HW 7.

2020년 9월 24일 목요일 오전 11:25

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Find a root of $N^+ + n_{int} e^{-\frac{\xi\phi}{k_B T}} - n_{int} e^{\frac{\xi\phi}{k_B T}} = 0$.

Let be a residue $r(\phi) = N^+ + n_{int} e^{-\frac{\xi\phi}{k_B T}} - n_{int} e^{\frac{\xi\phi}{k_B T}}$

$= N^+ - 2 n_{int} \sinh\left(\frac{\xi\phi}{k_B T}\right)$,

$\frac{dr(\phi)}{d\phi} = -2 n_{int} \frac{\xi}{k_B T} \cosh\left(\frac{\xi\phi}{k_B T}\right) \Rightarrow \phi_{new} = \phi_{old} - r(\phi_{old}) \Big/ \frac{dr(\phi)}{d\phi}\Big|_{\phi=\phi_{old}}$

$= \phi_{old} + \left[\frac{N^+ k_B T}{2 n_{int} \cosh\left(\frac{\xi\phi}{k_B T}\right) \xi} - \frac{k_B T}{\xi} \tanh\left(\frac{\xi\phi}{k_B T}\right)\right]$

$\equiv \delta\phi$

* Numerical algorithm : $|\delta\phi| < \varepsilon = 10^{-7}$ 이면 loop termination.

* Exact solution

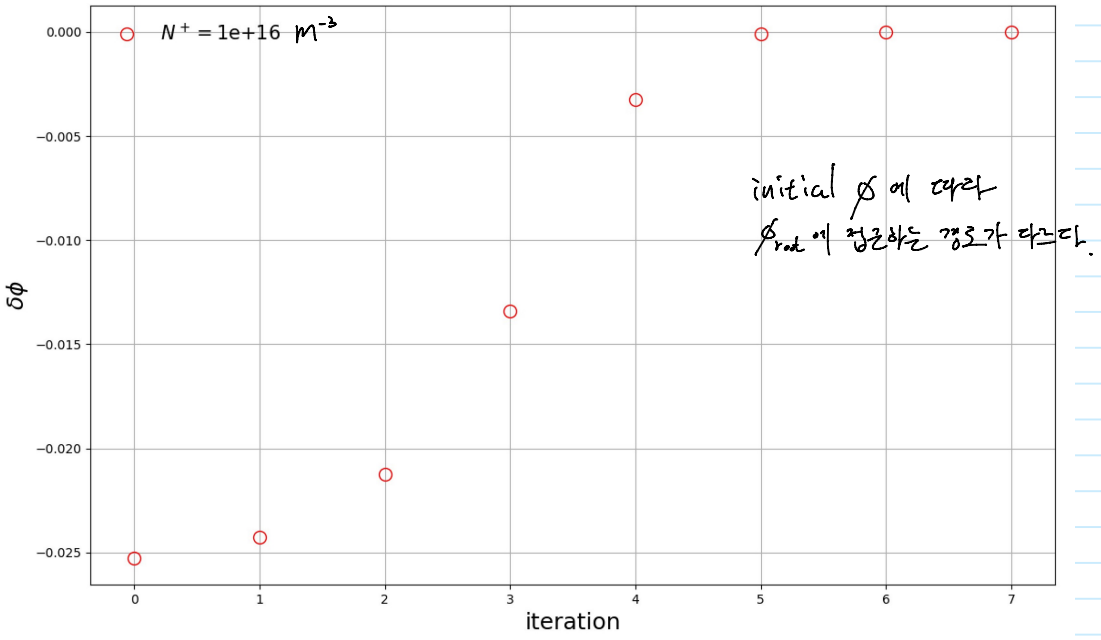
$\phi_{root} = \frac{k_B T}{\xi} \sinh^{-1}\left(\frac{N^+}{2 n_{int}}\right)$

Results.

