Special Topics on Basic EECS I
VLSI Devices
Lecture 22

Sung-Min Hong (smhong@gist.ac.kr)
Semiconductor Device Simulation Laboratory
School of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology

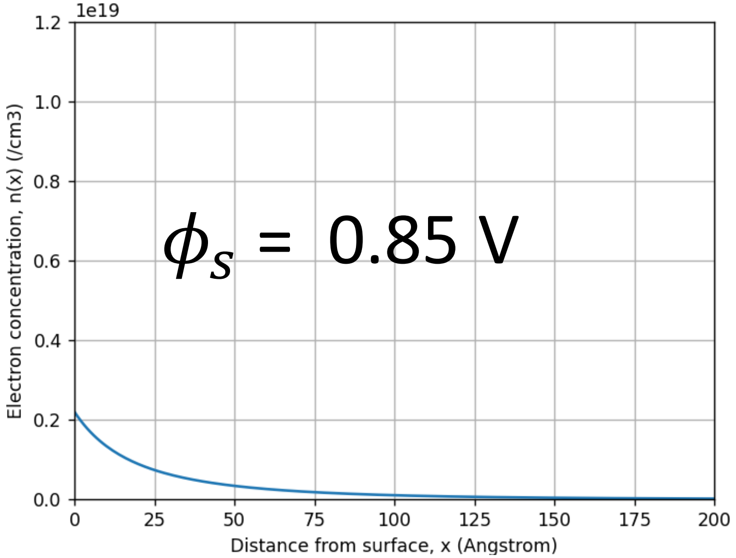
Strong inversion

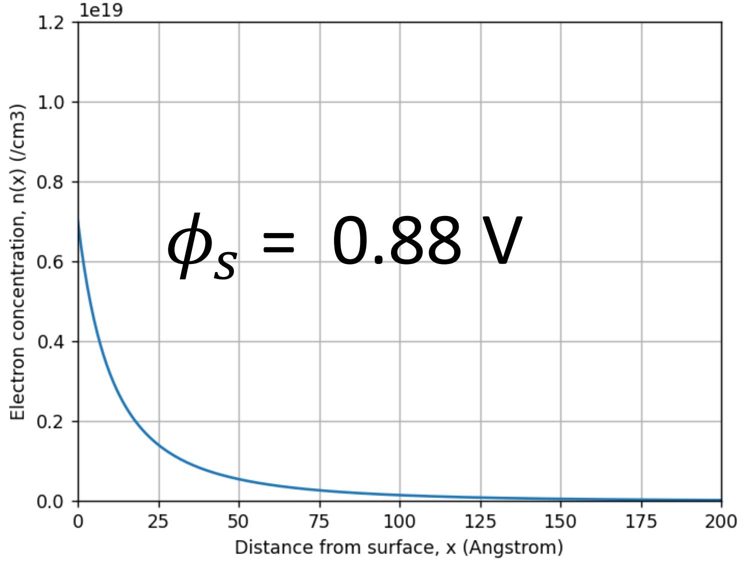
- Beyond strong inversion,

$$\frac{d\phi}{dx} \approx -\sqrt{\frac{2k_B T N_a}{\epsilon_{si}}\left(\frac{q\phi}{k_B T} + \frac{n_i^2}{N_a^2}\exp\left(\frac{q\phi}{k_B T}\right)\right)}$$

Taur, Eq. (2.191)

