Computational Microelectronics
Lecture 13 Poisson Equation

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Source-Free Poisson Equation

Laplace versus Poisson

- Laplace equation

$$\nabla^2\phi = \nabla\cdot(\nabla\phi) = 0$$

- Poisson equation

$$\nabla\cdot[\epsilon(\mathbf{r})\nabla\phi] = -\rho(\mathbf{r})$$

  - When the permittivity is a constant,

$$\nabla\cdot(\nabla\phi) = -\frac{\rho(\mathbf{r})}{\epsilon}$$

  - When there is no net charge,

$$\nabla\cdot(\nabla\phi) = 0$$

Discretization of $\nabla \cdot [\epsilon(\mathbf{r}) \nabla \phi]$

- 1D

$$\frac{d}{dx}\left[\epsilon(x)\frac{d\phi}{dx}\right]$$

  - Integration from $x_{i-0.5}$ to $x_{i+0.5}$

$$\int_{x_{i-0.5}}^{x_{i+0.5}} \frac{d}{dx}\left[\epsilon(x)\frac{d\phi}{dx}\right] dx = \epsilon(x_{i+0.5})\left.\frac{d\phi}{dx}\right|_{x_{i+0.5}} - \epsilon(x_{i-0.5})\left.\frac{d\phi}{dx}\right|_{x_{i-0.5}}$$

In 2D or 3D,

- Laplace equation
  - Discretized Laplace equation reads

$$\sum_j \frac{\phi_j - \phi_i}{|\mathbf{r}_j - \mathbf{r}_i|} A_{ij} = 0$$

- Poisson equation
  - Discretized Poisson equation reads

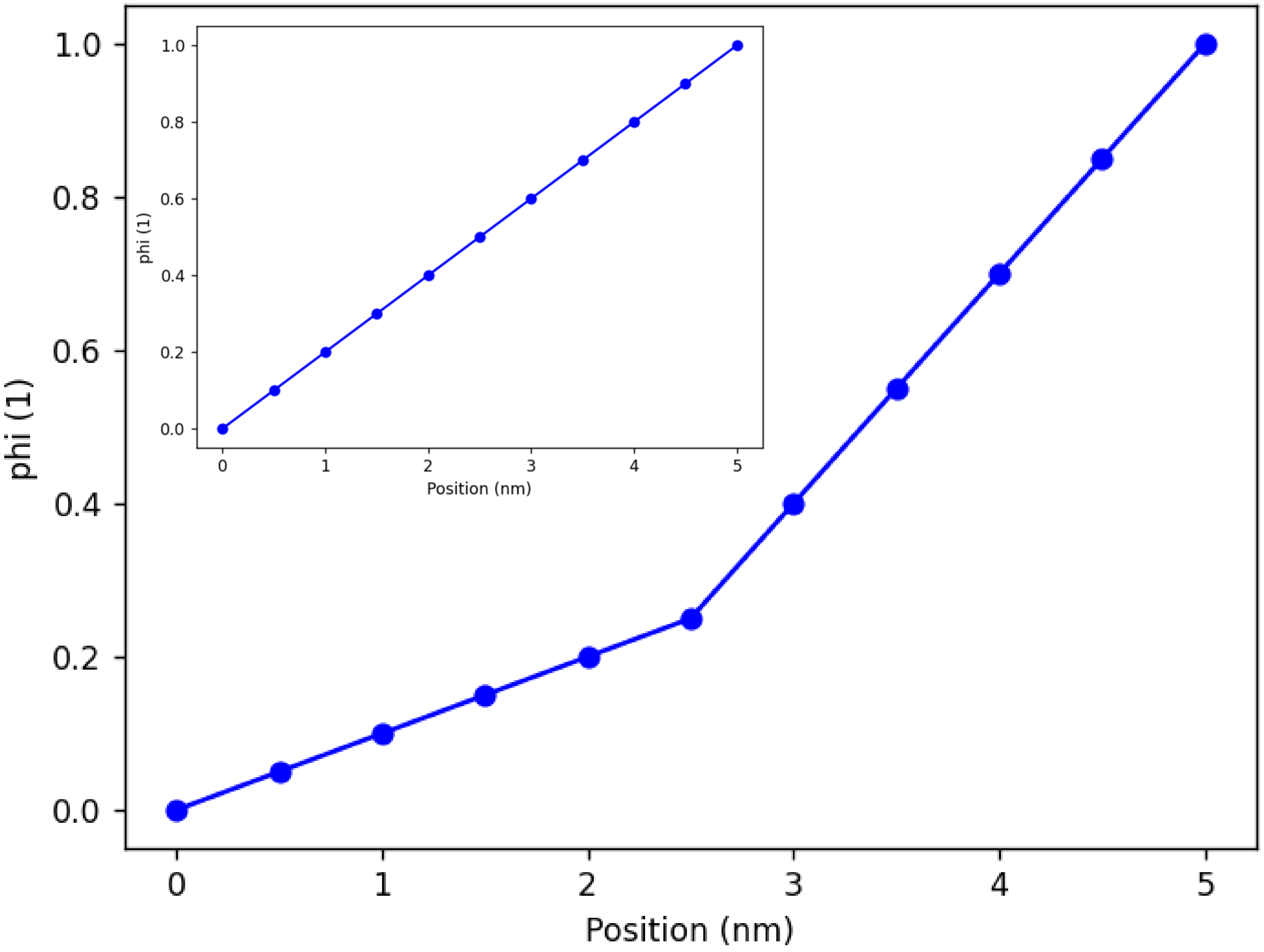
$$\sum_j \epsilon_{ij} \frac{\phi_j - \phi_i}{|\mathbf{r}_j - \mathbf{r}_i|} A_{ij} + \rho(\mathbf{r}_i)\Omega_i = 0$$

