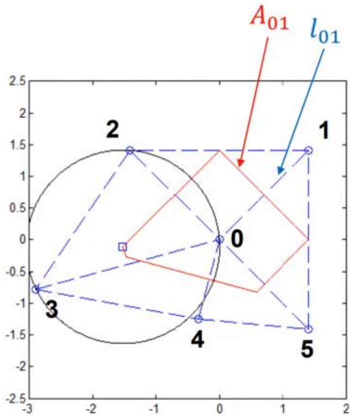
HW18

20202041 Park, Nuri

Finite Volume Method (box method)

Differential form : $\nabla \cdot F = s$ $\quad \rightarrow$ Integrated form $\oint_{\partial\Omega} F \cdot da = \int_{\Omega} s d^3x$

Ω is the Voronoi cell



$$\sum_{i}^{5} F_{0i} A_{0i} = \sum_{i}^{5} \epsilon \frac{\phi_i - \phi_0}{l_{0i}} A_{0i}$$

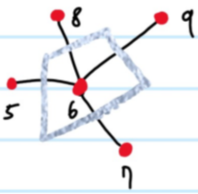
$A_{01}^2 + \left(\frac{l_{01}}{2}\right)^2 = R^2$, R is the radius of the circumcenter of the triangle.

Since we want to solve Laplace equation, we have equation

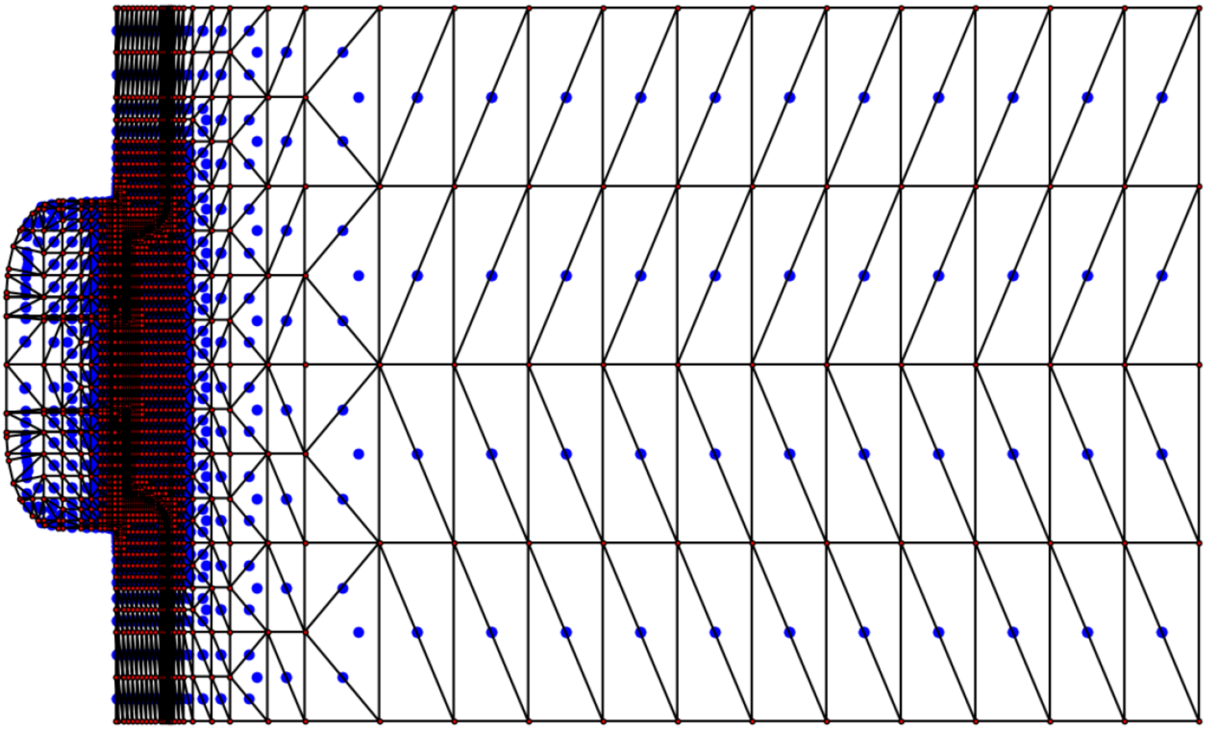
$$\sum_i C_{0i}\phi_i - \phi_0 \sum_i C_{0i} = 0, \qquad \left(\frac{A_{0i}}{l_{0i}} = C_{0i}\right)$$

