Sound Propagation in 3D Spaces Using Computer Graphics Techniques

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***Abstract*—Sound propagation in 3D spaces is governed by similar physical principles as light. As a result, sound rendering in a 3D virtual environment can benefit from methods developed for graphics rendering and vice versa. In this review, we provide an overview of methods used for sound rendering that share concepts and techniques with graphics rendering. Firstly we describe geometrical propagation techniques where the computations are based on ray theory similar to ray tracing techniques in computer graphics. Secondly, we review numerical techniques. These techniques, similar to the idea of radiosity, are based on the subdivision of the space into elements. Then we describe acceleration techniques that can be used in combination with other methods to speed up calculations. Lastly, for the sake of completeness, a quick overview is given of sound computation techniques that simulate specific sound effects that do not apply on illumination. The aim of this survey is to share knowledge among the two disciplines using familiar and known concepts.**

I. Introduction

Simulation of sound propagation in three dimensional Euclidian space is a field with a notable progress over the last few years and an increased research interest has been shown in many engineering fields, such as in video games development [1] [2], virtual reality [3] [4] [5], acoustics engineering [6] [7] [8] and other disciplines.

The mechanics behind sound propagation bear similar physical rules to the propagation of light. As a result, computation of sound rendering can take advantage of techniques similar to those used for graphics rendering. At the same time, it has been shown that sound propagation is an important component in the sense of immersion in interactive 3D applications [2] [9] [10] [11]. However, the computational cost of sound propagation is high and as a result real-time sound propagation in interactive applications is a challenging topic. This has led to an increased borrowing of techniques developed for real-time graphics rendering, for use in the domain of 3D sound rendering.

In the next section we are going to describe the physical rules that apply to sound and we will give the basic equations that describe the propagation of sound. Then we explain the classification of various techniques described in the sections that follow. For each one of the main categories a dedicated section is given, where related algorithms and techniques are explained. At the end, we give a comprehensive review of the latest research findings on the subject and we provide a starting point for further development in the field.

II. Calculation of Sound Propagation

Sound, as a wave phenomenon is described by the wave equation [12] [13], as follows:

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

The wave equation is a second-order linear partial differential equation which describes the propagation of waves in space.

Taking into account the spherical symmetry and omnidirectionality of the sound, based on D' Alembert solution [14] [12] the equation can take a simpler form as shown below:

$$p(r,t) = \frac{\rho_0}{4\pi r} Q(t - \frac{r}{c})$$

where p is the change in the atmospheric pressure at a point in space, ρ the atmospheric density at rest, Q the sound source strength, t the time after the wave has arrived at the receiver, r the distance from the sound source and c the speed of sound.

In the case of a harmonic excitation it becomes

$$p(r,t) = \frac{j\omega\rho_0 Q}{4\pi r} e^{j(\omega t - kr)}$$

where j the imaginary number, ω the wavenumber and k the angular frequency.

In the case of sound propagation in 3D spaces, the main target of any rendering algorithm is the calculation of the transfer function or impulse response. This calculation is then convolved with an anechoic recording of a sound. Thus, the equation of interest is reduced to the following function, which describes the transfer function of sound in a 3D space.

$$T(f) = e^{jkr}/r$$

where T is the transfer function and f is the frequency. Moreover, on this equation, other factors can be added to describe various sound phenomena, like reflections, diffractions, absorption etc. More information about fundamentals of acoustics can be found in [12] [13] .

III. SOUND VS LIGHT

Sound and light are physical phenomena that share many common properties. They are both waves that propagate throughout space as a result similar techniques can be used to trace and reproduce these propagations. They also have important differences that make different handling of each case a necessity [15]. The most significant difference is that light is an electromagnetic radiation while sound is the fluctuation of pressure. Therefore, sound needs a medium to travel while light does not. An important consequence of this difference is that the medium affects drastically important propagation properties of sound like speed and attenuation and this needs to be considered in each case. Another important difference is the wavelength. Sound wavelengths are orders of scale lower that those of light. Audible sound ranges from 20 to 20000 Hz while visible light ranges from 430 to 790 THz. This makes sound more sensitive to its wave nature and related phenomena like cancellation and diffraction. Another important difference is the speed of propagation. Sound propagates in air with a speed around 343 meters per second, depending on atmospheric conditions, while light travels in the vacuum about one hundred million times faster than that, at the speed of 299 794 458 meters per second. As a result, in contrast with light and the eye, the ear does not receive all relevant information at the same time making the temporal aspect of sound propagation important. These differences are important in sound propagation algorithms, as such algorithms should take into consideration diffraction phenomena and late reflections, in order to generate realistic results. As a result, proper modifications on techniques borrowed from graphics usually take place.

