Unit - I

1.) State and Prove Cauchy - Riemann equation in Polar form

2.) If $f(z) = u + iv$ is analytic, show that

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4\,|f'(z)|^2.$$

3.) Find the analytic function $f(z) = u + iv$ given $u - v = e^x [\cos y - \sin y]$.

4.) Show that $U = \left(r + \frac{1}{r}\right) \cdot \cos\theta$ is harmonic. Find its harmonic conjugate and also the corresponding analytic function

5.) Find the analytic function $f(z)$ whose imaginary part is $e^x [x \sin y + y \cos y]$.

~~6) State and Prove Cau~~

6.) Define (i.) Limit (ii) Continuity (iii) Differentiability

7.) Show that $U = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function $f(z)$

8.) Show that $f(z) = \sin z$ is analytic and hence find $f(z)$

9.) S.T $u = e^x \cos y + xy$ is harmonic and find its harmonic conjugate.

10.) Evaluate $\int_C \frac{\sin \pi z + \cos \pi z}{(z-1)(z-2)} dz$, where C is the circle $|z| = 4$

11.) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where C is (a) $|z| = 3$ (b.) $|z| = \frac{1}{2}$

12.) State and prove Cauchy's theorem.

13.) S.T $V = \cos x \sinh y$ is harmonic. find the conjugate harmonic function. Also find the corresponding analytic function

14.) Evaluate $\int_C \bar{z}\, dz$ where C represents the following paths (a) the straight line from $-i$ to i (b.) the right half of the unit circle $|z| = 1$ from $-i$ to i.

15.) Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along

(a.) the line $x = 2y$

(b.) the real axis upto 2 and then vertically to $2+i$

16.) Find the bilinear transformation that maps the points $-1, i, 1$ onto the points $1, i, -1$ respectively. Also find the fixed points of the transformation.

17.) Find the bilinear transformation that maps the points $0, -i, -1$ onto the points $i, 1, 0$ respectively. (2)

18.) Discuss the transformation of $w = e^z$

19.) Discuss the transformation of $w = z^2$

20.) If C_1 & C_2 are two simple curves with C_1 enclosing C_2 and $f(z)$ is analytic inside and on the boundary of the ~~angu~~ angular region b/w C_1 & C_2 the P.T $\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz$.

Unit - 2

1.) Laurents series

a.) $f(z) = \dfrac{1}{z(z^2-3z+2)}$ in regions (i.) $0 < |z| < 1$
(ii.) $1 < |z| < 2$

b.) $f(z) = \dfrac{1}{(z+1)(z+3)}$ in regions (i.) $|z| < 1$
(ii.) $1 < |z| < 3$

2.) Expand $f(z) = \dfrac{z-1}{z+1}$ as a Taylor's series about (i.) $z = 0$ (ii.) $z = 1$
(iii.) its Laurents series for the domain $1 < |z| < \infty$.

3.) Expand $\dfrac{(z-2)(z+2)}{(z+1)(z+4)}$ for

(i.) $|z|<1$ (ii.) $1<|z|<4$

4.) Expand $f(z)=\dfrac{z+3}{z(z^2-z-2)}$ in powers of z, where (i.) $|z|<1$ (ii.) $1<|z|<2$

5.) Find ~~#~~ zeros and poles of $\left(\dfrac{z+1}{z^2+1}\right)^2$

6.) what kind of singularity have the following functions (i.) $f(z)=(z-3)\sin\left(\dfrac{1}{z+2}\right)$ at $z=-2$

(ii.) $\tan\dfrac{1}{z}$ at $z=0$

7.) what kind of singularity have the following functions. (i) $\dfrac{\cot \pi z}{(z-a)^2}$ at $z=0, z=\infty$

(ii.) $\sin\left(\dfrac{1}{z-1}\right)$ at $z=1$

~~8.)~~ (iii.) $\sin z - \cos z$ at $z=\infty$

8.) S.T the function $\dfrac{z^2+4}{e^z}$ has an isolated essential singularity at $z=\infty$

