HW9 Laplace eqn in the 2D space

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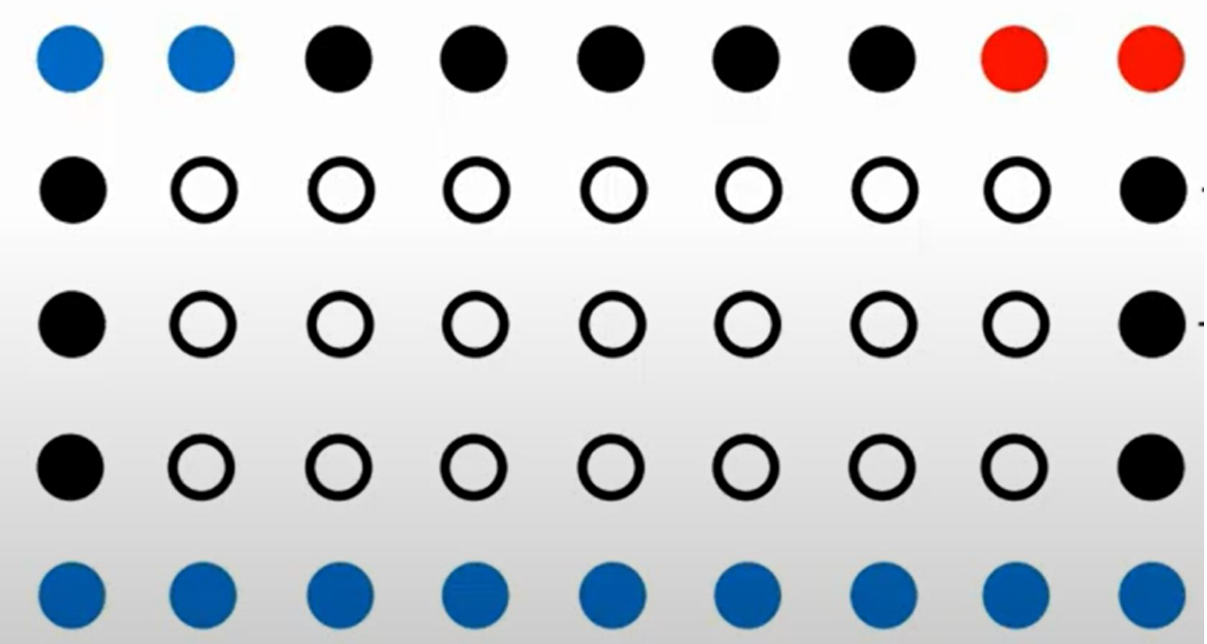
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi(x,y)=0$$

Dirichlet boundary condition : It use the value on the surface.

Neumann boundary condition : It use the value of normal derivative on the surface.

Example) $n_x = 9, n_y = 5$. **black : Neumann B.C.**, red : unity, blue : zero, empty : bulk nodes .



The integration of the Laplace eqn for a given node : $\int_V \nabla^2\phi dr = \oint_{surface} \nabla\phi\, da$

Discretization …

hollow points : $\phi_{x+1,y} + \phi_{x,y+1} - 4\phi_{x,y} + \phi_{x-1,y} + \phi_{x,y-1} = 0$

Black points : Neumann boundary condition.

ex) at the left boundary : $\phi_{1,y} - \phi_{0,y} + \frac{\phi_{0,y+1}}{2} - \frac{\phi_{0,y}}{2} + \frac{\phi_{0,y-1}}{2} - \frac{\phi_{0,y}}{2}$

