

Special Topics on Basic EECS I

VLSI Devices

Lecture 7

Sung-Min Hong (smhong@gist.ac.kr)

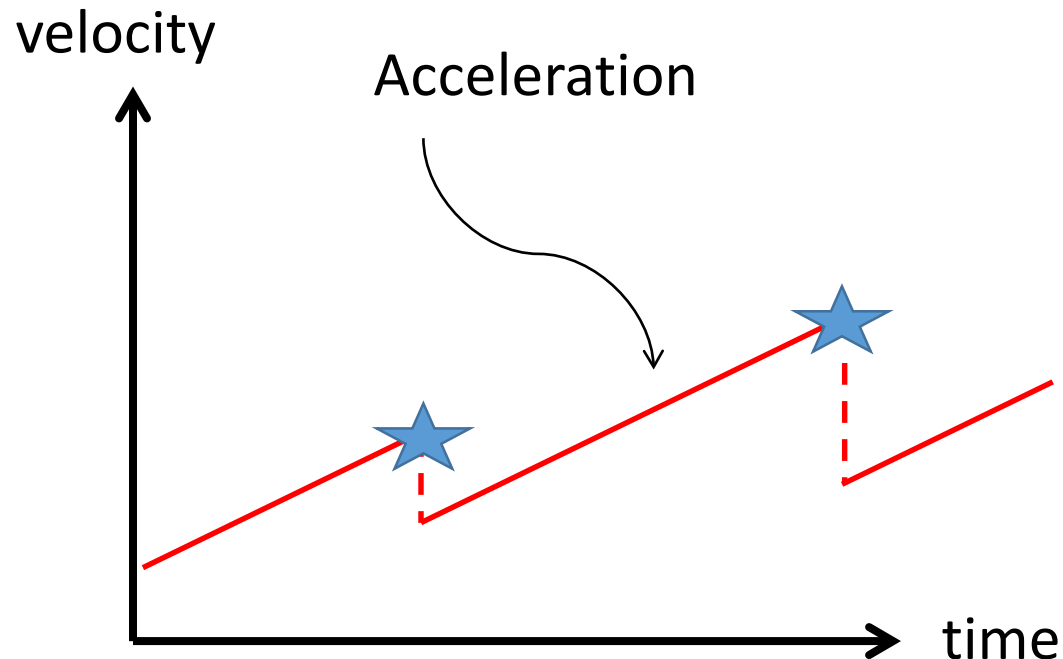
Semiconductor Device Simulation Laboratory
School of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology

Drift

- Net movement of charge due to an electric field
 - Since electrons/holes are charged particles, they are accelerated by an electric field. (V/cm)
 - For electrons, $\mathbf{F} = -q\mathbf{E}$ (For holes, $\mathbf{F} = +q\mathbf{E}$.)
 - According to Newton's 2nd law, the velocity satisfies $\frac{d\mathbf{v}}{dt} = -\frac{q\mathbf{E}}{m_n}$
 - Here, m_n is the conductivity effective mass of electrons.
 - Then, $\mathbf{v}(t) = \mathbf{v}(0) - \frac{q\mathbf{E}}{m_n}t$ (*Right?*)

Scattering

- The velocity of the carriers...
 - Does not increase indefinitely under the field acceleration. Why?
 - They are scattered frequently and lose the momentum.



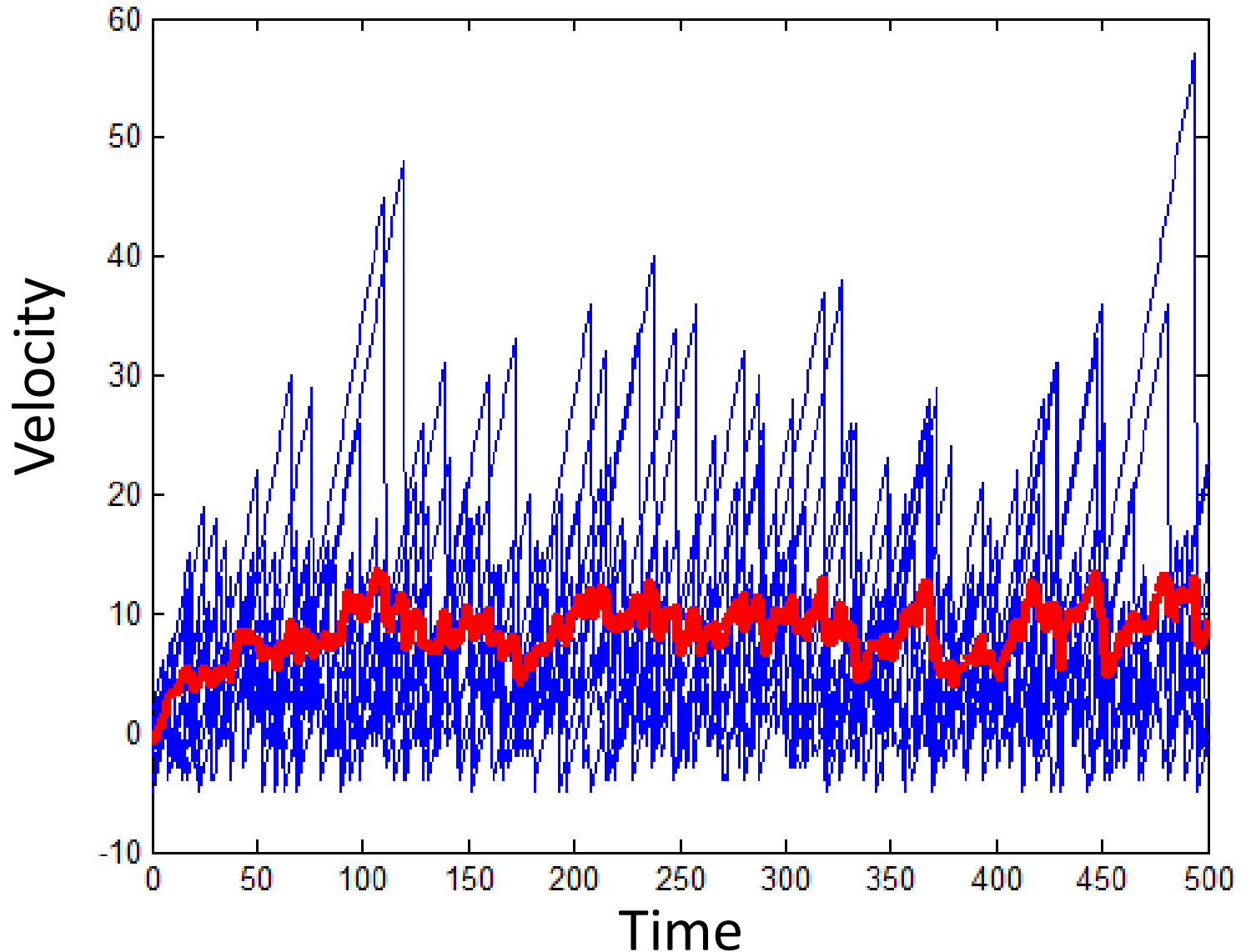
Velocity of an electron as a function of time, when scattering events are considered.

