Numerical calculation potential in MOS

Assignment #5

20202010 Hyunsuk Shin

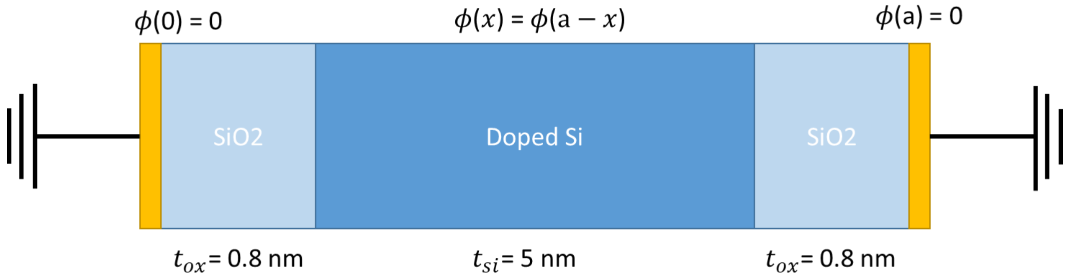

Figure 1 Schematic image for MOS structure

Analytic solution for Poisson's equation:

Oxide layer:

$$\epsilon_{\mathrm{ox}} \frac{d^2}{dx^2} \phi_{ox} = 0$$

$$\phi_{ox} = \frac{\mathrm{C}_1}{\epsilon_{ox}} x + C_2$$

$$\phi_{ox}(0) = C_2 = 0$$

$$\phi_{ox}(\mathrm{t_{ox}}) = \frac{\mathrm{C}_1}{\epsilon_{ox}} t_{ox} => \mathrm{C}_1 = \frac{\phi_{ox}(\mathrm{t_{ox}})\epsilon_{ox}}{t_{ox}}$$

$$\phi_{ox}(x) = \frac{\phi_{ox}(\mathrm{t_{ox}})}{t_{ox}} x \;\; where \;\; 0 < x < t_{ox}$$

Semiconducting layer

$$\epsilon_{\mathrm{si}} \frac{d^2}{dx^2} \phi_{si} = \mathrm{eN_{acc}}$$

$$\phi_{si} = \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}} x^2 + D_1 x + D_2$$

However, we know the solution has mirror symmetry from their structure, the solution must be

$$\phi_{si} = \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}} \left(x - \left(\frac{t_{si}}{2} + t_{ox} \right) \right)^2 + D'$$

$$\phi_{ox}(\mathrm{t_{ox}}) = \frac{\mathrm{C}_1}{\epsilon_{ox}} t_{ox} => \mathrm{C}_1 = \frac{\phi_{ox}(\mathrm{t_{ox}})\epsilon_{ox}}{t_{ox}} \;\; where \;\; t_{ox} < x < t_{ox} + t_{si}$$

Now we use boundary condition between oxide layer and semiconducting layer .

i) $\phi_{ox}(\mathrm{t_{ox}}) = \phi_{si}(t_{ox})$

$$\phi_{ox}(t_{ox}) = \phi_{si}(\mathrm{t_{ox}}) = \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}\left(\frac{t_{ox}}{2}\right)^2 + D'$$

ii) $\epsilon_{ox}\frac{d}{dx}\phi_{ox}(\mathrm{t_{ox}}) = \epsilon_{si}\frac{d}{dx}\phi_{si}(t_{ox})$

$$\phi_{ox}(t_{ox}) = \phi_{si}(\mathrm{t_{ox}}) = -\frac{\mathrm{eN_{acc}t_{ox}t_{si}}}{2\epsilon_{ox}}$$

$$\therefore \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}\left(\frac{t_{si}}{2}\right)^2 + D' = -\frac{\mathrm{eN_{acc}t_{ox}t_{si}}}{2\epsilon_{ox}}$$

$$D' = -\frac{\mathrm{eN_{acc}t_{ox}t_{si}}}{2\epsilon_{ox}} - \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}\left(\frac{t_{si}}{2}\right)^2$$

Then, the solution is as follows:

