Applied Linear Algebra: Chapter 1

MATH

26th September 2020

This chapter is the easiest among all the chapters. You just have to study some propositions for some **true or false** problems and practice some calculations like inverse matrix using Gauss-Jordan method, and LDU decomposition(I didn't wrote calculation problems since it's really a tedius task). This is almost all that can be covered in Chapter1.

1. Important proposition

(a) If A is nonsigular matrix, then there exists a permutation matrix P such that $PA = LU$ (or $PA = LDU$).

(b) If A has LDU decomposition, then its decomposition is unique. *i.e. If* $A = L_1D_1U_1 = L_2D_2U_2$*, then* $L_1 = L_2$*,* $D_1 = D_2$ *and* $U_1 = U_2$*.*

(c) TFAE(The followings are equivalent):

For given $Ax = b$,

- (a) There exists a unique solution for all $b \in \mathbb{R}$.
- (b) A is nonsingular.
- (c) For its LDU decomposition, the pivots(which is on D) are all nonzero.

(d) Assume $A = LDU$. If A is symmetric, than $A = LDL^T$ where L^T is the transpose matrix of L.

2. Problems: from exams

(a) Product of triangle matrices is a triangle matrix. (**False, from 2020-1 mid**)

sol) Consider (upper triangle matrix)*(lower triangle matrix).

(b) If D is an n by n digonal matrix, then $AD = DA$ for every n by n matrix A. (**False, from 2020-1 mid**)

sol) Put any matrix. In general, it doesn't hold.

(c) Every nonsingular symmetric square matrix A should have LDL^T factorization. (**False, from 2020-1 mid**)

sol) It's weird.. We have to consider P, a permutation matrix, when $a_{11} = 0$.

(d) Put $A = \begin{pmatrix} a & 0 & b \\ a & a & 4 \\ 0 & a & 2 \end{pmatrix}$.

For what values of a and b does the system $Ax = \begin{pmatrix} 2 \\ 4 \\ b \end{pmatrix}$ have

(a) a unique solution,

(b) a one-parameter solution,

(c) a two-parameter solution,

(d) no solution?

(**from 2019-2 mid**)

sol) It's worth to solve. Use Gaussian elimination.

(e) Consider a linear system $Ax = b$ where A is m by n matrix, x is n by 1 column, and b is m by 1 column. If there are more variables than equations ($m < n$), then there is always a solution.(**False, from 2018-2 mid**)

sol) We can't sure there are two parallel rows however the corresponding values of b are different.

(f) Matrices A, B, and C are given as

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{pmatrix}.$$

(a) (LU decomposition) Write $A = LU$ where L is lower triangular and U is upper triangular.(**from 2018-2 mid**)

sol) It's also very very frequent problems so I recommend you to solve it!