A simple game (1)

- We have ten players.
 - Initially, each of them has an integer velocity, $[-5, 5]$. (Uniform distribution)
 - After one time step, 1 is added to the velocity.
 - Randomly select a player. Then, the player's velocity is again randomly distributed over $[-5, 5]$.

A simple game (2)

- A realization
 - Blue: Ten trajectories
 - Red: Average



Average velocity

- The velocity of each carrier
 - An individual electron exhibits sharp transitions.
- The average velocity
 - However, the average velocity follows a much smoother trajectory.
 - Therefore, it would be better to write

$$\frac{d\langle \mathbf{v} \rangle}{dt} = -\frac{q\mathbf{E}}{m_n} - \frac{\langle \mathbf{v} \rangle}{\tau_n}$$

- Here, τ_n is the mean time between collisions.

Mobility

	μ_n	μ_p
Si	1350	480
GaAs	8500	400
Ge	3900	1900

- Mobility

- Conduction currents are the result of the drift motion of charge carriers under the influence of an applied electric field.
- Average drift velocity is directly proportional to the electric field intensity:

$$\langle \mathbf{v} \rangle = -\frac{q\tau_n}{m_n} \mathbf{E} = -\mu_n \mathbf{E}$$

Taur, Eq. (2.26)

Negative sign
due to polarity



- μ_n : Electron mobility in (cm²/V/sec)
- When the above relation is used, the drift current density becomes

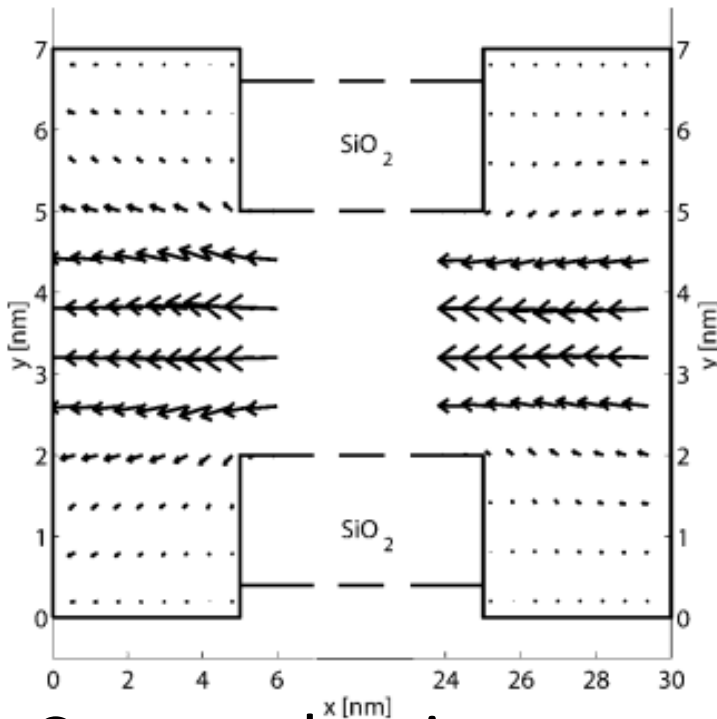
$$\mathbf{J} = q(\mu_n n + \mu_p p) \mathbf{E} = \sigma \mathbf{E}$$

Taur, Eq. (2.28)
and Eq. (2.30)

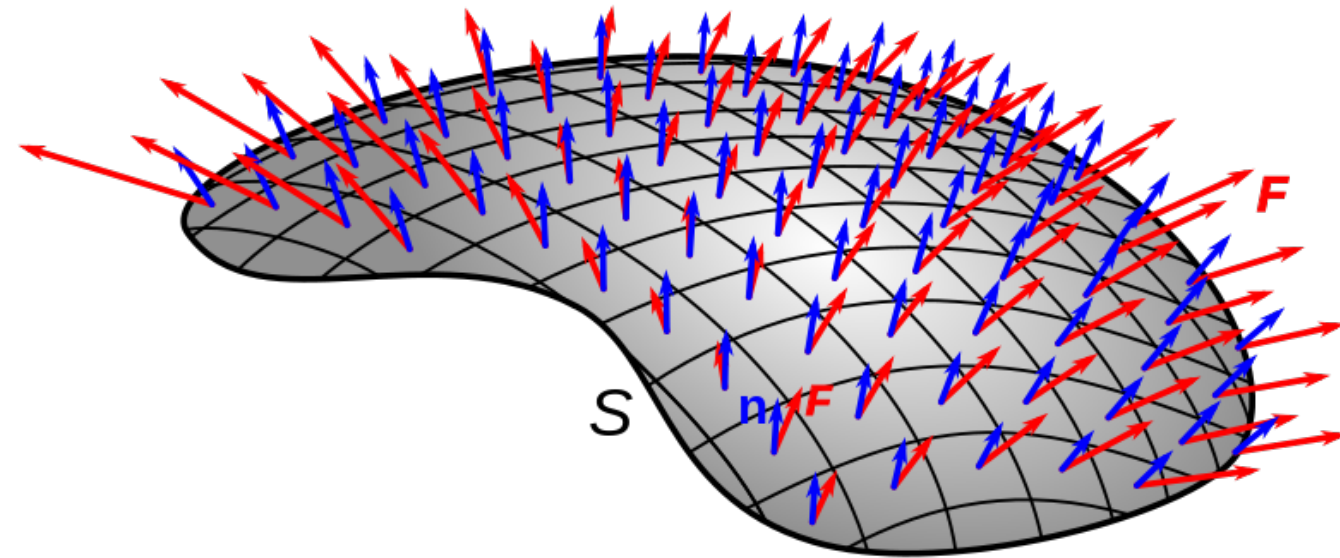
Terminal current vs. current density

- \mathbf{J} , current per area (defined everywhere)

$$I_{terminal} = - \int_{terminal} \mathbf{J} \cdot d\mathbf{a}$$



Current density vector (Luisier et al., JAP, vol. 100, p. 043713, 2006)



