



# Chain-Of-Thought Prompting And Beyond

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Hi!Paris Reading Group

# What is a language model?



Hi ChatGPT! What's up today?



Hello! I'm just here to help answer your questions and assist you with any information or tasks you need. What can I help you with today?



I want to draw a face. What are the steps?



Drawing a face can be a fun and creative activity. Here are some basic steps to help you get started:

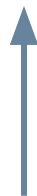
1. **Gather your materials:**
  - You'll need paper, a pencil, an eraser, and optional tools like colored pencils or markers if you want to add color to your drawing.
2. **Start with basic shapes:**
  - Begin by drawing the basic shapes that make up the head, like an oval for the face and a smaller circle for the cranium (top of the head).
3. **Divide the face:**

# A Probability Machine!

$$\mathbb{P}(w_0 w_1 \dots w_n)$$

# A Probability Machine!

$$\mathbb{P}(w_0 w_1 \dots w_n)$$



Probability of a sentence

# A Probability Machine!

$$\mathbb{P}(w_0 w_1 \dots w_n)$$

Hi! PARIS is the Center on Data Analytics and Artificial Intelligence for Science, Business and Society created by Institut Polytechnique de Paris.



The do make of my voiture regina.



Paris is the capital of Germany.

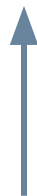


# What is a GENERATIVE language model?

$$\mathbb{P}(w_n \mid w_0 \dots w_{n-1})$$

# What is a GENERATIVE language model?

$$\mathbb{P}(w_n \mid w_0 \dots w_{n-1})$$



Probability of the next word  
in a sentence

# What is a GENERATIVE language model?

$$\mathbb{P}(w_n \mid w_0 \dots w_{n-1})$$

Paris 

The capital of France is

Berlin 



# We can now generate sentences!

$$\mathbb{P}(w_0) \quad \mathbb{P}(w_1 \mid w_0) \quad \mathbb{P}(w_2 \mid w_0 w_1) \quad \mathbb{P}(w_3 \mid w_0 w_1 w_2)$$

The

child

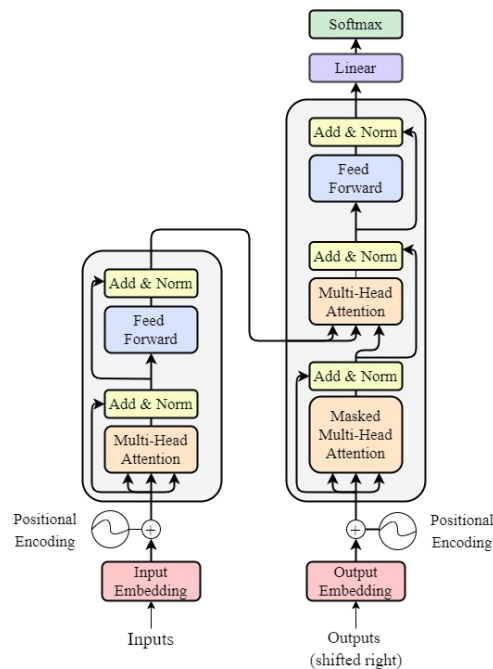
walks

happily

# How do we get this probability?

$$\mathbb{P}(w_n \mid w_1 \dots w_{n-1})$$

# How do we get this probability? Deep Learning!



## The Transformers Architecture

Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., ... & Polosukhin, I. (2017). Attention is all you need.

# How to use my language model?

Sentiment  
Analysis

Ticket  
Booking

Translation

Question  
Answering

Summarization

Story Writing

# First Solution: Finetuning

**Key Idea:** Add additional layer to your **pretrained** language model and train it to specialize it.



Example: Sentiment Analysis

# Second Solution: Zero-Shot With Prompting

**Key Idea:** If you describe a task or ask a question, **the most probable continuation** is the answer to your problem.



What is the capital of France?



The capital of France is Paris. Paris is not only the capital city but also the largest city in France and is known for its rich history, culture, art, and iconic landmarks such as the Eiffel Tower, Louvre Museum, Notre-Dame Cathedral, and many more.

Xian, Y., Lampert, C. H., Schiele, B., & Akata, Z. (2018). Zero-shot learning-a comprehensive evaluation of the good, the bad and the ugly.

# Second Solution: Zero-Shot With Prompting

**Key Idea:** If you describe a task or ask a question, **the most probable continuation** is the answer to your problem.



Q: Change the vowels of the word cameo by a dot.



To change the vowels of the word "cameo" to dots, you can replace the vowels (a and e) with dots. The result would be: "c.m.o."

o not transformed

# Second Solution Bis: Few-Shot With Prompting

**Key Idea:** Write prompts with examples of inputs/outputs.



Continue the sequence following the pattern.

Q: Change the vowels of the word movies by a dot

A: The answer is m.v..s

Q: Change the vowels of the word animal by a dot.

A: The answer is .n.m.l

Q: Change the vowels of the word oiseau by a dot.

A: The answer is ..s...

Q: Change the vowels of the word cameo by a dot



A: The answer is c.m..

Brown, T., Mann, B., Ryder, N., Subbiah, M., Kaplan, J. D., Dhariwal, P., ... & Amodei, D. (2020). Language models are few-shot learners.



# Second Solution Bis: Few-Shot With Prompting

**Key Idea:** Write prompts with examples of inputs/outputs.



Answer following the pattern given below.

Q: Take the last letters of the words in "Elon Musk" and concatenate them.

A: The answer is nk.

Q: Take the last letters of the words in "Larry Page" and concatenate them.

A: The answer is ye.

Q: Take the last letters of the words in "Sergey Brin" and concatenate them.

A: The answer is yn.

Q: Take the last letters of the words in "Bill Gates" and concatenate them.



The answer is ts.

Should be ls

# Chain-of-Thought Prompting Elicits Reasoning in Large Language Models

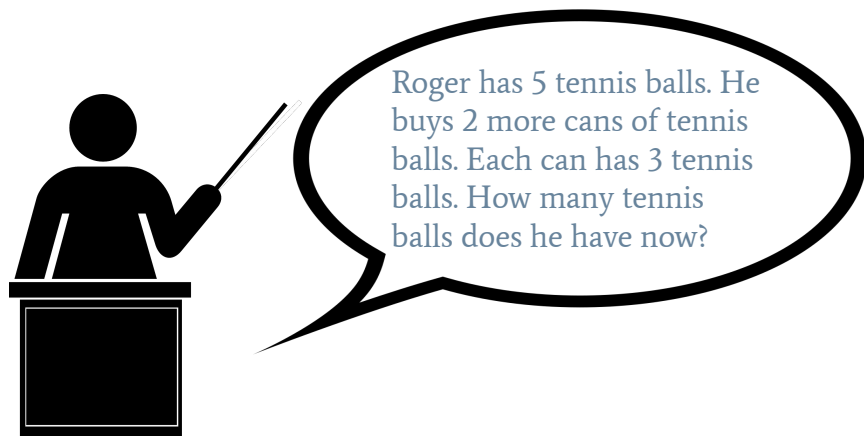
Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed Chi, Quoc Le, Denny Zhou  
Google Research, Brain Team

