Computational Microelectronics
Lecture 18 Continuity Equation

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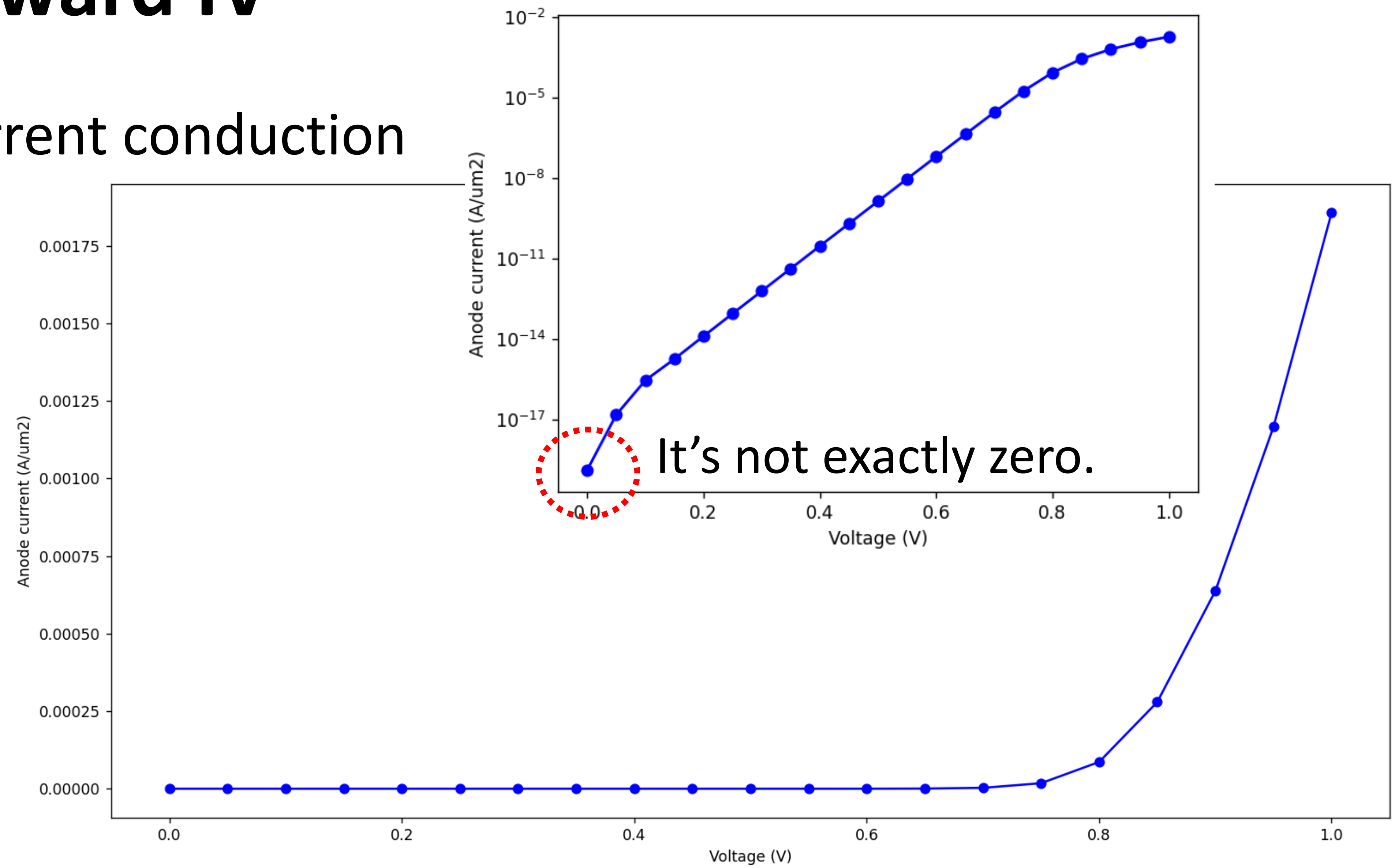
Bias Ramping

