HW 5.

2020년 9월 18일 금요일 오후 11:33

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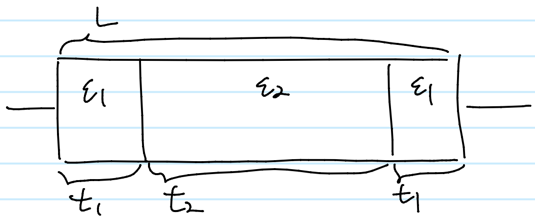


$$\frac{d}{dx}\left(\varepsilon(x)\frac{d\phi(x)}{dx}\right) = b(x)$$ 에서,

$$\varepsilon(x) = \begin{cases} \varepsilon_1 & \text{for } 0 \le x < t_1 \\ \varepsilon_2 & \text{for } t_1 \le x < t_1+t_2 \\ \varepsilon_1 & \text{for } t_1+t_2 \le x < L \end{cases}, \quad b(x) = \begin{cases} 0 & \text{for } 0 \le x < t_1 \\ qN_{acc} & \text{for } t_1 < x < t_1+t_2 \\ 0 & \text{for } t_1+t_2 < x < L \end{cases}$$

$$\phi(0) = \phi(L) = 0.$$

By finite difference,

$$\frac{d}{dx}\left(\varepsilon(x)\frac{d}{dx}\right) \rightarrow A = \begin{pmatrix} 1 & 0 & \cdots & & & & & & & \\ \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1 & & & & & & & \\ & \ddots & \ddots & \ddots & & & & & 0 & \\ & & \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1 & & & & & \\ & & & \varepsilon_1 & -\varepsilon_1-\varepsilon_2 & \varepsilon_2 & & & & \\ & & & & \varepsilon_2 & -2\varepsilon_2 & \varepsilon_2 & & & \\ & & & & & \ddots & \ddots & \ddots & & \\ & & & & & \varepsilon_2 & -2\varepsilon_2 & \varepsilon_2 & & \\ & & & & & & \varepsilon_2 & -\varepsilon_1-\varepsilon_2 & \varepsilon_1 & \\ & 0 & & & & & \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1 & \\ & & & & & & & \ddots & \ddots & \ddots \\ & & & & & & & \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1 \\ & & & & & & & \cdots & 0 & 1 \end{pmatrix}$$

$$b(x) = (\Delta x)^2 \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \frac{qN_{acc}}{2} \\ qN_{acc} \\ \vdots \\ qN_{acc} \\ \frac{qN_{acc}}{2} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{array}{l} \rightarrow \phi(0) \\ \\ \\ \rightarrow x = t_1 \\ \\ \\ \\ \rightarrow x = t_1+t_2 \\ \\ \\ \rightarrow \phi(L) \end{array}, \quad \phi(x) = \begin{pmatrix} \phi_1 \\ \vdots \\ \vdots \\ \phi_N \end{pmatrix}$$

* Exact solution for $t_1 = \frac{8}{66}L$, $t_2 = \frac{50}{66}L$.

