

# Special Topics on Basic EECS I

## VLSI Devices

### Lecture 24

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# Drain current

- Using the previous approximation,
  - We can obtain the following expression:

$$I_d = \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left( V_{gs} - V_{fb} + \frac{k_B T}{q} \right) \phi_s - \frac{1}{2} C_{ox} \phi_s^2 - \frac{2}{3} \sqrt{2 \epsilon_{si} q N_a} \phi_s^{1.5} + \frac{k_B T}{q} \sqrt{2 \epsilon_{si} q N_a} \phi_s \right\} \bigg|_{\phi_{s,s}}^{\phi_{s,d}}$$

Taur, Eq. (3.21)

- Only with  $\phi_{s,s}$  and  $\phi_{s,d}$ , we can evaluate the drain current.

# Let's evaluate it together! (1)

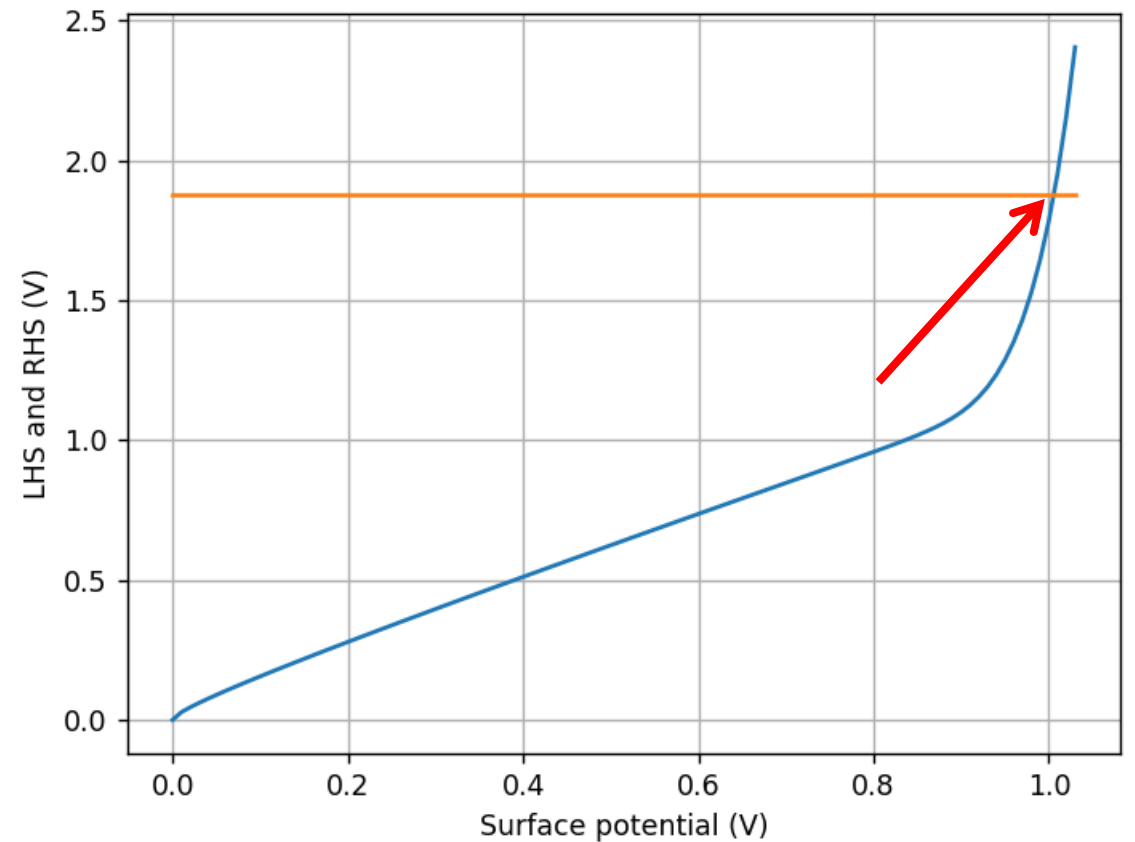
- Step-by-step

- Assume that  $N_a = 10^{17} \text{ cm}^{-3}$ ,  $t_{ox} = 10 \text{ nm}$ ,  $V_{gs} = 1.0 \text{ V}$ , and  $V_{fb} = -0.88 \text{ V}$ .
- Consider a case of  $V_{ds} = 0.1 \text{ V}$ .
- First, we must calculate  $\phi_{s,s}$ . How?