9.) Find each pole and its order and calculate residue at each of the pole of $f(z)=\dfrac{z^2}{(z-1)^2(z+2)}$

10.) What kind of singularity has the function:

③

$$\frac{e^z}{z^2+4}$$

11.) Find the expansion of $\frac{1}{(z^2+1)(z^2+2)}$ in powers of z when (i) $|z|<1$ (ii) $1<|z|<\sqrt{2}$

12.) Find the ~~of~~ Residue of

(a) $\frac{1}{(z^2+1)^2}$ at $z=i$ (b.) $\frac{z^2}{z^2+a^2}$ at $z=ia$

13.) Determine the order of each pole and calculate ~~a~~ residue at each of the pole of

$$f(z)=\frac{1-2z}{z(z-1)(z-2)}$$

14.) By Cauchy's residue theorem Evaluate

i) $\int_c \frac{z-3}{z^2+2z+5}\,dz$, where c is the circle

(a) $|z+1-i|=2$ (b) $|z+1+i|=2$

15) (ii) $\int_c \frac{z-3}{z^2+2z+5}\,dz$ where c is the circle

(a) $|z|=1$ (b) $|z+1-i|=2$

16.) ~~Expand $f(z) = z+3z$~~

Discuss the nature of singularities of the following functions (i.) $\tan z$ (ii.) $\frac{z}{1+z^4}$

Unit - 3

1.) Bisection Mtd

a) $x^3 - 2x - 5 = 0$ (b) $x^3 - x - 1 = 0$

2.) Dolittle Mtd

a) $5x - 2y + z = 0$, $7x + y - 5z = 8$, $3x + 7y - 4z = 10$

(b) $3x + 2y + 7z = 4$, $2x + 3y + z = 5$, $3x + 4y + z = 7$

3.) Smallest root : Ramanujun's Mtd

(a) $3x - \cos x - 1 = 0$ (b.) $x^3 - 9x^2 + 26x - 24$

4.) Newton - Raphson Mtd

(a.) $3x = \cos x + 1$ (b.) $\sqrt{12}$ (e.) $\sqrt[3]{24}$

(c.) $x e^x = 2$ (d.) $x \log_{10} x = 1.2$ near 2.5

5.) Cholesky Mtd

$x + 2y + 3z = 5$, $2x + 8y + 22z = 6$, $3x + 22y + 82z = -10$

6.) Gauss - Seidel Mtd.

a) $20x + y - 2z = 17$, $3x + 20y - z = -18$,
$2x - 3y + 20z = 25$

b) 7.) Gauss Elimination Mtd

a) $5x + y + z + w = 4$, $x + 7y + z + w = 12$,
$x + y + 6z + w = -5$, $x + y + z + 4w = -6$

b.) $2x + 2y + z = 12$, $3x + 2y + 2z = 8$,
$5x + 10y - 8z = 10$

c.) $x + 4y - z = -5$, $x + y - 6z = -12$,
$3x - y - z = 4$

8.) Gauss - Jordan Mtd

a) $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$

b.) $4x - y = 1$, $-x + 4y - z = 0$, $-z + 4w = 0$
$-y + 4z - w = 0$,

Unit - 4

1.) Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule

a) $\int_0^6 \frac{dx}{1+x^2}$

(b.) $\int_0^1 \frac{1}{1+x^2} dx$ $n = 5$
hence deduce an approximate value of π

c) $\int_{0.2}^{1.4} [\sin x - \log x + e^x] dx$

2.)

Year	1921	1931	1941	1951	1961	1971
Population	20	24	29	36	46	51

Estimate the increase in population during the period 1955 – 1961.

3.)

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No of students	~~35~~ 35	48	70	40	22

Find who secured marks not more than 45

4.) Lagrange's formula

a.) Find $f(5)$ & $f(6)$.

