# Lecture20

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# Revisiting our derivation

$n(x_{i+1}) = n_{i+1}, \quad n(x_i) = n_i$

$x_{i+1} \rightarrow x_i + (x_{i+1} - x_i)$

$$\left.\begin{aligned} n_{i+1} &= A\, e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_{i+1}} + B \\ n_i &= A\, e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i} + B \end{aligned}\right\}$$

$$n_{i+1} - n_i = \underbrace{A\, e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i}}\left(e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}(x_{i+1}-x_i)} - 1\right)$$

$$= \boxed{A\, e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i}} \times \left(e^{\frac{1}{V_T}\Delta\phi} - 1\right)$$

$$\boxed{\frac{dn}{dx} - \underbrace{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}}\, n = \frac{J_n}{q D_n}}$$

$$\boxed{\boxed{A\, e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i}} = \frac{n_{i+1} - n_i}{e^{\frac{\Delta\phi}{V_T}} - 1}}$$

$$\underbrace{n_i - \boxed{\frac{n_{i+1} - n_i}{e^{\frac{\Delta\phi}{V_T}} - 1}}} = \underline{B}$$

# Scharfetter-Gummel

- What happens if $|\phi_{i+1} - \phi_i| > 2V_T$?
  - One of two coefficients for the electron densities becomes negative. Unphysical!

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = n_{i+1}\left(1 - \frac{\phi_{i+1} - \phi_i}{2V_T}\right) - n_i\left(1 + \frac{\phi_{i+1} - \phi_i}{2V_T}\right)$$

