# Special Topics on Basic EECS I
# VLSI Devices
# Lecture 8

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# Homework#2

- Sheet resistivity
  - Assume the following electron distribution,

$$n(z) = n_s \exp\left(-\frac{z}{\Delta}\right),$$

  where the vertical coordinate, $z$, starts at 0.
  - Also assume a constant electron mobility, $\mu_n$.
  - (Neglect holes.)
  - Calculate the sheet resistivity, which satisfies $R = \frac{L}{W}\rho_{sh}$.

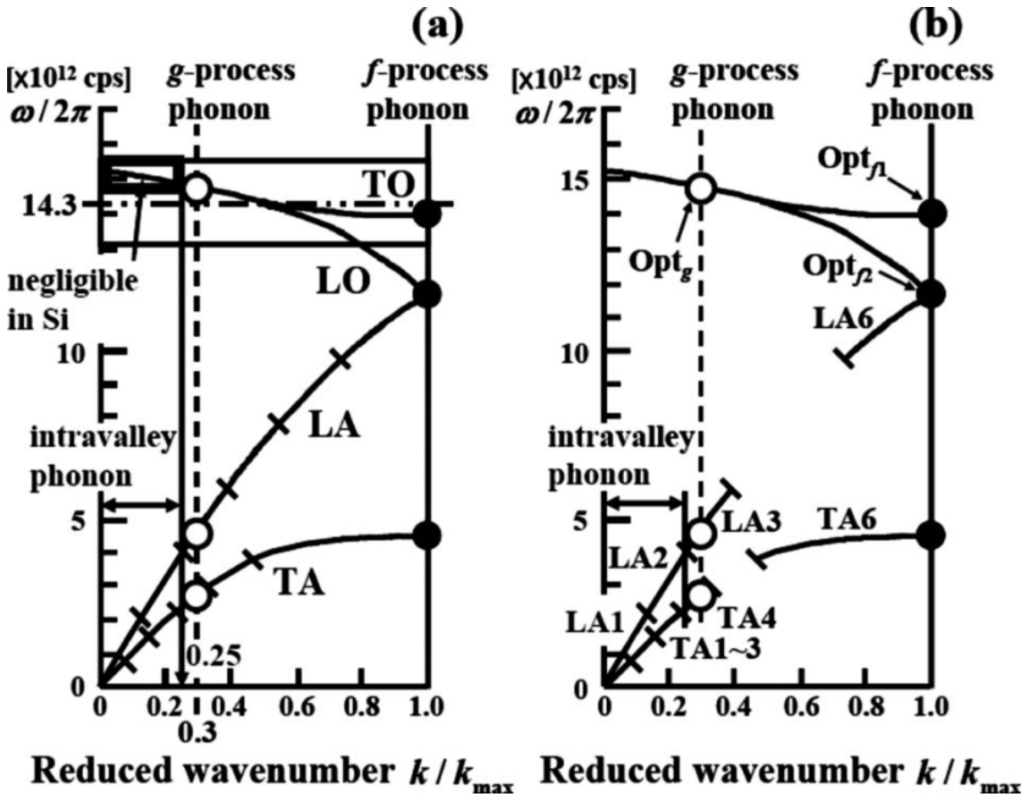
# Example 2-2 of Hu's book

- Hole mobility, $\mu_p = 470\ \mathrm{cm^2V^{-1}s^{-1}}$
  - When the electric field is $10^3\ \mathrm{V\ cm^{-1}}$, the drift velocity is $4.7 \times 10^5\ \mathrm{cm\ s^{-1}}$.
  - Momentum relaxation time (with $m_p = 0.39\ m_0$) is 0.1 psec.

# Phonon scattering

- Various phonon modes
  - Acoustic phonon : Low energy
  - Optical phonon : High energy, often treated as dispersion-less