Surface integral of a vector field (Wikipedia)

Resistivity & sheet resistivity

- Drift current density

$$\mathbf{J}_{n,drift} = q\mu_n n \mathbf{E}$$

Taur, Eq. (2.28)

$$\mathbf{J}_{p,drift} = q\mu_p p \mathbf{E}$$

Taur, Eq. (2.30)

- Then, the total resistivity is

$$\Omega \text{ cm} \longrightarrow \rho = \frac{1}{q\mu_n n + q\mu_p p}$$

Taur, Eq. (2.32)

- Sheet resistivity

- The resistance of a conductor of length, L , and width, W ,

$$R = \frac{L}{W} \rho_{sh} \quad \Omega/\square$$

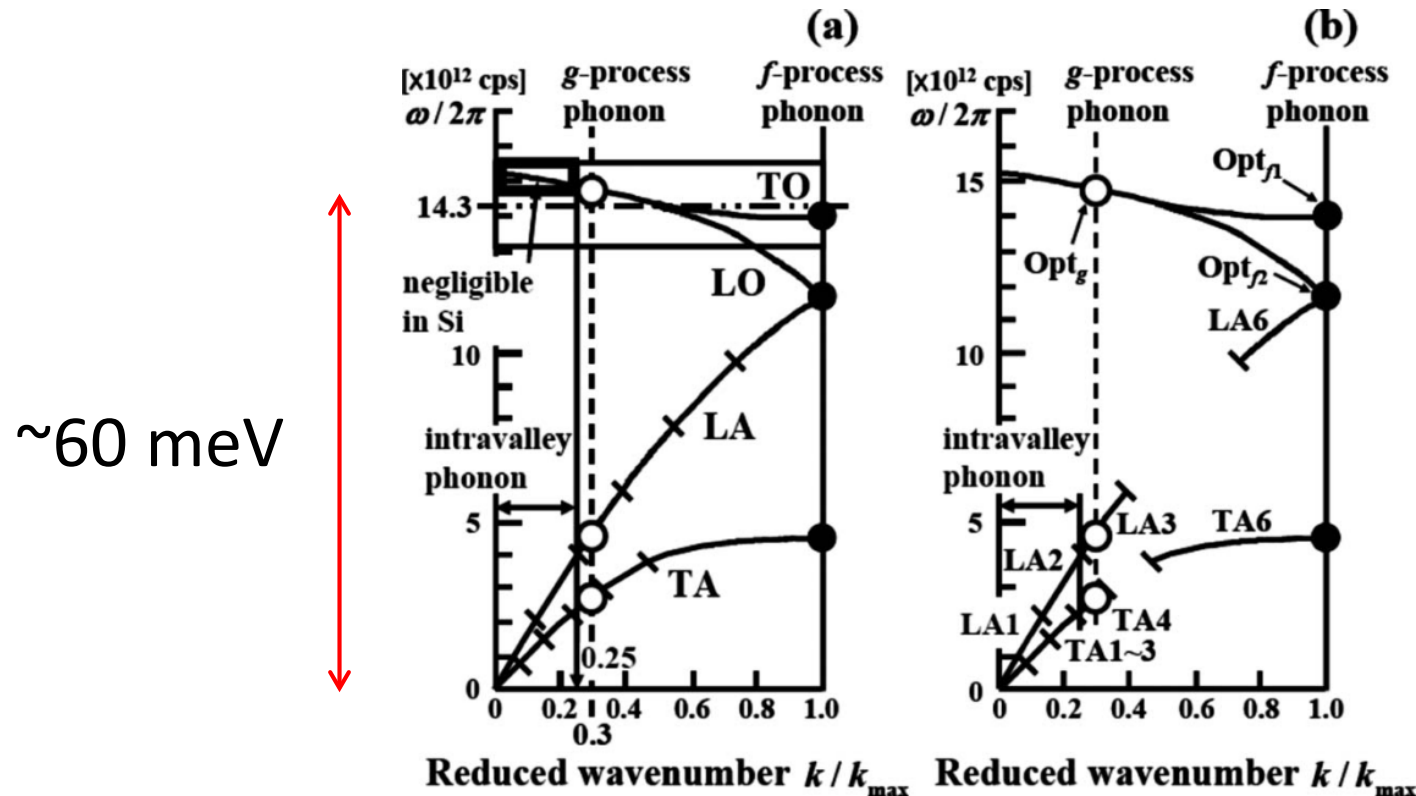
Taur, Eq. (2.35)

Example 2-2 of Hu's book

- Hole mobility, $\mu_p = 470 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$
 - When the electric field is 10^3 V cm^{-1} , the drift velocity is $4.7 \times 10^5 \text{ cm s}^{-1}$.
 - Momentum relaxation time (with $m_p = 0.39 m_0$) is 0.1 psec.

