Computational Microelectronics
Lecture 22 Transient

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RC Circuit

Euler schemes (Copied from L4)

- Forward Euler scheme
  - In the forward Euler scheme, the time derivative is calculated by taking the difference between the ***future*** and ***present*** values:

$$\left.\frac{\partial C(x_i)}{\partial t}\right|_{t_k} \approx \frac{C(t_{k+1}) - C(t_k)}{t_{k+1} - t_k}$$

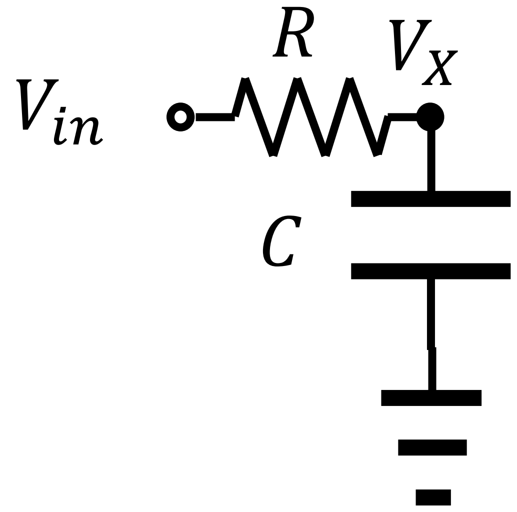
- Backward Euler scheme
  - In the backward Euler scheme, the time derivative is calculated by taking the difference between the ***present*** and ***past*** values:

$$\left.\frac{\partial C(x_i)}{\partial t}\right|_{t_k} \approx \frac{C(t_k) - C(t_{k-1})}{t_k - t_{k-1}}$$

RC circuit

- Consider a series RC circuit.
  - It is driven by a voltage source, $V_{in}(t)$. For $t < 0$, it was zero.

$$\frac{V_X - V_{in}(t)}{R} + C\frac{dV_X}{dt} = 0$$



Analytic solution

- First-order differential equation
  - At $t = 0, V_{in}(t)$ is suddenly ramped up to V_{DD},

$$V_X(t) = V_{DD}\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$$

Backward Euler

- Let us assume a constant time step, Δt.
  - At $t_i = i\Delta t \ (i \geq 1)$, the KCL can be discretized as

$$\frac{V_X(t_i) - V_{in}(t_i)}{R} + C\,\frac{V_X(t_i) - V_X(t_{i-1})}{\Delta t} = 0$$

  - After a simple manipulation,

$$\left(1 + \frac{\Delta t}{RC}\right) V_X(t_i) = V_X(t_{i-1}) + \frac{\Delta t}{RC} V_{in}(t_i)$$

  - For our present example, $V_{in}(t_i)$ is always V_{DD}.

