Computational Microelectronics L3 (Pre-recorded)

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Newton method

Kirchhoff's current law

- OUT node
  - Sum of two branch currents must vanish:

$$I_{Dn}(V_{in}, V_{out}) + I_{Dp}(V_{in}, V_{out}) = 0$$

A nonlinear function

Another nonlinear function

  - Nonlinearity is the root cause of difficulties.

Counterexample

- Consider a linear system, instead of a nonlinear system.
  - The KCL reads

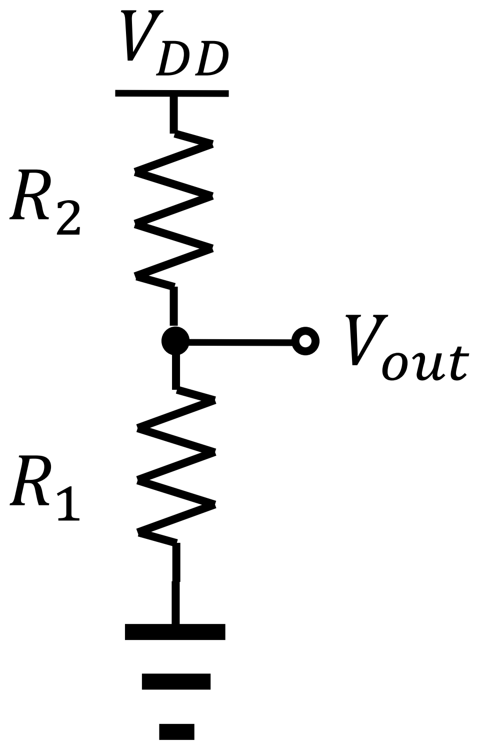
$$I_{R1}(V_{out}) = I_{R2}(V_{out})$$

  - For resistors, Ohm's law must be applied.

$$\frac{V_{out}}{R_1} = \frac{V_{DD} - V_{out}}{R_2}$$

  - Solution can be found immediately

$$V_{out} = \frac{R_1}{R_1 + R_2} V_{DD}$$



Discussion

- Remember our previous L2.
  - When V_{in} is 0.5 V and V_{out} is 0.7 V, we have I_{Dn} = 1.77 µA and I_{Dp} = -7.92 µA.
  - In L2, we increased V_{out} by 0.1 V.
  - Q) Why increased?
  - A) In order to balance two currents. (We know that I_{Dn} must be increased.)
  - Q) Why 0.1 V?
  - A) Just heuristically.
  - Can we find a better way to update V_{out}?

Our approach

- With an initial set of V_{in} and V_{out}, the KCL (usually) does not hold.

$$I_{Dn}(V_{in}, V_{out}) + I_{Dp}(V_{in}, V_{out}) \neq 0$$

  - We assert that a better solution, $V_{out} + \delta V_{out}$, satisfies the KCL.

$$I_{Dn}(V_{in}, V_{out} + \delta V_{out}) + I_{Dp}(V_{in}, V_{out} + \delta V_{out}) = 0$$

  - Now, we want to find an equation for δV_{out}.
  - The exact equation for δV_{out} is still very difficult to solve.
  - An appropriate approximation is needed.

Linear approximation

- Key step toward a practical method
  - An exact equation

$$I_{Dn}(V_{in}, V_{out} + \delta V_{out}) + I_{Dp}(V_{in}, V_{out} + \delta V_{out}) = 0$$

  - An approximate equation

$$I_{Dn}(V_{in}, V_{out}) + \left.\frac{\partial I_{Dn}}{\partial V_{out}}\right|_{V_{in}, V_{out}} \delta V_{out} + I_{Dp}(V_{in}, V_{out}) + \left.\frac{\partial I_{Dp}}{\partial V_{out}}\right|_{V_{in}, V_{out}} \delta V_{out} \approx 0$$

Calculation of update

- After simple manipulation,
  - An approximate equation for δV_{out} can be re-arranged as:

$$\left[\left.\frac{\partial I_{Dn}}{\partial V_{out}}\right|_{V_{in},V_{out}} + \left.\frac{\partial I_{Dp}}{\partial V_{out}}\right|_{V_{in},V_{out}}\right]\delta V_{out}$$

$$= -\left[I_{Dn}(V_{in}, V_{out}) + I_{Dp}(V_{in}, V_{out})\right]$$

  - Two terms:
  - Value of $I_{Dn} + I_{Dp}$ evaluated at the present solution set
  - Sensitivity of $I_{Dn} + I_{Dp}$ with respect to change of solution

