Special Topics on Basic EECS I
VLSI Devices
Lecture 12

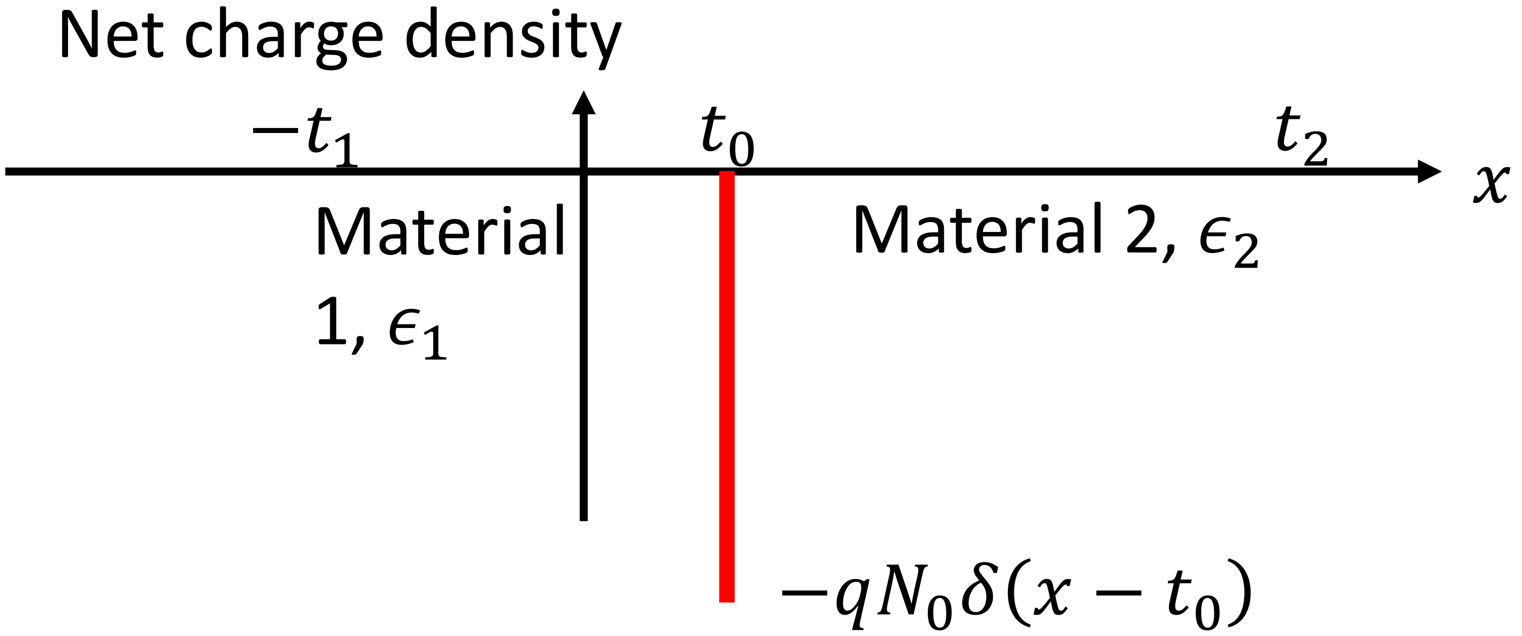
Sung-Min Hong (smhong@gist.ac.kr)
Semiconductor Device Simulation Laboratory
School of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology

Homework#3

- Solve the Poisson equation, when the interface occurs at $x = 0$.
  - Two boundary conditions

$$\phi(-t_1) = \phi_1, \qquad \phi(t_1) = \phi_2$$

  - Except for a sheet charge at $x = t_0$, no net charge



Continuity equations

- (Change of electron number) = (Net incoming-flux integrated over surface)
  - Therefore,