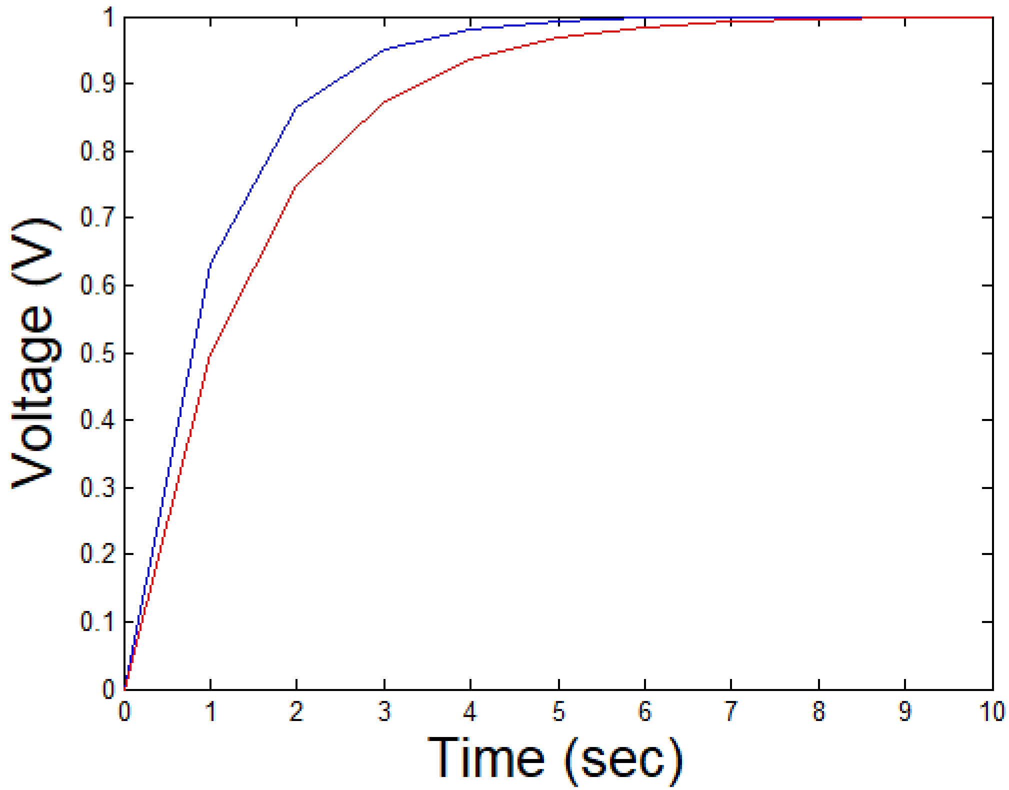
Impact of Time Step

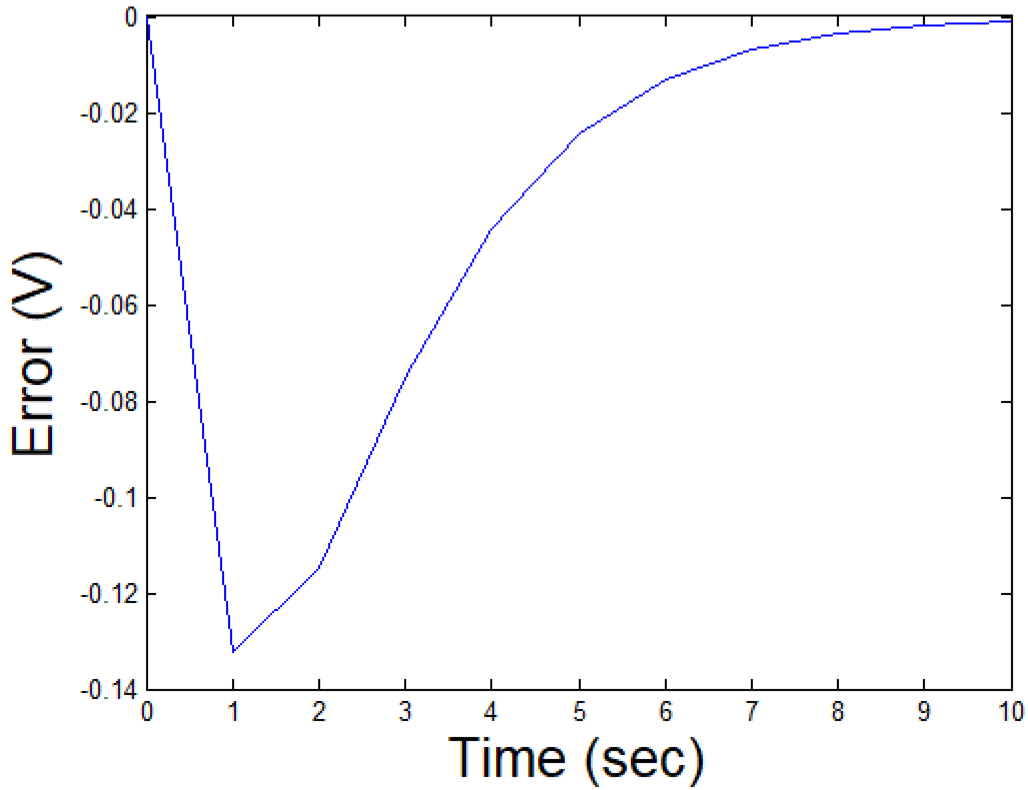
$R = C = V_{DD} = 1$ with their own dimensions

- It means that R is 1 Ω, C is 1 F, and V_{DD} is 1 V.
  - In this case, the equation is simply written as
$$(1 + \Delta t)V_X(t_i) = V_X(t_{i-1}) + \Delta t$$
  - Its analytic solution is
$$V_X(t_i) = 1 - \exp(-t_i)$$

Case study) $\Delta t = 1$

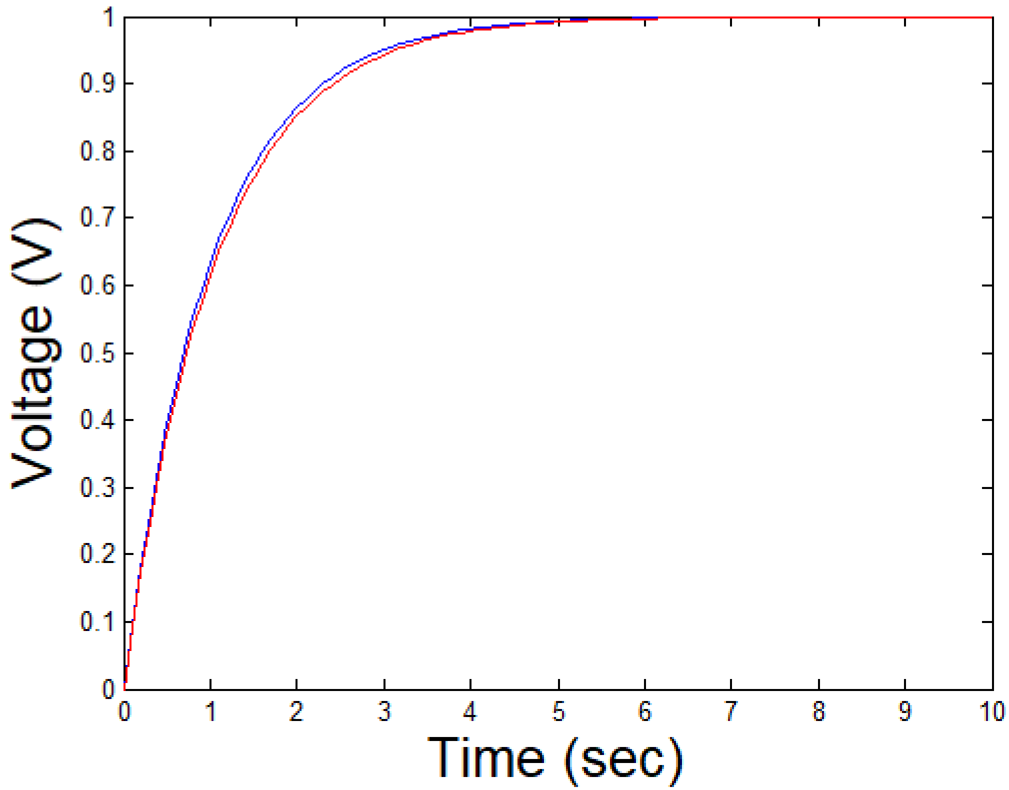
- Run the simulation up to 10 sec.
  - Exact solution (blue curve in left figure), numerical solution (red cruve in left figure), and error (right figure)

