HW 17.

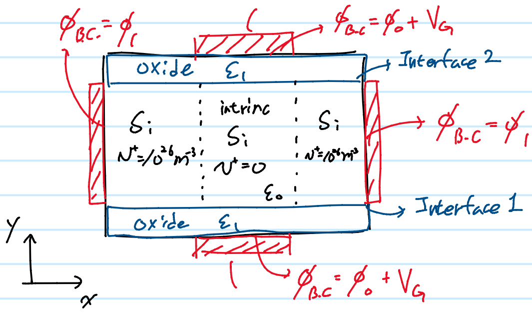
Thursday, November 19, 2020 7:11 PM

20184060 Jicheol Kim

* Scharfetter-Gummel eq. in 1D

$$J_{n,i+\frac{1}{2}} = \frac{q D_n}{\Delta x}\left[n_{i+1} B\left(\frac{\phi_{i+1}-\phi_i}{V_T}\right) - n_i B\left(\frac{\phi_i - \phi_{i+1}}{V_T}\right)\right] = \frac{\delta J_n(i+1,i)}{\delta x}$$

$$B(x) = \frac{x}{e^x - 1}, \quad \frac{dB(x)}{dx} = \frac{e^x - 1 - x e^x}{(e^x-1)^2}$$

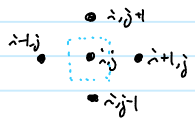


Hw 10. 에서는 $N_{acc} = 10^{26} m^{-3}$로 두었지만, Hw 17. 에선 $N^+ = -N_{acc} = 10^{26} m^{-3}$으로 하였다.

: Interface 에서 density는 0이다.

In 2D, $\vec{\nabla}\cdot\vec{J}_n = 0$,

i) Bulk



$$\int_v \nabla\cdot\vec{J}_n \, dz = \int_s \vec{J}_n \cdot d\vec{a} = 0 \qquad \text{y-dir Current는 0}$$

$$\Rightarrow \Delta\left(\cancelto{0}{J_n(i,j+\tfrac{1}{2})} + J_n(i+\tfrac{1}{2},j) - J_n(i-\tfrac{1}{2},j) - \cancelto{0}{J_n(i,j-\tfrac{1}{2})} \right) = 0 \quad (\Delta\text{는 grid spacing})$$

$$\Rightarrow \delta J_n(i,j+1;i,j) + \delta J_n(i+1,j;i,j) - \delta J_n(i,j;i-1,j) - \delta J_n(i,j;i,j-1) = 0$$

$$\therefore [R_n]_{i,j} = \begin{cases} \cancel{\delta J_n(i,j+1;i,j)} + \delta J_n(i+1,j;i,j) - \delta J_n(i,j;i-1,j) - \cancel{\delta J_n(i,j;i,j-1)} & \text{in silicon} \\ n_{i,j} & \text{in oxide} \end{cases}$$

For Poisson eq.,

$$[R_\phi]_{i,j} = \varepsilon(-4\phi_{i,j} + \phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}) - \Delta^2 q(-N^+ + n_{i,j})$$

$$\therefore \begin{cases} \partial_{\phi_{i,j}}[R_\phi]_{i,j} = -4\varepsilon, \ \partial_{\phi_{i+1,j}}[R_\phi]_{i,j} = \partial_{\phi_{i-1,j}}[R_\phi]_{i,j} = \partial_{\phi_{i,j+1}}[R_\phi]_{i,j} = \partial_{\phi_{i,j-1}}[R_\phi]_{i,j} = \varepsilon \\ \partial_{n_i}[R_\phi]_{i,j} = -\Delta^2 q \end{cases}$$

