Special Topics on Basic EECS I
VLSI Devices
Lecture 10

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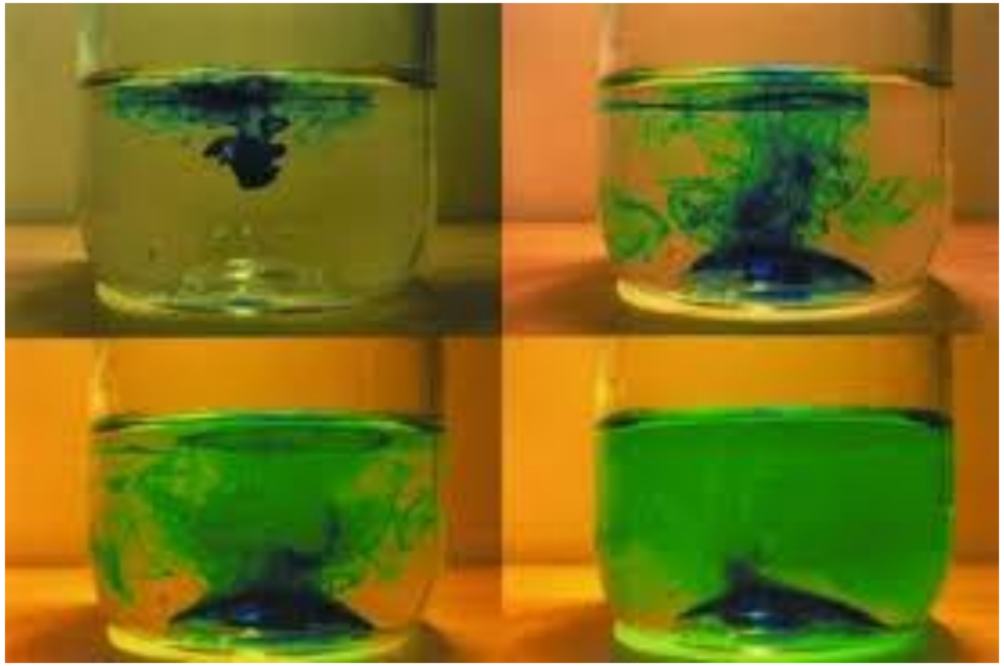
Diffusion

- It is not only for charged particles.
  - For example,



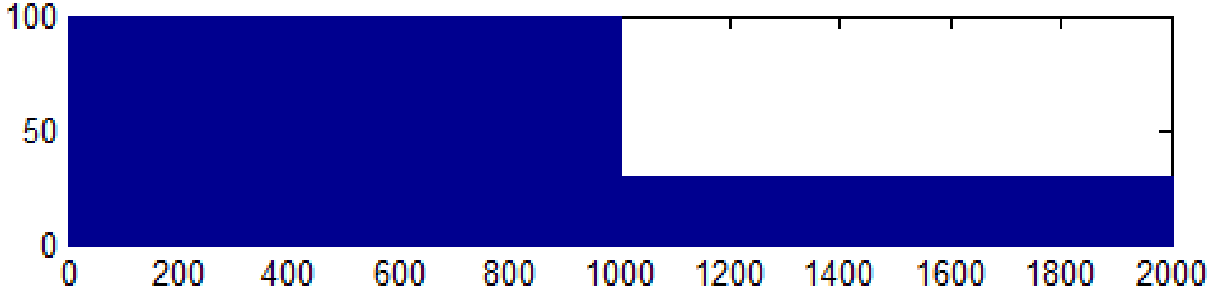
Diffusion of ink
(Google images)

  - Therefore, no polarity is expected.

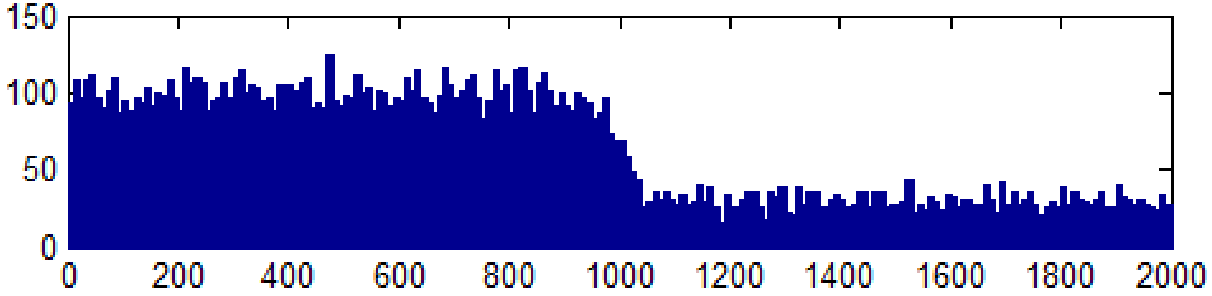
A simple game, again

- Random motion of balls in a 1D box
  - At each turn, they can move forward (+1) or backward (-1).