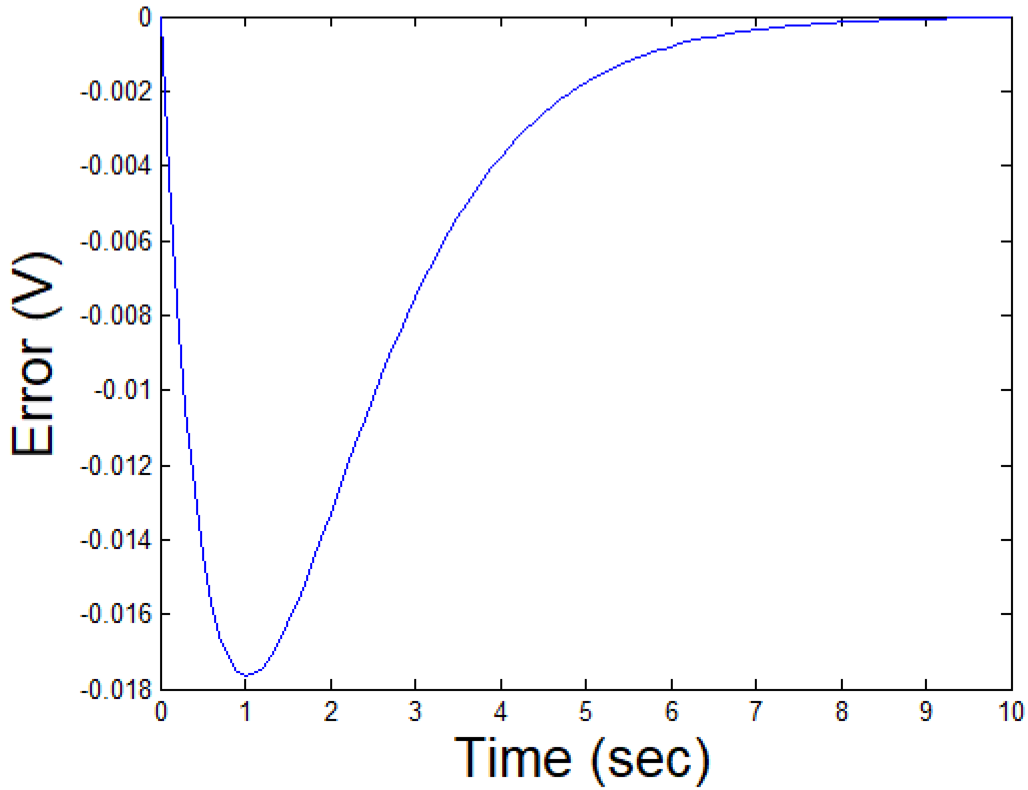




Case study) $\Delta t = 0.1$

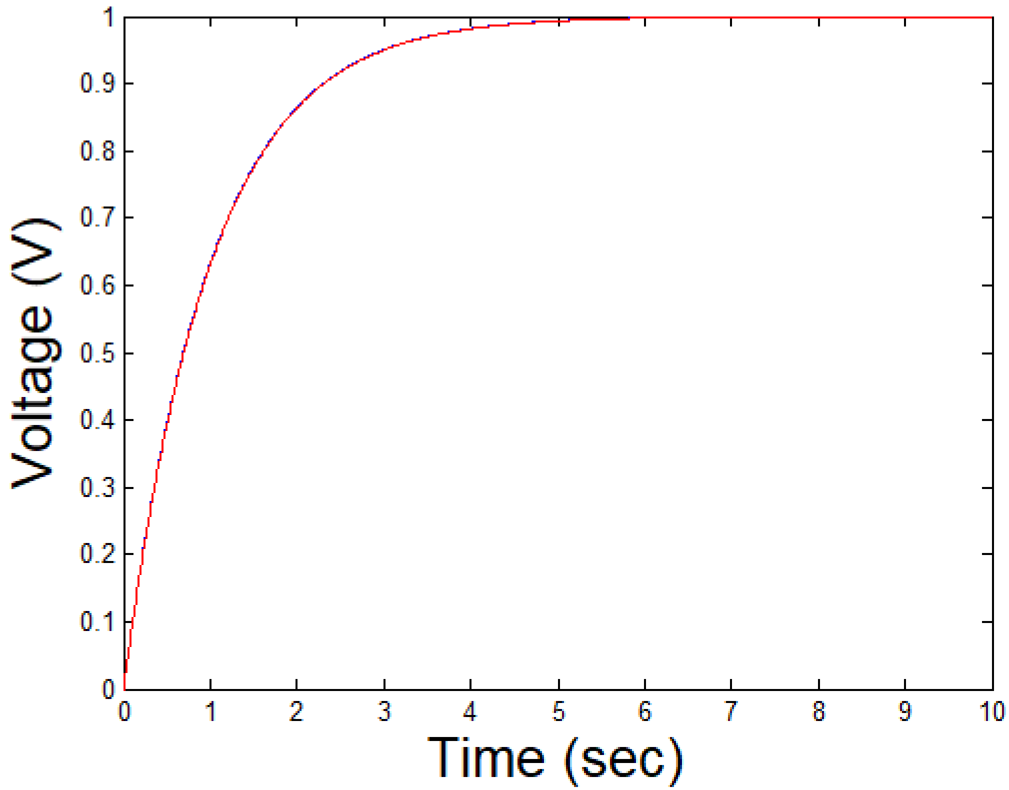
- Ten times shorter time spacing
  - It looks much better.
  - Still, we have the maximum difference larger than 0.01 V.