IV. CLASSIFICATION OF TECHNIQUES

In recent years, there has been a renewed interest in sound rendering especially for interactive applications, thus the literature is rich with sound propagation algorithms. Classification of existing methods has been previously done in [4] [13] [11] and [15], however none of these surveys has attempted a direct mapping of acoustics to graphics techniques.

In this state of the art report we classify sound propagation techniques in the following categories:

- Geometrical Propagation Techniques
- Numerical Techniques
- Hybrid Techniques
- Acceleration Techniques
- Non-graphics Techniques

The first two categories share concepts and ideas with the two main classical categories of global illumination in graphics, those of Ray Tracing and Radiosity respectively. In Geometrical Propagation Techniques computations are based in ray theory while Numerical Techniques solve the wave equation assuming that the environment is subdivided into elements.

In a similar way to illumination algorithms in graphics, sound propagation algorithms use various techniques to speed up computations. Acceleration techniques are described in the third category of this paper. One can see that techniques described in this section used to reduce computational time for sound, are also used in computer graphics.

In the last category we briefly mention sound propagation techniques not related to graphics due to differences between sound and light.

V. GEOMETRICAL PROPAGATION TECHNIQUES

In geometrical acoustics, sound is described as a ray phenomenon. Thus sound rays propagated within an environment are used for the estimation of a sound field at a given receiver position. Tracing methods are the methods that detect the propagation of such sound paths in a 3D space.

A. Deterministic Tracing

Deterministic methods are called the algorithms that will produce the same results when run multiple times. For example, a deterministic algorithm for detecting sound reflections in a given model will detect the exact same reflection paths up to a given order of termination each time executed. In sound propagation techniques, two are the most prominent and well known categories of such algorithms, the image source method and the beam tracing method.

1) Image Sources: Image source method for the computation of an impulse response within an enclosed space was first proposed by Alen and Berkley [16] for rectangular rooms and extended by Borish for arbitrary polyhedral[17]. Image source methods compute virtual sources by considering each polygonal surface in the environment as a reflector and mirroring in it, the location of the original source. Virtual sources can be used for the determination of reflection points, by finding the intersection of a line segment from the image source to the receiver. Then, the reflection points can be used for the construction of reflected sound paths. Virtual sources can be recursively mirrored resulting to new virtual sources of higher order, therefore representing higher order reflections. Image source method is a method that provides accurate results, as it detects all the possible sound reflections in a 3D environment, but suffers from poor performance. A simple image source algorithm has a growth of exponential complexity [1]. Mechel has proposed an improved image source method by placing criteria on the generation of the image sources [18]. Using his method, Mechel reported up to 8 times less effective image sources resulting to significant pruning of the image source tree. Savioja et al. introduced a hybrid time-domain model for simulating room acoustics where direct sound and early reflections are obtained using the image source method and late reflections are modeled as exponentially decaying random noise functions [19]. Schröder uses binary space partitioning to accelerate the image source method [20]. Even though image source is an expensive method, it is broadly used as standalone, as a part of other methods like beam and frustum tracing and also in hybrid implementations [14]

2) Beam Tracing: Beam tracing is a method of tracing the polyhedral beams within a 3D environment and then casting them to rays for the computation of the impulse response. Beam tracing is a method that has been borrowed from graphics [21] [22]. In this method, beams are casted throughout the 3D space. Each beam is intersected with each polygon in the environment in a front to back order. After the intersecting polygons are detected, the beam is clipped, removing the shadow region. Then, a transmission beam is constructed by matching the shadow region and a reflection beam is constructed by using the image source method described earlier and mirroring the transmission beam over the polygon's plane. It is also possible to form other types of beams to model diffraction and scattering [11]. Early implementations of beam tracing algorithms were based on the tracing of cones emitted from the source [23]. This method led to multiple detection of the same paths and resulted to sampling errors and artefacts. Therefore, new beam tracing algorithms were proposed using pyramids or other polyhedra to trace the propagation of sound. Beam tracing is currently considered as the fastest commonly used geometric room acoustics modeling technique [22]. In comparison with other methods, like ray tracing, beam tracing benefits from the fact that it is a deterministic method. Another benefit is that it can easily incorporate diffractions, as beams, in contrast with rays, can easily intersect with edges too. As a result it does not suffer from sampling problems [24]. Recent developments in this area include the development of priority based beam tracing [25], bidirectional beam tracing, amortized beam tracing [26], beam tracing using precomputed visibility diagrams [27], beam tracing using binary space partitioning [22], multi-threaded beam tracing [28], as a part of hybrid models [29] and with the inclusion of refraction effect [30].