Thus, the matrix we would make is,

$$\begin{pmatrix} (-C_{01} - C_{02} \cdots - C_{05}) & C_{01} & C_{02} & \cdots & C_{05} & 0 & 0 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{pmatrix} \begin{pmatrix} \phi_0 \\ \phi_1 \\ \phi_5 \\ \vdots \\ \phi_N \end{pmatrix}$$



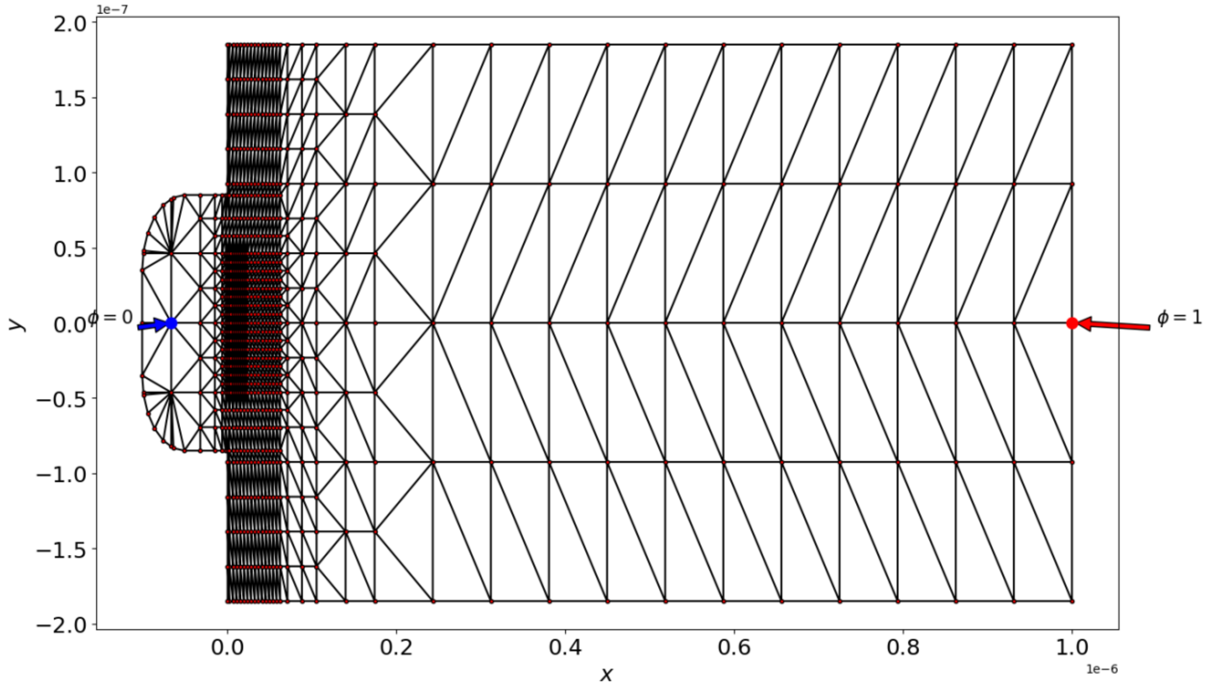
$$\begin{pmatrix} & & & & & & & & & \\ \cdots 0 & 0 & C_{65} & (-C_{65} - C_{67} \cdots - C_{69}) & C_{67} & C_{68} & C_{69} & 0 & 0 & \cdots \\ & & & & & & & & & \end{pmatrix} \begin{pmatrix} \vdots \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \vdots \end{pmatrix}$$



