>2.600±1.114</b>	6.400±0.800	3.400±0.800	5.200±1.166	2.900±2.071	8.900±0.300	8.100±0.300	4.600±2.289	2.900±1.513

(c) email-enron

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.772±0.058	0.732±0.058	0.795±0.065	0.752±0.053	<b>0.804±0.054</b>	0.578±0.069	0.492±0.058	0.590±0.062	0.749±0.068
	RF	0.775±0.056	0.712±0.058	0.780±0.071	0.773±0.056	<b>0.796±0.053</b>	0.626±0.069	0.562±0.073	0.592±0.063	0.754±0.066
	DT	0.696±0.064	0.654±0.057	<b>0.705±0.054</b>	0.689±0.071	0.705±0.069	0.551±0.069	0.528±0.073	0.542±0.068	0.650±0.076
	KNN	0.761±0.068	0.694±0.058	0.769±0.064	0.737±0.056	<b>0.777±0.055</b>	0.636±0.058	0.571±0.070	0.567±0.061	0.720±0.066
	MLP	<b>0.810±0.050</b>	0.731±0.054	0.808±0.062	0.751±0.053	0.805±0.055	0.639±0.073	0.551±0.080	0.588±0.064	0.721±0.070
	XGB	0.763±0.059	0.709±0.060	<b>0.780±0.064</b>	0.763±0.064	<b>0.775±0.060</b>	0.614±0.072	0.579±0.074	0.577±0.066	0.750±0.074
	LGBM	0.765±0.057	0.709±0.060	<b>0.775±0.060</b>	0.763±0.056	0.756±0.059	0.609±0.064	0.580±0.077	0.581±0.074	0.750±0.076
	HGNN	0.505±0.047	0.538±0.074	0.512±0.044	<b>0.543±0.063</b>	0.499±0.049	0.513±0.055	0.526±0.048	0.512±0.058	0.510±0.049
	FHGCN	0.587±0.077	<b>0.703±0.101</b>	0.598±0.079	0.651±0.090	0.693±0.117	0.536±0.061	0.566±0.076	0.550±0.069	0.680±0.096
	UGCNII	0.668±0.063	<b>0.727±0.046</b>	0.675±0.063	0.708±0.065	0.710±0.050	0.673±0.045	0.689±0.055	0.582±0.051	0.719±0.058
Max		<b>0.810±0.050</b>	0.732±0.058	<b>0.808±0.062</b>	0.773±0.056	0.805±0.055	0.673±0.045	0.689±0.055	0.592±0.063	0.754±0.066
Avg.		0.710±0.092	0.691±0.055	0.720±0.093	0.713±0.068	<b>0.732±0.087</b>	0.597±0.049	0.564±0.049	0.568±0.025	0.700±0.071
Rank Avg.		4.100±2.300	4.400±2.059	<b>2.800±1.939</b>	3.500±0.922	2.800±2.272	7.000±1.183	7.600±1.960	7.900±1.136	4.900±1.375
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.855±0.052	0.783±0.062	<b>0.870±0.059</b>	0.826±0.058	<b>0.883±0.049</b>	0.627±0.071	0.480±0.076	0.634±0.074	0.820±0.066
	RF	0.839±0.055	0.773±0.059	0.850±0.068	0.856±0.054	<b>0.880±0.047</b>	0.684±0.071	0.624±0.098	0.623±0.074	0.841±0.069
	DT	0.699±0.064	0.652±0.057	<b>0.708±0.056</b>	0.690±0.071	0.707±0.069	0.551±0.069	0.529±0.073	0.542±0.068	0.651±0.075
	KNN	0.827±0.059	0.745±0.060	0.829±0.067	0.810±0.048	<b>0.846±0.051</b>	0.685±0.073	0.597±0.089	0.591±0.073	0.788±0.065
	MLP	0.881±0.039	0.780±0.062	0.881±0.059	0.825±0.061	<b>0.883±0.047</b>	0.697±0.067	0.618±0.096	0.636±0.069	0.786±0.073
	XGB	0.833±0.049	0.765±0.067	<b>0.851±0.065</b>	0.847±0.057	<b>0.863±0.052</b>	0.666±0.082	0.632±0.097	0.610±0.077	0.831±0.074
	LGBM	0.840±0.051	0.769±0.063	<b>0.849±0.063</b>	0.847±0.060	0.842±0.056	0.655±0.081	0.635±0.094	0.612±0.085	0.837±0.070
	HGNN	0.507±0.064	<b>0.554±0.070</b>	0.521±0.053	0.543±0.070	0.505±0.052	0.532±0.070	<b>0.543±0.062</b>	0.532±0.080	0.505±0.060
	FHGCN	0.642±0.078	0.773±0.073	0.663±0.085	0.738±0.099	<b>0.804±0.102</b>	0.548±0.075	0.619±0.084	0.570±0.073	0.763±0.103
	UGCNII	0.722±0.066	<b>0.788±0.052</b>	0.724±0.066	0.767±0.065	0.787±0.048	0.706±0.045	0.739±0.056	0.606±0.059	0.784±0.070
Max		0.881±0.039	0.788±0.052	0.881±0.059	0.856±0.054	<b>0.883±0.049</b>	0.706±0.045	0.739±0.056	0.636±0.069	0.841±0.069
Avg.		0.764±0.114	0.738±0.072	0.775±0.112	0.775±0.093	<b>0.800±0.111</b>	0.635±0.063	0.601±0.067	0.596±0.035	0.761±0.100
Rank Avg.		4.400±1.685	4.500±2.110	3.100±1.814	3.400±0.800	<b>2.100±2.071</b>	7.200±0.980	7.300±2.100	8.000±1.483	5.000±1.612