$0 < x < t_{ox}$	$t_{ox} < x < t_{ox} + t_{si}$	$t_{ox} + t_{si} < x < 2t_{ox} + t_{si}$
$\phi_{ox}(x) = -\frac{\mathrm{eN_{acc}t_{si}}}{2\epsilon_{ox}}x$	$\phi_{si} = \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}\left(x - \left(\frac{t_{si}}{2} + t_{ox}\right)\right)^2 - \frac{\mathrm{eN_{acc}t_{ox}t_{si}}}{2\epsilon_{ox}} - \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}\left(\frac{t_{si}}{2}\right)^2$	$\phi_{ox}(x) = \frac{\mathrm{eN_{acc}t_{si}}}{2\epsilon_{ox}}(x - (2t_{ox} + t_{si}))$
$\phi_{ox}(x) = -C1x$	$\phi_{ox}(x) = C2(x - C3)^2 - C4$	$\phi_{ox}(x) = C1(x - a)$

Where ϵ_{ox} is permittivity of oxide, ϵ_{si} is permittivity of silicon(or semiconducting layer). Let's calculate the coefficients.

	$\mathrm{N_{acc}} = 10^{23}m^{-3}$	$\mathrm{N_{acc}} = 10^{24}m^{-3}$	$\mathrm{N_{acc}} = 10^{25}m^{-3}$
C1 = $\frac{\mathrm{eN_{acc}t_{si}}}{2\epsilon_{ox}}$	1.16×10^6 V/m	1.16×10^7 V/m	1.16×10^8 V/m
C2 = $\frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}$	$7.73 \times 10^{13}\ V/m^2$	$7.73 \times 10^{14}\ V/m^2$	7.73×10^{15} V/m^2
C3 = $\frac{t_{si}}{2} + t_{ox}$	3.3 nm	3.3 nm	3.3 nm
C4= $\frac{\mathrm{eN_{acc}t_{ox}t_{si}}}{2\epsilon_{ox}} + \frac{\mathrm{eN_{acc}}}{2\epsilon_{si}}\left(\frac{t_{si}}{2}\right)^2$	1.40×10^{-3} V	1.40×10^{-2} V	1.40×10^{-1} V

Numerical Results

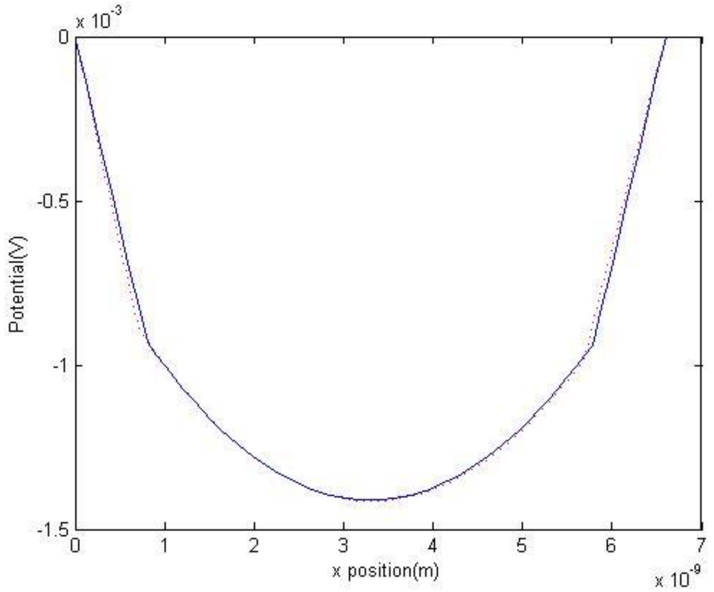

Figure 2. $\mathrm{N_{acc}} = 10^{23}m^{-3}$, N = 66 case. Blue line indicates analytic solution and red dotted line our numerical results.

