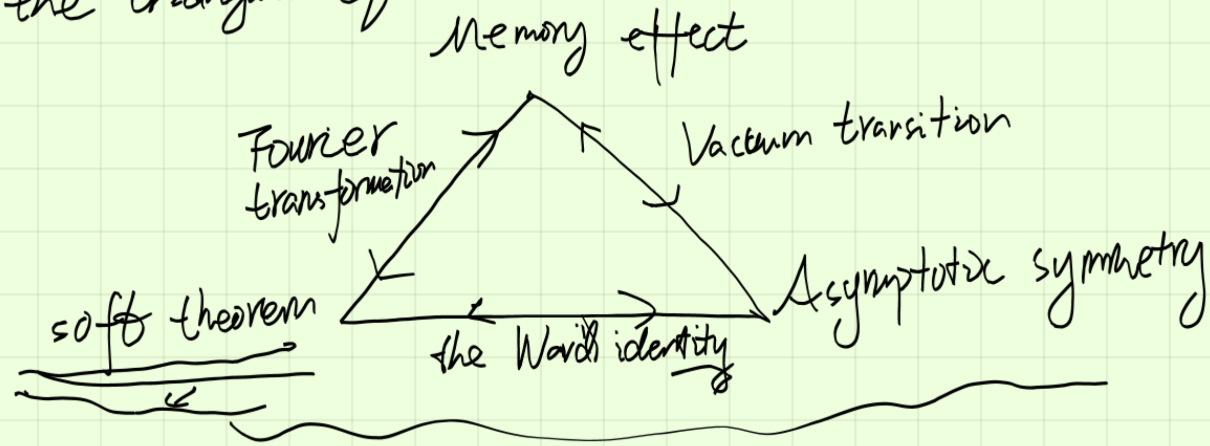
AdS/CFT → holography for negatively background.

Celestial CFT → holography for asymptotic flat spacetimes (AFS)

Evidence:

1. the triangular equivalence in the IR



2. Lorentz group $SO(3,1)$ ~ 2D global conformal group.

$SO(2,2)$

including gravitons

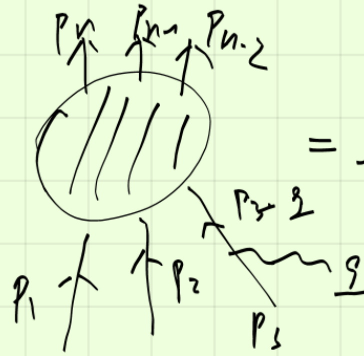
Lorentz group is enhanced to → 2D Virasoro algebra ~ 2D Virasoro algebra

Soft theorem:

1. soft photons (Scalar QED)

$= \mathcal{M}_n(p_i)$

$\underline{\mathcal{M}_{n+1}(p_i, q)}$



$= \mathcal{M}_{n+1}$

$$\mathcal{M}_{n+1} = \frac{(-iQ_k)\, i\,(p_3^\mu + (p_3-q)^\mu)\epsilon_\mu}{(p_3-q)^2 + m_3^2}\, \mathcal{M}_n(p_3-q, p_i)$$

soft limit: $q \to 0$:

$$\frac{2Q_k\, p_3\cdot\epsilon}{-2p_3\cdot q}\, \mathcal{M}_n(p_i)$$

Leading terms:

$$\mathcal{M}_{n+1}(p_i;q) \overset{q\to 0}{\approx} \left(\sum_{\substack{\text{outgoing}\\ k}} \frac{Q_k\, p_k\cdot\epsilon}{p_k\cdot q} - \sum_{\substack{\text{ingoing}\\ k}} \frac{Q_k\, p_k\cdot\epsilon}{p_k\cdot q} \right) \mathcal{M}_n(p_i)$$

Lorentz transformation:

$$\epsilon^\mu(q) \to \epsilon^\mu(q) + q^\mu$$

$$\delta\mathcal{M}_{n+1} \overset{q\to 0}{\approx} \left(\sum_k Q_k^{out} - \sum_k Q_k^{in} \right) \mathcal{M}_n = 0$$

