Special Topics on Basic EECS I
VLSI Devices
Lecture 6

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Fermi level in extrinsic silicon

- Charge neutrality
  - For an n-type bulk material at equilibrium,

$$p - n + N_d - N_d f_D(E_d) = 0$$

Taur, Eq. (2.17)

  - It is known that

$$f(E_d) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_f}{k_B T}\right)}$$

Taur, Eq. (2.18)

Due to the spin degeneracy

Discussion

- Let me try to explain the reason.
  - We must start from the Fermi-Dirac distribution…

Equation for the Fermi level

- Assume N_d and E_d are given. Then,

$$N_v \exp\left(\frac{E_v - E_f}{k_B T}\right) - N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) + \frac{N_d}{1 + 2\exp\left(-\frac{E_d - E_f}{k_B T}\right)} = 0$$

Taur, Eq. (2.19)

  - For shallow donor impurities,

$$-N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) + N_d = 0$$

$$E_c - E_f = k_B T \ln \frac{N_c}{N_d}$$

Taur, Eq. (2.20)

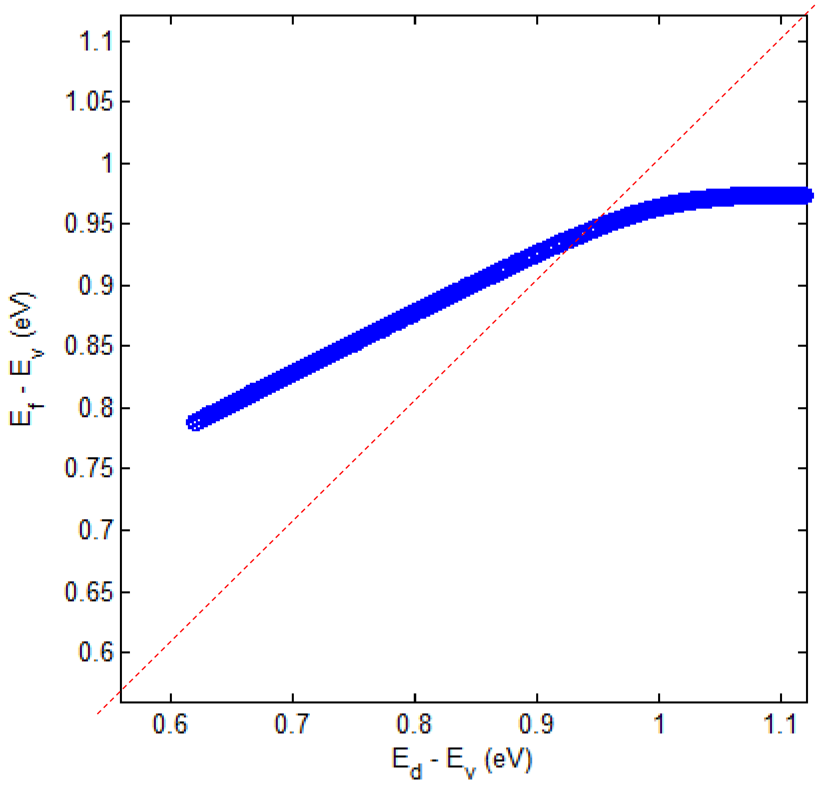
  - Hole density, $p = \frac{n_i^2}{N_d}$

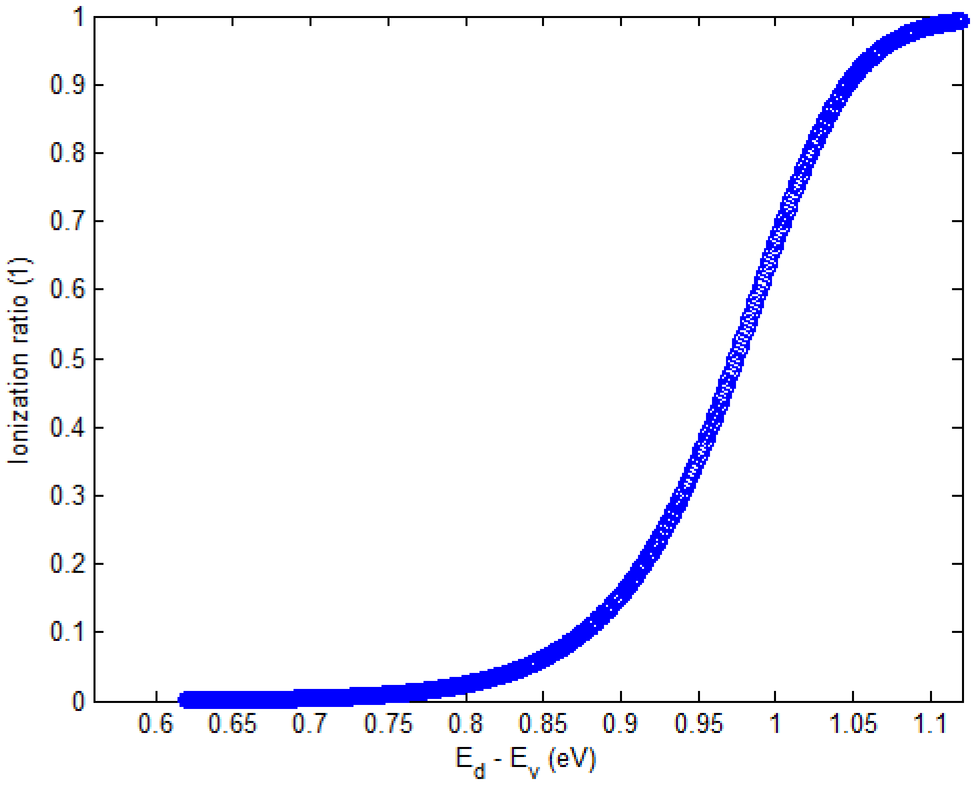
Consider a deep donor state.

