Computational Microelectronics
Lecture 25 Small-Signal Analysis

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Frequency Response

Transient simulation

- Consider a single device, PN junction. (A symmetric one with 10^{17} cm^{-3})
  - While its cathode is grounded, the anode voltage is
$$V_{anode}(t) = V_{anode,DC} + v_{amp,sin}\sin(2\pi f t)$$
  - We assume a small amplitude, v_{amp}. (How small?)
  - Then, observe the anode current, after a long time elapses.
$$I_{anode}(t) = I_{anode,DC} + i_{amp,sin}\sin(2\pi f t) + i_{amp,\cos}\cos(2\pi f t)$$

  - Question: Why do we have only $I_{anode,DC}$, $i_{amp,sin}\sin(2\pi f t)$, and $i_{amp,\cos}\cos(2\pi f t)$ terms? Is this device a linear device?

One way to get the frequency response

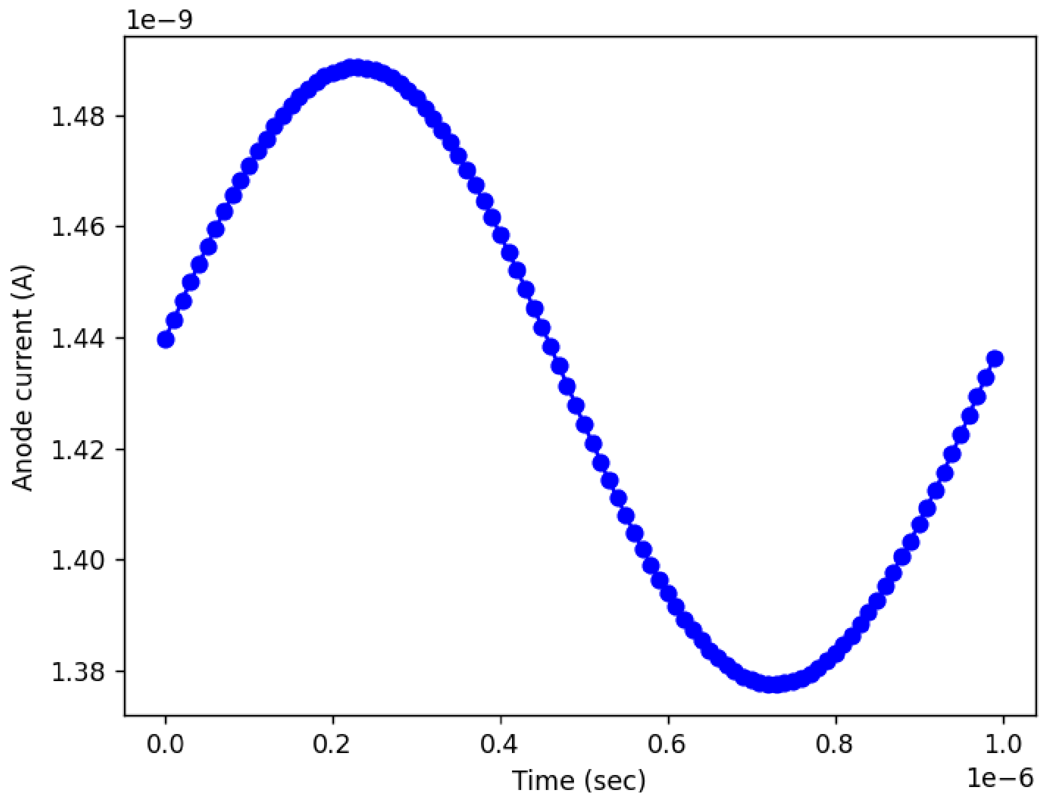
- Run a long transient simulation.
  - The initial response (due to introducing the time-varying excitation) will be diminished.
  - Then, two periods show the same response.
  - By using responses of the last period, calculate the frequency components. For example,