Phonon scattering

- Various phonon modes
 - Acoustic phonon : Low energy
 - Optical phonon : High energy, often treated as dispersion-less



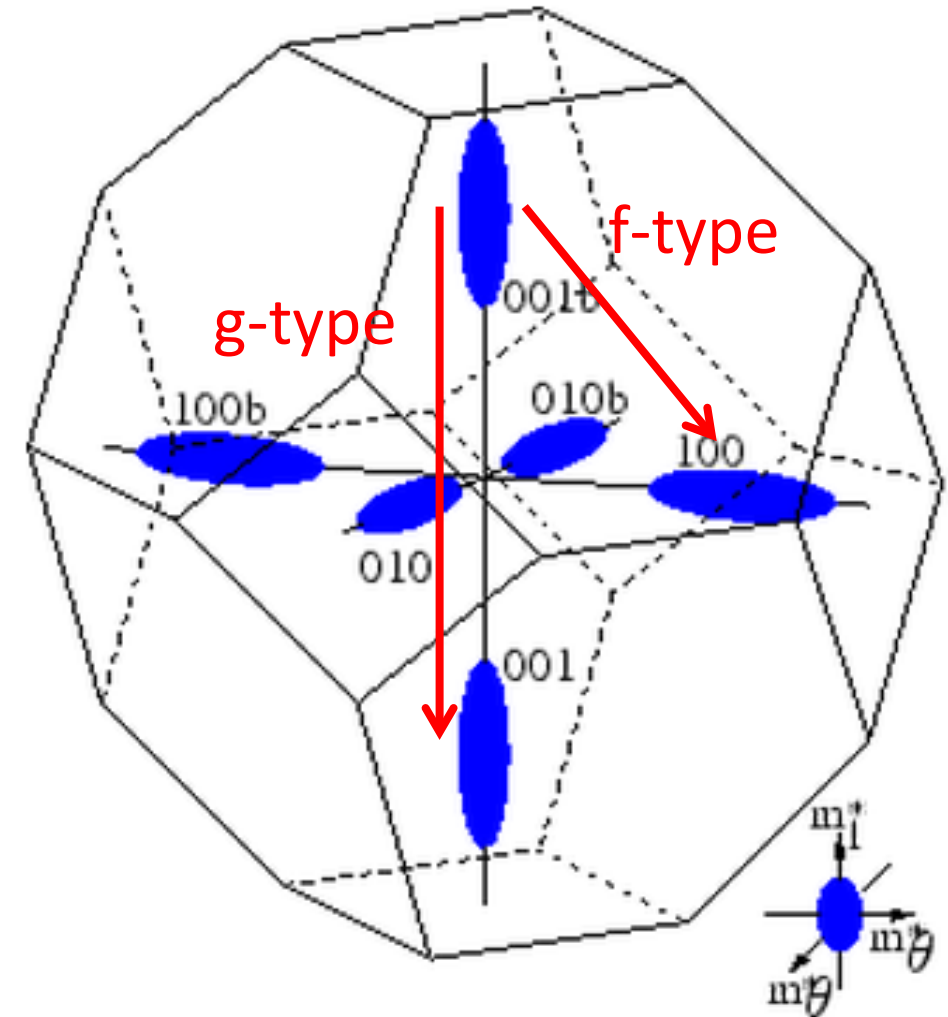
“Selection rule” matters.
Intravalley / f-process / g-process