~60 meV



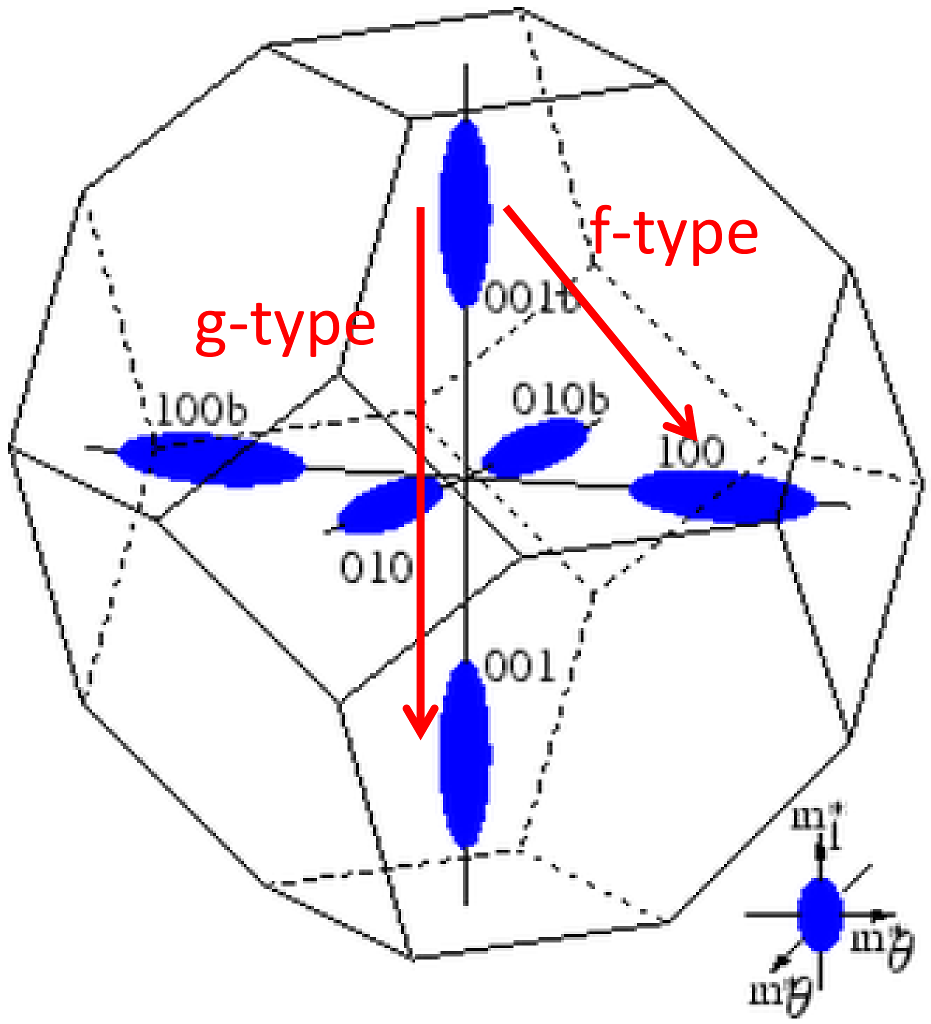

"Selection rule" matters.
Intravalley / f-process / g-process

# Typical parameters

- Various phonon modes
  - Acoustic phonon : Low energy

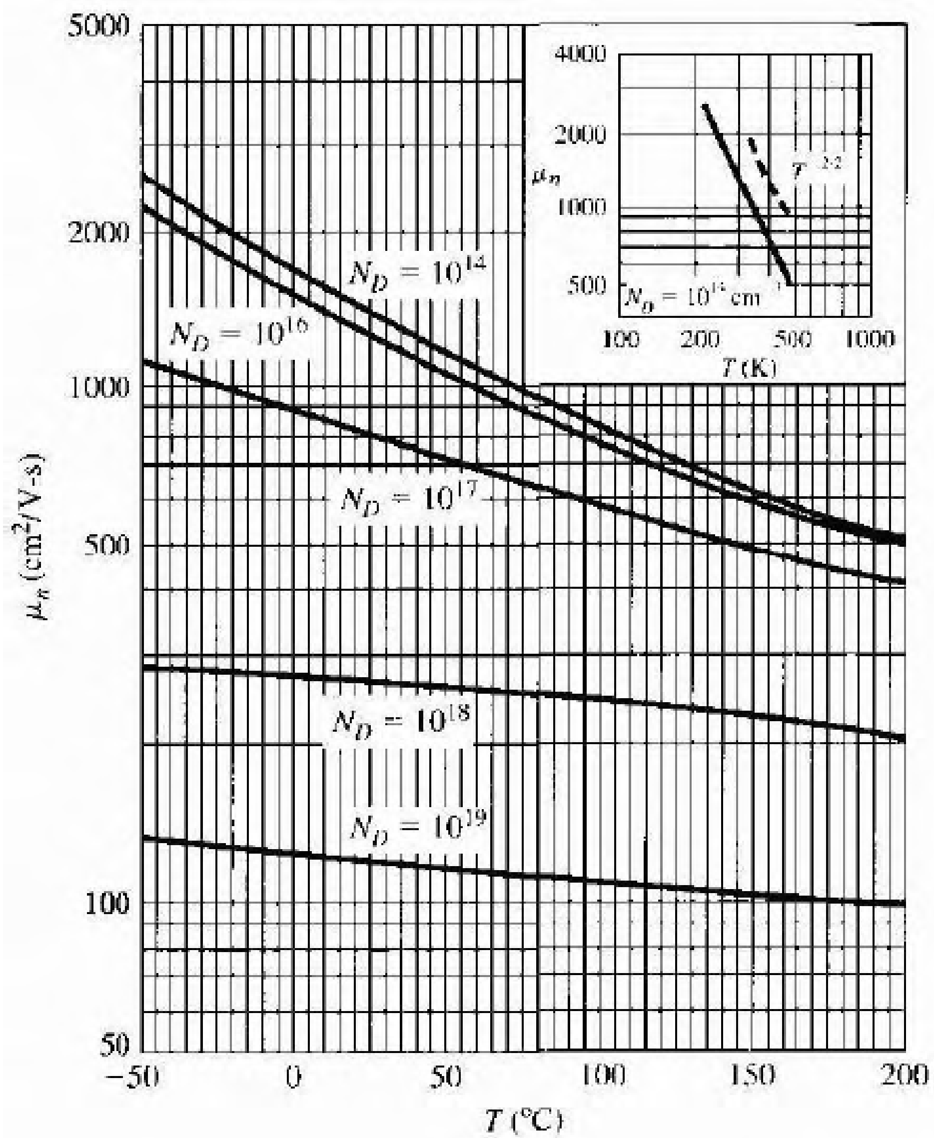
| Mode | $D_t K$ ($10^8$ eV/cm) | $\hbar\omega$ (meV) | Type |
|---|---|---|---|
| TA | 0.470 | 12.1 | g-type |
| LA | 0.740 | 18.5 | g-type |
| LO | 10.23 | 62.0 | g-type |
| TA | 0.280 | 19.0 | f-type |
| LA | 1.860 | 47.4 | f-type |
| TO | 1.860 | 58.6 | f-type |