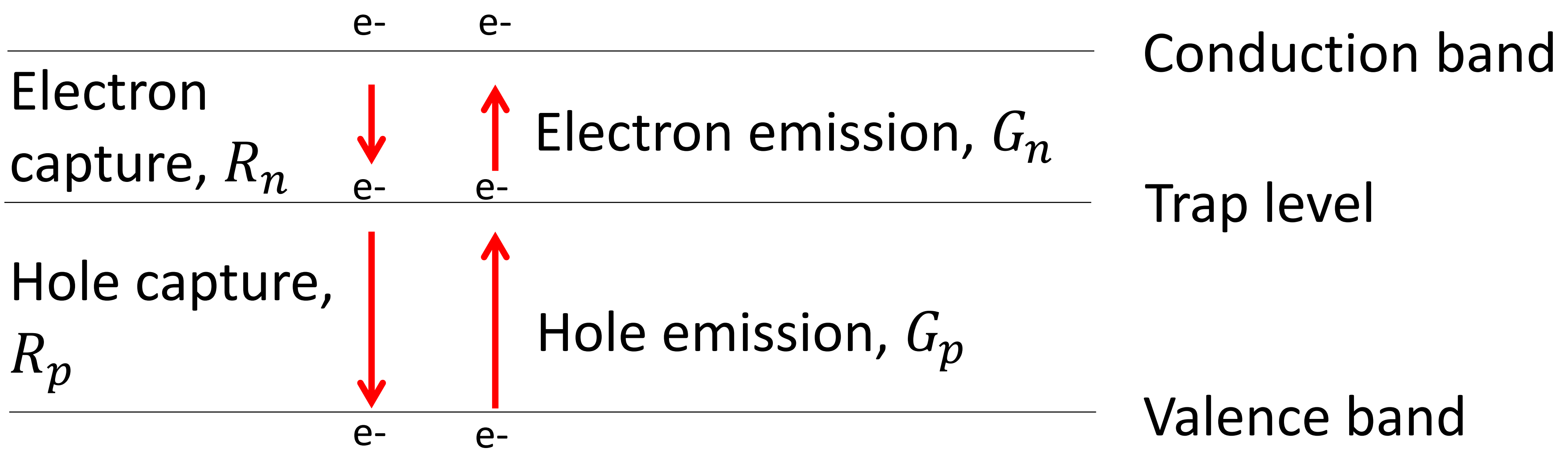
$$\frac{\partial n}{\partial t} = -\nabla \cdot \mathbf{F}_n$$

- When the generation and recombination processes are considered,
  - We need additional terms.

$$\frac{\partial n}{\partial t} = -\nabla \cdot \mathbf{F}_n + G_n - R_n$$

SRH (Shockley-Read-Hall) recombination (1)

- In silicon, a *radiative* or *Auger* process is very low due to its indirect bandgap.
  - Most of the recombination processes take place indirectly via a trap or a deep impurity level near the middle of the forbidden gap.



SRH (Shockley-Read-Hall) recombination (2)

- After some derivation steps,
  - We have the following expression:

$$R_n - G_n = R_p - G_p = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

Taur, Eq. (A5.16)

  - It vanishes at equilibrium.

Full set of equations

- Drift-diffusion model
  - Poisson equation

$$\nabla \cdot (\epsilon \mathbf{E}) = q(p - n + N_d^+ - N_a^-)$$

  - Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_n + G_n - R_n$$

Taur, Eq. (2.68)

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot \mathbf{J}_p + G_p - R_p$$

Taur, Eq. (2.69)

  - Current density

$$\mathbf{J}_n = q\mu_n n\mathbf{E} + qD_n \nabla n, \qquad \mathbf{J}_p = q\mu_p p\mathbf{E} - qD_p \nabla p$$