January 2022

# Key Ideas

It is very hard to answer to all questions immediately

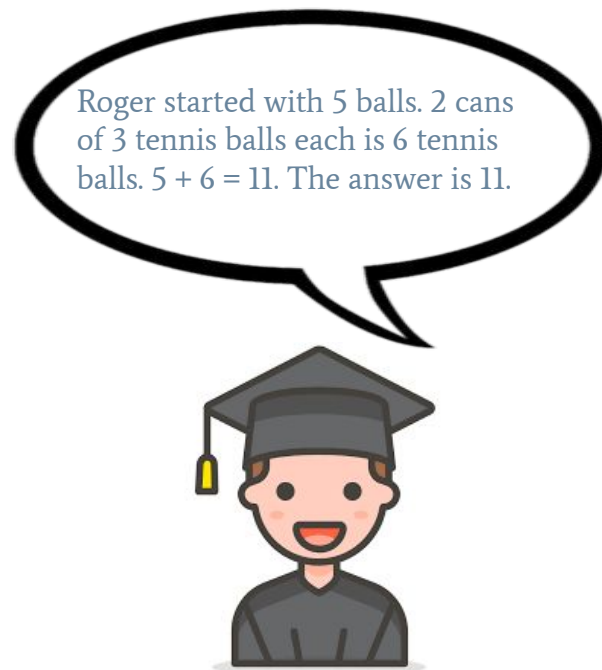
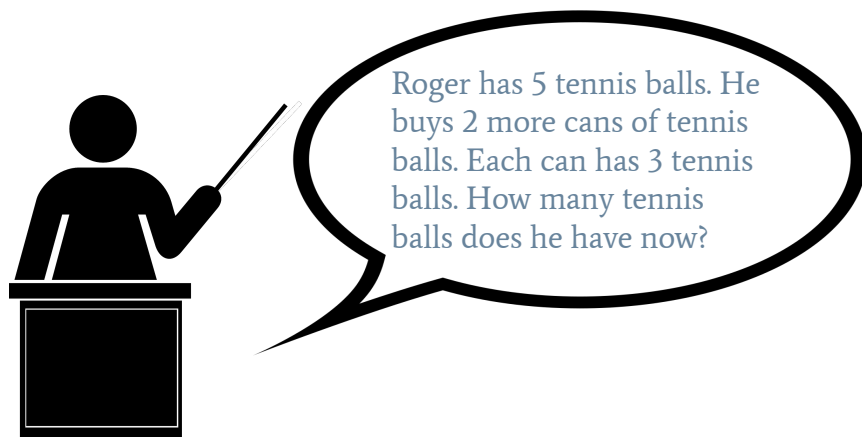
- We need time to think and reason
- Maybe language models too!



# Key Ideas

It is very hard to answer to all questions immediately

- We need time to think and reason
- Maybe language models too!



# How to force a reasoning for a language model?

## First Solution - Zero-shot chain-of-thought

Ask explicitly the network to give explanations



Q: Take the last letters of the words in "Bill Gates" and concatenate them.  
Explain your result.



The last letters of the words in "Bill Gates" are "l" and "s". If we concatenate them, we get the result "ls". This is a simple combination of the final letters of each word in the given name.

Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. Large language models are zero-shot reasoners

# How to force a reasoning for a language model?

## First Solution - Zero-shot chain-of-thought

Ask explicitly the network to give explanations

But...

- The result might be hard to parse.
- Like for few-shot learning, the language model could benefit from reasoning examples.

Takeshi Kojima, Shixiang Shane Gu, Machel Reid, Yutaka Matsuo, and Yusuke Iwasawa. Large language models are zero-shot reasoners

# How to force a reasoning for a language model?



Q: Olivia has \$23. She bought five bagels for \$3 each. How much money does she have left? Explain the result.



Olivia initially has \$23. She buys five bagels, and each bagel costs \$3. To find out how much money she spent on bagels, you multiply the cost per bagel by the number of bagels:

$$5 \text{ bagels} * \$3/\text{bagel} = \$15$$

Olivia spent \$15 on the bagels. To determine how much money she has left, you subtract the amount spent from the initial amount:

$$\$23 - \$15 = \$8$$

Therefore, Olivia has \$8 left after buying five bagels for \$3 each.

Result in the middle of a sentence

# Chain-of-Thought Prompting

- We improve the previous few-shot scenario with explanations
  - A serie of reasoning steps are called a **chain-of-thought**

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: The answer is 11.

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls.  $5 + 6 = 11$ .  
The answer is 11.



# Example



Q: Take the last letters of the words in "Elon Musk" and concatenate them.

A: The last letter of "Elon" is "n". The last letter of "Musk" is "k". Concatenating them is "nk".  
The answer is nk.

Q: Take the last letters of the words in "Larry Page" and concatenate them.

A: The last letter of "Larry" is "y". The last letter of "Page" is "e". Concatenating them is "ye".  
The answer is ye.

Q: Take the last letters of the words in "Sergey Brin" and concatenate them.

A: The last letter of "Sergey" is "y". The last letter of "Brin" is "n". Concatenating them is "yn".  
The answer is yn.

Q: Take the last letters of the words in "Bill Gates" and concatenate them.



The last letter of "Bill" is "l". The last letter of "Gates" is "s". Concatenating them is "ls". The answer is "ls".

# Chain-of-Thought Advantages

- Decomposition of problems
  - Hard problem get more resources



Q: Take the last letters of the words in "Elon Musk" and concatenate them.

A: The last letter of "Elon" is "n". The last letter of "Musk" is "k". Concatenating them is "nk".  
The answer is nk.

Q: Take the last letters of the words in "Larry Page" and concatenate them.

A: The last letter of "Larry" is "y". The last letter of "Page" is "e". Concatenating them is "ye".  
The answer is ye.

Q: Take the last letters of the words in "Sergey Brin" and concatenate them.

A: The last letter of "Sergey" is "y". The last letter of "Brin" is "n". Concatenating them is "yn".  
The answer is yn.

Q: Take the last letters of the words in "Bill Gates" and concatenate them.



The last letter of "Bill" is "l". The last letter of "Gates" is "s". Concatenating them is "ls". The answer is "ls".

# Chain-of-Thought Advantages

- Decomposition of problems
  - Hard problem get more resources



Q: John found that the average of 15 numbers is 40. If 10 is added to each number then the mean of the numbers is?

Answer Choices: (a) 50 (b) 45 (c) 65 (d) 78 (e) 64

A: If 10 is added to each number, then the mean of the numbers also increases by 10. So the new mean would be 50. The answer is (a).

Q: If  $a/b = 3/4$  and  $8a + 5b = 22$ , then find the value of  $a$ .

Answer Choices: (a)  $1/2$  (b)  $3/2$  (c)  $5/2$  (d)  $4/2$  (e)  $7/2$

A: If  $a/b = 3/4$ , then  $b = 4a/3$ . So  $8a + 5(4a/3) = 22$ . This simplifies to  $8a + 20a/3 = 22$ , which means  $44a/3 = 22$ . So  $a$  is equal to  $3/2$ . The answer is (b).

Q: A person is traveling at 20 km/hr and reached his destiny in 2.5 hr then find the distance?

Answer Choices: (a) 53 km (b) 55 km (c) 52 km (d) 60 km (e) 50 km

A: The distance that the person traveled would have been  $20 \text{ km/hr} * 2.5 \text{ hrs} = 50 \text{ km}$ . The answer is (e).

Q: How many keystrokes are needed to type the numbers from 1 to 500?

Answer Choices: (a) 1156 (b) 1392 (c) 1480 (d) 1562 (e) 1788

# Chain-of-Thought Advantages

- Decomposition of problems
  - Hard problem get more resources



To find the number of keystrokes needed to type the numbers from 1 to 500, you can count the number of digits in each number and then add them up.

Here's how you can calculate it:

1. Count the number of digits in each number.
2. Multiply the count of digits by the frequency of each digit (from 1 to 9).
3. Sum up the results for all numbers from 1 to 500.

Here's the calculation:

- For one-digit numbers (1-9):  $9 \times 1 = 9$  keystrokes
- For two-digit numbers (10-99):  $90 \times 2 = 180$  keystrokes
- For three-digit numbers (100-500):  $401 \times 3 = 1203$  keystrokes

Now, add these together:  $9 + 180 + 1203 = 1392$

So, the answer is (b) 1392 keystrokes needed to type the numbers from 1 to 500.