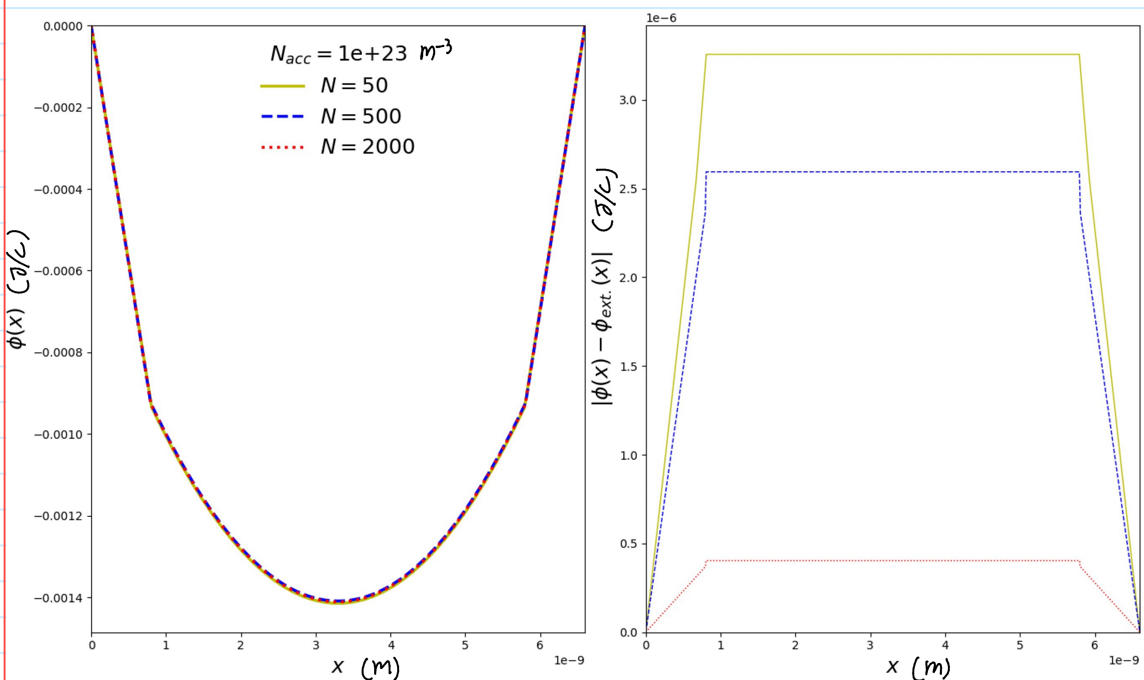
$$\phi(x) = \begin{cases} -\frac{qN_{acc}}{2\varepsilon_1} t_2 x & \text{for } 0 \le x < t_1 \\ \frac{qN_{acc}}{2\varepsilon_2}\left(x - t_1 - \frac{t_2}{2}\right)^2 - \frac{qN_{acc}}{2}\left(\frac{t_1 t_2}{\varepsilon_1} + \frac{1}{\varepsilon_2}\left(\frac{t_2}{2}\right)^2\right) & \text{for } t_1 \le x < t_1 + t_2 \\ \frac{qN_{acc}}{2\varepsilon_1} t_2 (x - L) & \text{for } t_1 + t_2 \le x < L \end{cases}$$

