

Special Topics on Basic EECS I

VLSI Devices

Lecture 20

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Surface potential, $\phi_s \equiv \phi(\mathbf{0}) - \phi(\infty)$

- A downward bending of bands in the p-type silicon near the surface

- It is important to note that

$$V_g - V_{fb} = \phi_s + V_{ox} \quad \text{Taur, Eq. (2.172)}$$

- At the silicon-oxide interface,

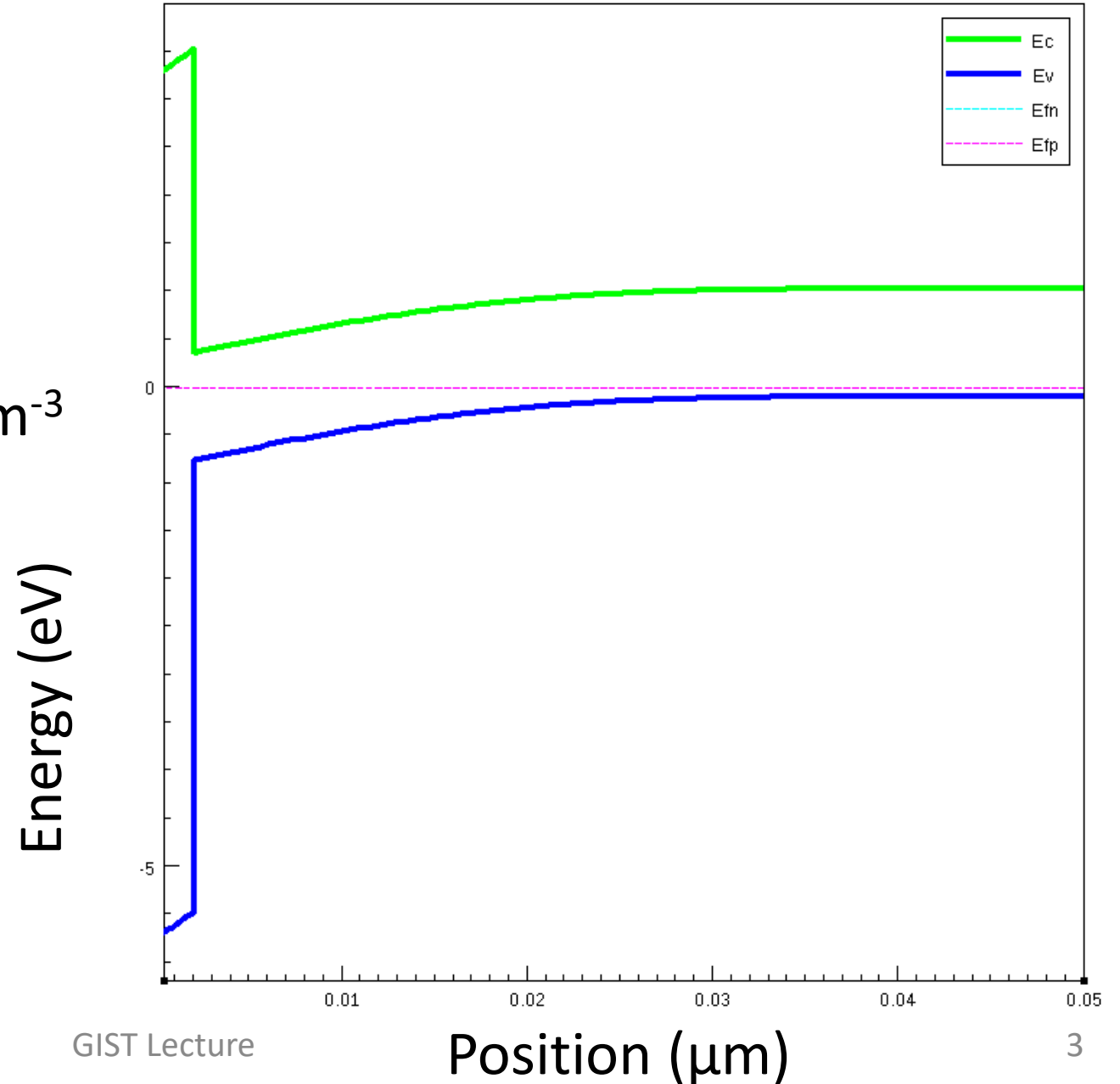
$$\epsilon_{ox} |\mathbf{E}_{ox}| = \epsilon_{si} |\mathbf{E}_{si}| \quad \text{Taur, Eq. (2.173)}$$