$n_{int} = 10^{16}\ m^{-3}$, $N^+ = 10^{16} m^{-3} \sim 10^{24} m^{-3}$, $T = 300k$

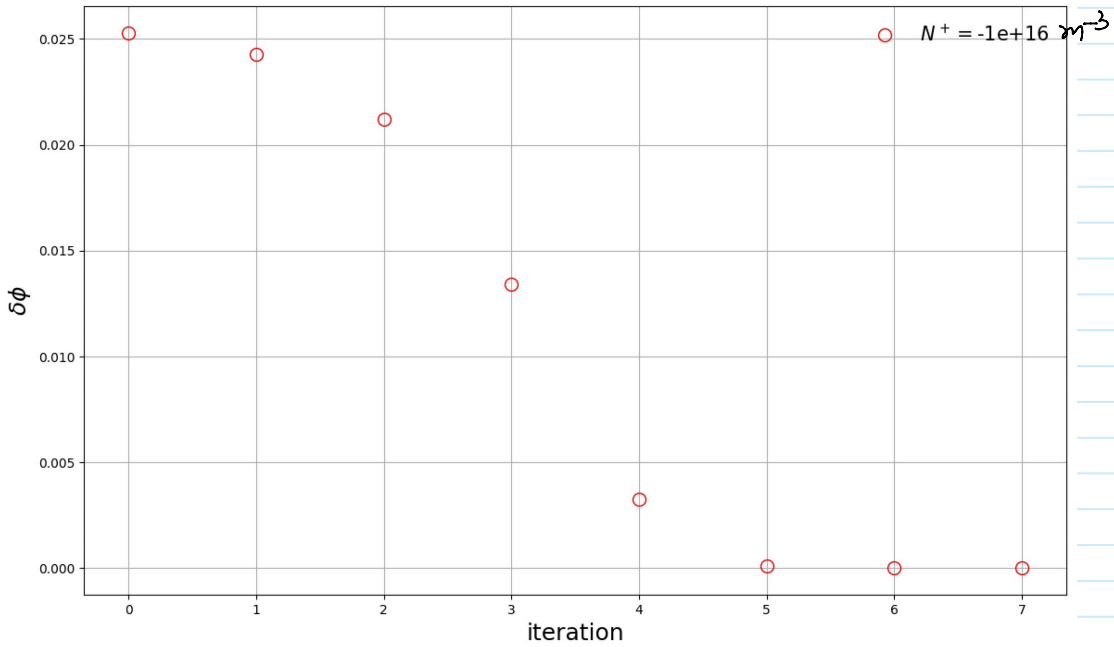
1) $N^+ = 10^{16}\ m^{-3}$

Exact ϕ : 0.012440283776141349
Numerical ϕ : 0.012440283776141349
, $\phi_0 = 0.1$



