Lecture9

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Laplace equation in 2D

- Laplacian operator in 2D (xy-plane)
  - A second-order differentiation

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

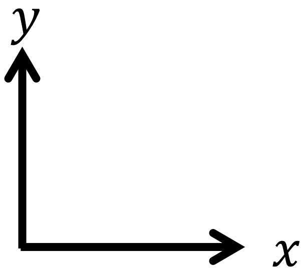
- Laplace equation in 2D (xy-plane)
  - For a function, $\phi(x, y)$, the Laplace equation reads

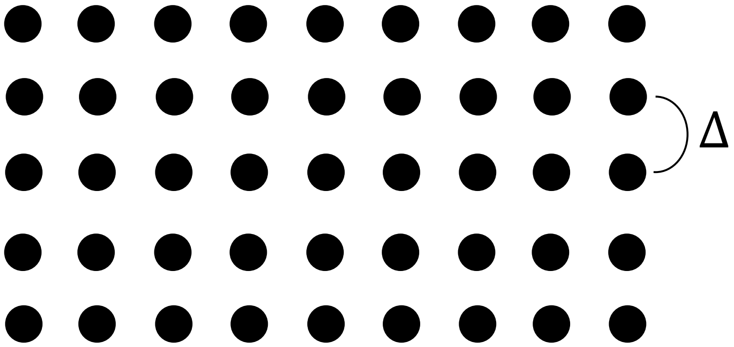
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi(x, y) = 0$$

  - Of course, we need boundary conditions.

An example

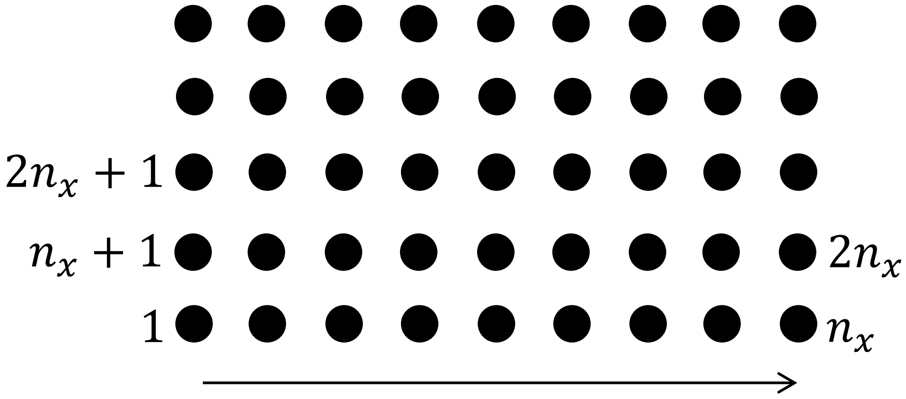
- Consider a rectangle.
  - A common spacing of Δ is assumed.
  - Along the x-direction, n_x points are assigned.
  - Along the y-direction, n_y points are assigned.
  - Mixed boundary condition will be considered later.





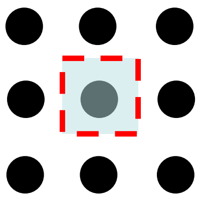
Solution vector

- The solution vector is a vector.
  - We must assign an integer to each node.
  - There can be many different ways to assign the index.
  - In this example, we change y more frequently.
  - The index is given by `(iy-1)*nx+ix`.



Discretization

- How to assign equations
  - Basically, the Laplace equation for a given node is integrated over its control volume.



$$\int_{Volume} \nabla^2 \phi d\mathbf{r} = \oint_{Surface} \nabla\phi \cdot d\mathbf{a}$$

  - The integrated form can be discretized as

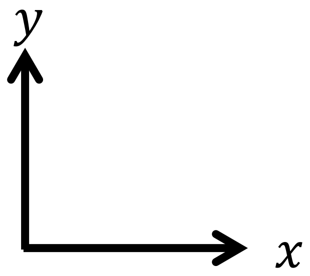
$$\phi_{i+1,j} + \phi_{i,j+1} - 4\phi_{i,j} + \phi_{i-1,j} + \phi_{i,j-1}$$