In silicon

$$\begin{cases} \partial_{\phi_{i,j}}[R_n]_{i,j} = qD_n\Big[\cancel{n_{i,j+1}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i,j+1}-\phi_{i,j}}{V_T}\right) - n_{i,j}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i,j}-\phi_{i,j+1}}{V_T}\right)} + n_{i+1,j}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i+1,j}-\phi_{i,j}}{V_T}\right) - n_{i,j}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i,j}-\phi_{i+1,j}}{V_T}\right) \\ \qquad - n_{i,j}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i,j}-\phi_{i-1,j}}{V_T}\right) + n_{i-1,j}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i-1,j}-\phi_{i,j}}{V_T}\right) - \cancel{n_{i,j}\partial_{\phi_{i,j}} B\left(\frac{\phi_{i,j}-\phi_{i,j-1}}{V_T}\right) + n_{i,j-1}\partial_{\phi_{i,j-1}} B\left(\frac{\phi_{i,j-1}-\phi_{i,j}}{V_T}\right)} \Big] \\ \partial_{\phi_{i,j+1}}[R_n]_{i,j} = \partial_{\phi_{i,j+1}}\left(\delta J_n(\cancel{i,j+1};i,j)\right), \ \partial_{\phi_{i+1,j}}[R_n]_{i,j} = \partial_{\phi_{i+1,j}}\left(\delta J_n(i+1,j;i,j)\right) \\ \partial_{\phi_{i-1,j}}[R_n]_{i,j} = -\partial_{\phi_{i-1,j}}\left(\delta J_n(i,j;i-1,j)\right), \ \partial_{\phi_{i,j-1}}[R_n]_{\cancel{i,j} = -}\partial_{\phi_{i,j-1}}\left(\delta J_n(i,j;i,j-1)\right) \end{cases}$$

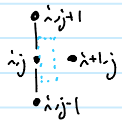
In silicon

$$\begin{cases} \partial_{n_{i,j}}[R_n]_{i,j} = qD_n\left[-\cancel{B\left(\frac{\phi_{i,j}-\phi_{i,j+1}}{V_T}\right)} - B\left(\frac{\phi_{i,j}-\phi_{i+1,j}}{V_T}\right) + B\left(\frac{\phi_{i-1,j}-\phi_{i,j}}{V_T}\right) + \cancel{B\left(\frac{\phi_{i,j-1}-\phi_{i,j}}{V_T}\right)} \right] \\ \partial_{n_{i,j+1}}[R_n]_{i,j} = qD_n\left[\cancel{B\left(\frac{\phi_{i,j+1}-\phi_{i,j}}{V_T}\right)}\right], \ \partial_{n_{i+1,j}}[R_n]_{i,j} = qD_n\left[B\left(\frac{\phi_{i+1,j}-\phi_{i,j}}{V_T}\right)\right] \\ \partial_{n_{i-1,j}}[R_n]_{i,j} = qD_n\left[B\left(\frac{\phi_{i-1,j}-\phi_{i,j}}{V_T}\right)\right], \ \partial_{n_{i,j-1}}[R_n]_{i,j} = qD_n\left[\cancel{B\left(\frac{\phi_{i,j-1}-\phi_{i,j}}{V_T}\right)}\right] \end{cases}$$

In Oxide $\partial_{n_{i,j}}[R_n]_{i,j} = 1$.

ii) Edge

Homogeneous Neumann B.C.,

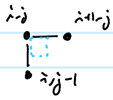


$$[R_n]_{i,j} = n_{i,j} \quad \text{or} \quad n_{i,j} - N^+ \ (\text{silicon layer 에서만})$$

$$[R_\phi]_{i,j} = \varepsilon\left(-2\phi_{i,j} + \phi_{i+1,j} + \tfrac{1}{2}\phi_{i,j+1} + \tfrac{1}{2}\phi_{i,j-1}\right)$$

$\therefore$ $\partial_{\phi_{i,j}}[R_\phi]_{i,j} = -2\varepsilon$, $\partial_{\phi_{i+1,j}}[R_\phi]_{i,j} = \varepsilon$, $\partial_{\phi_{i,j+1}}[R_\phi]_{i,j} = \partial_{\phi_{i,j-1}}[R_\phi]_{i,j} = \frac{1}{2}\varepsilon$

$\partial_{n_{i,j}}[R_n]_{i,j} = 1.$



$$[R_n]_{i,j} = n_{i,j}$$

$$[R_\phi]_{i,j} = \varepsilon\left(-\phi_{i,j} + \tfrac{1}{2}\phi_{i+1,j} + \tfrac{1}{2}\phi_{i,j-1}\right)$$

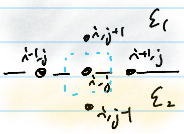
$\therefore$ $\partial_{\phi_{i,j}}[R_\phi]_{i,j} = -\varepsilon$, $\partial_{\phi_{i+1,j}}[R_\phi]_{i,j} = \partial_{\phi_{i,j-1}}[R_\phi]_{i,j} = \frac{\varepsilon}{2}$

$\partial_{n_{i,j}}[R_n]_{i,j} = 1.$

Dirichlet B.C.