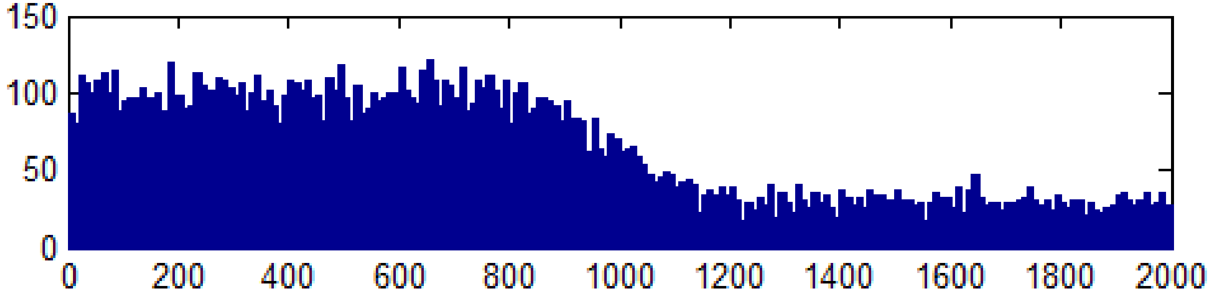
Initial condition



1 k turns



10 k turns



Equation

- Flux
  - The electron flux due to the diffusion mechanism is given by

$$\mathbf{F}_n = -D_n \nabla n$$

  where D_n is the electron diffusion coefficient in the unit of (cm^2/sec).
  - The diffusion current density is

$$\mathbf{J}_{n,diff} = qD_n \nabla n \quad \text{Taur, Eq. (2.36)}$$

- How about the hole?
  - The diffusion current density is

$$\mathbf{J}_{p,diff} = -qD_p \nabla p \quad \text{Taur, Eq. (2.37)}$$

An example

- Taken from Neamen's book
  - Over 1 mm, the electron density varies linearly from 1X10^{18} cm^{-3} to 7X10^{17} cm^{-3}.
  - The diffusion coefficient is D_n = 225 cm2/sec.
  - Calculate the current density.

$$J_n = +qD_n \frac{dn}{dx}$$

$$= (1.6 \times 10^{-19}\ \mathrm{C})(225\ \mathrm{cm^2/s}) \left(\frac{1 \times 10^{18}\ \mathrm{cm^{-3}} - 7 \times 10^{17}\ \mathrm{cm^{-3}}}{0.1\ \mathrm{cm}} \right)$$

$$= 108\ \mathrm{A/cm^2}$$

Revisit the total current density.

- Total current density
  - Electron current density

$$\mathbf{J}_n = q\mu_n n\mathbf{E} + qD_n \nabla n$$ Taur, Eq. (2.54)

  - Hole current density

$$\mathbf{J}_p = q\mu_p p\mathbf{E} - qD_p \nabla p$$ Taur, Eq. (2.55)

