Special Topics on Basic EECS I
VLSI Devices
Lecture 23

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Drain current

- Electron current density at a point (x, y)

$$J_n(x, y) = -q\mu_n n(x, y)\frac{dV}{dy}$$ Taur, Eq. (3.5)

  - (It includes both the drift and diffusion currents.)
  - When integrated from $x = 0$ to x_i,

$$I_d(y) = qW \int_0^{x_i} \mu_n n(x, y)\frac{dV}{dy}dx$$ Taur, Eq. (3.6)

Sign change due to convention of terminal current

z-directional width

Further simplification

- Electron current density at a point (x, y)

$$I_d(y) = qW \int_0^{x_i} \mu_n n(x,y) \frac{dV}{dy} dx = -\mu_{eff} W \frac{dV}{dy} \left(-q \int_0^{x_i} n(x,y) dx \right)$$

$$= -\mu_{eff} W \frac{dV}{dy} Q_i(y)$$

Taur, Eq. (3.8)

  - We introduce an effective mobility, μ_{eff}.
  - Since V is a function of y only, V is interchangeable with y.

$$Q_i(y) = Q_i(V)$$

  - Then,

$$I_d(y)dy = \mu_{eff} W[-Q_i(V)]dV$$

$I_d(y)$ is actually a constant.

- When integrated from $y = 0$ to L, (from $V = 0$ to V_{ds})

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV$$ Taur, Eq. (3.10)

  - Then, how can we find $Q_i(V)$? (We must perform the x-directional integration.)

It's small, but not zero.

$$Q_i = -q \int_0^{x_i} n(x, y) dx = -q \int_{\phi_s}^{\delta} n(\phi, V) \frac{dx}{d\phi} d\phi$$

$$= -q \int_{\delta}^{\phi_s} \frac{(n_i^2/N_a) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$ Taur, Eq. (3.12)

Then, how can we determine ϕ_s?

- For given V_{gs} and V, we can solve the MOS equation.

$$V_{gs} = V_{fb} + \phi_s - \frac{Q_s}{C_{ox}}$$

$$= V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right) \right]^{1/2}$$

Taur, Eq. (3.14)

Only two important terms are kept.

  - We can numerically solve the above equation to obtain ϕ_s.

$V(\phi_s)$?

- Recall that

$$V_{gs} = V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T}(\phi_s - V) \right) \right]^{1/2}$$

Taur, Eq. (3.14)

  - We can rearrange

$$\frac{C_{ox}^2 \left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} = \frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T}(\phi_s - V) \right)$$

  - Therefore,

$$V = \phi_s - \frac{k_B T}{q} \log\left\{ \frac{N_a^2}{n_i^2} \left[\frac{C_{ox}^2 \left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T} \right] \right\}$$

Taur, Eq. (3.18)

Pao-Sah double integral

- Finally, the expression for I_d reads

$$I_d = q\mu_{eff}\frac{W}{L}\int_0^{V_{ds}}\left[\int_\delta^{\phi_s}\frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)}d\phi\right]dV$$

Taur, Eq. (3.13)

  - It is the Pao-Sah double integral.
  - Rigorous within the GCA, but it is difficult to evaluate.

Charge-sheet model

- Simpler model with further approximations
  - Consider the previous method to calculate Q_i:

$$Q_i = -q \int_{\delta}^{\phi_s} \frac{\left(n_i^2/N_a\right) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$

  - A more simple way? Instead, Q_d is approximated as

$$Q_d = -qN_a W_d = -\sqrt{2\epsilon_{si} q N_a \phi_s}$$

Taur, Eq. (3.15)

  - Then, Q_i can be approximated as

$$Q_i = Q_s - Q_d = -C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si} q N_a \phi_s}$$

Taur, Eq. (3.16)

(Of course, it is not exact.)

Change of variable

- Now, Q_i can be a function of ϕ_s.
  - Variable change from V to ϕ_s:

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV = \mu_{eff} \frac{W}{L} \int_{\phi_{s,s}}^{\phi_{s,d}} [-Q_i(\phi_s)] \frac{dV}{d\phi_s} d\phi_s$$

Taur, Eq. (3.17)

Surface potentials at the two ends, $y = 0$ and L. They can be calculated by solving Taur, Eq. (3.14).

