Computational Microelectronics L19

Sung-Min Hong

smhong@gist.ac.kr

Semiconductor Device Simulation Laboratory, GIST

Small-signal analysis

Transient simulation

- Consider a single device, PN junction. (A symmetric one with 10^{17} cm^{-3})
  - While its cathode is grounded, the anode voltage is
  $$V_{anode}(t) = V_{anode,DC} + v_{amp,sin}\sin(2\pi f t)$$
  - We assume a small amplitude, v_{amp}. (How small?)
  - Then, observe the anode current, after a long time elapses.
  $$I_{anode}(t) = I_{anode,DC} + i_{amp,sin}\sin(2\pi f t) + i_{amp,cos}\cos(2\pi f t)$$
  - Question: Why do we have only $I_{anode,DC}$, $i_{amp,sin}\sin(2\pi f t)$, and $i_{amp,cos}\cos(2\pi f t)$ terms? Is this device a linear device?

Getting the frequency response

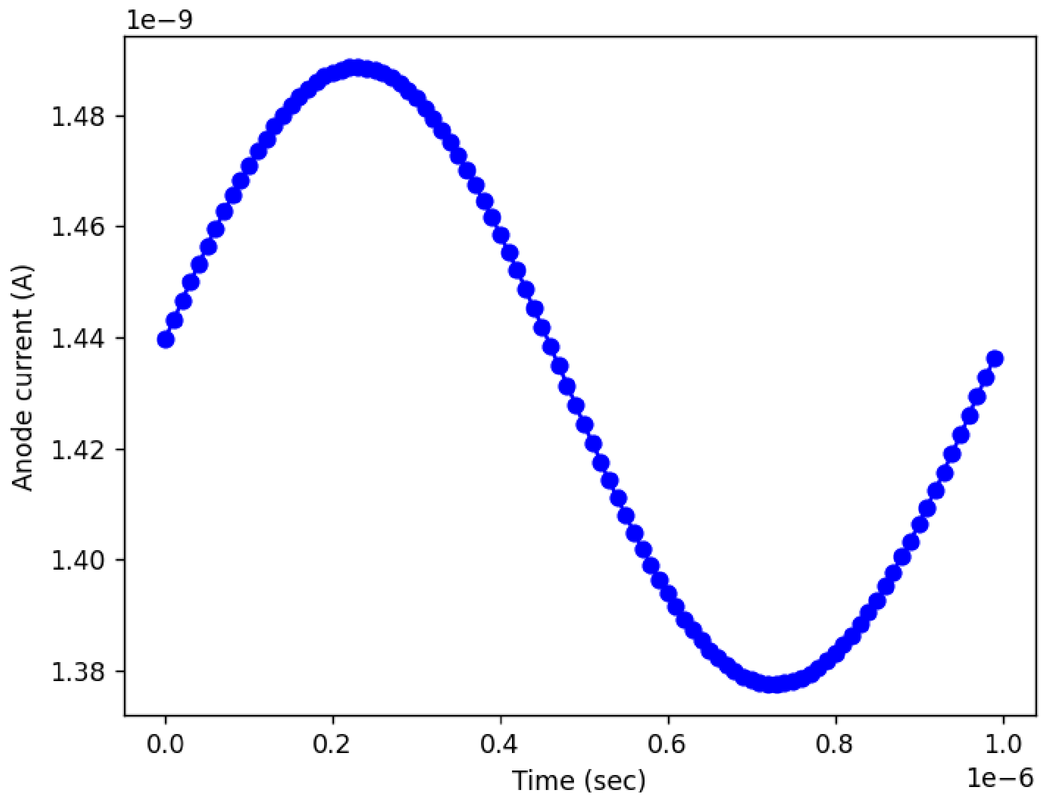
- Run a long transient simulation.
  - The initial response (due to introducing the time-varying excitation) will be diminished.
  - Then, two periods show the same response.
  - By using responses of the last period, calculate the frequency components. For example,

