Special Topics on Basic EECS I
VLSI Devices
Lecture 4

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A special case

- Sometimes, $f(\mathbf{k})$ depends on only the energy, $f(E)$.
  - In such a case, the electron density can be written as

$$n = \frac{1}{(2\pi)^3} \iiint\limits_{\substack{Entire \\ \mathbf{k}\,space}} f(\mathbf{k}) dk_x dk_y dk_z = \int_{E_C}^{\infty} N(E) f(E) dE$$

~ Taur, Eq. (2.7)

  - When do we have $f(E)$, instead of $f(\mathbf{k})$?
  - The equilibrium state is a typical example.

Fermi-Dirac distribution

- At equilibrium, the Fermi-Dirac distribution holds

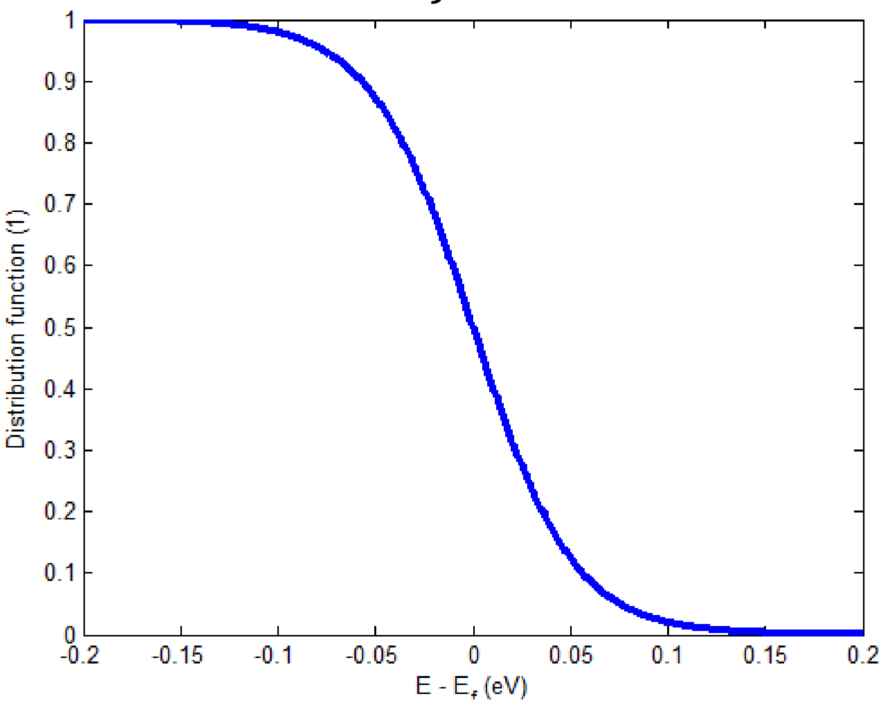
$$f_D(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{k_B T}\right)}$$

Taur, Eq. (2.4)