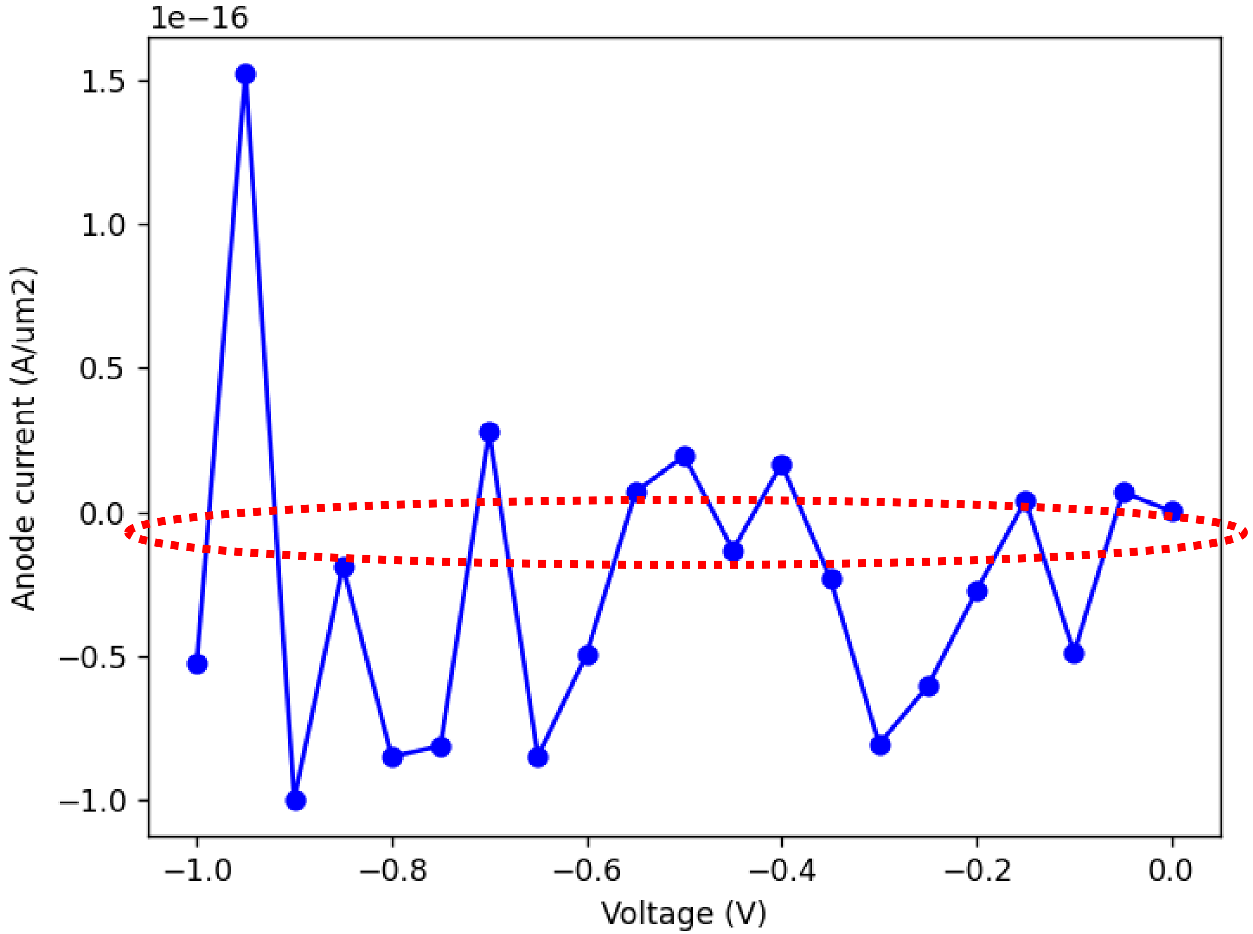
Forward IV

- Current conduction



Reverse IV

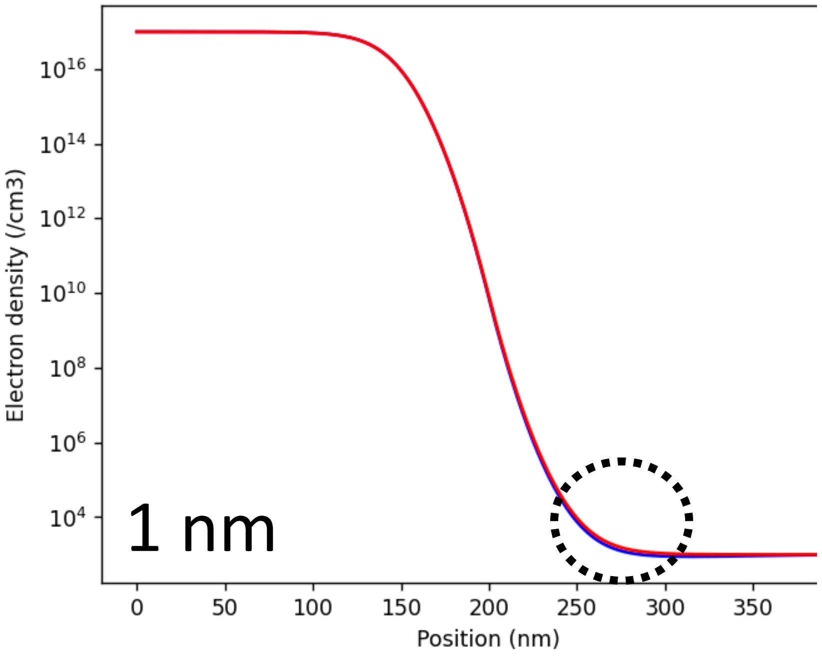
- No current conduction

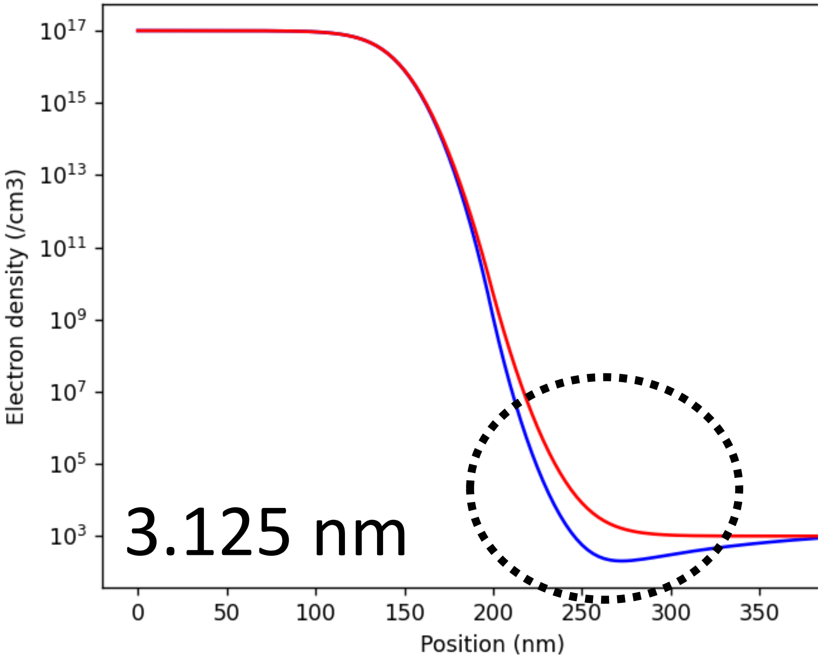