Soft gravitons:

$$\sqrt{8\pi G}\left(\sum_{\substack{k \\ \text{outgoing}}} \frac{\epsilon^{\mu\nu}_{(q)} p_{k\mu} p_{k\nu}}{p_k \cdot q} - \sum_{\substack{k \\ \text{incoming}}} \frac{\epsilon^{\mu\nu} p_{k\mu} p_{k\nu}}{p_k \cdot q}\right)$$ leading term.

$$\epsilon^{\mu\nu}_{(q)} \rightarrow \epsilon^{\mu\nu}_{(q)} + \Lambda^{\mu} q^{\nu}$$

$$\Rightarrow \sum_k p_k^{out} - \sum_k p_k^{in} = 0$$

$$\lim_{\underline{q}\to 0} \mathcal{M}_{n+1}(p_i, \underline{q}) = \left(S_n^{(0)} + S_n^{(1)} + O(\underline{q}) \right) \mathcal{M}_n(p_i)$$

photons: $$S^{(1)} = -i \sum_{k=1}^{n} \frac{Q_k \quad q \cdot J_k \cdot \epsilon(q)}{p_k \cdot q}$$

gravitons: $$-\frac{i\kappa}{2} \underbrace{\sum_{k=1}^{n}}\ \frac{\epsilon(q)\cdot p_k}{\underbrace{p_k \cdot \underline{q}}} q \cdot J_k \cdot \epsilon(q) \qquad \epsilon^{\mu\nu} = \epsilon^{\mu}\epsilon^{\nu}$$

Asymptotic symmetry group (ASG):

$$ASG = \frac{\text{allowed gauge symmetry}}{\text{trivial gauge symmetry}}$$

1 AFS

$$ds^2 = -du^2 - 2du\,dr + 2r^2\gamma_{z\bar{z}}\,dz\,d\bar{z} + \frac{2m_B}{r}du^2 + rC_{zz}\,dz^2 + rC_{\bar{z}\bar{z}}\,d\bar{z}^2 + 2g_{uz}\,du\,dz + 2g_{u\bar{z}}\,du\,d\bar{z} + \cdots$$

$(u, r, z, \bar{z})$

$u = t - r$ $\Rightarrow$ Bondi coordinates

ASG: the extended BMS group? → Virasoro

$Y(z) \sim z^n$

Ponocaré + supertranslation + superrotation

global

local

Superrotation:

$$Q = Q_S + Q_H$$

$$\langle out | Q^+ \hat{S} - \hat{S} Q^- | in \rangle = 0 \qquad Q^+ = Q^-$$

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)\langle out|\underline{a_-(\omega)}\hat{S}|in\rangle$$

$$= \sqrt{8\pi G}\, S^{(1)-}\underline{\langle out|\hat{S}|in\rangle}$$

$O_1(\omega, z\bar{z})$

$O_1(p)$

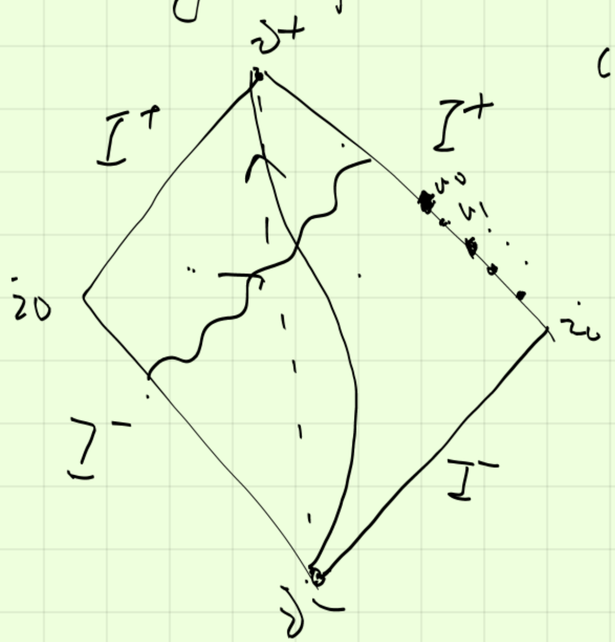
$$\Rightarrow \langle T_{zz}O_1\cdots O_n\rangle = \sum_{k=1}^{n}\left[\frac{\hat{h}_k}{(z-z_k)^2}+\frac{\Gamma^{z_k}_{z_kz_k}}{z-z_k}\hat{h}_k+\frac{1}{z-z_k}\partial_{z_k}\right]\langle O_1\cdots O_n\rangle$$