~ 0.02585 meV
@ 300 K

  - At $E = E_f$ (Fermi level),

$$f_D(E_f) = \frac{1}{2}$$

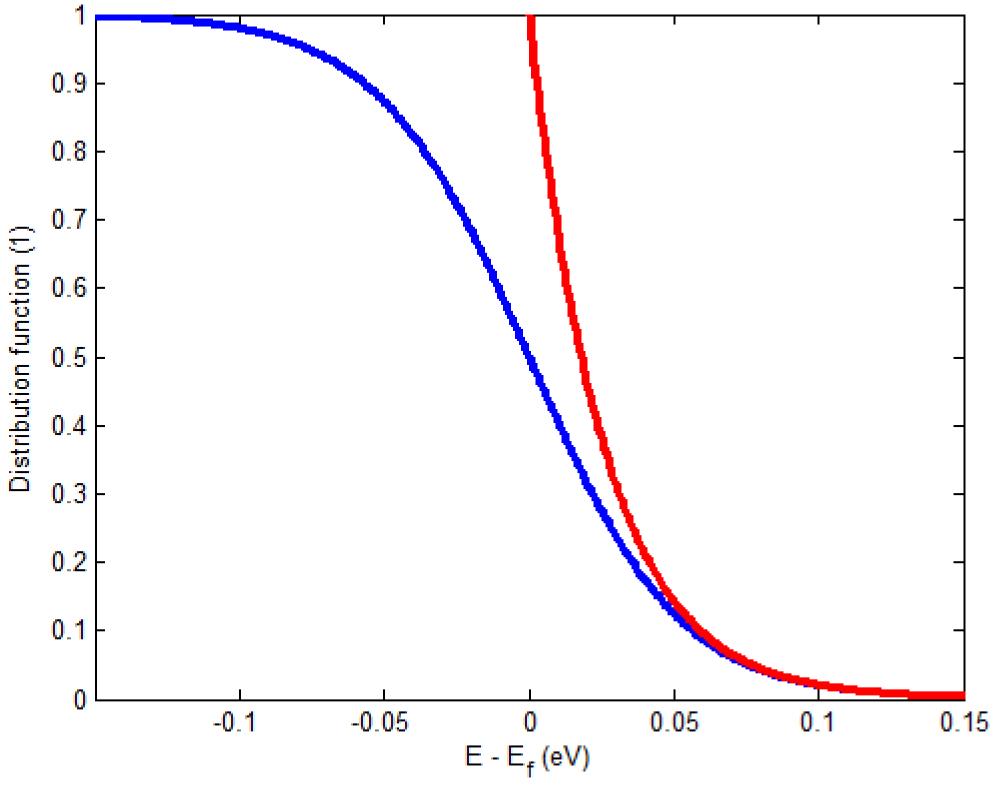


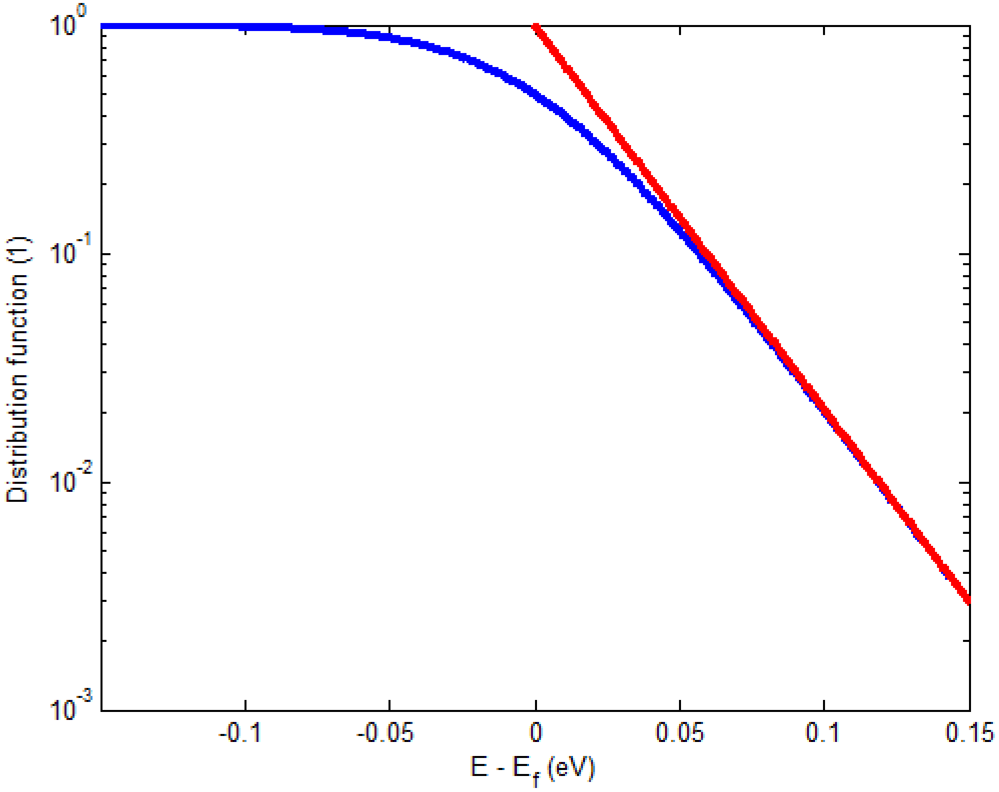
Boltzmann limit

- When $E > E_f$,

$$f_D(E) \approx \exp\left(-\frac{E - E_f}{k_B T}\right)$$

Taur, Eq. (2.5)