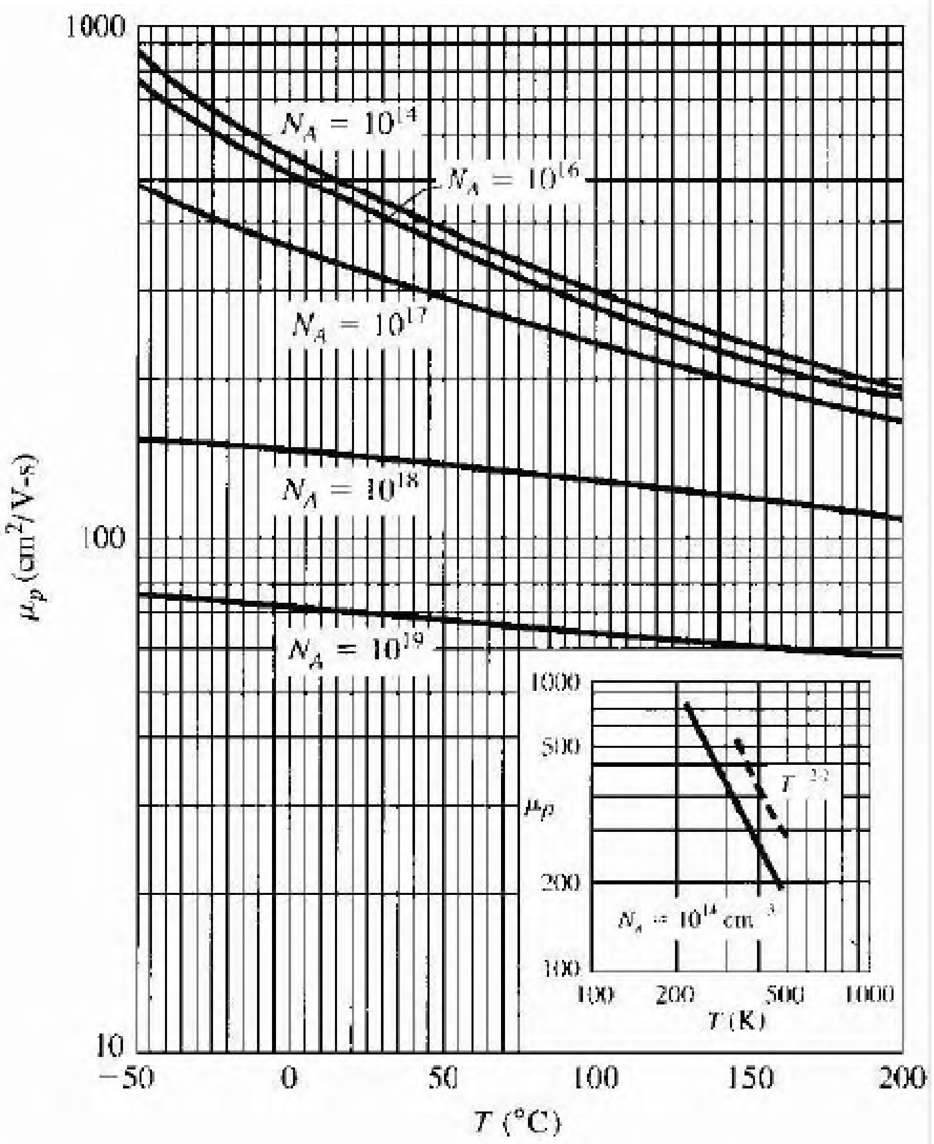
Parameters for inelastic phonon scatterings in the Si conduction band



# Temperature & doping

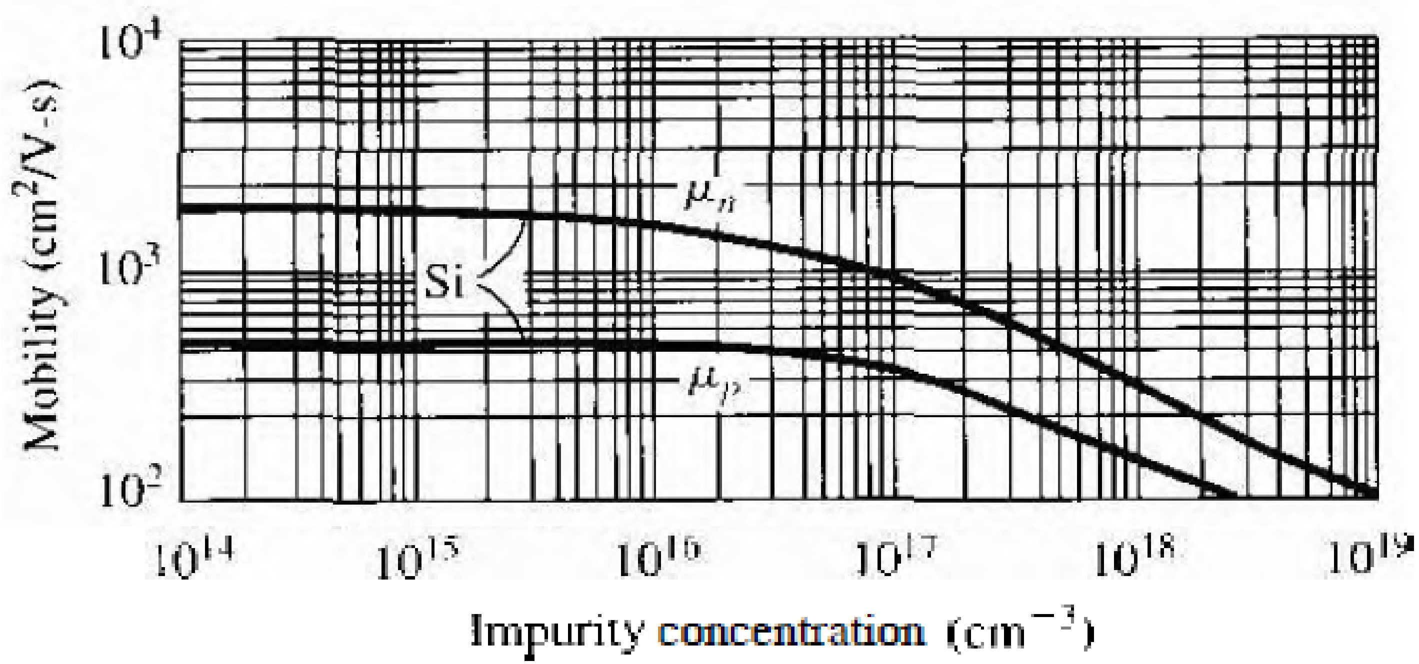
(Neamen's book)



# Impurity concentration

- It is modeled as an elastic scattering process.