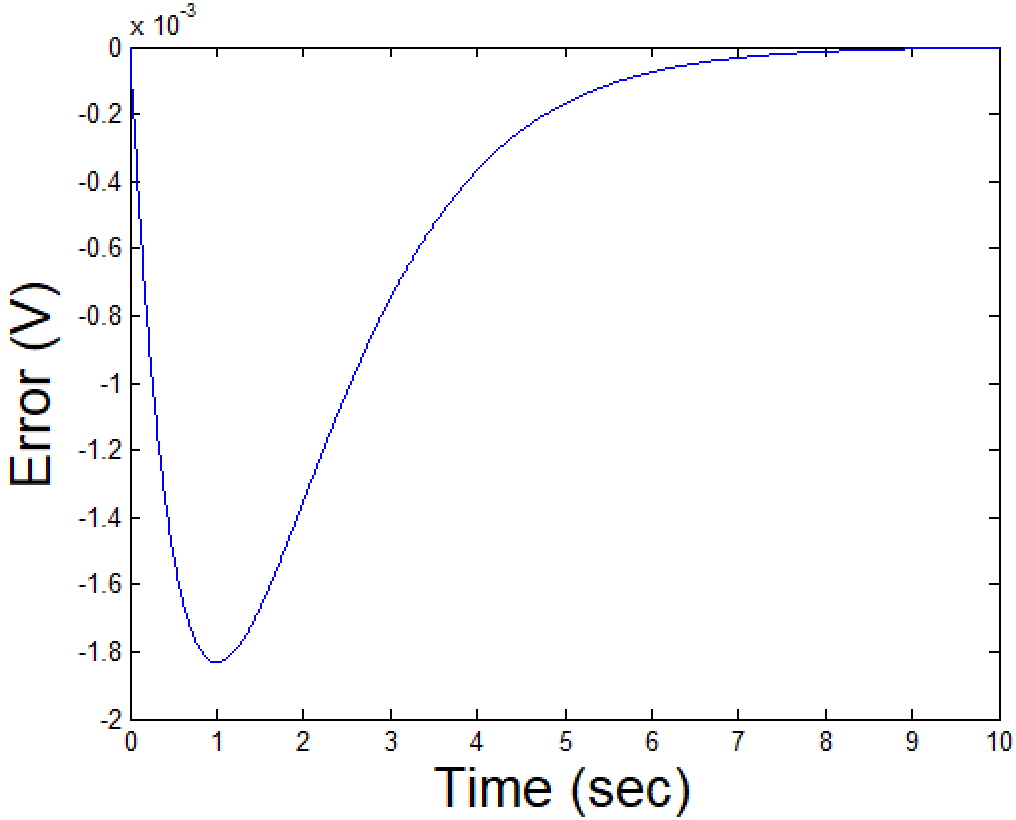




Case study) $\Delta t = 0.01$

- Ten times shorter time spacing, again
  - The difference becomes almost invisible.
  - Still, we have the maximum difference larger than 0.01 V.





Another method

- Gear's 2nd order method
  - Assume a constant time spacing, Δt.
  - Then, for $i \geq 2$,

$$\left.\frac{\partial f}{\partial t}\right|_{t_i} \approx \frac{1.5f(t_i) - 2f(t_{i-1}) + 0.5f(t_{i-2})}{\Delta t}$$

  - For $i = 1$, just use the backward Euler method.
  - It means that

$$\frac{V_X(t_i) - V_{in}(t_i)}{R} + C\,\frac{1.5V_X(t_i) - 2V_X(t_{i-1}) + 0.5V_X(t_{i-2})}{\Delta t} = 0$$