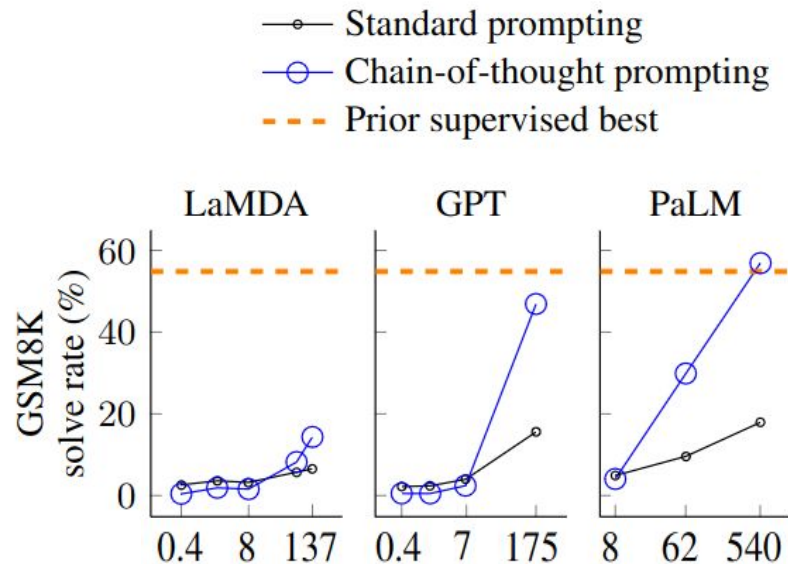
# Chain-of-Thought Advantages

- Decomposition of problems
  - Hard problem get more resources
- Interpretable
  - Is the language model says something right or wrong, we know why
  - Important for debugging
- Usable in many tasks
- Can be applied to all few-shot settings
  - Just add explanations in the prompts
  - Adding explanations is fast and cheap



# Experiments - Arithmetic Reasoning

- Is chain-of-thought prompting good for math problems?
  - We generally need a lot of reasoning

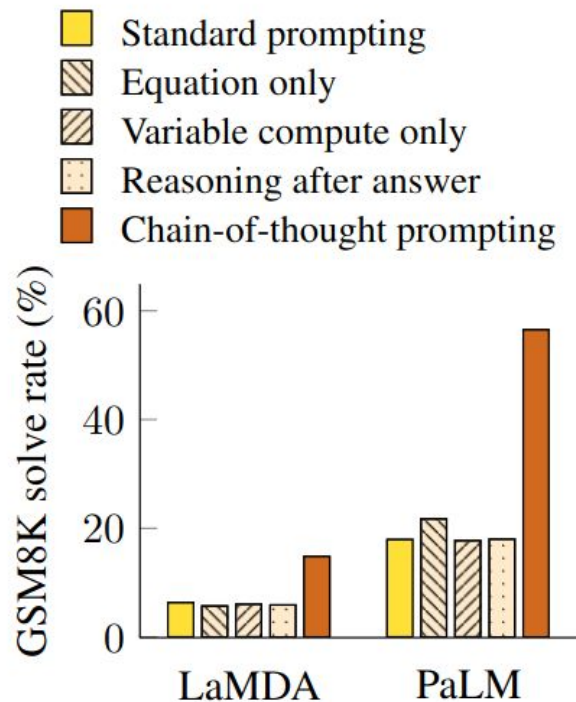


## Key takeaways

- The larger the model, the bigger the gain
- Might reach supervised settings
- Improvement faster than traditional prompting regarding model size

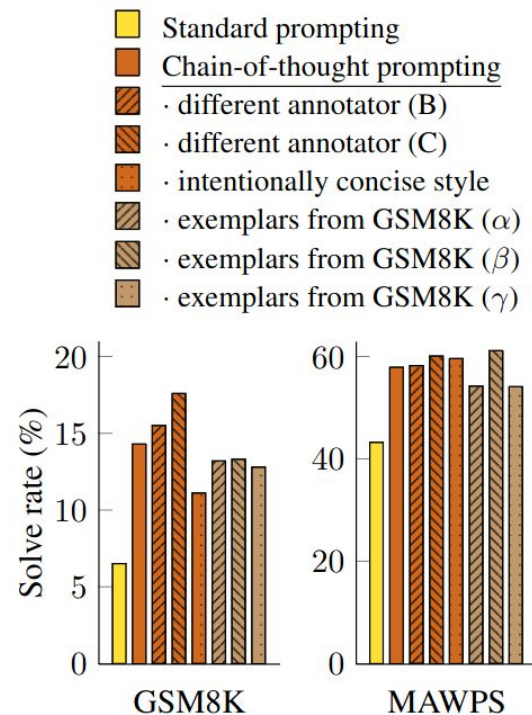
# Experiments - Arithmetic Reasoning - Ablation Study

- What is important to include in the prompts?
- Equation only = no text
- Variable compute only = express the length of the reasoning the dots (.....)
- Chain-of-thought after = Check if generating the explanation before helps



# Experiments - Arithmetic Reasoning - Robustness

- Prompts are known to be very sensitive
  - Small change can have huge impact on results
- Ask several prompt engineer to generate chain-of-thoughts
- Still some variance, but better than nothing





# Experiments - Other Settings

Similar results observed in different settings

- Commonsense reasoning
- Symbolic reasoning

In summary, chain-of-thought prompting helps when:

- The task is challenging and requires reasoning
- Large language models are used
- The scaling curve is relatively flat

# Self-Consistency Improves Chain-of-Thought Reasoning in Language Models

Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc Le, Ed Chi, Sharan Narang, Aakanksha Chowdhery, Denny Zhou  
Google Research, Brain Team

March 2022

# More Details on Sentence Generation

$$\mathbb{P}(w_0) \quad \mathbb{P}(w_1 \mid w_0) \quad \mathbb{P}(w_2 \mid w_0 w_1) \quad \mathbb{P}(w_3 \mid w_0 w_1 w_2)$$

The

child

walks

happily

# More Details on Sentence Generation - Greedy Decoding

Always take the best word.

$\mathbb{P}(w_0)$	The	→	0.0023
	A	→	0.0012
	My	→	0.0006
	I	→	0.0005
	...		

# More Details on Sentence Generation - Greedy Decoding

$\mathbb{P}(w_0)$

The

$\mathbb{P}(w_1 \mid w_0)$

child → 0.0036

man → 0.0025

kid → 0.0011

loop → 0.0007

...

# More Details on Sentence Generation - Greedy Decoding

$$\mathbb{P}(w_0) \quad \mathbb{P}(w_1 \mid w_0)$$

The

child

# More Details on Sentence Generation - Greedy Decoding

$$\mathbb{P}(w_0) \quad \mathbb{P}(w_1 \mid w_0) \quad \mathbb{P}(w_2 \mid w_0 w_1) \quad \mathbb{P}(w_3 \mid w_0 w_1 w_2)$$

The

child

walks

happily

# More Details on Sentence Generation - Greedy Decoding

$$\mathbb{P}(w_0) \quad \mathbb{P}(w_1 \mid w_0) \quad \mathbb{P}(w_2 \mid w_0 w_1) \quad \mathbb{P}(w_3 \mid w_0 w_1 w_2)$$