1D example

- Source-free Poisson equation
  - Consider a capacitor with two layers.
  - Each layer is 2.5-nm-thick.
  - The first layer (from 0 to 2.5 nm) has a relative permittivity of 11.7.
  - The second layer (from 2.5 nm to 5 nm) has a relative permittivity of 3.9.
  - 1 V is applied across the capacitor. Calculate the potential, ϕ.

Its solution

- Potential, ϕ



HW#12

- Due: AM08:00, October 18
- Problem#1
  - Draw the potential, when the capacitor has the following conditions:
  - The first layer (from 0 to 2.5 nm) has a relative permittivity of 3.9.
  - The second layer (from 2.5 nm to 5 nm) has a relative permittivity of 7.4.
  - From the numerical results, calculate its areal capacticance.

Poisson Equation with Fixed Charges

Poisson equation with fixed chrages

- Charges inside semiconductor

$$\rho(\mathbf{r}) = qp(\mathbf{r}) - qn(\mathbf{r}) + qN_{dop}^{+}(\mathbf{r})$$

  - Hole density, $p(\mathbf{r})$
  - Electron density, $n(\mathbf{r})$
  - Doping density, $N_{dop}^{+}(\mathbf{r})$
- Calculating $p(\mathbf{r})$ and $n(\mathbf{r})$ is the main goal of TCAD simulation!
  - It's not an easy task.
  - Without $p(\mathbf{r})$ and $n(\mathbf{r})$,

$$\nabla \cdot [\epsilon(\mathbf{r})\nabla\phi] = -qN_{dop}^{+}(\mathbf{r})$$

Discretization

- Integrated form in 1D

$$\epsilon(x_{i+0.5})\left.\frac{d\phi}{dx}\right|_{x_{i+0.5}} - \epsilon(x_{i-0.5})\left.\frac{d\phi}{dx}\right|_{x_{i-0.5}}$$
$$= -qN_{dop}^{+}(x_i)(x_{i+0.5} - x_{i-0.5})$$

