#### Lecture 22

Sung-Min Hong (<a href="mailto:smhong@gist.ac.kr">smhong@gist.ac.kr</a>)

Semiconductor Device Simulation Lab.
School of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology

### Finite volume method

- FEM is <u>NOT</u> used.
- The box method is used.
  - Differential form

$$\nabla \cdot \mathbf{F} = s$$

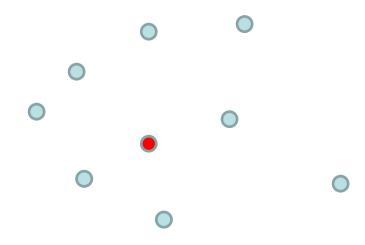
Integrated form

$$\oint_{\partial \Omega} \mathbf{F} \cdot d\mathbf{a} = \int_{\Omega} s d^3 x$$

- $\Omega$  is the Voronoi cell.
- Delaunay mesh is needed!

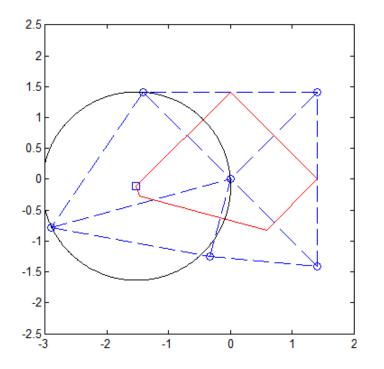
#### Voronoi volume

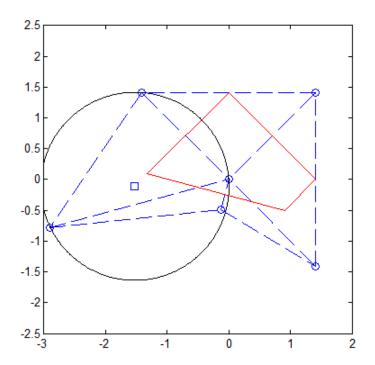
- Within the Voronoi volume of a center node, the closest node is the center node.
  - Calculating the plane (3D) or the line (2D), which is perpendicular to an edge and cross its mid-point



## Pentagon examples

Delaunay (Left) versus non-Delaunay (Right)



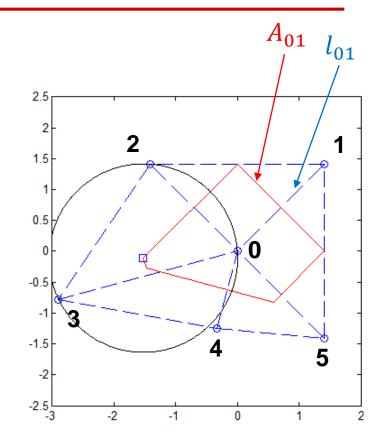


## Poisson equation

- LHS,  $\oint_{\partial\Omega} \mathbf{F} \cdot d\mathbf{a}$ 
  - It can be written as

$$\sum_{i=1}^{5} F_{0i} A_{0i} = -\sum_{i=1}^{5} \epsilon \frac{\phi_i - \phi_0}{l_{0i}} A_{0i}$$

- We must calculate  $l_{0i}$  and  $A_{0i}$ .



### Recipe

- For a 2D structure, it is not very difficult.
  - For each triangle whose side lengths are a, b, and c, calculate the circumradius, R.

$$Area = \frac{abc}{4R}$$

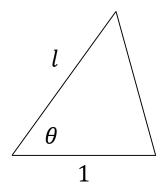
From the circumcenter, by using the Pythagorean theorem,

$$R^2 = \left(A_{01,triangle}\right)^2 + \left(\frac{l_{01}}{2}\right)^2$$

How about an obtuse triangle?

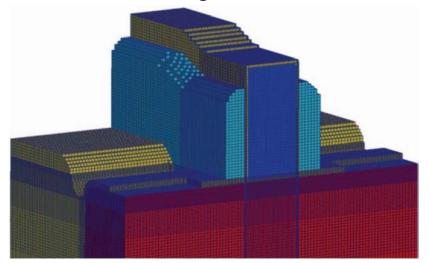
# Toy example

- Consider a triangle.
  - For an angle,  $\theta$ , the side lengths are 1 and l.
  - Calculate the coordinate of its circumcenter.

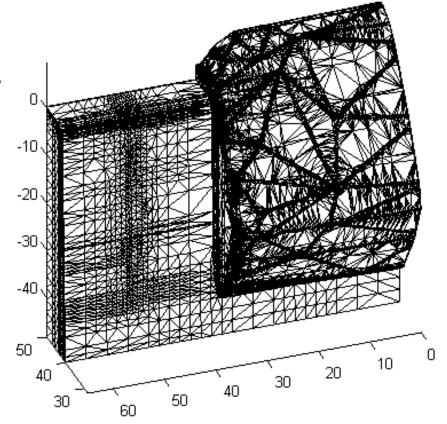


#### 3D mesh?

- Tetrahedron-based mesh
  - Difficult to calculate Voronoi cells
  - Structured grid cannot be used.



A structured grid (L. Wang et al., ULIS, 2014)



### Electric field

- Consider a triangle.
  - We know the electrostatic potential.
  - How can we calculate the electric field?

