HW 14.

2020년 10월 29일 목요일 오후 9:11

HW 14 에서는 Nonlinear poisson's eq. 과. Drift-Diffusion eq. 을 통해 electron 과 hole 의 carrier density를 계산해 보았다.

우선 poisson's eq. 은 다음과 같이 구해진다.

Poisson's equation.

$$\frac{d}{dx}\left(\epsilon(x)\frac{d}{dx}\phi\right) = -\rho(x)$$
$$= -q(p - n + N^+)$$

i) for the P-type silicon. the dopant itself negatively charged.

$$N^+_{dop} = -N_{acc}.$$

$$\frac{\epsilon_{si}(\phi_{i+1}+\phi_{i-1}) - 2\epsilon_{si}\phi_i}{(\Delta x)^2} = -q(p - N_{acc} - n)$$

$$\frac{\epsilon_{si}(\phi_{i+1}+\phi_{i-1}) - 2\epsilon_{si}\phi_i}{(\Delta x)^2} + q\left(n_i \exp(-\phi/V_T) - N_{acc} - n_i \exp(\phi/V_T)\right) = 0$$

$$\left(\because p = n_i \exp(-\phi/V_T),\ n = n_i \exp(\phi/V_T)\right)$$

$\therefore$

$$J_{i,i-1} = \epsilon_{si}$$
$$J_{i,i+1} = \epsilon_{si}$$
$$J_{i,i} = -2\epsilon_{si} + A\cdot\left(\frac{-1}{V_T} n_i \exp(-\phi/V_T) - \frac{n_i}{V_T}\exp(\phi/V_T)\right)$$

$$\text{residue} = \epsilon_{si}(\phi_{i+1} + \phi_{i-1} - 2\phi_i) + A\Big(\underbrace{n_i \exp(-\phi/V_T)}_{\text{hole.}} - N_{acc} - \underbrace{n_i \exp(\phi/V_T)}_{\text{electron}}\Big)$$

$$\left(\text{where } A \equiv \left((\Delta x)^2 \times q/\epsilon_0\right)\right)$$

ii) for the N-type silicon. the donor itself positively charged.

$$N^+_{dop} = N_{don}.$$

$$\frac{\epsilon_{si}(\phi_{i+1}+\phi_{i-1}) - 2\epsilon_{si}\phi_i}{(\Delta x)^2} - q(n - N_{don} - p) = 0$$

$$\frac{\epsilon_{si}(\phi_{i+1}+\phi_{i-1}) - 2\epsilon_{si}\phi_i}{(\Delta x)^2} - q\left(n_i \exp(\phi/V_T) - N_{don} - n_i \exp(-\phi/V_T)\right) = 0.$$

$$\left(\because n = n_i \exp(\phi/V_T)\right)$$

So

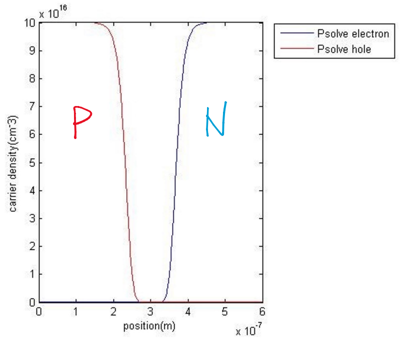
$$J_{i,i-1} = \epsilon_{si}$$
$$J_{i,i+1} = \epsilon_{si}$$
$$J_{i,i} = -2\epsilon_{si} - A\cdot\left(\frac{1}{V_T} n_i \exp(\phi/V_T) + \frac{1}{V_T} n_i \exp(-\phi/V_T)\right) = 0.$$

$$\text{residue} = \epsilon_{si}(\phi_{i+1} + \phi_{i-1} - 2\phi_i) - A\left(n_i \exp(\phi/V_T) - N_{don} - n_i \exp(-\phi/V_T)\right) = 0.$$