(d) email-eu

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.854±0.009	0.837±0.013	<b>0.855±0.009</b>	0.776±0.019	<b>0.869±0.010</b>	0.618±0.028	0.496±0.012	0.659±0.019	0.799±0.017
	RF	0.883±0.009	0.839±0.011	<b>0.890±0.008</b>	0.838±0.011	<b>0.907±0.010</b>	0.652±0.017	0.515±0.021	0.668±0.020	0.840±0.011
	DT	0.825±0.010	0.787±0.016	0.828±0.010	0.761±0.016	<b>0.849±0.012</b>	0.546±0.015	0.504±0.019	0.564±0.015	0.753±0.018
	KNN	0.867±0.009	0.838±0.011	0.868±0.008	0.780±0.014	<b>0.875±0.010</b>	0.573±0.009	0.556±0.040	0.677±0.013	0.795±0.013
	MLP	0.904±0.008	0.857±0.012	<b>0.911±0.007</b>	0.821±0.023	0.906±0.011	0.660±0.047	0.507±0.018	0.675±0.048	0.808±0.027
	XGB	0.885±0.008	0.854±0.011	0.891±0.008	0.831±0.012	<b>0.903±0.009</b>	0.654±0.018	0.522±0.017	0.656±0.022	0.840±0.011
	LGBM	0.889±0.009	0.856±0.011	0.895±0.008	0.839±0.010	<b>0.906±0.010</b>	0.645±0.027	0.512±0.010	0.645±0.026	0.846±0.011
	HGNN	0.521±0.015	0.520±0.015	0.523±0.016	0.523±0.020	<b>0.529±0.020</b>	0.512±0.018	0.505±0.014	0.513±0.017	0.526±0.021
	FHGCN	0.607±0.087	<b>0.790±0.136</b>	0.622±0.095	0.638±0.060	0.742±0.072	0.512±0.020	0.519±0.038	0.547±0.054	0.678±0.072
	UGCNII	0.772±0.014	<b>0.859±0.008</b>	0.788±0.013	0.726±0.009	0.783±0.013	0.724±0.014	0.740±0.020	0.706±0.013	0.762±0.011
Max		0.904±0.008	0.859±0.008	<b>0.911±0.007</b>	0.839±0.010	0.907±0.010	0.724±0.014	0.740±0.020	0.706±0.013	0.846±0.011
Avg.		0.801±0.125	0.804±0.098	0.807±0.124	0.753±0.097	<b>0.827±0.113</b>	0.610±0.067	0.538±0.069	0.631±0.062	0.765±0.093
Rank Avg.		3.600±1.020	3.700±1.487	2.400±1.114	5.400±1.114	<b>1.400±0.663</b>	8.100±0.300	8.600±0.917	7.200±0.600	4.600±1.200
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.922±0.009	0.876±0.009	0.926±0.008	0.838±0.015	<b>0.933±0.008</b>	0.691±0.013	0.494±0.018	0.722±0.014	0.868±0.013
	RF	0.941±0.007	0.901±0.009	<b>0.946±0.005</b>	0.921±0.007	<b>0.960±0.005</b>	0.737±0.016	0.529±0.032	0.770±0.015	0.925±0.007
	DT	0.827±0.010	0.785±0.017	0.830±0.010	0.762±0.016	<b>0.852±0.012</b>	0.546±0.015	0.504±0.019	0.564±0.015	0.754±0.018
	KNN	0.915±0.008	0.881±0.010	0.915±0.007	0.854±0.013	<b>0.920±0.008</b>	0.701±0.013	0.586±0.059	0.749±0.014	0.870±0.010
	MLP	0.960±0.005	0.911±0.010	<b>0.963±0.004</b>	0.909±0.013	0.962±0.006	0.790±0.014	0.539±0.052	0.802±0.017	0.887±0.021
	XGB	0.945±0.006	0.905±0.009	0.949±0.005	0.917±0.007	<b>0.961±0.005</b>	0.747±0.014	0.557±0.033	0.777±0.014	0.925±0.008
	LGBM	0.948±0.006	0.908±0.008	<b>0.952±0.005</b>	0.923±0.006	<b>0.963±0.005</b>	0.761±0.014	0.555±0.032	0.796±0.013	0.930±0.007
	HGNN	0.524±0.020	0.516±0.017	<b>0.524±0.020</b>	0.520±0.020	<b>0.529±0.028</b>	0.504±0.018	0.502±0.014	0.500±0.017	0.524±0.024
	FHGCN	0.724±0.082	<b>0.888±0.041</b>	0.745±0.080	0.724±0.051	0.849±0.054	0.520±0.029	0.550±0.057	0.614±0.070	0.768±0.062
	UGCNII	0.864±0.011	<b>0.912±0.007</b>	0.881±0.010	0.805±0.010	0.874±0.010	0.793±0.014	0.812±0.020	0.784±0.012	0.849±0.008
Max		0.960±0.005	0.912±0.007	<b>0.963±0.004</b>	0.923±0.006	0.963±0.005	0.793±0.014	0.812±0.020	0.802±0.017	0.930±0.007
Avg.		0.857±0.131	0.848±0.116	0.863±0.130	0.817±0.119	<b>0.880±0.125</b>	0.679±0.107	0.563±0.087	0.708±0.103	0.830±0.118
Rank Avg.		3.300±0.640	4.200±1.833	2.300±0.900	5.500±0.671	<b>1.400±0.663</b>	8.000±0.447	8.500±0.922	7.400±0.800	4.400±1.200

