HW 9.

Tuesday, October 6, 2020 4:39 PM

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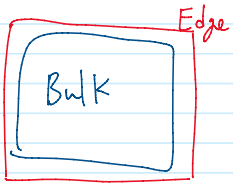
In 2D space, Laplace equation is

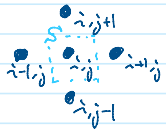
$$\nabla^2 \phi_{(x,y)} = 0 \Rightarrow (\partial_x^2 + \partial_y^2)\phi_{(x,y)} = 0 \quad \text{in the bulk}$$

Integrated form

$$\int_V dz\, \nabla^2 \phi_{(\vec{r})} = \int_S d\vec{a} \cdot \nabla \phi_{(\vec{r})} = 0$$

Discretization



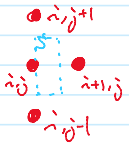
Bulk : 

$$\int d\vec{a} \cdot \nabla \phi_{(\vec{r})} = 0$$

$$-4\phi_{i,j} + \phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1} = 0$$

$$(j \cdot N_x + i)\text{th row} \leftarrow \begin{pmatrix} \ddots & & & & & & \\ & 1 & & 1 & -4 & 1 & \\ & & & & 1 & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \phi_{i,j-1} \\ \phi_{i-1,j} \\ \phi_{i,j} \\ \phi_{i+1,j} \\ \phi_{i,j+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix}$$

Edge : For Homogeneous Neumann B.C.,



$$\int d\vec{a} \cdot \nabla \phi_{(\vec{r})} = 0$$

$$-2\phi_{i,j} + \phi_{i+1,j} + \frac{1}{2}\phi_{i,j+1} + \frac{1}{2}\phi_{i,j-1} = 0$$

$$(j \cdot N_x + i)\text{th row} \leftarrow \begin{pmatrix} \ddots & & & & & \\ & \frac{1}{2} & & -2 & 1 & \frac{1}{2} & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \phi_{i,j-1} \\ \phi_{i,j} \\ \phi_{i+1,j} \\ \vdots \\ \phi_{i,j+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix}$$

For Dirichlet B.C.,

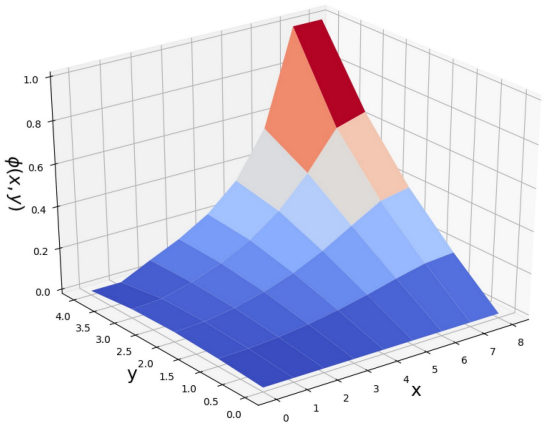
$$\phi_{i,j} = \phi_{B.C.}$$

$$(j \cdot N_x + i)\text{th row} \leftarrow \begin{pmatrix} \ddots & & \\ & 1 & \\ & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ \phi_{i,j} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ \phi_{B.C.} \\ \vdots \end{pmatrix}$$

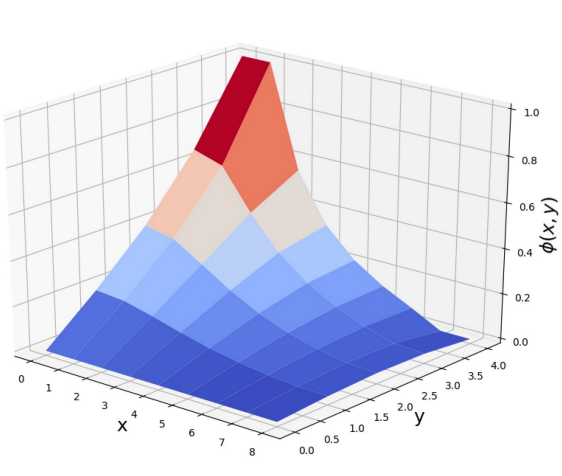
Results.

i) $N_x = 9,\ N_y = 5$

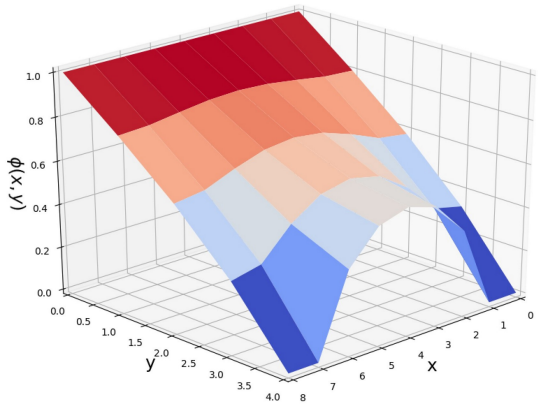
Case 1.



