Special Topics on Basic EECS I
VLSI Devices
Lecture 21

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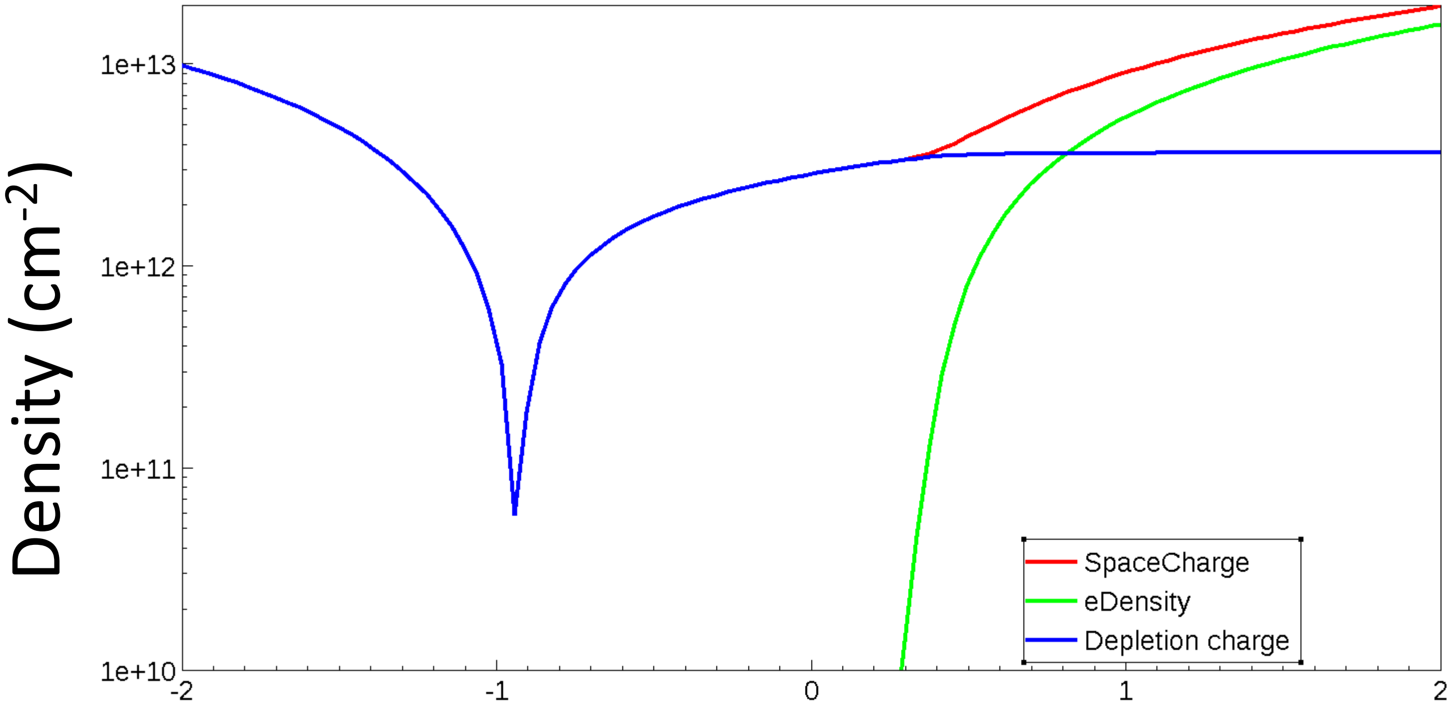
Maximum depletion width

- Therefore, maximum depletion width becomes

$$W_d = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_a}} = \sqrt{\frac{4\epsilon_{si}k_BT\ \ln(N_a/n_i)}{q^2N_a}}$$

Taur, Eq. (2.190)



Gate voltage (V)

← Depletion charge does not increase.

Beyond threshold voltage

It’s not perfectly fixed.