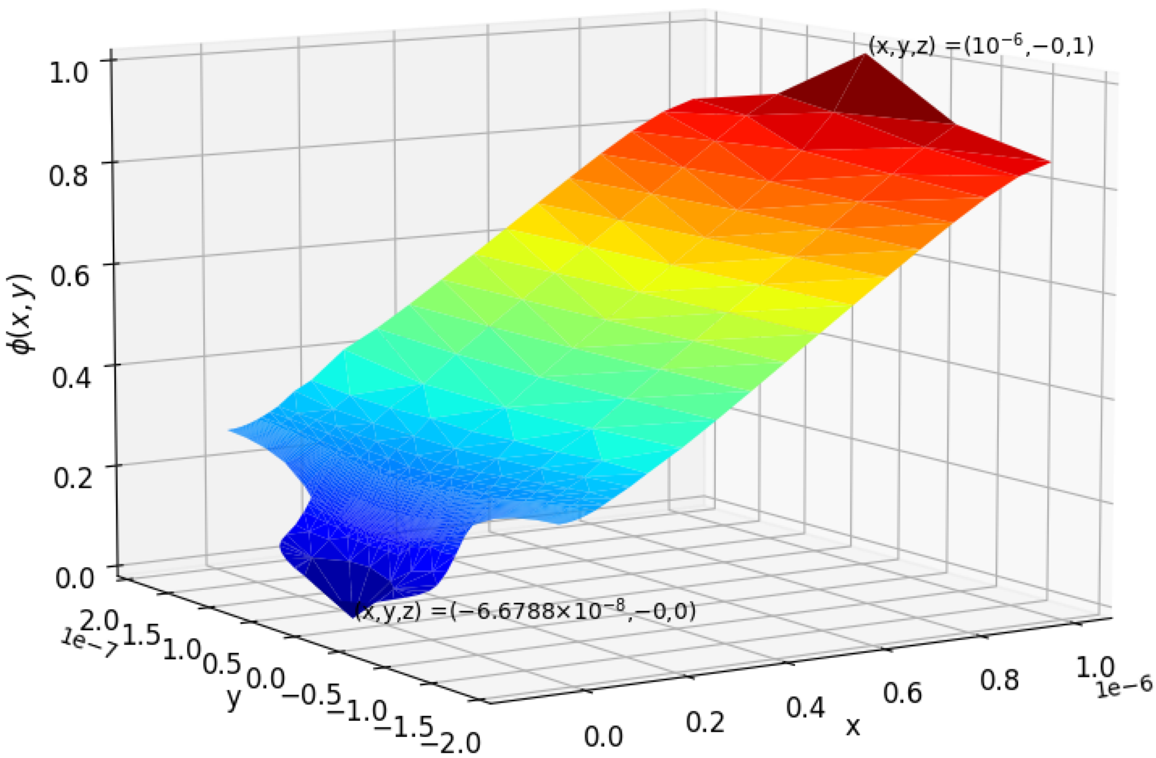
Upper figure shows triangular mesh. The blue circle is the circumcenter for each triangle. For each triangle, we would calculate R, A, l to make proper matrix which solve laplace matrix.

We give $\phi = 0$ on one point and $\phi = 1$ on another point. Then, each point of the triangle would have correspoding ϕ values. We now show the result of them.

Result 1.