- Since $\epsilon_{ox} = 3.9\epsilon_0$ and $\epsilon_{si} = 11.7\epsilon_0$,

$$|\mathbf{E}_{ox}| \approx 3 |\mathbf{E}_{si}|$$

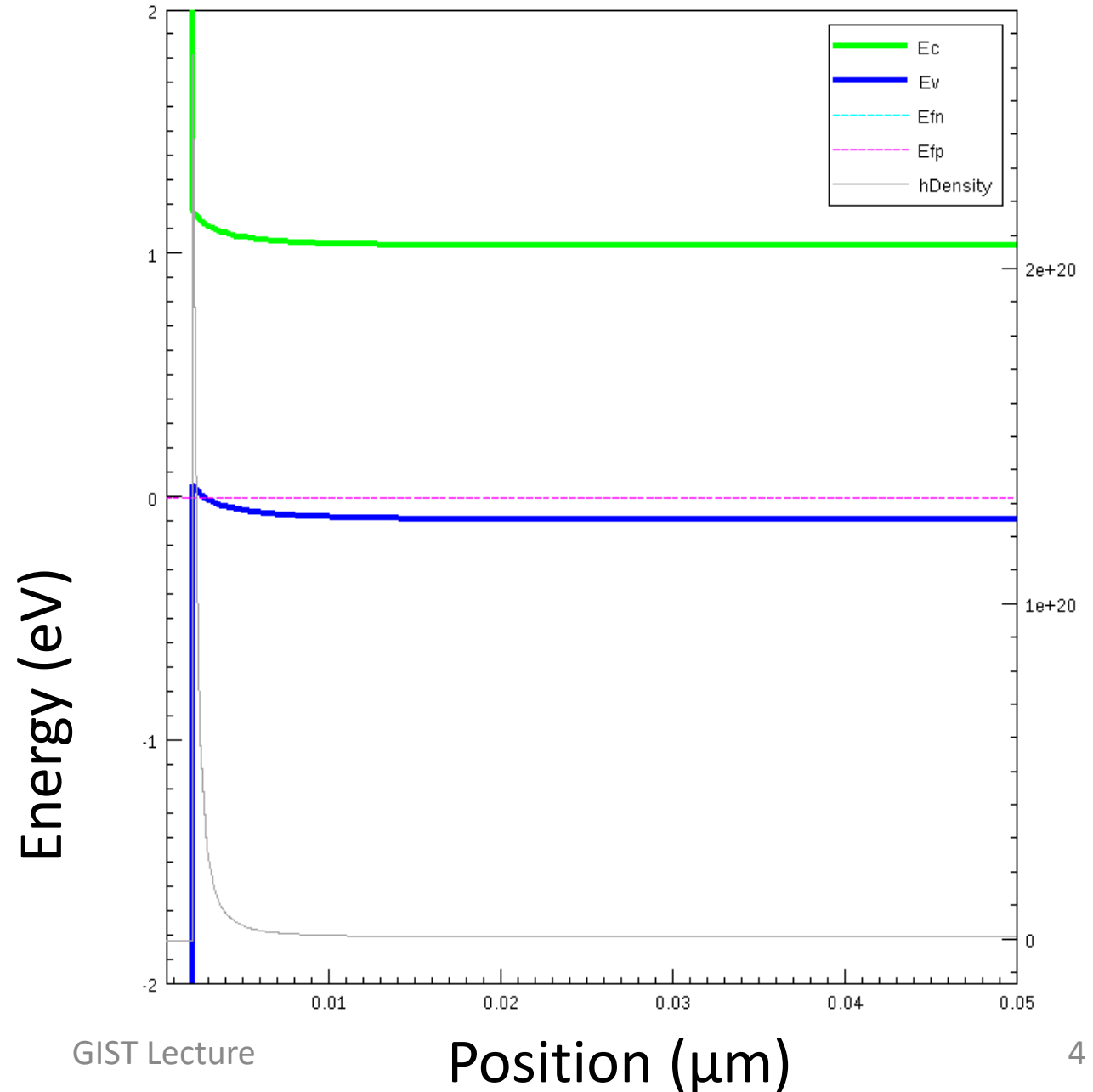
TCAD simulation

- Model parameter
 - Workfunction of 4.17 eV
 - Oxide thickness of 20 Å
 - P-type doping of $1 \times 10^{18} \text{ cm}^{-3}$