$$1.88 = \phi_{s,s} + \frac{\sqrt{2\epsilon_{si}k_B T N_a}}{C_{ox}} \left[ \frac{q\phi_{s,s}}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} \phi_{s,s}\right) \right]^{1/2}$$

# Let's evaluate it together! (2)

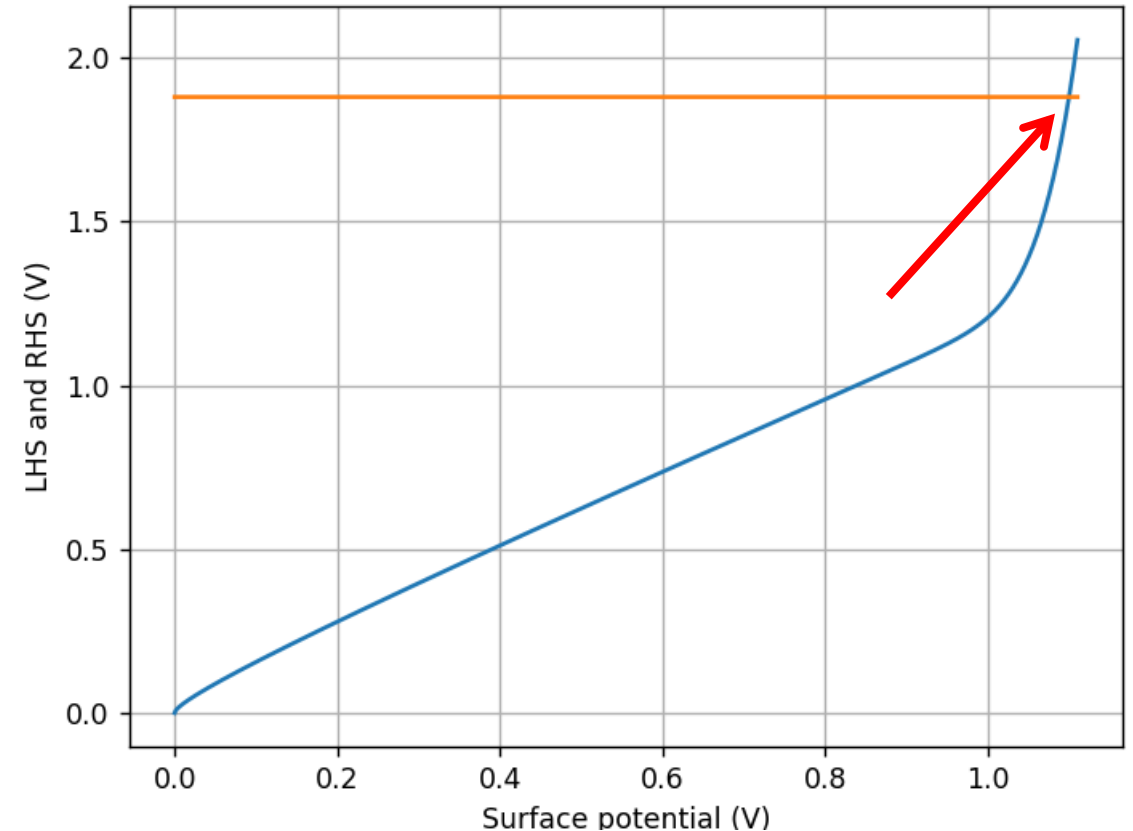
- Graphical solution
    - Draw the LHS and RHS.
- $\phi_{S,S} = 1.006 \text{ V}$



# Let's evaluate it together! (3)

- Now, for the drain end.
  - We must calculate  $\phi_{s,d}$ .

