HW17

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If $|\phi_{i+1} - \phi_i| > 2V_T$, one of coefficients for the electron densities becomes negative. We introduce Bernoulli function $B(x) = \frac{x}{e^x - 1}$

$$B = -n_{i+1}\frac{1}{e^{\frac{\Delta\phi}{V_T}} - 1} + n_i \frac{e^{\Delta\phi/V_T}}{e^{\Delta\phi/V_T} - 1}$$

$$J_n = -\frac{qD_n}{\Delta x}\frac{\Delta\phi}{V_T}B$$

Implementing Bernoulli function, you should be careful where x=0, $\frac{x}{e^x - 1}$ have to be 1.

$|\phi_{i+1} - \phi_i| \simeq 0$, original scheme is obtained.

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = n_{i+1}\left(1 - \frac{\phi_{i+1} - \phi_i}{2V_T}\right) - n_i\left(1 + \frac{\phi_{i+1} - \phi_i}{2V_T}\right)$$

$|\phi_{i+1} - \phi_i| \gg 0,$ by the Bernoulli function.

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = -n_i\left(\frac{\phi_{i+1} - \phi_i}{V_T}\right)$$

$$J_{n,i+0.5} = -q\mu_n n_i\left(\frac{\phi_{i+1} - \phi_i}{\Delta x}\right)$$

Poisson equation:

-Over the silicon layer

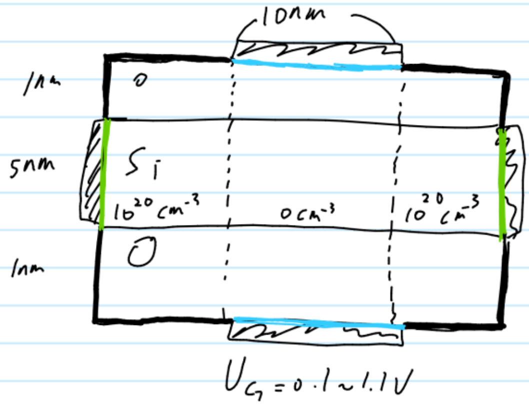
$$R_\phi = \frac{\epsilon_{i+1,j}\phi_{i+1,j} + \epsilon_{i-1,j}\phi_{i-1,j} + \epsilon_{i,j-1}\phi_{i,j-1} + \epsilon_{i,j+1}\phi_{i,j+1} - 4\epsilon_{i,j}\phi_{i,j}}{\epsilon_0} + \frac{\Delta \mathrm{x}^2 q}{\epsilon_0}(N^+ - n_i) = 0$$

$$\frac{\partial R_\phi}{\partial \phi_{i\pm1,j\pm1}} = \frac{\epsilon_{i\pm1,j\pm1}}{\epsilon_0}, \qquad \frac{\partial R_\phi}{\partial \phi_{i,j}} = -\frac{-4\epsilon_{i,j}}{\epsilon_0}, \qquad \frac{\partial R_\phi}{\partial n_i} = -\frac{\Delta x^2 q}{\epsilon_0}$$

- boundary example : left top

$$R_\phi = \frac{\epsilon_{0,1}\phi_{0,1} + \epsilon_{1,0}\phi_{1,0} - 2\epsilon_{0,0}\phi_{0,0}}{2\epsilon_0} = 0$$

$$\frac{\partial R_\phi}{\partial \phi_{0,1}} = \frac{\epsilon_{0,1}}{\epsilon_0}, \qquad \frac{\partial R_\phi}{\partial \phi_{1,0}} = \frac{\epsilon_{1,0}}{\epsilon_0}, \qquad \frac{\partial R_\phi}{\partial \phi_{0,0}} = -\frac{-\epsilon_{0,0}}{\epsilon_0}$$