Typical parameters

- Various phonon modes
 - Acoustic phonon : Low energy

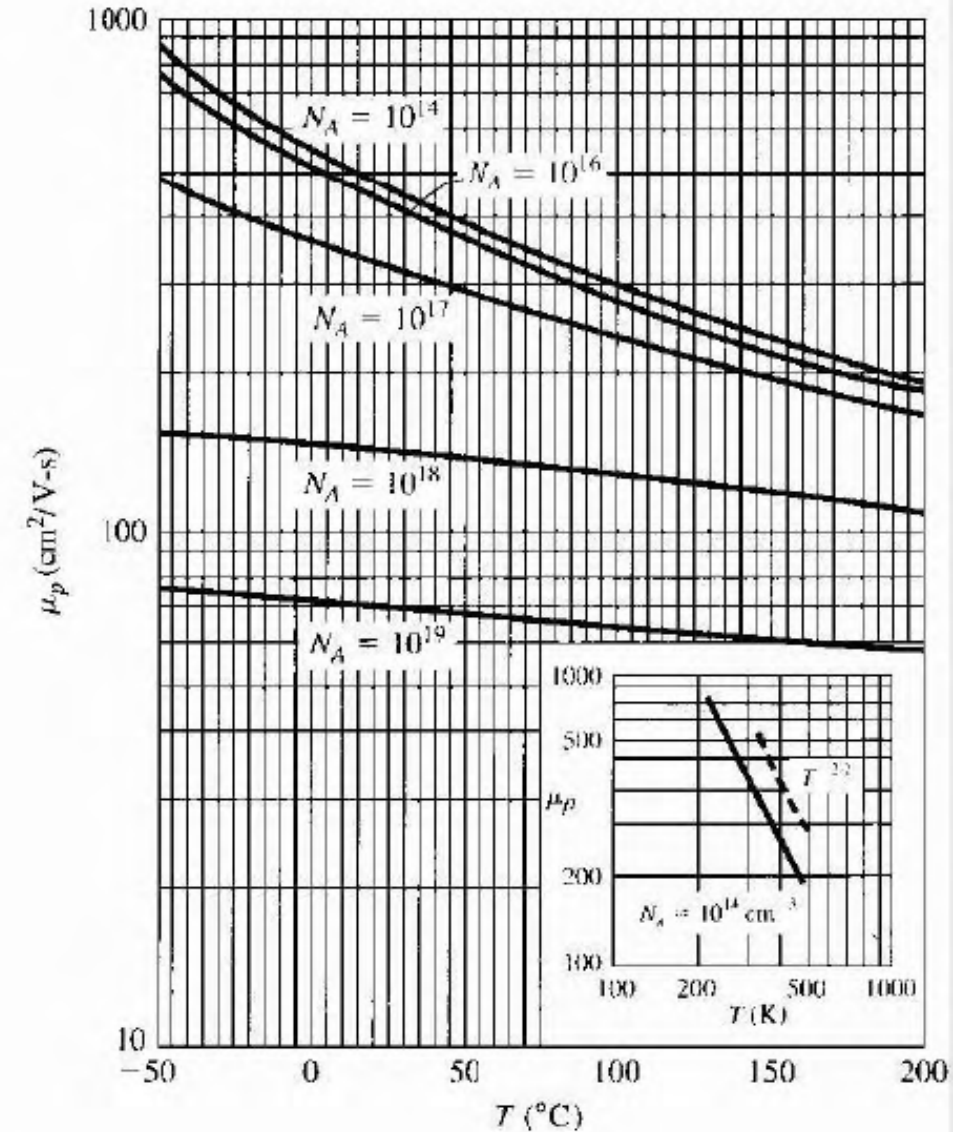
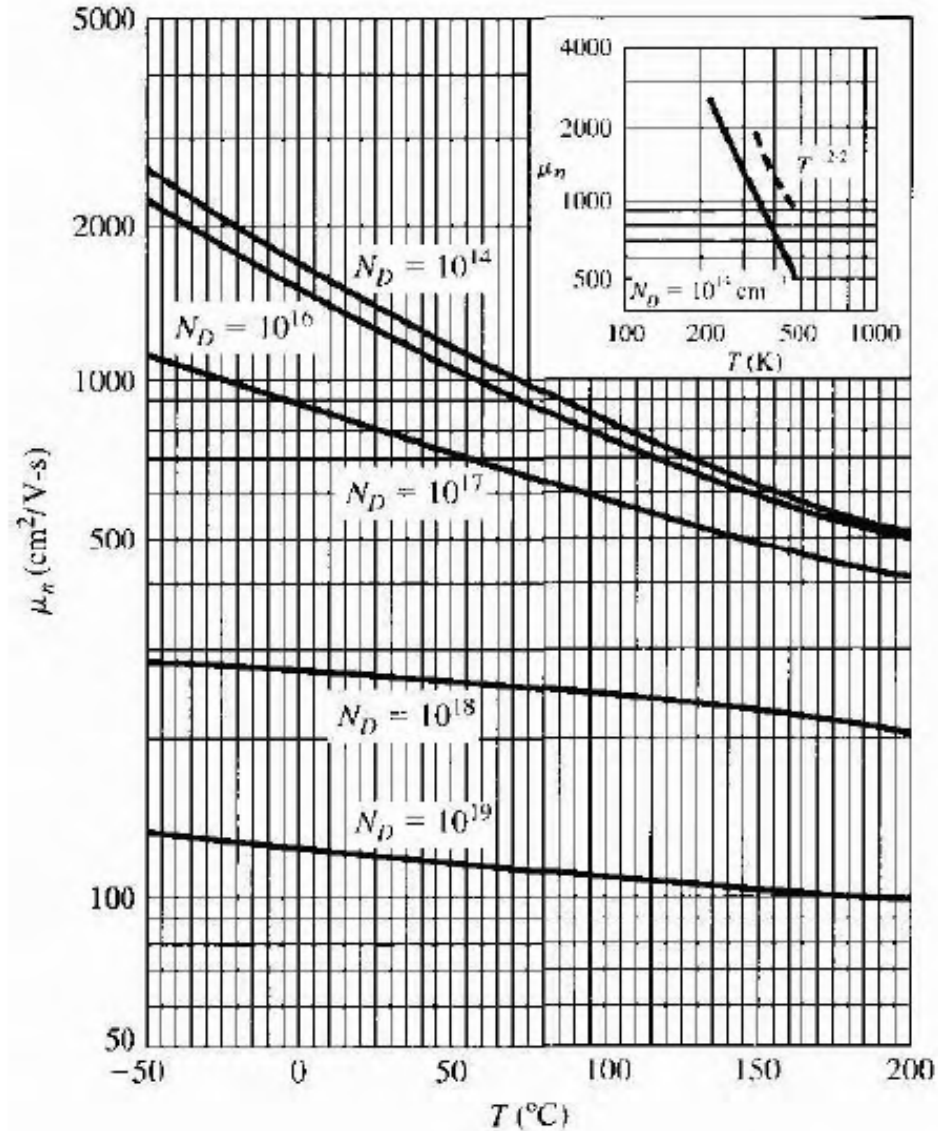
Mode	$D_t K$ (10^8 eV/cm)	$\hbar\omega$ (meV)	Type
TA	0.470	12.1	g-type
LA	0.740	18.5	g-type
LO	10.23	62.0	g-type
TA	0.280	19.0	f-type
LA	1.860	47.4	f-type
LO	1.860	58.6	f-type

Parameters for inelastic phonon scatterings in the Si conduction band



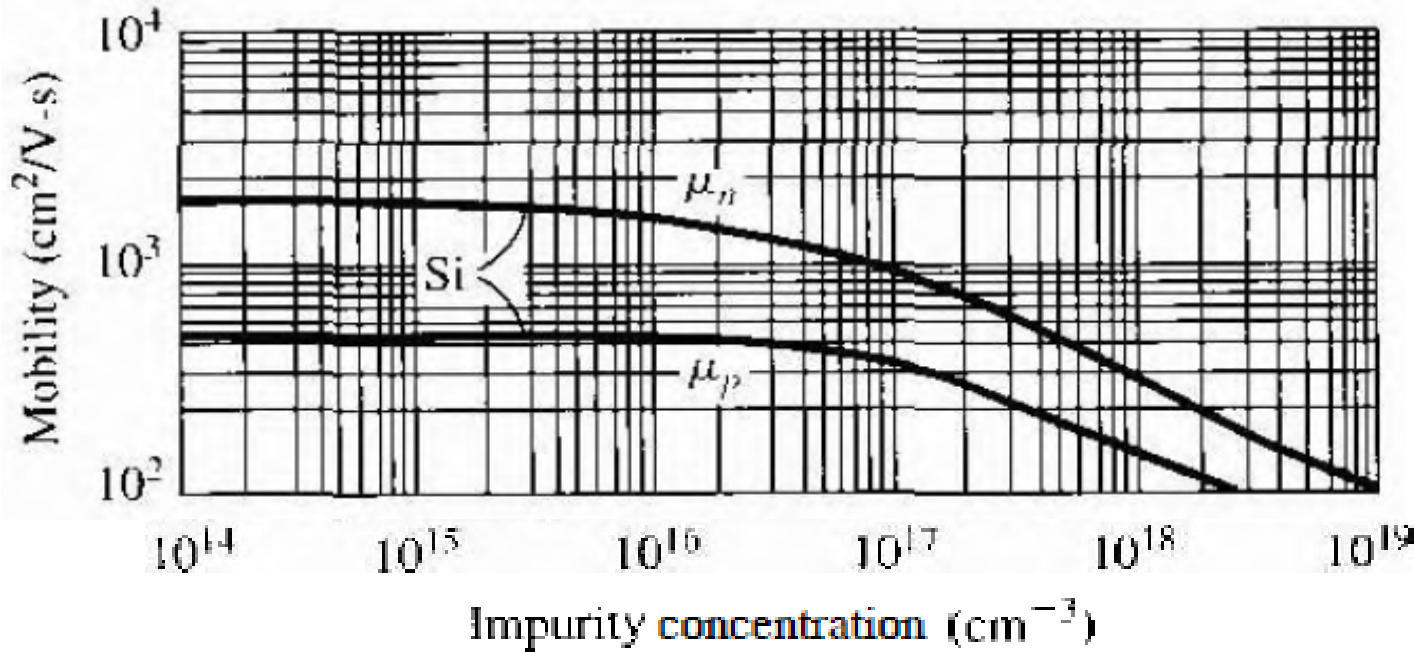
Temperature & doping

(Neamen's book)



Impurity concentration

- It is modeled as an elastic scattering process.