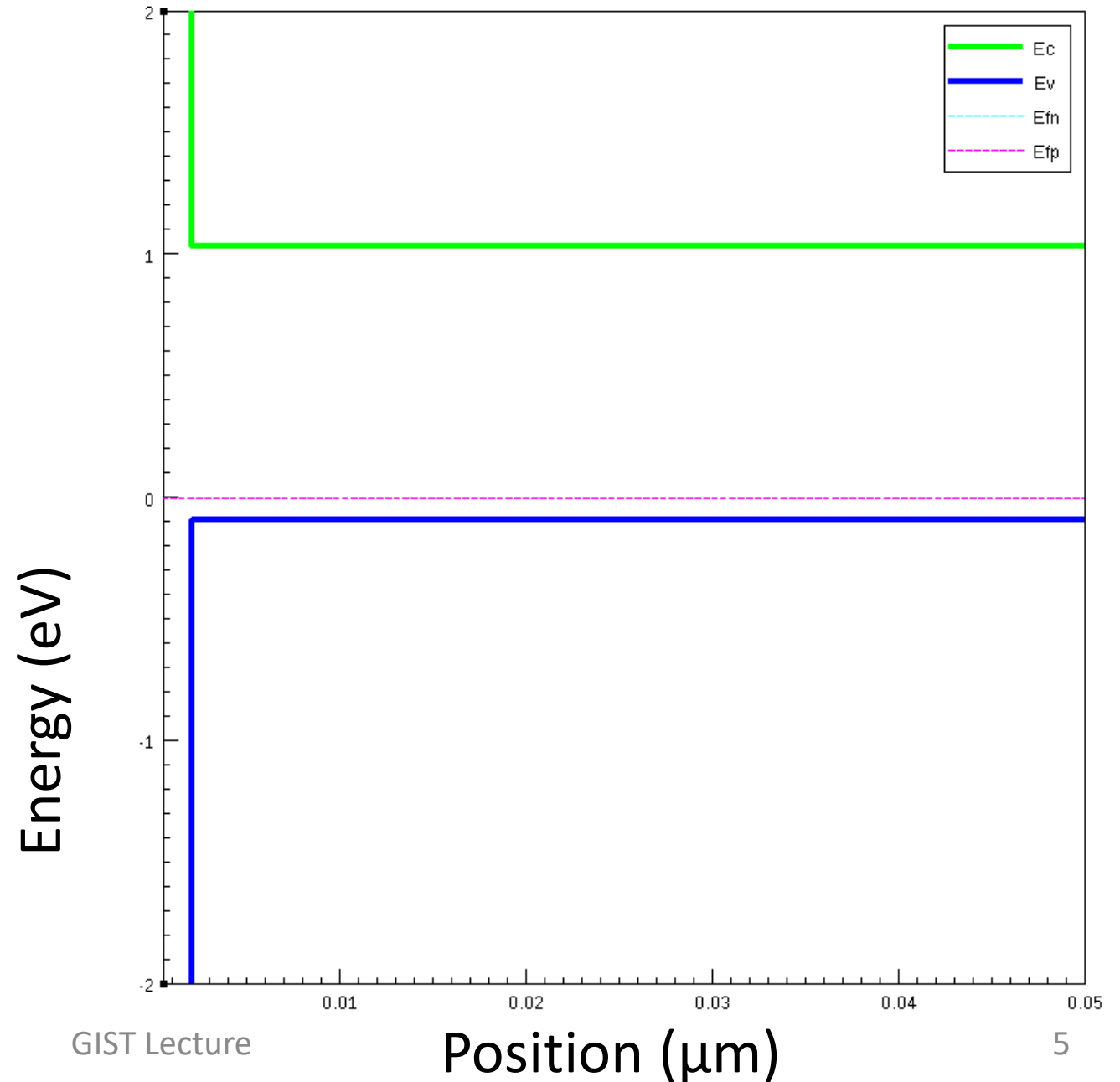
Case1

- $V_g = -2.0$ V
– Accumulation
- Hole density



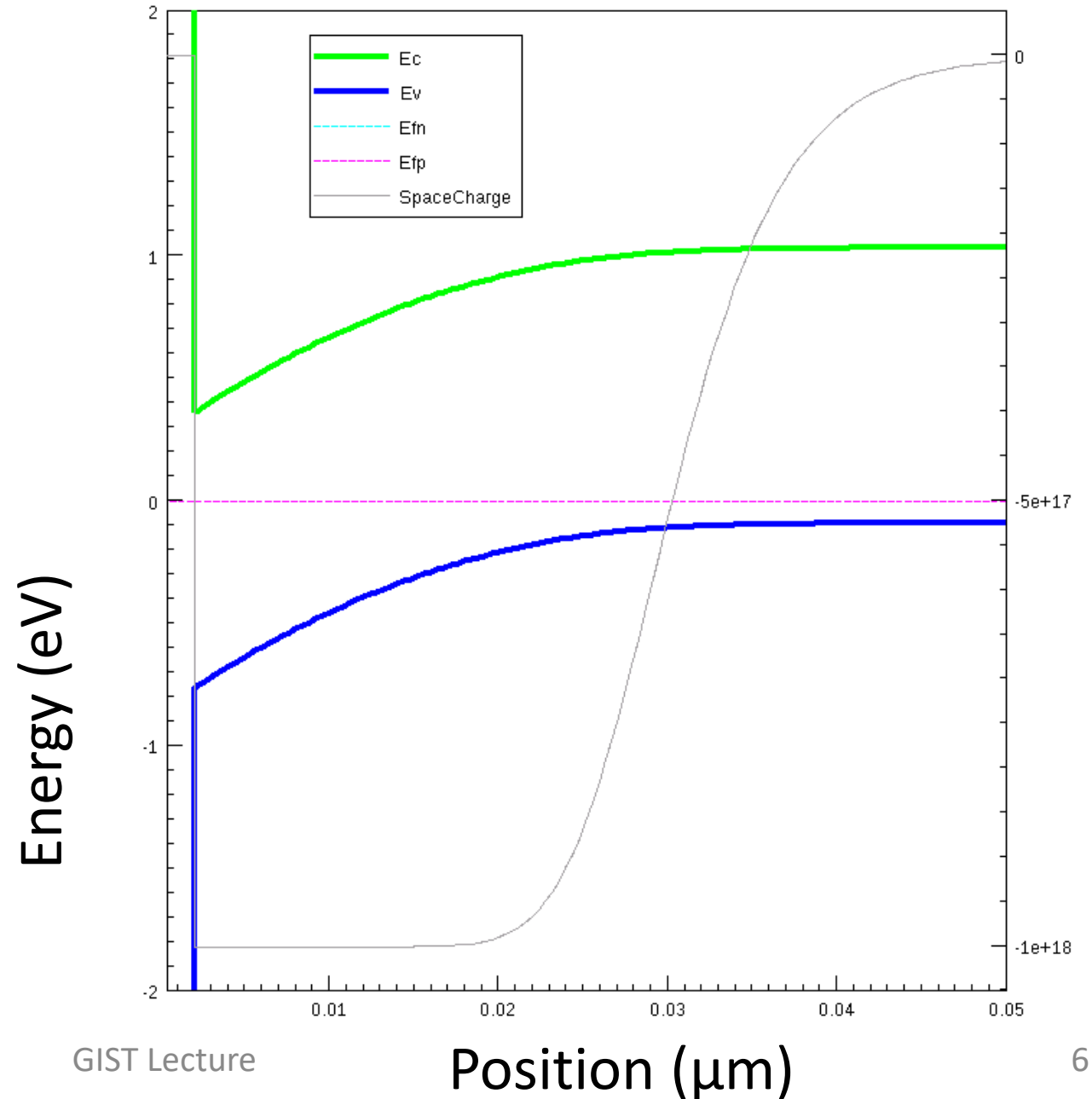
Case2

- $V_g = -0.94$ V