- The surface potential is *almost* fixed. (Surface potential pinning)
  - Small additional change in ϕ_s induces an exponential increase of the electron density.
  - Remember that $n = n_i \exp\left(\frac{q\phi}{k_B T}\right)$.
  - When $\phi_s = 2\phi_B$,

$$n(0) = n_i \exp\left(\frac{q\phi_B}{k_B T}\right) = p(\infty)$$

  - Additional potential ($\Delta\phi$) yields

$$n(0) = p(\infty) \exp\left(\frac{q\Delta\phi}{k_B T}\right)$$

It’s a high density.

General relation beyond depletion approx. (1)

- With the depletion approximation, we obtained

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si}qN_a\phi_s}$$

- We can do much better!
  - A generation relation for $Q_s = Q_d + Q_i$
  - The Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}[p(x) - n(x) - N_a]$$

Inversion charge

Taur, Eq. (2.175)

General relation beyond depletion approx. (2)

- Following Taur's notation,

For a while, $\phi(\infty) = -\phi_B$ is used as the reference value.
Therefore,

$$n(x) = n_i \exp\left(\frac{q\phi(x)}{k_B T}\right) \rightarrow n(x) = n(\infty)\exp\left(\frac{q\phi(x)}{k_B T}\right)$$ Taur, Eq. (2.178)

$$p(x) = n_i \exp\left(-\frac{q\phi(x)}{k_B T}\right) \rightarrow p(x) = p(\infty)\exp\left(-\frac{q\phi(x)}{k_B T}\right)$$ Taur, Eq. (2.177)

– The Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}\left[N_a\left(\exp\left(-\frac{q\phi}{k_B T}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_B T}\right)-1\right)\right]$$

Taur, Eq. (2.179)

General relation beyond depletion approx. (3)

- Multiplying $\frac{d\phi}{dx}dx$,
  - The Poisson equation

$$\frac{d\phi}{dx}d\left(\frac{d\phi}{dx}\right) = -\frac{q}{\epsilon_{si}}\left[N_a\left(\exp\left(-\frac{q\phi}{k_BT}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_BT}\right)-1\right)\right]d\phi$$

  - Integrate the above equation.

$$\int_0^{-E_x(x)}\frac{d\phi}{dx}d\left(\frac{d\phi}{dx}\right) = -\frac{q}{\epsilon_{si}}\int_0^{\phi(x)}\left[N_a\left(\exp\left(-\frac{q\phi}{k_BT}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_BT}\right)-1\right)\right]d\phi$$

Taur, Eq. (2.180)

General relation beyond depletion approx. (4)

- (Square of) Electric field
  - From $\frac{1}{2}E_x^2(x) = -\frac{q}{\epsilon_{si}}\left[-N_a\frac{k_BT}{q}\exp\left(-\frac{q\phi}{k_BT}\right) - N_a\phi + N_a\frac{k_BT}{q} - \frac{n_i^2}{N_a}\frac{k_BT}{q}\exp\left(\frac{q\phi}{k_BT}\right) + \frac{n_i^2}{N_a}\phi + \frac{n_i^2}{N_a}\frac{k_BT}{q}\right]$, we get

$$E_x^2(x) = \frac{2k_BTN_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi}{k_BT}\right) + \frac{q\phi}{k_BT} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi}{k_BT}\right) - \frac{q\phi}{k_BT} - 1\right)\right]$$

Taur, Eq. (2.181)

General relation beyond depletion approx. (5)

- At $x = 0$, we have $\phi(0) = \phi_s$.
  - Then,

$$E_s^2 = \frac{2k_B T N_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]$$

General relation beyond depletion approx. (6)