$$\rightarrow \phi_{1,y} + 0.5\phi_{0,y+1} - 2\phi_{0,y} + 0.5\phi_{1,y-1} = 0$$

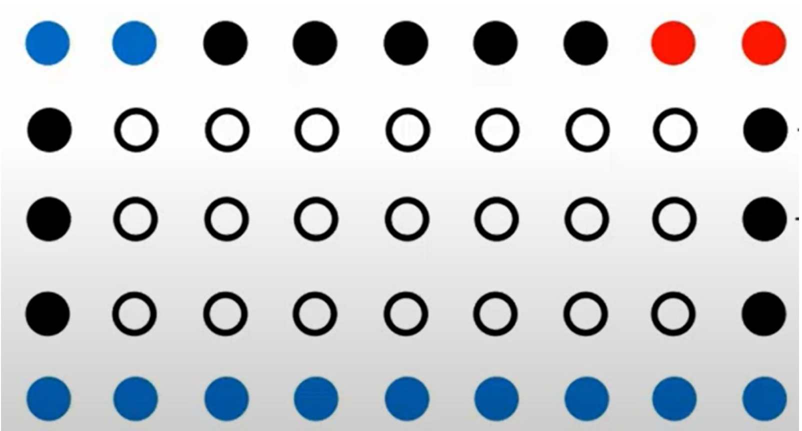
Blue point : $\phi_{x,y} = 0$