$$i_{amp,sin} = \frac{2}{T}\int_0^T I_{anode}(t)\sin(2\pi f t)\,dt$$

$$i_{amp,cos} = \frac{2}{T}\int_0^T I_{anode}(t)\cos(2\pi f t)\,dt$$

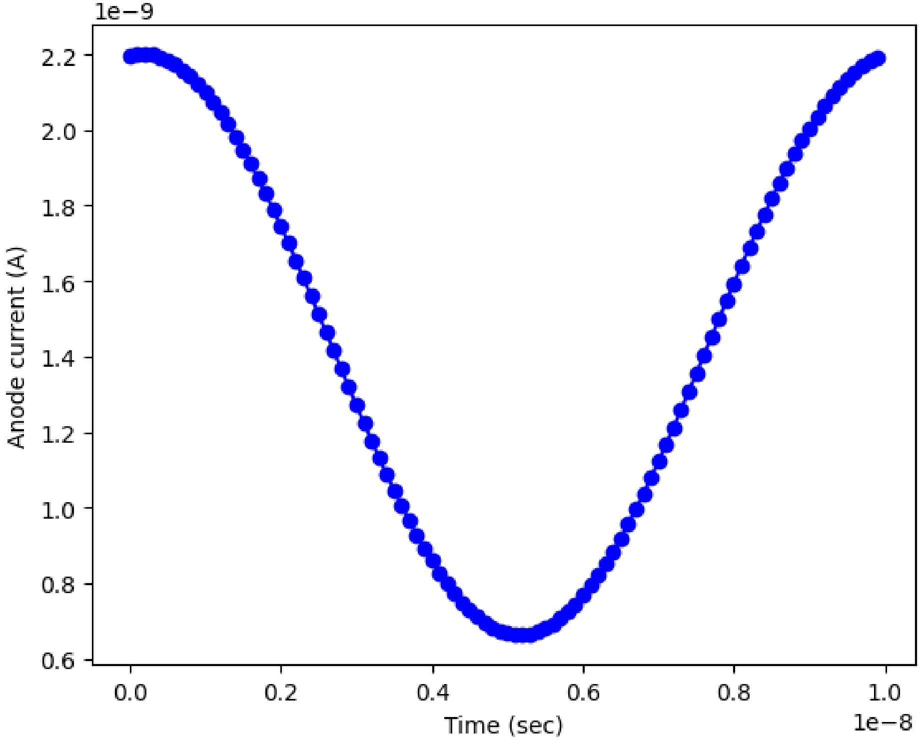
10^{th} period @ 0.5 V & 1 MHz

- Consider a 1 MHz signals, f is 10^6 Hz.
  - Amplitude is 1 mV.
  - For each period, 100 intervals are assigned.
  - Its DC value is 1.4326 nA.
  - Its sine amplitude is 54.958 pA.
  - Its cosine amplitude is 7.6511 pA.



10^{th} period @ 0.5 V & 100 MHz

- Consider a 100 MHz signals, f is 10^8 Hz.
  - The frequency is 100 times higher. Now it looks like a cosine function.
  - Its DC value is 1.4326 nA. (Not changed)
  - Its cosine amplitude is 765.10 pA. (100 times larger)



Response as a function of frequency

- For example, from 1 Hz to 100 GHz?
  - The number of frequencies is N_f.
- Okay, then, starting from the DC solution,
  - Run several transient simulations.
  - Estimate the computational efforts:

$$(Single\ DC\ calculation) + (\underline{Single\ transient\ calculation}) \times N_f$$

It takes long…

An efficient approach

- Can we avoid the time-consuming transient calculation to obtain the frequency respons?
  - The answer is “Yes!”
  - In this efficient approach, the computational efforts can be expressed as

$$(Single\ DC\ calculation) + (\underline{Single\ frequency\ calculation}) \times N_f$$

It is much faster!

Important assumption

- We assume that time-varying quantities are "small" in their amplitudes.
  - Variables are expressed as

$$\phi(t) = \phi_{DC} + \delta\phi(t)$$
$$n(t) = n_{DC} + \delta n(t)$$
$$p(t) = p_{DC} + \delta p(t)$$

  - Under this assumption, we can write the DD model as

$$\nabla \cdot [\epsilon \nabla \phi_{DC} + \epsilon \nabla \delta\phi\ ] + q p_{DC} + q\delta p - q n_{DC} - q\delta n + q N_{dop}^{+} = 0$$

$$-q \frac{\partial \delta n}{\partial t} + \nabla \cdot \left(\mathbf{J}_{n,DC} + \delta \mathbf{J}_n \right) = 0$$

$$q \frac{\partial \delta p}{\partial t} + \nabla \cdot \left(\mathbf{J}_{p,DC} + \delta \mathbf{J}_p \right) = 0$$

Current densities

- Nonlinearity comes from the current densities.
  - Remember that

$$\mathbf{J}_n = -q\mu_n n\nabla\phi + qD_n\nabla n$$
$$\mathbf{J}_p = -q\mu_p p\nabla\phi - qD_p\nabla p$$

  - With time-varying variables,

$$\delta\mathbf{J}_n = -q\mu_n n_{DC}\nabla\delta\phi - q\mu_n\delta n\nabla\phi_{DC} + qD_n\nabla\delta n$$
$$\delta\mathbf{J}_p = -q\mu_p p_{DC}\nabla\delta\phi - q\mu_p\delta p\nabla\phi_{DC} - qD_p\nabla\delta p$$

  - The second order terms are nelected!
  - Of course, in the practical implementation, the Scharfetter-Gummel discretization is employed.