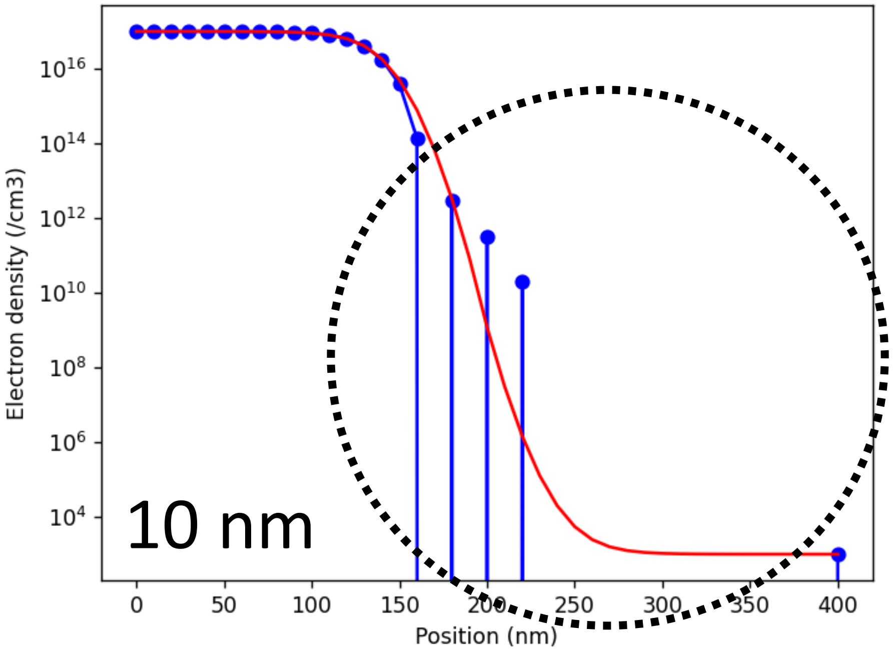
Failure of Naïve Implementation

Electron density at equilibrium

- Drift-diffusion result at 0 V
  - 401 mesh points, 129 mesh points, and 41 mesh points