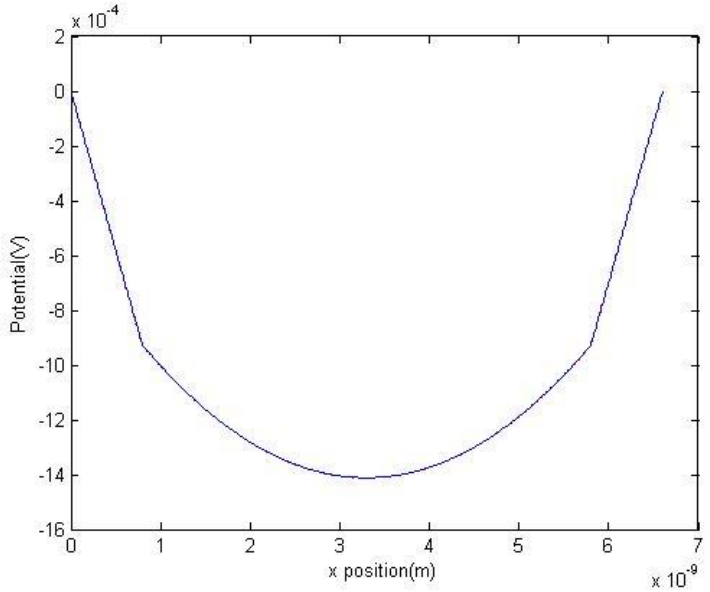

Figure 3. $\mathrm{N_{acc}} = 10^{23}m^{-3}$, N = 660 case. Blue line indicates analytic solution and red dotted line our numerical results.

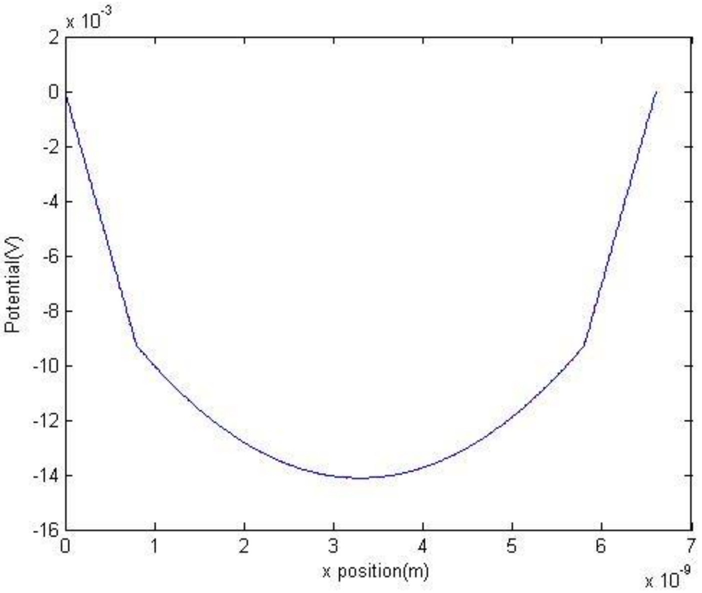

Figure 4. $\mathrm{N_{acc}} = 10^{24} m^{-3}$, N= 660 case.

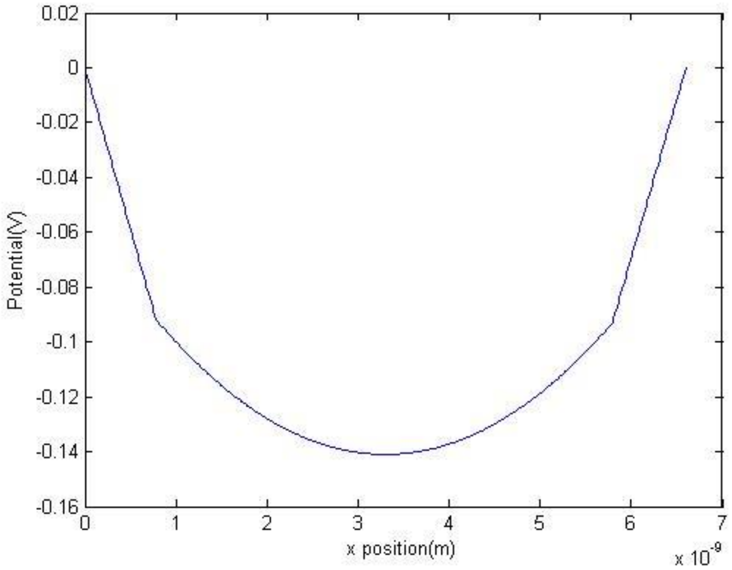

Figure 5. $\mathrm{N_{acc}} = 10^{25} m^{-3}$, N = 660 case.

Error analysis

	Intersection 1	midpoint	Intersection 2
N = 66	11.6 %	0.264 %	1.06 %
N = 660	0.480 %	0.0931%	0.209 %