- Boundaries
  - Black lines : Neumann Boundary conditions.
    ex) left top : $\frac{\epsilon_{ox}}{2}(\phi_{0,1}+\phi_{1,0}) - \epsilon_{ox}\phi_{0,0} = 0$.
  - Blue lines : $\phi = 0.33374 + V_G$, $0 \le V_G \le 1.1$
  - Green lines : $\phi = \mathrm{arcsinh}(N^+/2n_i)\,kT/q$

Continuity equation : we don't need to consider y-direction current.

$$J_{n,i+0.5,j} + \cancel{J_{n,i,j+0.5}} - J_{n,i-0.5,j} \cancel{- J_{n,i,j-0.5}} = 0$$

$|\phi_{i+1} - \phi_i| \simeq 0$:

$$\frac{(J_{n,i+0.5,j}+\cancel{J_{n,i,j+0.5}}-J_{n,i-0.5,j}-\cancel{J_{n,i,j-0.5}})}{qD_n}\Delta x \quad = n_{i+1,j}\left(1-\frac{\phi_{i+1,j}-\phi_{i,j}}{2V_T}\right) - n_{i,j}\left(1+\frac{\phi_{i+1,j}-\phi_{i,j}}{2V_T}\right) - n_{i,j}\left(1-\frac{\phi_{i,j}-\phi_{i-1,j}}{2V_T}\right) + n_{i-1,j}\left(1+\frac{\phi_{i,j}-\phi_{i-1,j}}{2V_T}\right) + \cancel{n_{i,j+1}\left(1-\frac{\phi_{i,j+1}-\phi_{i,j}}{2V_T}\right) - n_{i,j}\left(1+\frac{\phi_{i,j+1}-\phi_{i,j}}{2V_T}\right) - n_{i,j}\left(1-\frac{\phi_{i,j}-\phi_{i,j-1}}{2V_T}\right) + n_{i,j-1}\left(1+\frac{\phi_{i,j}-\phi_{i,j-1}}{2V_T}\right)}$$

$$\frac{\partial R_n}{\partial n_{i+1,j}} = \left(1-\frac{\phi_{i+1,j}-\phi_{i,j}}{2V_T}\right)\frac{qD_n}{\Delta x},$$
$$\frac{\partial R_n}{\partial n_{i-1,j}} = \left(1+\frac{\phi_{i,j}-\phi_{i-1,j}}{2V_T}\right)\frac{qD_n}{\Delta x},$$
$$\cancel{\frac{\partial R_n}{\partial n_{i,j+1}} = \left(1-\frac{\phi_{i,j+1}-\phi_{i,j}}{2V_T}\right)\frac{qD_n}{\Delta x}},$$
$$\cancel{\frac{\partial R_n}{\partial n_{i,j-1}} = \left(1-\frac{\phi_{i,j}-\phi_{i,j-1}}{2V_T}\right)\frac{qD_n}{\Delta x}}$$
$$\frac{\partial R_n}{\partial n_{i,j}} = -\left(4+\frac{\phi_{i+1,j}+\phi_{i-1,j}+\cancel{\phi_{i,j+1}+\phi_{i,j-1}}-2\phi_{i,j}}{2V_T}\right)\frac{qD_n}{\Delta x},$$

$$\frac{\partial R_n}{\partial \phi_{i+1,j}} = -\left(\frac{n_{i+1,j}+n_{i,j}}{2V_T}\right)\frac{qD_n}{\Delta x},$$
$$\frac{\partial R_n}{\partial \phi_{i-1,j}} = -\left(\frac{n_{i,j}+n_{i-1,j}}{2V_T}\right)\frac{qD_n}{\Delta x},$$
$$\cancel{\frac{\partial R_n}{\partial \phi_{i,j+1}} = -\left(\frac{n_{i,j}+n_{i,j+1}}{2V_T}\right)\frac{qD_n}{\Delta x}},$$
$$\cancel{\frac{\partial R_n}{\partial \phi_{i,j-1}} = -\left(\frac{n_{i,j}+n_{i,j-1}}{2V_T}\right)\frac{qD_n}{\Delta x}}$$
$$\frac{\partial R_n}{\partial \phi_{i,j}} = \left(\frac{n_{i+1,j}+n_{i-1,j}+\cancel{n_{i,j+1}+n_{i,j-1}}+2n_{i,j}}{2V_T}\right)\frac{qD_n}{\Delta x},$$