$$\phi_{s,d} = 1.100 \text{ V}$$



$$1.88 = \phi_{s,d} + \frac{\sqrt{2\epsilon_{si}k_B T N_a}}{C_{ox}} \left[ \frac{q\phi_{s,d}}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_{s,d} - 0.1)\right) \right]^{1/2}$$

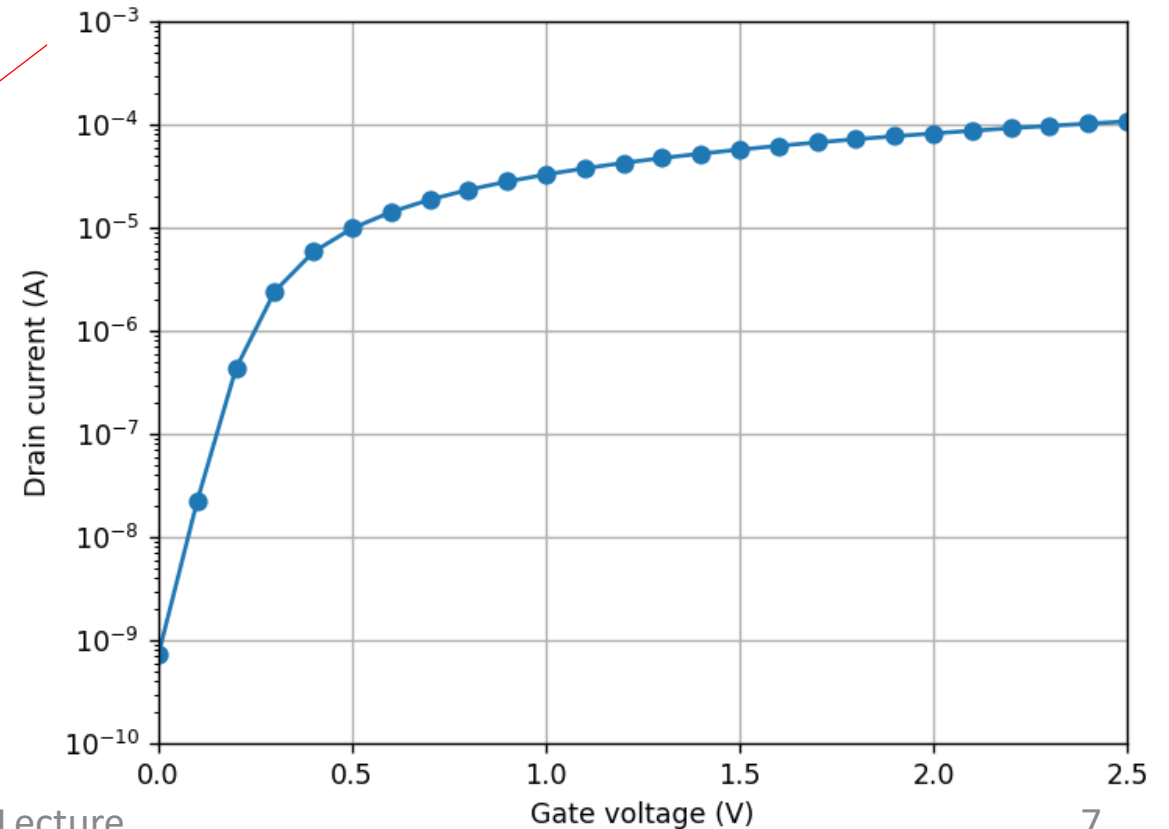
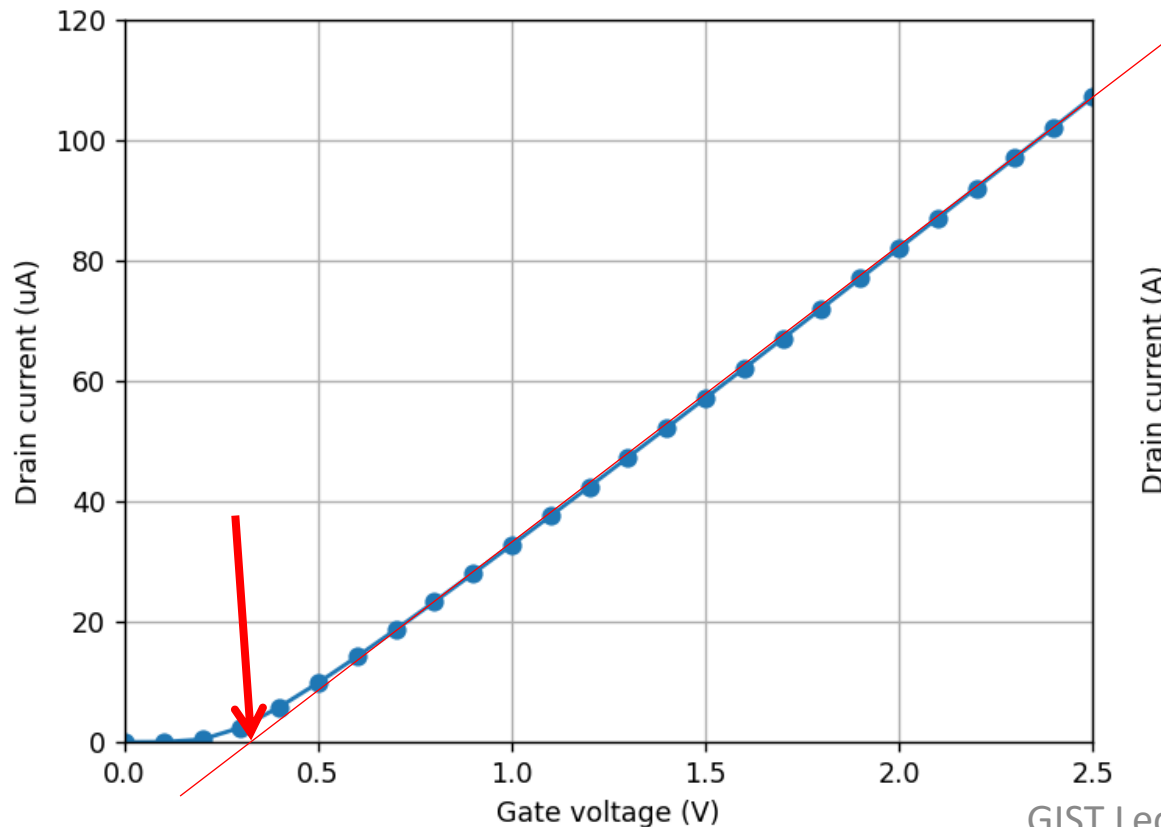
# Let's evaluate it together! (4)