Given

x	1	2	3	~~4~~ 7
$f(x)$	2	4	8	128

b.) Find $f(x) = 15$

x	5	6	9	11
$f(x)$	12	13	14	16

c.) Find $f(3)$

x	0	1	2	5
$f(x)$	2	3	12	147

d.) Find $f(7)$

Age	0	2	5	8
weights	6	10	12	16

5.) Find $f'(2.2)$ & $f''(2.2)$ given the following table

x	1.4	1.6	1.8	2	2.2
$f(x)$	4.0552	4.9530	6.0496	7.3891	9.0250

6.) Find $\cos 10^\circ$

θ	0	10°	20°	30°	40°
$\sin\theta$	0	0.1736	0.3420	0.5	0.6427

7.) Find y' & y'' at $x=1$

x	1	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0725	1.0954	1.1180	1.1401

8.) Newton's Divided difference formula

a.) Find $f(9)$

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

b.) find $f(-2)$ & $f(3)$

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

c.) $f(18)$ & $f(15)$

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

9.) Forward difference table

$f(x) = x^3 + x^2 - 2x + 1$

$x = 0(1)5$

Find $f(6)$

10.) Interpolating polynomial

a.) Find $f(0.75)$ & $f(-0.5)$

x	0	1	2	3
$f(x)$	1	2	1	10

b.) 11.) Find $t(0.28)$

x	0.1	0.15	0.2	0.25
$y = \tan x$	0.1003	0.1511	0.2027	0.2553

12.)

Unit - 5

1.) Picard's Mtd : (a) $y' = x^2 + y^2$ $y(0) = 0$ find $y(0.1)$

b.) $\frac{dy}{dx} = 1 + xy$, $y(0) = 0$ find $y(0.2)$ upto 3rd approximation

c.) $y' = x + y$, $y(0) = 1$ ~~At~~ find $y(0.1)$, $y(0.2)$

d.) $y' = x - y^2$, $y(0) = 1$. Find $y(0.1)$. Correct to four decimal places.

2.) Runge Kutta Mtd

a) $y' = x + y$, $y(0) = 1$. find $y(0.2)$

b.) $y' = y - x$, $y(0) = 2$ find $y(0.1), y(0.2)$

R-K 2nd order

c.) $y' = 3x + \frac{y}{2}$ $y(0) = 1$. find $y(0.2)$

$h = 0.2$

d.) $y' = xy$ $y(1) = 2$ ~~at~~ find $y(1.2)$

e.) $y' = \frac{y - x}{y + x}$ $y(0) = 1$, $h = 0.2$, $y(0.2) = ?$

3.) Adam's Bashforth Mtd

a.) $y' = x - y^2$ $y(0) = 1$, $y(0.1) = 0.9117$,

$y(0.2) = 0.8499$, $y(0.3) = 0.8061$. find $y(0.4)$

b.) $y' = x - y^2$ at $x = 0.8$

$y(0) = 0$, $y(0.2) = 0.0200$ $y(0.4) = 0.0795$

$y(0.6) = 0.1762$

c.) $y' = x^2(1 + y)$ $y(1) = 1$, $y(1.1) = 1.233$

$y(1.2) = 1.548$, $y(1.3) = 1.979$. find $y(1.4)$

4.) Modified Eulers Mtd

a.) $y' = y - x$ $y(0) = 1.5$ ~~for~~ $y(0.1) = ?$

$h = 0.1$

~~b.) $y' = y - x,$ $y(0) = 1.$~~

b.) $y' = x^2 + y$ $y = 0.94$ when $x = 0$

Find $y(0.1)$

c.) $y' = x + y$ $y(0) = 1$. Find $y(0.05)$

d.) ~~$y' = x^2(1+y)$ $y(1) = 1$ $y(1.1)$~~

Explain modified Eulers Mtd

Taylors Series

1.) $xy' = x - y$ $y(2) = 2$ at $x = 2.1$

2.) $y' = x^2 + y^2$ $y(0) = 1$. Find $y(0.1)$

3.) $y' = x^2y - 1$ $y(0) = 1$ Find $y(0.1)$

& $y(0.2)$