  - (Time-dependent) displacement current density

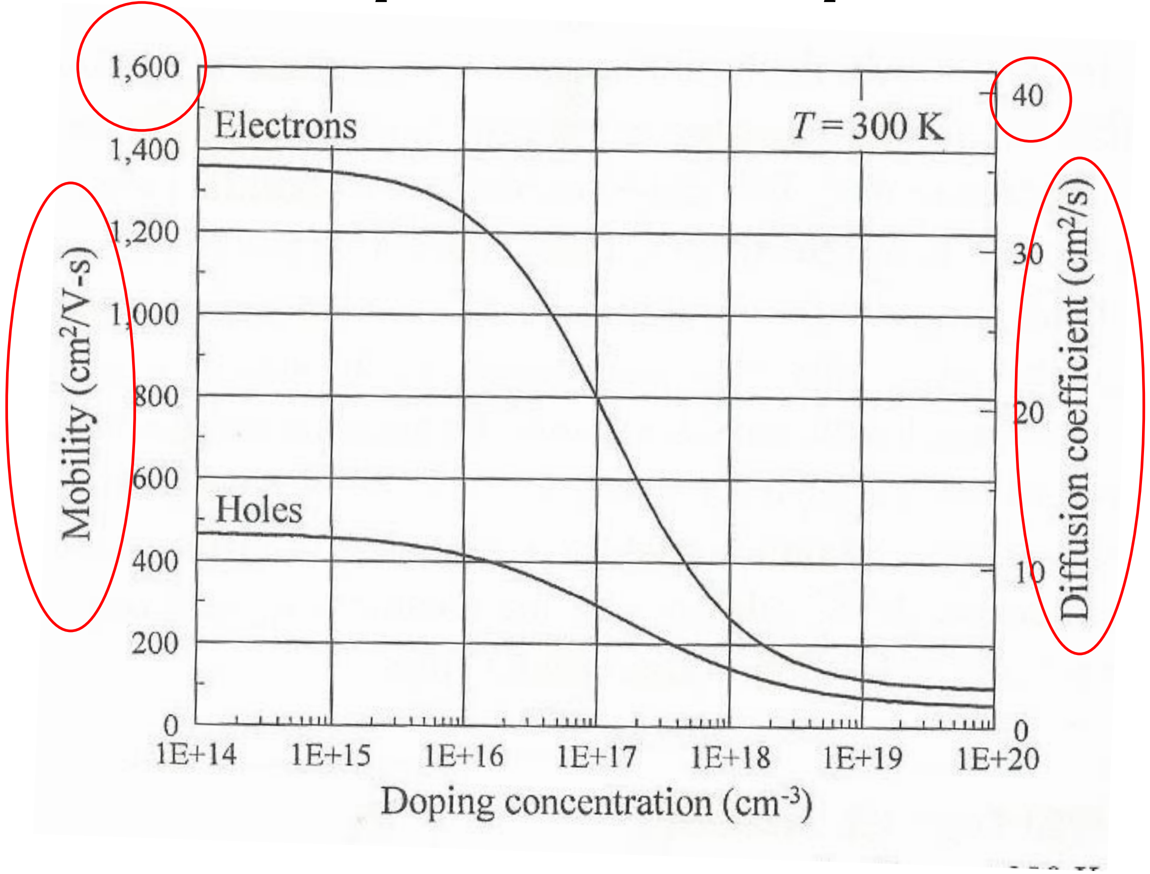
$$\mathbf{J}_{displacement} = \frac{\partial}{\partial t}(\epsilon\mathbf{E})$$

Einstein relation

- At equilibrium, we have the following relations:

$$D_n = \frac{k_B T}{q}\mu_n, D_p = \frac{k_B T}{q}\mu_p$$

Taur, Eq. (2.38) and Eq. (2.39)



(Park's book)

Current density with Einstein relation

- Alternative forms
  - Electron current density

$$\mathbf{J}_n = -q\mu_n n\left[\nabla\phi - \frac{k_B T}{q}\frac{1}{n}\nabla n\right]$$

Taur, Eq. (2.56)

  - Hole current density

$$\mathbf{J}_p = -q\mu_p p\left[\nabla\phi + \frac{k_B T}{q}\frac{1}{p}\nabla p\right]$$

Taur, Eq. (2.57)

Revisiting carrier concentrations

$$E_f = -q\phi_f$$

- Carrier densities are expressed as

$$n = n_i \exp\left(-\frac{E_i - E_f}{k_B T}\right) = n_i \exp\left(\frac{\phi - \phi_f}{k_B T/q}\right)$$ Taur, Eq. (2.49)

$$p = n_i \exp\left(\frac{E_i - E_f}{k_B T}\right) = n_i \exp\left(-\frac{\phi - \phi_f}{k_B T/q}\right)$$ Taur, Eq. (2.50)

  – These relations are generally applicable in the presence of net charge and band bending.

Extension to non-equilibrium cases

- Of course, at non-equilibrium cases, we cannot define E_f.
  - However, we *introduce* ϕ_n and ϕ_p to satisfy:

$$n = n_i \exp\left(\frac{\phi - \phi_n}{k_B T/q}\right)$$

Taur, Eq. (2.61)

$$p = n_i \exp\left(-\frac{\phi - \phi_p}{k_B T/q}\right)$$

Taur, Eq. (2.62)

  - They are called quasi-Fermi potentials. (Electron quasi-Fermi potential and hole quasi-Fermi potential)

Current density with $\nabla\phi_n$ and $\nabla\phi_p$

- From the expression for the quasi-Fermi potential,
  - It can be written as

$$\phi_n = \phi - \frac{k_B T}{q} \log \frac{n}{n_i} \qquad \text{Taur, Eq. (2.65)}$$

  - Taking the gradient,

$$\nabla\phi_n = \nabla\phi - \frac{k_B T}{q} \frac{\nabla n}{n}$$

  - Using this expression,

$$\mathbf{J}_n = -\, q\mu_n n \left[\nabla\phi - \frac{k_B T}{q} \frac{1}{n} \nabla n \right] = -\, q\mu_n n \nabla\phi_n \qquad \text{Taur, Eq. (2.63)}$$

$$\mathbf{J}_p = -q\mu_p p \nabla\phi_p \qquad \text{Taur, Eq. (2.64)}$$

Gradient of quasi-Fermi potential

- We have the following relations,

$$\mathbf{J}_n = -\, q\mu_n n \nabla \phi_n$$

$$\mathbf{J}_p = -q\mu_p p \nabla \phi_p$$

  - The gradient of electron quasi-Fermi potential drives the electron current.
  - The gradient of hole quasi-Fermi potential drives the hole current.