- Now, we can calculate  $I_d$ .

- Evaluate  $C_{ox} \left( V_{gs} - V_{fb} + \frac{k_B T}{q} \right) \phi_s - \frac{1}{2} C_{ox} \phi_s^2 - \frac{2}{3} \sqrt{2 \epsilon_{si} q N_a} \phi_s^{1.5} + \frac{k_B T}{q} \sqrt{2 \epsilon_{si} q N_a} \phi_s$  twice with  $\phi_{s,s} = 1.006$  V and  $\phi_{s,d} = 1.100$  V.
- Values are  $1.3442 \times 10^{-6}$  C V cm<sup>-2</sup> and  $1.4096 \times 10^{-6}$  C V cm<sup>-2</sup>.
- When  $\mu_{eff} \frac{W}{L}$  is 500 cm<sup>2</sup> V<sup>-1</sup> sec<sup>-1</sup>,  $I_d$  is about 32.76  $\mu$ A.

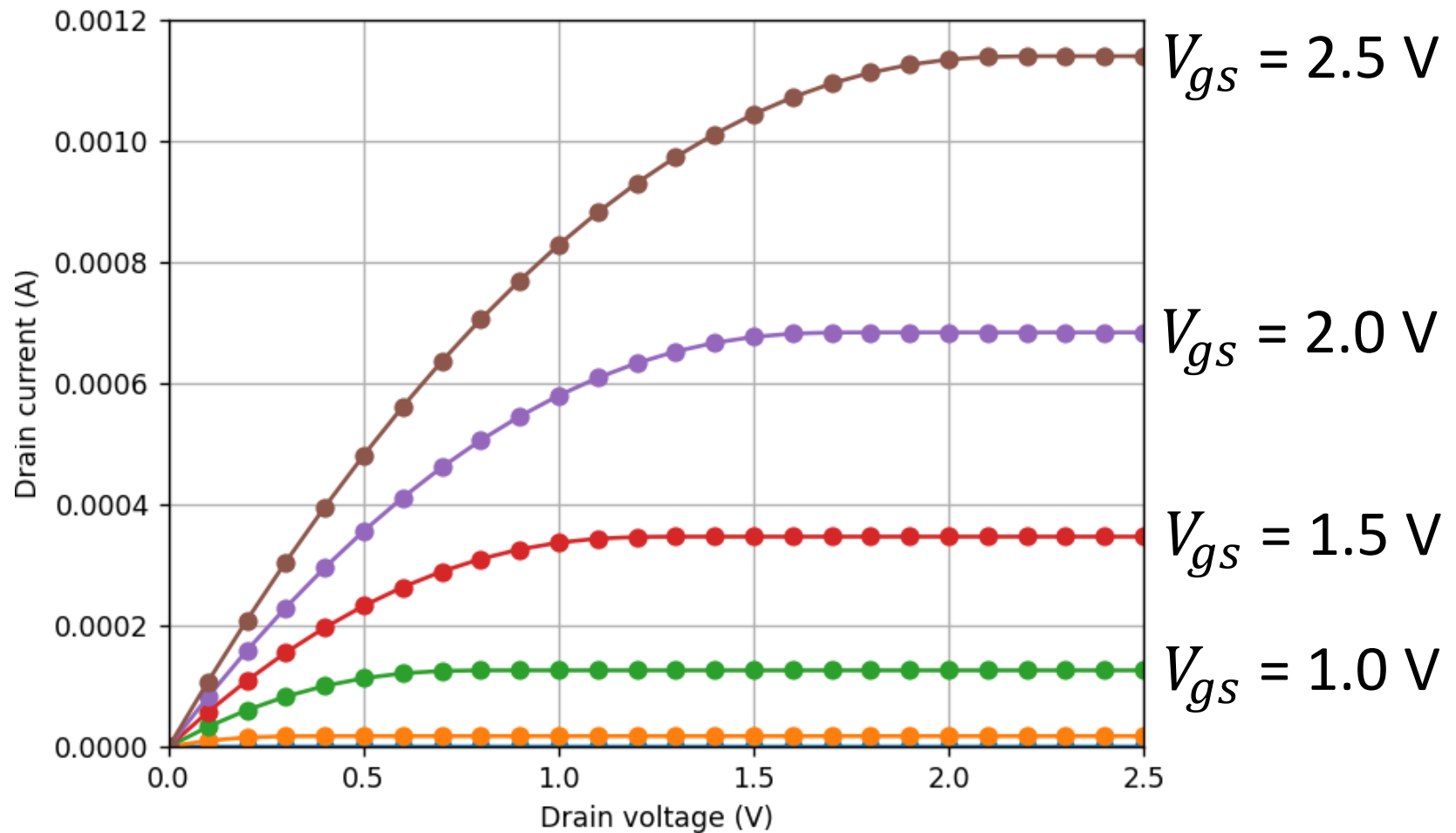
# Input characteristics (at a low $V_{ds}$ , 0.1 V)

- Increase  $V_{gs}$  up to 2.5 V.
  - Linear scale and semi-log scale



# Output characteristics

- Increase  $V_{ds}$  up to 2.5 V at various  $V_{gs}$  values.