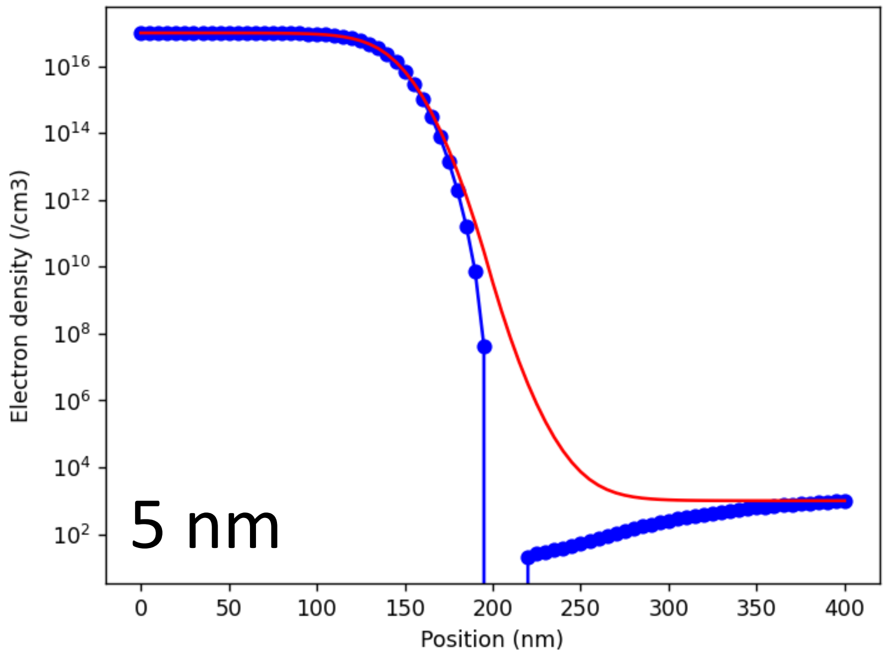


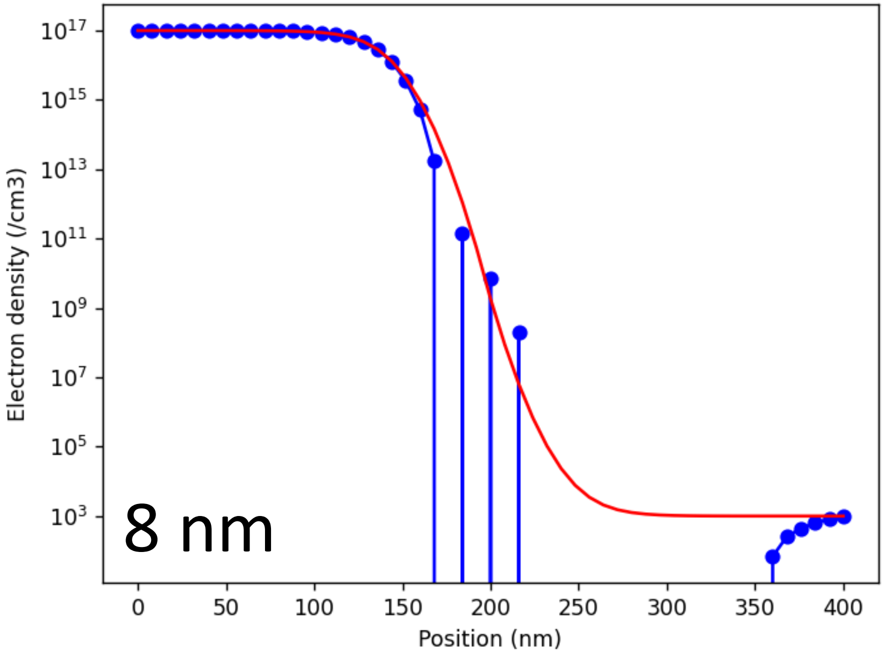




Negative electron density?

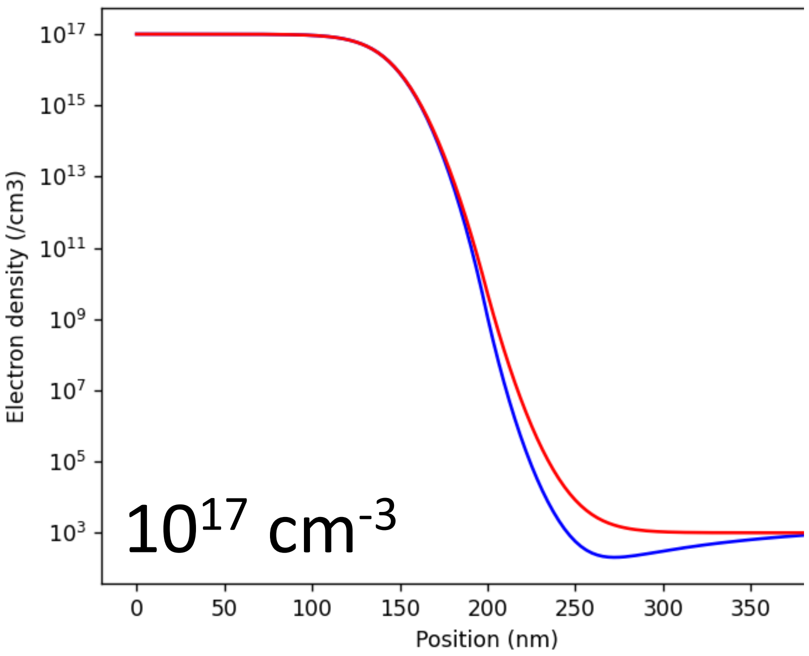
- It's non-sense.
  - Failure of our discretization scheme with a coarse grid
  - 81 mesh points and 51 mesh points