Linearization

- Note that ϕ_{DC}, n_{DC}, and p_{DC} are the DC solutions.
  - Without the time-dependent terms, they satisfy the DD model, too:

$$\nabla \cdot [\epsilon \nabla \phi_{DC} + \epsilon \nabla \delta\phi\ ] + q p_{DC} + q\delta p - q n_{DC} - q\delta n + q N_{dop}^{+} = 0$$

$$-q \frac{\partial \delta n}{\partial t} + \nabla \cdot \left(\mathbf{J}_{n,DC} + \delta \mathbf{J}_n \right) = 0$$

$$q \frac{\partial \delta p}{\partial t} + \nabla \cdot \left(\mathbf{J}_{p,DC} + \delta \mathbf{J}_p \right) = 0$$

  - In other words,

$$\nabla \cdot [\epsilon \nabla \delta\phi\ ] + q\delta p - q\delta n = 0$$

$$-q \frac{\partial \delta n}{\partial t} + \nabla \cdot \left(\delta \mathbf{J}_n \right) = 0$$

$$q \frac{\partial \delta p}{\partial t} + \nabla \cdot \left(\delta \mathbf{J}_p \right) = 0$$

Relation to Jacobian

- The linearized system is closely related to the Jacobian matrix.
  - The product between the Jacobian matrix and the update vector ($\mathbf{Ax}$) describes the following functions:

$$\nabla \cdot [\epsilon \nabla \delta\phi] + q\delta p - q\delta n$$
$$\nabla \cdot (\delta \mathbf{J}_n)$$
$$\nabla \cdot (\delta \mathbf{J}_p)$$

  - Simply speaking, the DC Jacobian matrix is related to the small-signal analysis at the zero frequency (or the low-frequency limit).
  - Of course, we must consider $-q\frac{\partial \delta n}{\partial t}$ and $q\frac{\partial \delta p}{\partial t}$ for general cases.

Non-zero frequency

- In the frequency domain, let us introduce the time dependence of $e^{j2\pi ft}$.
  - Then, the time derivation terms can be expressed as

$$-q\frac{\partial \delta n e^{j2\pi ft}}{\partial t} = -q(j2\pi f)\delta n e^{j2\pi ft}$$

$$q\frac{\partial \delta p e^{j2\pi ft}}{\partial t} = q(j2\pi f)\delta p e^{j2\pi ft}$$