# Regional approximations

- After the onset of inversion but before saturation,

- The surface potential,  $\phi_s(y)$ , can be approximated by

$$\phi(0, y) = V(y) + 2\phi_B \quad \text{Taur, Eq. (3.3)}$$

- It means that

$$\phi_{s,s} = 2\phi_B$$

$$\phi_{s,d} = 2\phi_B + V_{ds}$$

- In this case,  $\frac{dV}{d\phi_s} = 1$ . We must calculate the following term for  $\phi_{s,d}$ :

$$C_{ox}(V_{gs} - V_{fb})(2\phi_B + V_{ds}) - \frac{1}{2}C_{ox}(2\phi_B + V_{ds})^2 \\ - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}(2\phi_B + V_{ds})^{1.5}$$

# A simpler form of $I_d$

- By taking the difference, we can find a simpler form:

$$I_d = \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left( V_{gs} - V_{fb} - 2\phi_B - \frac{1}{2} V_{ds} \right) V_{ds} - \frac{2}{3} \sqrt{2\epsilon_{si} q N_a} [(2\phi_B + V_{ds})^{1.5} - (2\phi_B)^{1.5}] \right\} \quad \text{Taur, Eq. (3.22)}$$

- For a given  $V_{gs}$ ,  $I_d$  first increases linearly with  $V_{ds}$ , then gradually levels off to a saturated value.

# Linear (triode) region

- When  $V_{ds}$  is small, we may keep only up to the first order.

$$\begin{aligned} I_d &= \mu_{eff} \frac{W}{L} \left\{ C_{ox} (V_{gs} - V_{fb} - 2\phi_B) V_{ds} - \frac{2}{3} \sqrt{2\epsilon_{si} q N_a} \left[ \frac{3}{2} (2\phi_B)^{0.5} V_{ds} \right] \right\} \\ &= \mu_{eff} \frac{W}{L} C_{ox} \left( V_{gs} - V_{fb} - 2\phi_B - \frac{\sqrt{4\epsilon_{si} q N_a \phi_B}}{C_{ox}} \right) V_{ds} \\ &= \mu_{eff} \frac{W}{L} C_{ox} (V_{gs} - V_t) V_{ds} \end{aligned} \quad \text{Taur, Eq. (3.23)}$$

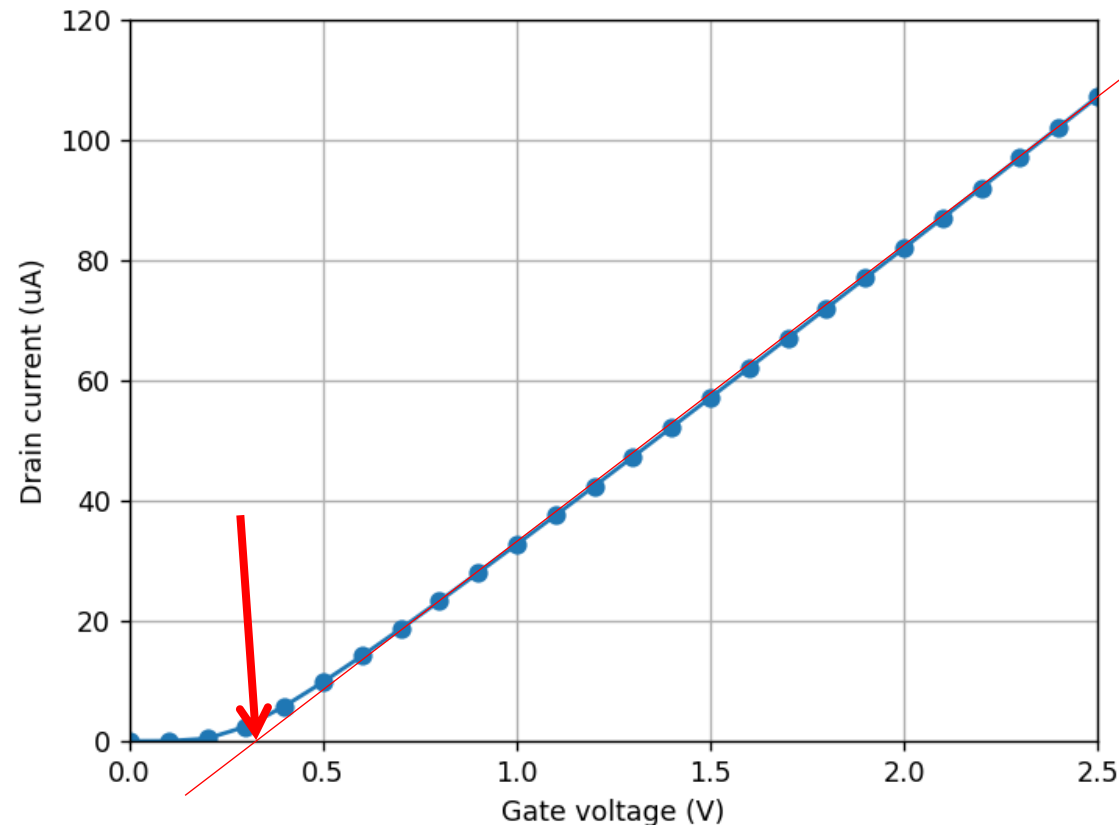
– The threshold voltage,  $V_t$ , is given by