(Neamen's book)

- Which one is dominant? Phonon or impurity?

– Matthiessen's rule, $\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$

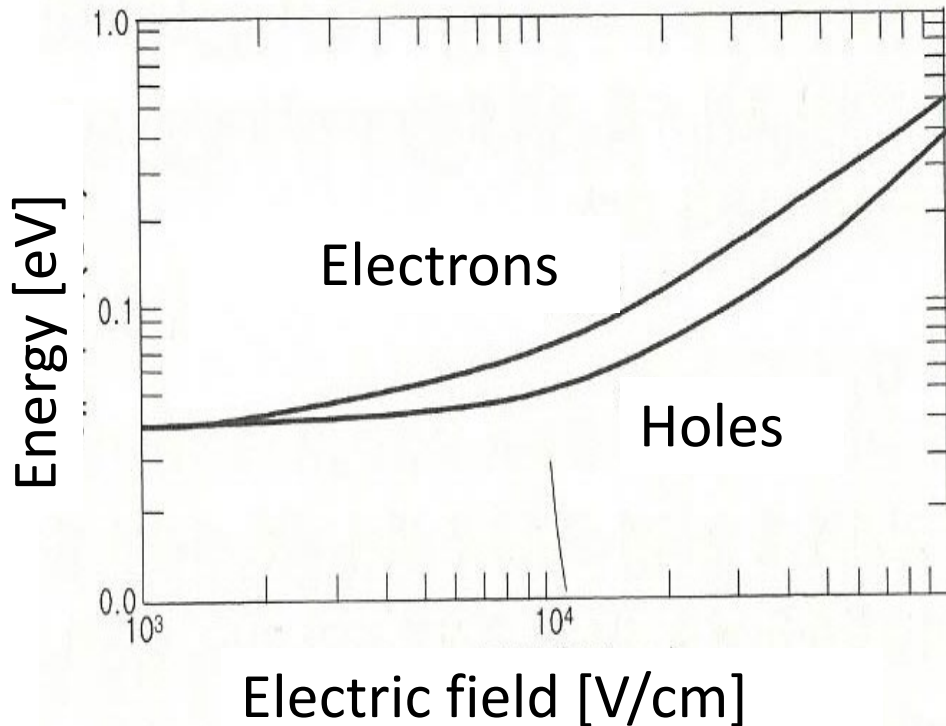
Taur, Eq. (2.27)

Matthiessen's rule

- When there are multiple contributions to the mobility,
 - (For example, phonon-limited mobility / impurity-limited mobility)
 - The overall collision rate is given by sum of all contributions.
 - $\frac{1}{\tau_m} = \frac{1}{\tau_{mL}} + \frac{1}{\tau_{mI}} + \dots$
 - (The above relation holds exactly only in the microscopic level.)
 - When recalling $\mu = \frac{q\tau_m}{m_n}$, it means $\frac{1}{\mu_m} = \frac{1}{\mu_{mL}} + \frac{1}{\mu_{mI}} + \dots$
 - It is very useful.

Hot electron

- Not only velocity, but also energy...
 - Increases when the electric field increases.
 - Increase of energy is a reason of the velocity saturation. *Why?*



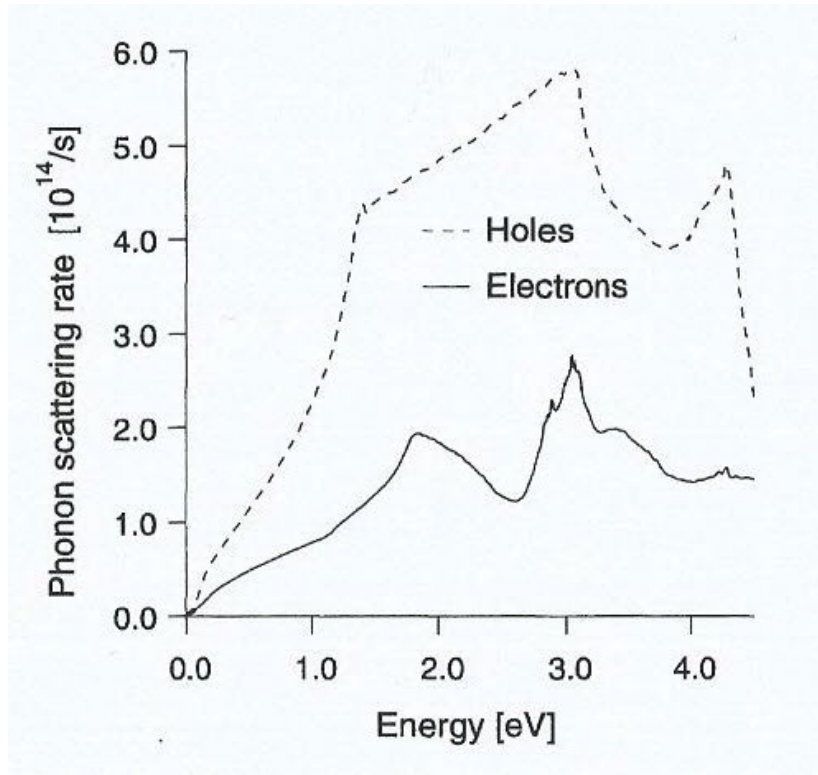
Up to 1 kV/cm, average energy is almost the same with the lattice energy.

Above 10 kV/cm, average energy significantly deviates from the lattice energy.