(Neamen's book)

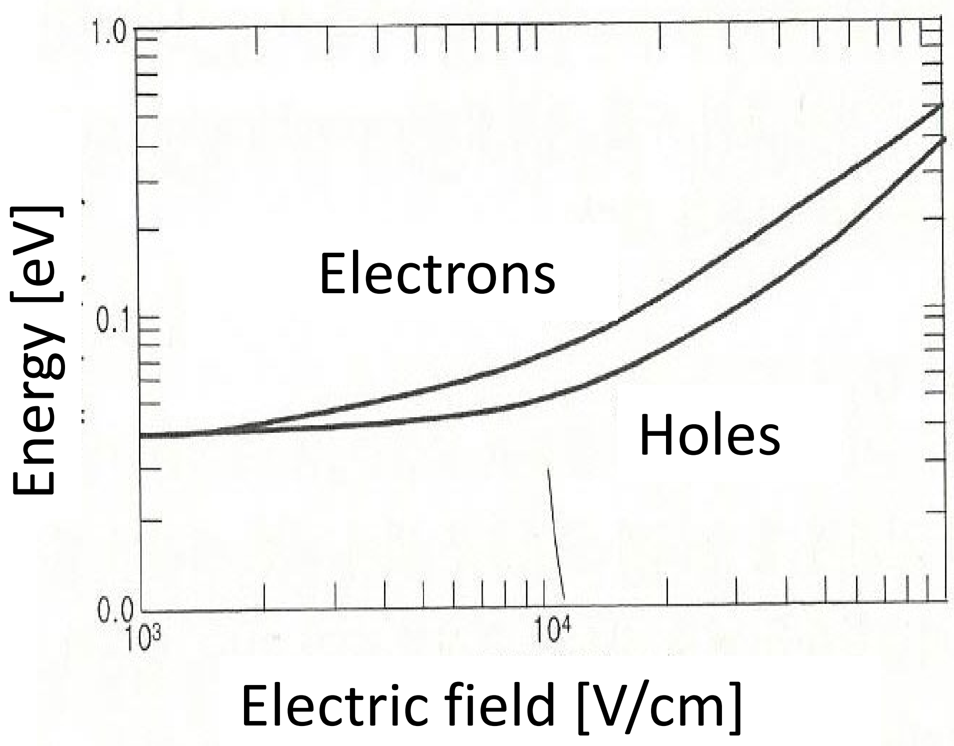
- Which one is dominant? Phonon or impurity?
  - Matthiessen's rule, $\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$ Taur, Eq. (2.27)

# Matthiessen's rule

- When there are multiple contributions to the mobility,
  - (For example, phonon-limited mobility / impurity-limited mobility)
  - The overall collision rate is given by sum of all contributions.
  - $\frac{1}{\tau_m} = \frac{1}{\tau_{mL}} + \frac{1}{\tau_{mI}} + \cdots$
  - (The above relation holds exactly only in the microscopic level.)
  - When recalling $\mu = \frac{q\tau_m}{m_n}$, it means $\frac{1}{\mu_m} = \frac{1}{\mu_{mL}} + \frac{1}{\mu_{mI}} + \cdots$
  - It is very useful.

# Hot electron

- Not only velocity, but also energy…
  - Increases when the electric field increases.
  - Increase of energy is a reason of the velocity saturation. *Why?*



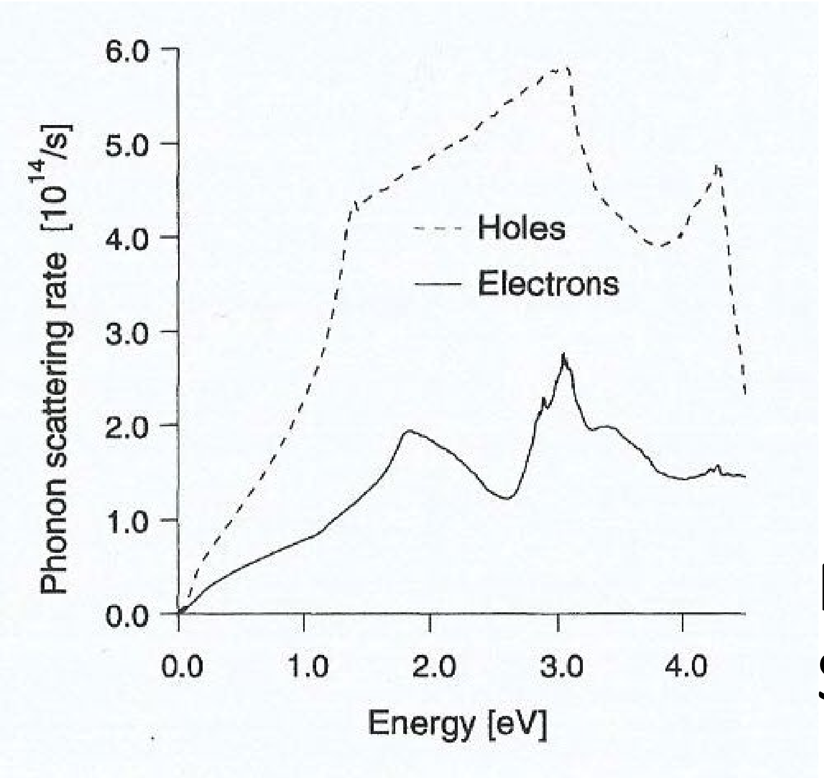

Up to 1 kV/cm, average energy is almost the same with the lattice energy.

Above 10 kV/cm, average energy significantly deviates from the lattice energy.