$p\geq 0$ $\quad \hat{h}_k = -\frac{1}{2}\omega_k\partial_{\omega_k}$ $\quad e^{ip\cdot x}$ $\quad \omega(1+z^2)x^0+\cdots$

$p_1=\omega(1+z\bar{z},\ z+\bar{z},\ -i(z-\bar{z}),\ 1-z\bar{z})$

Celestial sphere:

1. Penrose diagram of Minkowski space: $(u, r, z, \bar{z}) \longrightarrow (u', r', z', \bar{z}')$



$(u, r, z, \bar{z})$

$u = r - t$

$\underline{(u_0, +\infty, z, \bar{z}) \sim CS^2}$

$$r' = F(z,\bar{z})\, r + O(1)$$

$$u' = \frac{u}{F(z,\bar{z})} + O\left(\frac{1}{r}\right)$$

$$z' = \frac{az+b}{cz+d} + O\left(\frac{1}{r}\right) \quad ad-bc=1$$

Celestial CFT

1. Conformal primary wave functions: (scalars)

- Satisfies Klein-Gordon equations.
- transforms as a scalar conformal primary operator under 2D conformal transformation.

$$\tilde{\mathcal{O}}^{\Delta}(x) \sim \int d^d y \, \frac{\mathcal{O}^{\Delta}(y)}{(x-y)^{d-\Delta}} \qquad \Delta = \frac{d}{2} + i\nu$$

massive:

$\Delta = 1 + i\lambda$
$\lambda \geq 0$

$$\phi^{\pm}_{\Delta}(X^{\mu}, \vec{z}) = \int_{H_3 \to \hat{p}^2 = 1} [d\hat{p}] \, \underline{G_{\Delta}(\hat{p}, \vec{z})} \, e^{\pm i m \hat{p} \cdot X}$$

$$G_{\Delta}(\hat{p}, q) = \frac{1}{(-\hat{p} \cdot q)^{\Delta}}$$

massless:

$$\phi^{\pm}_{\Delta}(X^{\mu}, \vec{z}) = \int_0^{\infty} d\omega \, \omega^{\Delta - 1} \, \underline{e^{\pm i \omega \hat{q} \cdot X}}$$

$\Rightarrow \Delta = 1 + i\lambda$
$\lambda \in \mathbb{R}$

$$4pt \Rightarrow \underset{z}{\underline{\qquad}} \cdots f(z,\bar{z})$$

$$\square f = 0$$

$$D f = 0$$

$$f(z,\bar{z}) = \int_{-i\infty}^{i\infty} d\Delta \, \Psi_{\frac{d}{2}+i\nu}(a(\Delta))$$

