Special Topics on Basic EECS I
VLSI Devices
Lecture 11

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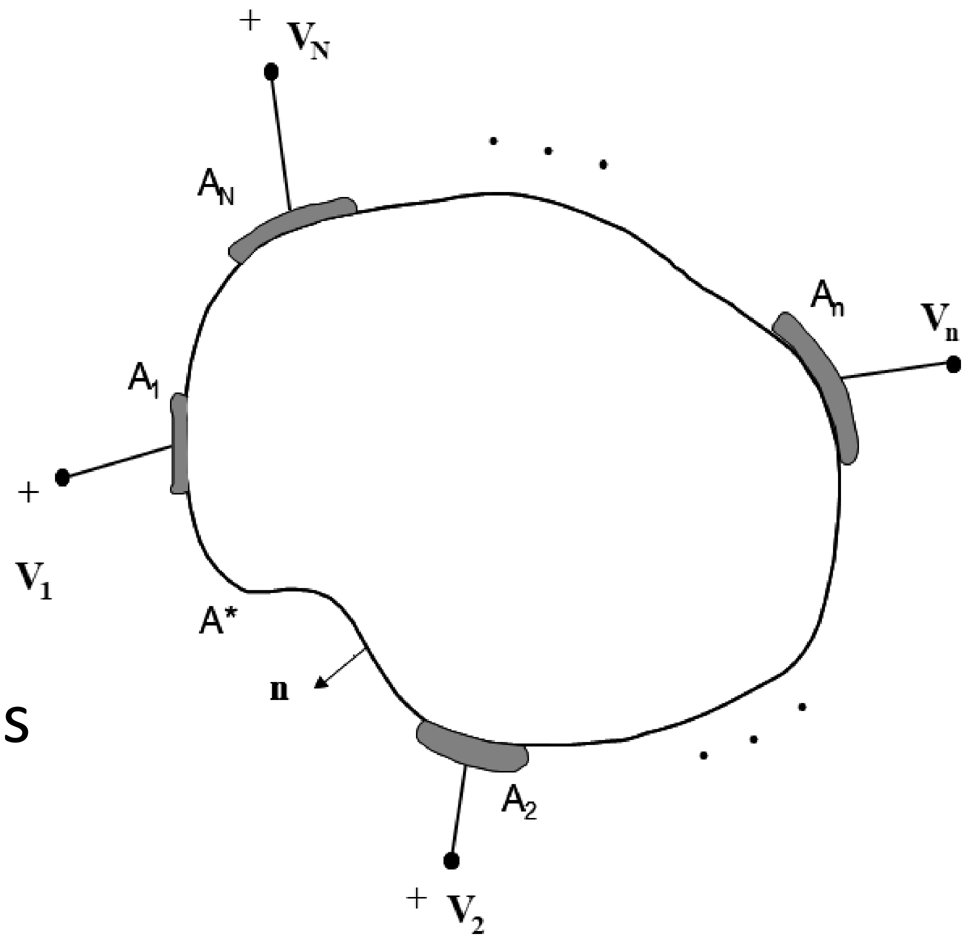
Boundary conditions for ϕ_n and ϕ_p

- Dirichlet boundary condition
  - At terminals,
    $\phi_n = \phi_p = V_{app}$
- Neumann boundary condition
  - No current through non-contact surfaces
    $\nabla\phi_n \cdot \mathbf{n} = \nabla\phi_p \cdot \mathbf{n} = 0$



Poisson equation

- Electrostatic potential, ϕ (In Taur, it is denoted as ψ_i.)
  - Conventionally, it is defined in terms of the intrinsic Fermi level,

$$E_i = -q\phi \qquad \text{Taur, Eq. (2.40)}$$

- Electric field, $\mathbf{E}$
  - It is equal to the negative gradient of ϕ,

$$\mathbf{E} = -\nabla\phi \qquad \text{Taur, Eq. (2.41)}$$

- Poisson equation

$$\rho = q(p - n + N_d^+ - N_a^-)$$

$$\nabla \cdot \mathbf{D} = \nabla \cdot (\epsilon\mathbf{E}) = \rho$$

  - One dimensional and homogeneous system

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon} \qquad \text{Taur, Eq. (2.42)}$$

