HW 12

20221059 정승목

- The derivation of Scharfetter - Gummel

1) J_n

$$J_n = q\mu_n n E + qD_n \nabla n = -q\mu_n n \nabla\phi + qD_n \nabla n$$

— The current density was discretized as

$$-\frac{J_{n,i+0.5}}{q\mu_n} = \frac{n_{i+1}+n_i}{2}\cdot\frac{\phi_{i+1}-\phi_i}{\Delta x} - V_T\frac{n_{i+1}-n_i}{\Delta x}$$

μ_n과 D_n은 Einstein 관계식을 만족하므로 $D_n = \frac{k_BT}{q}\mu_n = V_T\mu_n$ 을 이용하면

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = -\frac{n_{i+1}+n_i}{2}\,\frac{\phi_{i+1}-\phi_i}{V_T} + n_{i+1} - n_i$$

n_i 와 n_{i+1} 항으로 정리하면

$$\frac{J_{n,i+0.5}}{qD_n}\Delta x = n_{i+1}\left(1-\frac{\phi_{i+1}-\phi_i}{2V_T}\right) + n_i\left(1+\frac{\phi_{i+1}-\phi_i}{2V_T}\right)$$

만약 $|\phi_{i+1}-\phi_i| > 2V_T$, 즉 전자 농도의 계수중 하나가 음수가 되면 이는 맞지 않다.

$$\Rightarrow J_{n,i+0.5} = n_{i+1}(A) - n_i(B)$$

A.B.C.D ≥ 0

$$J_{n,i-0.5} = n_{i+1}(C) - n_i(D)$$

$$\frac{dJ_n}{dx} = 0 \Rightarrow J_{n,i+0.5} - J_{n,i-0.5} = 0$$

$$\Rightarrow n_{i-1}(D) - n_i(B+C) + n_{i+1}(A) = 0$$

$$\frac{dJ_n}{dx} = 0 \quad , \quad J_n = \text{Const.}$$

$$J_n = -q\mu_n n \frac{d\phi}{dx} + qD_n \frac{dn}{dx}$$

· 1차원 구조에서 전류밀도식은

$$\underbrace{\frac{dn}{dx} - \frac{1}{V_T}\frac{d\phi}{dx} n}_{} = \underbrace{\frac{J_n}{qD_n}}_{\text{Const.}}$$

$$\Downarrow$$

$$-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} \quad \Delta\phi = \phi_{i+1} - \phi_i$$

$$\Rightarrow \frac{dn}{dx} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x} n = \frac{J_n}{qD_n}$$

• Homogeneous Solution.

$$n = \underbrace{A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x}}_{\text{homogeneous}} + B$$

· $n(x_{i+1}) = n_{i+1}$, $n(x_i) = n_i$

$x_{i+1} = x_i + (x_{i+1} - x_i)$

$$n_{i+1} = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_{i+1}} + B$$

$$n_i = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i} + B$$

$$\Rightarrow n_{i+1} - n_i = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i}\left(e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}(x_{i+1} - x_i)} - 1\right)$$

$$= A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i} \times \left(e^{\frac{1}{V_T}\Delta\phi} - 1\right)$$

$$\Rightarrow A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i} = \frac{n_{i+1} - n_i}{e^{\frac{\Delta\phi}{V_T}} - 1} \quad \cdots ①$$

$n_i = A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i} + B$ 를 이용하면

$$\Rightarrow B = n_i - A e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x} x_i} \quad \cdots ②$$

② 식에 ①을 대입하면

$$B = n_i - \frac{n_{i+1} - n_i}{e^{\frac{\Delta\phi}{V_T}} - 1}$$

B 식을 n_i 와 n_{i+1} 로 다시 정리하면

$$B = -n_{i+1}\frac{1}{e^{\frac{\Delta\phi}{V_T}} - 1} + n_i \frac{e^{\frac{\Delta\phi}{V_T}} - 1 + 1}{e^{\frac{\Delta\phi}{V_T}} - 1} = -n_{i+1}\frac{1}{e^{\frac{\Delta\phi}{V_T}} - 1} + n_i \frac{e^{\frac{\Delta\phi}{V_T}}}{e^{\frac{\Delta\phi}{V_T}} - 1}$$

$$\Rightarrow J_n = -\frac{qD_n}{\Delta x}\frac{\Delta\phi}{V_T} B = +\frac{qD_n}{\Delta x}\frac{\Delta\phi}{V_T} n_{i+1} \cdot \frac{1}{e^{\frac{\Delta\phi}{V_T}} - 1} - \frac{qD_n}{\Delta x}\frac{\Delta\phi}{V_T} n_i \frac{e^{\frac{\Delta\phi}{V_T}}}{e^{\frac{\Delta\phi}{V_T}} - 1}$$

$$= \frac{qD_n}{\Delta x}\left[n_{i+1} \frac{\frac{\Delta\phi}{V_T}}{e^{\frac{\Delta\phi}{V_T}} - 1} - n_i \frac{\frac{\Delta\phi}{V_T} e^{\frac{\Delta\phi}{V_T}}}{e^{\frac{\Delta\phi}{V_T}} - 1} \right]$$