Average energy of electrons/holes in Si at 300K (Park’s book)

# Velocity saturation

- Electron with higher energy
  - Has a higher chance to be scattered by phonons. (Higher DOS)
  - More frequent scattering : Smaller $\tau_m$

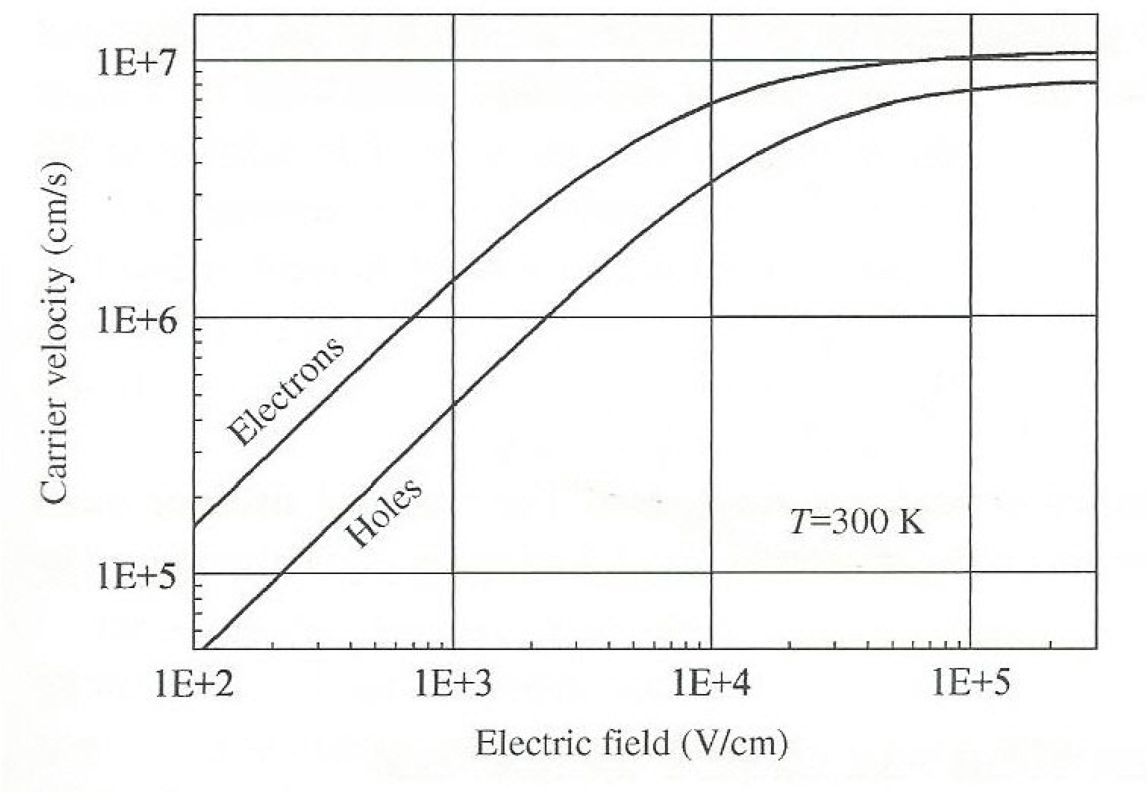



Phonon scattering rate in Si (Jungemann's book)

Phonon scattering rate in Si resembles the Density-Of-States.

# Velocity vs. electric field

- At low electric fields, the linear relationship is valid.
  - At high electric fields, the velocity saturation starts to occur. The saturation velocity of Si is about $10^7$ (cm/sec).




Velocity-field relationship in Si at 300K (Taur's book)

# Caughey-Thomas relation

- For silicon,
  - Electron velocity can be approximated by

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{0.5}}$$

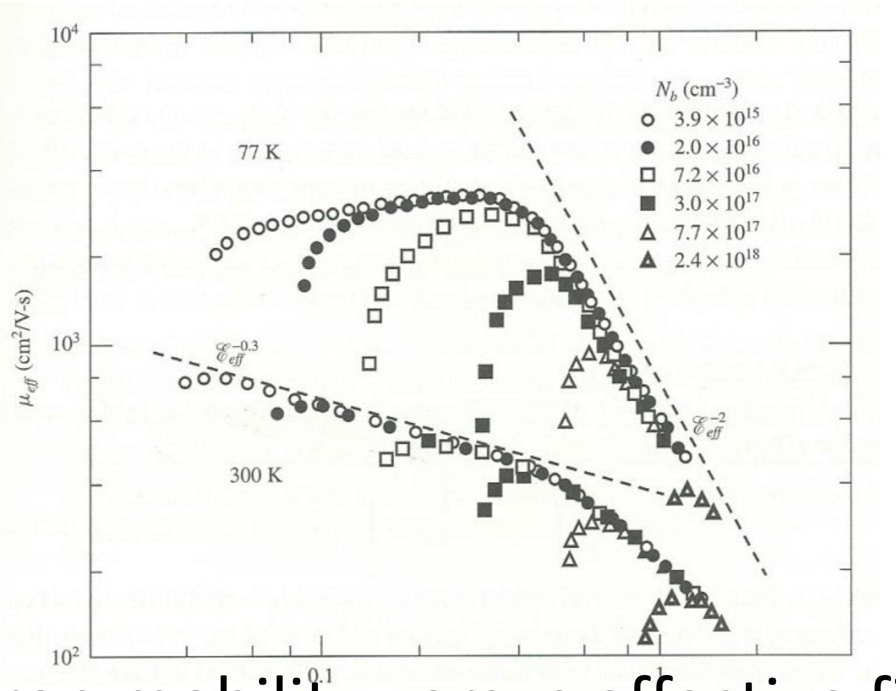
  - Hole velocity

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)\right]}$$

  - Why are they different?

# Other scattering mechanisms

- We have discussed about the bulk mobility.
  - Other scattering mechanisms (alloy scattering & impact ionization)
  - Surface scattering severely reduces the inversion mobility.