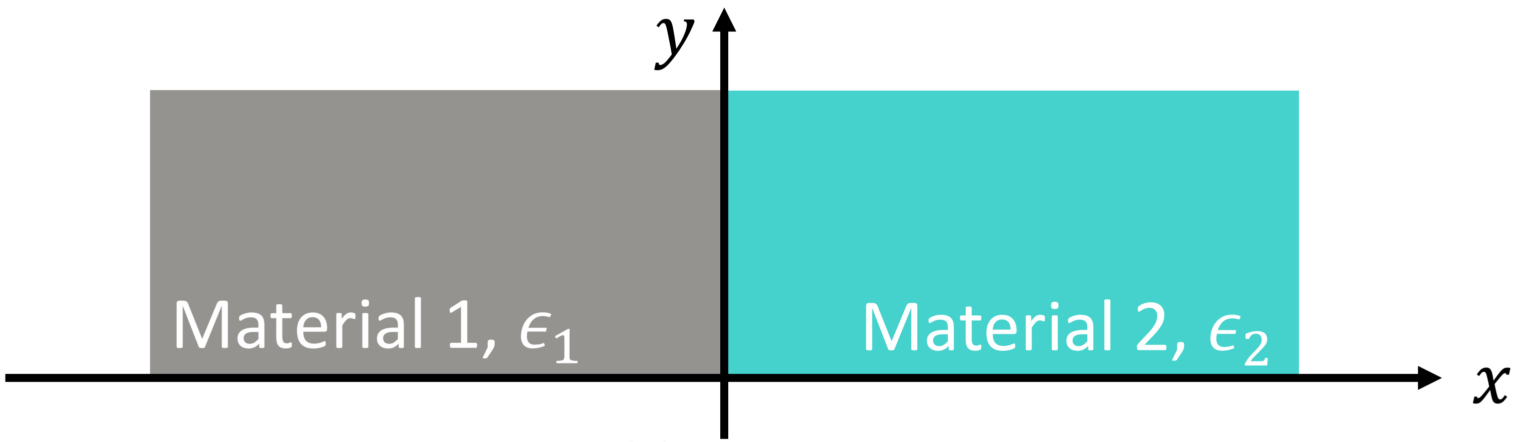
Boundary condition

- Tangential field
  - Tangential fields are continuous.

$$E_{1y}(0, y) = E_{2y}(0, y)$$ Taur, Eq. (2.46)

- Normal field
  - Perpendicular component of is continuous.

$$\epsilon_1 E_{1x}(0, y) = \epsilon_2 E_{2x}(0, y)$$ Taur, Eq. (2.47)



Debye length (1)

- Consider an n-type silicon. (Neglect holes and acceptors)

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon}\left[N_d(x) - n_i \exp\left(\frac{\phi - \phi_f}{k_B T/q}\right)\right] \qquad \text{Taur, Eq. (2.51)}$$

  – For a uniformly doped background, N_d,

$$0 = -\frac{q}{\epsilon}\left[N_d - n_i \exp\left(\frac{\phi - \phi_f}{k_B T/q}\right)\right]$$

  – By introducing an incremental change, $\Delta N_d(x)$, the corresponding change in the electrostatic potential, $\Delta\phi(x)$, is given as

$$\frac{d^2\Delta\phi}{dx^2} = -\frac{q}{\epsilon}\left[\Delta N_d(x) - N_d \frac{\Delta\phi}{k_B T/q}\right] \qquad \text{Taur, Eq. (2.52)}$$

Debye length (2)

- With the Debye length,

$$L_D = \sqrt{\frac{\epsilon k_B T}{q^2 N_d}}$$

Taur, Eq. (2.51)

  - The equation can be written as

$$\frac{d^2\Delta\phi}{dx^2} - \frac{\Delta\phi}{L_D^2} = -\frac{q}{\epsilon}\Delta N_d(x)$$

  - Its solution takes the form of $\exp\left(-\frac{x}{L_D}\right)$.
  - It keas a distance on the order of L_D for the silicon bands to respond to an abrupt change in N_d.