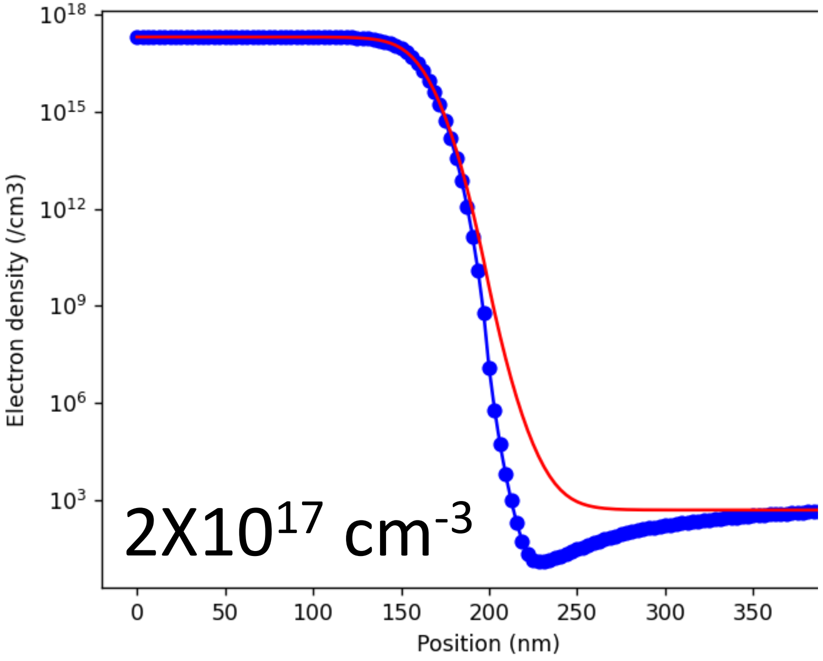


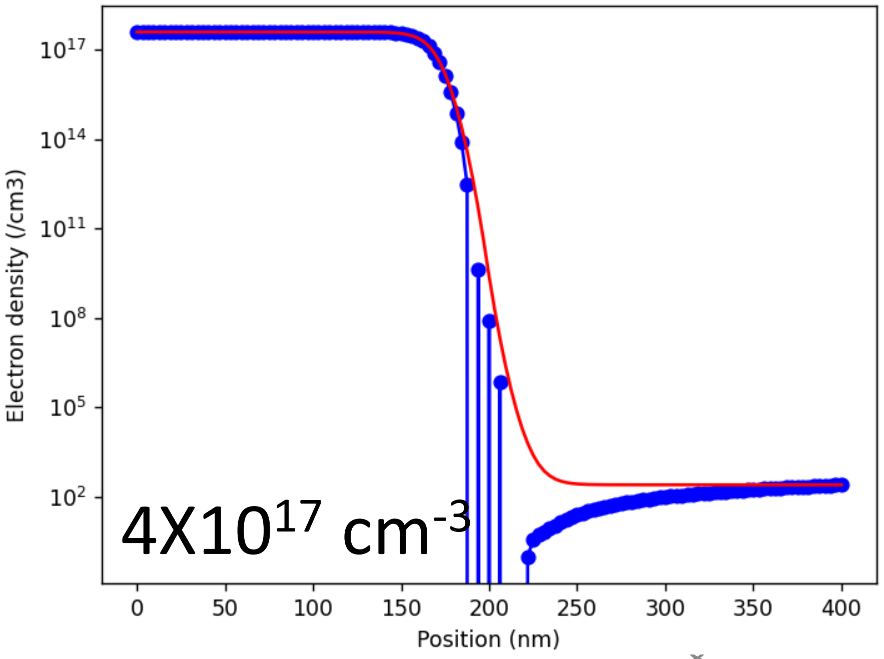


Doping dependance

- Spacing of 3.125 nm
  - 10^{17} cm^{-3}, $2X10^{17}$ cm^{-3} , and $4X10^{17}$ cm^{-3}
  - Worse result for a higher doping density