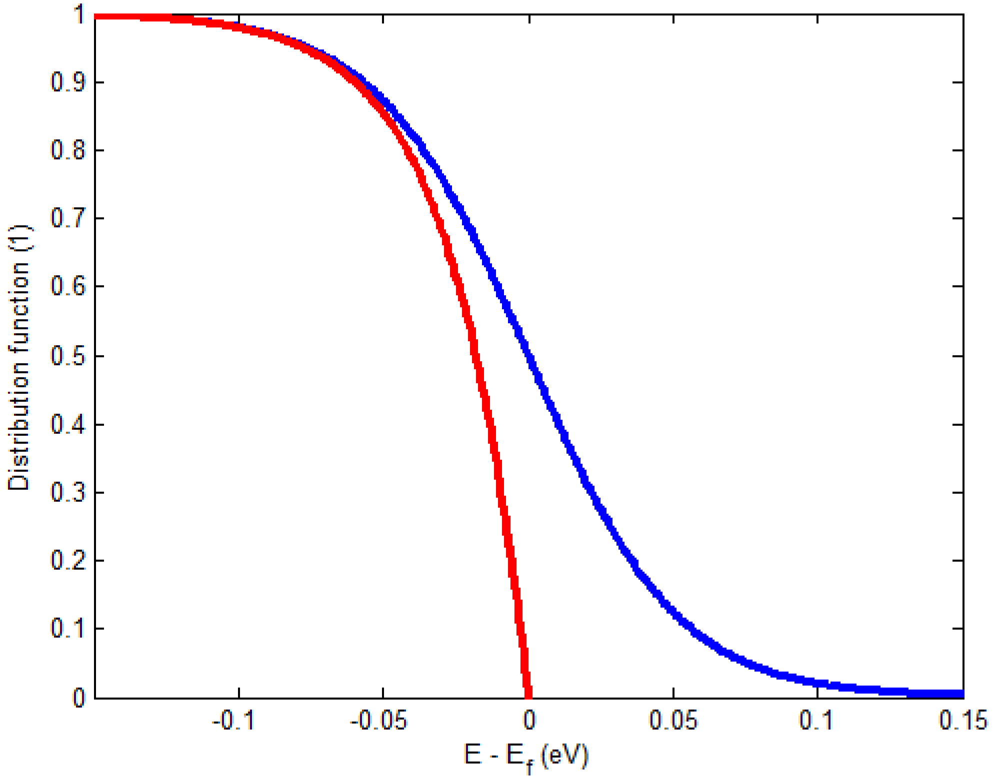


Another Boltzmann limit

- When $E < E_f$,

$$f_D(E) \approx 1 - \exp\left(\frac{E - E_f}{k_B T}\right)$$

Taur, Eq. (2.6)



Carrier concentration (Electron)

- Recall that

Spin and valley degeneracy

$$n = \int_{E_c}^{\infty} N(E) f(E) dE$$

$$N(E) = 2g \frac{4\pi}{h^3} (2m_l m_t^2)^{0.5} (E - E_c)^{0.5}$$

$$f_D(E) = \exp\left(-\frac{E - E_f}{k_B T}\right)$$

- Collecting them all,

$$n = \frac{8\pi g}{h^3} (2m_l m_t^2)^{0.5} \int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_f}{k_B T}\right) dE$$

Taur, Eq. (2.8)

Manipulation

- It is found that

$$n = \frac{8\pi g}{h^3}(2m_l m_t^2)^{0.5} \exp\left(-\frac{E_c - E_f}{k_B T}\right) \times \int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_c}{k_B T}\right) dE$$