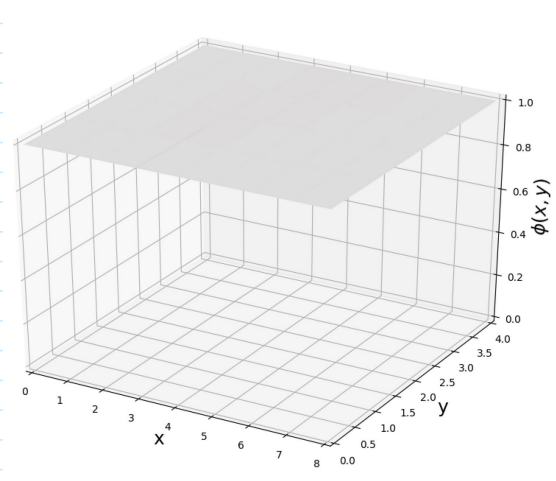
Case 2.



Case 3.

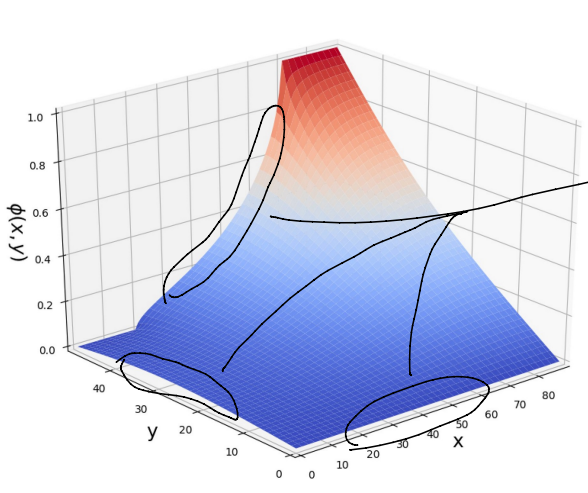


Case 4.



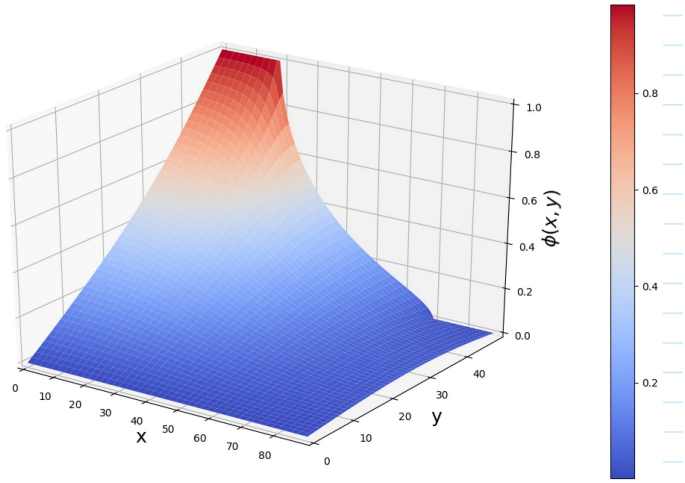
ii) $N_x = 90,\ N_y = 50$

Case 1.

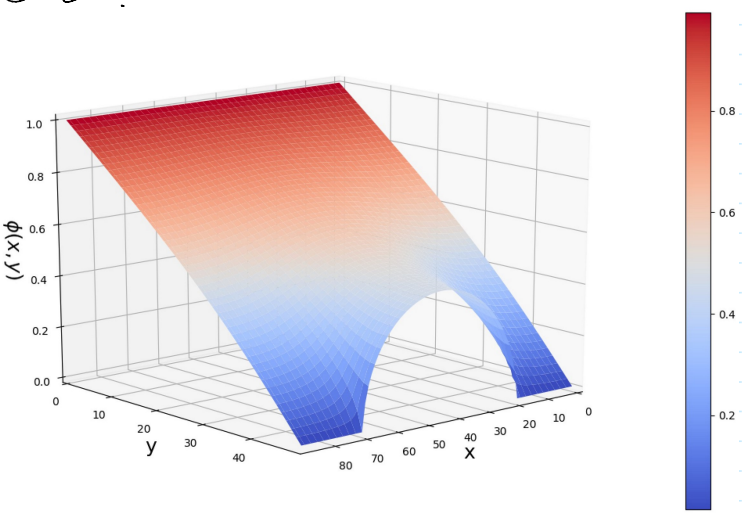


→ Homogeneous Neumann B.C.
이번 $\nabla\phi = 0$ 인걸 알 수 있다.

Case 2.



Case 3.



Case 4.