– Flatband condition



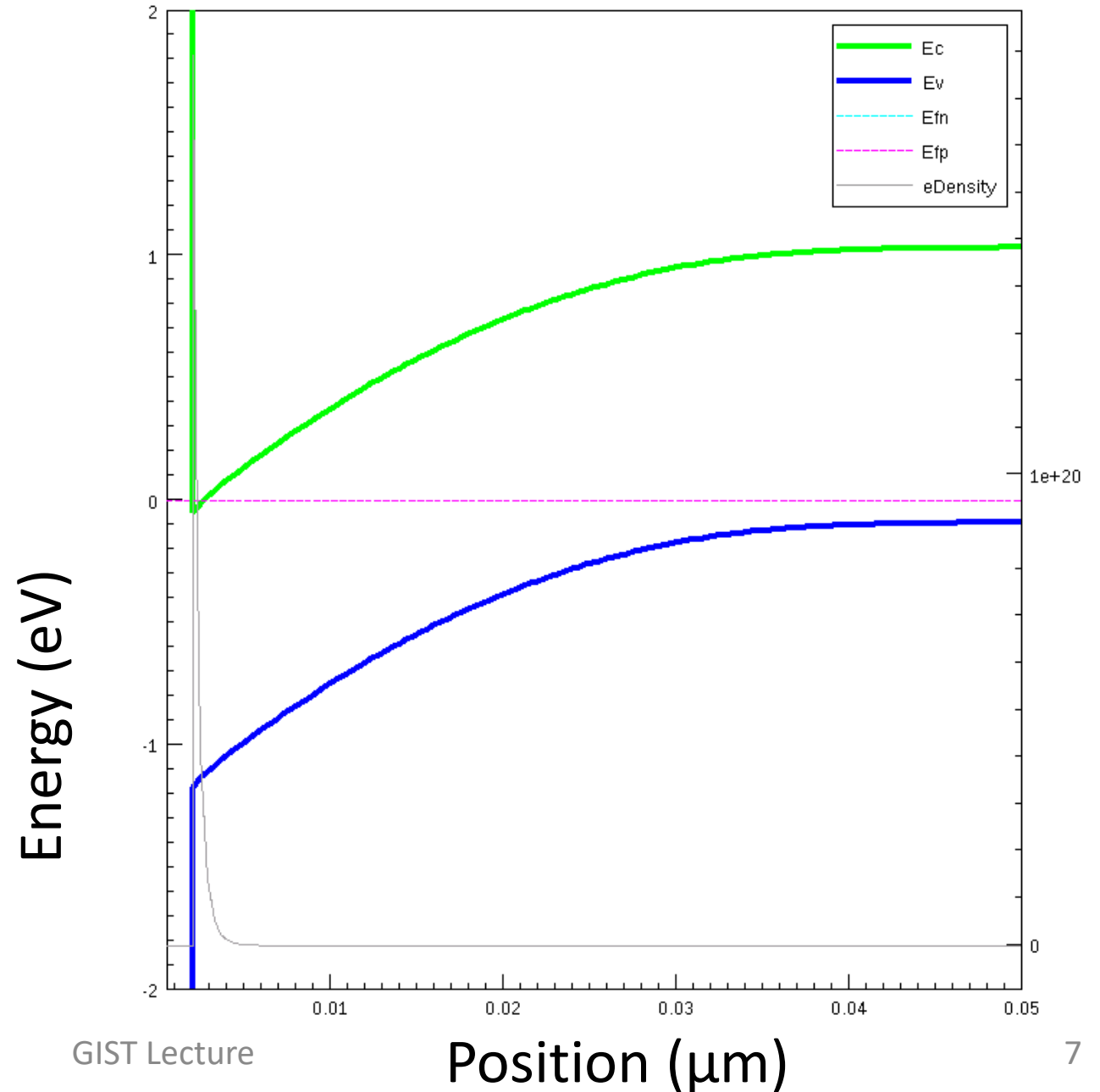
Case3

- $V_g = 0.0$ V
– Depletion
- Space charge



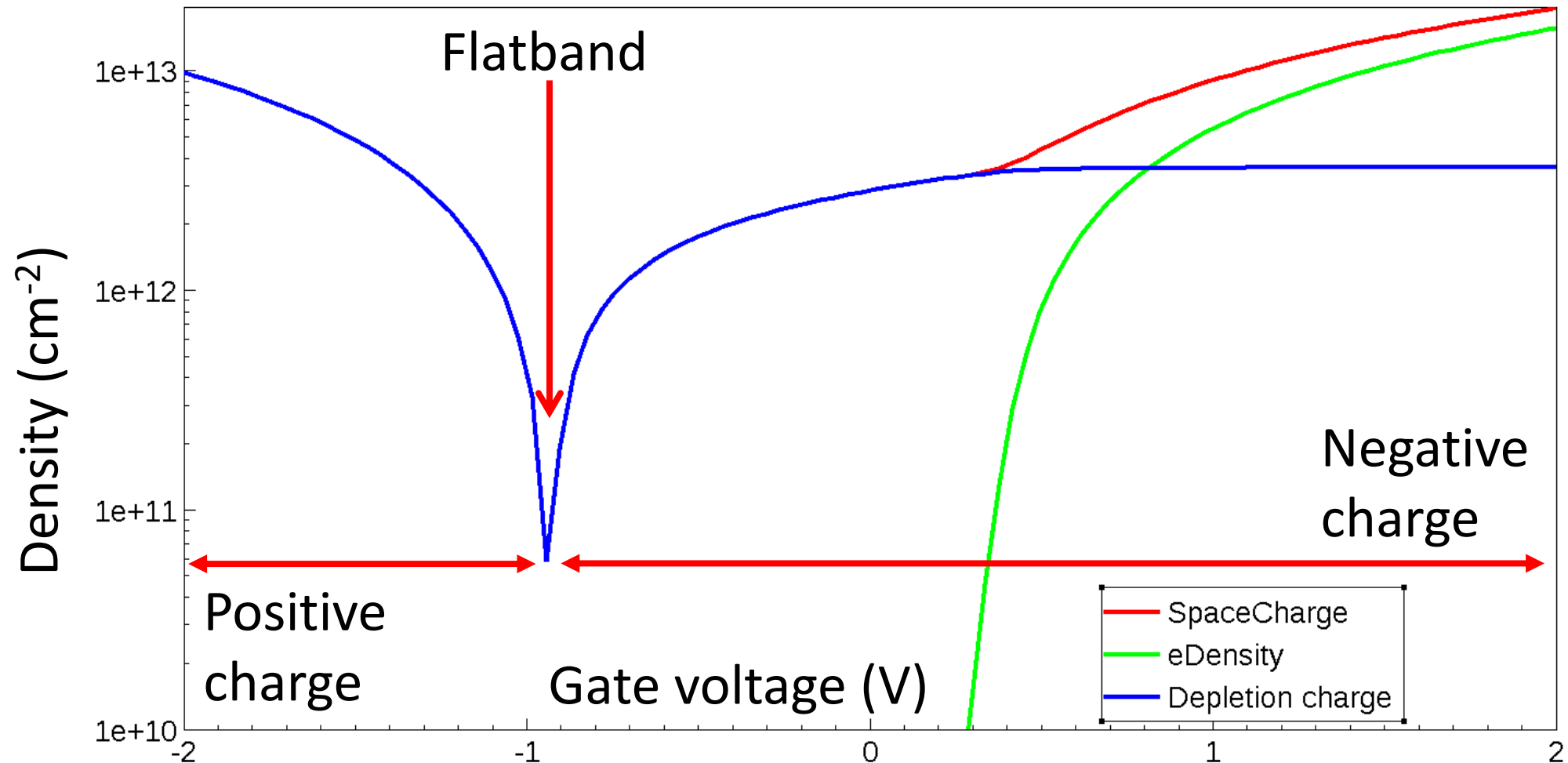
Case4