The

child

walks

happily

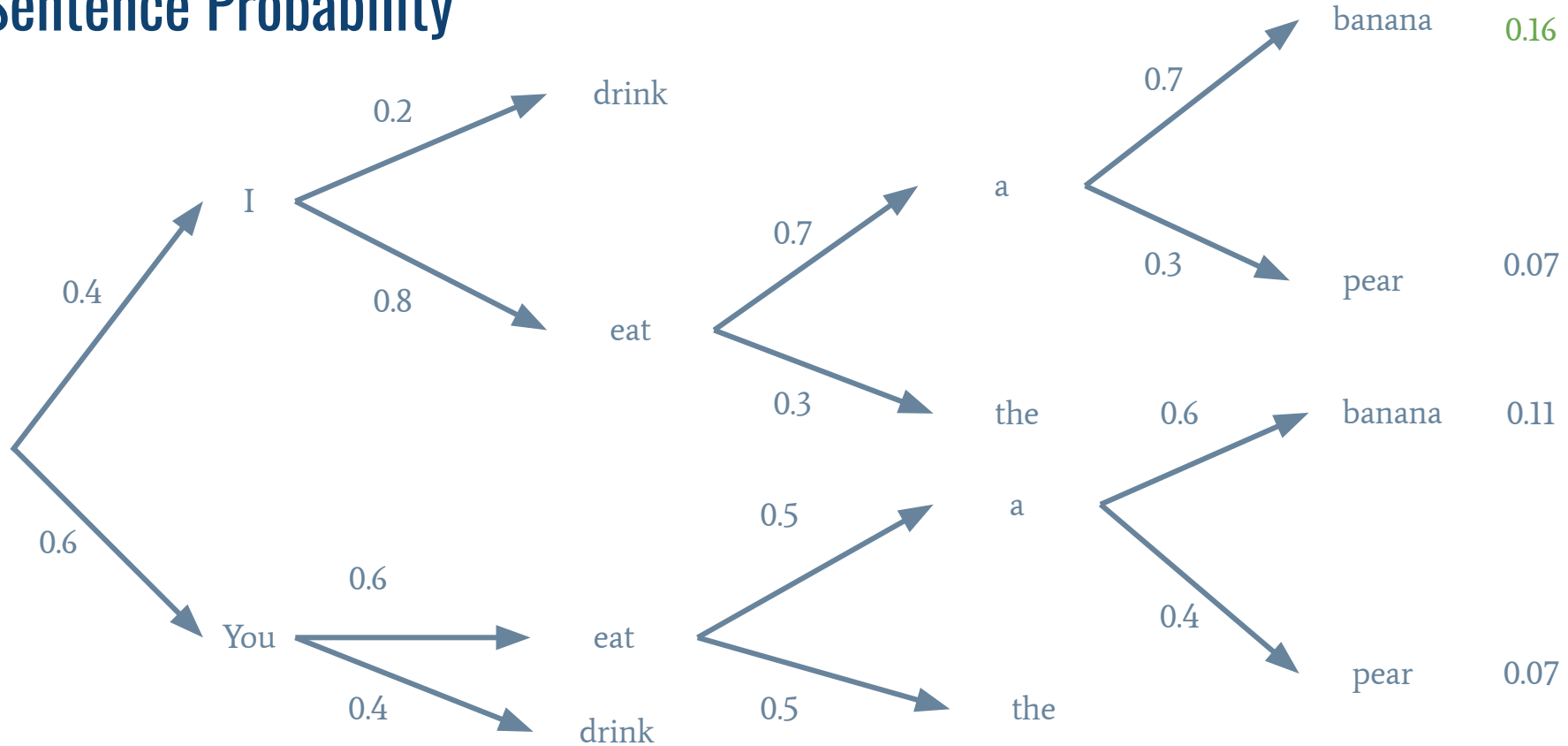
- Not always the best sentence probability