$\frac{dV}{d\phi_s}$? (1)

- Recall that

$$V = \phi_s - \frac{k_B T}{q} \log\left\{\frac{N_a^2}{n_i^2}\left[\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}\right]\right\}$$

Taur, Eq. (3.18)

  - Therefore,

$$\frac{dV}{d\phi_s} = 1 - \frac{k_B T}{q} \frac{-\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)}{\epsilon_{si} k_B T N_a} - \frac{q}{k_B T}}{\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}}$$

$\frac{dV}{d\phi_s}$? (2)

- Simple rearrange yields

$$\frac{dV}{d\phi_s} = 1 + \frac{2k_BT}{q}\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right) + \epsilon_{si}qN_a}{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2 - 2\epsilon_{si}qN_a\phi_s}$$

Taur, Eq. (3.19)

  - It is still very complicated…

Integrand

- When multiplied with $-Q_i(\phi_s)$,

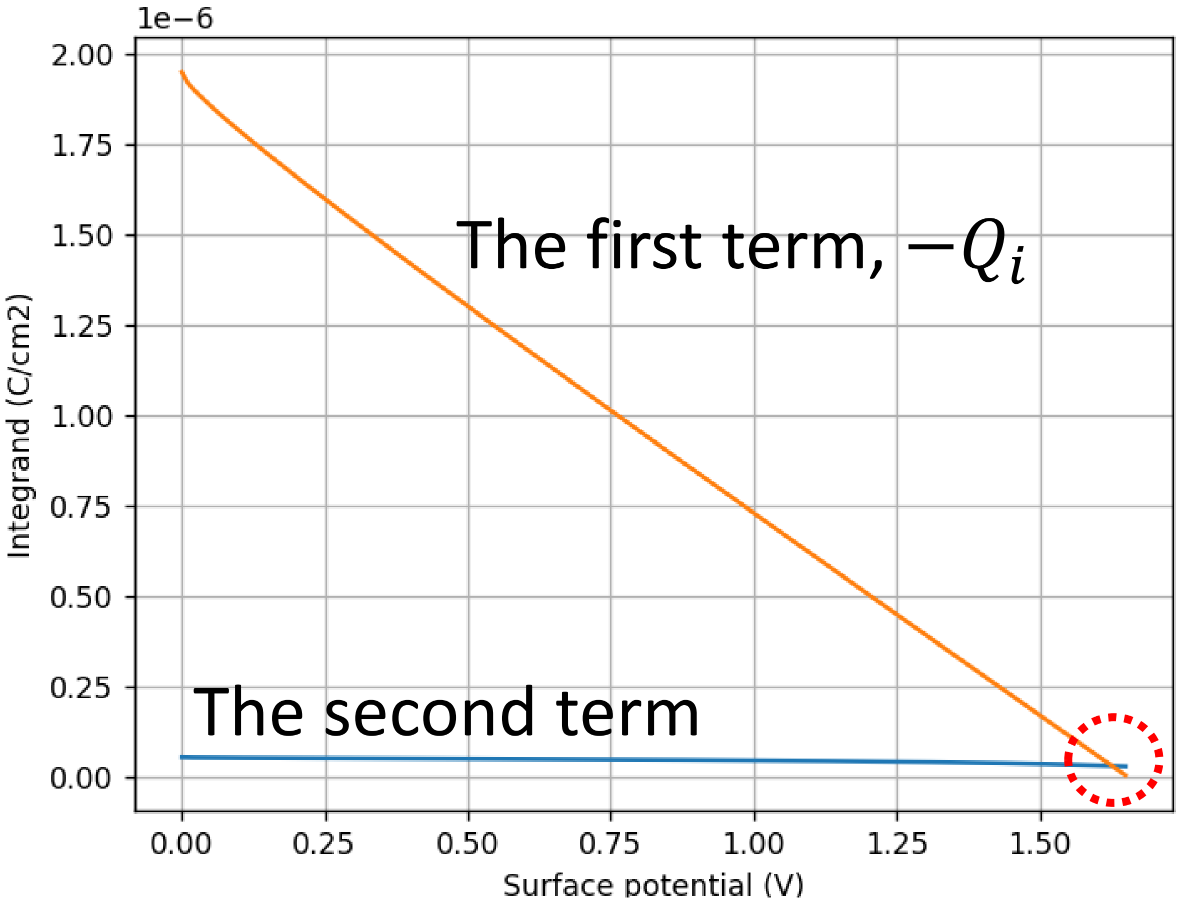
$$\left(-Q_i(\phi_s)\right)\frac{dV}{d\phi_s}$$

$$= -Q_i(\phi_s) + \frac{2k_BT}{q}\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right) + \epsilon_{si}qN_a}{C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si}qN_a\phi_s}}$$

  – The second term is still very complicated…
  – *Is it really important?*

Comparison between two terms

- Let's draw two terms.
  - Assume that N_a = 10^{17} cm^{-3}, t_{ox} = 10 nm, V_{gs} = 1.0 V, and V_{fb} = -0.88 V.
  - The second term is small.
  - It is meaningful only when $Q_i \approx 0$.
  - This is corresponding to $C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) = \sqrt{2\epsilon_{si} q N_a \phi_s}$.