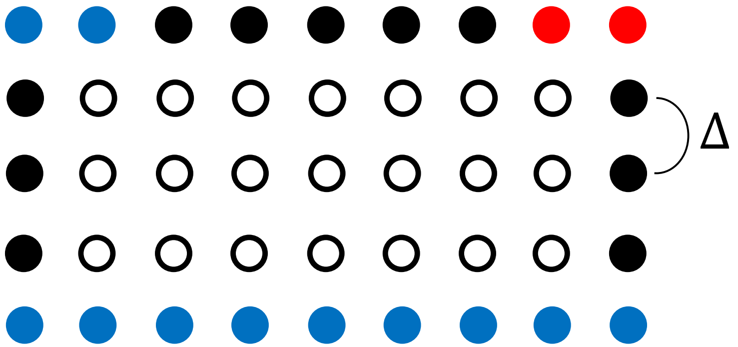
(Thickness along the x-direction is assumed to be unity)

  - Special care for the boundary nodes is required.

Our example

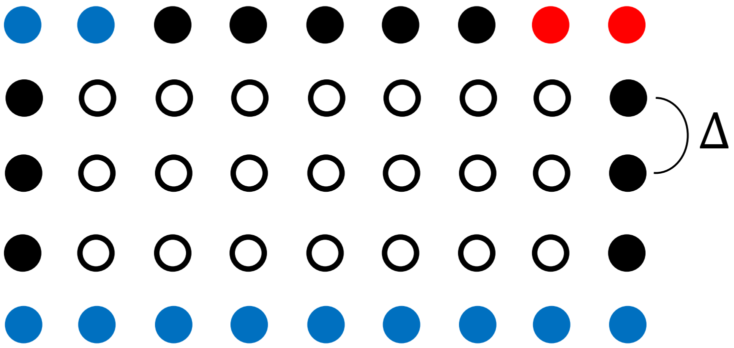
- Consider a toy problem.
  - Here, $n_x = 9$ and $n_y = 5$.
  - Empty circles: Bulk nodes. Their discretization is already studied.
  - Black circles: Homogeneous Neumann boundary condition
  - Blue circles: The function is zero.
  - Red circles: The function is unity.





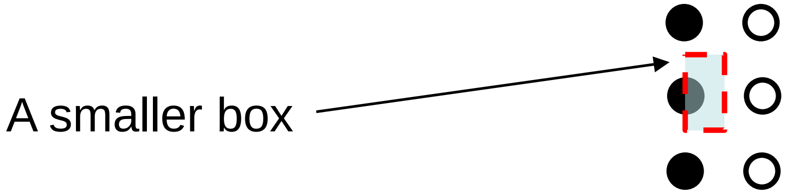
Boundary condition (1)

- Dirichlet boundary condition
  - It is not difficult to consider the Dirichlet boundary condition.
  - For a (ix, iy) node, we simply have $\phi_{ix,iy} = 0$ (for blue circles) or $\phi_{ix,iy} = 1$ (for red circles).



Boundary condition (2)

- Consider a node with a black circle (left boundary)
  - Now, the integrated form of the Laplacian operator reads
$$\phi_{2,j} + 0.5\ \phi_{1,j+1} - 2\phi_{1,j} + 0.5\ \phi_{1,j-1}$$
  - Similar expressions hold for other boundary nodes.
  - When you build the Jacobian matrix, you should be careful.
  - An edge-wise construction would be beneficial.



Homework#9

- Due: AM08:00, October 7 (This Wednesday)
- Problem#1
  - Solve our toy problem. (Laplace equation in the 2D space)
  - Consider four cases:
  - 1) The red circles are located in the original position.
  - 2) The red circles are located in the top/left position.
  - 3) The red circles are located in the bottom position.
  - 4) At all three positions above, the function is unity.
  - Draw 3D graphs for each of them.