Dielectric relaxation time

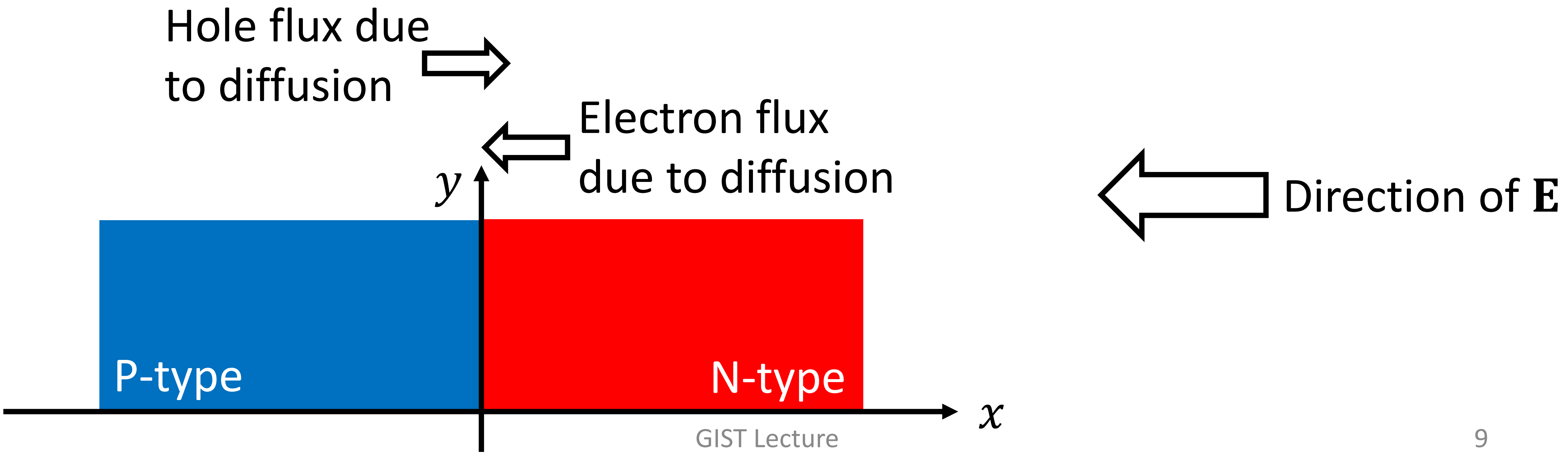
- Consider a homogeneous n-type sample. (Neglect g-r processes.) Assume $\Delta n(\mathbf{r})$.
  - In the first order,
  $$\Delta \mathbf{J}_n = \cancel{q\mu_n(\Delta n)\mathbf{E}} + q\mu_n n(\Delta \mathbf{E}) + \cancel{qD_n\nabla(\Delta n)}$$
  - Electron continuity equation
  $$\frac{\partial}{\partial t}\Delta n = \frac{1}{q}\nabla \cdot \Delta \mathbf{J}_n \approx \nabla \cdot [\mu_n n(\Delta \mathbf{E})] = \mu_n n \frac{(-q)\Delta n}{\epsilon} \quad \text{Taur, Eq. (2.70)}$$
  - The dielectric relaxation time, $\frac{\epsilon}{q\mu_n n}$
  - The majority-carrier response time in silicon is typically on the order of 10^{-12} sec, which is shorter than most device switching times.

PN junction

- Where can we find PN junctions?

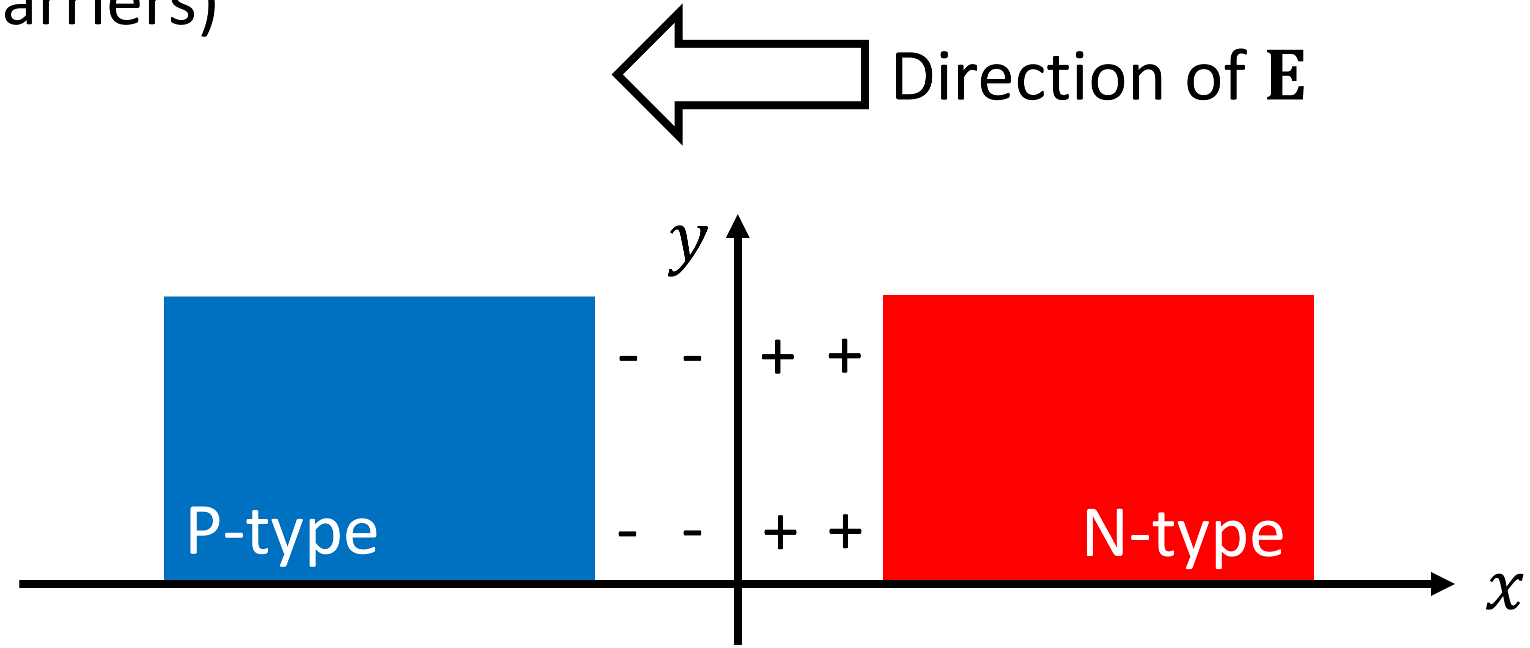
Qualitative description (1)