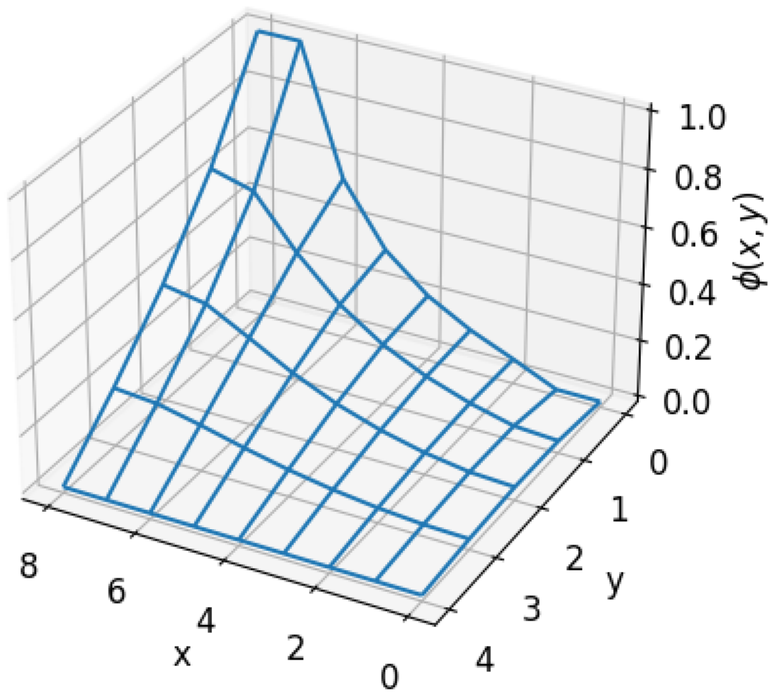
Red point : $\phi_{x,y} = 1$

$$H * \phi_{x,y} = \phi_{boundary}$$

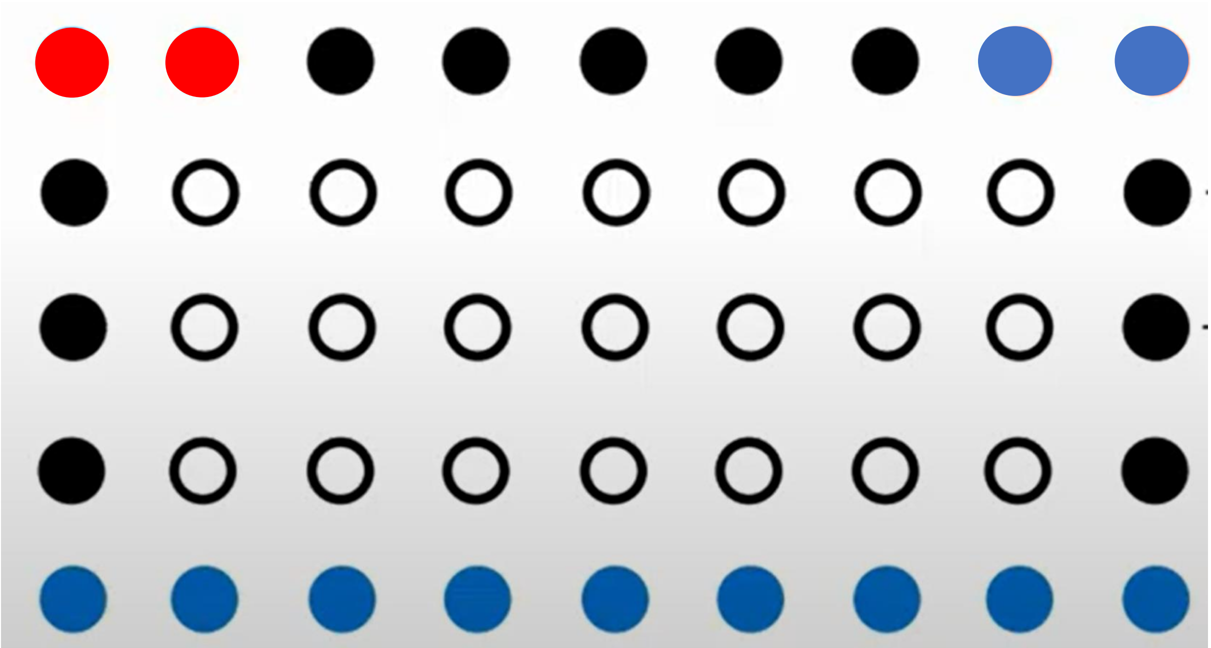
Result