$$i_{amp,sin} = \frac{2}{T}\int_0^T I_{anode}(t)\sin(2\pi f t)\,dt$$

$$i_{amp,cos} = \frac{2}{T}\int_0^T I_{anode}(t)\cos(2\pi f t)\,dt$$

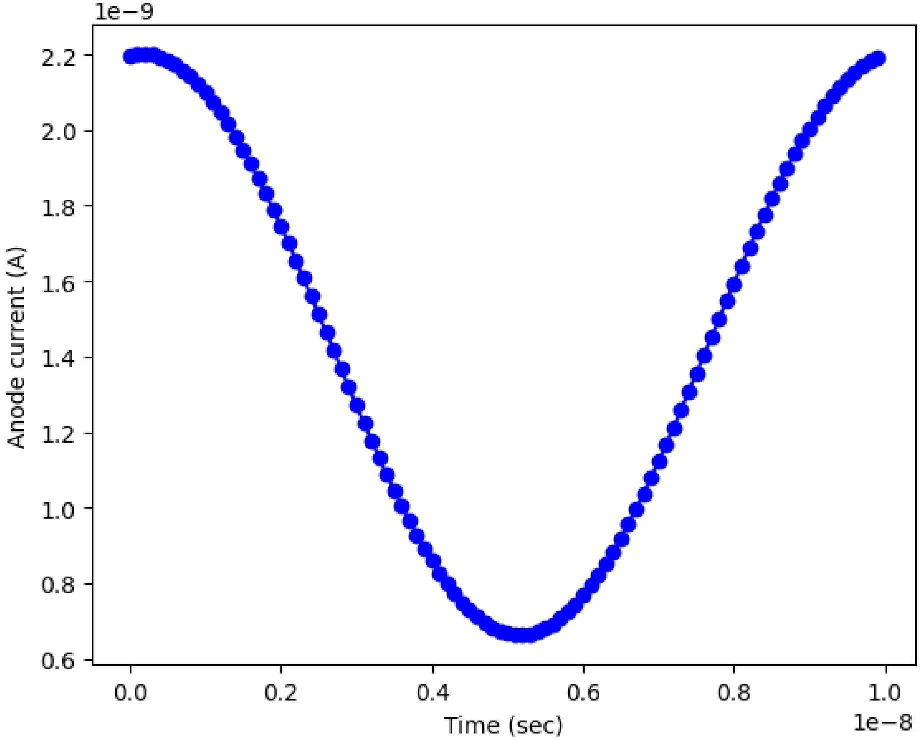
10^{th} period @ 0.5 V & 1 MHz

- Consider a 1 MHz signals, f is 10^6 Hz.
  - Amplitude is 1 mV.
  - For each period, 100 intervals are assigned.
  - Its DC value is 1.4326 nA.
  - Its sine amplitude is 54.958 pA.
  - Its cosine amplitude is 7.6511 pA.



10^{th} period @ 0.5 V & 100 MHz

- Consider a 100 MHz signals, f is 10^8 Hz.
  - The frequency is 100 times higher. Now it looks like a cosine function.
  - Its DC value is 1.4326 nA. (Not changed)
  - Its cosine amplitude is 765.10 pA. (100 times larger)



Small-Signal Analysis

Draw the response as a function of frequency.

- For example, from 1 Hz to 100 GHz?
  - The number of frequencies is N_f.
- Okay, then, starting from the DC solution,
  - Run several transient simulations.
  - Estimate the computational efforts:

$$(Single\ DC\ calculation) + \underline{(Single\ transient\ calculation)} \times N_f$$

It takes long…

An efficient approach

- Can we avoid the time-consuming transient calculation to obtain the frequency responst?
  - The answer is "Yes!"
  - In this efficient approach, the computational efforts can be expressed as

$$(Single\ DC\ calculation) + \underline{(Single\ frequency\ calculation}) \times N_f$$

It is much faster!

