HW 4.

2020년 9월 15일 화요일 오후 2:46

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$\frac{d}{dx}\left(\varepsilon(x)\frac{d}{dx}\right)\phi(x) = b(x)$ 에서,

$$\varepsilon(x) = \begin{cases} \varepsilon_1 & \text{for } 0 \le x < d \\ \varepsilon_2 & \text{for } d \le x < L \end{cases} \quad (d < L)$$

By finite difference (continuous $x \rightarrow \{x_1, x_2 \cdots x_N\}$),

$$\frac{d}{dx}\left(\varepsilon(x)\frac{d}{dx}\right) \Rightarrow A = \begin{pmatrix} 1 & 0 & 0 & \cdots & & & & \\ \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1 & & & & 0 & \\ & \varepsilon_1 & -2\varepsilon_1 & \varepsilon_1 & & & & \\ & & \ddots & & & & & \\ & & \varepsilon_1 & -\varepsilon_1-\varepsilon_2 & \varepsilon_2 & & & \\ & & & \ddots & & & & \\ & 0 & & \varepsilon_2 & -2\varepsilon_2 & \varepsilon_2 & & \\ & & & & \varepsilon_2 & -2\varepsilon_2 & \varepsilon_2 & \\ & & & \cdots & & 0 & 0 & 1 \end{pmatrix}$$

$$b(x) \Rightarrow \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \quad \phi(x) = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{pmatrix}$$

* Exact solution for $d = \frac{5}{24}L$ and $\phi(0) = 0$, $\phi(L) = 1$

$$\phi(x) = \begin{cases} \frac{C_n}{\varepsilon_1} x & \text{for } 0 \le x < d \\ \frac{C_n}{\varepsilon_2} x + \frac{\left(\frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2}\right)}{\left(\frac{1}{\varepsilon_1} + \frac{19}{5\varepsilon_2}\right)} & \text{for } d \le x < L \end{cases}$$

where $C_n = \frac{1}{\left(\frac{d}{\varepsilon_1} + \frac{(L-d)}{\varepsilon_2}\right)}$.

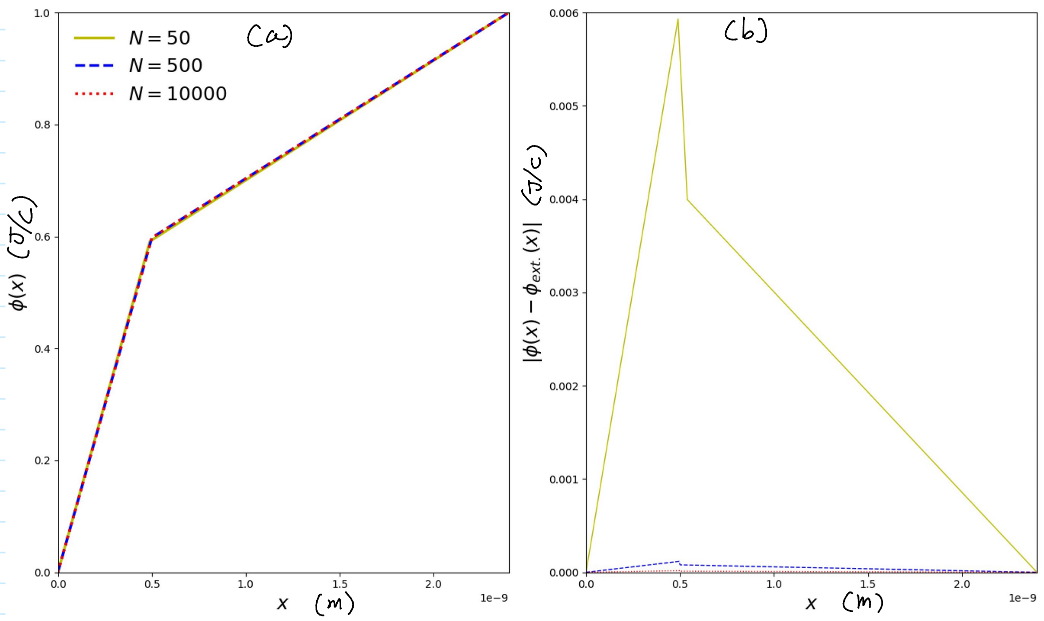
Capacitance/area $C/A = 1 / \left(\frac{d}{\varepsilon_1} + \frac{(L-d)}{\varepsilon_2}\right)$ that is the same with C_n.

Numerically, we can take C/A by slope of $\phi(x)$,

$$C/A = \varepsilon_1 \frac{\Delta\phi_1(x)}{\Delta x} = \varepsilon_2 \frac{\Delta\phi_2(x)}{\Delta x}$$ where $\phi_1(x) = \phi(0 \le x < d)$, $\phi_2(x) = \phi(d \le x < L)$

Results.

$L = 2.4\,nm$, $\varepsilon_1 = 3.9\varepsilon_0$, $\varepsilon_2 = 22.0\varepsilon_0$



Error $\max(|\phi(x) - \phi_{ext.}(x)|) \sim 10^{-3}$. That is, the numerical calculation approximates well to the exact solution.

Figure (b) shows that the error is reduced as N increases.

* Capacitance per Area

Exact : 0.0412650 F/m^2

Numerical solution with $N = 50$: 0.0408325 F/m^2

Numerical solution with $N = 500$: 0.0412734 F/m^2

Numerical solution with $N = 10000$: 0.0412638 F/m^2

The Capacitance / Area goes to the exact value as N increases.