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Homework assignments Week 13 (Students should submit their homework before 21 p.m. on December 25, 2021.)

Programming work

5. Variable selection via lasso estimation (6%)

將 Dataset (y, X) 分為 m 個部分如： $\{(y_k, X_k)\}_{k=1}^{m}$ ，所以：

$$\frac{1}{2}||y - X\beta||_2^2 = \frac{1}{2}\sum_{k=1}^{m}||y_k - X_k\beta||_2^2$$

並且可將 original problem reformulate 為：

$$\underset{\theta_k's,\beta}{min} \quad \frac{1}{2}\sum_{k=1}^{m}||y_k - X_k\theta_k||_2^2 + \lambda||\beta||_1$$

Iterative scheme：

$$\theta_k^{r+1} = arg\underset{\theta_k}{min}\left\{\frac{1}{2}\left||y_k - X_k\theta_k|\right|_2^2 + \frac{\rho}{2}\left||\theta_k - \beta^r + \alpha_k^r|\right|_2^2\right\}$$

$$= (X_k^T X_k + \rho I_{p\times p})^{-1}[(X_k^T y_k + \rho(\beta^r - \alpha_k^r)] \quad for\ k = 1,2,\dots,m$$

$$\beta^{r+1} = arg\underset{\beta}{min}\left\{\lambda\left||\beta|\right|_1 + \frac{m\rho}{2}\left\|\beta - \frac{1}{m}\sum_{k=1}^{m}\theta_k^{r+1} - \alpha_k^r\right\|_2^2\right\}$$

$$= ST_{\lambda/(m\rho)}o[\frac{1}{m}\sum_{k=1}^{m}\theta_k^{r+1} + \alpha_k^r]$$

$$\alpha_k^{r+1} = \alpha_k^r + \theta_k^{r+1} - \beta^{r+1} \quad for\ k = 1,2,\dots,m$$

Soft-thresholding operator：$ST_{\lambda/(m\rho)}$(a) = sign(a)(| a |–λ/(mϱ)) $_+$