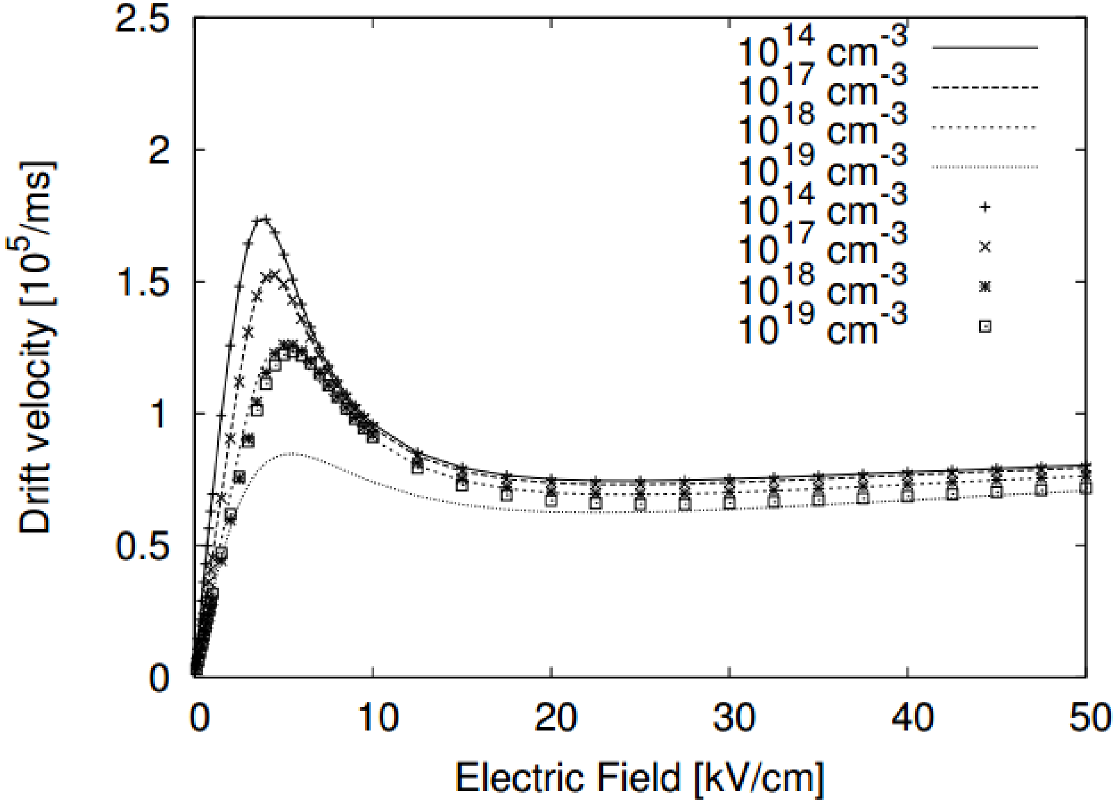

So-called "universal" mobility curve in the Si inversion layer.

Two difference contributions are clearly visible.

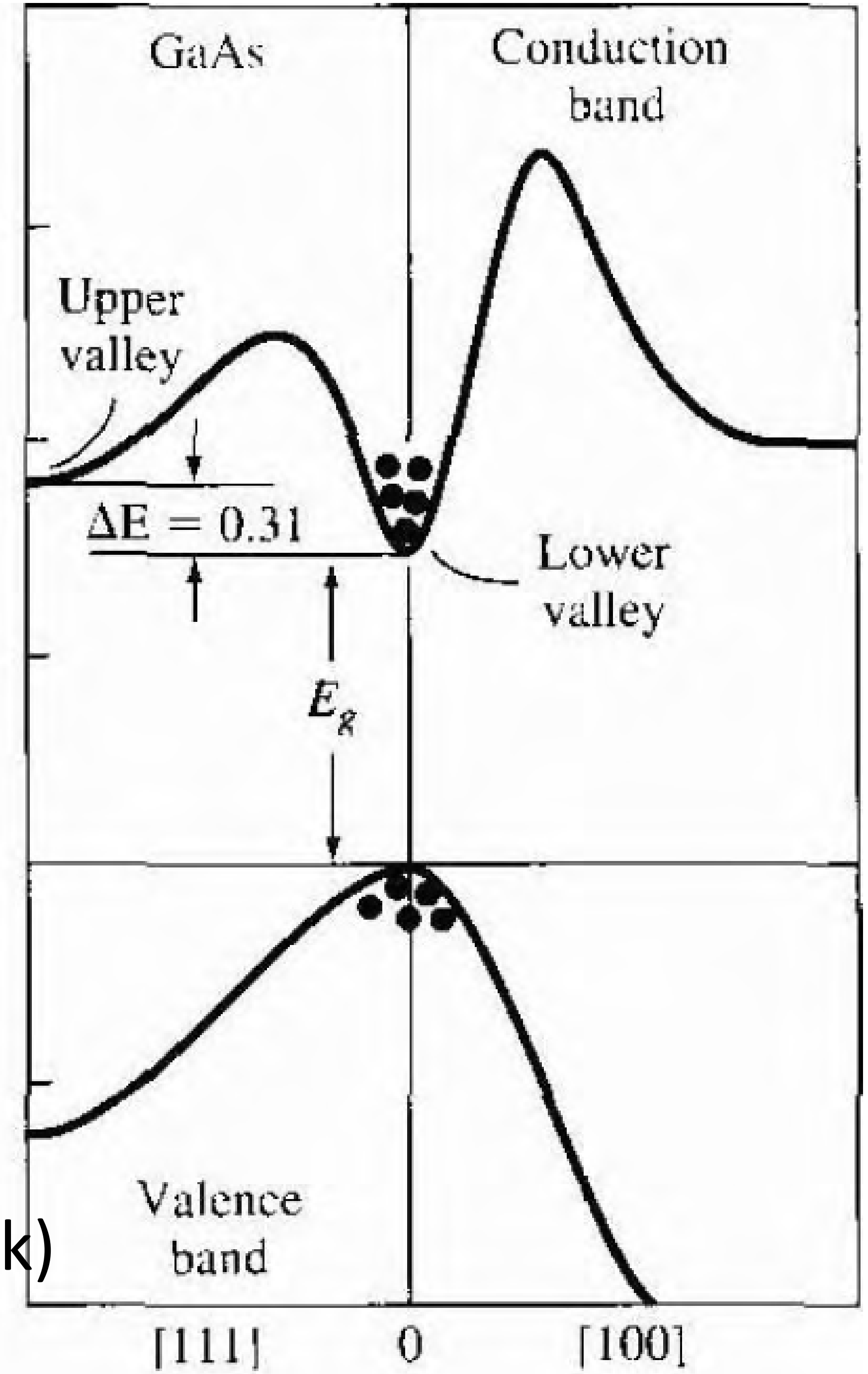
Electron mobility versus effective field for several doping concentrations (Takagi's paper)

# GaAs case

- Negative differential mobility
  - Its *L* valley is heavy.