- $V_g = 1.0$ V
– Inversion
- Electron density



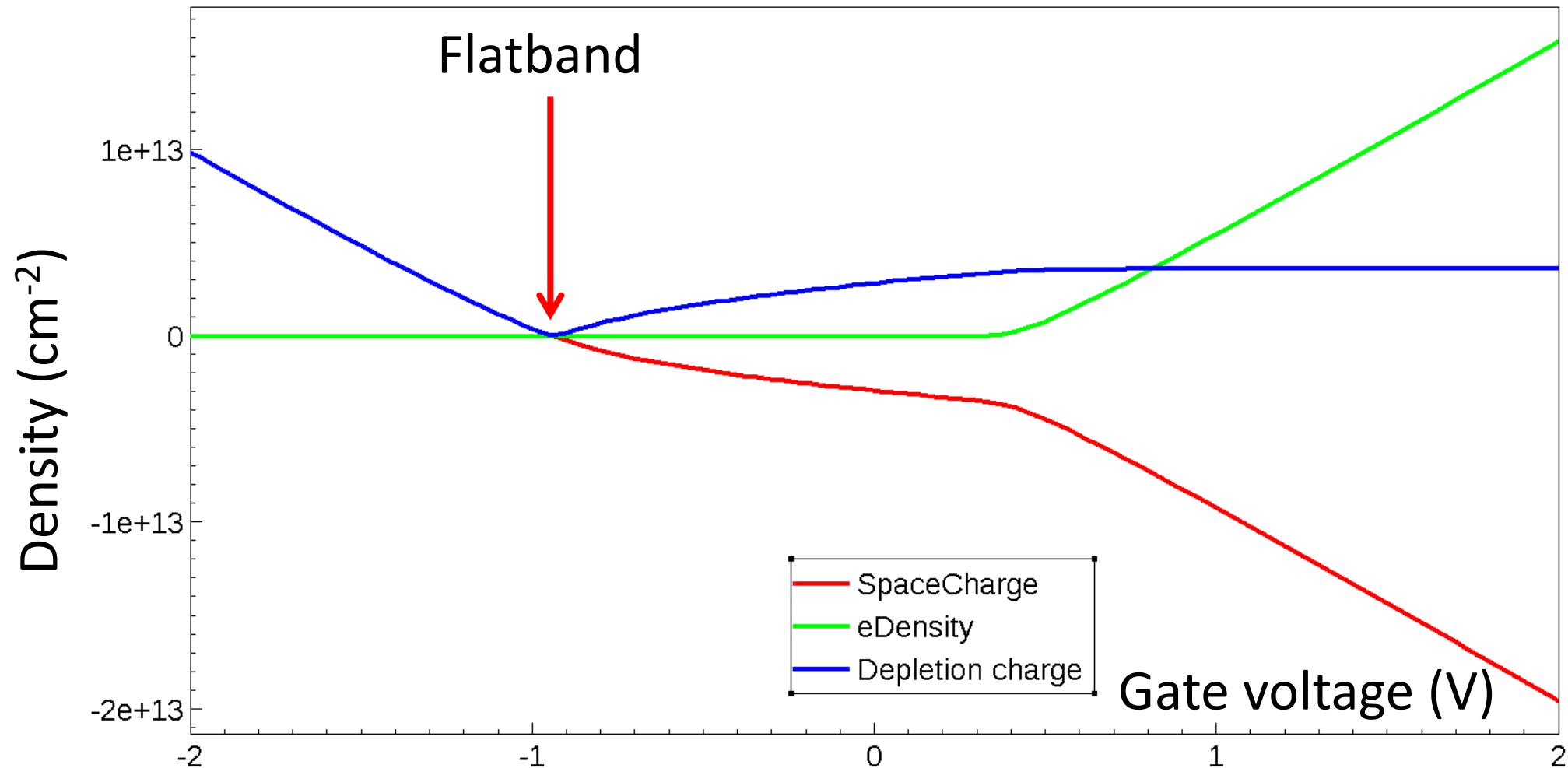
Threshold voltage (1)

- Draw quantities as functions of the gate voltage.