- Built-in electric field
  - Strong diffusion current density
  - At equilibrium, the net flux must vanish.
  - An electric field is required.



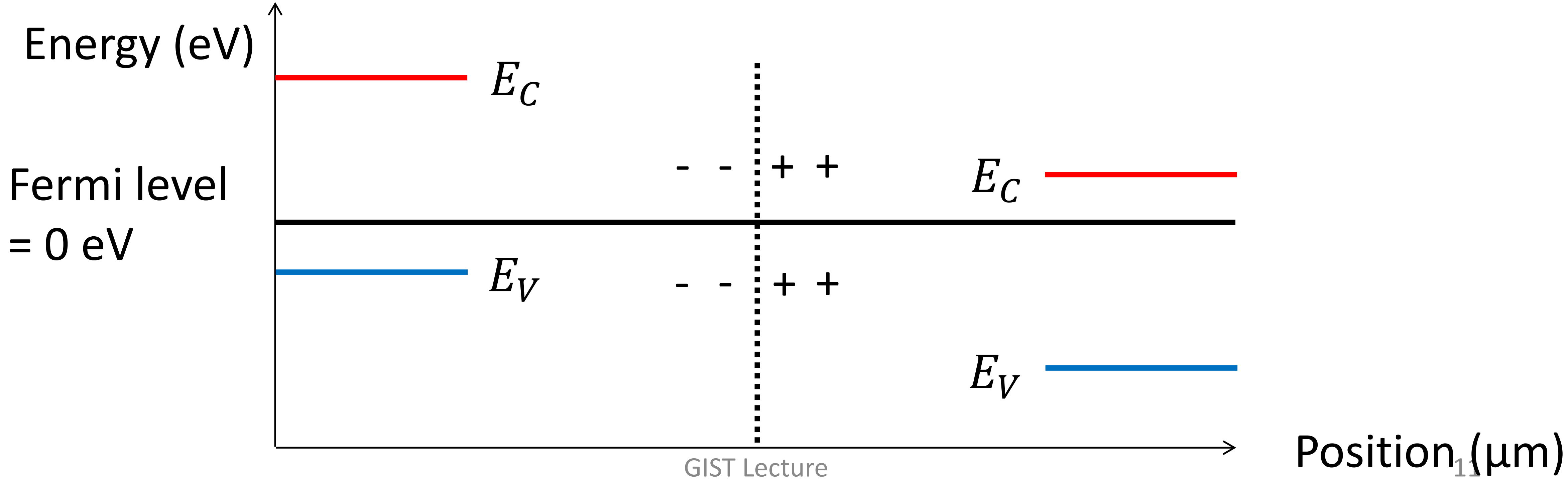
Qualitative description (2)

- How to build an electric field
  - We need the net charge density, ρ.
  - Positive charges in the n-type region, negative p-type
  - These charges can be supplied by the depletion layer. (Eliminating charge carriers)