+ Fast to run

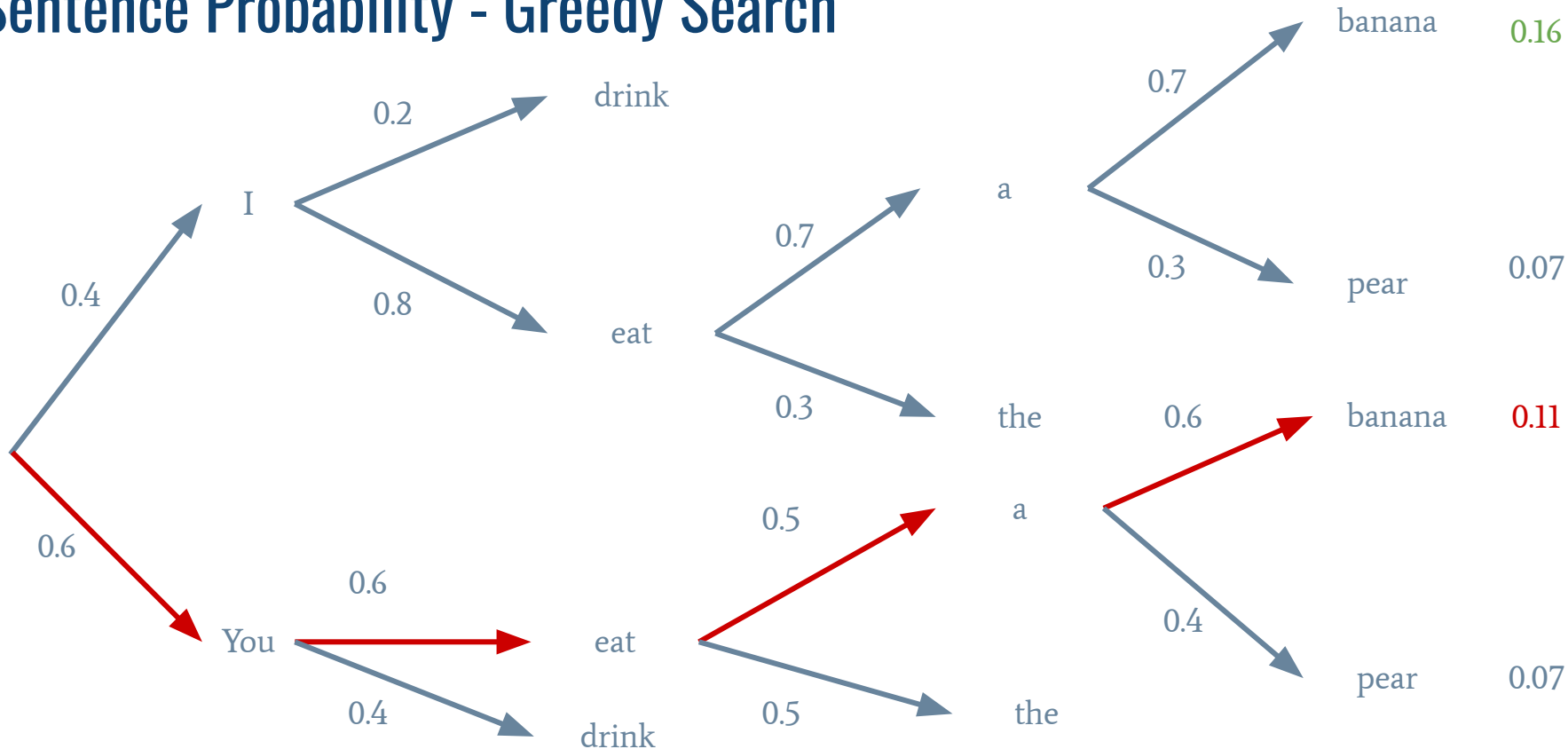
- Generates only one sentence



# Sentence Probability



# Sentence Probability - Greedy Search

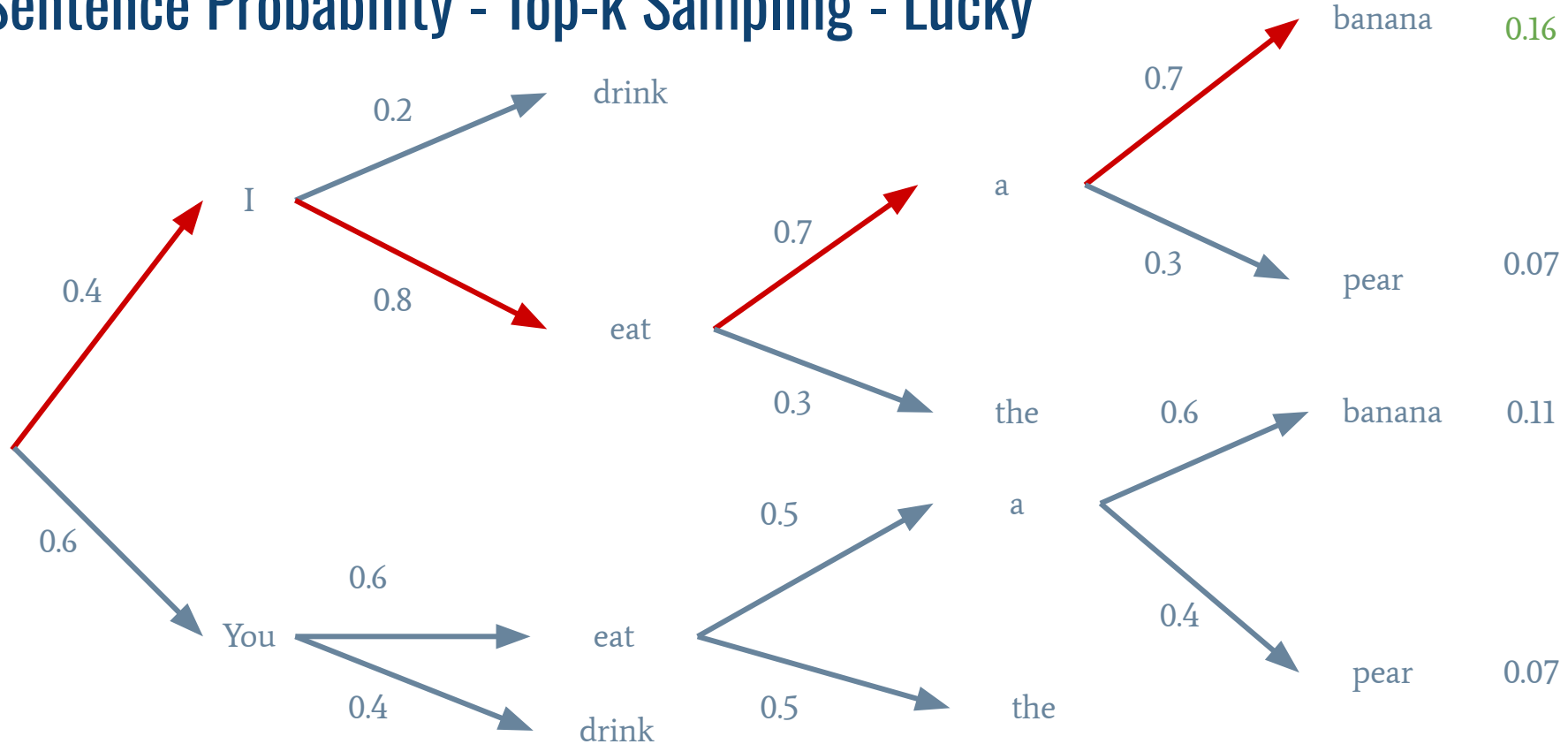


# More Details on Sentence Generation

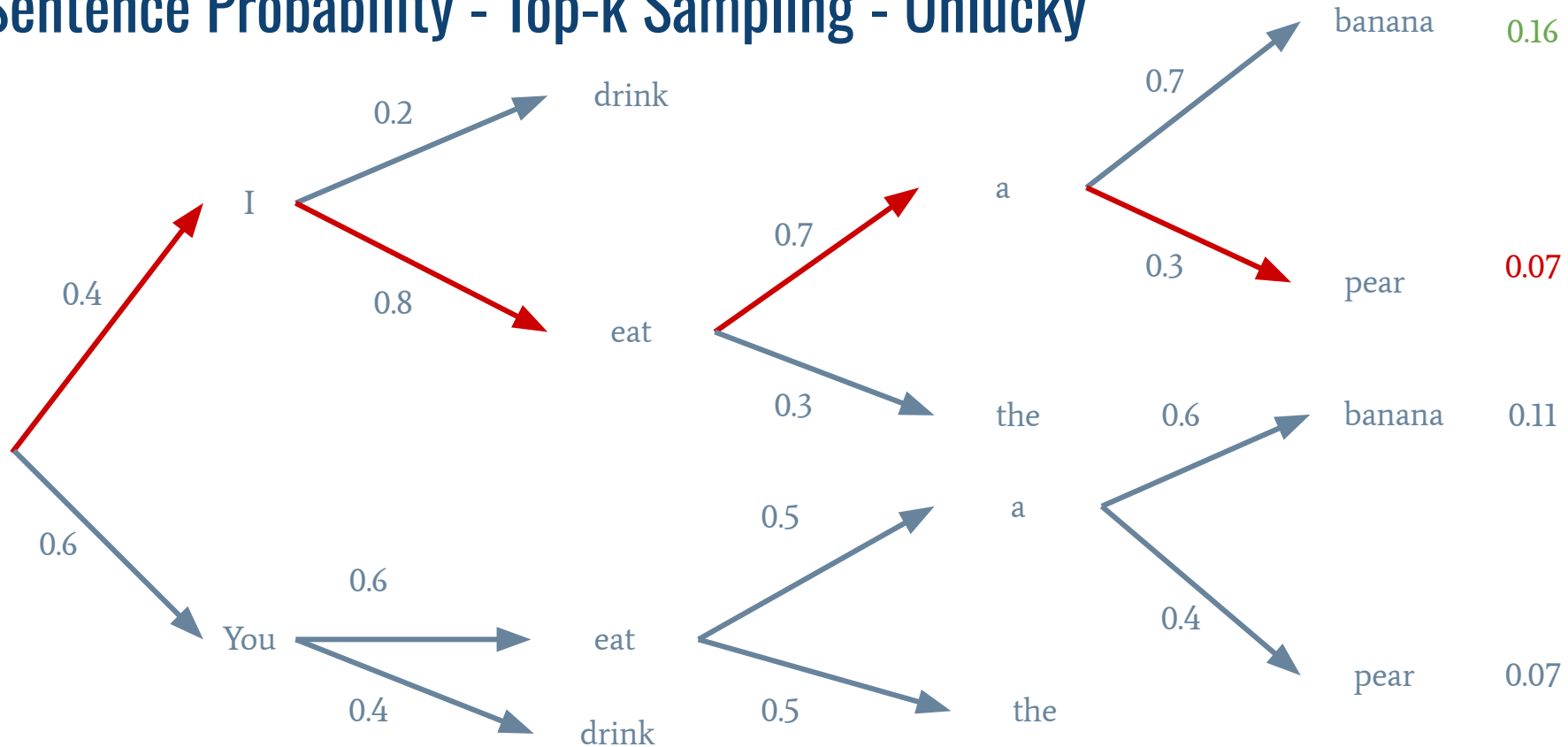
It is impossible to find the best probability

- More than 100k words for ChatGPT
  - A tree of depth 10 contains  $10^{50}$  nodes!
- We need to use heuristic to explore the tree
  - Top-k sampling: Sample words from the top k probabilities
    - + Fast to run
    - + Diversity of the output
    - - Not necessary the best sentence probability
  - Beam search: At each depth of the exploration, only remember the N best generations
    - + Find high probability sentences
    - - Lack diversity
    - - Long to run

# Sentence Probability - Top-k Sampling - Lucky



# Sentence Probability - Top-k Sampling - Unlucky



# What is the link with chain-of-thought?

- There might be more than one correct reasoning path towards the solution

Q: Janet's ducks lay 16 eggs per day. She eats three for breakfast every morning and bakes muffins for her friends every day with four. She sells the remainder for \$2 per egg. How much does she make every day?

She has  $16 - 3 - 4 = 9$  eggs left. So she makes  $\$2 * 9 = \$18$  per day.

She eats 3 for breakfast, so she has  $16 - 3 = 13$  left. Then she bakes muffins, so she has  $13 - 4 = 9$  eggs left. So she has  $9 \text{ eggs} * \$2 = \$18$ .

# What is the link with chain-of-thought?

- There might be more than one correct reasoning path towards the solution
- If the language model generates many reasoning paths that lead to the correct answer:
  - It is **self-consistent**
  - The language model is more sure of the result
- The final result is the most common answer.

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 1 / 10 >



A: Olivia initially had \$23. She bought 5 bagels for \$3 each, so she spent  $5 * \$3 = \$15$ . After buying bagels, she had  $\$23 - \$15 = \$8$  left. She received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . In total, she had  $\$8 + \$10 + \$20 = \$38$ . She gave a quarter of her money to each charity, which is  $\$38 / 4 = \$9$ . Olivia gave \$9 to charities, so she has  $\$38 - \$9 = \$29$  left. Despite wanting another bagel, no more bagels are left, so she doesn't spend any more money. Olivia has \$29 left. The answer is \$29.

29	1



# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

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Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 2 / 10 >



A: Olivia started with \$23. She spent 5 bagels \* \$3 each = \$15 on bagels. After that, she received \$10 from her mother and twice as much from her aunt, which is  $\$10 * 2 = \$20$ . In total, she had  $\$23 - \$15 + \$10 + \$20 = \$38$ . She gave a quarter to two charities, which is  $2 * (\$38 / 4) = \$19$ . Now, Olivia has  $\$38 - \$19 = \$19$  left. Although she is sad because there are no more bagels left, she still has \$19. The answer is \$19.