(e) citation-data-science

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.907±0.008	0.602±0.018	0.918±0.007	0.751±0.039	<b>0.921±0.011</b>	0.527±0.017	0.504±0.008	0.593±0.022	0.837±0.027
	RF	0.952±0.006	0.644±0.022	0.952±0.008	0.855±0.017	<b>0.977±0.004</b>	0.548±0.015	0.500±0.012	0.599±0.023	0.923±0.007
	DT	0.914±0.011	0.583±0.018	0.906±0.015	0.777±0.026	<b>0.963±0.005</b>	0.511±0.016	0.497±0.015	0.539±0.015	0.848±0.015
	KNN	0.922±0.007	0.594±0.016	<b>0.925±0.007</b>	0.787±0.025	0.917±0.010	0.558±0.013	0.514±0.021	0.592±0.014	0.844±0.012
	MLP	0.967±0.005	0.637±0.026	<b>0.971±0.005</b>	0.822±0.035	0.969±0.005	0.545±0.024	0.502±0.010	0.633±0.037	0.888±0.018
	XGB	0.958±0.005	0.639±0.025	<b>0.961±0.006</b>	0.843±0.024	<b>0.975±0.004</b>	0.547±0.018	0.505±0.011	0.618±0.024	0.927±0.011
	LGBM	0.957±0.006	0.652±0.022	<b>0.960±0.006</b>	0.849±0.020	<b>0.977±0.004</b>	0.546±0.020	0.504±0.010	0.601±0.026	0.930±0.009
	HGNN	0.572±0.010	0.534±0.013	<b>0.585±0.012</b>	0.543±0.015	<b>0.595±0.009</b>	0.542±0.012	0.535±0.017	0.519±0.013	0.554±0.017
	FHGCN	0.593±0.093	0.556±0.067	0.621±0.095	0.597±0.087	<b>0.754±0.091</b>	0.505±0.015	0.504±0.013	0.502±0.008	0.622±0.099
	UGCNII	0.923±0.009	0.657±0.012	<b>0.927±0.009</b>	0.798±0.013	<b>0.932±0.005</b>	0.769±0.016	0.630±0.028	0.541±0.011	0.823±0.010
Max		0.967±0.005	0.657±0.012	<b>0.971±0.005</b>	0.855±0.017	<b>0.977±0.004</b>	0.769±0.016	0.630±0.028	0.633±0.037	0.930±0.009
Avg.		0.866±0.143	0.610±0.041	<b>0.873±0.136</b>	0.762±0.102	<b>0.898±0.119</b>	0.560±0.072	0.519±0.038	0.574±0.042	0.820±0.123
Rank Avg.		3.000±0.775	6.300±0.640	<b>2.000±0.632</b>	4.900±0.300	<b>1.300±0.640</b>	7.500±0.806	8.600±0.663	7.600±0.917	3.800±0.600

(f) citation-software

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.908±0.005	0.662±0.015	<b>0.912±0.004</b>	0.767±0.007	<b>0.919±0.007</b>	0.541±0.018	0.508±0.013	0.625±0.019	0.829±0.010
	RF	0.966±0.003	0.702±0.019	<b>0.968±0.003</b>	0.866±0.022	<b>0.984±0.002</b>	0.568±0.018	0.501±0.012	0.621±0.022	0.941±0.004
	DT	0.945±0.004	0.623±0.012	0.944±0.004	0.783±0.045	<b>0.974±0.002</b>	0.519±0.010	0.498±0.016	0.548±0.009	0.875±0.012
	KNN	<b>0.947±0.004</b>	0.652±0.013	0.945±0.003	0.834±0.013	0.941±0.005	0.578±0.010	0.521±0.026	0.606±0.006	0.884±0.008
	MLP	0.978±0.002	0.699±0.018	<b>0.979±0.002</b>	0.844±0.029	<b>0.980±0.002</b>	0.575±0.032	0.512±0.022	0.652±0.041	0.919±0.011
	XGB	0.971±0.002	0.704±0.018	<b>0.973±0.002</b>	0.850±0.037	<b>0.984±0.002</b>	0.562±0.023	0.506±0.008	0.636±0.024	0.943±0.007
	LGBM	0.969±0.003	0.713±0.016	<b>0.971±0.002</b>	0.860±0.032	<b>0.984±0.002</b>	0.560±0.028	0.502±0.006	0.622±0.025	0.946±0.006
	HGNN	0.560±0.009	0.529±0.008	<b>0.562±0.009</b>	0.534±0.011	0.555±0.007	0.553±0.009	<b>0.568±0.009</b>	0.521±0.006	0.543±0.007
	FHGCN	0.561±0.074	0.594±0.089	0.572±0.074	0.609±0.086	<b>0.738±0.098</b>	0.503±0.013	0.502±0.009	0.503±0.009	0.630±0.084
	UGCNII	0.917±0.005	0.718±0.010	<b>0.921±0.005</b>	0.739±0.010	0.917±0.006	0.827±0.016	0.823±0.009	0.578±0.012	0.792±0.006
Max		0.978±0.002	0.718±0.010	0.979±0.002	0.866±0.022	<b>0.984±0.002</b>	0.827±0.016	0.823±0.009	0.652±0.041	0.946±0.006
Avg.		0.872±0.157	0.660±0.058	<b>0.875±0.155</b>	0.769±0.108	<b>0.898±0.134</b>	0.579±0.086	0.544±0.095	0.591±0.049	0.830±0.132
Rank Avg.		2.900±1.221	6.200±1.077	2.300±1.005	5.200±1.077	<b>1.700±1.100</b>	7.200±1.400	7.800±2.561	7.500±0.806	4.200±1.077

Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.977±0.003	0.722±0.015	<b>0.979±0.002</b>	0.890±0.008	<b>0.980±0.003</b>	0.584±0.010	0.516±0.008	0.688±0.008	0.943±0.005
	RF	0.993±0.001	0.777±0.012	<b>0.994±0.001</b>	0.945±0.012	<b>0.999±0.000</b>	0.611±0.015	0.502±0.017	0.739±0.013	0.986±0.001
	DT	0.945±0.004	0.623±0.012	0.944±0.004	0.783±0.045	<b>0.974±0.002</b>	0.519±0.010	0.498±0.016	0.548±0.009	0.875±0.012
	KNN	<b>0.976±0.002</b>	0.702±0.015	<b>0.976±0.002</b>	0.899±0.010	0.974±0.003	0.632±0.011	0.531±0.034	0.659±0.007	0.940±0.006
	MLP	<b>0.997±0.001</b>	0.775±0.016	<b>0.997±0.001</b>	0.930±0.018	0.996±0.002	0.671±0.019	0.544±0.048	0.806±0.009	0.964±0.008
	XGB	0.995±0.001	0.781±0.014	0.996±0.001	0.939±0.019	<b>0.999±0.000</b>	0.620±0.017	0.522±0.020	0.786±0.009	0.991±0.001
	LGBM	0.995±0.001	0.792±0.012	0.996±0.001	0.946±0.017	<b>0.999±0.000</b>	0.626±0.019	0.525±0.020	0.807±0.008	0.991±0.001
	HGNN	0.584±0.012	0.510±0.012	<b>0.589±0.011</b>	0.509±0.023	0.587±0.006	0.562±0.009	<b>0.590±0.012</b>	0.508±0.007	0.551±0.014
	FHGCN	0.651±0.094	0.714±0.054	0.659±0.085	0.718±0.050	<b>0.852±0.044</b>	0.513±0.025	0.511±0.024	0.505±0.013	0.732±0.045
	UGCNII	0.972±0.002	0.792±0.011	<b>0.973±0.003</b>	0.812±0.010	0.971±0.003	0.899±0.016	0.895±0.007	0.609±0.015	0.868±0.005
Max		0.997±0.001	0.792±0.012	<b>0.997±0.001</b>	0.946±0.017	<b>0.999±0.000</b>	0.899±0.016	0.895±0.007	0.807±0.008	0.991±0.001
Avg.		0.909±0.147	0.719±0.086	<b>0.910±0.145</b>	0.837±0.132	<b>0.933±0.123</b>	0.624±0.103	0.564±0.113	0.666±0.113	0.884±0.134
Rank Avg.		2.900±1.300	6.400±1.020	2.200±1.077	5.300±1.269	<b>1.800±0.980</b>	7.200±1.400	7.700±2.532	7.300±1.187	4.200±1.077

(g) qna-math

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.606±0.010	0.553±0.013	0.607±0.010	0.579±0.011	0.604±0.009	0.500±0.005	0.504±0.006	0.566±0.009	<b>0.615±0.012</b>
	RF	0.640±0.008	0.620±0.012	0.641±0.008	0.613±0.017	<b>0.673±0.010</b>	0.503±0.010	0.502±0.010	0.581±0.024	0.656±0.015
	DT	0.598±0.009	0.572±0.010	0.595±0.009	0.547±0.016	<b>0.617±0.011</b>	0.502±0.008	0.502±0.011	0.547±0.013	0.574±0.017
	KNN	<b>0.607±0.008</b>	0.553±0.011	0.605±0.007	0.576±0.011	0.601±0.010	0.504±0.009	0.506±0.015	0.523±0.008	0.603±0.010
	MLP	0.639±0.017	0.590±0.014	<b>0.640±0.016</b>	0.598±0.015	<b>0.679±0.010</b>	0.505±0.009	0.503±0.005	0.611±0.043	0.626±0.020
	XGB	0.643±0.008	0.629±0.011	0.644±0.009	0.607±0.019	<b>0.681±0.010</b>	0.502±0.010	0.508±0.008	0.601±0.027	0.668±0.016
	LGBM	0.651±0.008	0.641±0.012	0.652±0.009	0.618±0.019	<b>0.690±0.009</b>	0.504±0.009	0.506±0.008	0.577±0.027	0.686±0.015
	HGNN	<b>0.552±0.010</b>	0.529±0.008	<b>0.552±0.009</b>	0.525±0.011	0.549±0.011	0.520±0.011	0.546±0.007	0.545±0.008	0.548±0.008
	FHGCN	0.501±0.006	<b>0.516±0.029</b>	0.501±0.004	0.503±0.007	0.512±0.022	0.500±0.003	0.502±0.008	0.507±0.016	0.508±0.017
	UGCNII	0.613±0.005	0.599±0.010	0.614±0.004	0.583±0.009	0.607±0.006	0.615±0.010	<b>0.637±0.008</b>	<b>0.743±0.013</b>	0.609±0.008
Max		0.651±0.008	0.641±0.012	0.652±0.009	0.618±0.019	0.690±0.009	0.615±0.010	0.637±0.008	<b>0.743±0.013</b>	0.686±0.015
Avg.		0.605±0.044	0.580±0.041	0.605±0.045	0.575±0.037	<b>0.621±0.057</b>	0.515±0.034	0.521±0.040	0.580±0.063	0.609±0.052
Rank Avg.		3.500±1.628	5.600±1.855	3.100±1.814	6.200±1.249	<b>2.500±1.910</b>	8.200±1.778	7.100±2.071	5.700±1.847	3.100±1.375
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.653±0.008	0.580±0.014	<b>0.655±0.010</b>	0.620±0.011	0.652±0.011	0.499±0.009	0.514±0.010	0.600±0.009	<b>0.666±0.013</b>
	RF	0.692±0.009	0.663±0.015	0.695±0.009	0.657±0.020	0.734±0.011	0.505±0.012	0.504±0.014	<b>0.767±0.020</b>	0.716±0.019
	DT	0.602±0.009	0.569±0.010	0.599±0.009	0.547±0.016	<b>0.621±0.011</b>	0.502±0.008	0.502±0.011	0.547±0.013	0.574±0.017
	KNN	<b>0.647±0.009</b>	0.570±0.012	0.644±0.009	0.601±0.013	0.638±0.012	0.506±0.011	0.511±0.019	0.561±0.012	0.638±0.013
	MLP	0.696±0.015	0.634±0.015	0.700±0.016	0.649±0.015	0.737±0.013	0.514±0.013	0.509±0.010	<b>0.834±0.011</b>	0.679±0.022
	XGB	0.700±0.009	0.677±0.014	0.702±0.009	0.660±0.020	0.744±0.011	0.504±0.012	0.513±0.011	<b>0.823±0.010</b>	0.736±0.016
	LGBM	0.708±0.008	0.694±0.015	0.711±0.010	0.679±0.016	0.755±0.010	0.505±0.014	0.513±0.012	<b>0.844±0.009</b>	0.758±0.013
	HGNN	<b>0.576±0.010</b>	0.535±0.012	0.575±0.008	0.520±0.011	0.570±0.013	0.519±0.012	0.566±0.009	0.551±0.010	0.568±0.009
	FHGCN	0.504±0.011	<b>0.545±0.044</b>	0.502±0.008	0.507±0.010	0.538±0.031	0.501±0.005	0.507±0.020	0.521±0.028	0.525±0.028
	UGCNII	0.666±0.008	0.642±0.011	0.667±0.007	0.620±0.011	0.658±0.007	0.661±0.012	<b>0.689±0.007</b>	<b>0.817±0.014</b>	0.659±0.009
Max		0.708±0.008	0.694±0.015	0.711±0.010	0.679±0.016	0.755±0.010	0.661±0.012	0.689±0.007	<b>0.844±0.009</b>	0.758±0.013
Avg.		0.644±0.062	0.611±0.055	0.645±0.064	0.606±0.058	0.665±0.072	0.521±0.047	0.533±0.055	<b>0.687±0.133</b>	0.652±0.073
Rank Avg.		3.700±1.847	5.900±1.814	3.500±1.688	6.600±1.200	<b>2.900±1.578</b>	8.400±1.200	7.000±2.145	3.500±2.617	3.500±1.360