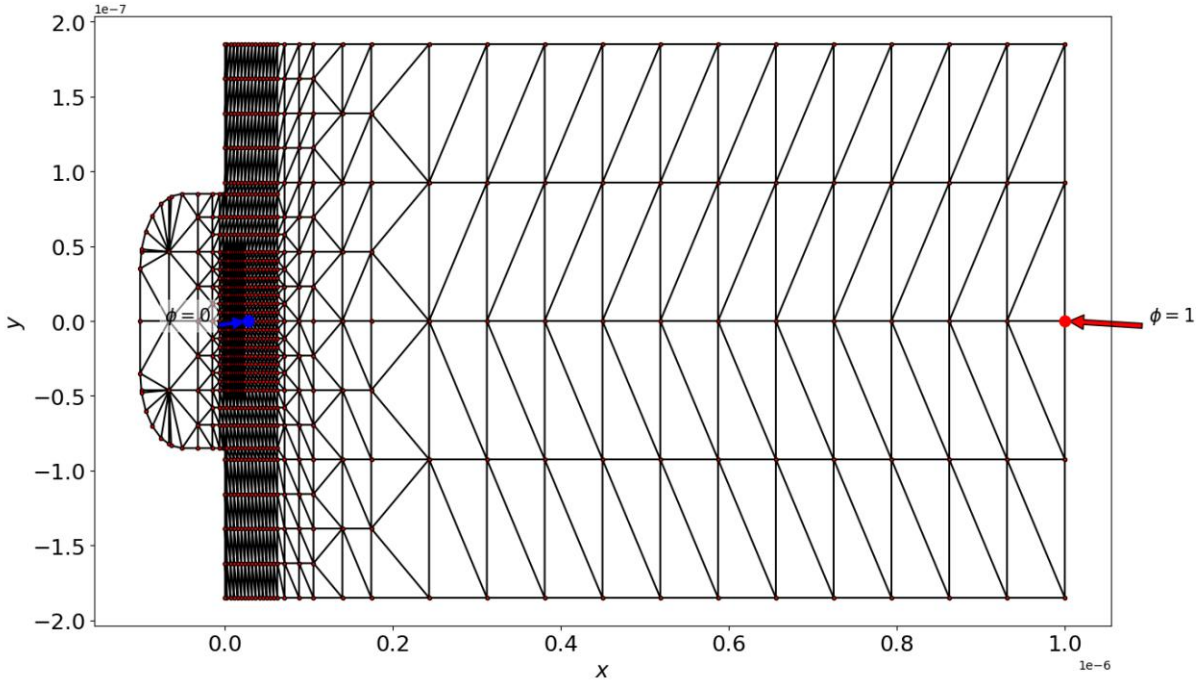


1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
$\phi = 0$
$\phi = 1$
0.0
0.2
0.4
0.6
0.8
1.0
1e−6
x