Case) Saturation region

- Function value
  - It is straightforward to evaluate the currents:

$$I_{Dn} = \frac{KP_n}{2}\frac{W_{effn}}{L_{effn}}(1 + LAMBDA_n \times V_{out})(V_{in} - V_{tn})^2$$

$$I_{Dp} = -\frac{KP_p}{2}\frac{W_{effp}}{L_{effp}}\left(1 - LAMBDA_p \times (V_{out} - V_{DD})\right)\left(V_{in} - V_{DD} - V_{tp}\right)^2$$

- (We can derive similar expressions in the linear region.)

Case) Saturation region

- Sensitivity
  - Partial derivative with respect to V_{out} is taken:

$$\frac{\partial I_{Dn}}{\partial V_{out}} = \frac{KP_n}{2}\frac{W_{effn}}{L_{effn}} LAMBDA_n (V_{in} - V_{tn})^2$$

$$\frac{\partial I_{Dp}}{\partial V_{out}} = \frac{KP_p}{2}\frac{W_{effp}}{L_{effp}} LAMBDA_p \left(V_{in} - V_{DD} - V_{tp}\right)^2$$

- (We can derive similar expressions in the linear region.)

Numeric values

- When V_{in} = 0.5 V, the correct value of V_{out} is about 1.157565 V.
  - Let us assume that we start from V_{out} = 1.1 V.
  - Then, the function value is -2.189 μA.
  - The sensitivity is 33.75 μA/V.
  - The calculated update is 0.064859 V.

  - Try again with V_{out} = 1.164859 V.
  - The calculated update is -0.007204 V.

  - Try again with V_{out} = 1.157655 V.
  - The calculated update is -0.000089 V.

Newton method

- We want to solve the following equation:

$$f(x) = 0$$

  - Assume an approximate solution, x_n.
  - Evaluate the function value, $f(x_n)$. (It should vanish, but it does not.)
  - Evaluate the sensitivity, $\left.\frac{df}{dx}\right|_{x_n}$.
  - Calculate the update, $\delta x = -f(x_n)/\left(\left.\frac{df}{dx}\right|_{x_n}\right)$.
  - Repeat the above procedure with $x_{n+1} = x_n + \delta x$.

Application of Taylor's theorem

- True solution is α.

$$f(\alpha) = 0$$

  - By using the Taylor's theorem,

$$f(\alpha) = f(x_n) + \left.\frac{df}{dx}\right|_{x_n} (\alpha - x_n) + \frac{1}{2}\left.\frac{d^2 f}{dx^2}\right|_{\xi_n} (\alpha - x_n)^2$$

(Note that ξ_n is between x_n and α.)

  - After simple manipulation,

$$x_n - f(x_n) \Big/ \left(\left.\frac{df}{dx}\right|_{x_n}\right) - \alpha = \frac{1}{2}\left(\left.\frac{d^2 f}{dx^2}\right|_{\xi_n}\right) \Big/ \left(\left.\frac{df}{dx}\right|_{x_n}\right) (x_n - \alpha)^2$$

Quadratic convergence

- The LHS can be further simplified.
  - Remember that

$$x_{n+1} = x_n + \delta x = x_n - f(x_n) \Big/ \left(\left. \frac{df}{dx} \right|_{x_n} \right)$$

  - Therefore,

$$x_{n+1} - \alpha = \frac{1}{2} \left(\left. \frac{d^2 f}{dx^2} \right|_{\xi_n} \right) \Big/ \left(\left. \frac{df}{dx} \right|_{x_n} \right) (x_n - \alpha)^2$$

  - Quadratic convergence

Caution for practical application

- The Newton method is sensitive to x_n.
  - Let us assume that we start from V_{out} = 1.0 V. (Still, V_{in} = 0.5 V)
  - Then, the update becomes 0.263083 V.
  - It suggests that our next solution is 1.263083 V, which exceeds V_{DD}.

- It is very important to start from a reasonable initial solution.
  - When we have too large update, it should be damped.
  - For example, we can limit the (absolute update) up to 0.1 V.
  - Also, V_{out} is bounded within [0, V_{DD}].