  - Therefore, in the frequency domain, the linearized system reads

$$\nabla \cdot [\epsilon \nabla \delta\phi] + q\delta p - q\delta n = 0$$

$$-j2\pi f q\delta n + \nabla \cdot (\delta \mathbf{J}_n) = 0$$

$$j2\pi f q\delta p + \nabla \cdot \left(\delta \mathbf{J}_p\right) = 0$$

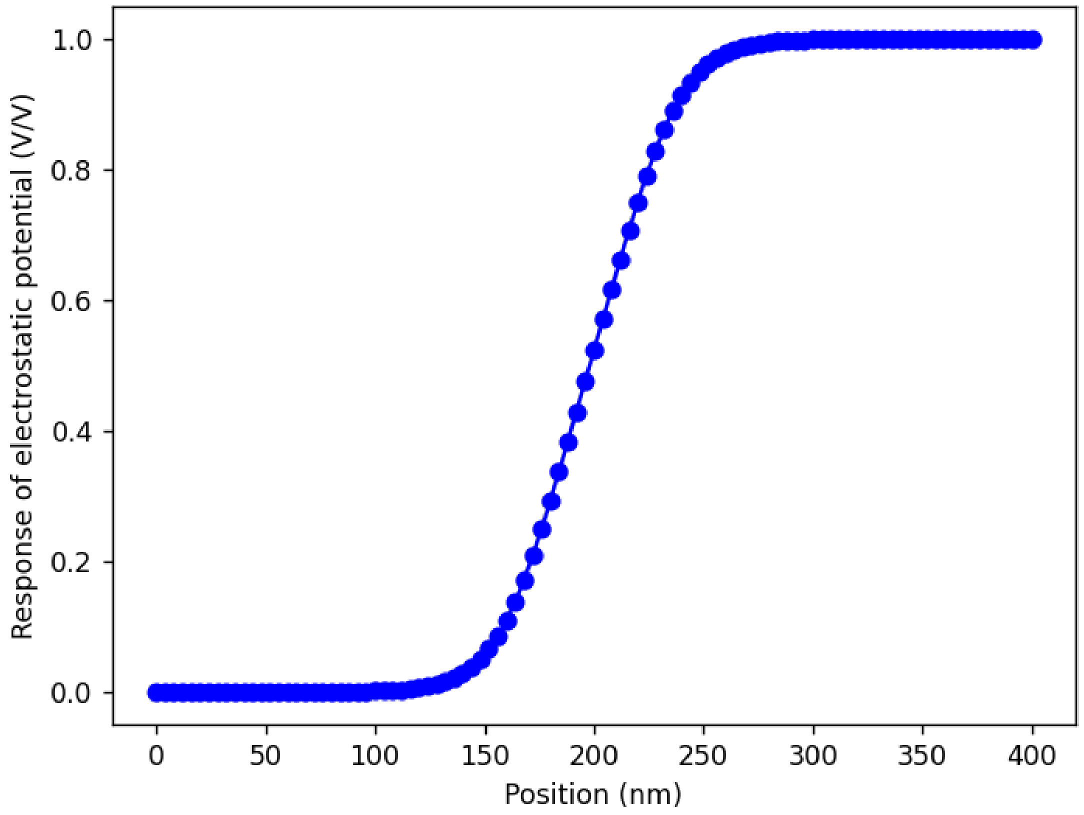
Calculate dI_{anode}/dV_{anode}.

- First, prepare the DC solution.
  - Since it is the solution (or very close to the solution), the residue vector is a null vector (or very close to a null vector).
$$\mathbf{A\delta} \approx 0$$
  - Then, re-use the Jacobian matrix, $\mathbf{A}$.
  - The boundry condition for the anode potential is given as
$$\phi_{anode} - \phi_{anode,0} - V_{anode} = 0$$
  - Now, the response, $\delta\phi_{anode}$, must satisfy
$$\delta\phi_{anode} = 1$$
  - Therefore, the RHS vector is modified to include 1 for the adode potential.

1 means the unit perturbation.