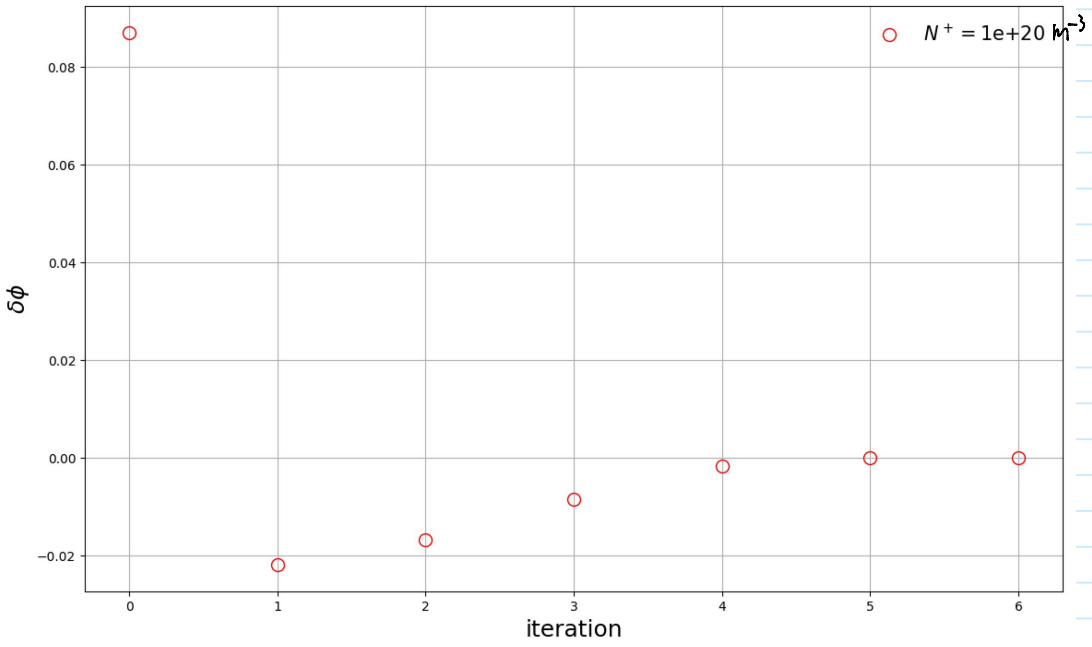
$N^+ = -10^{16}\ m^{-3}$

Exact ϕ : -0.012440283776141349
Numerical ϕ : -0.012440283776141349 , $\phi_0 = -0.1$