$$\text{where } A \equiv \left((\Delta x)^2 \times q/\epsilon_0\right)$$

Jaco와 residue를 Code로 나타내면 다음과 같다.

```
Ndop = zeros(N,1);
Ndop(:,1) = 1e23;
res = zeros (N,1);
Jaco = sparse(N,N);
phi = zeros(N,1);
phi(1:j1,1) = -V_T*log(Ndop(1:j1,1)/ni); %p-type
phi(j1+1:N,1) = V_T*log(Ndop(j1+1:N,1)/ni); %n-type

for nt = 1:10
    res(1,1) = phi(1,1) + V_T*log(Ndop(1,1)/ni);
    Jaco(1,1) = 1.0;


    for ii = 2:N-1
        res(ii,1) = eps_si*(phi(ii+1,1)-2*phi(ii,1)+phi(ii-1,1));
        Jaco(ii,ii+1) = eps_si;
        Jaco(ii,ii) = -2*eps_si;
        Jaco(ii,ii-1) = eps_si;
    end

    res(N,1) = phi(N,1) - V_T*log(Ndop(1,1)/ni);
    Jaco(N,N) = 1.0;

    for ii = 2:N-1
        if (ii < j1+1)
            res(ii,1) = res(ii,1) + coef*(ni*exp(-phi(ii,1)/V_T) - ni*exp(phi(ii,1)/V_T) - Ndop(ii,1));
            Jaco(ii,ii) = Jaco(ii,ii) - coef*(ni*exp(-phi(ii,1)/V_T)/V_T + ni*exp(phi(ii,1)/V_T)/V_T);

        else
            res(ii,1) = res(ii,1) + coef*(ni*exp(-phi(ii,1)/V_T) - ni*exp(phi(ii,1)/V_T) + Ndop(ii,1));
            Jaco(ii,ii) = Jaco(ii,ii) - coef*(ni*exp(-phi(ii,1)/V_T)/V_T + ni*exp(phi(ii,1)/V_T)/V_T);
        end
    end

   update = Jaco\(-res);
   phi = phi+update;
```



연속 방정식.

electron 에서.

$$\frac{\partial n}{\partial t} = -\nabla\cdot F_n = \frac{1}{q}\nabla\cdot J_n = 0. \ (\text{Steady state})$$

$$\nabla\cdot J_n = \nabla\cdot(q\mu_n n E + qD_n\nabla n) = 0$$

1-D case로 줄여보면.

$$\frac{d}{dx}\left(-q\mu_n n\frac{d\phi}{dx} + qD_n\frac{dn}{dx}\right) = 0. \ (\text{1-D})$$

where $D_n = \underbrace{V_T}_{\text{thermal voltage}}\mu_n$ (μ_n → electron mobility.) (Einstein relation)

$x_{i-0.5}$ 에서 $x_{i+0.5}$ 까지 적분해 보면.

$$J_{n,i+0.5} - J_{n,i-0.5} = 0.$$

$$J_{n,i+0.5} = -q\mu_n n_{i+0.5}\left.\frac{d\phi}{dx}\right|_{i+0.5} + qD_n\left.\frac{dn}{dx}\right|_{i+0.5} = -q\mu_n\left(n_{i+0.5}\left.\frac{d\phi}{dx}\right|_{i+0.5} - V_T\left.\frac{dn}{dx}\right|_{i+0.5}\right)$$

$$J_{n,i-0.5} = -q\mu_n n_{i-0.5}\left.\frac{d\phi}{dx}\right|_{i+0.5} + qD_n\left.\frac{dn}{dx}\right|_{i-0.5} = -q\mu_n\left(n_{i-0.5}\left.\frac{d\phi}{dx}\right|_{i-0.5} - V_T\left.\frac{dn}{dx}\right|_{i-0.5}\right)$$

$$\therefore J_{n,i+0.5} - J_{n,i-0.5}$$
$$= n_{i+0.5}\left.\frac{d\phi}{dx}\right|_{i+0.5} - n_{i-0.5}\left.\frac{d\phi}{dx}\right|_{i-0.5}$$
$$- V_T\left(\left.\frac{dn}{dx}\right|_{i+0.5} - \left.\frac{dn}{dx}\right|_{i-0.5}\right) = 0.$$

$$n_{i+0.5} = \frac{1}{2}(n_{i+1} + n_i) \qquad \left.\frac{dn}{dx}\right|_{i+0.5} = \frac{n_{i+1} - n_i}{\Delta x} \qquad \frac{dp}{dx}$$

$$\left.\frac{d\phi}{dx}\right|_{i+0.5} = \frac{1}{\Delta x}(\phi_{i+1} - \phi_i)$$

$J_{n,i+0.5}$ 의 Jacobian 에 주는 영향.