Threshold voltage (2)

- The same graph, in the linear scale,



Depletion approximation

- Consider a depleted MOS structure.

- With the depletion width, W_d ,

$$\phi_s = \frac{1}{2} W_d \left(q \frac{N_a}{\epsilon_{si}} W_d \right)$$

- Then,

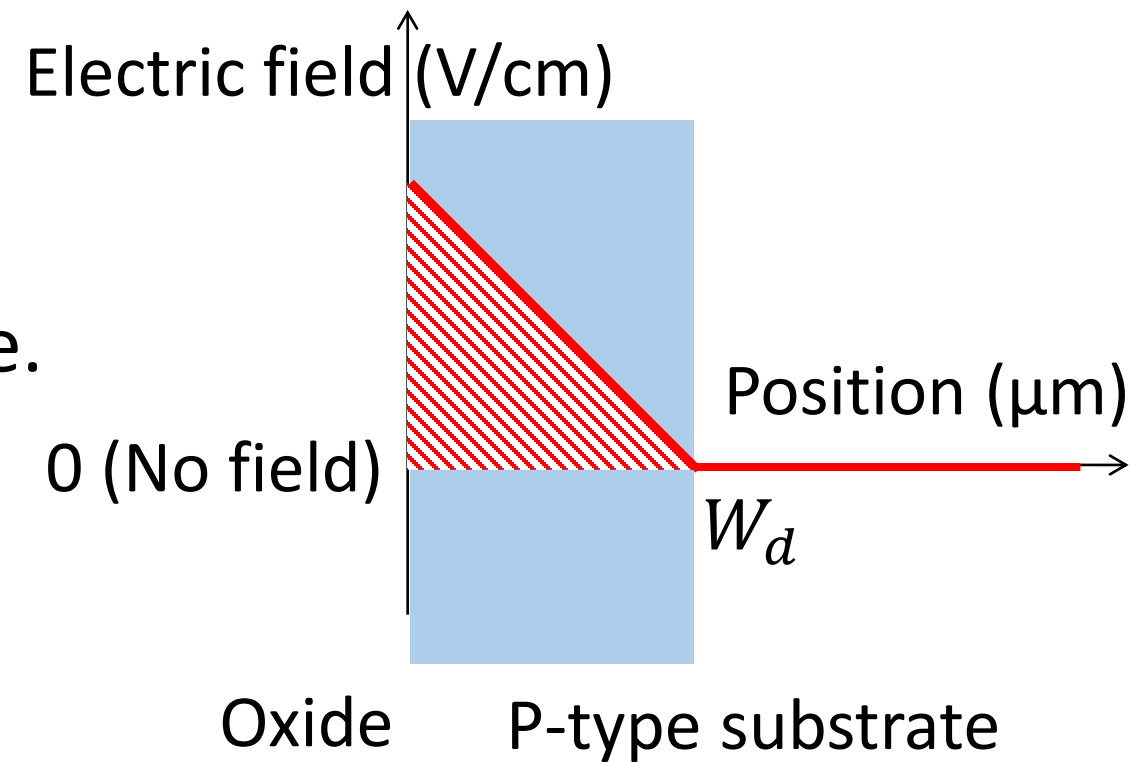
$$W_d = \sqrt{\frac{2\epsilon_{si}\phi_s}{qN_a}}$$

Taur, Eq. (2.188)