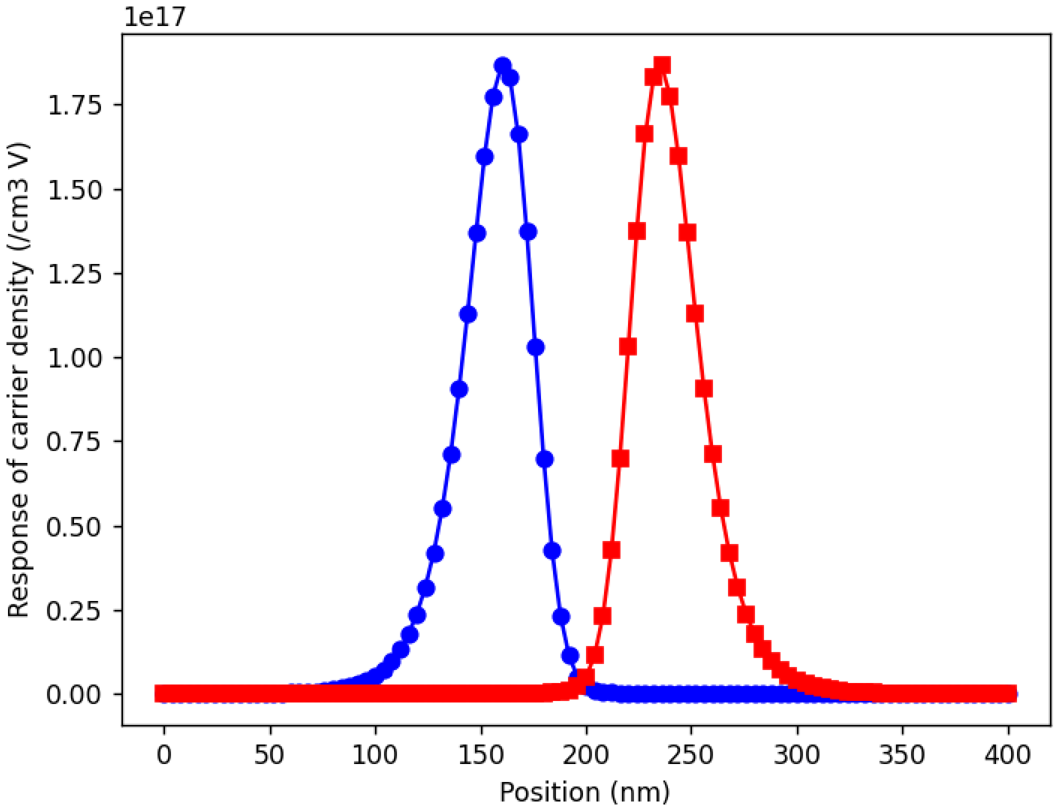
An example for PN junction

- Once again, consider a PN junction at 0.5 V.
  - The response of electrostatic potential is drawn. (line)
  - Difference between 0.5 V and 0.501 V (X 1,000) are drawn, too. (symbol)
  - They agree well.



Responses of carrier densities

- At the same bias, δn and δp are drawn.
  - The depletion region gets narrower.
  - Symbols are obtained from the finite difference.



Response of terminal current

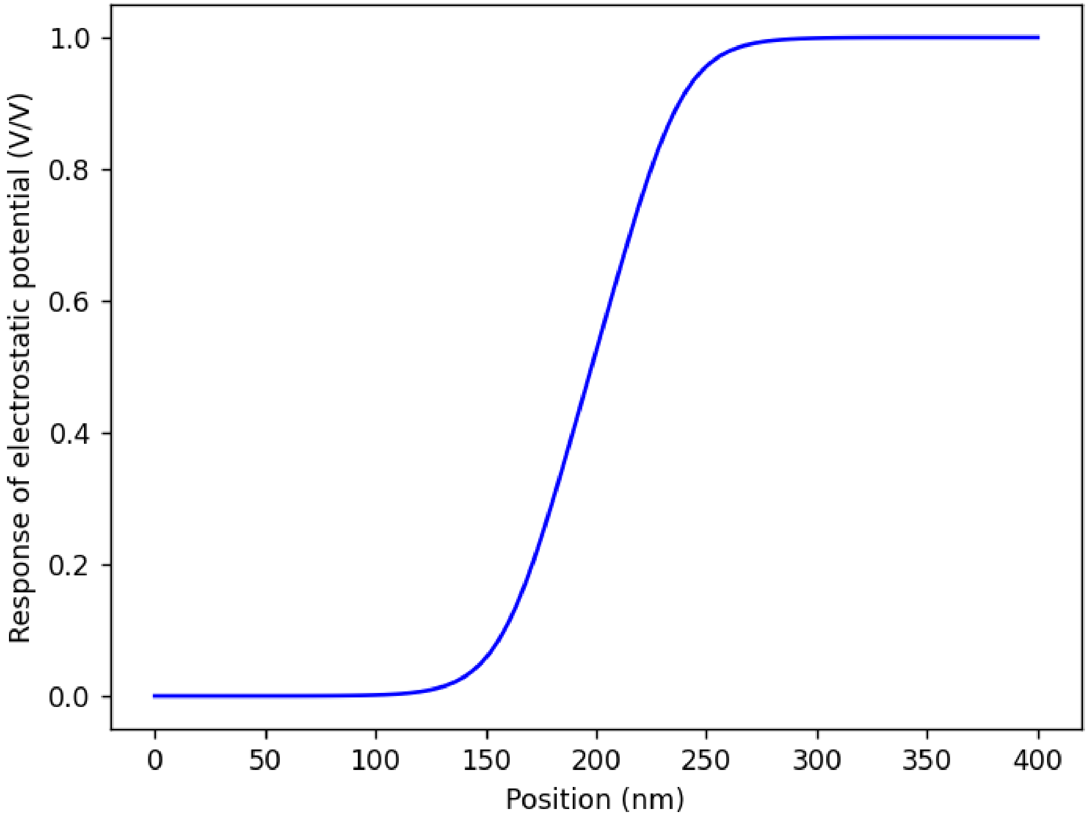
- In a similar way, we can calculate the terminal current.
  - It is 54.708 nA/V.
  - From the difference between 0.5 V and 0.501 V, we obtain 55.766 nA/V, which is quite close.
  - From the difference between 0.5 V and 0.500001 V, we obtain 54.774 nA/V, which is even closer to the small-signal value.