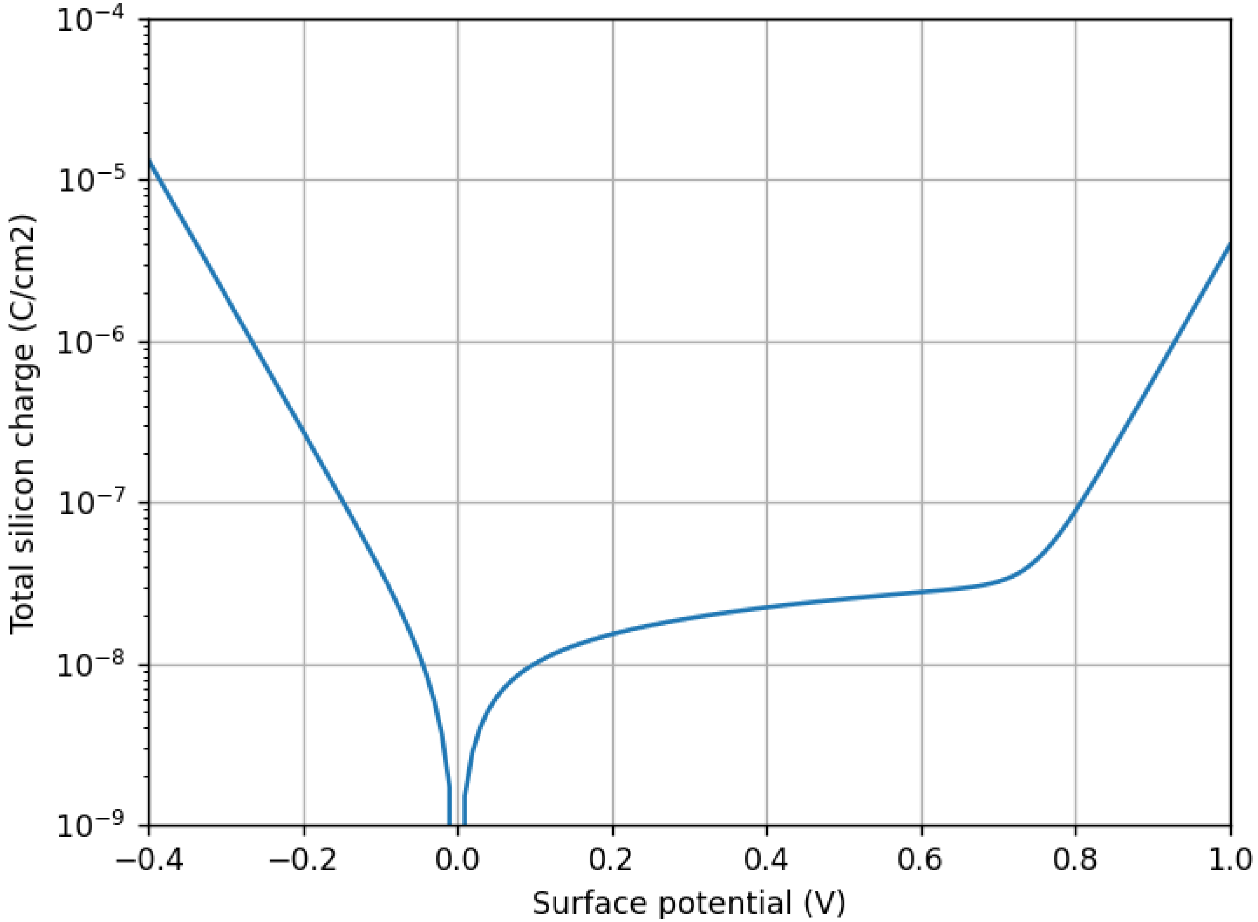
- At $x = 0$, we have $\phi(0) = \phi_s$.
  - From $Q_s = -\epsilon_{si} E_s$,

$$Q_s = \pm\sqrt{2\epsilon_{si} k_B T N_a}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]^{1/2}$$

Taur, Eq. (2.182)

Homework#4

- Draw $|Q_s|$ as a function of ϕ_s.
  - Assume that N_a is 4X10^{15} cm^{-3}. ϕ_s varies from -0.4 V to 1.0 V.



