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Homework assignments Week 12 (Students should submit their homework before 10 a.m. on December 15, 2021.)

3. Derivation of the Lipschitz constant for the logistic loss for regression estimation (2%)

Ans: (c)

Programming work

4. The gradient algorithm for logistic regression estimation (6%)

In this programming work we will build a gradient algorithm to find an estimate of regression coefficients in logistic regression model. To run such an algorithm, you need to:

1. Construct an iterative scheme based on the gradient algorithm.
2. Specify a **stepsize** for the iterative scheme. Here you are allowed to use whatever way you like to specify the **stepsize**.
3. Specify (a) a **stopping criterion**, (b) a **tolerance** for the error and (c) the **maximum number of iterations** for stopping the iterative scheme. Here you are allowed to use whatever way you like to specify the **stopping criterion** like the following one:

   Some measure on error $\leq$ tol OR The number of iterations $>$ max_iter.

   However the tolerance for the error and the maximum number of iterations should be

$$
\begin{aligned}
\text{tol} &= 5 \times 10^{-6}, \\
\text{max\_iter} &= 10,000.
\end{aligned}
$$

Data generation: We let the number of observations $n = 200$ and the number of covariates $p = 10$. We use the following model to generate the data:

$$
\begin{aligned}
\boldsymbol{\beta}^{\text{true}} &= (-1, 1, -1, 1, -1, 1, -1, 1, -1, 1), \\
\mathbf{x}_i &= (x_{i1}, x_{i2}, \cdots, x_{i10}), \\
x_{i1} &= 1 \text{ for } i = 1, 2, \cdots, 200, \\
x_{ij} &\sim \text{Normal}(0, 1) \text{ for } i = 1, 2, \cdots, 200 \text{ and } j = 2, 3, \cdots, 10, \\
y_i &\sim \text{Bernoulli}\left(\frac{\exp(\mathbf{x}_i^T \boldsymbol{\beta}^{\text{true}})}{1 + \exp(\mathbf{x}_i^T \boldsymbol{\beta}^{\text{true}})}\right) \text{ for } i = 1, 2, \cdots, 200.
\end{aligned}
$$

- 生成初始訓練與預測資料：(X 與 y)
  - X訓練資料：

```
import numpy as np
df = np.random.normal(0,1,(200,9)) #建立一個200*9陣列，內容為常態分佈的亂數值 平均0，標準差1
ones = np.ones((200,1) ,dtype=float) ## 第一直行的資料 (皆為1)
d = np.hstack((ones,df)) ## 合併上面兩個array
d
```

[13] ✓ 0.3s

```
... array([[ 1.        , -1.28579043, -0.92080805, ...,  0.21481575,
         1.422975  , -0.42182831],
       [ 1.        ,  1.4834412 , -1.74003085, ..., -1.69301932,
         0.41148259, -0.98027462],
       [ 1.        ,  1.51703484,  0.39417184, ..., -1.67434396,
        -0.13920761,  0.20002216],
       ...,
       [ 1.        ,  1.5053925 ,  0.48730986, ..., -1.03570155,
         1.12063432,  0.5888981 ],
       [ 1.        ,  0.34625933, -2.13407995, ...,  1.40676536,
        -0.37636965, -0.11023283],
       [ 1.        , -1.36226615,  0.13096245, ..., -1.034229  ,
         0.61139356, -0.46840271]])
```

  - Y 預測值：

```
import math
beta = [-1,1,-1,1,-1,1,-1,1,-1,1]
y = []
for i in range(200):
    y.append(  (math.exp(sum(d[i]*beta)) ) / ( 1+math.exp(sum(d[i]*beta)))  )

y_final = [] ## 建立 y 預測值 (Bernoulli分佈)
for i in y:
    if i>=0.5:
        y_final.append(1)
    else:
        y_final.append(0)

y_final_ = np.array(y_final)
```

[14] ✓ 0.6s

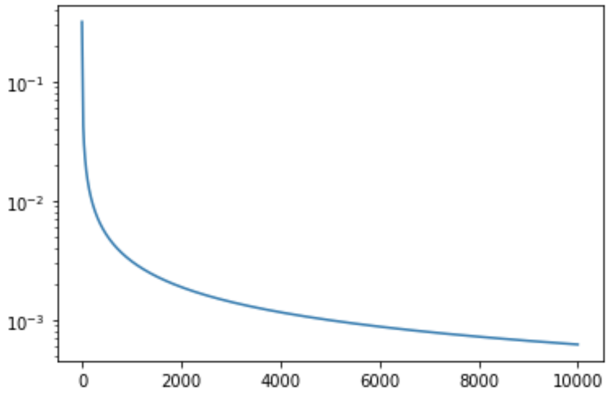
Tasks: Report line plots of the following two settings:

1. The x-axis is the number of iterations r and the y-axis is $||\nabla l(\boldsymbol{\beta}^r)||_2$, the Euclidean norm of the gradient of the loss function you use in your iterative scheme;

```
# 1 #log norm
import matplotlib.pyplot as plt
import seaborn as sn
%matplotlib inline
plt.yscale("log")
plt.plot( range(num_iterations), (norms))
# plt.plot( range(num_iterations), np.log(norms))
```

[166] ✓ 1.3s

```
[<matplotlib.lines.Line2D at 0x7fc765d61fd0>]
```



本圖有經過 log base with 10 處理後繪製。

2. The x-axis is the number of iterations r and the y-axis is $l(\boldsymbol{\beta}^r) - l(\boldsymbol{\beta}^*)$, the difference between the loss functions evaluated at the current update $\boldsymbol{\beta}^r$ and the *optimizer* $\boldsymbol{\beta}^*$. The optimizer $\boldsymbol{\beta}^*$ can be obtained from functions or software for carrying out logistic regression estimation available in your programming environment.

```python
#2 #loss
loss_diff = []
for i in range(len(loss)):
    loss_diff.append(loss[i]-loss_lg)

plt.plot( range(num_iterations), loss_diff )
```

[159] ✓ 1.4s