Nonlinear equation solving

Assignment #6

20202010 Hyunsuk Shin

Analytical solution

$$N^+ + n_{int}e^{-\frac{\phi}{V_T}} - n_{int}e^{\frac{\phi}{V_T}} = 0$$

$$\frac{N^+}{n_{int}} = 2\,sinh\left(\frac{\phi}{V_T}\right)$$

$$\therefore \phi = V_T\,sinh^{-1}\left(\frac{N^+}{n_{int}}\right)$$

Numerical methods (Newton-Raphson method)

1. Assume that F(ϕ) = $N^+ + n_{int}e^{-\frac{\phi}{V_T}} - n_{int}e^{\frac{\phi}{V_T}}$

2. Set trial solution ϕ_0 .

$$N^+ + n_{int}e^{-\frac{\phi_0}{V_T}} - n_{int}e^{\frac{\phi_0}{V_T}} = -r$$

3. Set Jacobian matrix

$$\mathrm{J} = \partial_{\phi}(N^+ + n_{int}e^{-\frac{\phi}{V_T}} - n_{int}e^{\frac{\phi}{V_T}}) = (1/V_T)\cdot(-n_{int}e^{-\frac{\phi}{V_T}} + n_{int}e^{\frac{\phi}{V_T}})$$

$$\text{Then, } \mathrm{J}\delta\phi = -\mathrm{r}$$

4. Construct ϕ_1 and repeat the steps.

$$\phi_1 = \phi_0 + \delta\phi$$

Following calculation, we set $\phi_0 = 1$.

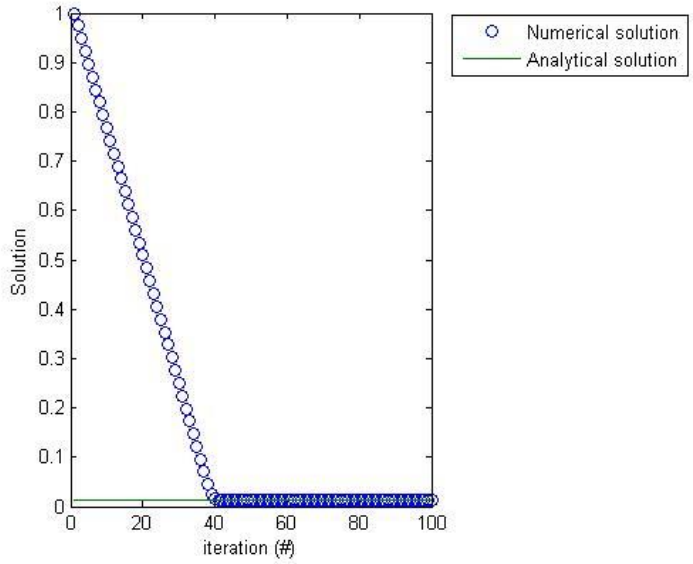

Figure 1. $N^+ = 10^{10} cm^{-3}$ case. Blue-dotted line correspond to Numerical solution which is approached to analytical solution with respect to number of repeats.

By changing the N^+ value, the results are shown in figure 2.

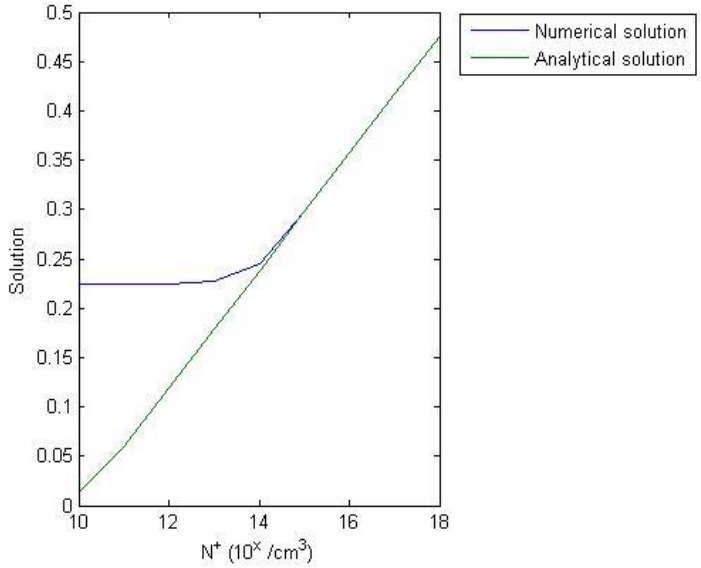

Figure 2. 30 $times\ iteration$ case. Blue-dotted line correspond to Numerical solution which is approached to analytical solution with respect to increasing N+.

When N+ is increased the results is closed to their analytical solution. This is because our first guess solution is closer to 1 when N+ is large. However if we increasing the number of iteration, all the results are approached to analytical solution. (figure 3)

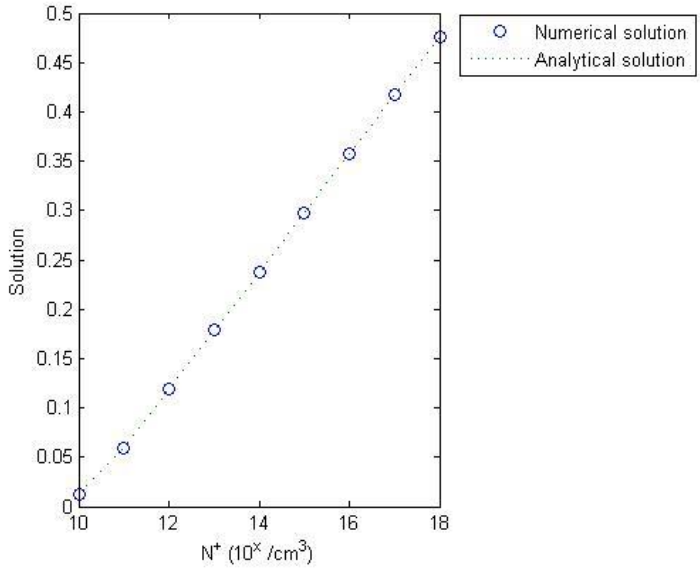

Figure 3. 50 *times iteration* case. Blue-circles correspond to Numerical solution which is approached to analytical solution with respect to increasing N+.