After re-arranging terms,

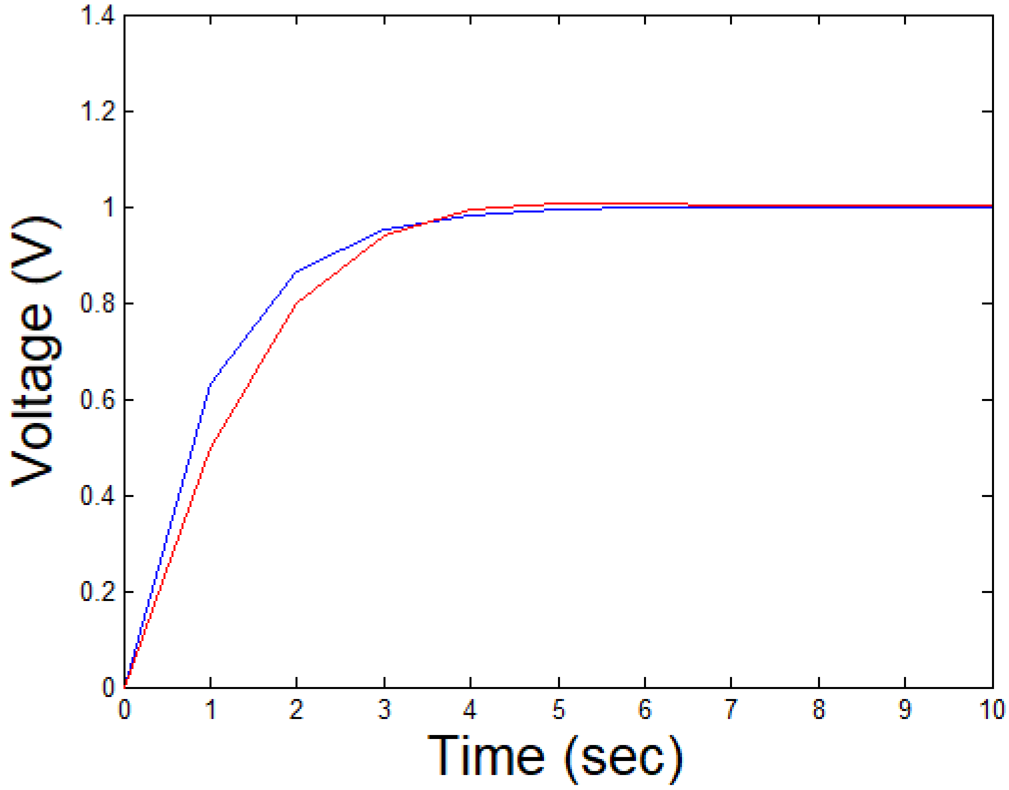
- A slightly different expression for $V_X(t_i)$
  - Compare it against the backward Euler:

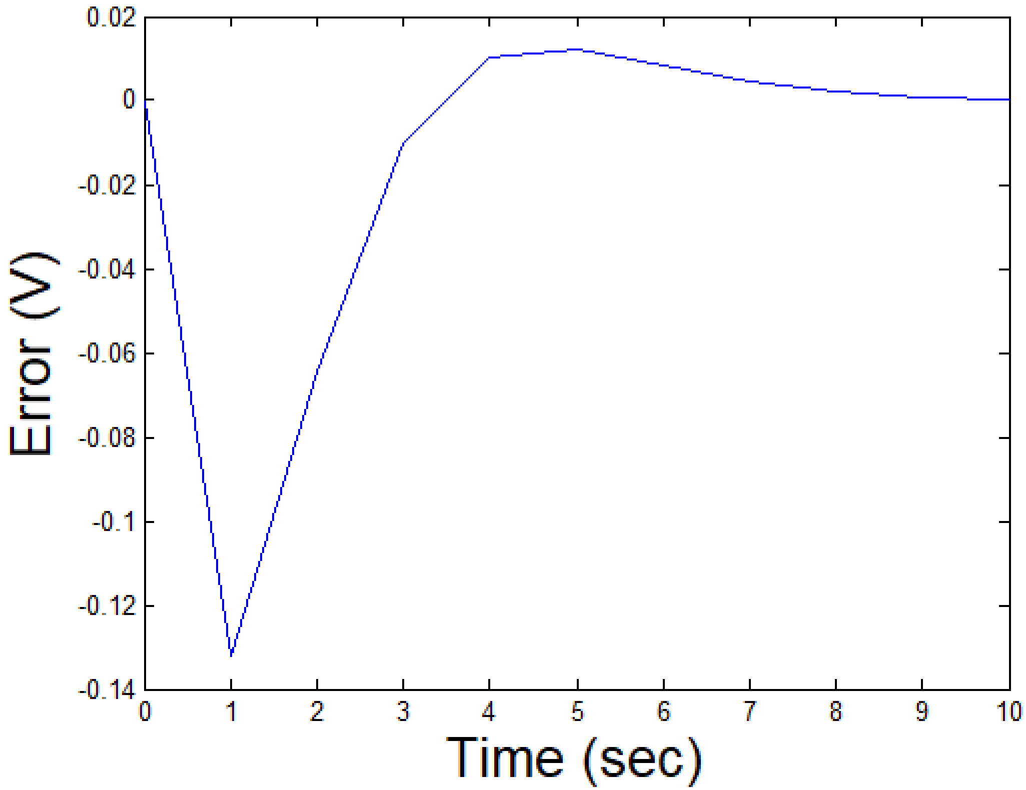
$$(1.5 + \Delta t)V_X(t_i) = 2V_X(t_{i-1}) - 0.5V_X(t_{i-2}) + \Delta t$$