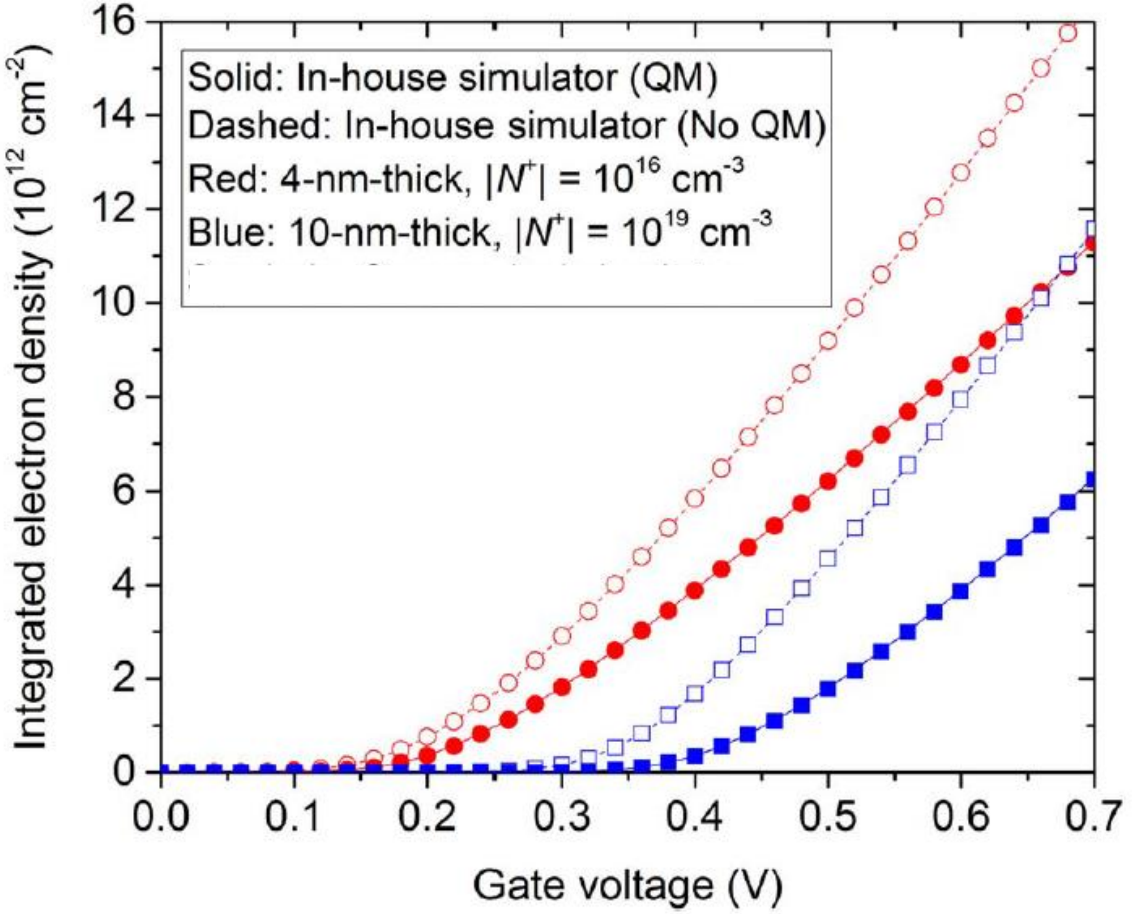
  – The electrons are distributed extremely close to the surface with an inversion-layer width less than 50 Å.





Strong inversion

- Quantum confinement effect
  - A peak distribution 10~20 Å away from the surface



MOS equation

Total silicon charge per unit area

- Up to now, $Q_s(\phi_s)$ is found. We can control only V_g.
  - Relation between V_g and ϕ_s

$$V_g - V_{fb} = V_{ox} + \phi_s = -\frac{Q_s}{C_{ox}} + \phi_s$$

Taur, Eq. (2.195)

$\frac{\epsilon_{ox}}{t_{ox}}$, oxide capacitance per unit area

  - In general, $Q_s(\phi_s)$ is known. We can solve the above equation.