29	1
19	1

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

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< 3 / 10 >



A: Olivia initially had \$23. She bought 5 bagels for \$3 each, so she spent  $5 * \$3 = \$15$  on bagels. After buying bagels, she received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . So, in total, she had  $\$23 + \$10 + \$20 = \$53$ .

After giving a quarter of her money to two charities, she gave away  $2 * (\$53 / 4) = \$26.50$ . Now, subtracting the amount given to charities from the total, Olivia has  $\$53 - \$26.50 = \$26.50$  left. Olivia is sad because no more bagels are left at the store, but she has \$26.50 remaining. The answer is \$26.50.

29	1
19	1
26.50	1

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 4 / 10 >



A: Olivia initially had \$23. She bought 5 bagels for \$3 each, which is a total of  $5 * \$3 = \$15$ . After buying bagels, she had  $\$23 - \$15 = \$8$  left. Then, she received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . In total, she received  $\$10 + \$20 = \$30$ . Adding this to her remaining money, Olivia had  $\$8 + \$30 = \$38$ . After donating a quarter of her money to two charities, she gave away  $2 * (\$38 / 4) = \$19$ . So, Olivia had  $\$38 - \$19 = \$19$  left. Although she is sad because there are no more bagels left at the store, she still has \$19. The answer is \$19.

29	1
19	2
26.50	1

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 5 / 10 >



A: Olivia initially had \$23. She spent 5 bagels \* \$3 each = \$15 on bagels, leaving her with \$23 - \$15 = \$8. After receiving \$10 from her mother and twice as much from her aunt, Olivia got \$10 + \$10 \* 2 = \$30. In total, she had \$8 + \$30 = \$38. She donated a quarter of her money to each charity, which is  $\$38 / 4 = \$9.50$  each. After donations, she has  $\$38 - \$9.50 - \$9.50 = \$19$  left. Olivia is sad because she doesn't have enough for another bagel. The answer is \$19.

29	1
19	3
26.50	1

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 6 / 10 >



A: Olivia had \$23. She spent  $5 * \$3 = \$15$  on bagels. After buying bagels, she had  $\$23 - \$15 = \$8$  left. She received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . So, in total, she had  $\$8 + \$10 + \$20 = \$38$ . She gave a quarter to each charity, which is  $1/4 * \$38 = \$9.50$ . After donating, she had  $\$38 - \$9.50 - \$9.50 = \$19$  left. Olivia wanted another bagel, but no more were left, so she is sad. The answer is \$19.

29	1
19	4
26.50	1



# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 7 / 10 >



A: Olivia started with \$23. She bought 5 bagels for \$3 each, which is a total of  $5 * \$3 = \$15$ . After buying bagels, she had  $\$23 - \$15 = \$8$  left. Then, she received \$10 from her mother, and twice as much from her aunt, which is  $2 * \$10 = \$20$ . In total, Olivia had  $\$8 + \$10 + \$20 = \$38$ . She gave a quarter of her money to each of the two charities, which is  $2 * (\$38 / 4) = \$19$ . After donating, she had  $\$38 - \$19 = \$19$  left. Olivia is sad because no more bagels are left, but she still has \$19. The answer is \$19.

29	1
19	5
26.50	1

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 8 / 10 >



A: Olivia had \$23. She bought 5 bagels for \$3 each, which cost her  $5 * \$3 = \$15$ . After buying the bagels, she had  $\$23 - \$15 = \$8$  left. Then, she received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . In total, she received  $\$10 + \$20 = \$30$ . Adding this to the \$8 she had left, Olivia had  $\$8 + \$30 = \$38$ . After giving a quarter of her money to each of the two charities, she gave away  $2 * (\$38 / 4) = \$19$ . Olivia's remaining money is  $\$38 - \$19 = \$19$ . Although she is sad because there are no more bagels left, Olivia has \$19 left. The answer is \$19.

29	1
19	6
26.50	1

# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 9 / 10 >



A: Olivia initially had \$23. She spent 5 bagels \* \$3 each = \$15 on bagels. After buying bagels, she had  $\$23 - \$15 = \$8$  left. Then, she received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . In total, she had  $\$8 + \$10 + \$20 = \$38$ . She gave a quarter to each charity, which is  $\$38 / 4 = \$9.5$  each. After donating to charities, she has  $\$38 - \$9.5 - \$9.5 = \$19$  left. Olivia wanted another bagel, but no more are left, so she remains with \$19. The answer is \$19.

29	1
19	7
26.50	1



# Example

Q: Michael had 58 golf balls. On tuesday, he lost 23 golf balls. On wednesday, he lost 2 more. How many golf balls did he have at the end of wednesday?

A: Michael initially had 58 balls. He lost 23 on Tuesday, so after that he has  $58 - 23 = 35$  balls. On Wednesday he lost 2 more so now he has  $35 - 2 = 33$  balls. The answer is 33.

Q: Olivia has \$23. She bought five bagels for \$3 each. Then, she received 10 dollars from her mother and twice as much from her aunt. After that, she gave to two charities a quarter of her money each. No more bagels are left at the store, so Olivia is sad because she wants another bagel. How much money does she have left?

< 10 / 10 >



A: Olivia started with \$23. She bought 5 bagels for \$3 each, which is  $5 * \$3 = \$15$ . After buying bagels, she had  $\$23 - \$15 = \$8$  left. She received \$10 from her mother and twice as much from her aunt, which is  $2 * \$10 = \$20$ . Now she has  $\$8 + \$10 + \$20 = \$38$ . She gave a quarter to each charity, which is  $1/4 * \$38 = \$9.5$ . After donating to charities, she has  $\$38 - \$9.5 - \$9.5 = \$19$  left. Olivia is sad because she doesn't have enough money for another bagel. The answer is \$19.

29	1
19	8
26.50	1

# Example

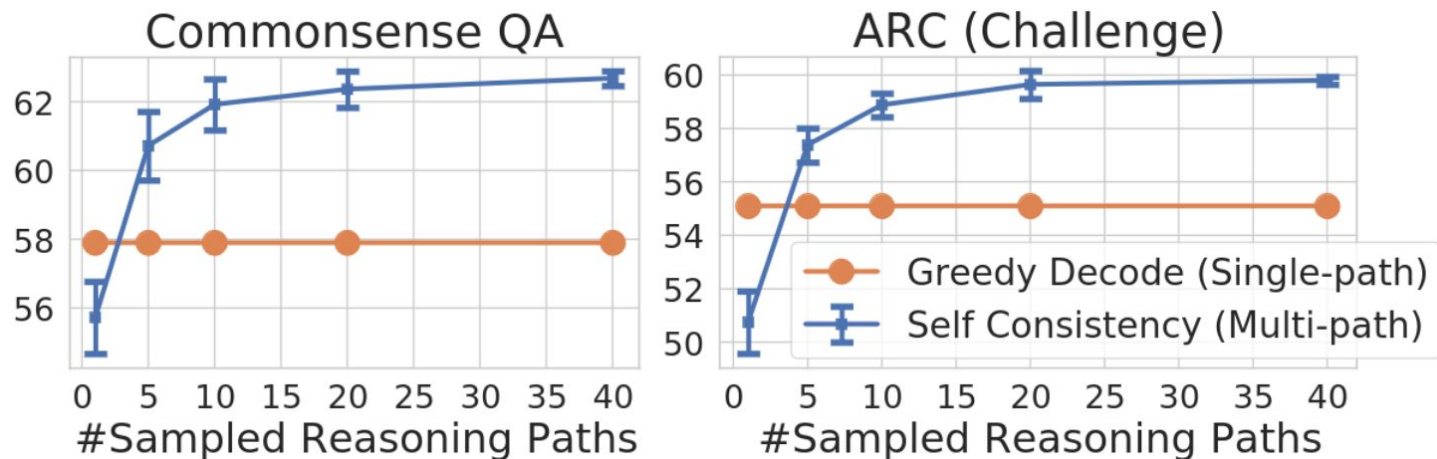
There are mistakes, but the best answer wins!