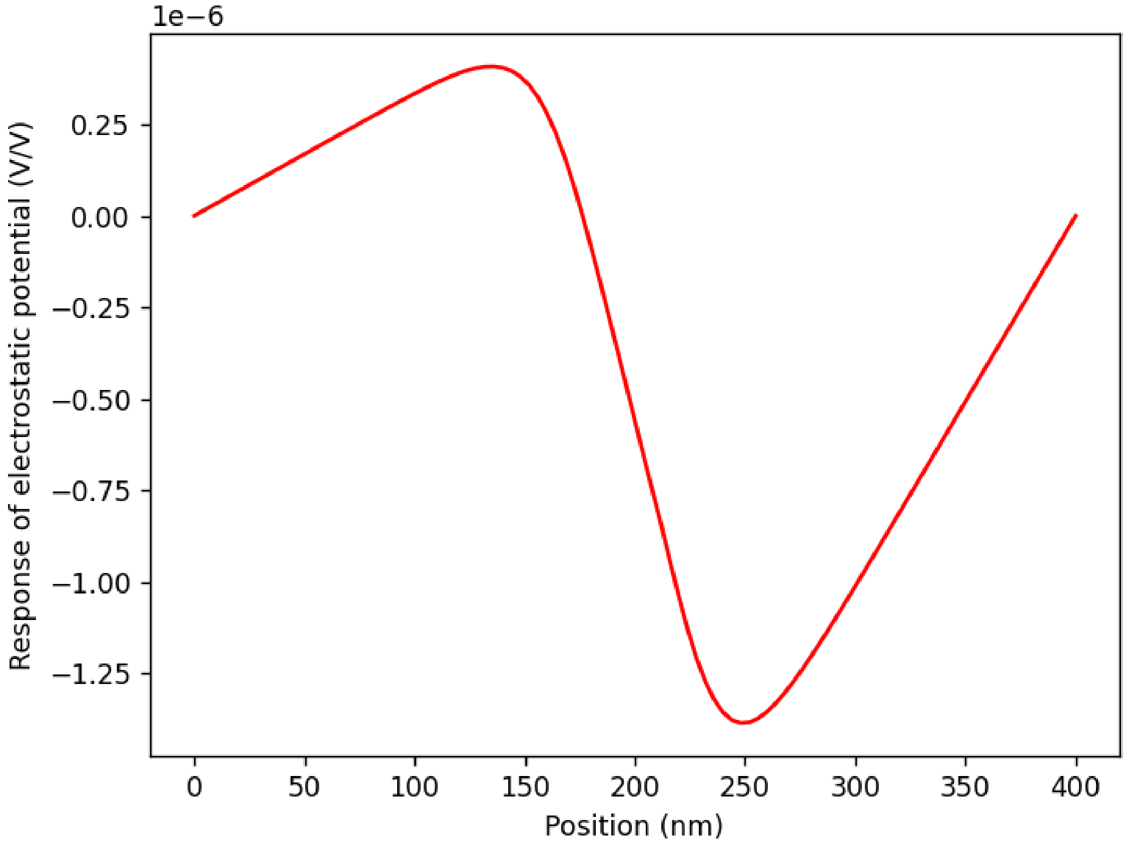
Nonzero frequency

- Frequency-dependent terms should be added.
  - It is noted that $\frac{\partial \delta n}{\partial t}$ contributes $(j2\pi f)\delta n$.
- Displacement current
  - Just like the transient simulation, we must include the displacement current.