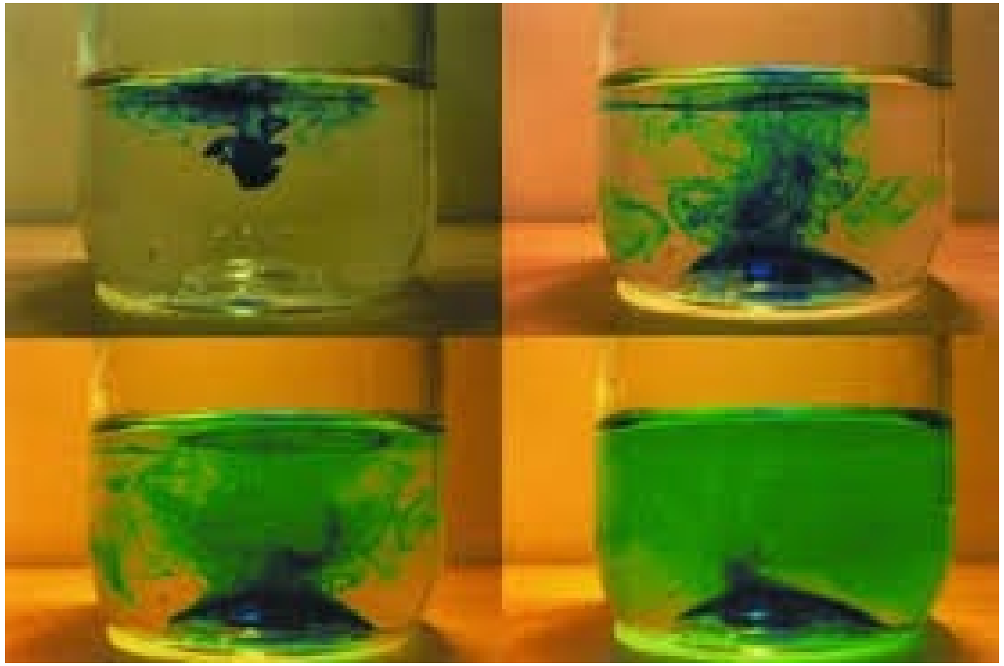

(J. Bieder et al., IWCE 2010)




(Neamen's book)

# Diffusion

- It is not only for charged particles.
  - For example,



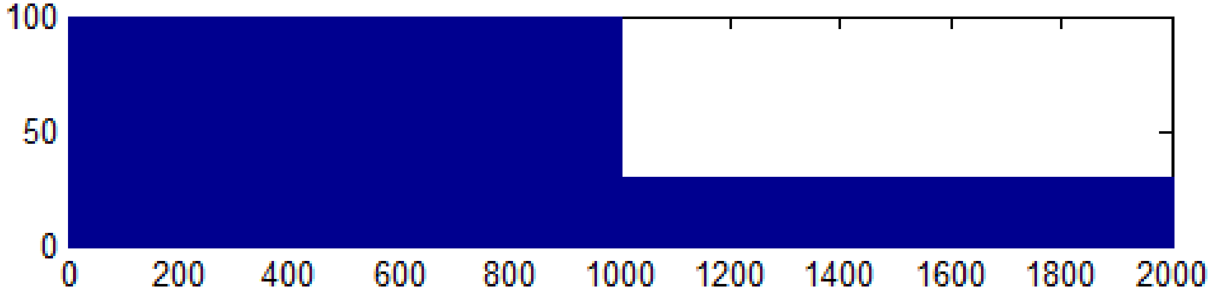
Diffusion of ink
(Google images)

  - Therefore, no polarity is expected.

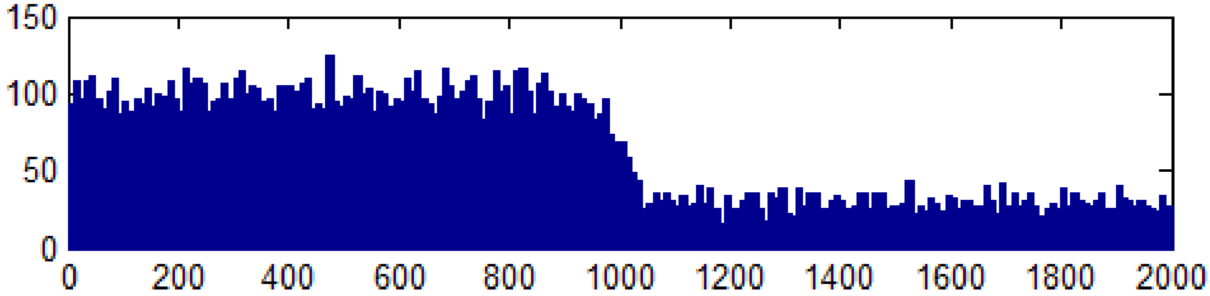
# A simple game, again

- Random motion of balls in a 1D box
  - At each turn, they can move forward (+1) or backward (-1).