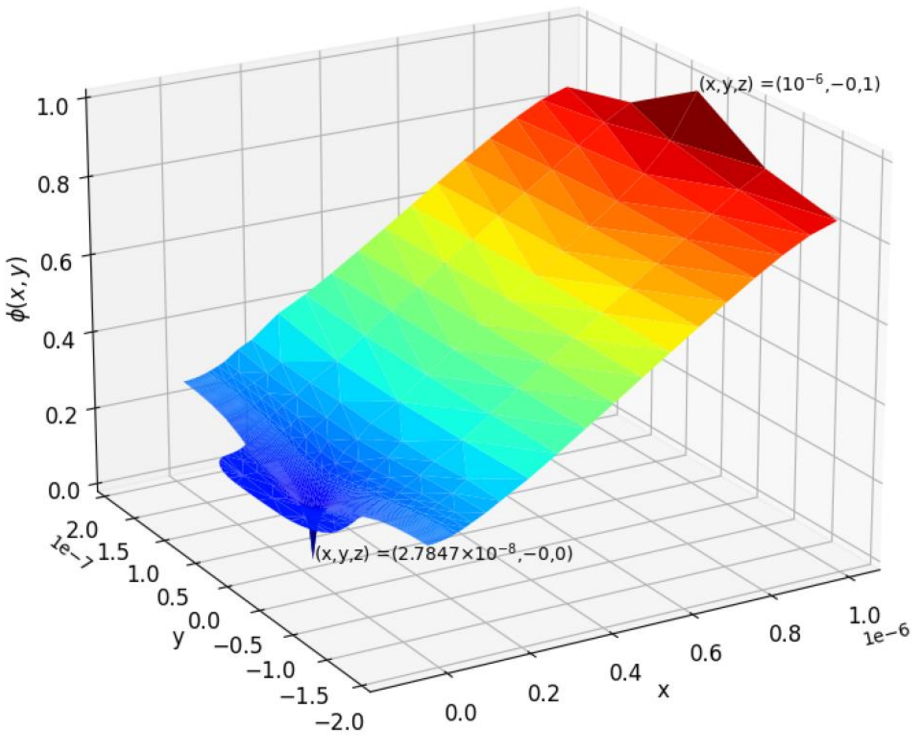


(x,y,z) =(10^{-6},−0,1)
(x,y,z) =(-6.6788×10^{-8},−0,0)
$\phi(x, y)$
1.0
0.8
0.6
0.4
0.2
0.0
1e−7
y
2.0 1.5 1.0 0.5 0.0 −0.5 −1.0 −1.5 −2.0
0.0
0.2
0.4
0.6
0.8
1.0
1e−6
x

Result 2.

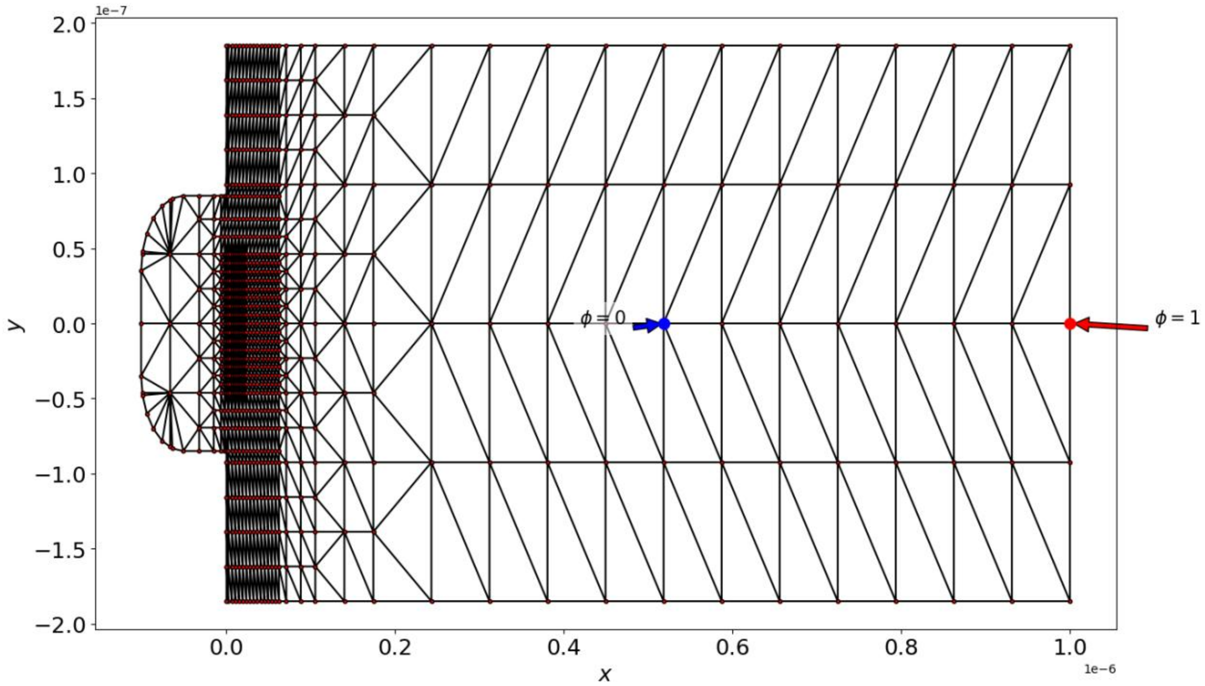


1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
$\phi = 0$
$\phi = 1$
0.0
0.2
0.4
0.6
0.8
1.0
1e−6
x