Poisson equation

- Electrostatic potential, ϕ (In Taur, it is denoted as ψ_i.)
  - Conventionally, it is defined in terms of the intrinsic Fermi level,

$$E_i = -q\phi$$

Taur, Eq. (2.40)

- Electric field, $\mathbf{E}$
  - It is equal to the negative gradient of ϕ,

$$\mathbf{E} = -\nabla\phi$$

Taur, Eq. (2.41)

- Poisson equation

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\epsilon \mathbf{E}) = \rho$$

$$\rho = q(p - n + N_d^+ - N_a^-)$$

  - One dimensional and homogeneous system

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon}$$

Taur, Eq. (2.42)

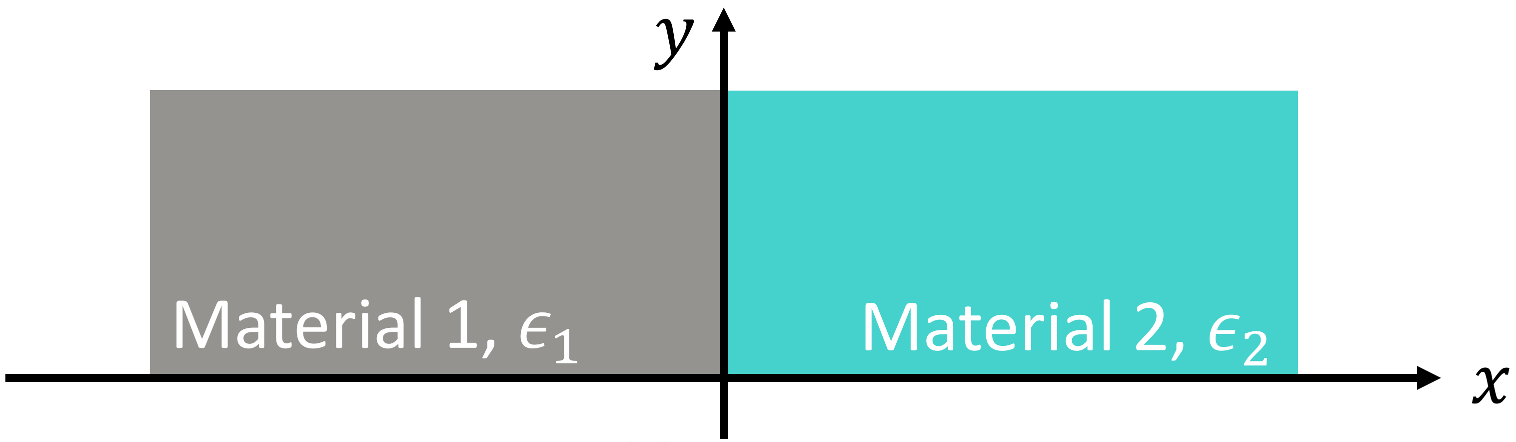
Boundary condition

- Tangential field
  - Tangential fields are continuous.

$$E_{1y}(0, y) = E_{2y}(0, y)$$ Taur, Eq. (2.46)

- Normal field
  - Perpendicular component of is continuous.

$$\epsilon_1 E_{1x}(0, y) = \epsilon_2 E_{2x}(0, y)$$ Taur, Eq. (2.47)



Debye length (1)

- Consider an n-type silicon. (Neglect holes and acceptors)

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon}\left[N_d(x) - n_i \exp\left(\frac{\phi - \phi_f}{k_B T/q}\right)\right] \qquad \text{Taur, Eq. (2.51)}$$

  - For a uniformly doped background, N_d,

$$0 = -\frac{q}{\epsilon}\left[N_d - n_i \exp\left(\frac{\phi - \phi_f}{k_B T/q}\right)\right]$$

  - By introducing an incremental change, $\Delta N_d(x)$, the corresponding change in the electrostatic potential, $\Delta\phi(x)$, is given as

$$\frac{d^2\Delta\phi}{dx^2} = -\frac{q}{\epsilon}\left[\Delta N_d(x) - N_d \frac{\Delta\phi}{k_B T/q}\right] \qquad \text{Taur, Eq. (2.52)}$$

Debye length (2)

- With the Debye length,

$$L_D = \sqrt{\frac{\epsilon k_B T}{q^2 N_d}}$$ Taur, Eq. (2.51)

  - The equation can be written as

$$\frac{d^2 \Delta\phi}{dx^2} - \frac{\Delta\phi}{L_D^2} = -\frac{q}{\epsilon} \Delta N_d(x)$$

  - Its solution takes the form of $\exp\left(-\frac{x}{L_D}\right)$.
  - It keas a distance on the order of L_D for the silicon bands to respond to an abrupt change in N_d.

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Thank you!