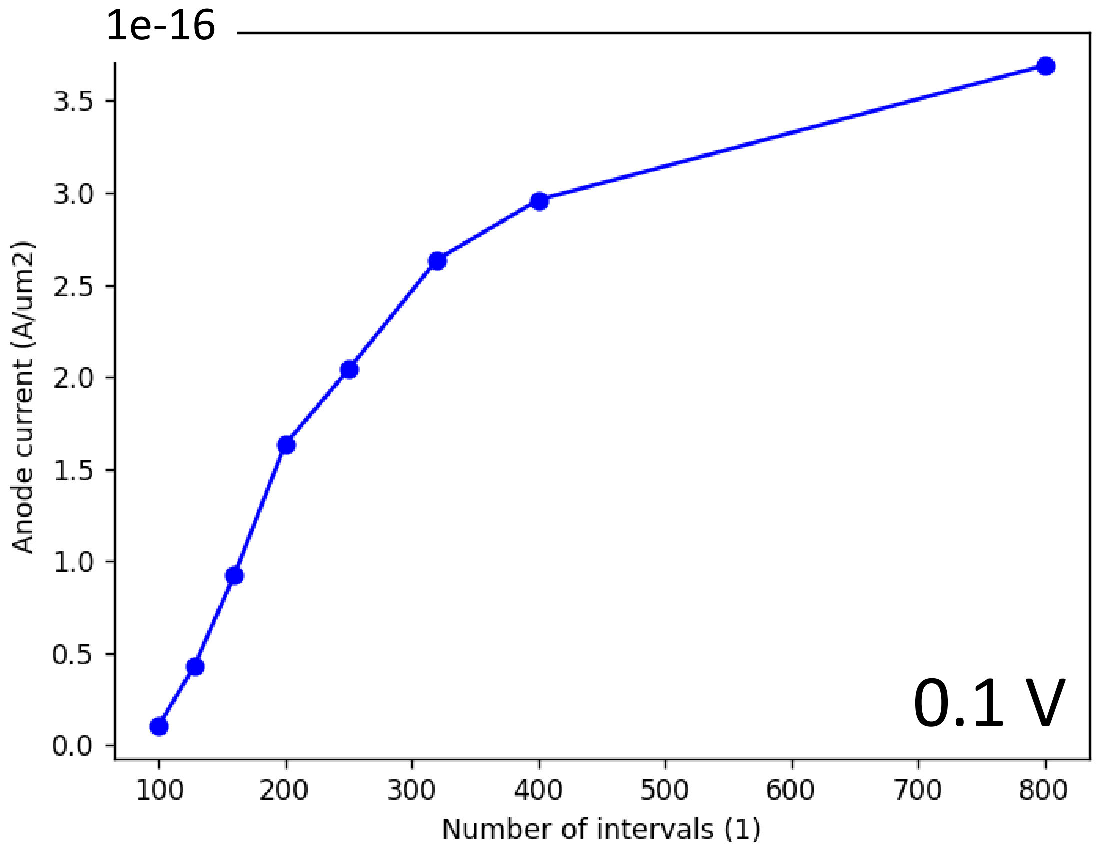


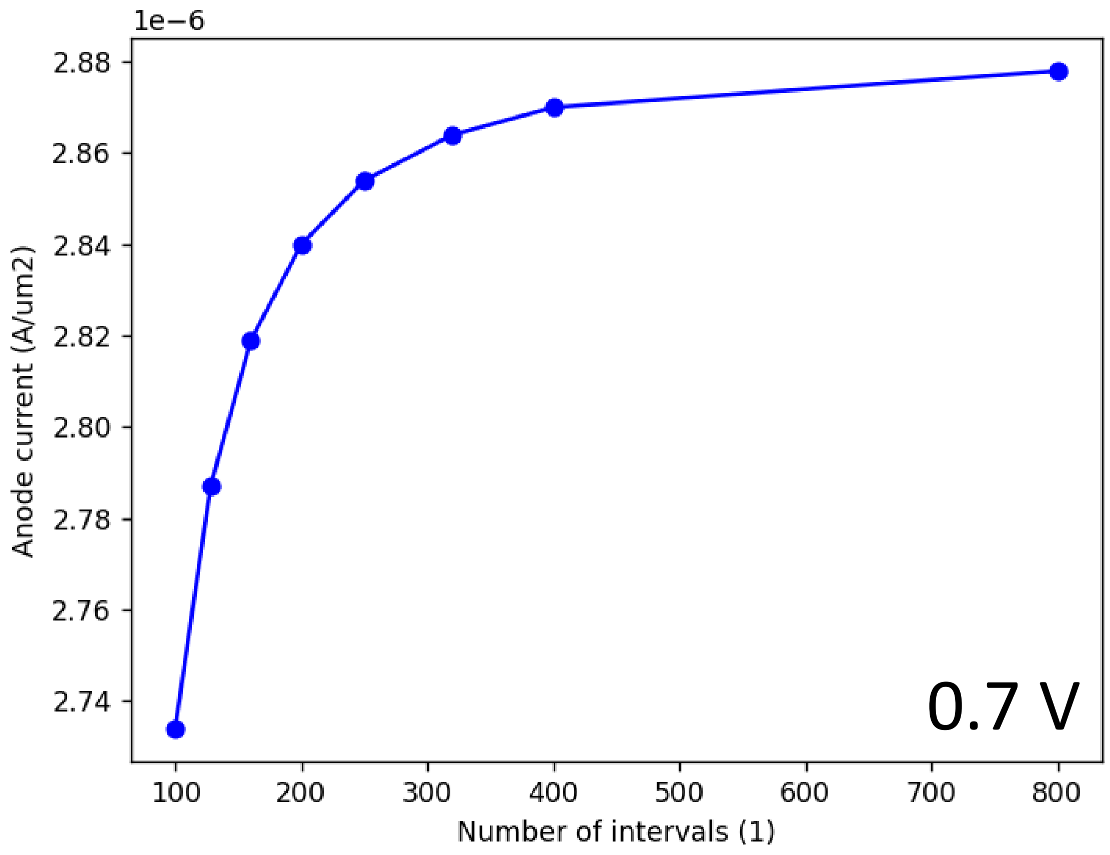




Forward IV

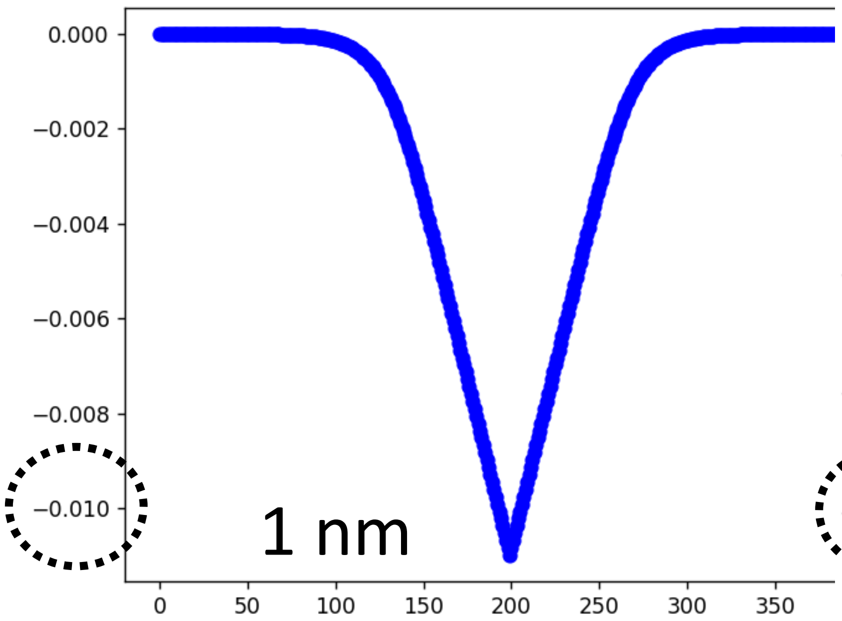
- Anode current at V_{anode} = 0.1 V and 0.7 V
  - 101, 129, 161, 201, 251, 321, 401, and 801 mesh points