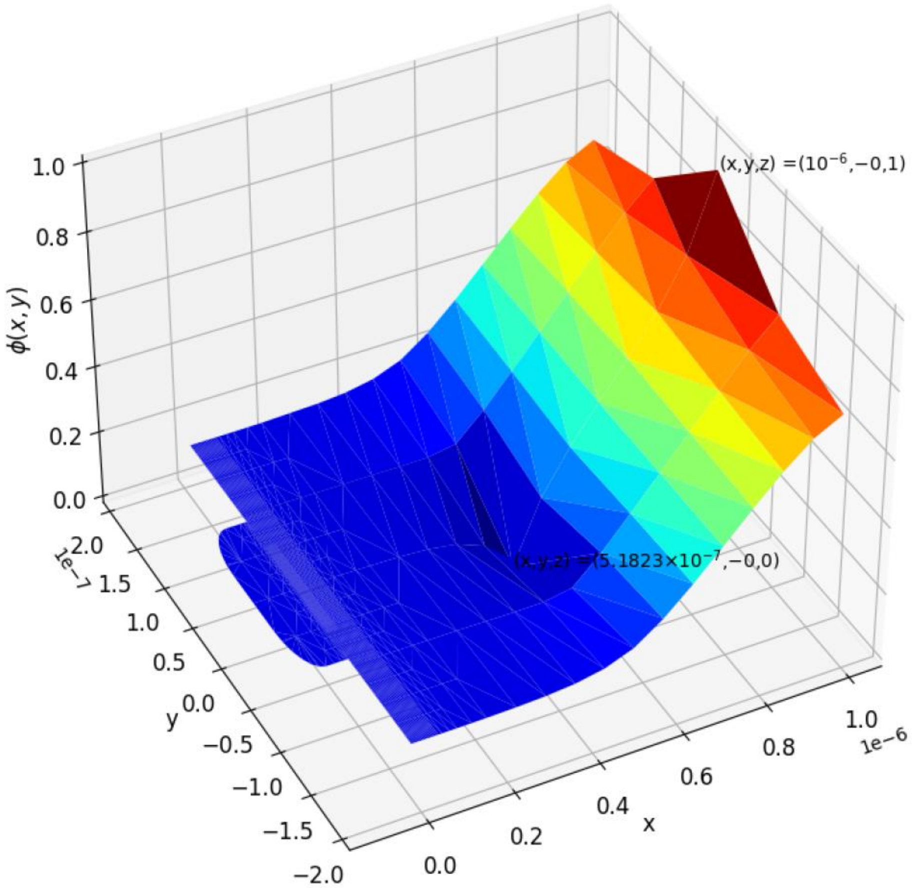


(x,y,z) =(10^{−6},−0,1)
(x,y,z) =(2.7847×10^{−8},−0,0)
$\phi(x, y)$
1.0
0.8
0.6
0.4
0.2
0.0
1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
0.0
0.2
0.4
0.6
0.8
1.0
1e−6
x

Result 3.

