Hw7 : Newton's method

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Simple quadratic equation

The example equation is $x^2 - 1 = 0$. We can define the distance from the real solution as $r(x_0) = x_0^2 - 1$. The next solution will follow $(x_0 + \delta x)^2 - 1 = 0$. We could calculate δx with the approximation of $\delta^2 \simeq 0$. Thus we could use iterative method to estimate the solution.

$$2x_0\delta x + (\delta x)^2 \simeq 2x_0\delta x = -(x_0^2 - 1), \qquad \delta x \simeq -\frac{x_0^2 - 1}{2x_0}$$

We will earn electrostatic potential ϕ with Newton method.

$$N^+ + n_{int}e^{-\phi/V_T} - n_{int}e^{\phi/V_T} = 0$$

$$n_{int} = 10^{10}cm^{-3}, T = 300K, |N^+| = 10^{10}cm^{-3} \sim 10^{18}cm^{-3}$$

Let's say ϕ_0 is the first guessing value. Then,

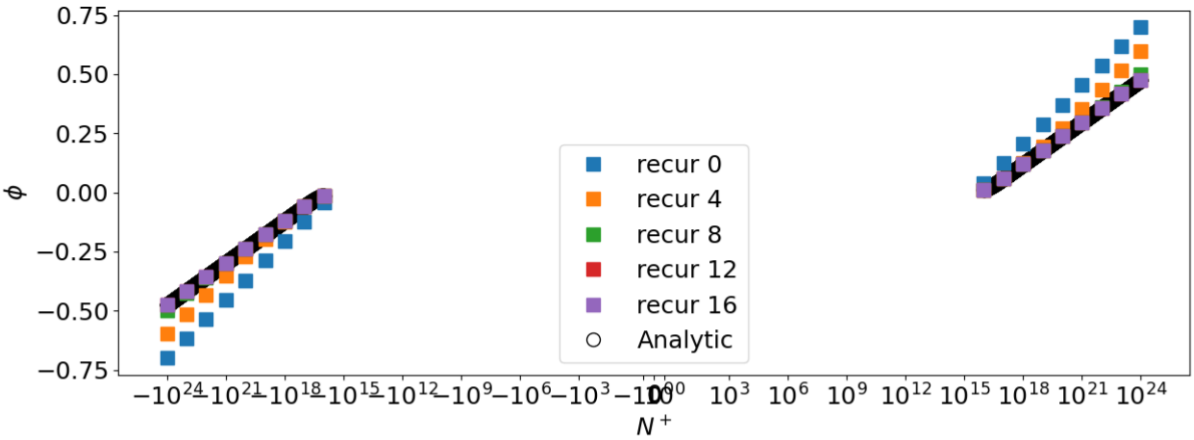
$$F(\phi) = N^+ + n_{int}e^{-\phi/V_T} - n_{int}e^{\frac{\phi}{V_T}} = 0, F(\phi_0) + \frac{\partial F}{\partial \phi_0}\delta\phi_0 = 0 \ldots \phi_1 = \phi_0 + \delta\phi_0$$

$$\delta\phi_0 = -\frac{F(\phi_0)}{\partial F/\partial \phi_0}$$

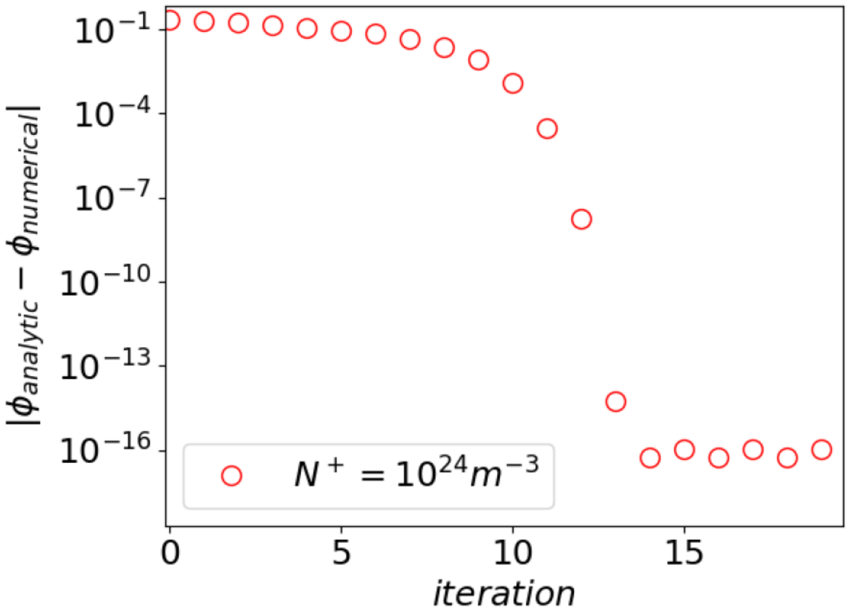
Its analytic solution is :

$$\phi = V_T arcsinh(N^+/2n_{int})$$

Result



This is the result of Newton method to estimate ϕ. ϕ is recursively calculated with Newton method. As the iteration proceeds, we could find that they converge to the analytic results. The next figure is how the numerical result converge to the analytic result where $N^{+} = 10^{24}$.



As the iteration is larger, the difference bet analytic result and numerical result similar. It goes down to 10^{-16} scale.