$|\phi_{i+1} - \phi_i| \gg 0:$

$$\frac{J_{n,i+0.5,j} + \cancel{J_{n,i,j+0.5}} - J_{n,i-0.5,j} - \cancel{J_{n,i,j-0.5}}}{qD_n}\Delta x$$

$$= -n_{i,j}\left(\frac{\phi_{i+1,j}-\phi_{i,j}}{V_T}\right) + n_{i-1,j}\left(\frac{\phi_{i,j}-\phi_{i-1,j}}{V_T}\right) \cancel{- n_{i,j}\left(\frac{\phi_{i,j+1}-\phi_{i,j}}{V_T}\right)} \cancel{+ n_{i,j-1}\left(\frac{\phi_{i,j}-\phi_{i,j-1}}{V_T}\right)}$$

$$\frac{\partial R_n}{\partial n_{i,j}} = -\left(\frac{\phi_{i+1,j} \cancel{+ \phi_{i,j+1}} - \phi_{i,j}}{V_T}\right)\frac{qD_n}{\Delta x},$$

$$\frac{\partial R_n}{\partial n_{i-1,j}} = \left(\frac{\phi_{i,j}-\phi_{i-1,j}}{V_T}\right)\frac{qD_n}{\Delta x},$$

$$\cancel{\frac{\partial R_n}{\partial n_{i,j-1}} = \left(\frac{\phi_{i,j}-\phi_{i,j-1}}{V_T}\right)\frac{qD_n}{\Delta x}}$$

$$\frac{\partial R_n}{\partial \phi_{i+1,j}} = -\left(\frac{n_{i,j}}{V_T}\right)\frac{qD_n}{\Delta x},$$

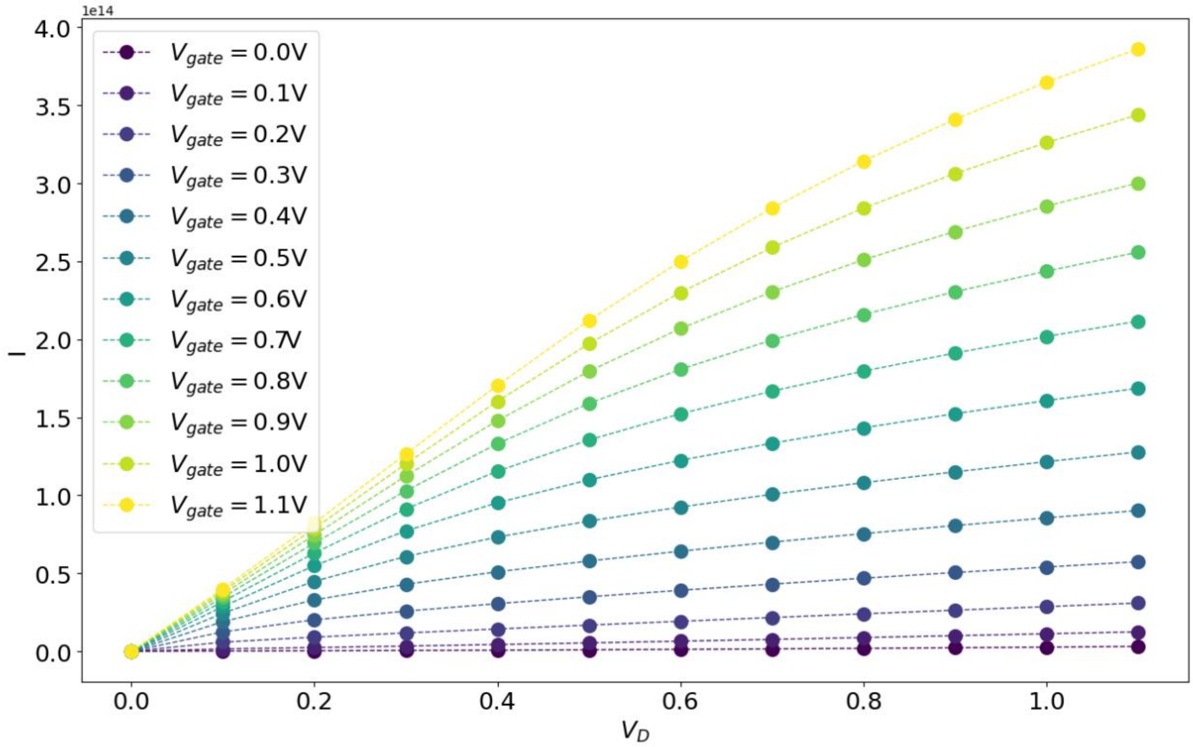
$$\frac{\partial R_n}{\partial \phi_{i-1,j}} = -\left(\frac{n_{i-1,j}}{V_T}\right)\frac{qD_n}{\Delta x}$$