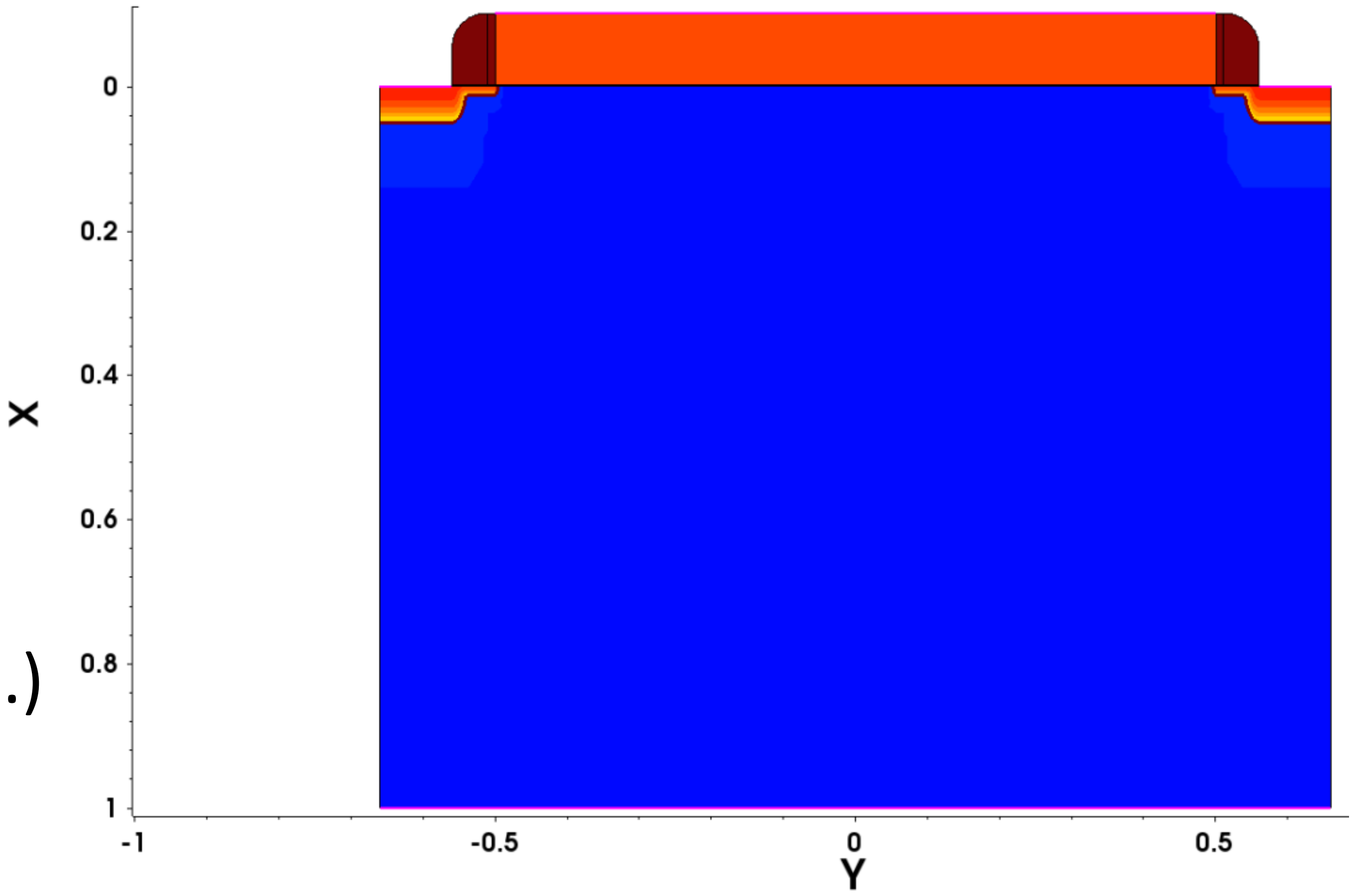
Taur, Eq. (2.182)

Homework#5

- Draw ϕ_s as a function of V_g (> 0 V).
  - Assume that N_a = 10^{17} cm^{-3}, t_{ox} = 10 nm, and V_{fb} = -0.88 V.
  - Hint: Calculate V_g using ϕ_s by using Taur, Eq. (2.195).