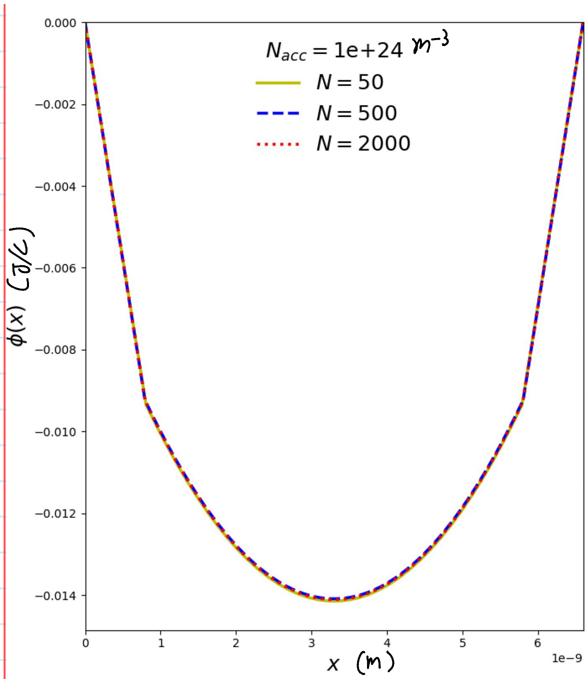
Results

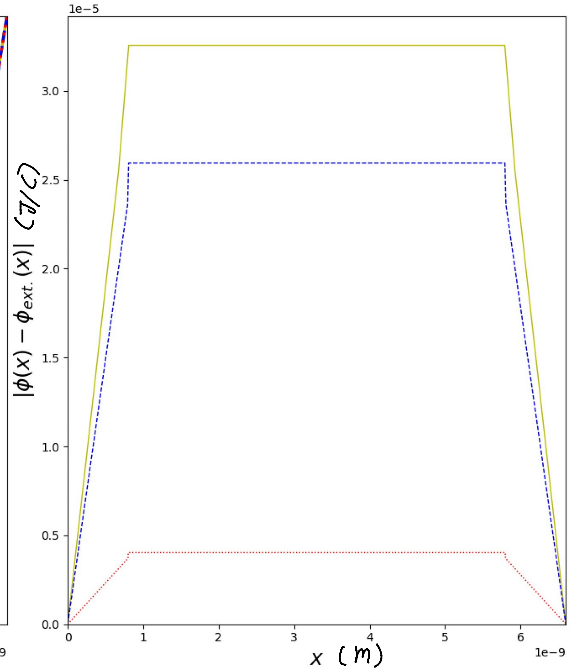
$N_{acc} = 10^{23}\,m^{-3}, 10^{24}\,m^{-3}, 10^{25}\,m^{-3}$

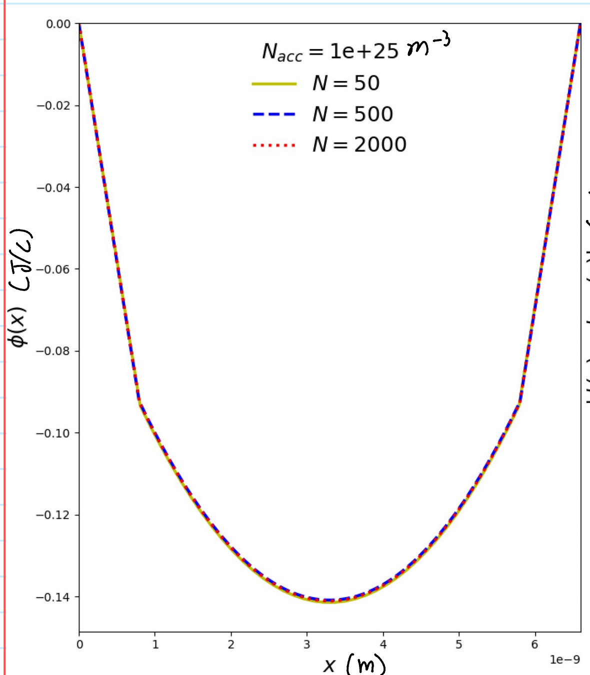
$N = 50, 500, 20000$

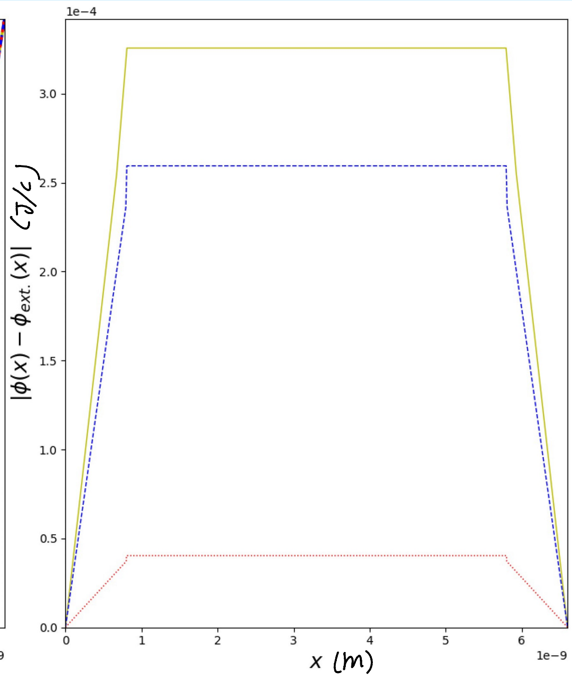
$\varepsilon_1 = 3.9\varepsilon_0$, $\varepsilon_2 = 11.7\varepsilon_0$, $L = 6.6\,nm$











Error $|\phi(x) - \phi_{ext.}(x)| \sim \begin{cases} 10^{-6} \text{ for } N_{acc} = 10^{23}\, m^{-3} \\ 10^{-5} \text{ for } N_{acc} = 10^{24}\, m^{-3} \\ 10^{-4} \text{ for } N_{acc} = 10^{25}\, m^{-3} \end{cases}$

The error is proportional to $\phi(x)$.

However, the error is negligible in comparison with $\phi(x)$.