R10946013 劉謦瑄

Homework assignments Week 14 (Students should submit their homework before 21 p.m. on January 1, 2022.)

6. The proximal operator of the l_0-norm (2%):

Let $\mathbb{I}\{\mathcal{A}\}$ be an indicator function such that $\mathbb{I}\{\mathcal{A}\} = 1$ if $\mathcal{A}$ is true, and $\mathbb{I}\{\mathcal{A}\} = 0$ otherwise. The l_0-norm of a p-dimensional vector $\boldsymbol{\beta} = (\beta_1, \beta_2, \cdots, \beta_p)$ is defined by

$$||\boldsymbol{\beta}||_0 = \sum_{j=1}^{p} \mathbb{I}\{\beta_j \neq 0\},$$

i.e. the number of non-zero valued elements in $\boldsymbol{\beta}$. The proximal operator of l_0-norm on vector $\mathbf{x} = (x_1, x_2, \cdots, x_p)$ is defined by

$$\mathrm{HT}_\alpha(\mathbf{x}) = \arg\min_{\boldsymbol{\beta}} \left\{ \alpha||\boldsymbol{\beta}||_0 + \frac{1}{2}||\boldsymbol{\beta} - \mathbf{x}||_2^2 \right\}.$$

Here the proximal operator $\mathrm{HT}_\alpha(\mathbf{x})$ is a p-dimensional vector, and its jth element is denoted by $[\mathrm{HT}_\alpha(\mathbf{x})]_j$.

Which of the following statements are *true*?

ANS: (a)

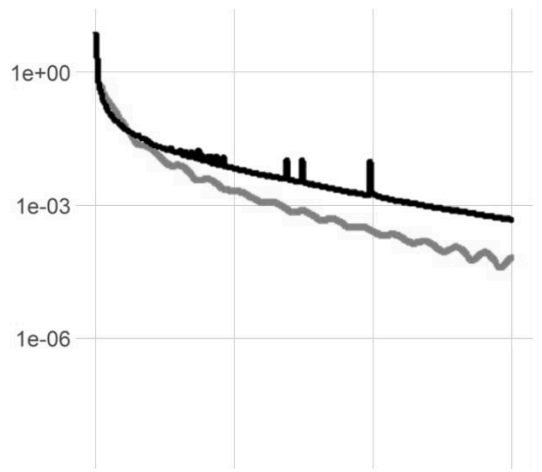
a. The proximal operator of l_0-norm is also called the **hard-thresholding** operator since it can be expressed as

$$[\mathrm{HT}_\alpha(\mathbf{x})]_j = x_j \mathbb{I}\{|x_j| \geq \sqrt{2\alpha}\}.$$

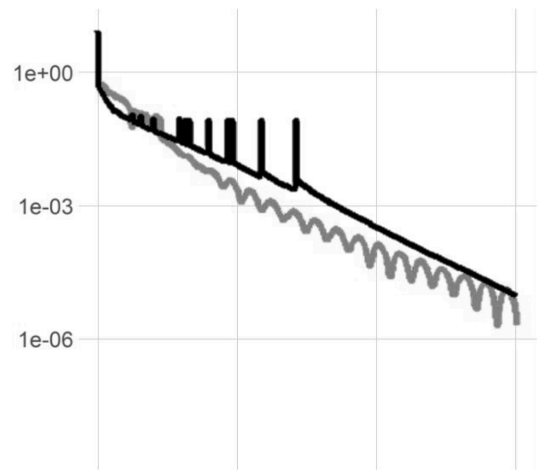
n	n^{test}	$\tilde{\alpha}^*$	MSE	Err^{train}	Err^{test}
200	2000	0.005	0.043	1.465	1.632

Black Line: Generic	X-axis: Number of iterations
Gray Line: Accelerated	Y-axis: Iteration Error

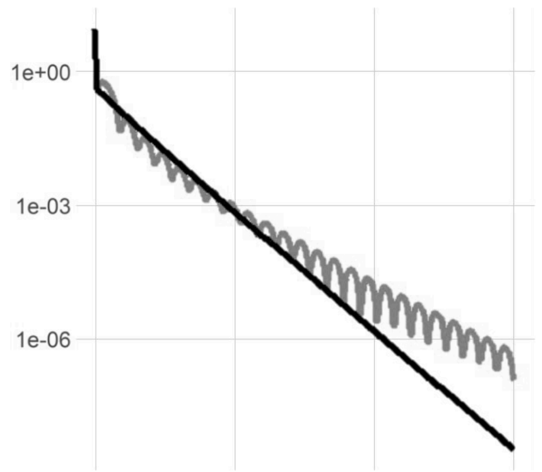
Alpha = 0.013*10^-3



Alpha = 1.362*10^-3



Alpha = 3.997*10^-3



Alpha = 7.294*10^-3