(h) qna-server

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.556±0.007	0.530±0.011	<b>0.557±0.007</b>	0.533±0.009	<b>0.561±0.007</b>	0.502±0.004	0.509±0.008	0.556±0.007	0.553±0.007
	RF	0.643±0.005	0.590±0.012	0.640±0.004	0.565±0.012	<b>0.661±0.004</b>	0.500±0.006	0.503±0.005	0.565±0.018	0.631±0.010
	DT	0.615±0.005	0.572±0.012	0.614±0.005	0.528±0.008	<b>0.626±0.006</b>	0.500±0.007	0.501±0.007	0.545±0.009	0.571±0.011
	KNN	<b>0.628±0.005</b>	0.566±0.010	0.626±0.005	0.538±0.007	0.619±0.006	0.504±0.007	0.502±0.009	0.576±0.005	0.562±0.008
	MLP	<b>0.670±0.005</b>	0.603±0.014	<b>0.670±0.005</b>	0.544±0.013	0.668±0.005	0.509±0.006	0.506±0.011	0.598±0.040	0.584±0.014
	XGB	0.658±0.005	0.612±0.016	0.656±0.005	0.575±0.013	<b>0.679±0.005</b>	0.503±0.005	0.503±0.007	0.585±0.019	0.647±0.010
	LGBM	0.666±0.004	0.624±0.013	0.665±0.005	0.585±0.011	<b>0.688±0.004</b>	0.504±0.006	0.503±0.007	0.567±0.018	0.657±0.009
	HGNN	0.588±0.005	0.546±0.005	0.588±0.005	0.539±0.005	0.579±0.006	0.535±0.007	0.572±0.006	0.571±0.005	<b>0.591±0.005</b>
	FHGCN	0.502±0.008	<b>0.515±0.027</b>	0.502±0.007	0.505±0.014	<b>0.530±0.041</b>	0.501±0.003	0.501±0.005	0.502±0.009	0.514±0.030
	UGCNII	0.657±0.006	0.605±0.005	0.656±0.006	0.595±0.007	0.713±0.005	0.563±0.005	0.653±0.006	<b>0.752±0.007</b>	0.652±0.005
Max		0.670±0.005	0.624±0.013	0.670±0.005	0.595±0.007	0.713±0.005	0.563±0.005	0.653±0.006	<b>0.752±0.007</b>	0.657±0.009
Avg.		0.619±0.052	0.576±0.035	0.617±0.051	0.551±0.027	<b>0.632±0.057</b>	0.512±0.020	0.525±0.047	0.582±0.062	0.596±0.046
Rank Avg.		2.500±1.432	5.100±1.513	3.200±1.400	6.700±1.100	<b>1.800±1.077</b>	8.600±0.490	7.800±1.470	4.900±1.700	4.400±1.497
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.596±0.005	0.558±0.009	<b>0.597±0.006</b>	0.553±0.010	<b>0.598±0.006</b>	0.512±0.006	0.528±0.006	0.586±0.006	0.587±0.009
	RF	0.704±0.005	0.637±0.017	0.701±0.005	0.595±0.016	0.728±0.005	0.501±0.008	0.504±0.009	<b>0.748±0.017</b>	0.692±0.013
	DT	<b>0.619±0.005</b>	0.571±0.010	0.618±0.005	0.524±0.008	<b>0.630±0.006</b>	0.500±0.007	0.501±0.007	0.545±0.009	0.571±0.011
	KNN	<b>0.681±0.005</b>	0.599±0.013	<b>0.678±0.005</b>	0.552±0.009	0.669±0.006	0.505±0.008	0.503±0.013	0.610±0.008	0.586±0.010
	MLP	0.719±0.006	0.630±0.014	<b>0.719±0.006</b>	0.573±0.013	0.717±0.006	0.511±0.006	0.525±0.013	<b>0.815±0.010</b>	0.631±0.015
	XGB	0.713±0.006	0.653±0.021	0.711±0.005	0.612±0.016	<b>0.745±0.005</b>	0.504±0.007	0.507±0.011	<b>0.801±0.007</b>	0.712±0.013
	LGBM	0.719±0.005	0.669±0.018	0.719±0.005	0.628±0.014	<b>0.753±0.005</b>	0.506±0.009	0.513±0.010	<b>0.813±0.007</b>	0.729±0.010
	HGNN	0.629±0.006	0.563±0.006	<b>0.629±0.005</b>	0.543±0.006	0.619±0.008	0.534±0.008	0.599±0.007	0.585±0.009	<b>0.631±0.009</b>
	FHGCN	0.508±0.016	0.546±0.049	0.507±0.016	0.528±0.025	<b>0.593±0.041</b>	0.502±0.007	0.503±0.012	0.511±0.018	0.557±0.045
	UGCNII	0.731±0.007	0.649±0.007	0.728±0.006	0.636±0.008	0.645±0.005	0.594±0.010	0.713±0.006	<b>0.824±0.008</b>	0.719±0.007
Max		0.731±0.007	0.669±0.018	0.728±0.006	0.636±0.008	<b>0.753±0.005</b>	0.594±0.010	0.713±0.006	<b>0.824±0.008</b>	0.729±0.010
Avg.		0.662±0.069	0.607±0.043	0.661±0.068	0.575±0.039	<b>0.670±0.058</b>	0.517±0.027	0.540±0.064	<b>0.684±0.121</b>	0.641±0.063
Rank Avg.		3.000±1.265	5.600±1.020	3.500±1.628	6.900±1.044	<b>2.700±1.792</b>	8.900±0.300	7.500±1.285	3.100±2.166	3.800±1.400