B. Stochastic Tracing

Stochastic methods for sound propagation in 3D spaces are a class of Monte Carlo methods which use random sampling to achieve approximate representation of the sound field at the listener's location. In contrast with deterministic methods, stochastic methods provide approximate results which may vary between executions and suffer from sampling problems, but enjoy faster execution times. These methods are based on tracing the propagation of 3D objects in an environment and their interaction with other 3D entities like triangles, faces and edges. The propagated objects most commonly used are rays, particles and frusta.

1) Ray Tracing: In acoustics, ray tracing is used as a technique for generating an impulse response by tracing the path of sound throughout a three dimensional environment and calculating the various effects that occur in when a ray encounters an obstacle. Acoustical ray tracing is based on the principles of geometrical acoustics. Here it is important to highlight that acoustical ray tracing does not map exactly to ray tracing used in graphics but it more similar to path finding techniques used for graphics rendering. Ray tracing techniques are split into two major categories [24]. The first category includes the methods where the rays are carriers of energy information. The second category includes algorithms solely used to trace valid sound propagation paths, which then are translated to impulse responses using analytical equations. Ray tracing is the most widely used acceleration technique in acoustics and it is used by a number of commercial applications [7]. Thus, it has been used extensively in the field of interactive sound rendering. The main strength of ray tracing techniques is their simplicity [11]. Ray tracing is based on the detection of rays throughout the environment by calculating ray surface intersections, a relatively easy task to implement. Rays are emitted from a point in a direction obtained, either by an equal distribution of points on a sphere with a center the source point [31] or by a statistical random distribution [24], and traced throughout the virtual environment until they reach the receiver. Ray tracing has been used in acoustics since 1958 [32]. Krodstad has proposed the first pioneering work using ray tracing to calculate impulse responses in rooms [31]. Kulowski presented an improved algorithm for ray tracing which handles arbitrary room shapes [33]. Vorlander used a combination of ray tracing and image source model to calculate acoustical impulse responses for rooms [14]. Svensson outlines a brief history of the use of ray tracing techniques for sound propagation [34]. The major disadvantages of ray tracing techniques are that the discrete number of rays traced and the arbitrary shape of rooms might lead to significant paths lost or paths counted multiple times [11] [24]. As a result, nowadays ray tracing in acoustics is often used in combination with other techniques. The most recent developments in ray tracing for sound rendering include the development of hybrid algorithms combining ray tracing with frustum tracing and methods for artificial reverb estimation [35], algorithms for the calculation of sound diffraction [36], ray tracing using multi-view ray casting [37], ray tracing using acceleration structures [38] and ray tracing for higher order diffractions and diffused reflections [39].

2) Particle Tracing: Particle Tracing is a variation of the ray tracing technique [40]. In literature, it is presented as phonon tracing and sonel mapping. Phonon tracing is inspired by the photon mapping technique used in computer graphics rendering [41]. In this technique, instead of rays, particles, which are called "phonons", are traced throughout the scene. Kapralos implements a similar technique even though he names it "sonel mapping" [42] [43]. Phonon tracing or sonel mapping are techniques used to trace the energy propagation from the sound sources through the environment while recording the interaction with any surfaces this energy may encounter in the phonon map. The recorder information can be reused to estimate the energy density at any point within the map without the need of a new tracing recomputation [42].

3) Frustum Tracing: Frustum tracing is an approach that uses a simple volumetric representation based on a four-sided convex frustum, for which efficient algorithms are described that perform hierarchy traversal, intersection and specular reflection and transmission interactions at the geometric primitives [44]. Frustum tracing is an approach similar to beam tracing, with the difference that it performs an approximate clipping by subdividing to sub-frusta, opposed to beam tracing which performs accurate clipping of beams. As a result, frustum tracing combines the efficiency of interactive ray tracing with the accuracy of tracing a volumetric representation. Lauterbach et al. [44] presented the first frustum tracing algorithms applied in sound propagation, discussing the advantages and disadvantages of this approach. The main advantage is that the algorithm is much faster than a beam tracing approach. On the other hand, the algorithm is prone to sampling errors and aliasing but in a much lower degree than traditional ray tracing. Chandak et al. [45] proposed an improved version of frustum tracing called adaptive frustum tracing which adaptively refines the quadtree in order to perform accurate intersection computations with the primitives in the scene and generate new frusta. Taylor et al. [46] use frustum tracing to calculate sound diffraction in complex environments.