Continuity equations

- (Change of electron number) = (Net incoming-flux integrated over surface)
  - Therefore,

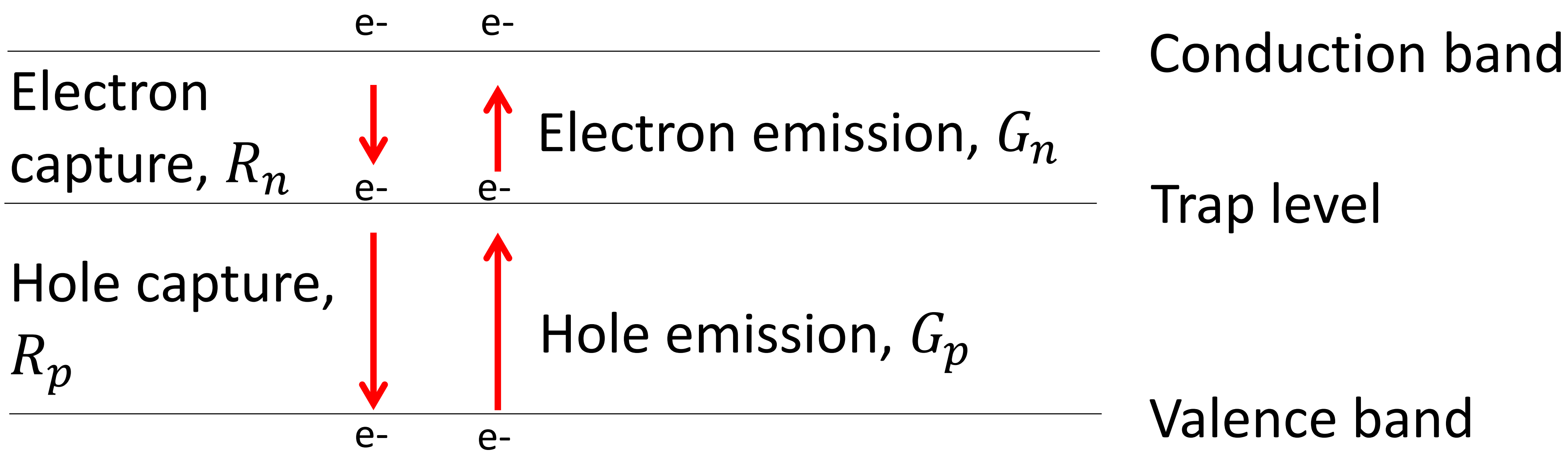
$$\frac{\partial n}{\partial t} = -\nabla \cdot \mathbf{F}_n$$

- When the generation and recombination processes are considered,
  - We need additional terms.

$$\frac{\partial n}{\partial t} = -\nabla \cdot \mathbf{F}_n + G_n - R_n$$

SRH (Shockley-Read-Hall) recombination (1)

- In silicon, a *radiative* or *Auger* process is very low due to its indirect bandgap.
  - Most of the recombination processes take place indirectly via a trap or a deep impurity level near the middle of the forbidden gap.



SRH (Shockley-Read-Hall) recombination (2)

- After some derivation steps,
  - We have the following expression:

$$R_n - G_n = R_p - G_p = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

Taur, Eq. (A5.16)

  - It vanishes at equilibrium.

Full set of equations

- Drift-diffusion model
  - Poisson equation

$$\nabla \cdot (\epsilon \mathbf{E}) = q(p - n + N_d^+ - N_a^-)$$

  - Continuity equations

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_n + G_n - R_n$$

Taur, Eq. (2.68)

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot \mathbf{J}_p + G_p - R_p$$

Taur, Eq. (2.69)

  - Current density

$$\mathbf{J}_n = q\mu_n n\mathbf{E} + qD_n \nabla n, \qquad \mathbf{J}_p = q\mu_p p\mathbf{E} - qD_p \nabla p$$

Dielectric relaxation time

- Consider a homogeneous n-type sample. (Neglect g-r processes.) Assume $\Delta n(\mathbf{r})$.
  - In the first order,
$$\Delta \mathbf{J}_n = \cancel{q\mu_n(\Delta n)\mathbf{E}} + q\mu_n n(\Delta \mathbf{E}) + \cancel{qD_n \nabla(\Delta n)}$$
  - Electron continuity equation
$$\frac{\partial}{\partial t}\Delta n = \frac{1}{q}\nabla \cdot \Delta \mathbf{J}_n \approx \nabla \cdot [\mu_n n(\Delta \mathbf{E})] = \mu_n n \frac{(-q)\Delta n}{\epsilon}$$ Taur, Eq. (2.70)
  - The dielectric relaxation time, $\frac{\epsilon}{q\mu_n n}$
  - The majority-carrier response time in silicon is typically on the order of 10^{-12} sec, which is shorter than most device switching times.

Thank you!