- Explicitly,

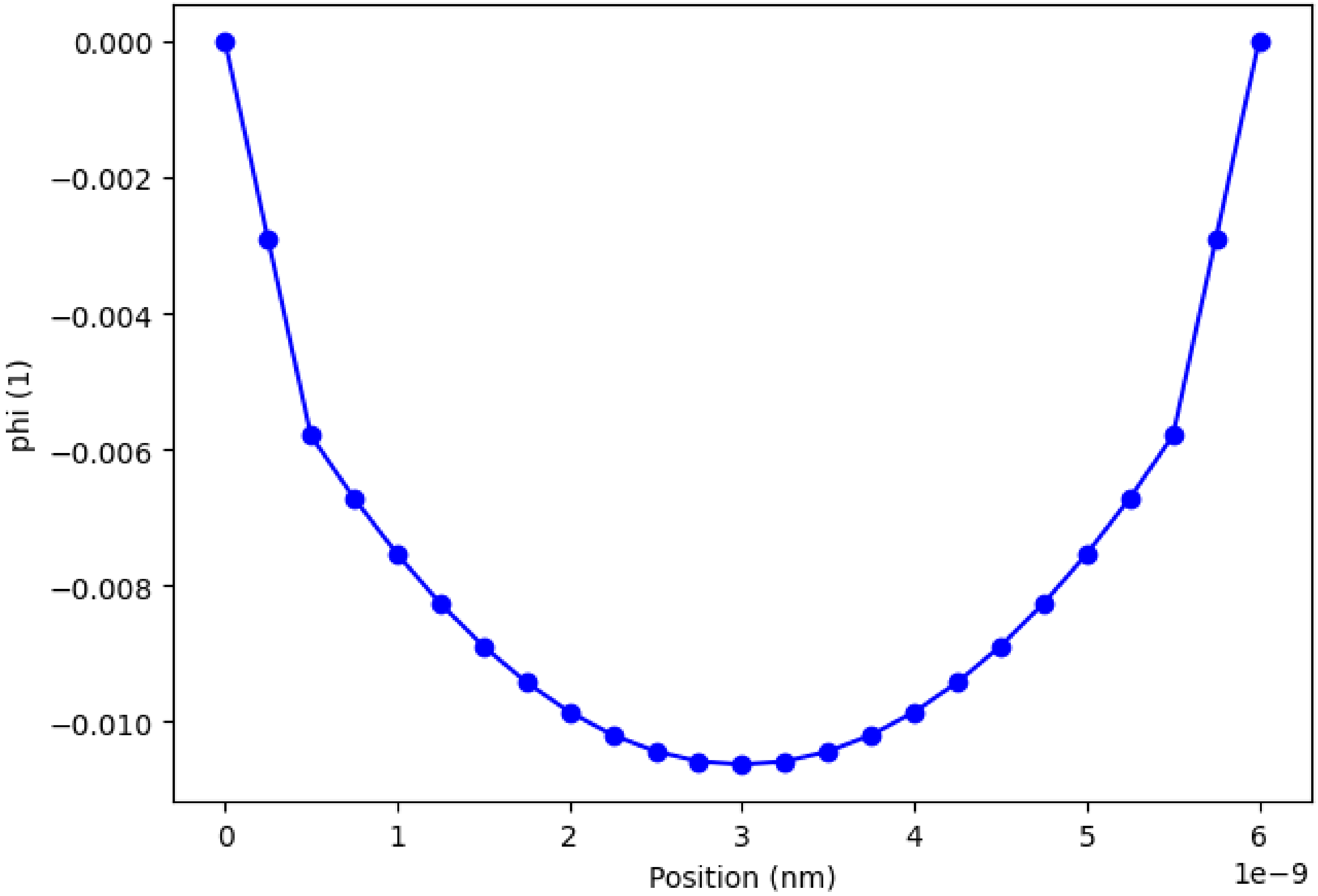
$$\epsilon(x_{i+0.5})\frac{\phi(x_{i+1}) - \phi(x_i)}{x_{i+1} - x_i} - \epsilon(x_{i-0.5})\frac{\phi(x_i) - \phi(x_{i-1})}{x_i - x_{i-1}}$$
$$= -qN_{dop}^{+}(x_i)(x_{i+0.5} - x_{i-0.5})$$

Double-gate

- 5-nm-thick Si substrate, 0.5-nm-thick oxide layers
  - Si permittivity: 11.7
  - SiO_2 permittivty: 3.9
  - P-type substrate: 10^{18} cm^{-3}

Its solution

- Potential, ϕ



Thank you!