  - Integral can be evaluated as

$$\int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_c}{k_B T}\right) dE = (k_B T)^{1.5} \int_{0}^{\infty} z^{0.5} \exp(-z)\, dz$$

$$= (k_B T)^{1.5} \frac{\sqrt{\pi}}{2}$$

Effective DOS

	N_c (cm^{-3})	N_v (cm^{-3})
Silicon	2.8x10^{19}	1.04x10^{19}
Gallium arsenide	4.7x10^{17}	7.0x10^{18}
Germanium	1.04x10^{19}	6.0x10^{18}

N_c and N_v @ 300 K (Hu's book)

- Now we know that

$$n = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5} (m_l m_t^2)^{0.5} \exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

  - With the effective DOS,

Dimension?

$$N_c = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5} (m_l m_t^2)^{0.5}$$

Taur, Eq. (2.10)

  - The electron density can be simply written as

$$n = N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

Taur, Eq. (2.9)

  - Following a similar derivation, $p = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$

Taur, Eq. (2.11)

Intrinsic carrier concentration

- In this case, $n = p$. Then, what is E_f?

$$N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$$

  - From the above equation,

$$E_f = \frac{E_c + E_v}{2} - \frac{k_B T}{2} \ln \frac{N_c}{N_v}$$ Taur, Eq. (2.12)

  - This energy level is called the intrinsic Fermi level, E_i.
  - In this case,

$$n = p = n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_c - E_v}{k_B T}\right)$$ Taur, Eq. (2.13)

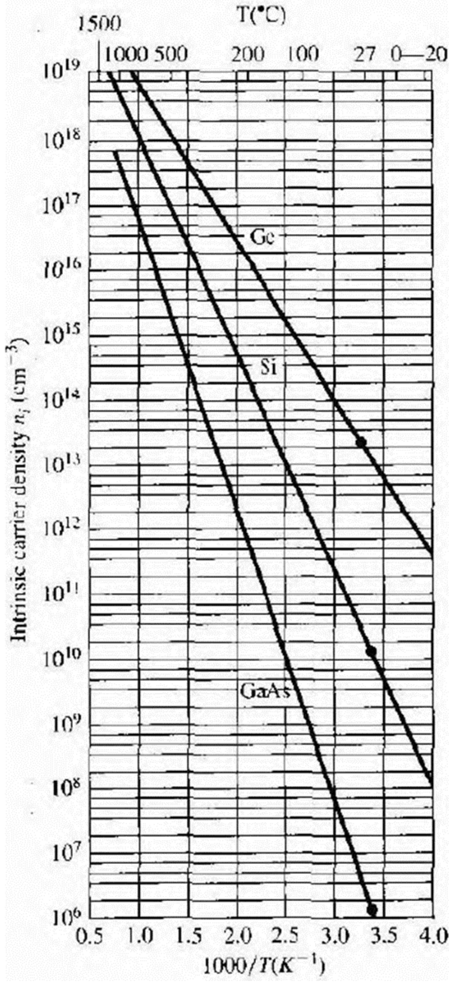
Its temperature dependence

- Recall that $N_c = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5}(m_l m_t^2)^{0.5}$. ($N_v$ has a similar form.)
  - Therefore,

$$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_c - E_v}{k_B T}\right)$$

It is dominant.

$T^{1.5}$, but it is not dominant.



Intrinsic carrier density (Neamen's book)

Using the intrinsic carrier density,

- Carrier densities are expressed as

$$n = n_i \exp\left(-\frac{E_i - E_f}{k_B T}\right)$$ Taur, Eq. (2.14)

$$p = n_i \exp\left(\frac{E_i - E_f}{k_B T}\right)$$ Taur, Eq. (2.15)

  - A useful, general relationship is that the product

$$np = n_i^2$$ Taur, Eq. (2.16)

in equilibrium is a constant, independent of the Fermi level position.