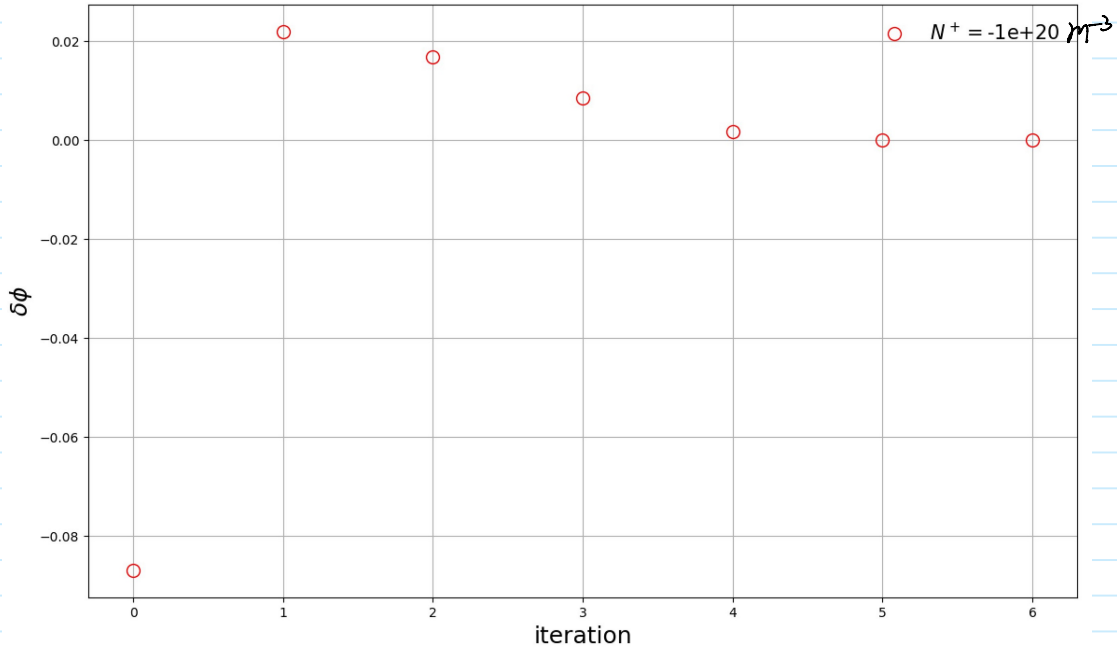
2) $N^+ = 10^{20}\ m^{-3}$

Exact ϕ : 0.23810563676910038
Numerical ϕ : 0.23810563676919358 , $\phi_0 = 0.2$



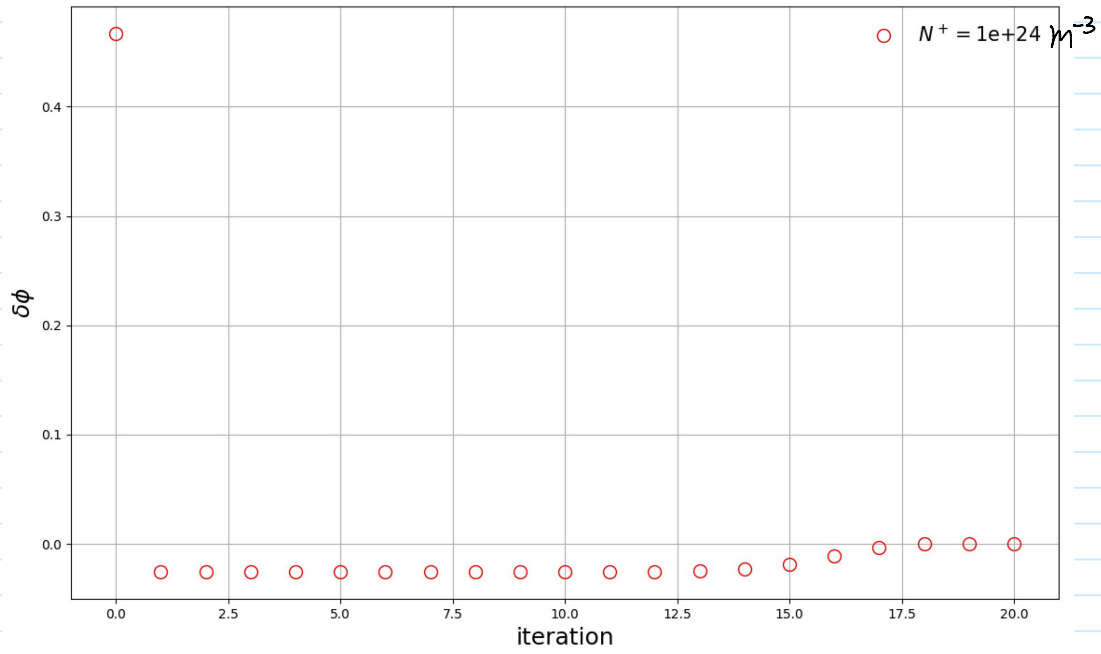
$N^+ = -10^{20}\ m^{-3}$

Exact ϕ : -0.23810563676910038
Numerical ϕ : -0.23810563676919358 , $\phi_0 = -0.2$