$$\cancel{\frac{\partial R_n}{\partial \phi_{i,j+1}} = -\left(\frac{n_{i,j}}{V_T}\right)\frac{qD_n}{\Delta x}}$$

$$\cancel{\frac{\partial R_n}{\partial \phi_{i,j-1}} = -\left(\frac{n_{i,j-1}}{V_T}\right)\frac{qD_n}{\Delta x}}$$

$$\frac{\partial R_n}{\partial \phi_{i,j}} = \left(\frac{n_{i,j} + n_{i-1,j} \cancel{+ n_{i,j-1}}}{V_T}\right)\frac{qD_n}{\Delta x},$$

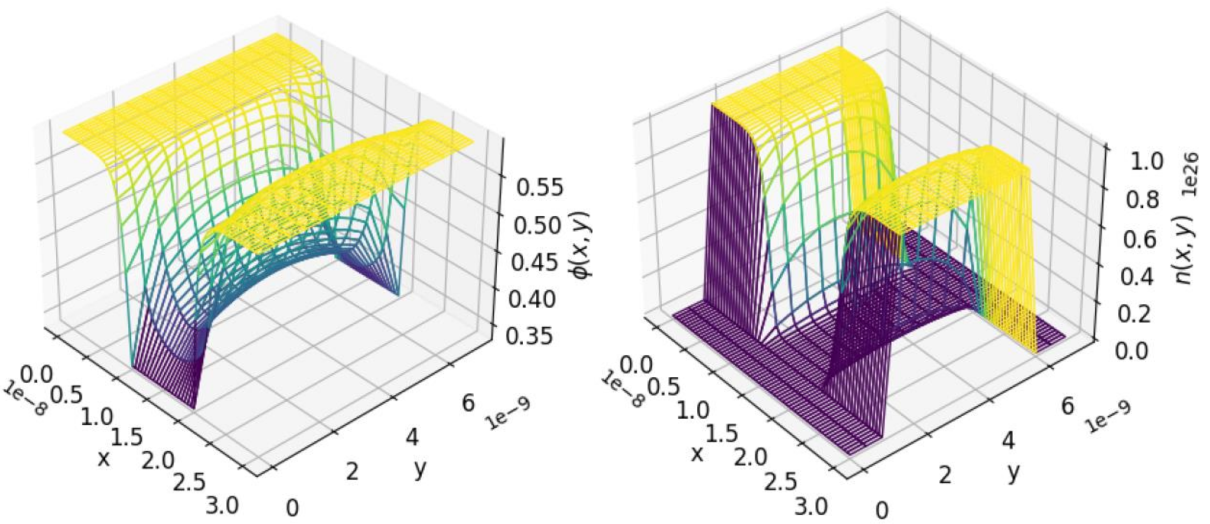
Result



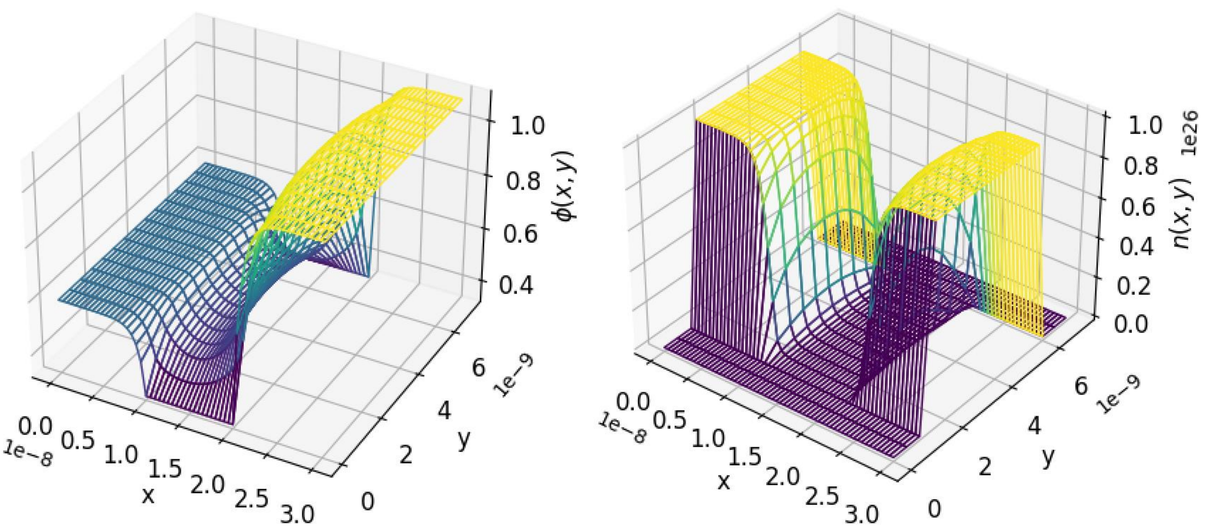
Higher the value of the V_{Gate} and V_{Drain}, higher the value of the terminal current.