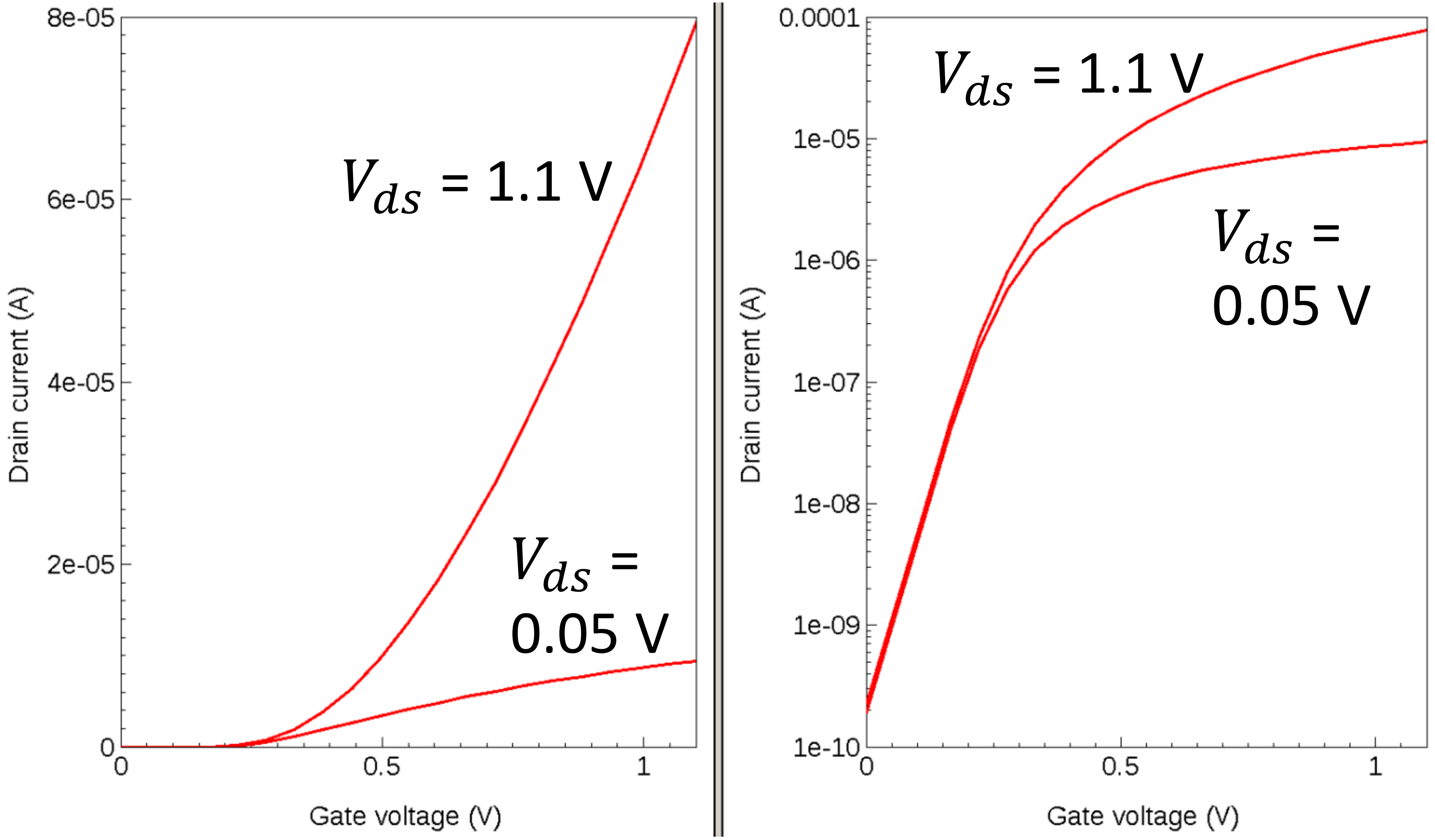
TCAD simulation of a long-channel MOSFET

- Channel length, 1 μm
  - V_{fb} is -1.08 V.
  - t_{ox} is 1.2 nm.
  - C_{ox} is 2.88X10^{-6} F/cm^2.
  - V_{DD} of 1.1 V
  - (Estimate its technology node.)