(i) bitcoin-2014

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.552±0.003	0.524±0.002	0.581±0.006		<b>0.626±0.022</b>	0.527±0.027	0.527±0.014	0.580±0.027	
	RF	0.914±0.003	0.731±0.029	0.919±0.003		<b>0.927±0.010</b>	0.540±0.013	0.517±0.005	0.608±0.039	
	DT	0.886±0.003	0.703±0.027	0.891±0.003		<b>0.896±0.017</b>	0.507±0.010	0.503±0.005	0.549±0.011	
	KNN	<b>0.879±0.003</b>	0.699±0.014	0.878±0.003		0.838±0.016	0.577±0.011	0.537±0.010	0.639±0.004	
	MLP	0.763±0.011	0.612±0.049	0.838±0.015		<b>0.922±0.002</b>	0.555±0.031	0.538±0.023	0.629±0.039	
	XGB	0.902±0.004	0.760±0.036	0.912±0.003		<b>0.935±0.003</b>	0.554±0.014	0.520±0.012	0.615±0.032	
	LGBM	0.882±0.005	0.765±0.036	0.896±0.004	O.O.T.*	<b>0.935±0.002</b>	0.560±0.018	0.526±0.016	0.606±0.034	O.O.T.*
	HGNN	0.730±0.003	0.633±0.003	<b>0.731±0.003</b>		0.727±0.003	0.632±0.004	0.628±0.003	0.600±0.003	
	FHGCN	0.517±0.027	0.623±0.090	0.516±0.025		<b>0.705±0.103</b>	0.504±0.009	0.504±0.010	0.505±0.009	
	UGCNII	0.853±0.003	0.771±0.003	<b>0.855±0.002</b>		0.845±0.003	0.773±0.008	0.711±0.005	0.652±0.003	
Max		0.914±0.003	0.771±0.003	0.919±0.003		<b>0.935±0.003</b>	0.773±0.008	0.711±0.005	0.652±0.003	
Avg.		0.788±0.139	0.682±0.077	0.802±0.137		<b>0.836±0.106</b>	0.573±0.075	0.551±0.063	0.598±0.042	
Rank Avg.		2.700±0.781	4.300±1.187	2.000±0.775		<b>1.600±0.917</b>	5.700±0.640	6.600±0.663	5.100±1.136	
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.559±0.013	0.616±0.007	0.674±0.021		<b>0.693±0.014</b>	0.569±0.003	0.559±0.002	0.640±0.003	
	RF	0.971±0.001	0.799±0.034	0.972±0.001		<b>0.976±0.004</b>	0.562±0.015	0.524±0.008	0.729±0.022	
	DT	0.885±0.004	0.704±0.031	0.890±0.003		<b>0.900±0.014</b>	0.507±0.010	0.503±0.005	0.549±0.011	
	KNN	<b>0.941±0.002</b>	0.744±0.016	0.941±0.003		0.905±0.015	0.606±0.013	0.556±0.012	0.694±0.005	
	MLP	0.854±0.013	0.679±0.053	0.917±0.010		<b>0.968±0.002</b>	0.622±0.029	0.577±0.031	0.771±0.009	
	XGB	0.966±0.002	0.841±0.033	0.971±0.002		<b>0.980±0.003</b>	0.575±0.015	0.549±0.009	0.784±0.005	
	LGBM	0.954±0.003	0.847±0.030	0.962±0.002	O.O.T.*	<b>0.980±0.002</b>	0.583±0.018	0.558±0.011	0.789±0.005	O.O.T.*
	HGNN	0.815±0.002	0.663±0.004	<b>0.816±0.003</b>		0.808±0.002	0.664±0.004	0.664±0.003	0.607±0.002	
	FHGCN	0.536±0.044	0.714±0.055	0.540±0.046		<b>0.822±0.028</b>	0.510±0.021	0.506±0.015	0.506±0.012	
	UGCNII	0.932±0.003	0.854±0.003	<b>0.933±0.002</b>		0.926±0.003	0.855±0.008	0.790±0.005	0.713±0.003	
Max		0.971±0.001	0.854±0.003	0.972±0.001		<b>0.980±0.003</b>	0.855±0.008	0.790±0.005	0.789±0.005	
Avg.		0.841±0.155	0.746±0.081	0.862±0.138		<b>0.896±0.090</b>	0.605±0.095	0.579±0.083	0.678±0.094	
Rank Avg.		3.100±1.513	4.200±0.980	1.900±0.539		<b>1.600±0.917</b>	5.500±0.671	6.400±0.917	5.300±1.269	