Integrand, again

- Within this condition,

$$\left(-Q_i(\phi_s)\right)\frac{dV}{d\phi_s} \approx -Q_i(\phi_s) + \frac{k_B T}{q}\frac{C_{ox}\sqrt{2\epsilon_{si} q N_a \phi_s} + \epsilon_{si} q N_a}{\sqrt{2\epsilon_{si} q N_a \phi_s}}$$

$$= -Q_i(\phi_s) + \frac{k_B T}{q} C_{ox} + \frac{k_B T}{q}\frac{\sqrt{2\epsilon_{si} q N_a}}{2\sqrt{\phi_s}}$$

Its integration yields $\frac{k_B T}{q} C_{ox} \phi_s$.

Its integration yields $\frac{k_B T}{q}\sqrt{2\epsilon_{si} q N_a \phi_s}$.

Drain current

- Using the previous approximation,
  - We can obtain the following expression:

$$I_d = \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs} - V_{fb} + \frac{k_BT}{q}\right)\phi_s - \frac{1}{2}C_{ox}\phi_s^2 - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}\phi_s^{1.5} + \frac{k_BT}{q}\sqrt{2\epsilon_{si}qN_a\phi_s}\right\}\Bigg|_{\phi_{s,s}}^{\phi_{s,d}}$$

Taur, Eq. (3.21)

  - Only with $\phi_{s,s}$ and $\phi_{s,d}$, we can evaluate the drain current.

Let's evaluate it together! (1)

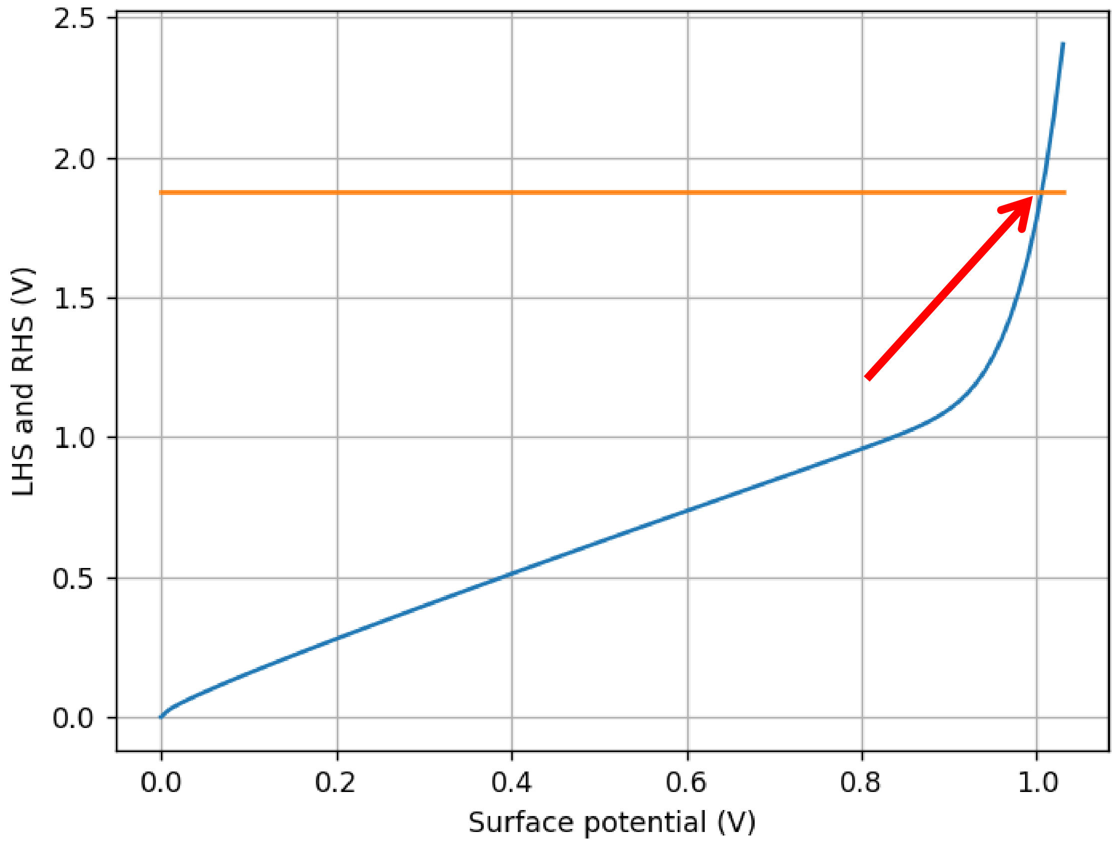
- Step-by-step
  - Assume that N_a = 10^{17} cm^{-3}, t_{ox} = 10 nm, V_{gs} = 1.0 V, and V_{fb} = -0.88 V.
  - Consider a case of V_{ds} = 0.1 V.
  - First, we must calculate $\phi_{s,s}$. How?