29	1
<b>19</b>	<b>8</b>
26.50	1

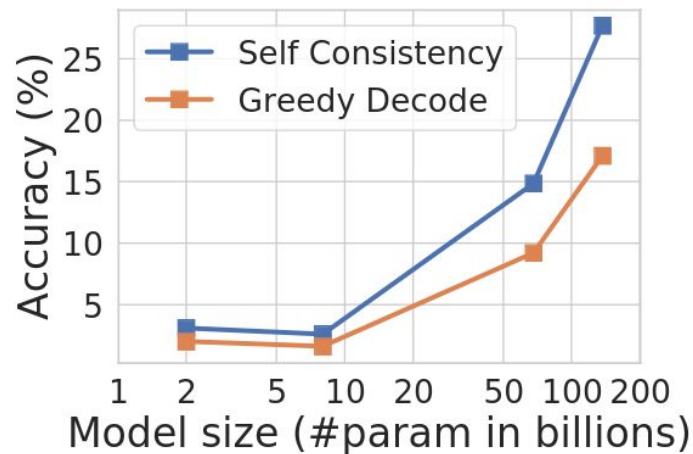
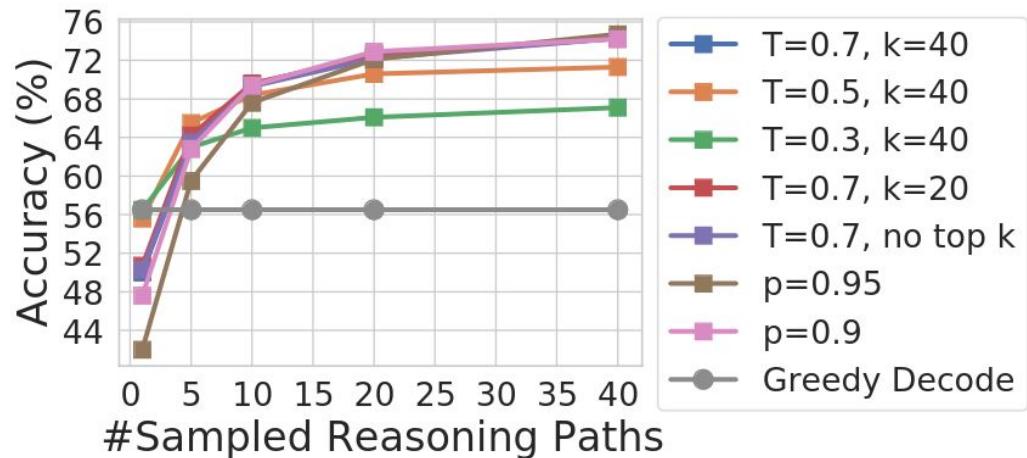
# Experiments

- Similar experiments as chain-of-thought
  - Arithmetic, commonsense, and symbolic reasoning
- Self-consistency significantly improves the results

# Experiments - Impact of the Number of Reasoning Paths



# Experiments - Self-Consistency is Robust to Sampling Strategies and Scaling



# Experiments - Self-Consistency Improves Robustness to Imperfect Prompts

LaMDA-137B	Prompt with correct chain-of-thought	17.1
	Prompt with imperfect chain-of-thought + Self-consistency (40 paths)	14.9 <b>23.4</b>
	Prompt with equations + Self-consistency (40 paths)	5.0 <b>6.5</b>
PaLM-540B	Zero-shot CoT (Kojima et al., 2022) + Self-consistency (40 paths)	43.0 <b>69.2</b>

# Experiments - Self-Consistency Works for Non-Natural-Language Reasoning Paths and Zero-shot Chain-of-Thought

LaMDA-137B	Prompt with correct chain-of-thought	17.1
	Prompt with imperfect chain-of-thought	14.9
	+ Self-consistency (40 paths)	<b>23.4</b>
PaLM-540B	Prompt with equations	5.0
	+ Self-consistency (40 paths)	<b>6.5</b>
	Zero-shot CoT (Kojima et al., 2022)	43.0
	+ Self-consistency (40 paths)	<b>69.2</b>

# Tree-of-Thoughts: Deliberate Problem Solving with Large Language Models

Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L. Griffiths,  
Yuan Cao, Karthik Narasimhan  
Princeton University, Google DeepMind

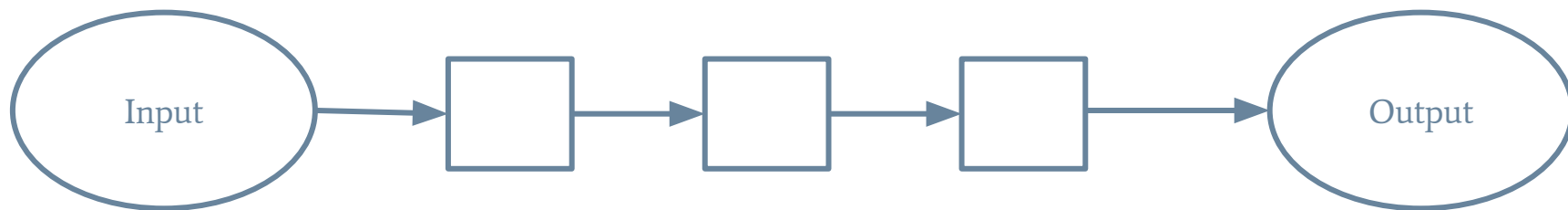
May 2023



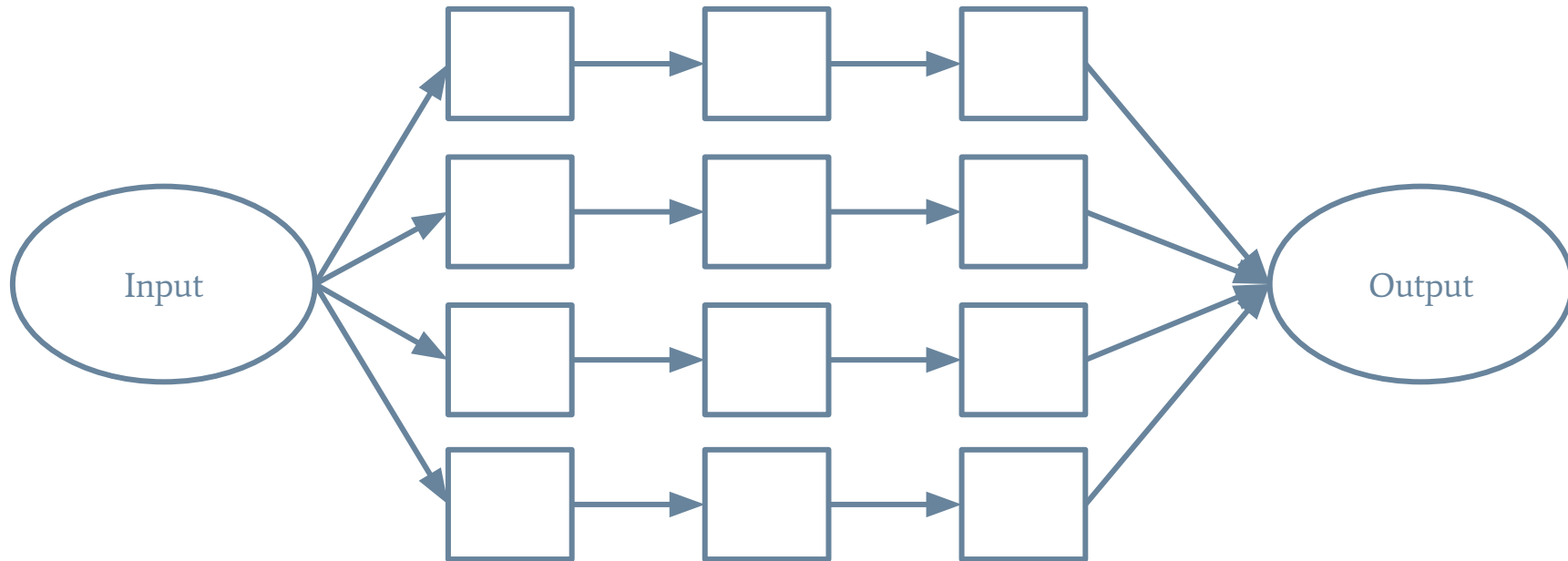
# Summary So Far... Zero/Few-Shot



# Summary So Far... Chain-of-Thought



# Summary So Far... Self-Consistency With Chain-of-Thought



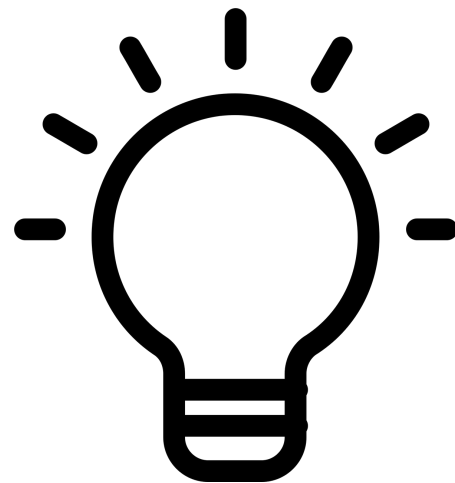
# Limitations of Self-Consistency With Chain-of-Thought

