HW 12

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1. The Derivation of Scharfetter-Gummel Method

① J_n (1D problem)

steady-state $\frac{dJ_n}{dx} = 0 \rightarrow J_{n,i+0.5} - J_{n,i-0.5} = 0$

$J_n = -q\mu_n n \frac{d\phi(x)}{dx} + qD_n \frac{dn(x)}{dx}$: a differential equation for n..

$$\frac{J_n}{qD_n} = \frac{dn}{dx} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x} n, \quad \Delta\phi = \phi_{i+1} - \phi_i, \quad \Delta x = x_{i+1} - x_i$$

$$n(x) = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x} + C \quad \cdots\cdots (i)$$

$$\frac{J_n}{qD_n} = \frac{1}{V_T}\frac{\Delta\phi}{\Delta x} \cdot A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x}\left(A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x} + C\right)$$

$$= -\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} C \quad \longrightarrow \therefore J_n = -\frac{qD_n}{V_T}\frac{\Delta\phi}{\Delta x} C$$

from (i)

$$\underset{(=n_{i+1})}{n(x_{i+1})} = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_{i+1}} + C, \quad \underset{(=n_i)}{n(x_i)} = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} + C$$

$$n_{i+1} - n_i = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_{i+1}} - A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i}$$

$$= A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i}\left(e^{\frac{\Delta\phi}{V_T}} - 1\right) \rightarrow \underline{A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} = \frac{n_{i+1} - n_i}{e^{\frac{\Delta\phi}{V_T}} - 1}}$$

$$C = n_i - A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x}$$

$$= n_i - \frac{n_{i+1} - n_i}{e^{\Delta\phi/V_T} - 1} = -n_{i+1}\frac{1}{e^{\Delta\phi/V_T} - 1} + n_i \frac{e^{\Delta\phi/V_T} - 1 + 1}{e^{\Delta\phi/V_T} - 1}$$

$$J_n = -\frac{qD_n}{V_T}\frac{\Delta\phi}{\Delta x}C = -\frac{qD_n}{V_T}\frac{\Delta\phi}{\Delta x}\left(-n_{i+1}\frac{1}{e^{\Delta\phi/V_T}-1} + n_i\frac{e^{\Delta\phi/V_T}}{e^{\Delta\phi/V_T}-1}\right)$$

$$= \frac{qD_n}{\Delta x}\left[n_{i+1}\underbrace{\frac{\Delta\phi/V_T}{e^{\Delta\phi/V_T}-1}}_{(ii)} - n_i\underbrace{\frac{\Delta\phi/V_T\, e^{\Delta\phi/V_T}}{e^{\Delta\phi/V_T}-1}}_{(iii)}\right]$$

if $x = \Delta\phi/V_T$

(ii) $\dfrac{x}{e^x-1}$: $B(x)$: Bernoulli's function.

$$\therefore J_n = \frac{qD_n}{\Delta x}\left[n_{i+1}B\left(\frac{\Delta\phi}{V_T}\right) - n_i\underbrace{\frac{x e^x}{e^x-1}}_{(iv)}\right]$$

$$(iv)\times\frac{e^{-x}}{e^{-x}} = n_i\frac{x}{1-e^{-x}} = \frac{-x}{e^{-x}-1} = B(-x)$$

$$\therefore J_n = \frac{qD_n}{\Delta x}\left[n_{i+1}B\left(\frac{\Delta\phi}{V_T}\right) - n_i B\left(-\frac{\Delta\phi}{V_T}\right)\right], \quad \Delta\phi = \phi_{i+1}-\phi_i$$

② J_p (1D problem)

steady-state $\dfrac{dJ_p}{dx}=0$, $J_p = -q\mu_p p\dfrac{d\phi(x)}{dx} - qD_n\dfrac{dP(x)}{dx}$

$$\frac{J_p}{qD_p} = -\frac{dP}{dx} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x}P, \quad \Delta\phi = \phi_{i+1}-\phi_i, \quad \Delta x = x_{i+1}-x_i.$$

$$P(x) = D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x} + E \quad \cdots\cdots (v)$$

$$\frac{J_p}{qD_p} = +\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}De^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x}\left(De^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x}+E\right)$$

$$= -\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}E \rightarrow \therefore J_p = -\frac{qD_p}{V_T}\times\frac{\Delta\phi}{\Delta x}E$$

from (V)

$$P(x_{i+1}) = P_{i+1} = D \cdot e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_{i+1}} + E$$

$$P(x_i) = P_i = D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} + E$$

$\ominus$.

$$P_{i+1} - P_i = D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_{i+1}} - D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i}.$$

$$= D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i}\left(e^{-\frac{\Delta\phi}{V_T}} - 1\right)$$

$$\rightarrow \quad D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} = \frac{P_{i+1} - P_i}{e^{-\frac{\Delta\phi}{V_T}} - 1}, \qquad E = P_i - D e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x}$$

$$\therefore \quad E = P_i - \frac{P_{i+1} - P_i}{e^{-\frac{\Delta\phi}{V_T}} - 1} = -P_{i+1}\frac{1}{e^{-\frac{\Delta\phi}{V_T}} - 1} + P_i \frac{e^{-\frac{\Delta\phi}{V_T}} - 1 + 1}{e^{-\frac{\Delta\phi}{V_T}} - 1}$$

$$J_p = -\frac{qD_p}{V_T}\frac{\Delta\phi}{\Delta x}E = -\frac{qD_p}{V_T}\frac{\Delta\phi}{\Delta x}\left(-P_{i+1}\frac{1}{e^{-\frac{\Delta\phi}{V_T}} - 1} + P_i \frac{e^{-\Delta\phi/V_T}}{e^{-\frac{\Delta\phi}{V_T}} - 1}\right)$$

$$= -\frac{qD_p}{\Delta x}\left(P_{i+1}\frac{-\Delta\phi/V_T}{e^{-\Delta\phi/V_T} - 1} - P_i \cdot \frac{-\Delta\phi/V_T \, e^{-\Delta\phi/V_T}}{e^{-\Delta\phi/V_T} - 1}\right)$$

if) $x = -\Delta\phi/V_T$

$$J_p = -\frac{qD_p}{\Delta x}\left(P_{i+1}\frac{x}{e^x - 1} - P_i \frac{x \cdot e^{-x}}{e^x - 1}\right)$$

$$= -\frac{qD_p}{\Delta x}\left(P_{i+1} B\left(-\frac{\Delta\phi}{V_T}\right) - P_i B\left(\frac{\Delta\phi}{V_T}\right)\right)$$

$$\therefore \quad J_p = -\frac{qD_p}{\Delta x}\left[P_{i+1} B\left(-\frac{\Delta\phi}{V_T}\right) - P_i B\left(\frac{\Delta\phi}{V_T}\right)\right]$$

$$\Delta\phi = \phi_{i+1} - \phi_i$$