VI. Numerical Techniques

One way to solve the sound propagation problem in 3D spaces is by using numerical techniques. The most prominent numerical techniques for solving the wave equation are the Finite Element Method (FEM) [47], the Boundary Element Method (BEM) [48] and the Finite Difference Time Domain (FDTD) [13] and radiant approaches [49]. Numerical techniques for sound propagation use the same idea with radiosity method, that of subdividing the environment into elements. Numerical techniques can be used to compute energy decay characteristics in a given environment as well as for the reconstruction of an impulse response for auralization [11].

In general, numerical techniques are perceived as too slow for real time sound rendering. On the other hand, they are known to yield more accurate results than other techniques. As a result, there is considerable research taking place in accelerating the aforementioned methods. [50] provides an overview of finite element methods for time-harmonic acoustics. Raghuvanshi proposes a faster method for FTDT based on Adaptive Rectangular Decomposition [51]. [52] presents a real time 3D FDTD of low and mid frequencies using GPU acceleration. Mehra et al. [53] propose an efficient GPU-based time domain solver for the acoustic wave equation. Mehra et al. also propose the Equivalent Source Method for real-time calculations in outdoor spaces [54] [55] [56].

In addition to the above, one category of numerical techniques that can be directly linked with graphics rendering, is the radiant techniques category which equivalent to radiosity in graphics. Radiosity in graphics is a type of a finite element method which solves the rendering equation for diffusely reflecting surfaces. In a similar way, acoustic radiant methods solve the acoustic rendering equation for sound reflecting on surfaces. Siltanen et al. propose the acoustic rendering equation [49]. Lewers uses acoustic radiant exchange combined with beam tracing for room acoustics modeling [57]. Nosal et al. investigate the use of acoustical radiosity for sound field prediction and use it in arbitrary polyhedral rooms [58] [59]. Hodgson et al. use acoustical radiosity for sound field prediction in cubic rooms [60]. Tsingos also applies radiant exchanges for acoustic simulation [61]. Antani et al. precompute compact acoustic transfer operators using acoustic radiance transfer [62]

VII. Hybrid Techniques

Hybrid techniques are techniques that synthesize a variety of approaches for the generation of the impulse responses required for proper auralization of the 3D space. Hybrid techniques attempt to find a balance between numerical and geometrical techniques to achieve the most accurate real time result possible. These techniques can be divided into main categories, techniques that are based on frequency decomposition and techniques that are based on spatial decomposition. Frequency decomposition is the method where the frequency spectrum is divided to low and high frequencies. Low frequencies are modeled using numerical techniques and high frequencies are modeled using geometrical acoustics. On the other hand, spatial decomposition splits the space in two discrete areas, an area close to the source and an area far from the source. Respectively, the area close to the source is modeled using numerical techniques and the area far from the source using geometrical acoustics.

Frequency decomposition hybrid methods are limited to small scale areas, as they need to perform numerical computations over the entire space domain. Murphy et al. propose the RenderAIR system which is a hybrid implementation that combines 3D Digital Waveguide Mesh(DMW) for the early part impulse response calculation, 2D DMW for the late reverberation tail and ray tracing for high frequency calculations [63]. Southenrn et al. demonstrate a hybrid method which combines FDTD, beam tracing and acoustic radiance transfer methods [64]. Aretz combines FEM, image source method and stochastic ray tracing for the determination of impulse responses determination [65]

Spatial decomposition methods decompose the simulation domain to different regions, near-object regions are handled by numerical acoustic techniques to simulate wave effects, while far-field regions are handled by geometric acoustic techniques. Barbone et al. developed a framework for the calculation of scattering coefficients using FEM and ray tracing [66]. Hampel et al. combine BEM and ray tracing using a spatial decomposition approach [67]. Yeh et al. propose a e two-way pressure coupling technique at the interface of near object and far-field regions [68]

VIII. Acceleration Techniques

The most intensive function of 3D sound propagation algorithms is the tracing of sound propagation paths. The computation times required for tracing paths is directly related with the complexity of the 3D model and the hardware capabilities of the computer. As a result, various supplementary techniques have been developed that can be used in parallel with tracing techniques. These techniques deal with the reduction of the environment complexity, like visibility computations and preprocessing, as well as the exploitation of the hardware capabilities, like GPU acceleration.