Detailed View

-$\phi(x,y)$ *and* $n(x,y)$*with* $(V_{Gate}, V_{Drain}) = (0,0)$

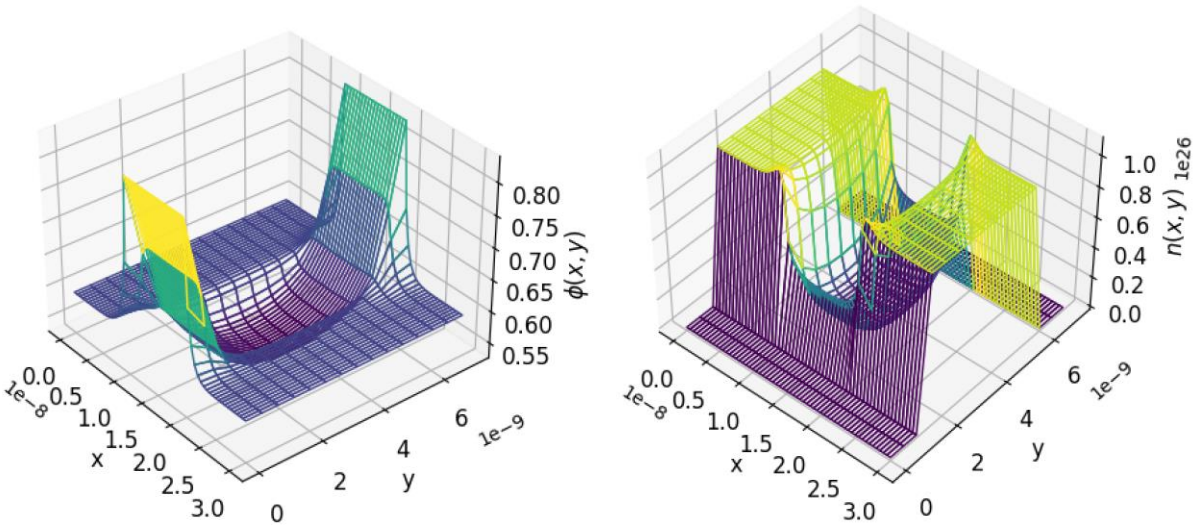


-$\phi(x,y)$ *and* $n(x,y)$ *with* $(V_{Gate}, V_{Drain}) = (0,0.5)$



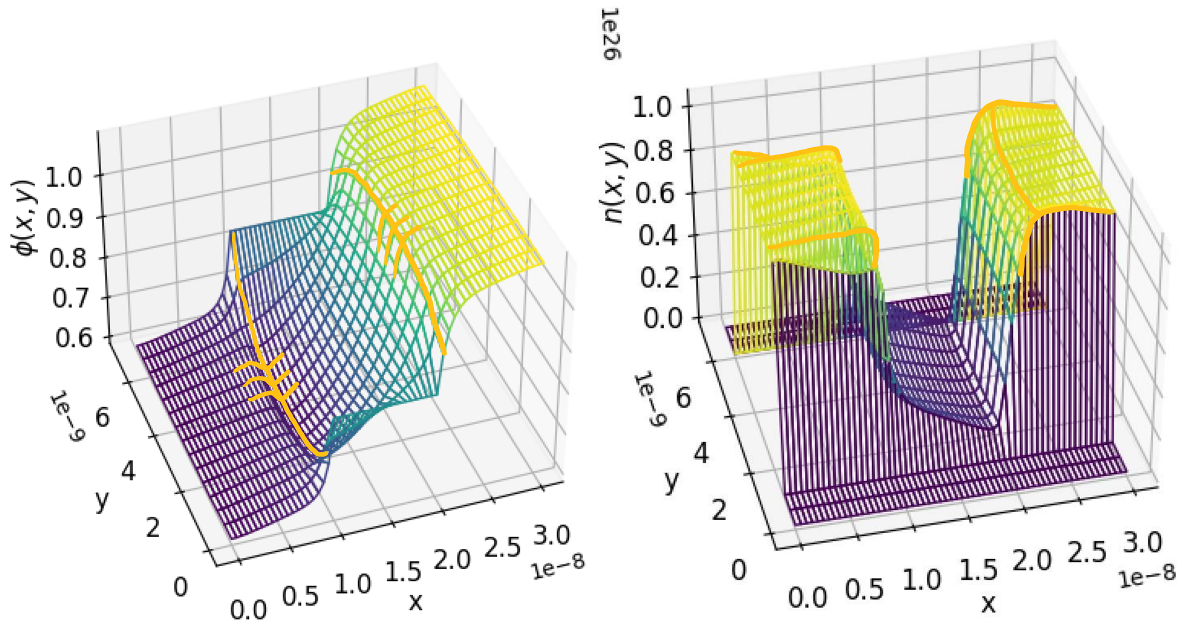
We find that drain voltage (0.5) makes $\phi(x,y)$ unsymmectric. And $n(x,y)$ is unchanged. This is because even though the absolute value of the $\phi(x,y)$ is getting higher with V_{Drain}, the gradient of $\phi(x,y)$ is similar. Thus, it could not affect the form of $n(x,y)$.

$-\phi(x,y)$ *and* $n(x,y)$ *with* $(V_{Gate}, V_{Drain}) = (0.5, 0)$



Now we give $V_{Gate} = 0.5$. Not only the $\phi(x,y)$, $n(x,y)$ also changed. Note that $n(x,y)$ could exist where $N_{acc} = 0$ which means $x = (10nm : 20nm)$ in the silicon layer.

$-\phi(x,y)$ *and* $n(x,y)$ *with* $(V_{Gate}, V_{Drain}) = (0.5, 0.5)$



As we see above figures, the difference of the gradient of $\phi(x,y)$ makes result of different form of $n(x,y)$.