$$(1 + \Delta t)V_X(t_i) = V_X(t_{i-1}) + \Delta t$$

Case study) $\Delta t = 1$ with Gear's method

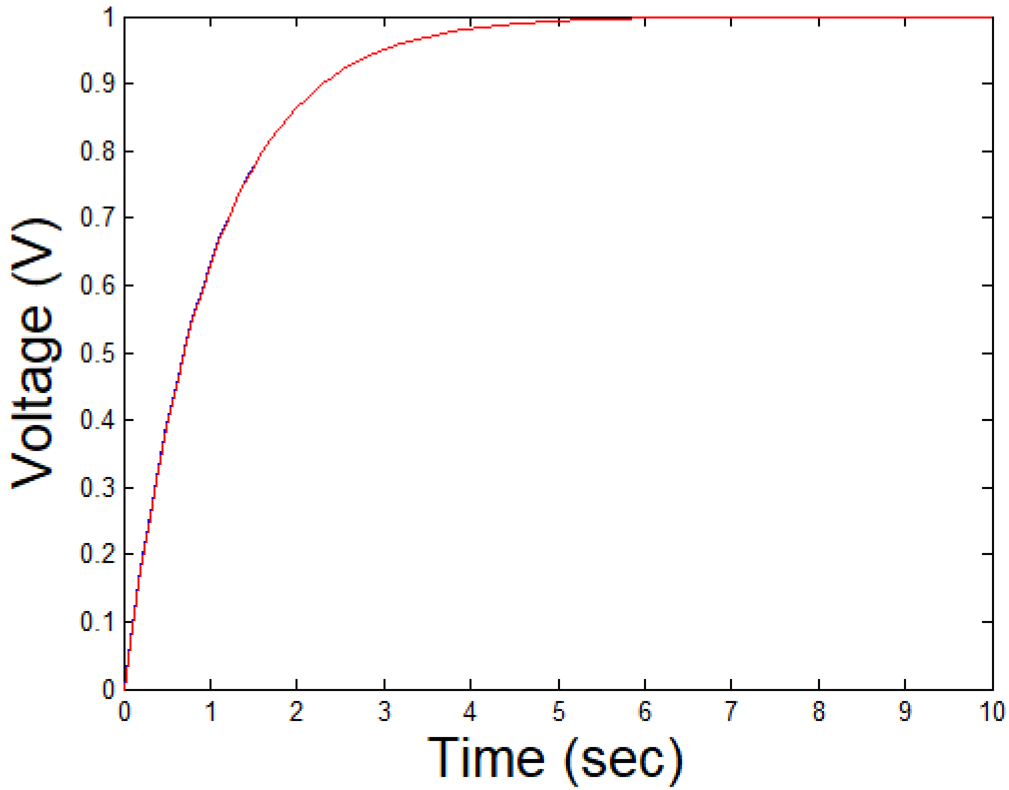
- Run the simulation up to 10 sec, again.
  - The maximum error is determined by the first backward Euler step.