A. Visibility Computations

Visibility computations have been widely used in graphics for the reduction of the 3D primitives to be rendered. Nirenstein proposes algorithms for visibility culling [69]. Cohen et al. [70] provide a review of the fundamental issues in visibility and conduct an overview of the latest visibility culling techniques developed. In the case of sound propagation, the performance of all proposed propagation methods, like image source, ray tracing and volume tracing algorithms is linked with the number of primitives under consideration. Hence, visibility computations are important for the reduction of the considered primitives. Chandak, Antani et al. [71] [72] [73] highlight the connection between these propagation techniques and the research on visibility computation in computer graphics and computational geometry, and also give a brief overview of visibility algorithms and apply some of these methods to accelerate geometrical acoustics.

B. Precomputations

Preprocessing information before the actual real-time visual rendering begins, has been a popular technique in computer graphics, as it allows the reduction of required operations during run-time. As a result, similar techniques have been adopted in the domain of audio rendering too. Precomputation is used for the calculation of perceptual characteristics of the environment, for the reduction of the environment's complexity and the calculation of transfer factors. Tsingos [74] presents a method for the precomputation and perceptual assessment of spectral features of the input signals and also for precomputing geometry based reverberation effects [75]. Foale et al. [76] use precomputations for caching offline sound propagation calculations based on the portal subdivision method. Siltanen et al. [77] use precomputation for the reduction of the model's geometrical complexity. Raguvanshi et al. precompute impulse responses for complex scenes and interpolate in real time for moving sources and receivers [51].

Antani et al. [78] precompute the acoustic transfer operators using a technique similar to precomputed light transport. Geometry reduction of complex 3D models to simpler ones containing only the acoustically relevant information is also another preprocessing function that can speed up calculations [79]. Stavrakis et al. precompute transport operators between coupled spaces connected by a portal to compute reverberation decay envelopes at interactive rates [80]. Mehra et al. also precompute transfer operators based on equivalent sources [56]. The main disadvantage of these methods is that they apply mostly for static scenes and in the case of dynamic environments, the preprocessing step needs to be repeated each time the environment changes.

C. Hardware Acceleration

Another approach in improving the performance of sound propagation techniques is by taking advantage of the latest developments in hardware. For example, advancements in GPU technology have allowed the use of GPUs for general purpose computing, also known as GPGPU. Hamidi and Kapralos [81] as well as Tsingos and Jiang [82] provide an extended overview of the use of GPUs for spatial sound in virtual environments and games. More specifically, GPU technology has been used extensively for geometrical acoustics calculations [83] [84]. Tsingos and Gascuel [85] use GPU for sound visibility calculations. Moreover, Tsingos exploit hardware capabilities to efficiently calculate sound scattering [86]. Rober et al. map acoustic equations to graphics rendering equations to take advantage of graphics programming technologies [84]. Cowan and Kapralos [87] use GPU acceleration for fast acoustical occlusion modeling. Raghuvanshi [51] and Saviola [52] also accelerate their FDTD algorithms using graphics cards programming. Besides GPU acceleration, other hardware acceleration techniques are also used, like the use of SSE instructions for Intel processors [88].

IX. Non-graphics Techniques

An important difference between graphics rendering and audio rendering is that in the case of light, propagation delay of light can be ignored, while in the case of sound, propagation delays are perceptible to humans, therefore late sound phenomena like late reflections and late diffractions are significant to the perception of sound. Late sound phenomena are usually reflections or diffractions of higher order that cannot be computed in real time due to the exponential growth of these algorithms [1]. As a result, artificial methods have been developed to estimate late reverberations and accelerate the overall calculation time. There are several methods proposed for the estimation of an artificial reverberant tail. Lehmann and Johansson [89] [90] predict energy decay curves in image-source simulations which approximate well real reverberation tails. Chandak [88] uses Eyring's model to estimate the energy decay (1930).

X. Conclusions

Even though graphics rendering and sound rendering have important differences, they have also many similarities in the way they are computed. Therefore, common techniques can be used in both disciplines. In this survey, we have presented techniques that accelerate the calculation of sound propagation and can be used for efficient and real time sound rendering. Most of these techniques are also used in the domain of graphics rendering, a fact that shows the close relationship between the two topics. Concluding, the main purpose of this paper was to provide a mapping of the most significant research taking place in this field and share knowledge among the two areas which can be used as basis for further development in both disciplines.

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