Average energy of electrons/holes in Si at 300K (Park's book)

Velocity saturation

- Electron with higher energy
 - Has a higher chance to be scattered by phonons. (Higher DOS)
 - More frequent scattering : Smaller τ_m

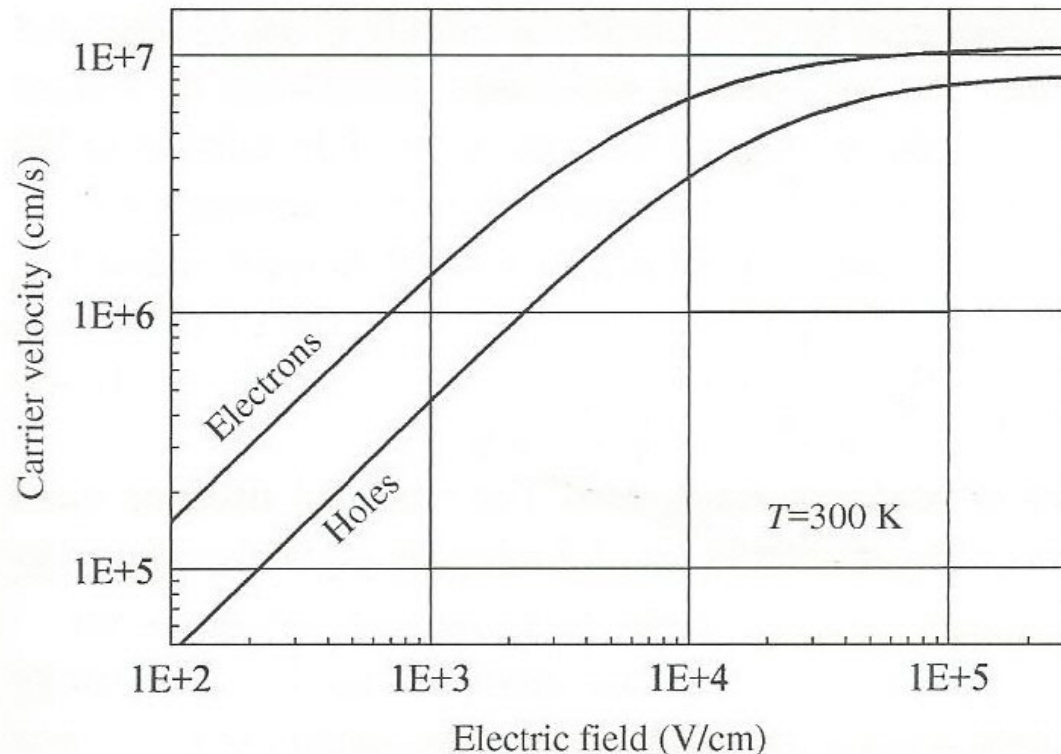


Phonon scattering rate in Si resembles the Density-Of-States.

Phonon scattering rate in Si (Jungemann's book)

Velocity vs. electric field

- At low electric fields, the linear relationship is valid.
 - At high electric fields, the velocity saturation starts to occur. The saturation velocity of Si is about 10^7 (cm/sec).



Velocity-field relationship
in Si at 300K
(Taur's book)

Caughey-Thomas relation

- For silicon,
 - Electron velocity can be approximated by

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{0.5}}$$

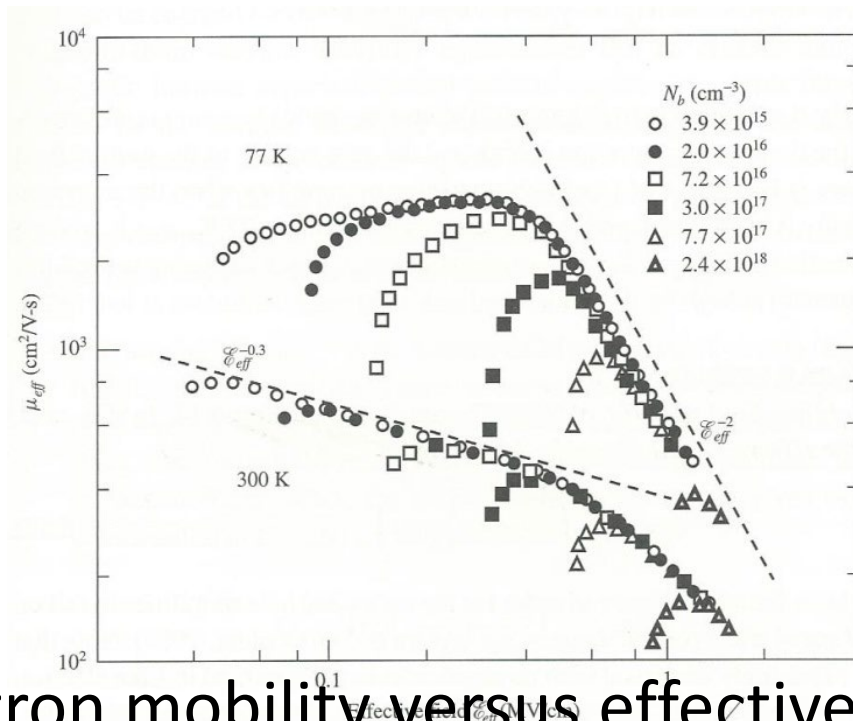
- Hole velocity

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)\right]}$$

- Why are they different?

Other scattering mechanisms

- We have discussed about the bulk mobility.
 - Other scattering mechanisms (alloy scattering & impact ionization)
 - Surface scattering severely reduces the inversion mobility.