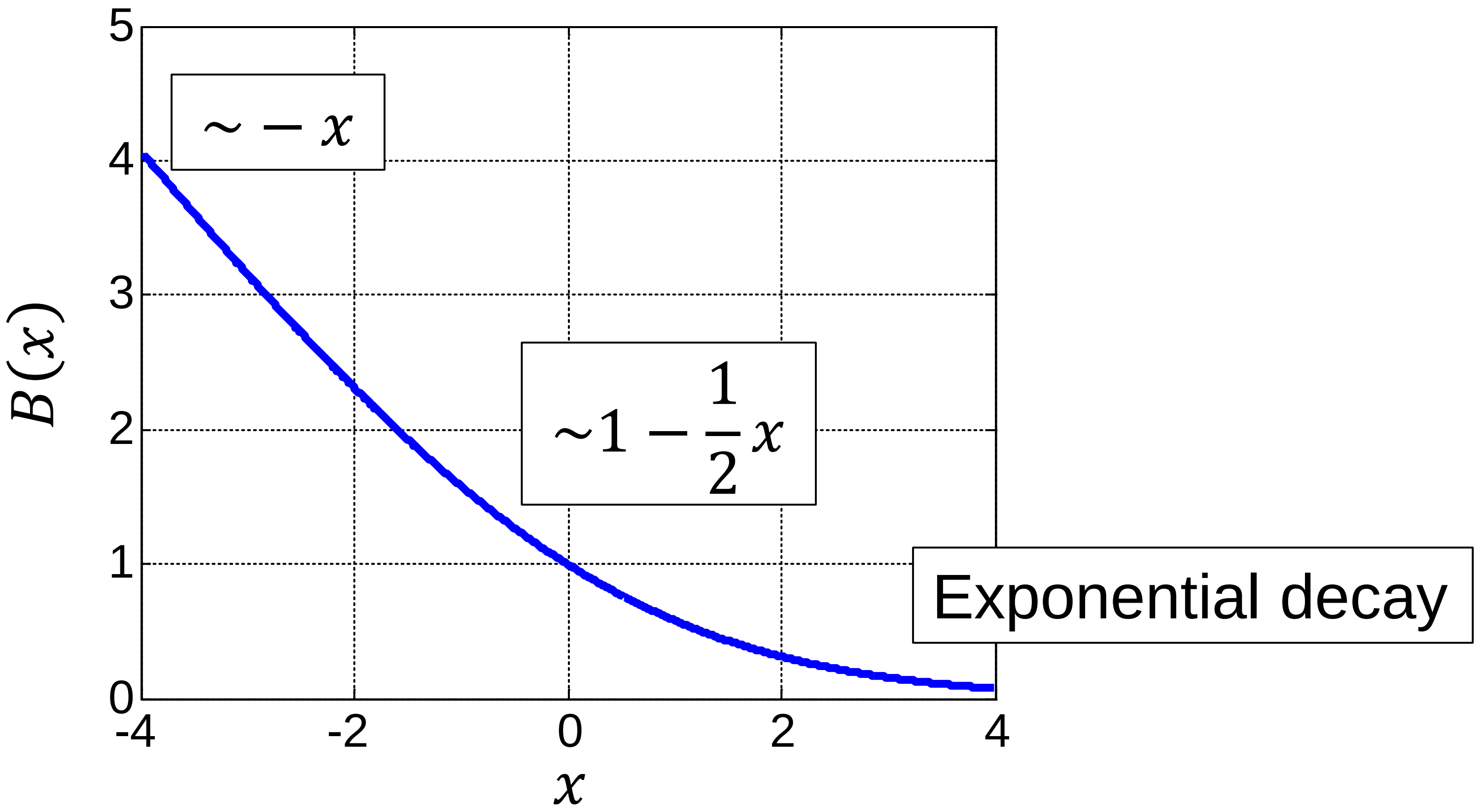
  - The Scharfetter-Gummel scheme

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = n_{i+1}B\left(\frac{\phi_{i+1} - \phi_i}{V_T}\right) - n_i B\left(\frac{\phi_i - \phi_{i+1}}{V_T}\right)$$

  - Here, the Bernoulli function is

$$B(x) = \frac{x}{e^x - 1}$$

# Bernoulli function



# Two limits

- When $|\phi_{i+1} - \phi_i| \approx 0$,
  - Our original scheme is obtained.

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = n_{i+1}\left(1 - \frac{\phi_{i+1} - \phi_i}{2V_T}\right) - n_i\left(1 + \frac{\phi_{i+1} - \phi_i}{2V_T}\right)$$

- When $|\phi_{i+1} - \phi_i| \gg 0$,
  - (Without loss of generality) when $\phi_{i+1} - \phi_i \gg 0$,

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = -n_i\frac{\phi_{i+1} - \phi_i}{V_T}$$

$$J_{n,i+0.5} = -q\mu_n n_i\frac{\phi_{i+1} - \phi_i}{\Delta x}$$

# Implementation

- Near zero, the direct evaluation causes a problem.
  - Piecewise implementation

```
double c = fabs(x);
if (c < 2.502000000000000e-02) {
  double sxd = x * x;
  B = ( 1.0-x/2.0+sxd/12.0*(1.0-sxd/60.0*(1.0-sxd/42.0)) );
  dBdx = ( -0.5 + x/6.0*(1.0-sxd/30.0*(1.0-sxd/28.0)) );
} else if (c < 1.500000000000000e-01) {
  double sxd = x * x;
  B = ( 1.0-x/2.0+sxd/12.0*(1.0-sxd/60.0*(1.0-sxd/42.0*(1.0-sxd/40.0*(1.0-0.02525252525252525252525*sxd)))) );
  dBdx = ( -0.5 + x/6.0*(1.0-sxd/30.0*(1.0-sxd/28.0*(1.0-sxd/30.0*(1.0-0.031565656565656565657*sxd)))) );
} else if (x > 150.01) {
  double inv_expx = exp(-x);
  B = x * inv_expx;
  dBdx = ( inv_expx - B );
} else {
  double inv_expx_1 = 1.0/(exp(x)-1.0);
  B = x * inv_expx_1;
  dBdx = ( inv_expx_1 - B*(inv_expx_1+1.0) );
}
```