Hw4 Two capacitors

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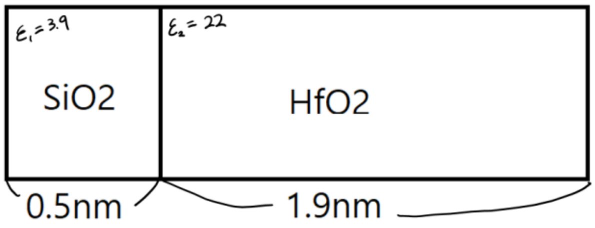


$$\nabla \cdot [\epsilon(r)\nabla\phi(r)] = 0 \;\rightarrow\; \frac{d}{dx}\left[\epsilon(x)\frac{d}{dx}\phi(x)\right] = 0, \qquad (1d\ case)$$

In the range of same ϵ, $0 < x < 0.5nm$, $0.5nm < x < 2.4nm$, the equation is just reduced to

$$\epsilon\frac{d^2}{dx^2}\phi(x) = 0$$

The main problem is that the range of equation is over the interface of two capacitors, $x \simeq 0.5$. In this case, we could earn the result by integrating the first equation. $\left[\epsilon(x)\frac{d\phi}{dx}\right]_{0.5-\Delta x}^{0.5+\Delta x} = 0$.

Thus this equation tells us that $\epsilon_2\frac{d\phi(0.5+\Delta x)}{dx} - \epsilon_1\frac{d\phi(0.5-\Delta x)}{dx} = 0$. Let the boundary condition : $\phi(0) = 0, \phi(2.4nm) = 1$.

Therefore, analytic solution is …

$$Analytic\ solution, \qquad \phi(x \le 0.5nm) = \frac{2200}{1841}x$$

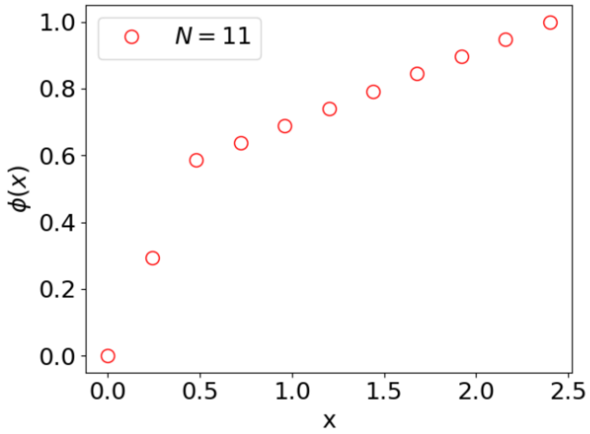
$$\phi(0.5nm < x < 2.4nm) = \frac{390}{1841}x + \frac{905}{1841}$$

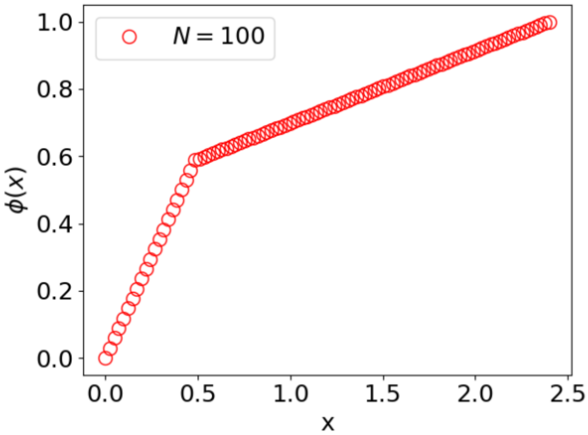
Plus, we must note that the slope is related with the capacitance as $\frac{C}{\epsilon_1} = \frac{2200}{1841}, \frac{C}{\epsilon_2} = \frac{390}{1841}$.

Now we need to earn numerical solutions. With above equations, we got matrix form of non-homogeneous laplace equations. Here is the example of 11 by 11 matrix.

```
[[  1.    0.    0.    0.    0.    0.    0.    0.    0.    0.    0. ]
 [  3.9  -7.8   3.9   0.    0.    0.    0.    0.    0.    0.    0. ]
 [  0.    3.9 -25.9  22.    0.    0.    0.    0.    0.    0.    0. ]
 [  0.    0.   22.  -44.   22.    0.    0.    0.    0.    0.    0. ]
 [  0.    0.    0.   22.  -44.   22.    0.    0.    0.    0.    0. ]
 [  0.    0.    0.    0.   22.  -44.   22.    0.    0.    0.    0. ]
 [  0.    0.    0.    0.    0.   22.  -44.   22.    0.    0.    0. ]
 [  0.    0.    0.    0.    0.    0.   22.  -44.   22.    0.    0. ]
 [  0.    0.    0.    0.    0.    0.    0.   22.  -44.   22.    0. ]
 [  0.    0.    0.    0.    0.    0.    0.    0.   22.  -44.   22. ]
 [  0.    0.    0.    0.    0.    0.    0.    0.    0.    0.    1. ]]
```

Numerical result :





As we expected, there are two linear functions and the slope is change at $x = 0.5$. And now we can check numerical capacitances with the slope.

```
Analytic C =  0.041265036085727326
N =  10
C_numerical =  0.05354952873624795
N =  100
C_numerical =  0.04188903116755923
N =  1000
C_numerical =  0.041277222923351425
```

We found that more steps approach analytical result. We also show absolute difference of the capacitance with analytical and numerical results.