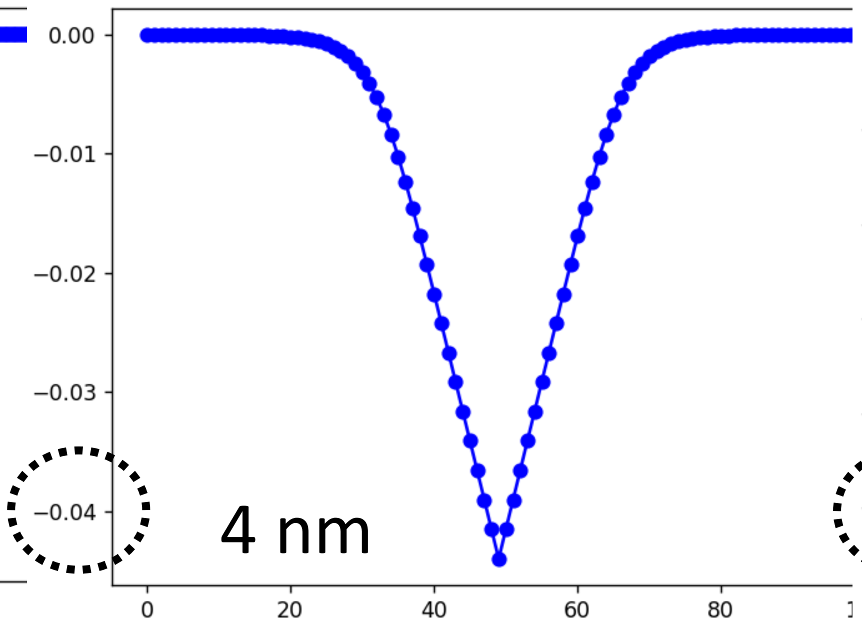


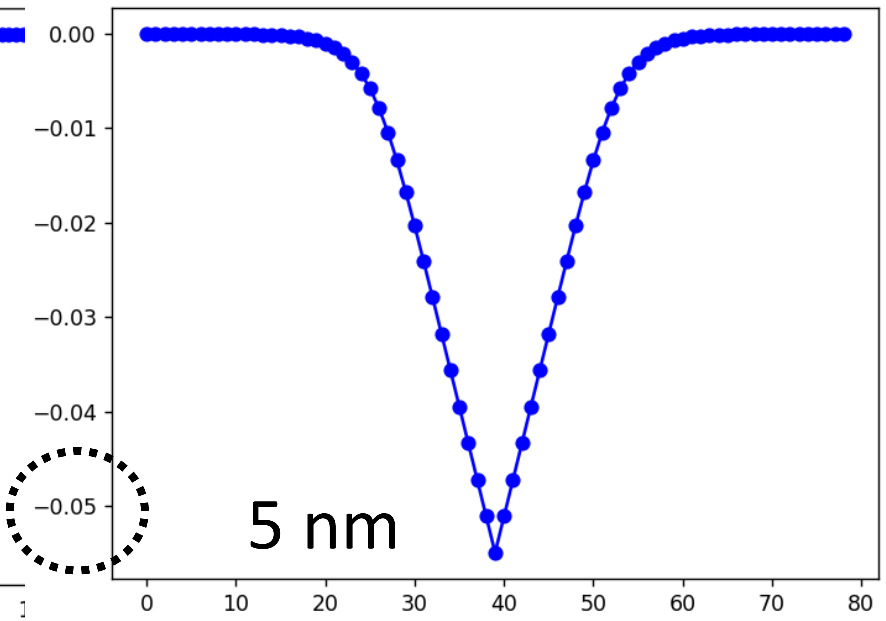


Potential difference at equilibrium

- `plt.plot(phi[1:N]-phi[0:N-1],'bo-')`
  - Maximum potential difference (absolute value) of 0.05 V seems to be a threshold for the negative density .







Scharfetter-Gummel Scheme

Importance of S-G scheme

- “The equation that started it all”
  - M. Lundstrom, SISPAD 2015 presentation

SISPAD 2015, September 9-11, 2015, Washington, DC, USA

Drift-Diffusion and computational electronics – Still going strong after 40 years!

Reflections on computational electronics and the equation that started it all

Derivation (1)

- The electron current density in 1D
  - It is treated as a differential equation for n.

$$J_n = -q\mu_n n \frac{d\phi}{dx} + qD_n \frac{dn}{dx}$$

  - Assumption: J_n is a constant. (Current continuity, $\frac{dJ_n}{dx} = 0$)
  - Assumption: $\frac{d\phi}{dx} \approx \frac{\Delta\phi}{\Delta x}$
  - After simple manipulation,

$$\frac{dn}{dx} - \frac{1}{V_T}\frac{\Delta\phi}{\Delta x} n = \frac{J_n}{qD_n}$$

Treated as contants

Derivation (2)

$$J_n = -\frac{qD_n}{V_T}\frac{\Delta\phi}{\Delta x}C_2$$

- First-order differential equation
  - The solution has the following form:

$$n(x) = \underbrace{C_1 \exp\left(\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x\right)}_{\text{Homogeneous solution}} + C_2$$

  - We must find out two contants, C_1 and C_2, to satisfy

$$n(x_i) = n_i$$
$$n(x_{i+1}) = n_{i+1}$$

Derivation (3)

- Boundary values
  - At two boundaries,

$$n_i = C_1 \exp\left(\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i\right) + C_2$$

$$n_{i+1} = C_1 \exp\left(\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_{i+1}\right) + C_2$$

  - Taking the difference,

$$n_{i+1} - n_i = C_1 \exp\left(\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i\right) \times \left(\exp\frac{\Delta\phi}{V_T} - 1\right)$$

  - Now, we know C_1.

Derivation (4)

- Calculate C_2.
  - Recall that

$$C_2 = n_i - C_1 \exp\left(\frac{1}{V_T}\frac{\Delta\phi}{\Delta x}x_i\right)$$

  - By using C_1,

$$C_2 = n_i - \frac{n_{i+1} - n_i}{\exp\frac{\Delta\phi}{V_T} - 1} = n_i \frac{\exp\frac{\Delta\phi}{V_T}}{\exp\frac{\Delta\phi}{V_T} - 1} - n_{i+1}\frac{1}{\exp\frac{\Delta\phi}{V_T} - 1}$$

  - From $J_n = -\frac{qD_n}{V_T}\frac{\Delta\phi}{\Delta x}B$,

Derivation (5)

- We are almost there.
  - From $J_n = -\frac{qD_n}{V_T}\frac{\Delta\phi}{\Delta x}C_2$,

$$J_n = \frac{qD_n}{\Delta x}\left(n_{i+1}\frac{\frac{\Delta\phi}{V_T}}{\exp\frac{\Delta\phi}{V_T} - 1} - n_i\frac{\frac{\Delta\phi}{V_T}\exp\frac{\Delta\phi}{V_T}}{\exp\frac{\Delta\phi}{V_T} - 1} \right)$$

Thank you!