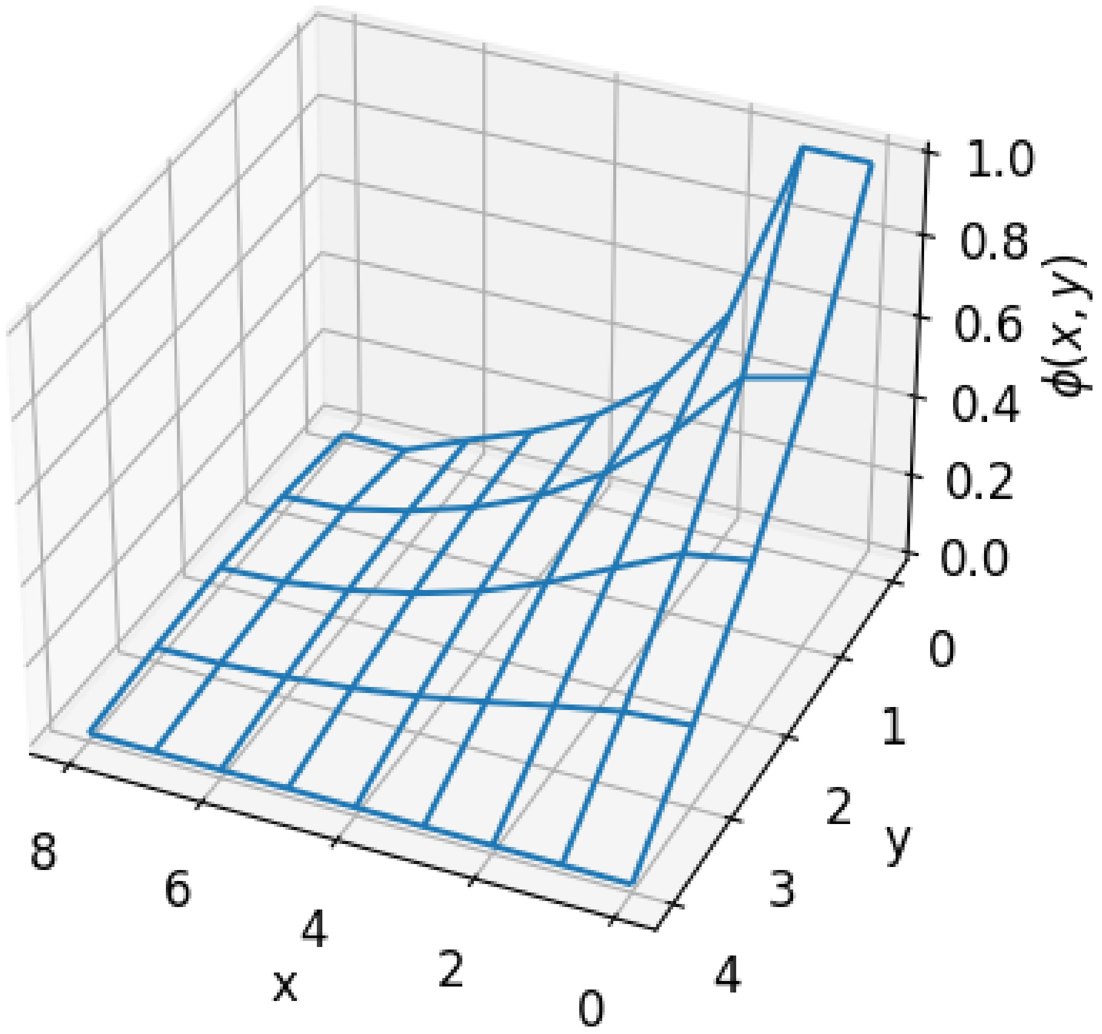
1. Top right



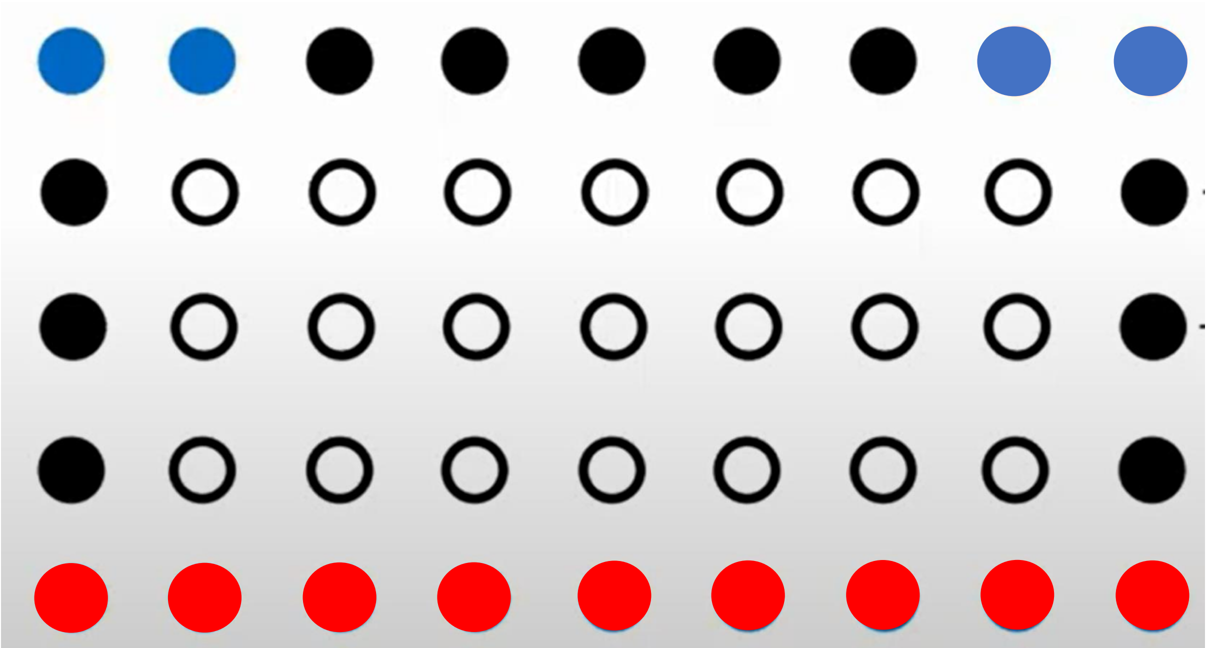


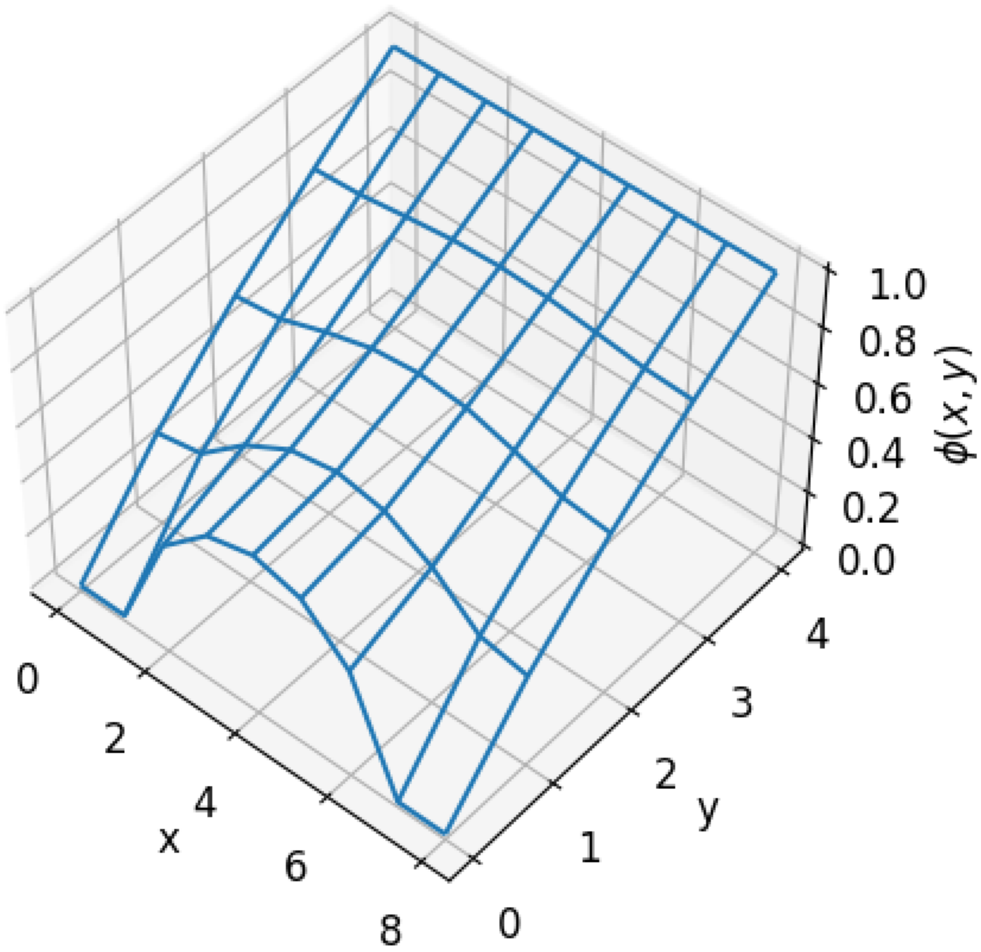
2.Top left





2. Bottom





4.All