Relatively low frequency, 1 MHz

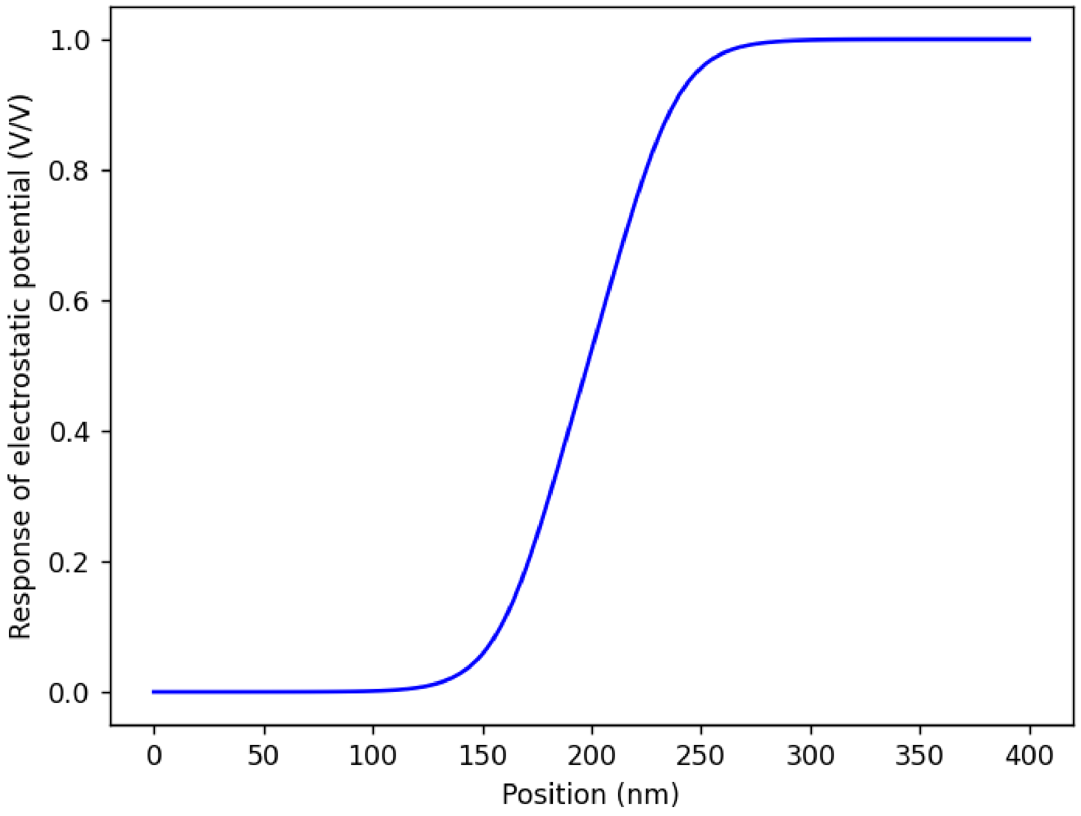
- Real (blue) and imaginary (red) parts of $\delta\phi$
  - We have a non-vanishing imaginary part.