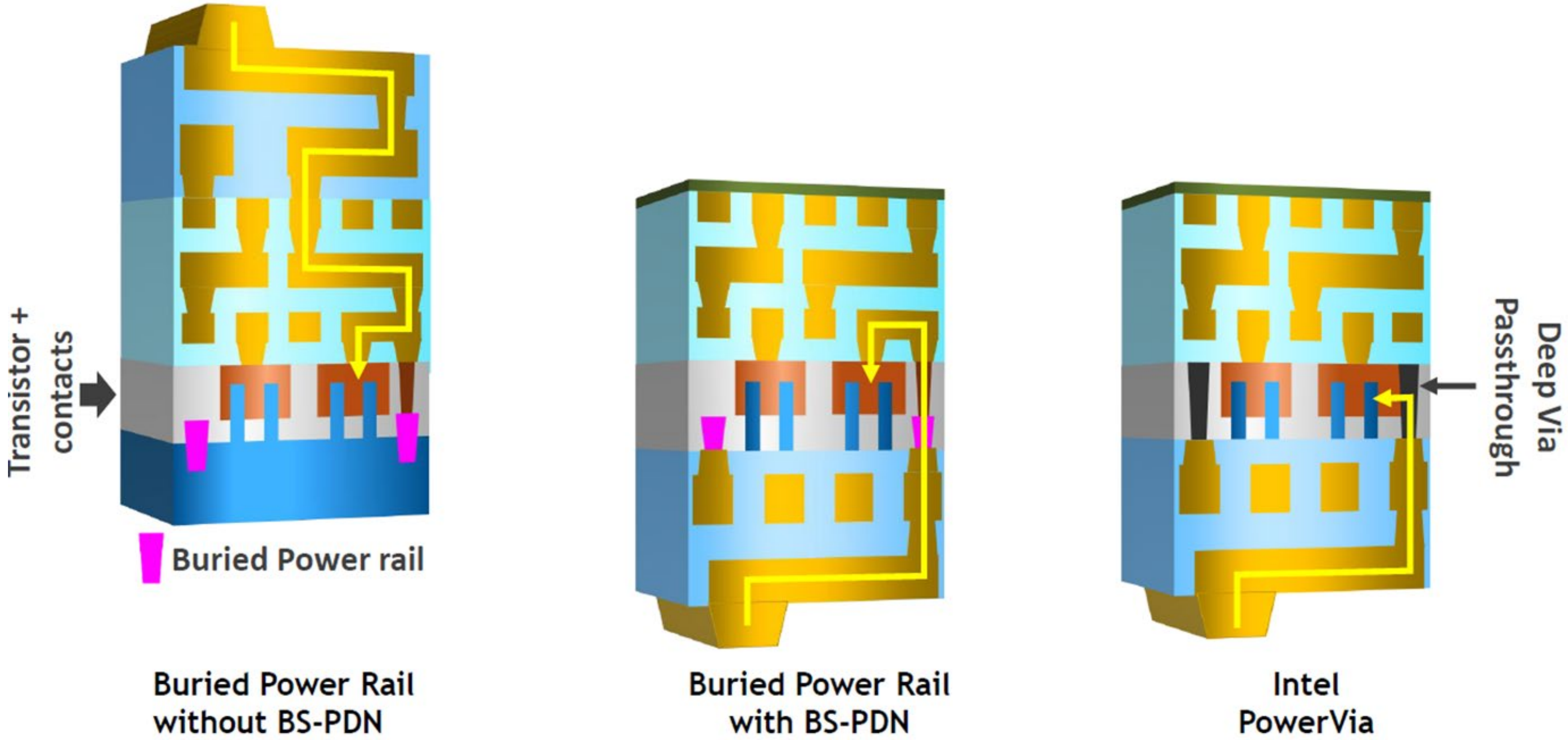
Recall that

- We have 7 X 10^{23} electrons/cm^3 in Si.
  - At 300 K, only ~ 1.4 X 10^{10} electrons/cm^3 can be found in the conduction band. Only a single elelctron among 5 X 10^{13} electrons occupies the conduction band.
  - There are 4 moonwalkers among 8.1 X 10^9 people.