$$V_t = V_{fb} + 2\phi_B + \frac{\sqrt{4\epsilon_{si} q N_a \phi_B}}{C_{ox}} \quad \text{Taur, Eq. (3.24)}$$

– It is the gate voltage when the surface potential reaches at  $2\phi_B$ .

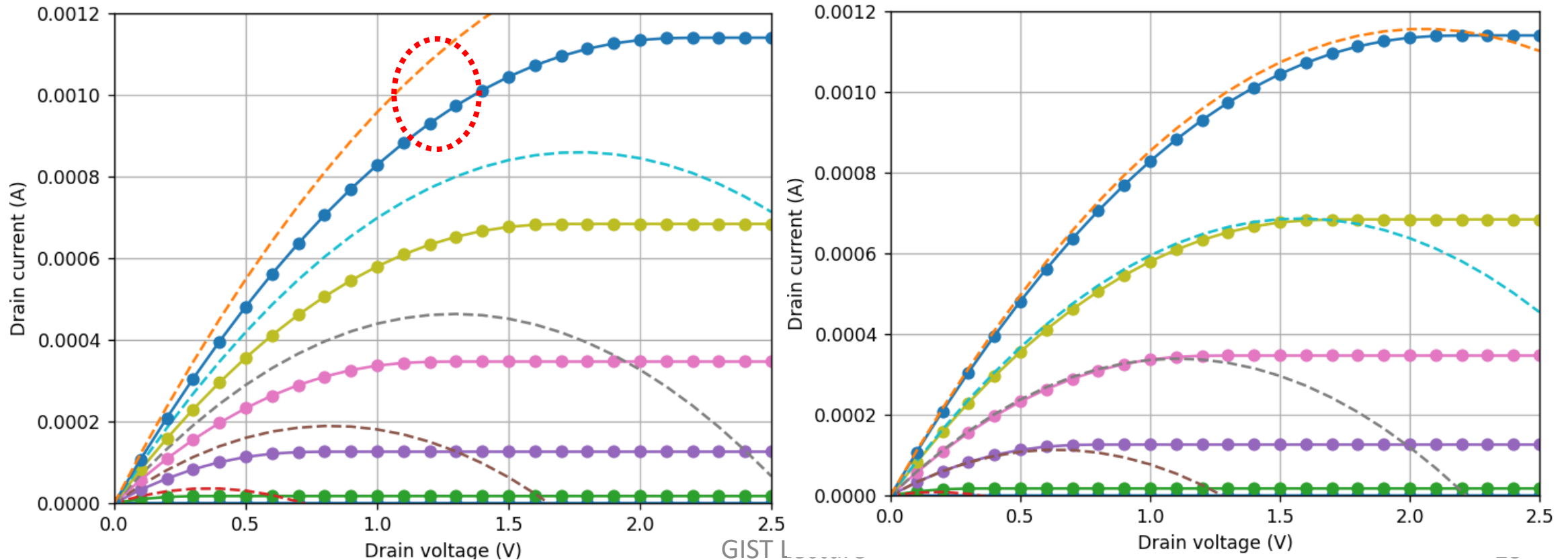
# In our previous example,

- The threshold voltage is about 0.11 V.
  - In reality, a *linearly extrapolated threshold voltage* is slightly higher than the “ $2\phi_B$ ”  $V_t$ .