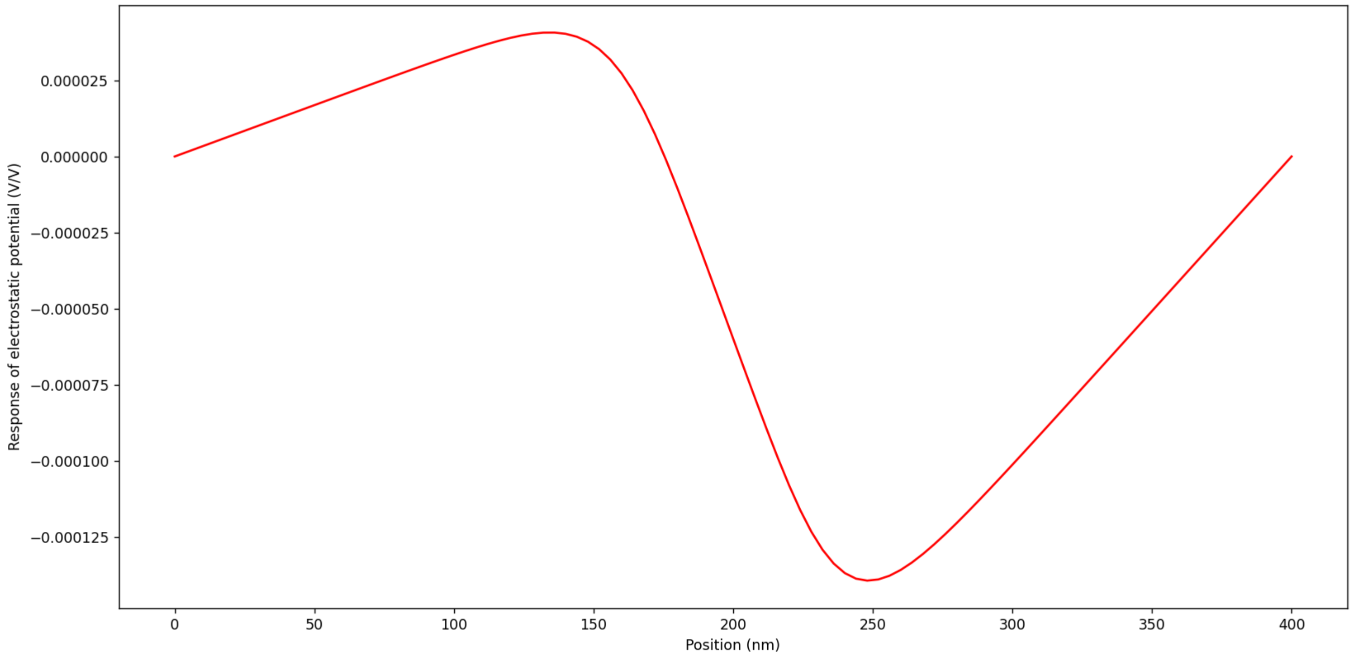




Relatively high frequency, 100 MHz

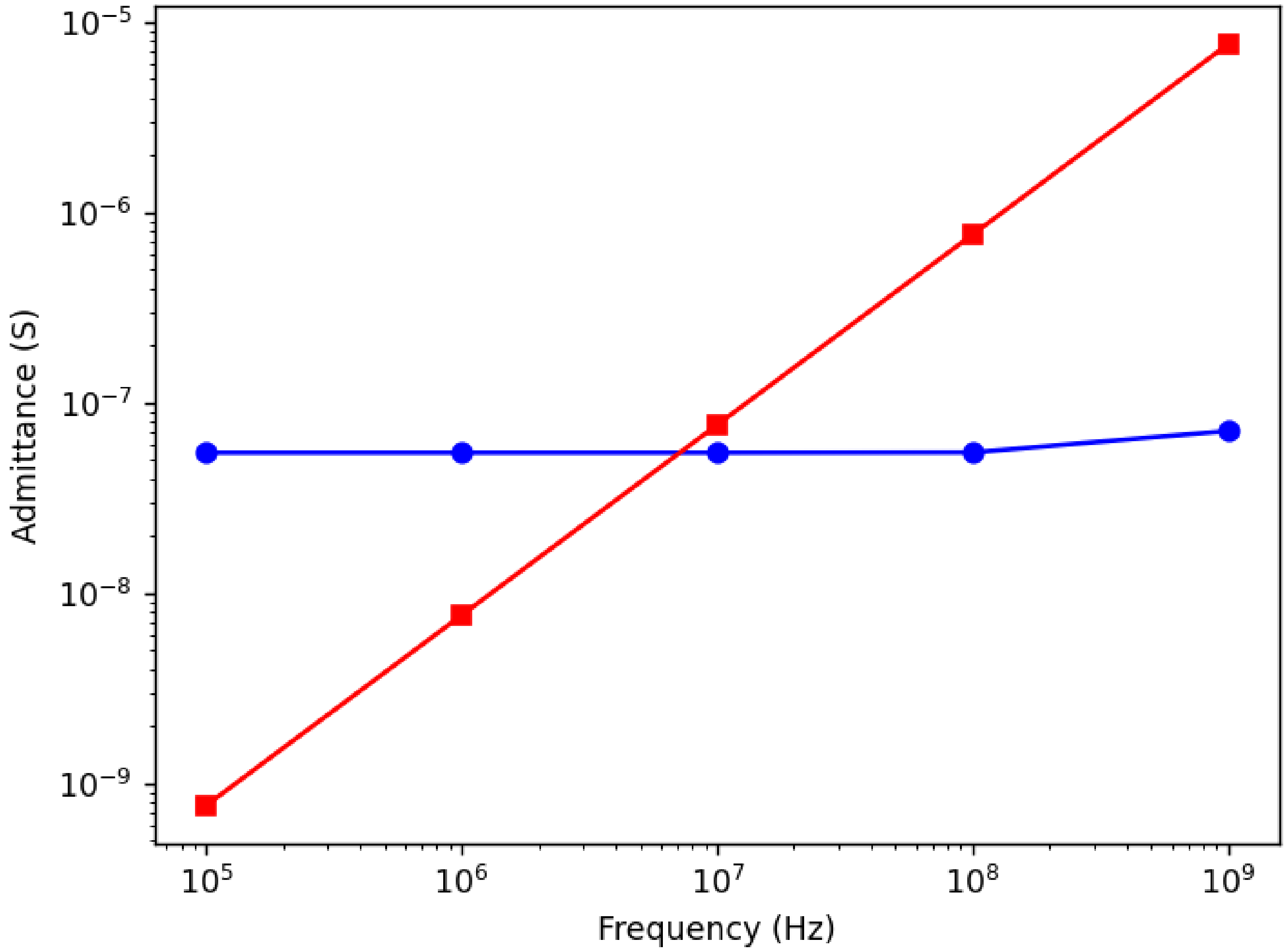
- Real (blue) and imaginary (red) parts of $\delta\phi$
  - Its real part looks similar, but the imaginary one is much larger (x100).





Admittance as a function of frequency

- 100 kHz, 1 MHz, 10 MHz, 100 MHz, and 1 GHz
  - Real (blue)
  - Imaginary (red)



- R and C

Homework#19

- Due: AM08:00, November 26
- Problem#1
  - Implement a small-signal DD solver. Check a valid range of the small-signal approximation.

Thank you for your attention!