Moonwalkers (NASA)

Copper (A good conductor)

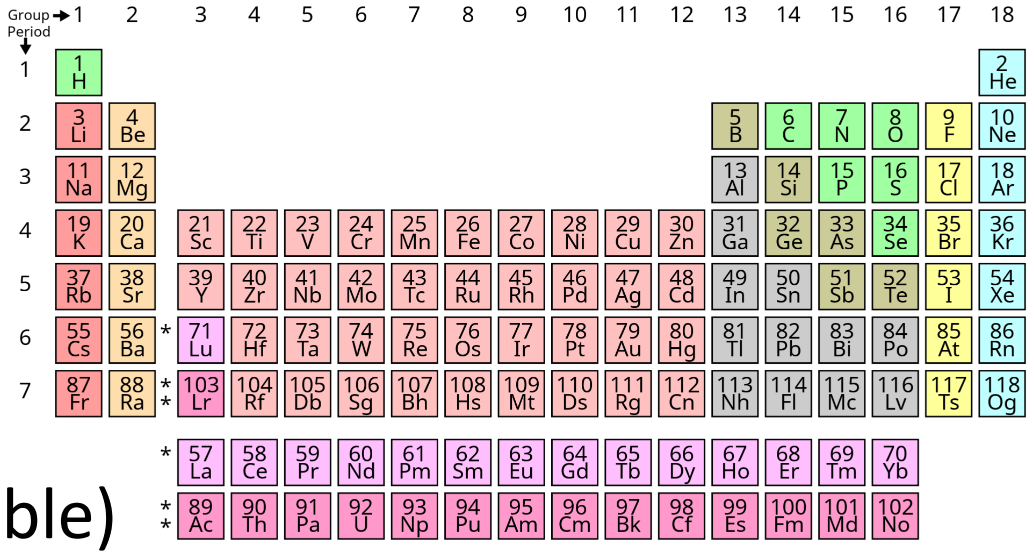
- How many conduction electrons in 1 cm^3?
  - Cu: ~ 8.5×10^{22} cm^{-3}
  - Best for interconnect

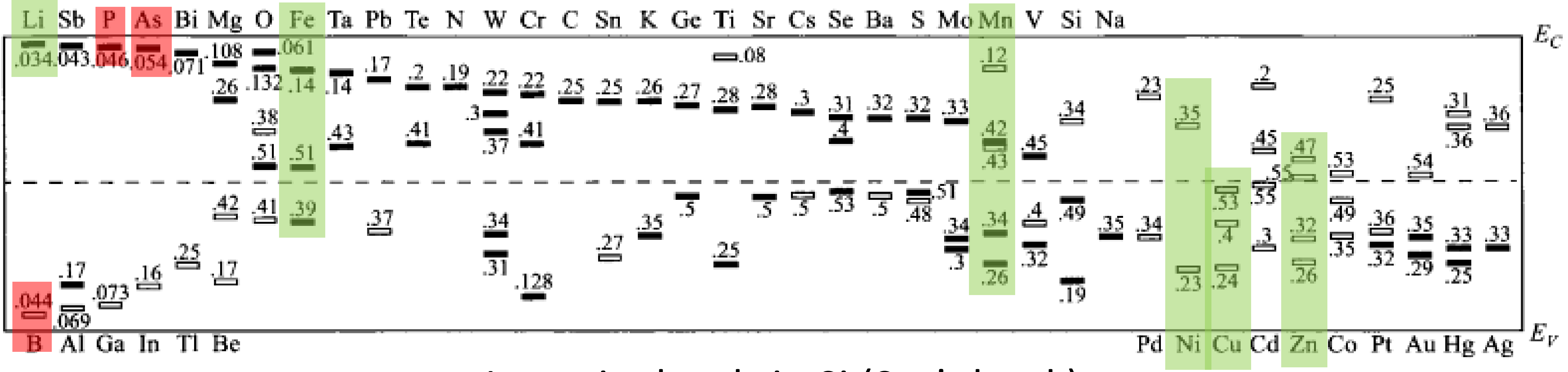


Various ways to supply the power to transistors (Intel, VLSI 2023)

Dopants

- 5 X 10^{20} impurities / cm^3 is 1 % of Si.
  - Find As, P, and B.
  - Find Fe, Cu, Li, Zn, Mn, and Ni. (Undesirable)





Impurity levels in Si (Sze's book)

Thank you!