\* O.O.T: out-of-time (&gt; 1 day).

(j) bitcoin-2015

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.539±0.016	0.539±0.006	0.561±0.003		<b>0.566±0.002</b>	0.528±0.029	0.532±0.013	<b>0.569±0.027</b>	
	RF	0.928±0.003	0.686±0.034	<b>0.929±0.003</b>		0.926±0.012	0.552±0.012	0.520±0.006	0.598±0.037	
	DT	0.904±0.004	0.664±0.036	<b>0.905±0.003</b>		0.889±0.021	0.512±0.009	0.504±0.005	0.543±0.011	
	KNN	<b>0.906±0.002</b>	0.658±0.028	0.895±0.003		0.839±0.025	0.592±0.008	0.539±0.009	0.628±0.004	
	MLP	0.825±0.018	0.635±0.027	0.854±0.014		<b>0.920±0.015</b>	0.571±0.029	0.553±0.023	0.619±0.039	
	XGB	0.918±0.005	0.721±0.042	0.923±0.004		<b>0.939±0.005</b>	0.571±0.012	0.519±0.011	0.606±0.028	
	LGBM	0.901±0.007	0.725±0.038	0.908±0.006	O.O.T.*	<b>0.941±0.003</b>	0.574±0.019	0.526±0.015	0.596±0.030	O.O.T.*
	HGNN	0.748±0.003	0.636±0.003	<b>0.750±0.003</b>		<b>0.750±0.003</b>	0.641±0.006	0.630±0.004	0.603±0.002	
	FHGCN	0.545±0.063	0.599±0.098	0.574±0.067		<b>0.699±0.112</b>	0.506±0.013	0.503±0.008	0.503±0.006	
	UGCNII	0.866±0.003	0.773±0.005	<b>0.867±0.002</b>		0.858±0.002	0.782±0.005	0.717±0.003	0.651±0.003	
Max		0.928±0.003	0.773±0.005	0.929±0.003		<b>0.941±0.003</b>	0.782±0.005	0.717±0.003	0.651±0.003	
Avg.		0.808±0.142	0.664±0.064	0.817±0.134		<b>0.833±0.118</b>	0.583±0.076	0.554±0.064	0.592±0.041	
Rank Avg.		2.800±1.077	4.000±0.775	<b>1.800±0.748</b>		2.000±0.894	5.600±0.917	6.700±0.458	5.100±1.578	
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.552±0.008	0.612±0.010	0.692±0.005		<b>0.696±0.012</b>	0.592±0.003	0.563±0.003	0.627±0.003	
	RF	0.977±0.001	0.755±0.040	<b>0.978±0.001</b>		0.977±0.004	0.578±0.014	0.528±0.008	0.715±0.023	
	DT	0.903±0.004	0.672±0.035	<b>0.904±0.004</b>		0.893±0.019	0.512±0.009	0.504±0.005	0.543±0.011	
	KNN	<b>0.957±0.002</b>	0.705±0.032	0.952±0.002		0.906±0.021	0.627±0.012	0.558±0.011	0.681±0.006	
	MLP	0.917±0.014	0.690±0.020	0.938±0.008		<b>0.967±0.005</b>	0.639±0.032	0.596±0.030	0.761±0.013	
	XGB	0.974±0.002	0.797±0.046	0.977±0.002		<b>0.981±0.004</b>	0.596±0.012	0.553±0.009	0.776±0.007	
	LGBM	0.965±0.003	0.806±0.040	0.969±0.003		<b>0.983±0.002</b>	0.597±0.012	0.561±0.009	0.782±0.007	O.O.T.*
	HGNN	0.832±0.003	0.663±0.003	<b>0.834±0.003</b>		0.828±0.002	0.681±0.006	0.666±0.005	0.605±0.002	
	FHGCN	0.592±0.079	0.712±0.070	0.623±0.077		<b>0.823±0.040</b>	0.510±0.018	0.504±0.011	0.503±0.008	
	UGCNII	0.940±0.002	0.857±0.006	<b>0.941±0.002</b>		0.935±0.002	0.863±0.005	0.795±0.004	0.713±0.004	
Max		0.977±0.001	0.857±0.006	0.978±0.001		<b>0.983±0.002</b>	0.863±0.005	0.795±0.004	0.782±0.007	
Avg.		0.861±0.150	0.727±0.072	0.881±0.120		<b>0.899±0.088</b>	0.619±0.095	0.583±0.084	0.671±0.093	
Rank Avg.		2.900±1.578	4.200±0.980	<b>1.700±0.640</b>		2.000±1.000	5.400±0.800	6.500±0.671	5.300±1.269	

\* O.O.T: out-of-time (&gt; 1 day).