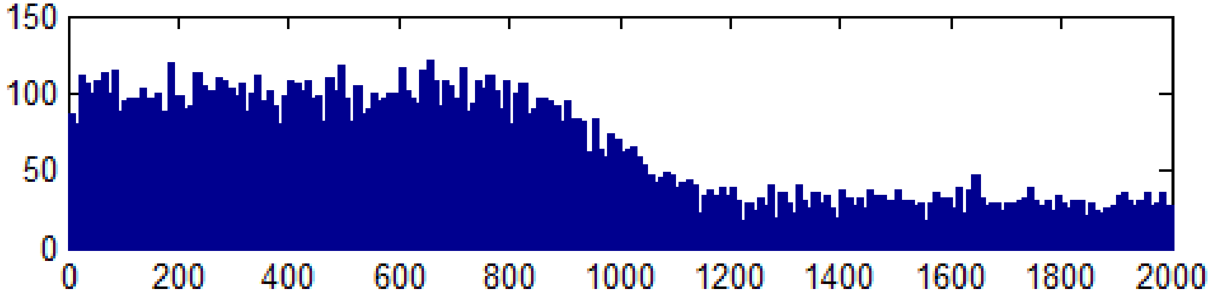
Initial condition




1 k turns




10 k turns



# Equation

- Flux
  - The electron flux due to the diffusion mechanism is given by

$$\mathbf{F}_n = -D_n \nabla n$$

  where $D_n$ is the electron diffusion coefficient in the unit of ($cm^2$/sec).
  - The diffusion current density is

$$\mathbf{J}_{n,diff} = qD_n \nabla n \qquad \text{Taur, Eq. (2.36)}$$

- How about the hole?
  - The diffusion current density is

$$\mathbf{J}_{p,diff} = -qD_p \nabla p \qquad \text{Taur, Eq. (2.37)}$$

# An example

- Taken from Neamen's book
  - Over 1 mm, the electron density varies linearly from 1X10$^{18}$ cm$^{-3}$ to 7X10$^{17}$ cm$^{-3}$.
  - The diffusion coefficient is $D_n$ = 225 cm2/sec.
  - Calculate the current density.

$$J_n = +qD_n \frac{dn}{dx}$$

$$= (1.6 \times 10^{-19}\ \mathrm{C})(225\ \mathrm{cm^2/s}) \left( \frac{1 \times 10^{18}\ \mathrm{cm^{-3}} - 7 \times 10^{17}\ \mathrm{cm^{-3}}}{0.1\ \mathrm{cm}} \right)$$

$$= 108\ \mathrm{A/cm^2}$$

# Revisit the total current density.

- Total current density
  - Electron current density

$$\mathbf{J}_n = q\mu_n n\mathbf{E} + qD_n \nabla n$$

Taur, Eq. (2.54)

  - Hole current density

$$\mathbf{J}_p = q\mu_p p\mathbf{E} - qD_p \nabla p$$

Taur, Eq. (2.55)

  - (Time-dependent) displacement current density

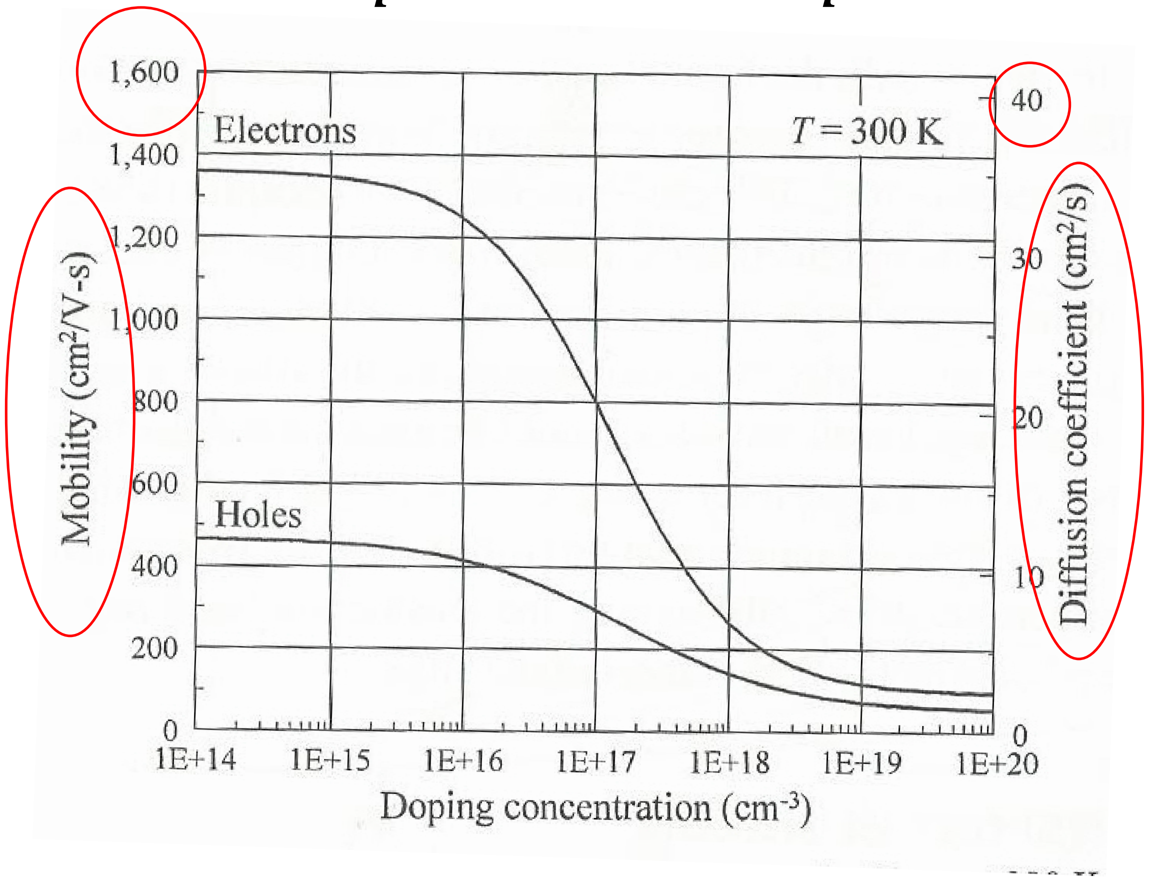
$$\mathbf{J}_{displacement} = \frac{\partial}{\partial t}(\epsilon\mathbf{E})$$

# Einstein relation

- At equilibrium, we have the following relations:

$$D_n = \frac{k_B T}{q}\mu_n, D_p = \frac{k_B T}{q}\mu_p$$

Taur, Eq. (2.38) and Eq. (2.39)




(Park's book)

# Thank you!