$$-\frac{1}{q\mu_n}\frac{\partial J_{n,i+0.5}}{\partial n_{i+1}} = \frac{1}{2}\left.\frac{d\phi}{dx}\right|_{i+0.5} - V_T\frac{1}{\Delta x}$$

$$-\frac{1}{q\mu_n}\frac{\partial J_{n,i+0.5}}{\partial n_{i+1}} = \frac{1}{2}\frac{d\phi}{dx}\Big|_{i+0.5} - V_T\frac{1}{\Delta x}$$

$$-\frac{1}{q\mu_n}\frac{\partial J_{n,i+0.5}}{\partial n_i} = \frac{1}{2}\frac{d\phi}{dx}\Big|_{i+0.5} + V_T\frac{1}{\Delta x}$$

∴ n-type region 에서,

continuity eq. 은. 다음과 같이 정리된다.

$$\frac{d}{dx}\left(-q\mu_n n\frac{d\phi}{dx} + qD_n\frac{dn}{dx}\right) = 0.$$

$$\Rightarrow \frac{d}{dx}\left(n\frac{d\phi}{dx} - V_T\frac{dn}{dx}\right) = 0.$$

$$\Rightarrow J_{n,i+0.5} - J_{n,i-0.5} = n_{i+0.5}\frac{d\phi}{dx}\Big|_{i+0.5} - n_{i-0.5}\frac{d\phi}{dx}\Big|_{i-0.5} - V_T\left(\frac{dn}{dx}\Big|_{i+0.5} - \frac{dn}{dx}\Big|_{i-0.5}\right) = 0.$$

※. hole 의 경우는 다음과 같다.

$$\frac{d}{dx}\left(-q\mu_p p\frac{d\phi}{dx} - qD_p\frac{dp}{dx}\right) = 0.$$

$$p\frac{d\phi}{dx} + V_T\frac{dp}{dx} = 0.$$

$$J_{p,i+0.5} - J_{p,i-0.5} = p_{i+0.5}\frac{d\phi}{dx}\Big|_{i+0.5} + V_T\left(\frac{dp}{dx}\Big|_{i+0.5}\right) - p_{i-0.5}\frac{d\phi}{dx}\Big|_{i-0.5} - V_T\frac{dp}{dx}\Big|_{i-0.5}$$

위 식을 이용하여 elec, hole 의 Jaco, residue 를 numerical 하게 나타내면. 다음과 같다.

```
res_elec = zeros(N,1);
Jaco_elec = sparse(N,N);
res_hole = zeros(N,1);
Jaco_hole = sparse(N,N);

%boundary condition


%construct Jaco and res

for ii = 1:N-1
    n_av = 0.5*(elec(ii+1,1)+elec(ii,1));
    h_av = 0.5*(hole(ii+1,1)+hole(ii,1));

    dphidx = (phi(ii+1,1)-phi(ii,1))/Deltax;
    delecdx = (elec(ii+1,1) - elec(ii,1))/Deltax;
    dholedx = (hole(ii+1,1)- hole(ii,1))/Deltax;


    Jn = n_av*dphidx - V_T*delecdx;
    Jp = h_av*dphidx +V_T*dholedx;

    res_elec(ii,1) = res_elec(ii,1) + Jn;
    Jaco_elec(ii,ii+1) = Jaco_elec(ii,ii+1)+0.5*dphidx-V_T/Deltax;
    Jaco_elec(ii,ii)   = Jaco_elec(ii,ii) +0.5*dphidx+V_T/Deltax;

    res_hole(ii,1) = res_hole(ii,1) + Jp;
    Jaco_hole(ii,ii+1) = Jaco_hole(ii,ii+1)+0.5*dphidx+V_T/Deltax;
    Jaco_hole(ii,ii)   = Jaco_hole(ii,ii) +0.5*dphidx-V_T/Deltax;

    res_elec(ii+1,1) = res_elec(ii+1,1) - Jn;
    Jaco_elec(ii+1,ii+1) = Jaco_elec(ii+1,ii+1)-0.5*dphidx+V_T/Deltax;
    Jaco_elec(ii+1,ii)   = Jaco_elec(ii+1,ii) -0.5*dphidx-V_T/Deltax;

    res_hole(ii+1,1) = res_hole(ii+1,1) - Jp;
    Jaco_hole(ii+1,ii+1) = Jaco_hole(ii+1,ii+1)-0.5*dphidx-V_T/Deltax;
    Jaco_hole(ii+1,ii)   = Jaco_hole(ii+1,ii) -0.5*dphidx+V_T/Deltax;


end
```

최종적으로 update 된 그래프는, 아래와 같다.