Energy band diagram at equilibrium

- At thermal equilibrium, the Fermi level must remain flat across the entire pn junction.
  - Away from the junction, the energy bands are flat.
  - Near the junction, a smooth transition of E_C and E_V



Built-in potential

- Assume that N_d is the donor density and N_a is the acceptor density.
  - In the n-type boundary,

$$N_d = n_i \exp\frac{\phi}{k_BT/q}, \qquad \phi = \frac{k_BT}{q}\ln\left(\frac{N_d}{n_i}\right)$$ Taur, Eq. (2.71)

  - In the p-type boundary,

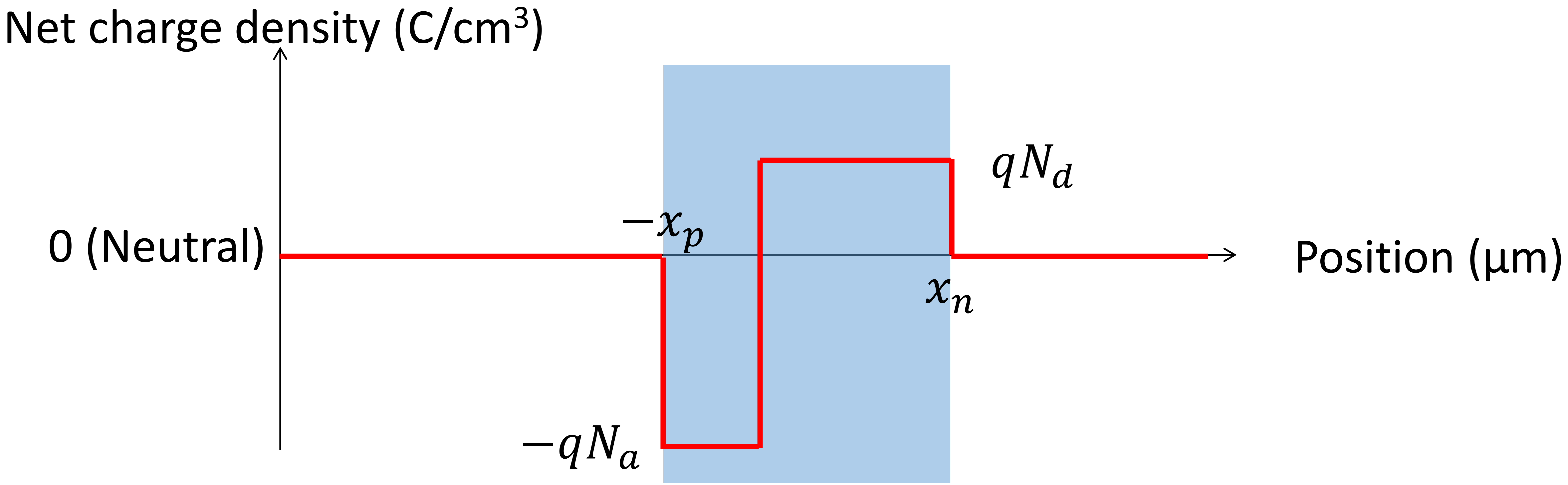
$$N_a = n_i \exp\frac{-\phi}{k_BT/q}, \qquad \phi = -\frac{k_BT}{q}\ln\left(\frac{N_a}{n_i}\right)$$ Taur, Eq. (2.72)

  - Their difference, the built-in potential (ϕ_{bi}) is

$$\phi_{bi} = \frac{k_BT}{q}\ln\left(\frac{N_dN_a}{n_i^2}\right)$$ Taur, Eq. (2.73)

Depletion approximation

- Electron and hole densities are assumed to be negligible in the entire band-bending region.
  - Depletion region(/layer) from $-x_p$ to x_n. $N_a x_p = N_d x_n$



Analytic solution

- The Poisson equation in the depletion region

$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon} N_a$$

Taur, Eq. (2.77)

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon} N_d$$

Taur, Eq. (2.76)

  – Assuming vanishing electric fields at $-x_p$ and x_n,

$$\left|\frac{d\phi}{dx}\right|_{x=0} = \frac{q}{\epsilon} N_d x_n = \frac{q}{\epsilon} N_a x_p$$

Taur, Eq. (2.78)

Thank you!