Number of iterations

- When we have a voltage change smaller than 1 µV, the calculation stops.
  - Start at $\frac{V_{DD}}{2}$.
  - Maximum (absolute) change is limited to 0.1 V.
  - 9 iterations at V_{in} = 0.5 V

```
1 0.7 0.1
2 0.7999999999999999 0.1
3 0.8999999999999999 0.1
4 0.9999999999999999 0.1
5 1.0999999999999999 0.1
6 1.1648592592592593 0.06485925925925938
7 1.1576546605493248 -0.007204598709934521
8 1.1575659813078383 -8.86792414864831e-05
9 1.1575659678253083 -1.3482529929426225e-08
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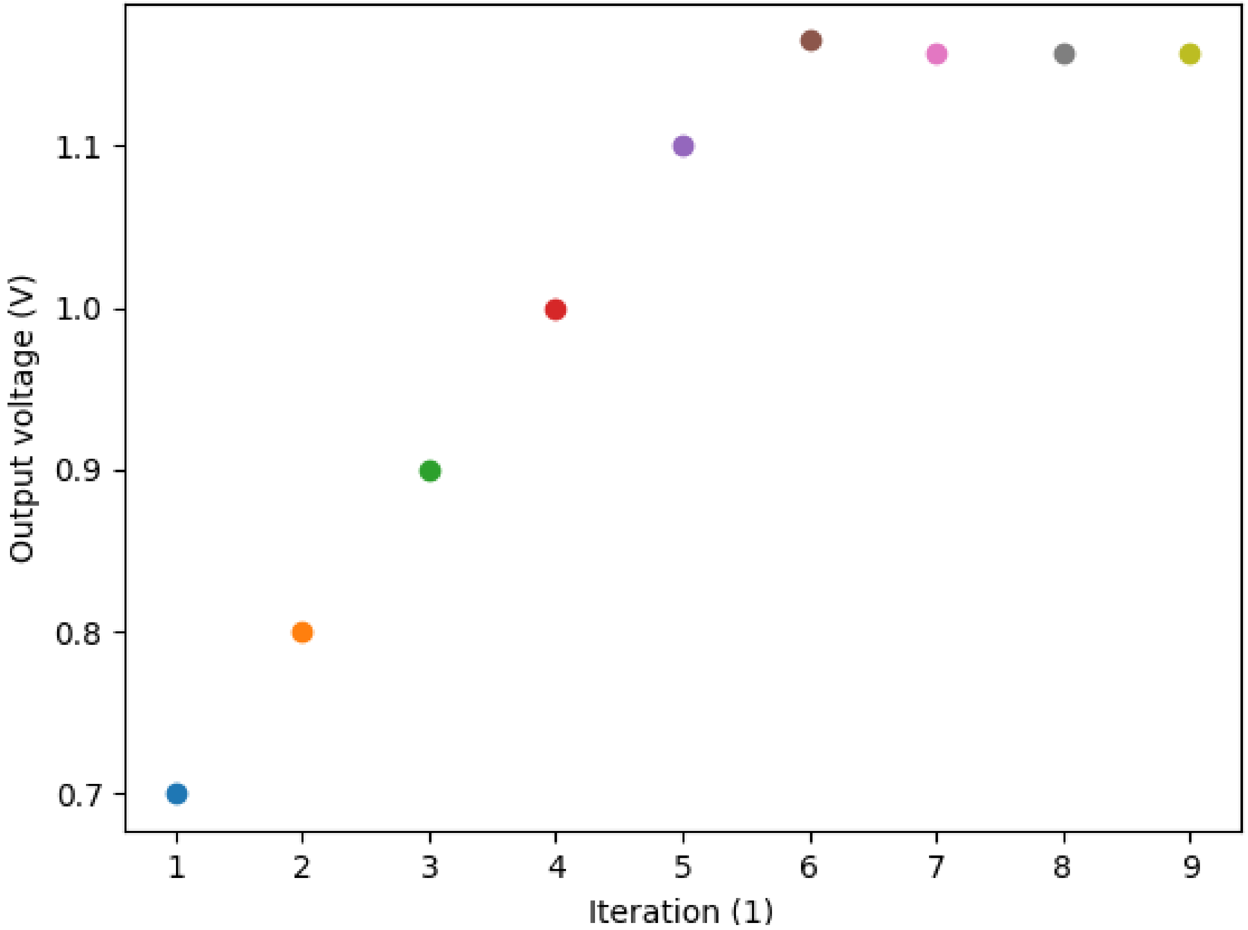

Homework#3

- Due: AM08:00, September 19
- Problem#1
  - Calculate the voltage transfer curve, once again. But, in this time, solve the problem with the Newton method.

Thank you for your attention!