(k) bitcoin-2016

Dimension		13		26		91				431
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
ACC	LR	0.545±0.002	0.549±0.022	0.551±0.002		0.556±0.003	0.527±0.022	0.530±0.015	<b>0.569±0.028</b>	
	RF	0.915±0.002	0.694±0.030	<u>0.921±0.001</u>		<b>0.934±0.007</b>	0.529±0.010	0.521±0.007	0.609±0.039	
	DT	0.888±0.002	0.671±0.029	<u>0.894±0.002</u>		<b>0.907±0.016</b>	0.509±0.008	0.504±0.005	0.547±0.011	
	KNN	0.888±0.001	0.668±0.028	<b>0.890±0.001</b>		0.866±0.020	0.567±0.010	0.536±0.009	0.626±0.003	
	MLP	0.825±0.010	0.591±0.030	<u>0.830±0.011</u>		<b>0.923±0.003</b>	0.549±0.027	0.545±0.026	0.630±0.040	
	XGB	0.897±0.002	0.722±0.038	<u>0.913±0.002</u>		<b>0.938±0.005</b>	0.544±0.012	0.522±0.013	0.621±0.030	
	LGBM	0.875±0.002	0.733±0.041	<u>0.897±0.002</u>	O.O.T.*	<b>0.937±0.003</b>	0.551±0.016	0.529±0.016	0.610±0.032	O.O.T.*
	HGNN	0.750±0.002	0.633±0.003	<b>0.751±0.002</b>		0.749±0.003	0.627±0.005	0.629±0.003	0.597±0.004	
	FHGCN	0.551±0.064	0.608±0.100	<u>0.553±0.067</u>		<b>0.712±0.110</b>	0.504±0.011	0.504±0.010	0.504±0.011	
	UGCNII	0.862±0.004	0.762±0.005	<b>0.862±0.003</b>		0.854±0.003	0.771±0.010	0.720±0.004	0.646±0.004	
Max		0.915±0.002	0.762±0.005	<u>0.921±0.001</u>		<b>0.938±0.005</b>	0.771±0.010	0.720±0.004	0.646±0.004	
Avg.		0.800±0.133	0.663±0.064	<u>0.806±0.135</u>		<b>0.838±0.121</b>	0.568±0.075	0.554±0.065	0.596±0.041	
Rank Avg.		3.000±0.894	4.000±0.775	<u>1.900±0.700</u>		<b>1.700±0.900</b>	6.000±0.775	6.400±0.800	5.000±1.612	
Measure	Model	DHG-13	triad	DHG-26	h-motif	DHG	n2v	h2v	deep-h	3h-motif
AUROC	LR	0.622±0.006	0.612±0.009	0.630±0.011		<b>0.654±0.012</b>	0.564±0.003	0.567±0.003	<b>0.635±0.002</b>	
	RF	0.971±0.001	0.766±0.035	<u>0.974±0.001</u>		<b>0.978±0.003</b>	0.550±0.012	0.529±0.009	0.725±0.021	
	DT	0.885±0.002	0.681±0.032	<u>0.893±0.002</u>		<b>0.909±0.016</b>	0.509±0.008	0.504±0.005	0.547±0.011	
	KNN	<b>0.948±0.001</b>	0.716±0.029	<b>0.948±0.001</b>		0.927±0.014	0.592±0.014	0.553±0.011	0.678±0.006	
	MLP	0.909±0.006	0.655±0.021	<u>0.915±0.008</u>		<b>0.968±0.002</b>	0.600±0.026	0.587±0.033	0.770±0.007	
	XGB	0.964±0.001	0.801±0.041	<u>0.971±0.001</u>		<b>0.982±0.003</b>	0.562±0.012	0.550±0.011	0.783±0.005	
	LGBM	0.951±0.001	0.814±0.044	<u>0.963±0.001</u>	O.O.T.*	<b>0.982±0.002</b>	0.563±0.011	0.558±0.010	0.788±0.006	O.O.T.*
	HGNN	0.833±0.002	0.659±0.003	<b>0.834±0.002</b>		0.827±0.002	0.659±0.006	0.663±0.004	0.596±0.003	
	FHGCN	0.598±0.074	0.712±0.062	<u>0.597±0.078</u>		<b>0.825±0.049</b>	0.508±0.020	0.507±0.017	0.505±0.015	
	UGCNII	0.937±0.003	0.847±0.006	<b>0.937±0.002</b>		0.931±0.002	0.856±0.010	0.801±0.004	0.704±0.005	
Max		0.971±0.001	0.847±0.006	<u>0.974±0.001</u>		<b>0.982±0.003</b>	0.856±0.010	0.801±0.004	0.788±0.006	
Avg.		0.862±0.132	0.726±0.074	<u>0.866±0.133</u>		<b>0.898±0.099</b>	0.596±0.096	0.582±0.085	0.673±0.095	
Rank Avg.		2.700±0.781	4.300±1.005	<u>2.100±0.831</u>		<b>1.600±0.917</b>	5.700±0.781	6.400±0.917	5.200±1.470	

\* O.O.T: out-of-time (&gt; 1 day).