HW 3.

2020년 9월 9일 수요일 오후 5:43

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$$A = \frac{1}{(\Delta x)^2}\begin{pmatrix} (\Delta x)^2 & 0 & \cdots & \cdots & 0 \\ 1 & -2 & 1 & & \vdots \\ \vdots & 1 & -2 & 1 & \vdots \\ \vdots & & \ddots & \ddots & \vdots \\ & & 1 & -2 & 1 \\ 0 & \cdots & \cdots & 0 & (\Delta x)^2 \end{pmatrix}, \quad X = \begin{pmatrix} \phi(x_1) \\ \vdots \\ \vdots \\ \phi(x_n) \end{pmatrix}$$

⇒ $AX = b$ 꼴로 Linear solver를 통해서 X 계산.

Problem 1.

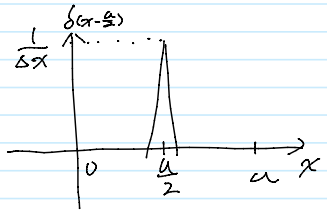
$$b = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \\ -1 \end{pmatrix}$$

Problem 2.

$$b = \begin{pmatrix} \delta(x_1 - \frac{a}{2}) \\ \vdots \\ \vdots \\ \delta(x_N - \frac{a}{2}) \end{pmatrix}$$

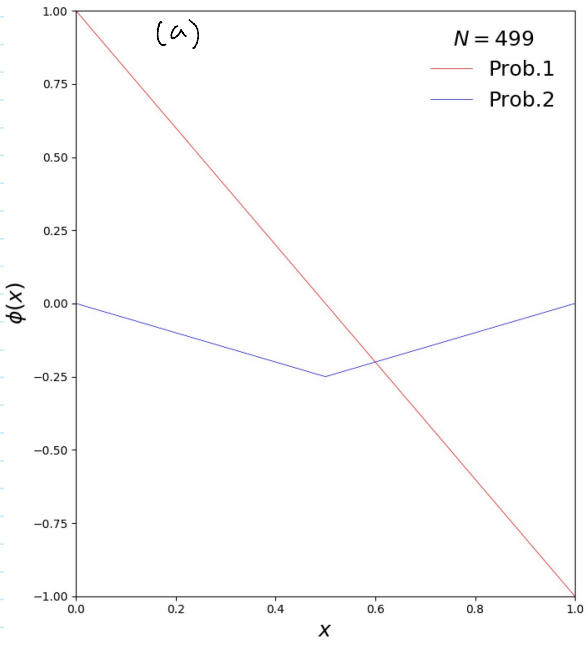
⇒ Numerically, $\delta(x - \frac{a}{2}) = \begin{cases} 0 & (x \neq x_{\frac{N+1}{2}}) \\ \frac{1}{\Delta x} & (x = x_{\frac{N+1}{2}}) \end{cases}$ where N is odd.

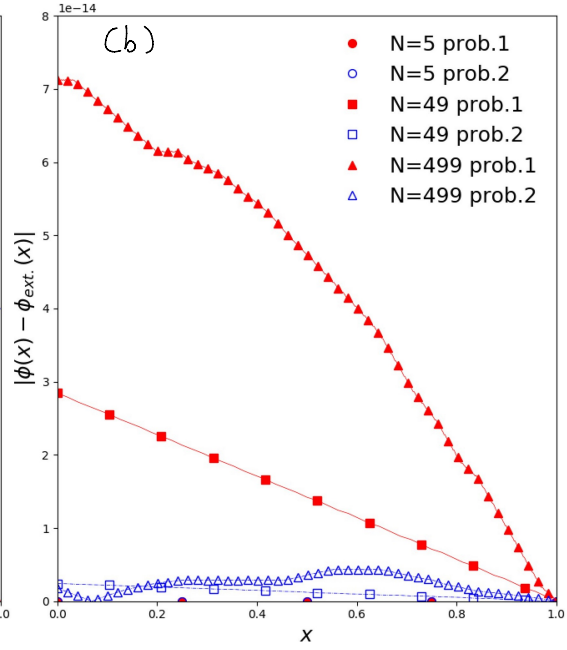
로 정의했다. $\int_0^a dx\, \delta(x - \frac{a}{2}) = 1$ 을 만족한다.



Results.

$N = 499$, $a = 1$, $\Delta x = \frac{a}{N-1}$





Exact solutions

: problem 1. $\phi_{ext.}(x) = -2x + 1$

problem 2. $\phi_{ext.}(x) = \frac{1}{2}|x - \frac{1}{2}| - \frac{1}{4}$ $\left(\because \left.\frac{d\phi(x)}{dx}\right|_{x=0} - \left.\frac{d\phi(x)}{dx}\right|_{x=a} = 1\right)$

(a)는 numerical results 고 (b)는 numerical과 exact solution의 차이를 보여준다.

(b)를 보면 $|\phi(x) - \phi_{ext.}(x)| \sim 10^{-14}$ 로 $\phi(x)$와 $\phi_{ext.}(x)$ 차이가 거의 없음을 알 수 있다.