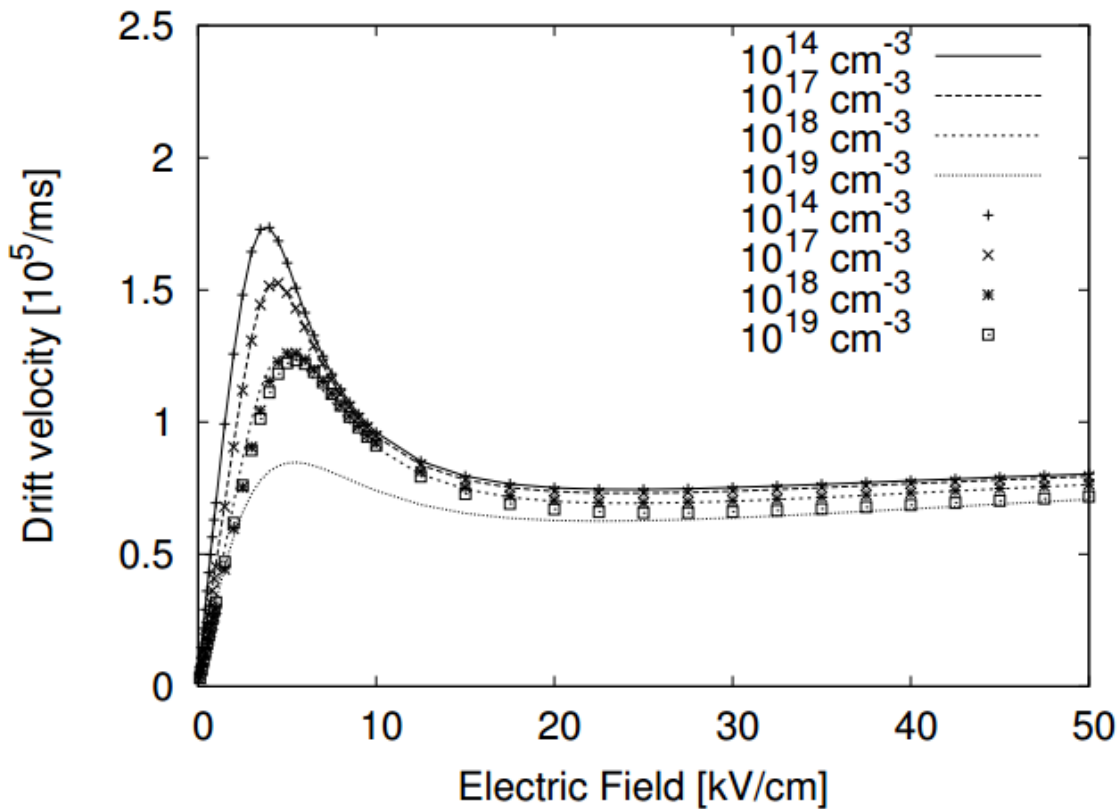
So-called “universal” mobility curve in the Si inversion layer.

Two different contributions are clearly visible.

Electron mobility versus effective field for several doping concentrations (Takagi's paper)

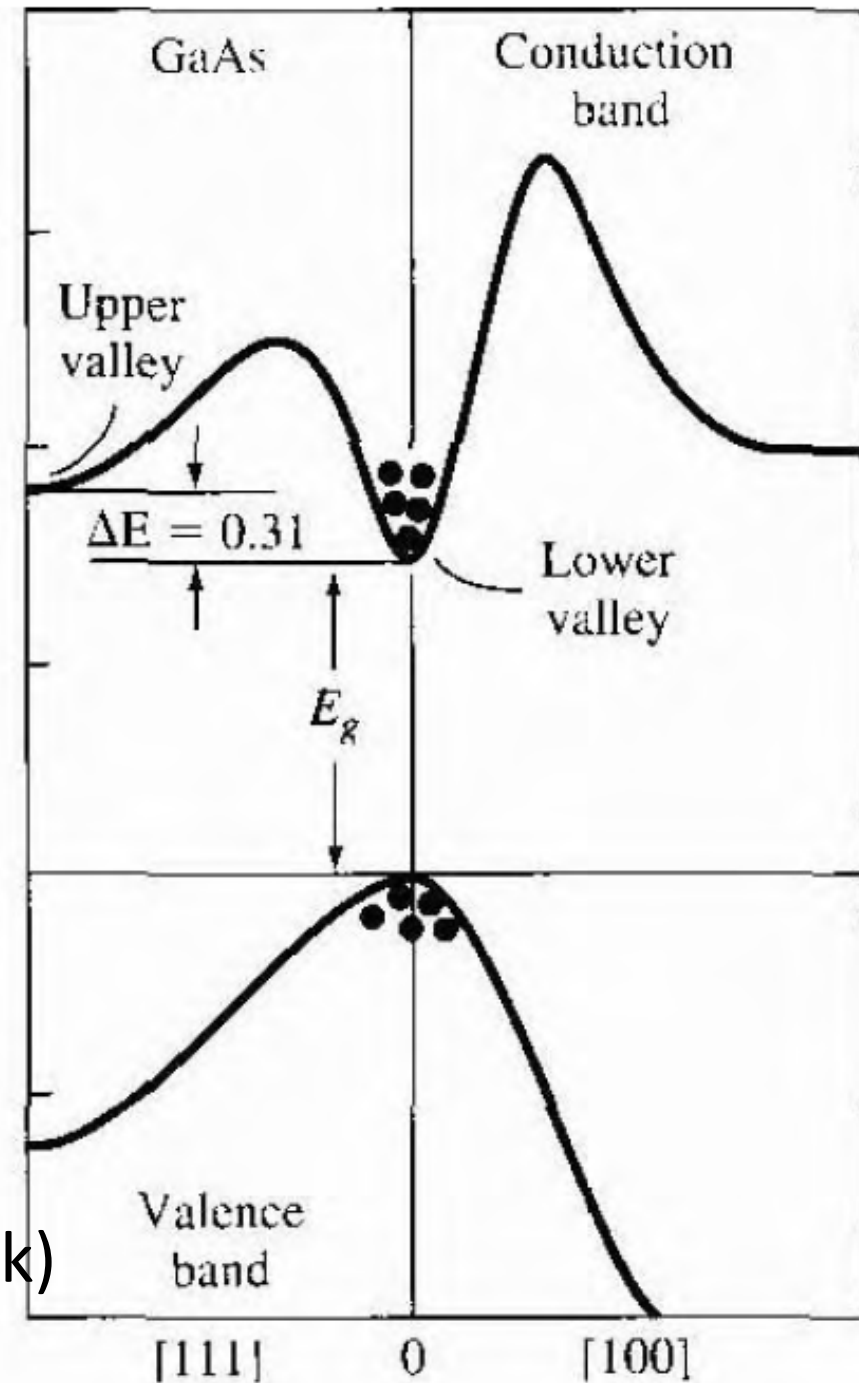
GaAs case

- Negative differential mobility
 - Its L valley is heavy.



(J. Bieder et al., IWCE 2010)

(Neamen's book)



Thank you!