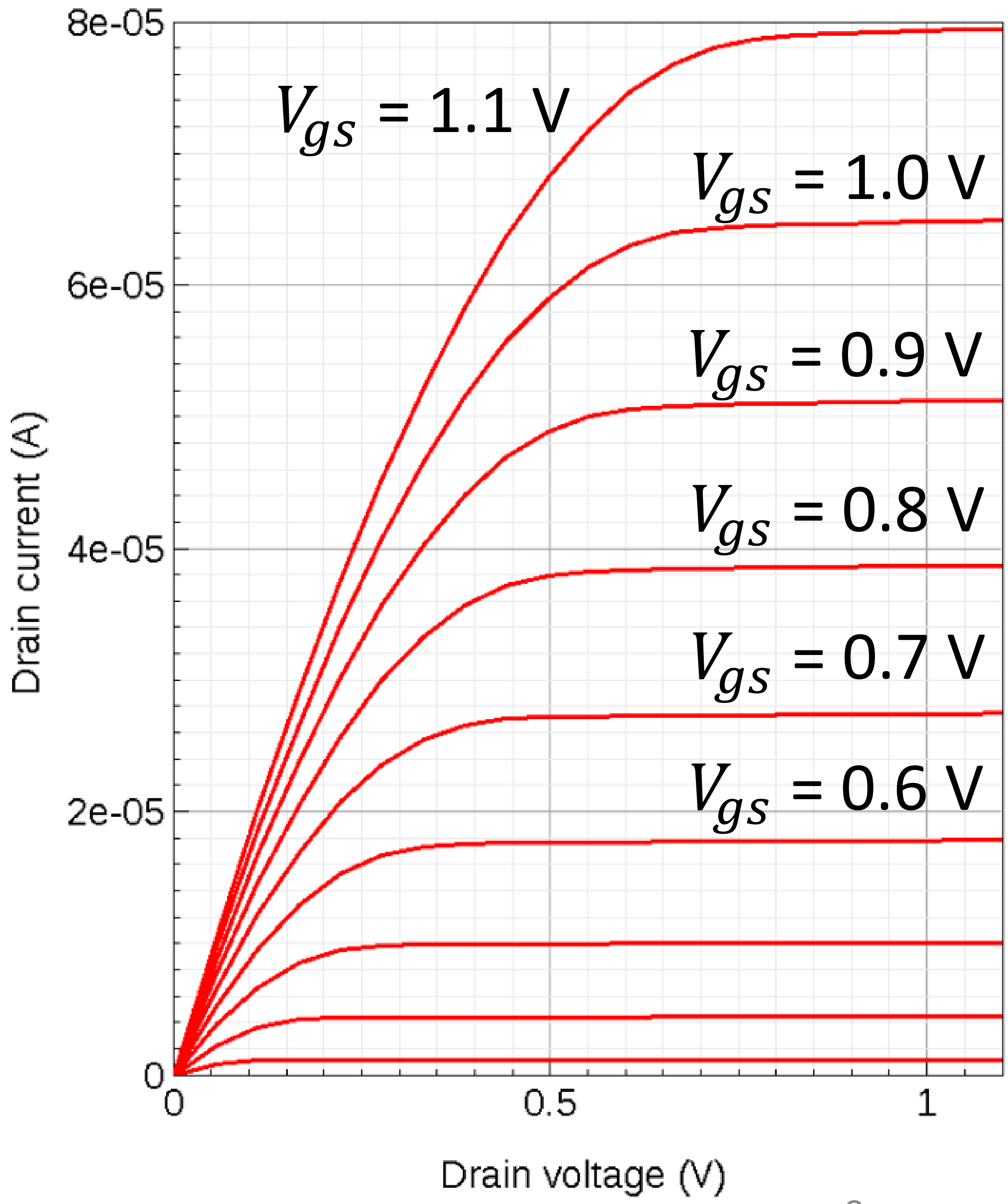
I_d versus V_{gs}

- Input characteristics



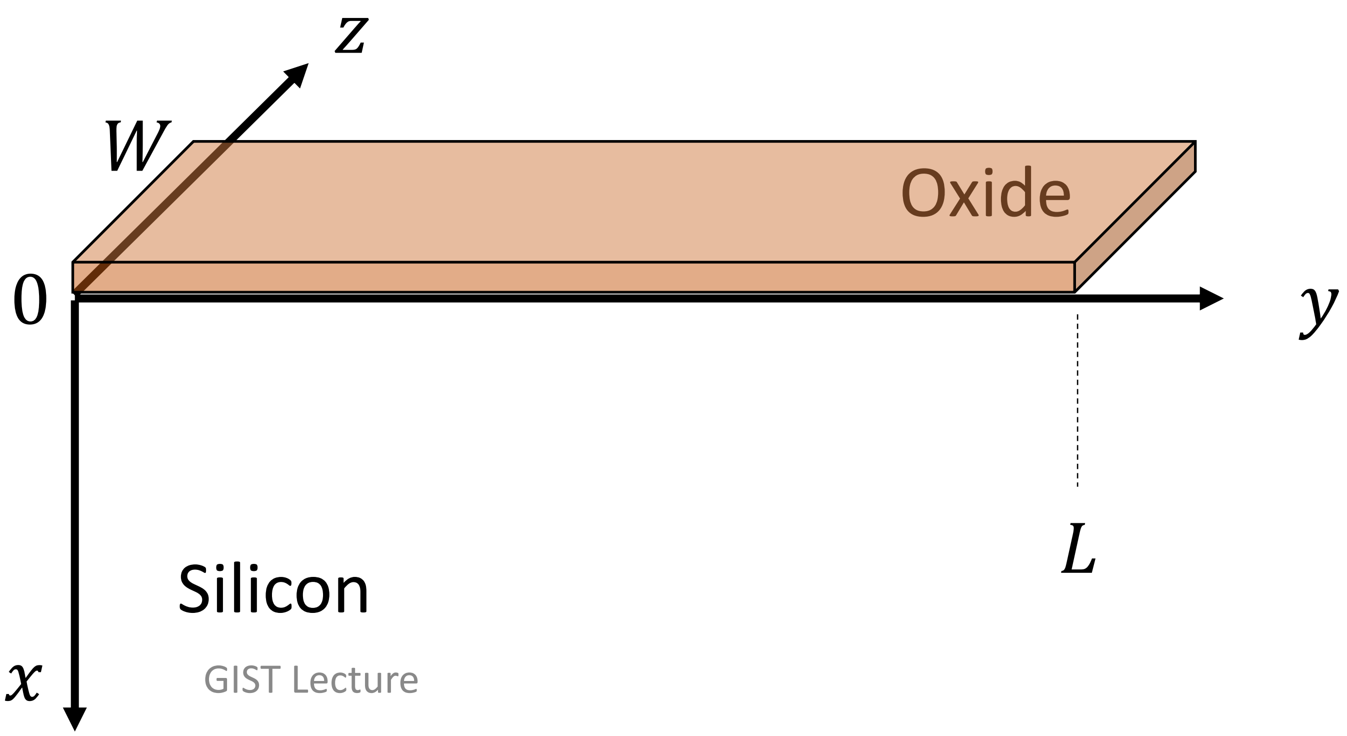
I_d versus V_{ds}

- Output characteristics
  - Triode mode
  - Saturation mode



Schematic

- $x = 0$ at silicon surface
  - $y = 0$ at the source and $y = L$ at the drain
  - Source and substrate are grounded.
  - Uniform p-type substrate



Gradual channel approximation

- Variation of the electric field in the y-direction is much less than the corresponding variation in the x-direction.

$$\left|\frac{\partial^2 \phi}{\partial x^2}\right| \gg \left|\frac{\partial^2 \phi}{\partial y^2}\right|$$

  - Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\frac{q}{\epsilon_{si}}[p(x,y) - n(x,y) - N_a]$$

  - Poisson equation under the GCA

$$\frac{d^2 \phi}{d x^2} = -\frac{q}{\epsilon_{si}}[p(x,y) - n(x,y) - N_a] \qquad \text{Taur, Eq. (2.175)}$$

Electron quasi-Fermi potential, $\boldsymbol{V(y)}$

It was written as ϕ_n in Taur, Eq. (2.61).