# Comparison

- Output characteristics by Taur, Eq. (3.22) & Taur, Eq. (3.23)
  - Difference in  $V_t$  ( $\sim 0.2$  V)



# Parabolic region

- We must keep up to the second order.

$$\begin{aligned}
 I_d &= \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left( V_{gs} - V_{fb} - 2\phi_B - \frac{1}{2} V_{ds} \right) V_{ds} \right. \\
 &\quad \left. - \frac{2}{3} \sqrt{2\epsilon_{si} q N_a} \left[ \frac{3}{2} (2\phi_B)^{0.5} V_{ds} + \frac{3}{8} (2\phi_B)^{-0.5} V_{ds}^2 \right] \right\} \\
 &= \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left( V_{gs} - V_t - \frac{1}{2} V_{ds} \right) V_{ds} - \frac{1}{4} \sqrt{2\epsilon_{si} q N_a} [(2\phi_B)^{-0.5} V_{ds}^2] \right\} \\
 &= \mu_{eff} \frac{W}{L} \left\{ C_{ox} (V_{gs} - V_t) V_{ds} - \frac{1}{2} C_{ox} \left[ 1 + \frac{\sqrt{\epsilon_{si} q N_a / (4\phi_B)}}{C_{ox}} \right] V_{ds}^2 \right\}
 \end{aligned}$$

Taur, Eq. (3.25)

# Let's introduce a factor, $m$ .

- It is given as

$$m = 1 + \frac{\sqrt{\epsilon_{si} q N_a / (4 \phi_B)}}{C_{ox}} \quad \text{Taur, Eq. (3.26)}$$

- From the maximum depletion width,

$$W_{dm} = \sqrt{\frac{4 \epsilon_{si} \phi_B}{q N_a}} \quad \text{Taur, Eq. (2.190)}$$

- An alternative form is available,

$$m = 1 + \frac{C_{dm}}{C_{ox}} = 1 + \frac{3 t_{ox}}{W_{dm}} \quad \text{Taur, Eq. (3.27)}$$

- In our previous example? It was about 1.1. (Due to a low  $N_a$ )

# Its physical meaning

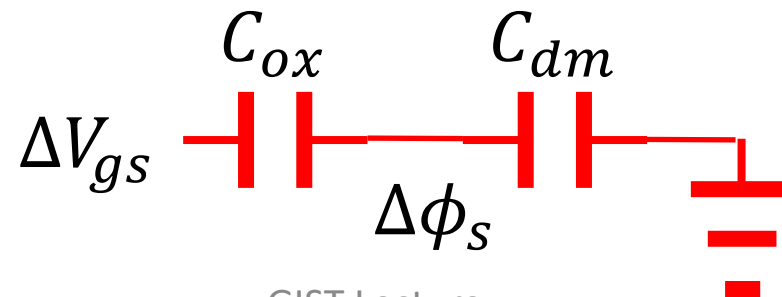
- Serial capacitors give  $\frac{C_{ox}C_{dm}}{C_{ox}+C_{dm}}$ .
  - Charge across the oxide capacitor:

$$\frac{C_{ox}C_{dm}}{C_{ox} + C_{dm}} \Delta V_{gs} = C_{dm} \Delta \phi_s$$

- Therefore,

$$m = \frac{C_{ox} + C_{dm}}{C_{ox}} = \frac{\Delta V_{gs}}{\Delta \phi_s}$$

- $m$  should be kept close to one.





# Saturation current

- Maximum value of  $I_d$  at a given  $V_{gs}$

– Recall that

$$I_d = \mu_{eff} \frac{W}{L} \left\{ C_{ox} (V_{gs} - V_t) V_{ds} - \frac{m}{2} C_{ox} V_{ds}^2 \right\} \quad \text{Taur, Eq. (3.25)}$$

– When  $V_{ds} = V_{dsat} = \frac{V_{gs} - V_t}{m}$ ,

$$I_d = I_{dsat} = \mu_{eff} C_{ox} \frac{W}{L} \frac{(V_{gs} - V_t)^2}{2m} \quad \text{Taur, Eq. (3.28)}$$

# Thank you!