* Bernoulli 함수 $B(x) = \frac{x}{e^x + 1}$ 을 도입하면

$$J_n = \frac{qD_n}{\Delta x}\left[n_{i+1} B\left(\frac{\Delta\phi}{V_T}\right) - n_i \underbrace{\frac{x e^x}{e^x - 1}}_{\times \frac{e^{-x}}{e^{-x}}} \right]$$

$$\frac{x}{1 - e^{-x}} = \frac{-x}{e^{-x} + 1} = B\left(\frac{\Delta\phi}{V_T}\right)$$

$$\therefore J_n = \frac{qD_n}{\Delta x}\left[n_{i+1} B\left(\frac{\Delta\phi}{V_T}\right) - n_i B\left(-\frac{\Delta\phi}{V_T}\right) \right]$$

2) J_p

- $J_p = -q\mu_p p \frac{d\phi}{dx} - qD_n \frac{dp(x)}{dx}$, $\frac{dJ_p}{dx} = 0$

$$\frac{J_p}{qD_p} = -\frac{dp}{dx} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x}p$$

- Homogeneous Solution.

$$p = \underbrace{F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x}}_{\text{homogeneous}} + G$$

- $p(x_{i+1}) = p_{i+1}$, $p(x_i) = p_i$

$x_{i+1} = x_i + (x_{i+1} - x_i)$

$$p_{i+1} = F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_{i+1}} + G$$
$$p_i = F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} + G$$

$$\Rightarrow p_{i+1} - p_i = F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i}\left(e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}(x_{i+1}+x_i)} - 1\right)$$

$$= F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} \times \left(e^{-\frac{1}{V_T}\Delta\phi} - 1\right)$$

$$\Rightarrow F e^{\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} = \frac{p_{i+1} - p_i}{e^{\frac{\Delta\phi}{V_T}} - 1} \quad \cdots ③$$

$p_i = F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} + G$ 을 이용하면

$$\Rightarrow G = n_i - F e^{-\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i} \quad \cdots ④$$

④ 식에 ③ 을 대입하면

$$G = p_i - \frac{p_{i+1} - p_i}{e^{-\frac{\Delta\phi}{V_T}} - 1}$$

B식을 p_i 와 p_{i+1} 로 다시 정리하면

$$G = -p_{i+1} \frac{1}{e^{-\frac{\Delta\phi}{V_T}} - 1} + p_i \frac{e^{-\frac{\Delta\phi}{V_T}} - 1 + 1}{e^{-\frac{\Delta\phi}{V_T}} - 1} = -p_{i+1} \frac{1}{e^{-\frac{\Delta\phi}{V_T}} - 1} + p_i \frac{e^{-\frac{\Delta\phi}{V_T}}}{e^{-\frac{\Delta\phi}{V_T}} - 1}$$

$$\Rightarrow J_p = -\frac{qD_p}{\Delta x} \frac{\Delta\phi}{V_T} G = -\frac{qD_p}{\Delta x} \frac{\Delta\phi}{V_T} \cdot -p_{i+1} \cdot \frac{1}{e^{-\frac{\Delta\phi}{V_T}} - 1} - \frac{qD_p}{\Delta x} \frac{\Delta\phi}{V_T} p_i \frac{e^{-\frac{\Delta\phi}{V_T}}}{e^{-\frac{\Delta\phi}{V_T}} - 1}$$

$$= -\frac{qD_p}{\Delta x} \left[p_{i+1} \frac{-\frac{\Delta\phi}{V_T}}{e^{-\frac{\Delta\phi}{V_T}} - 1} - p_i \frac{-\frac{\Delta\phi}{V_T} e^{-\frac{\Delta\phi}{V_T}}}{e^{-\frac{\Delta\phi}{V_T}} - 1} \right]$$

* Bernoulli 함수 $B(x) = \frac{x}{e^x + 1}$ 를 도입하면

$$\therefore J_p = -\frac{qD_n}{\Delta x} \left[n_{i+1} B\left(-\frac{\Delta\phi}{V_T}\right) - n_i B\left(\frac{\Delta\phi}{V_T}\right) \right]$$