Criterion to stop the iterative scheme：

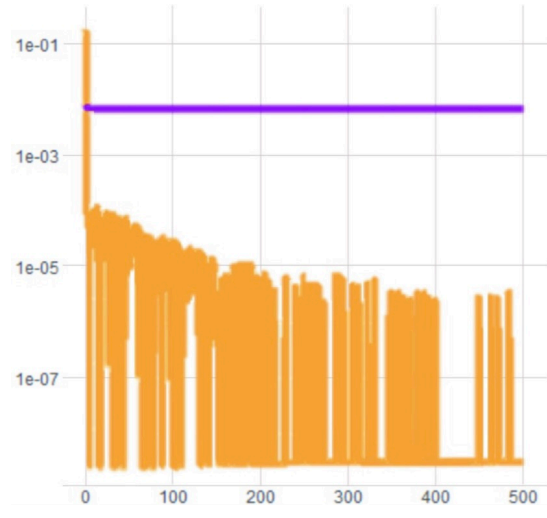
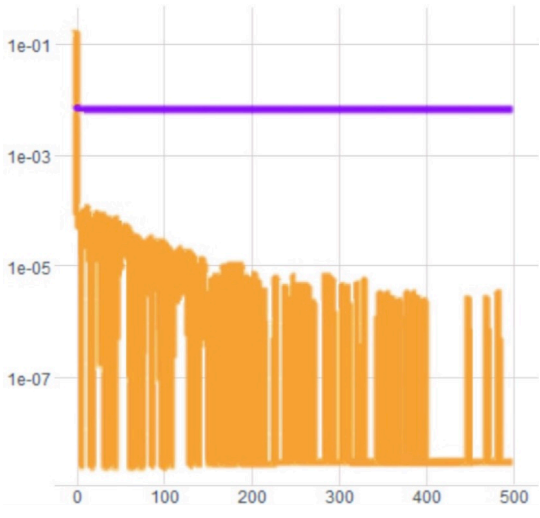
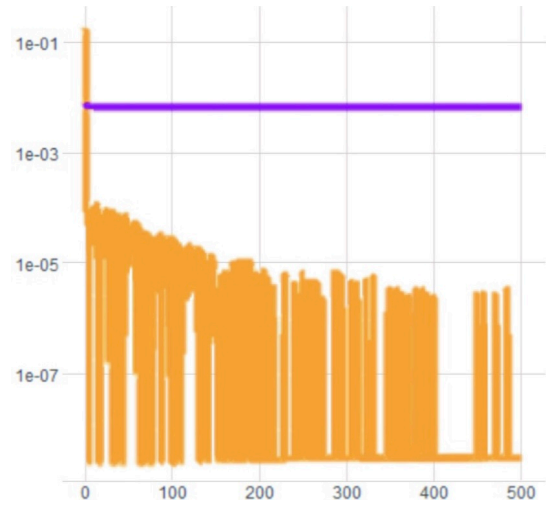
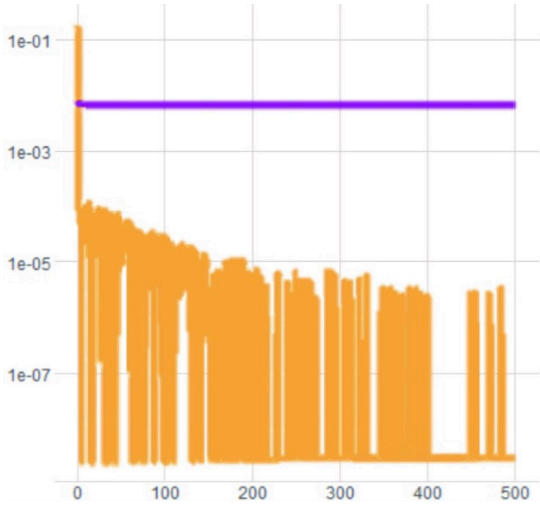
$$\sum_{k=1}^{m}\left(\frac{\left||\theta_k^r - \beta^r|\right|_2}{\sqrt{mp}}\right) + \frac{\rho\left||\beta^r - \beta^{r-1}|\right|_2}{\sqrt{p}} \le 5\times 10^{-3} \quad or \quad r > 500$$

Tasks: Report plots of the following two settings:

1. Fix tuning parameter λ at 4 different values you like and run the iterative scheme under the 4 different values of λ separately. Produce 4 plots according to the 4 different values of λ with the following format: The x-axis is the number of iterations r and the y-axis is the Euclidean norm of the **primal residual** and **dual residual** of the iterative scheme;

Purple Line: Primal Error X-axis: Number of iterations

Yellow Line: Dual Error Y-axis: Iteration Error

Lambda = 0.001	Lambda = 0.5
	
Lambda = 1	Lambda = 3
	

2. Select 20 different values of λ from the interval $[0.001, 5]$ and run the iterative scheme under the 20 different values of λ separately. Collect the values of $\widehat{\boldsymbol{\beta}}^{\text{lasso}}(\lambda)$. Produce a trace plot of $\widehat{\boldsymbol{\beta}}^{\text{lasso}}(\lambda)$ with the following format: The x-axis is the value of λ and the y-axis is $\widehat{\boldsymbol{\beta}}^{\text{lasso}}(\lambda)$. Since we have $p = 500$, there should be 500 such trace lines for $\widehat{\boldsymbol{\beta}}^{\text{lasso}}(\lambda)$. Use red color to draw the trace lines for the 100th, 200th, 300th, 400th and 500th elements of $\widehat{\boldsymbol{\beta}}^{\text{lasso}}(\lambda)$, and gray color to draw the trace line for the rest of elements in $\widehat{\boldsymbol{\beta}}^{\text{lasso}}(\lambda)$.

Dark Gray Line: None-Zero Value X-axis: Lambda / Lambda max

Gray Line: Zero Value Y-axis: Beta