- It is assumed that V is independent of x.
  - *Why?* MOSFET current flows predominantly along the y-direction.
  - Since $\mathbf{J}_n = -q\mu_n n \nabla \phi_n$, V varies mainly along the y-direction.
  - Boundary conditions:

$$V(y=0) = V_s = 0$$
$$V(y=L) = V_d = V_{ds}$$

- Electron density, $n(x, y)$

$$n(x,y) = \frac{n_i^2}{N_a} \exp\left(\frac{q}{k_B T}(\phi - V)\right)$$

Taur, Eq. (3.1)

Still, ϕ_B is the reference value.

Our previous expressions (1)

- They are modified by V. (Terms related with the electron density)
  - Electric field

$$E_x^2(x,y) = \left(\frac{d\phi}{dx}\right)^2$$
$$= \frac{2k_B T N_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi}{k_B T}\right) + \frac{q\phi}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(-\frac{qV}{k_B T}\right)\right)\left(\exp\left(\frac{q\phi}{k_B T}\right) - 1\right) - \frac{q\phi}{k_B T}\right)\right]$$

Taur, Eq. (3.2)

Our previous expressions (2)

- They are modified by V. (Terms related with the electron density)
  - Surface inversion

$$\phi(0, y) = V(y) + 2\phi_B \qquad \text{Taur, Eq. (3.3)}$$

  - Maximum depletion layer width

$$W_{dm} = \sqrt{\frac{2\epsilon_{si}[V(y) + 2\phi_B]}{qN_a}} \qquad \text{Taur, Eq. (3.4)}$$

- Summary: With the GCA, our MOS expressions are re-used only with modification by V.

Drain current

- Electron current density at a point (x, y)

$$J_n(x, y) = -q\mu_n n(x, y)\frac{dV}{dy} \qquad \text{Taur, Eq. (3.5)}$$

  - (It includes both the drift and diffusion currents.)
  - When integrated from $x = 0$ to x_i,

$$I_d(y) = qW\int_0^{x_i} \mu_n n(x, y)\frac{dV}{dy}dx \qquad \text{Taur, Eq. (3.6)}$$

Sign change due to convention of terminal current

z-directional width

Further simplification

- Electron current density at a point (x, y)

$$I_d(y) = qW\int_0^{x_i} \mu_n n(x,y)\frac{dV}{dy}dx = -\mu_{eff}W\frac{dV}{dy}\left(-q\int_0^{x_i} n(x,y)dx\right)$$

$$= -\mu_{eff}W\frac{dV}{dy}Q_i(y)$$

Taur, Eq. (3.8)

 – We introduce an effective mobility, μ_{eff}.
 – Since V is a function of y only, V is interchangeable with y.

$$Q_i(y) = Q_i(V)$$

 – Then,

$$I_d(y)dy = \mu_{eff}W[-Q_i(V)]dV$$

$I_d(y)$ is actually a constant.

- When integrated from $y = 0$ to L, (from $V = 0$ to V_{ds})

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV$$

Taur, Eq. (3.10)

  - Then, how can we find $Q_i(V)$? (We must perform the x-directional integration.)

$$Q_i = -q \int_0^{x_i} n(x, y) dx = -q \int_{\phi_s}^{\delta} n(\phi, V) \frac{dx}{d\phi} d\phi$$

$$= -q \int_{\delta}^{\phi_s} \frac{(n_i^2/N_a) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$

Taur, Eq. (3.12)

Then, how can we determine ϕ_s?

- For given V_{gs} and V, we can solve the MOS equation.

$$V_{gs} = V_{fb} + \phi_s - \frac{Q_s}{C_{ox}}$$

$$= V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right) \right]^{1/2}$$

Taur, Eq. (3.14)

  – We can numerically solve the above equation to obtain ϕ_s.

Pao-Sah double integral

- Finally, the expression for I_d reads

$$I_d = q\mu_{eff}\frac{W}{L}\int_0^{V_{ds}}\left[\int_\delta^{\phi_s}\frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi,V)}d\phi\right]dV$$

Taur, Eq. (3.13)

  - It is the Pao-Sah double integral.
  - Rigorous within the GCA, but it is difficult to evaluate.

Thank you!