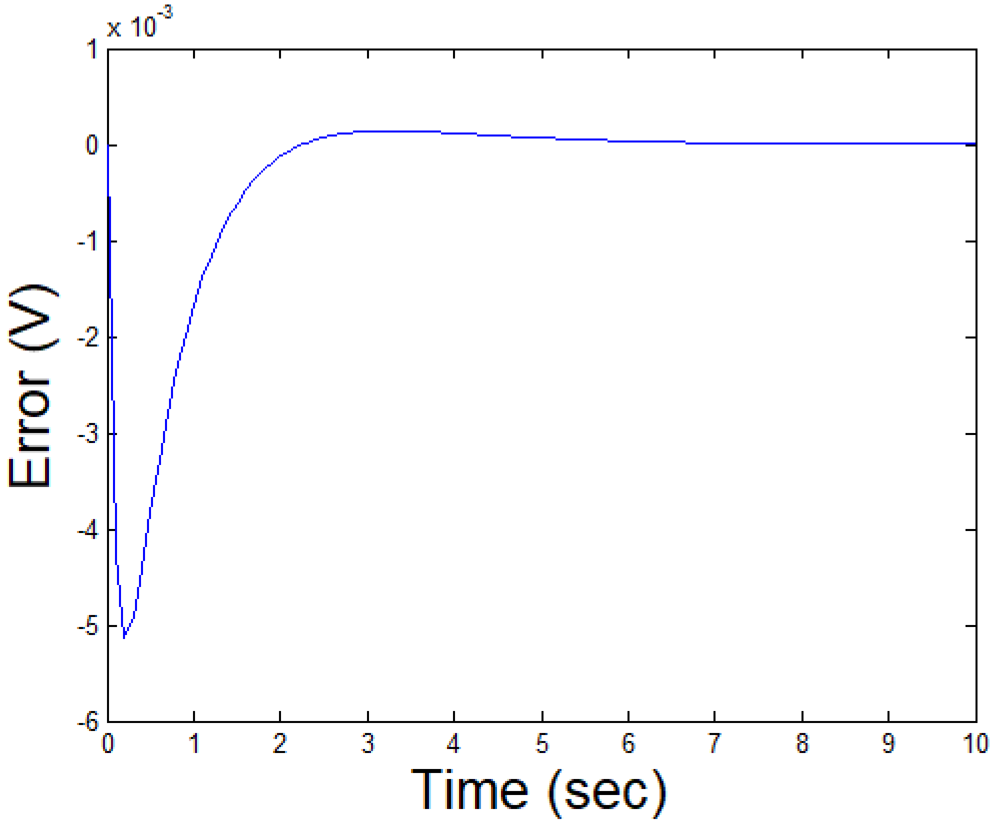




Case study) $\Delta t = 0.1$ with Gear's method

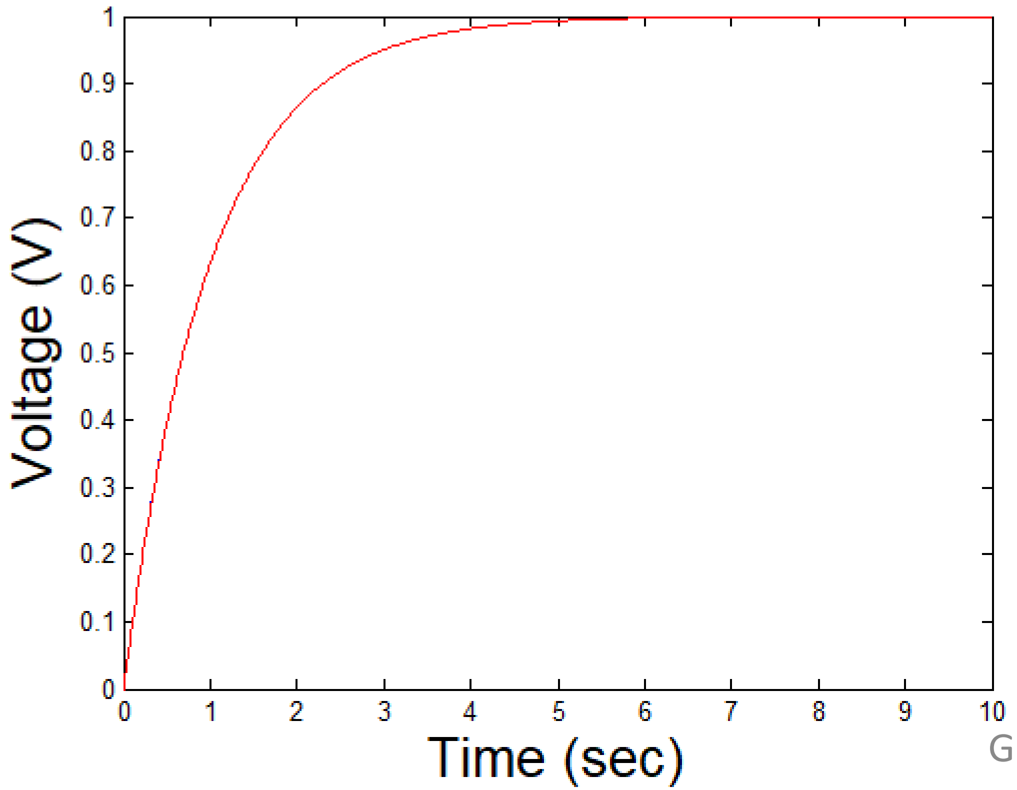
- Ten times shorter time spacing
  - The difference becomes almost invisible.
  - The maximum difference is smaller than that of the backward Euler.

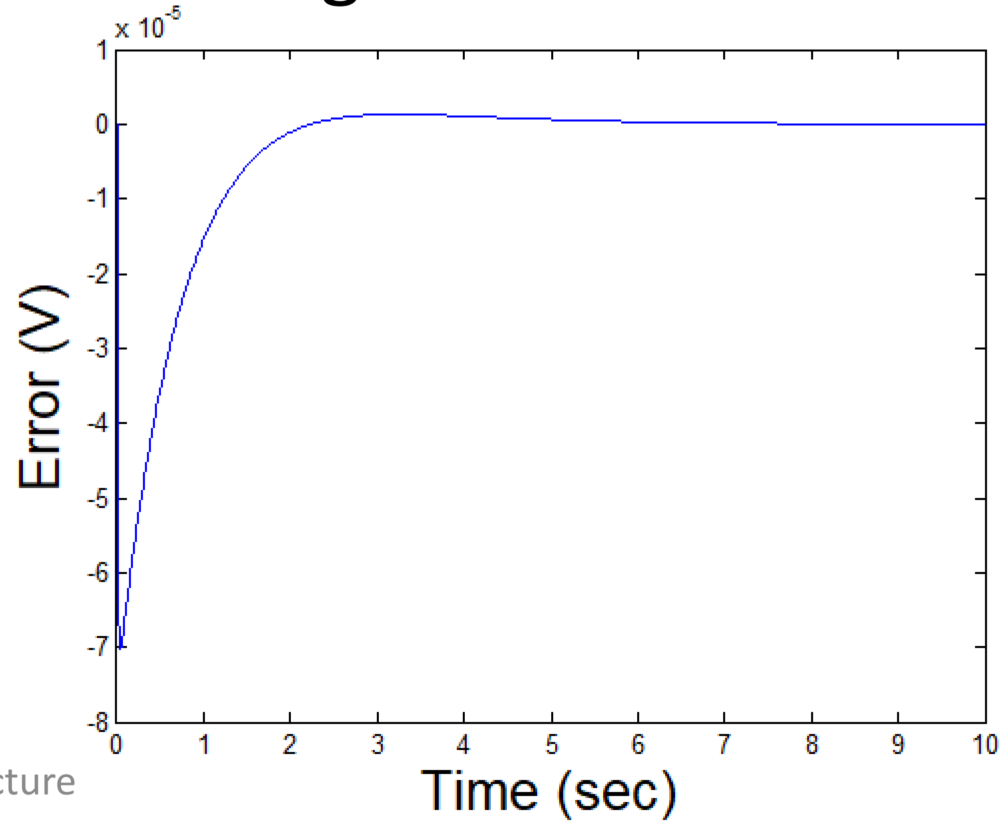




Case study) $\Delta t = 0.01$

- Ten times shorter time spacing, again
  - Much smaller difference is obtained.
- In summary, the error is rapidly decreasing with a smaller Δt.