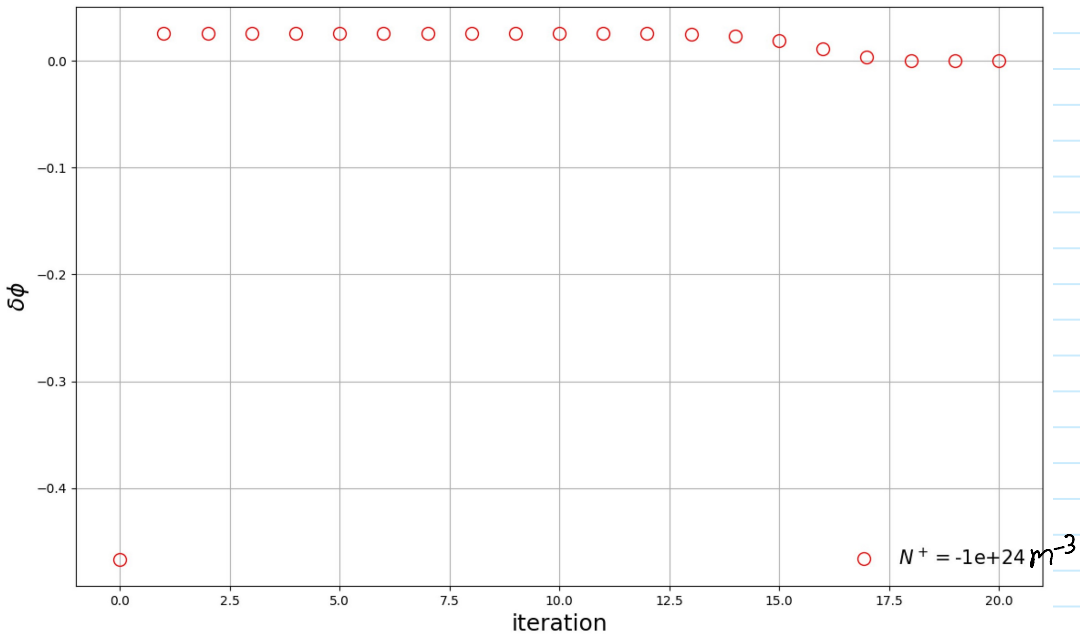
3) $N^+ = 10^{24}\ m^{-3}$

Exact ϕ : 0.47621127302116095
Numerical ϕ : 0.4762112730211609 , $\phi_0 = 0.4$



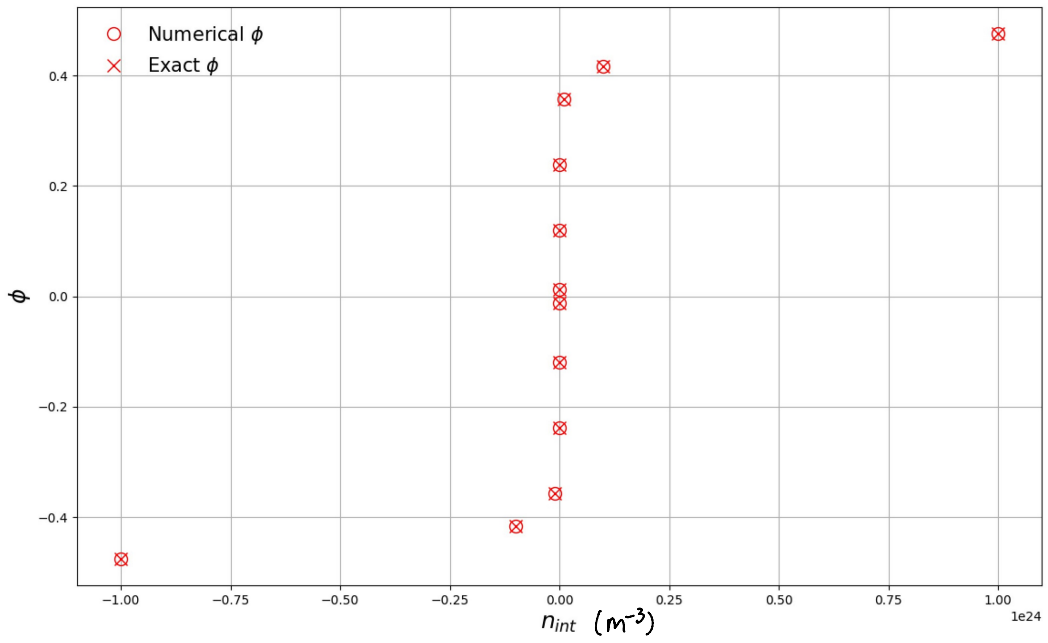
$N^+ = -10^{24}\ m^{-3}$

Exact ϕ : -0.47621127302116095
Numerical ϕ : -0.4762112730211609 , $\phi_0 = -0.4$



전체적으로 Newton method가 정확한 값으로 빠르게 수렴이 됨을 알 수 있다.

4) $N^+ = -10^{24} \sim 10^{24}\ m^{-3}$



Numerically 구한 $\phi_{root}(N^+)$가 $\sinh(N^+)$을 따르는 것을 알 수 있다.