2-D Laplace equation

Assignment #9

20202010 Hyunsuk Shin

In this assignment, we solve the 2-D Laplace equation

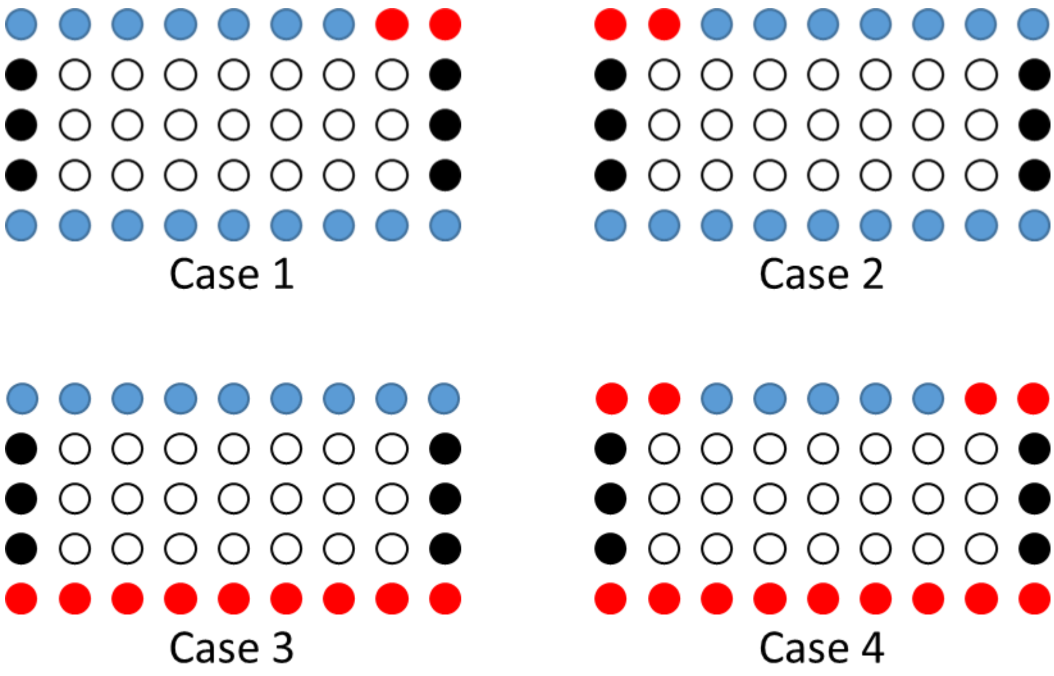

Figure 1. 4 cases of boundary condition

In the figure 1, red circles indicates $\phi_{i,j} = 1$, blue circles indicates $\phi_{i,j} = 0$, black circles indicates homogeneous Neumann boundary conditions, and empty circles indicates bulk nodes which follows the laplace equation.

For the empty circles, the laplace equation becomes as follows:

$$\int \nabla^2\phi \;\; d\boldsymbol{r} = \oint \boldsymbol{\nabla}\phi \cdot d\boldsymbol{a}$$

$$\boldsymbol{\nabla}\phi \cdot d\boldsymbol{a} = 0$$

$$\phi_{i+1,j} + \phi_{i,j+1} - 4\phi_{i,j} + \phi_{i-1,j} + \phi_{i,j-1} = 0$$

For the black circles, the homogeneous Neumann boundary conditions is (left hand side)

$$\phi_{2,j} + 0.5\phi_{i,j+1} - 2\phi_{1,j} + 0.5\phi_{i,j-1} = 0$$

From those equation, we can construct the 2D Laplacian matrix.

The figure 2 shows our results.

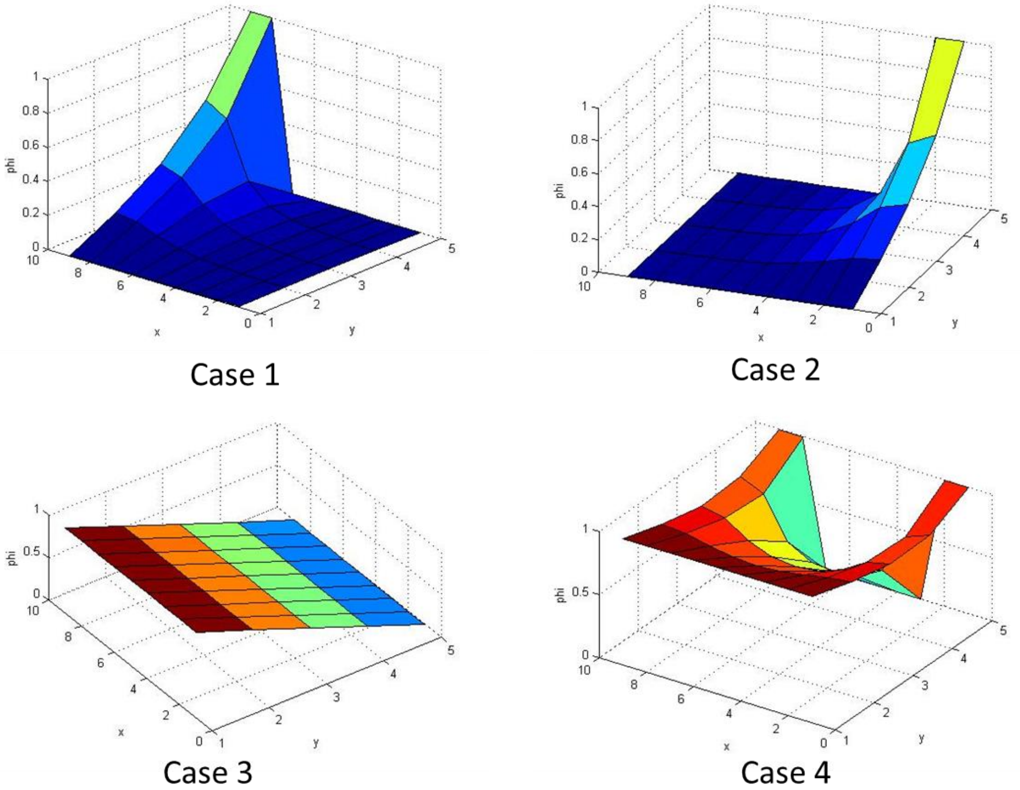

Figure 2. The results 3D plot.