How about the changing the N+ has a negative value? Figure 4 shows same condition but N+ has negative value.

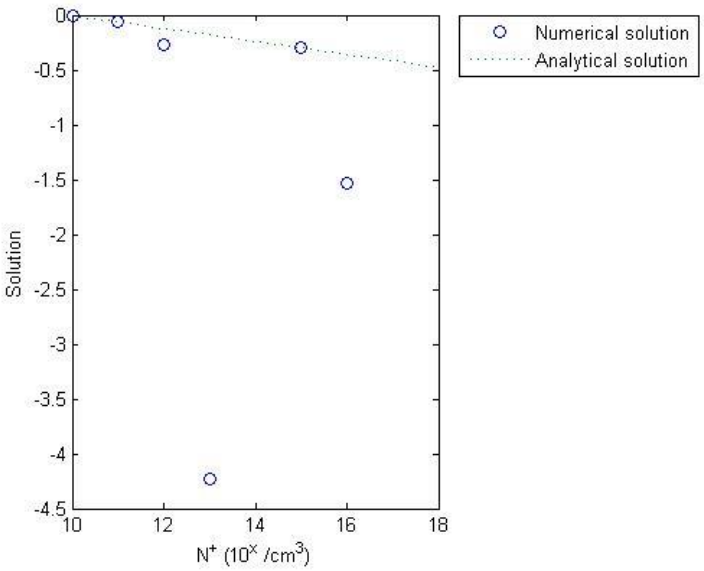

Figure 4. 50 *times iteration* case. Blue-circles correspond to Numerical solution which is approached to analytical solution with respect to increasing N+.

It has larger difference between the analytical solution and numerical solution for some cases, so for getting precise value, we have 2 choices. First, adjust the ϕ_0 from 1 to -1. Then we get the figure 5.

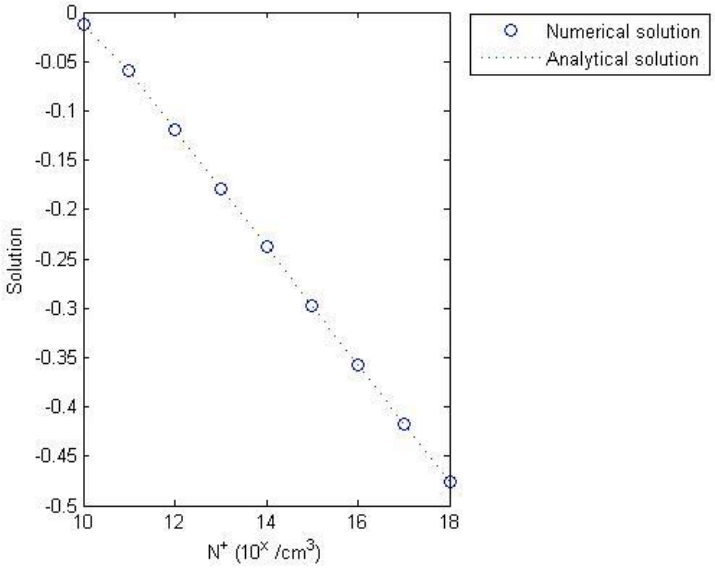

Figure 5. 50 *times iteration* case. N+ has negative sign in this graph. With ϕ_0 = -1 Blue-circles correspond to Numerical solution which is approached to analytical solution with respect to increasing N+.

Or we can just do more iteration.

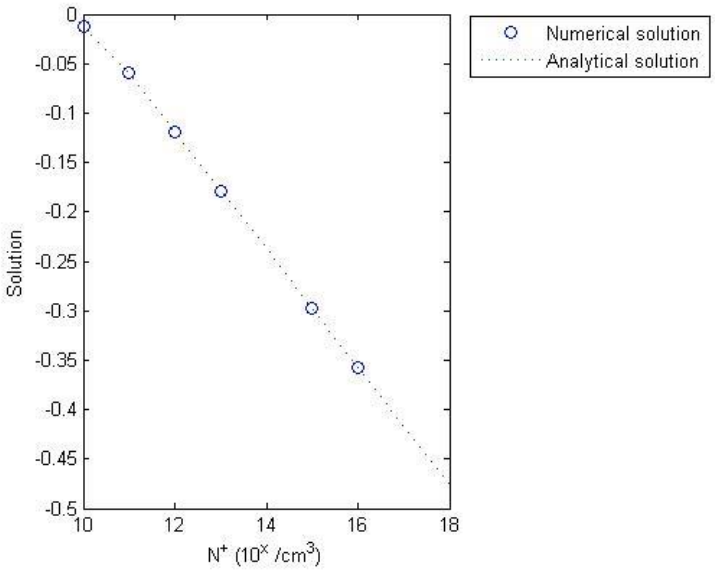

Figure 6. 500 *times iteration* case. N+ has negative sign in this graph. With ϕ_0 = 1 Blue-circles correspond to Numerical solution which is approached to analytical solution with respect to increasing N+.