$[R_\phi]_{i,j} = \phi_{i,j} - \phi_{B.C.}$, $[R_n]_{i,j} = n_{i,j} - N^+$

iii) Interface



$$[R_n]_{i,j} = n_{i,j}$$

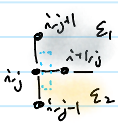
$$[R_\phi]_{i,j} = -2(\varepsilon_1+\varepsilon_2)\phi_{i,j} + \frac{\varepsilon_1+\varepsilon_2}{2}\phi_{i+1,j} + \frac{\varepsilon_1+\varepsilon_2}{2}\phi_{i-1,j} + \varepsilon_1\phi_{i,j+1} + \varepsilon_2\phi_{i,j-1} - \Delta^2 \xi\,(-N^+ + n_{i,j})$$

$\therefore$ $\partial_\phi[R_n]_{i,j}$ 와 $\partial_n[R_n]_{i,j}$ 는 i)과 동일.

$\partial_{\phi_{i,j}}[R_\phi]_{i,j} = -2(\varepsilon_1+\varepsilon_2)$, $\partial_{\phi_{i+1,j}}[R_\phi]_{i,j} = \partial_{\phi_{i-1,j}}[R_\phi]_{i,j} = \frac{\varepsilon_1+\varepsilon_2}{2}$

$\partial_{\phi_{i,j+1}}[R_\phi]_{i,j} = \varepsilon_1$, $\partial_{\phi_{i,j-1}}[R_\phi]_{i,j} = \varepsilon_2$

$\partial_{n_{i,j}}[R_\phi]_{i,j} = -\Delta^2 \xi$



$$[R_n]_{i,j} = n_{i,j}$$

$$[R_\phi]_{i,j} = -(\varepsilon_1+\varepsilon_2)\phi_{i,j} + \frac{(\varepsilon_1+\varepsilon_2)}{2}\phi_{i+1,j} + \frac{\varepsilon_1}{2}\phi_{i,j+1} + \frac{\varepsilon_2}{2}\phi_{i,j-1}$$

$\partial_\phi[R_n]_{i,j}$ 와 $\partial_n[R_n]_{i,j}$ 는 ii)과 동일.

$\partial_{\phi_{i,j}}[R_\phi]_{i,j} = -(\varepsilon_1+\varepsilon_2)$, $\partial_{\phi_{i+1,j}}[R_\phi]_{i,j} = \frac{\varepsilon_1+\varepsilon_2}{2}$

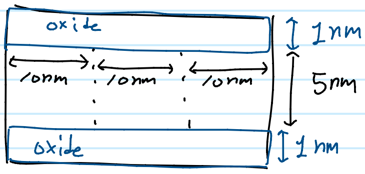
$\partial_{\phi_{i,j+1}}[R_\phi]_{i,j} = \frac{\varepsilon_1}{2}$, $\partial_{\phi_{i,j-1}}[R_\phi]_{i,j} = \frac{\varepsilon_2}{2}$

* Algorithm은 Hw 15.와 동일

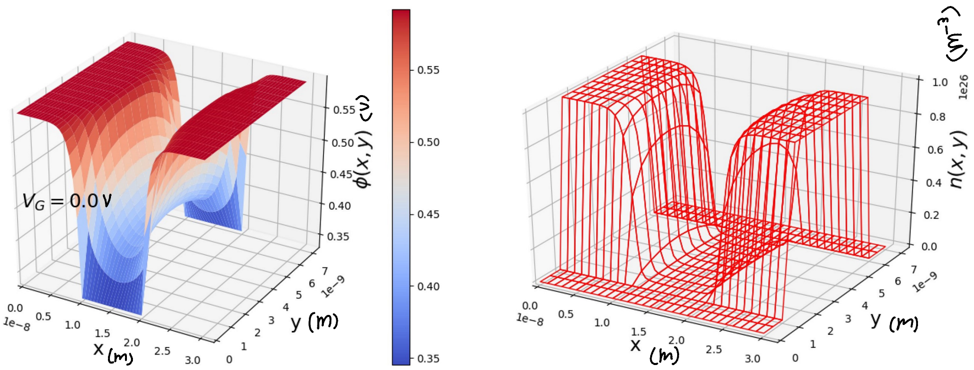
Resulcs.