$$1.88 = \phi_{s,s} + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_{s,s}}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} \phi_{s,s} \right) \right]^{1/2}$$

Let's evaluate it together! (2)

- Graphical solution
  - Draw the LHS and RHS.

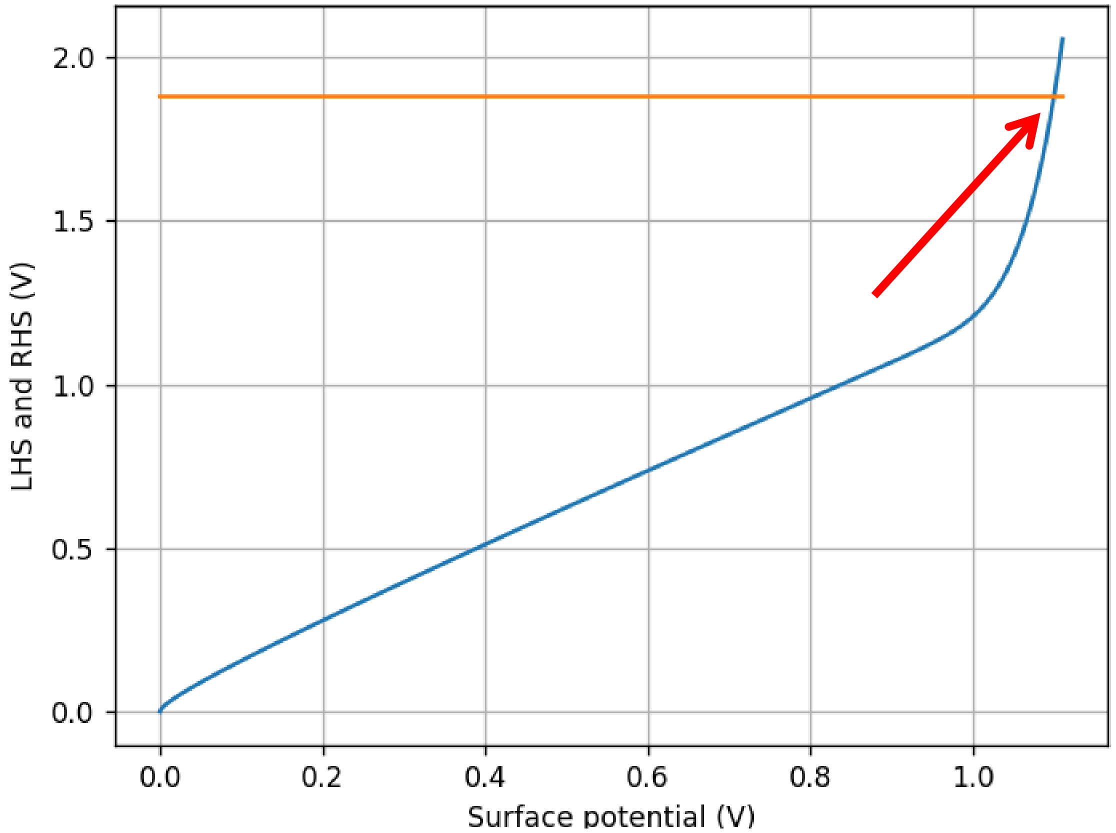
  $\phi_{s,s}$ = 1.006 V



Let's evaluate it together! (3)

- Now, for the drain end.
  - We must calculate $\phi_{s,d}$.

  $\phi_{s,d}$ = 1.100 V



$$1.88 = \phi_{s,d} + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_{s,d}}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_{s,d} - 0.1) \right) \right]^{1/2}$$

Thank you!