Important assumption

- We assume that time-varying quantities are "small" in their amplitudes.
  - Variables are expressed as

$$\phi(t) = \phi_{DC} + \delta\phi(t)$$
$$n(t) = n_{DC} + \delta n(t)$$
$$p(t) = p_{DC} + \delta p(t)$$

  - Under this assumption, we can write the DD model as

$$\nabla \cdot [\epsilon \nabla \phi_{DC} + \epsilon \nabla \delta\phi\ ] + q p_{DC} + q\delta p - q n_{DC} - q\delta n + q N_{dop}^{+} = 0$$

$$-q \frac{\partial \delta n}{\partial t} + \nabla \cdot \left(\mathbf{J}_{n,DC} + \delta \mathbf{J}_n \right) = 0$$

$$q \frac{\partial \delta p}{\partial t} + \nabla \cdot \left(\mathbf{J}_{p,DC} + \delta \mathbf{J}_p \right) = 0$$

Current densities

- Nonlinearity comes from the current densities.
  - Remember that
$$\mathbf{J}_n = -q\mu_n n \nabla\phi + qD_n \nabla n$$
$$\mathbf{J}_p = -q\mu_p p \nabla\phi - qD_p \nabla p$$
  - With time-varying variables,
$$\delta\mathbf{J}_n = -q\mu_n n_{DC} \nabla\delta\phi - q\mu_n \delta n \nabla\phi_{DC} + qD_n \nabla\delta n$$
$$\delta\mathbf{J}_p = -q\mu_p p_{DC} \nabla\delta\phi - q\mu_p \delta p \nabla\phi_{DC} - qD_p \nabla\delta p$$
  - The second order terms are nelected!
  - Of course, in the practical implementation, the Scharfetter-Gummel discretization is employed.

Linearization

- Note that ϕ_{DC}, n_{DC}, and p_{DC} are the DC solutions.
  - Without the time-dependent terms, they satisfy the DD model, too:

$$\nabla \cdot [\epsilon \nabla \phi_{DC} + \epsilon \nabla \delta \phi\ ] + q p_{DC} + q \delta p - q n_{DC} - q \delta n + q N_{dop}^{+} = 0$$

$$-q \frac{\partial \delta n}{\partial t} + \nabla \cdot \left(\mathbf{J}_{n,DC} + \delta \mathbf{J}_n \right) = 0$$

$$q \frac{\partial \delta p}{\partial t} + \nabla \cdot \left(\mathbf{J}_{p,DC} + \delta \mathbf{J}_p \right) = 0$$

  - In other words,

$$\nabla \cdot [\epsilon \nabla \delta \phi\ ] + q \delta p - q \delta n = 0$$

$$-q \frac{\partial \delta n}{\partial t} + \nabla \cdot (\delta \mathbf{J}_n) = 0$$

$$q \frac{\partial \delta p}{\partial t} + \nabla \cdot \left(\delta \mathbf{J}_p \right) = 0$$

Relation to Jacobian

- The linearized system is closely related to the Jacobian matrix.
  - The product between the Jacobian matrix and the update vector (Ax) describes the following functions:

$$\nabla \cdot [\epsilon \nabla \delta\phi\ ] + q\delta p - q\delta n$$
$$\nabla \cdot (\delta \mathbf{J}_n)$$
$$\nabla \cdot \left(\delta \mathbf{J}_p\right)$$

  - Simply speaking, the DC Jacobian matrix is related to the small-signal analysis at the zero frequency (or the low-frequency limit).
  - Of course, we must consider $-q\frac{\partial \delta n}{\partial t}$ and $q\frac{\partial \delta p}{\partial t}$ for general cases.

Non-zero frequency

- In the frequency domain, let us introduce the time dependence of $e^{j2\pi f t}$.
  - Then, the time derivation terms can be expressed as

$$-q\frac{\partial \delta n e^{j2\pi f t}}{\partial t} = -q(j2\pi f)\delta n e^{j2\pi f t}$$

$$q\frac{\partial \delta p e^{j2\pi f t}}{\partial t} = q(j2\pi f)\delta p e^{j2\pi f t}$$

  - Therefore, in the frequency domain, the linearized system reads

$$\nabla \cdot [\epsilon \nabla \delta\phi] + q\delta p - q\delta n = 0$$

$$-j2\pi f q\delta n + \nabla \cdot (\delta \mathbf{J}_n) = 0$$

$$j2\pi f q\delta p + \nabla \cdot (\delta \mathbf{J}_p) = 0$$

Thank you!