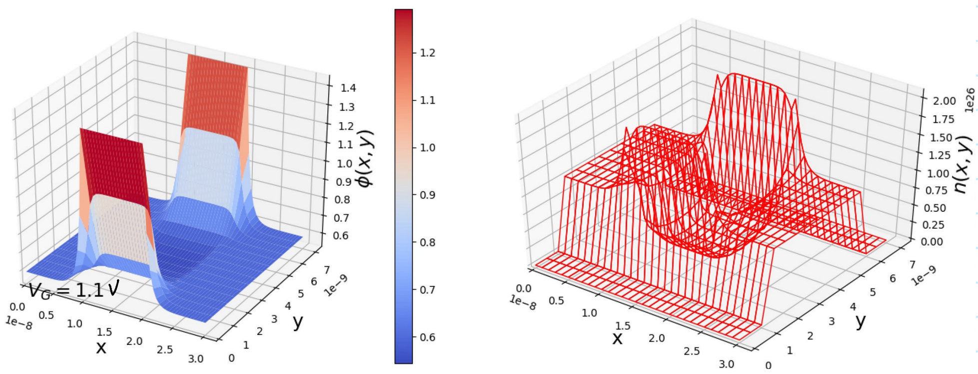
Results.

$\Delta = 0.5\,nm,\ L_x = 30\,nm,\ L_y = 7\,nm$

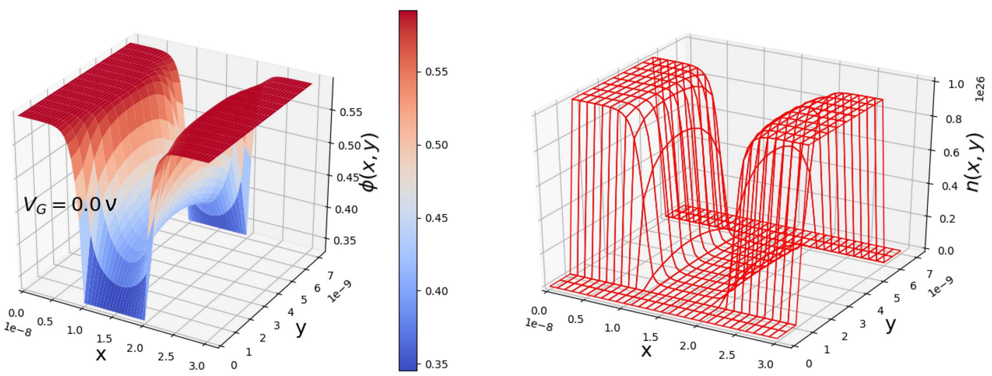


1) $\phi(x,y)$ & $n(x,y)$ with Hw10. method.