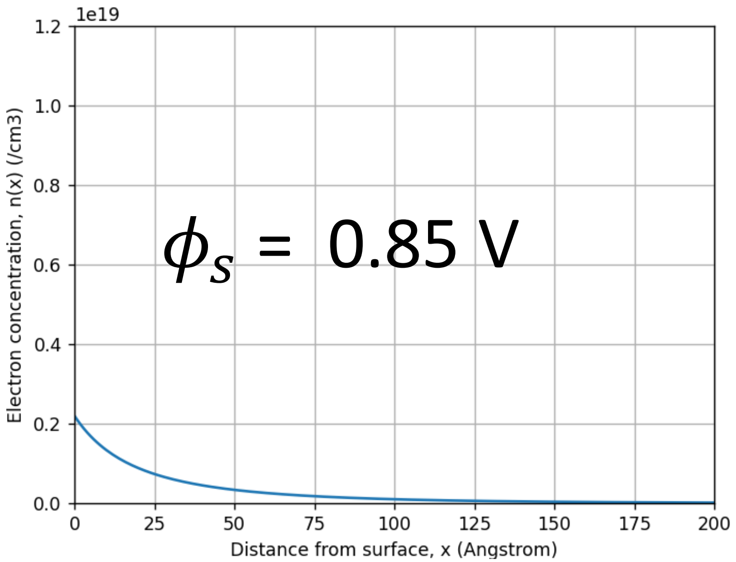
Strong inversion

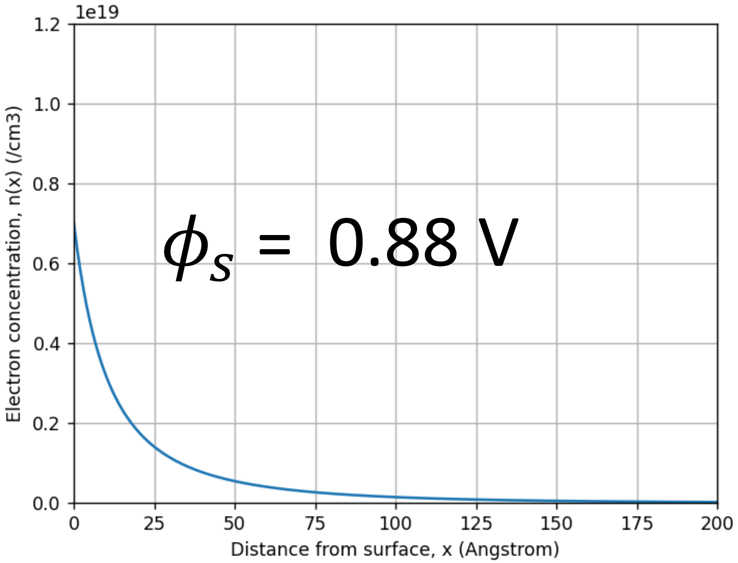
- Beyond strong inversion,

$$\frac{d\phi}{dx} \approx -\sqrt{\frac{2k_BTN_a}{\epsilon_{si}}\left(\frac{q\phi}{k_BT} + \frac{n_i^2}{N_a^2}\exp\left(\frac{q\phi}{k_BT}\right)\right)}$$

Taur, Eq. (2.191)

  – The electrons are distributed extremely close to the surface with an inversion-layer width less than 50 Å.





MOS equation

- Up to now, $Q_s(\phi_s)$ is found. We can control only V_g.
  - Relation between V_g and ϕ_s

Total silicon charge per unit area

$$V_g - V_{fb} = V_{ox} + \phi_s = -\frac{Q_s}{C_{ox}} + \phi_s$$

Taur, Eq. (2.195)

$\frac{\epsilon_{ox}}{t_{ox}}$, oxide capacitance per unit area

  - In general, $Q_s(\phi_s)$ is known. We can solve the above equation.

Taur, Eq. (2.182)

Thank you!