- Total depletion charge in silicon, Q_d , is

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si}qN_a\phi_s}$$

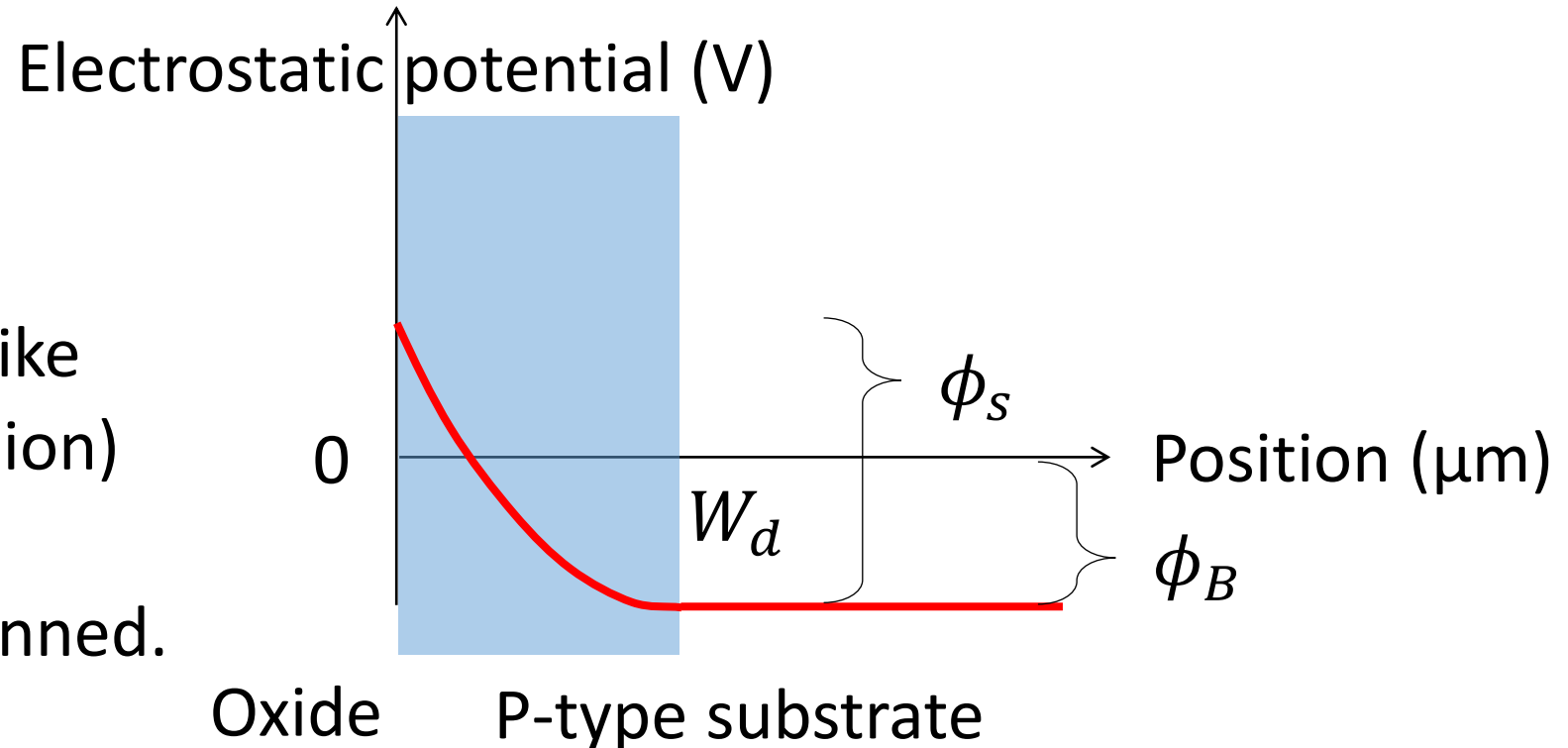
Taur, Eq. (2.189)



Potential profile

- A parabolic potential profile
 - The depletion region cannot grow indefinitely.

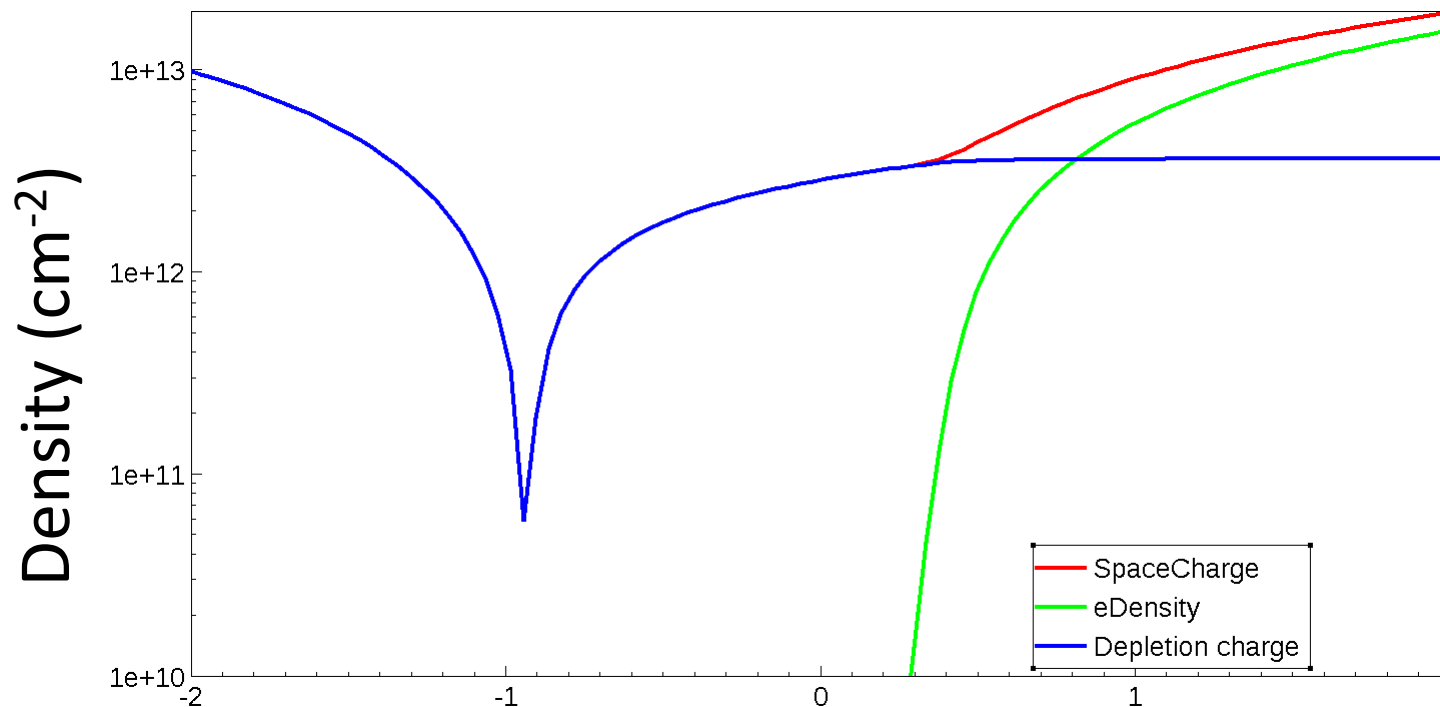
- When $\phi_s = 2\phi_B$,
 $n(0) = p(\infty)$
- The surface behaves like n-type material. (Inversion)
- ϕ_s is approximately pinned.