→ conformal partial wave

$$G_\Delta + G_{d-\Delta}$$

$$= \sum_{\Delta} G_\Delta a_\Delta$$

Celestial amplitudes: $\mathcal{A}$

$$\mathcal{A}(O_i, z_i, \bar{z}_i) = \prod_{i=1}^{n} \int_{H_3} d^3[\hat{p}_i]\, G_{O_i}(\hat{p}_i, z_i, \bar{z}_i) \prod_{i=1}^{m} \int_0^\infty d\omega_i\, \omega_i^{O_i - 1} \underbrace{M(p_i)}_{p_i(z, \bar{z}, \omega_i)}$$

$$M(p_i) = \int dx\, e^{ip\cdot x} M(x) \to \text{QFT}$$

$$G_O(\hat{p}, q) = \frac{1}{(-\hat{p}\cdot q)^O} \qquad \hat{q}(z, \bar{z})$$

$$A_2^{\circ}(O_i, \vec{z}, \vec{w}) = c\, g^{(n)}(z-w)$$

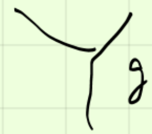
$\langle \text{outlin} \rangle$ shadow $\dfrac{1}{(z-w)^{2\Delta}}$

$$\langle O_1 O_2 O_3 O_4 \rangle = \int d_\Delta \, \langle O_1 O_2 \tilde{O}_k \rangle \langle O_k O_3 O_4 \rangle$$

$$\sum |n\rangle\langle n| = 1$$

$$\sum_k |\tilde{O}_k\rangle\langle O_k| = 1$$

2 massless and 1 massive scalars.



$$M^0(m_3\hat{p}_3, \omega_1\hat{q}_1, \omega_2\hat{q}_2) = g\,\delta^{(4)}(\epsilon_3 m_3 \hat{p}_3 + \epsilon_1\omega_1\hat{q}_1 + \epsilon_2\omega_2\hat{q}_2)$$

$$A(O_i, z_i, \bar{z}_i) = \frac{C(O_1, O_2, O_3)}{|z_1 - z_2|^{O_1+O_2-O_3}\,|z_1 - z_3|^{O_1+O_3-O_2}\,|z_2 - z_3|^{O_2+O_3-O_1}}$$

$$C(O_1, O_2, O_3) = g\,\frac{m_3^{O_1+O_2-4}}{2^{O_1+O_2-1}}\,B\left(\frac{O_{12}+O_3}{2}, \frac{-O_{12}+O_3}{2}\right)$$

massless.

translation

$$\hat{P}_\mu \mathcal{M}(\epsilon\omega\hat{q}) = \epsilon\omega\hat{q}\,\mathcal{M}(\epsilon\omega\hat{q})$$

$$\hat{P}_{k,\mu} A(\Delta_i, z_i, \bar{z}_i) = \prod_{i=1}^{n}\left(\int_0^\infty d\omega_i\, \omega_i^{\Delta_i-1}\right)\epsilon_k\omega_k\hat{q}_k\,\mathcal{M}(q_i)$$

$$= \epsilon_k\hat{q}_k A(\Delta_1, \cdots \Delta_k+1, \cdots \Delta_n)$$

$$\sum_{k=1}^{n}\epsilon_k\hat{q}_k A(\Delta_1, \cdots \Delta_k+1, \cdots \Delta_n) = 0$$

4pt.

Poincaré symmetry implies that any celestial scalar 4-point function can be put onto the form:

$$A_4^0 = \delta(\underline{z-\bar{z}})\, f^{h_i \bar{h}_i}(z,\bar{z}) \prod_{1\le i<j\le 4} z_{ij}^{h/3 - h_i - h_j}\, \bar{z}_{ij}^{\bar{h}/3 - \bar{h}_i - \bar{h}_j}$$

$$z = \frac{z_{13} z_{24}}{z_{12} z_{34}} \qquad \bar{z} = \frac{\bar{z}_{13} \bar{z}_{24}}{\bar{z}_{12} \bar{z}_{34}}$$

$$z = -\frac{t}{s} \qquad s = -(p_1+p_2)^2,\ t = -(p_1+p_3)^2$$

log

$SO(3,1) \sim$ Euclidean CFT

$SO(2,2) \sim SO(1,1) \to$ Light ray operator

$P_1 = \omega_1(z,\bar{z})$

$P_2 = \omega_2(w,\bar{w})$

$P_2 = \omega_2(z,\bar{z})$