- No local exploration of promising branches
  - Maybe one beginning is better than other and the end can be improved
- No evaluation of the reasoning steps
  - All branches are the same

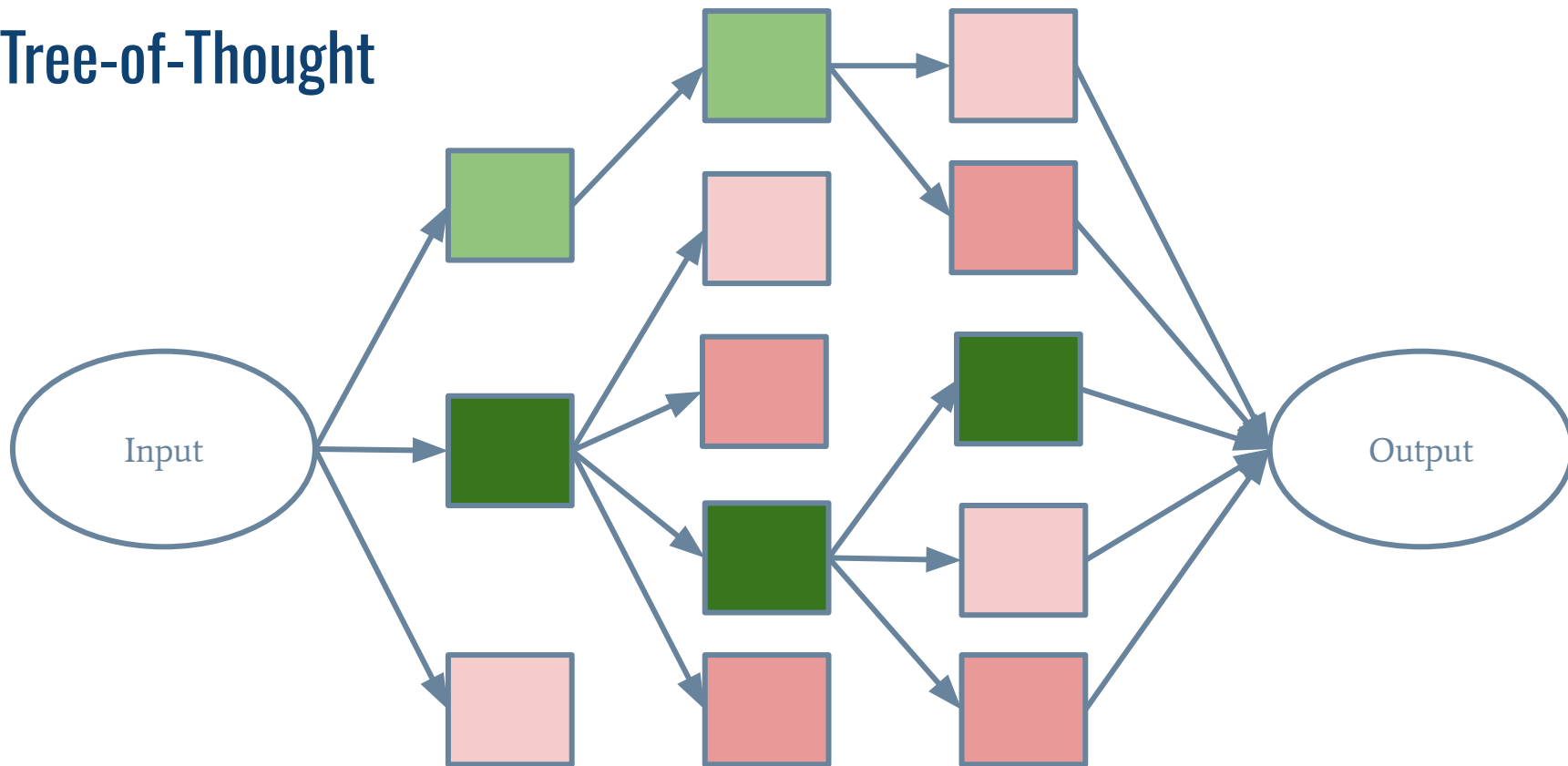
# Introducing Tree-of-Thoughts

Key ideas:

- Decompose the reasoning path into explicit reasoning steps
- Generation of thoughts of each steps
- Heuristic evaluation of each step
- Exploration using a search algorithm



# Tree-of-Thought



# Decomposition And Generation Of Thoughts

- We need a function that can generate possible next thoughts
- Can be done with chain-of-thought/few-shot

## Example: Game of 24

- Reach 24 using  $+/ - / *$  from a list of numbers
- Each number must be used exactly once
  - Input: 4 4 6 8
  - Answer:  $(4 + 8) * (6 - 4) = 24$

Input: 2 8 8 14  
Possible next steps:  
 $2 + 8 = 10$  (left: 8 10 14)  
 $8 / 2 = 4$  (left: 4 8 14)  
 $14 + 2 = 16$  (left: 8 8 16)  
 $2 * 8 = 16$  (left: 8 14 16)  
 $8 - 2 = 6$  (left: 6 8 14)  
 $14 - 8 = 6$  (left: 2 6 8)  
 $14 / 2 = 7$  (left: 7 8 8)  
 $14 - 2 = 12$  (left: 8 8 12)  
Input: {input}  
Possible next steps:

# Heuristic Evaluation of Each Step

- We need a function to evaluate a thought
- Can also be done with chain-of-thought/few-shot
- We can use a natural language scale (e.g.: impossible/likely/sure)



10 14  
 $10 + 14 = 24$   
sure  
11 12  
 $11 + 12 = 23$   
 $12 - 11 = 1$   
 $11 * 12 = 132$   
 $11 / 12 = 0.91$   
impossible



# Exploration Using a Search Algorithm

- We need an algorithm to explore a tree-of-thought
  - Breadth-first search: At each depth, we take the K best thoughts
  - Depth-first search: Only consider thoughts evaluated above a given threshold



# Experiments - Game of 24

Method	Success
IO prompt	7.3%
CoT prompt	4.0%
CoT-SC (k=100)	9.0%
ToT (ours) (b=1)	45%
ToT (ours) (b=5)	<b>74%</b>

# Experiments - 5x5 Crosswords

- Find the words to complete a crossword

Method	Success Rate (%)		
	Letter	Word	Game
IO	38.7	14	0
CoT	40.6	15.6	1
ToT (ours)	<b>78</b>	<b>60</b>	<b>20</b>

Input:

h1. A lunar valley

h2. A fatty oil

h3. To entice

h4. To lower; to reduce

h5. A solitary person

v1. According to the roster

v2. Another name for Port-Francqui

v3. An illicit lover; a European lake

v4. To lisp

v5. To come in

Output:

R I L L E

O L E I N

T E M P T

A B A S E

L O N E R

# Limitations

- For some tasks, language models are very good and tree-of-thought do not help
- Requires more resources
- Prompt engineering required to generate thoughts and evaluate them



# Conclusion

- How language models can be used to solve tasks in few-shot scenarios
- We focused on chain-of-thought prompting
  - Chain-of-thought = serie of reasoning steps
  - Help models produce better results
  - Can be forced through prompting
- Extensions over chain-of-thought
  - Self-consistency = ask the model for multiple explanations
  - Tree-of-thought = more control in the exploration of thoughts

# Questions?

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