Maximum depletion width

- Therefore, maximum depletion width becomes

$$W_d = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_a}} = \sqrt{\frac{4\epsilon_{si}k_B T \ln(N_a/n_i)}{q^2 N_a}} \quad \text{Taur, Eq. (2.190)}$$



← Depletion charge does not increase.

Gate voltage (V)

Beyond threshold voltage

It's not perfectly fixed.

- The surface potential is almost fixed. (Surface potential pinning)
 - Small additional change in ϕ_s induces an exponential increase of the electron density.

- Remember that $n = n_i \exp\left(\frac{q\phi}{k_B T}\right)$.

- When $\phi_s = 2\phi_B$,

$$n(0) = n_i \exp\left(\frac{q\phi_B}{k_B T}\right) = p(\infty)$$

- Additional potential ($\Delta\phi$) yields

$$n(0) = p(\infty) \exp\left(\frac{q\Delta\phi}{k_B T}\right)$$

It's a high density.

General relation beyond depletion approx. (1)

- With the depletion approximation, we obtained

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si}qN_a\phi_s}$$

- We can do much better!

- A generation relation for $Q_s = Q_d + Q_i$

Inversion
charge



- The Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}} [p(x) - n(x) - N_a]$$

Taur, Eq. (2.175)

General relation beyond depletion approx. (2)

- Following Taur's notation,

For a while, $\phi(\infty) = -\phi_B$ is used as the reference value.
Therefore,

$$n(x) = n_i \exp\left(\frac{q\phi(x)}{k_B T}\right) \Rightarrow n(x) = n(\infty) \exp\left(\frac{q\phi(x)}{k_B T}\right) \quad \text{Taur, Eq. (2.178)}$$

$$p(x) = n_i \exp\left(-\frac{q\phi(x)}{k_B T}\right) \Rightarrow p(x) = p(\infty) \exp\left(-\frac{q\phi(x)}{k_B T}\right) \quad \text{Taur, Eq. (2.177)}$$

– The Poisson equation

$$\frac{d^2 \phi}{dx^2} = -\frac{q}{\epsilon_{si}} \left[N_a \left(\exp\left(-\frac{q\phi}{k_B T}\right) - 1 \right) - \frac{n_i^2}{N_a} \left(\exp\left(\frac{q\phi}{k_B T}\right) - 1 \right) \right] \quad \text{Taur, Eq. (2.179)}$$

General relation beyond depletion approx. (3)

- Multiplying $\frac{d\phi}{dx} dx$,

– The Poisson equation

$$\frac{d\phi}{dx} d\left(\frac{d\phi}{dx}\right)$$

$$= -\frac{q}{\epsilon_{si}} \left[N_a \left(\exp\left(-\frac{q\phi}{k_B T}\right) - 1 \right) - \frac{n_i^2}{N_a} \left(\exp\left(\frac{q\phi}{k_B T}\right) - 1 \right) \right] d\phi$$

– Integrate the above equation.

$$\int_0^{-E_x(x)} \frac{d\phi}{dx} d\left(\frac{d\phi}{dx}\right)$$

Taur, Eq. (2.180)

$$= -\frac{q}{\epsilon_{si}} \int_0^{\phi(x)} \left[N_a \left(\exp\left(-\frac{q\phi}{k_B T}\right) - 1 \right) - \frac{n_i^2}{N_a} \left(\exp\left(\frac{q\phi}{k_B T}\right) - 1 \right) \right] d\phi$$

General relation beyond depletion approx. (4)

- (Square of) Electric field

– From $\frac{1}{2} E_x^2(x) = -\frac{q}{\epsilon_{si}} \left[-N_a \frac{k_B T}{q} \exp\left(-\frac{q\phi}{k_B T}\right) - N_a \phi + N_a \frac{k_B T}{q} - \frac{n_i^2}{N_a} \frac{k_B T}{q} \exp\left(\frac{q\phi}{k_B T}\right) + \frac{n_i^2}{N_a} \phi + \frac{n_i^2}{N_a} \frac{k_B T}{q} \right]$, we get

$$\begin{aligned} E_x^2(x) &= \frac{2k_B T N_a}{\epsilon_{si}} \left[\left(\exp\left(-\frac{q\phi}{k_B T}\right) + \frac{q\phi}{k_B T} - 1 \right) \right. \\ &\quad \left. + \frac{n_i^2}{N_a^2} \left(\exp\left(\frac{q\phi}{k_B T}\right) - \frac{q\phi}{k_B T} - 1 \right) \right] \end{aligned}$$

Taur, Eq. (2.181)

General relation beyond depletion approx. (5)

- At $x = 0$, we have $\phi(0) = \phi_s$.
 - Then,

$$\begin{aligned} E_s^2 &= \frac{2k_B T N_a}{\epsilon_{si}} \left[\left(\exp \left(-\frac{q\phi_s}{k_B T} \right) + \frac{q\phi_s}{k_B T} - 1 \right) \right. \\ &\quad \left. + \frac{n_i^2}{N_a^2} \left(\exp \left(\frac{q\phi_s}{k_B T} \right) - \frac{q\phi_s}{k_B T} - 1 \right) \right] \end{aligned}$$

General relation beyond depletion approx. (6)

- At $x = 0$, we have $\phi(0) = \phi_s$.

– From $Q_s = -\epsilon_{si}E_s$,

$$Q_s = \pm \sqrt{2\epsilon_{si}k_B T N_a} \left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1 \right) + \frac{n_i^2}{N_a^2} \left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1 \right) \right]^{1/2}$$

Taur, Eq. (2.182)

Homework#4

- Draw $|Q_s|$ as a function of ϕ_s .
 - Assume that N_a is $4 \times 10^{15} \text{ cm}^{-3}$.
 - ϕ_s varies from -0.4 V to 1.0 V.

Thank you!