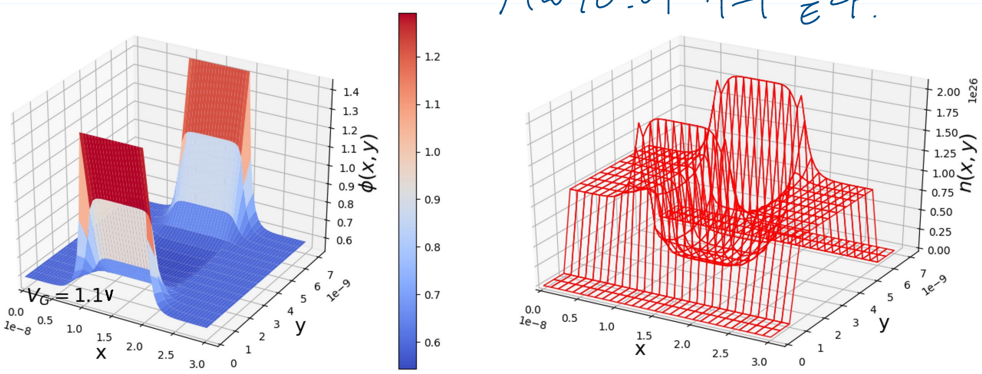




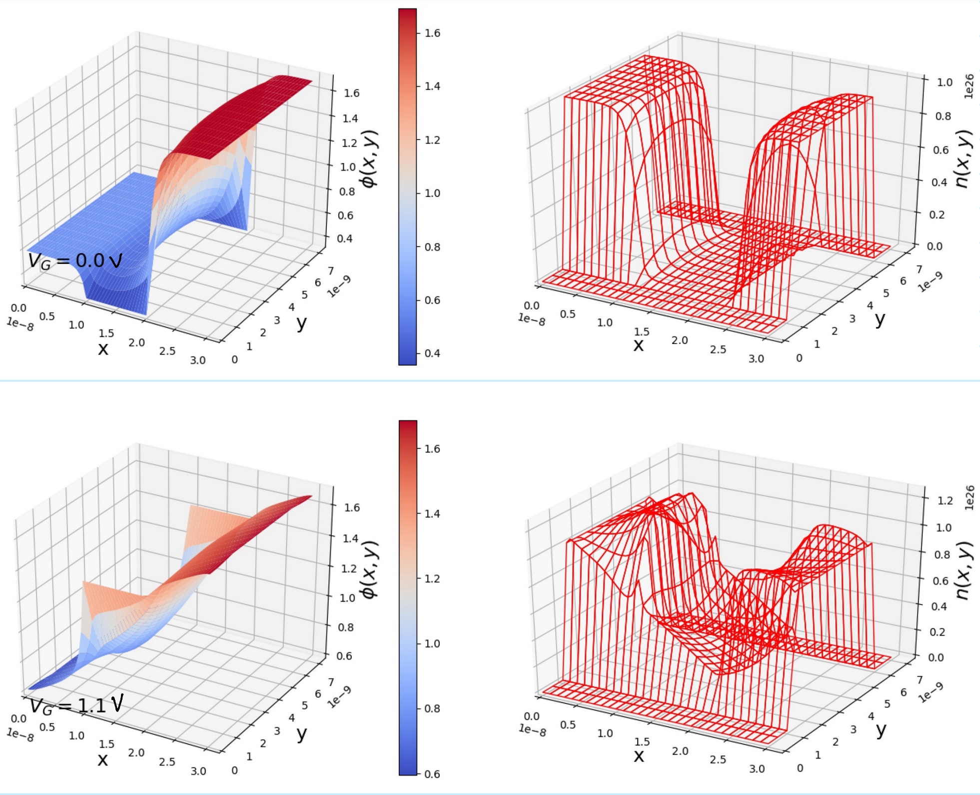
2) $\phi(x,y)$ & $n(x,y)$ with Hw17. method at $V_D = 0V$



Hw 10.과 거의 같다.

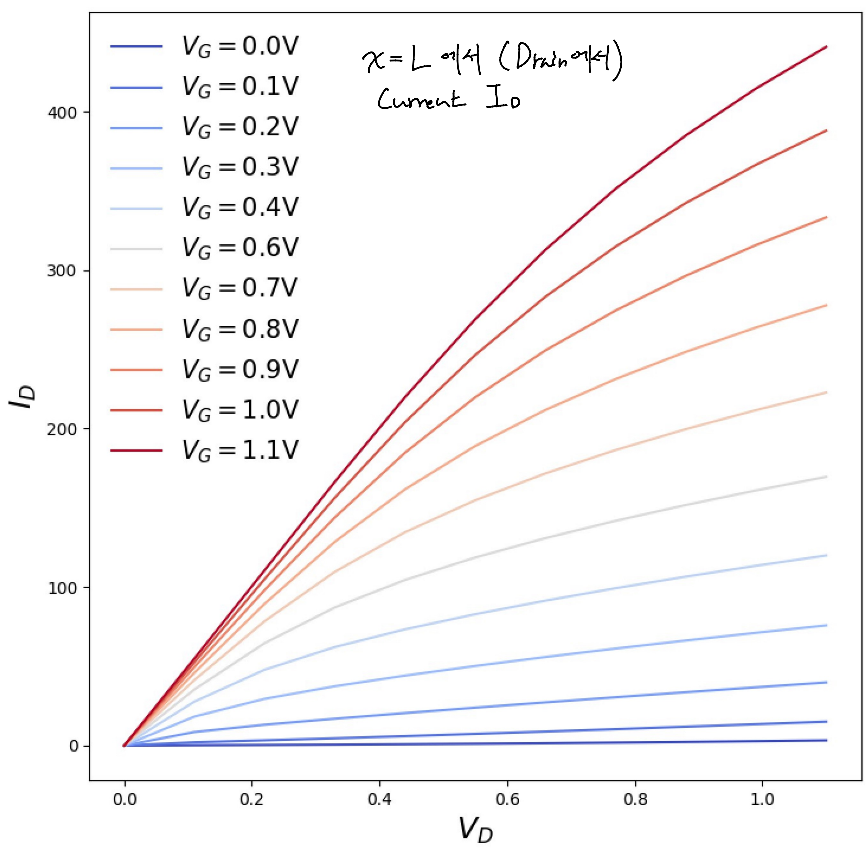


3) $\phi_{(x,y)}$ & $n_{(x,y)}$ with HW 17. method at $V_D = 1.1\,V$



4) I_D or I_S vs. V_D

$\mu = 1500\,cm^2/V\cdot s$. $Area = 10^{-12}\,m^2$



V_D를 올릴 수록 I_D가 커지는 결론이 나왔다. $\phi_{(x,y)}$의 경향과 잘 부합하는 결과이다.