Sinusoidal voltage source

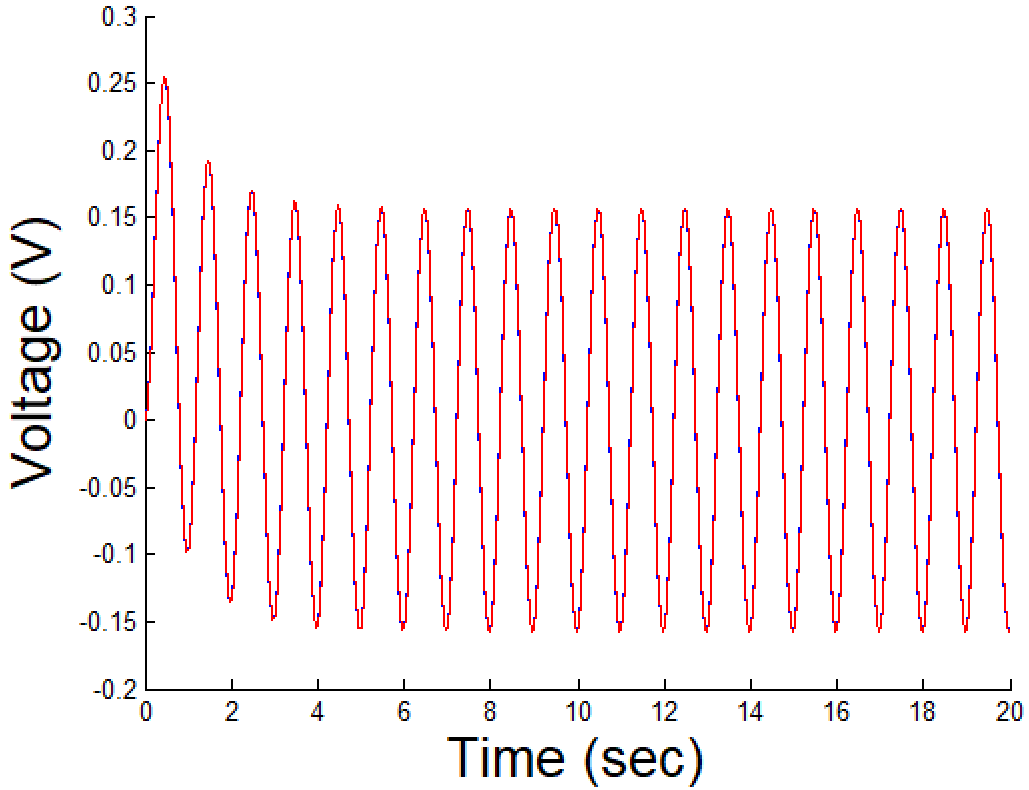
- Apply a sine wave with its amplitude of 1 V.
  - The frequency is 1 Hz.
  - Then, the amplitude of V_X should be $\frac{1}{\sqrt{1+(2\pi)^2}}$ V. (0.15718 V)
  - Investigate the amplitude with various Δt values.

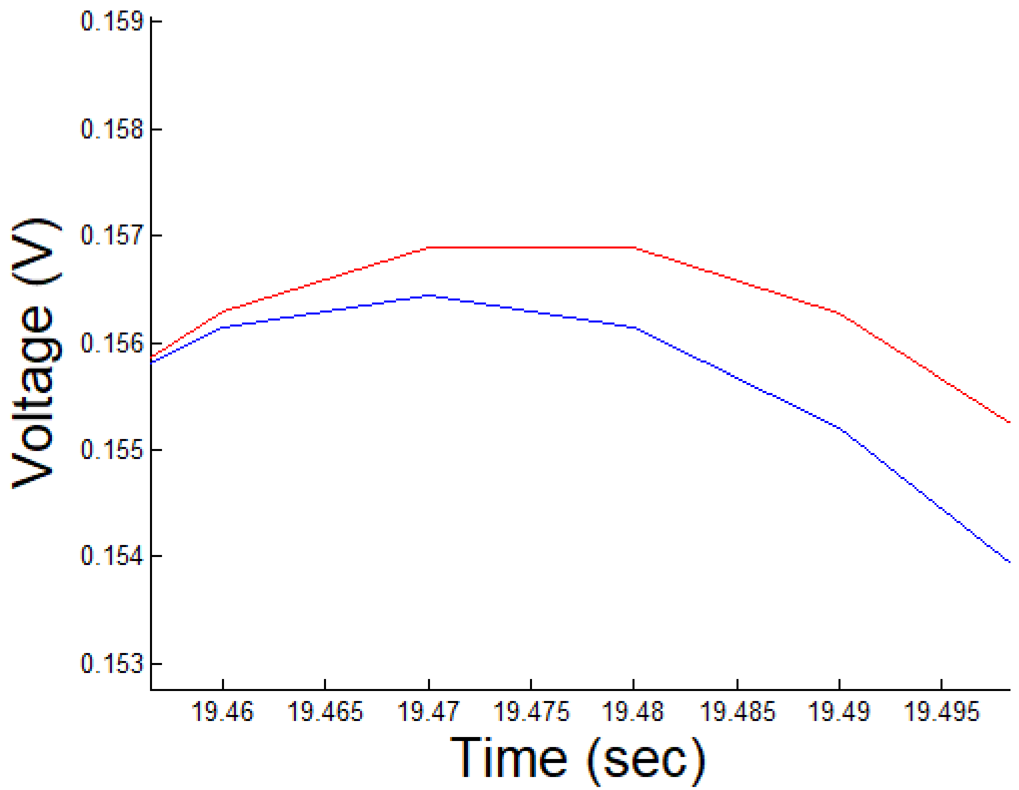
$$(1.5 + \Delta t)V_X(t_i) = 2V_X(t_{i-1}) - 0.5V_X(t_{i-2}) + \Delta t \sin 2\pi t$$

$$(1 + \Delta t)V_X(t_i) = V_X(t_{i-1}) + \Delta t \sin 2\pi t$$

Case study) $\Delta t = 0.01$

- Only ten points in a period
  - 20 cycles are passed. (Sufficiently stabilized)
  - Gear's method predicts a larger amplitude, close to 0.15718 V.





Thank you!