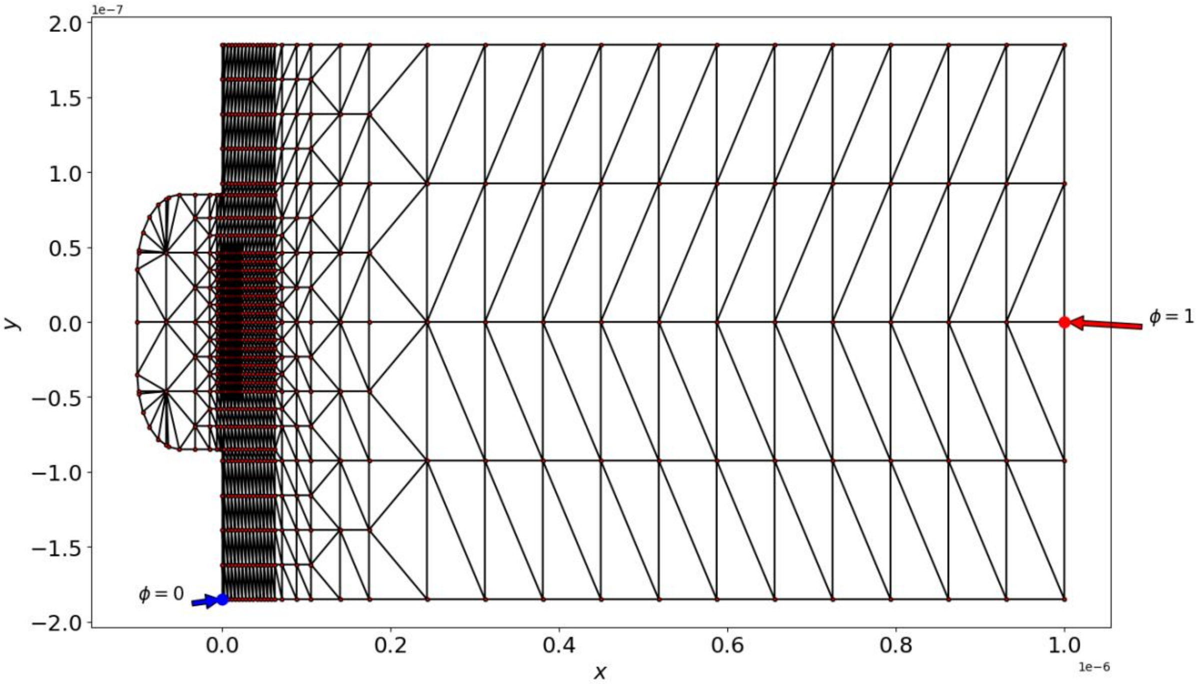


1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
$\phi = 0$
$\phi = 1$
0.0
0.2
0.4
0.6
0.8
1.0
x
1e−6

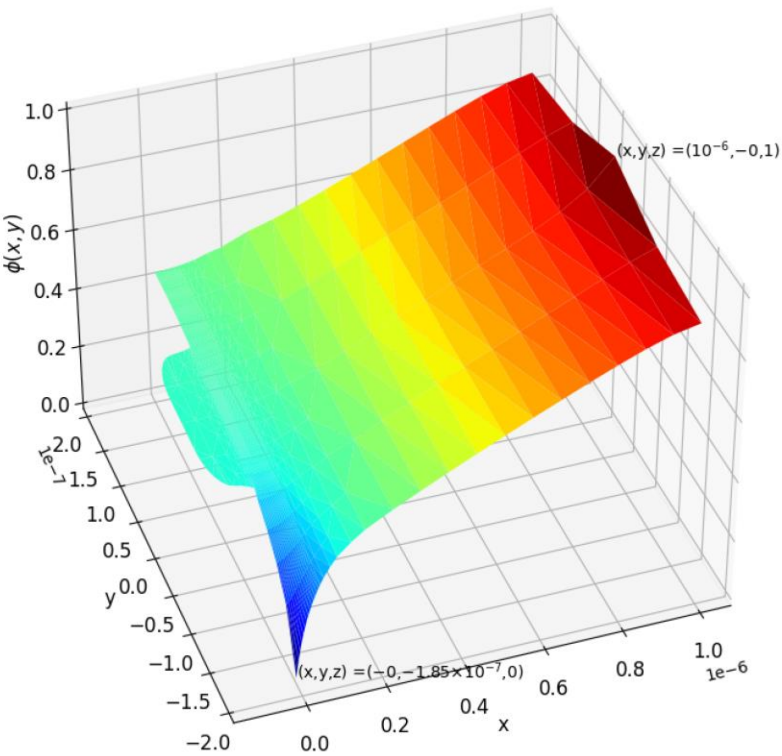


(x,y,z) =(10^{-6},−0,1)
(x,y,z) =(5.1823×10^{-7},−0,0)
$\phi(x, y)$
1.0
0.8
0.6
0.4
0.2
0.0
1e−7
y
x
1e−6

Result 4.



1e−7
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
$\phi = 1$
$\phi = 0$
0.0
0.2
0.4
0.6
0.8
1.0
1e−6
x



$\phi(x, y)$
1.0
0.8
0.6
0.4
0.2
0.0
(x,y,z) =(10^{-6},−0,1)
(x,y,z) =(−0,−1.85×10^{-7},0)
1e−7
2.0
1.5
1.0
0.5
y 0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.2
0.4
0.6
0.8
1.0
1e−6
x