	N_c (cm^{-3})	N_v (cm^{-3})
Silicon	2.8×10^{19}	1.04×10^{19}

- Assume N_d is 10^{17} cm^{-3} and T is 300 K.
  - Now, draw E_f and $1 - f_D(E_d)$ as a function of E_d.

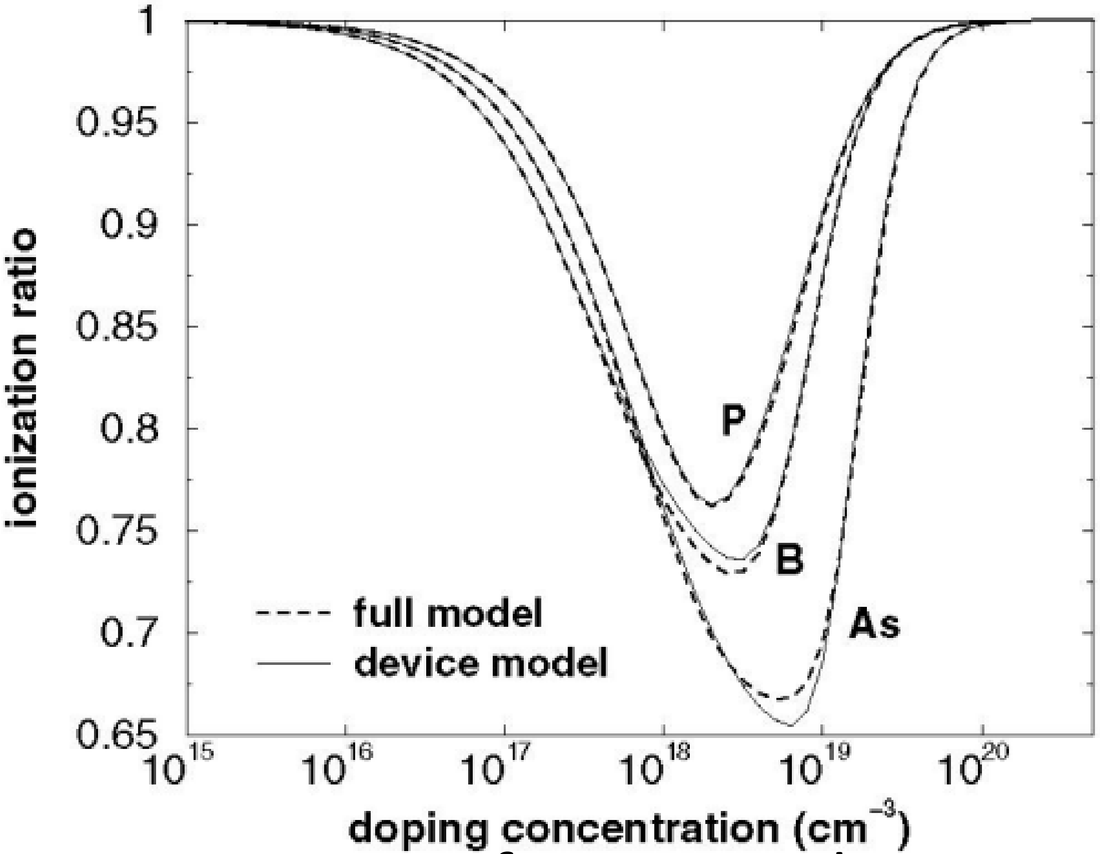
Ionization ratio



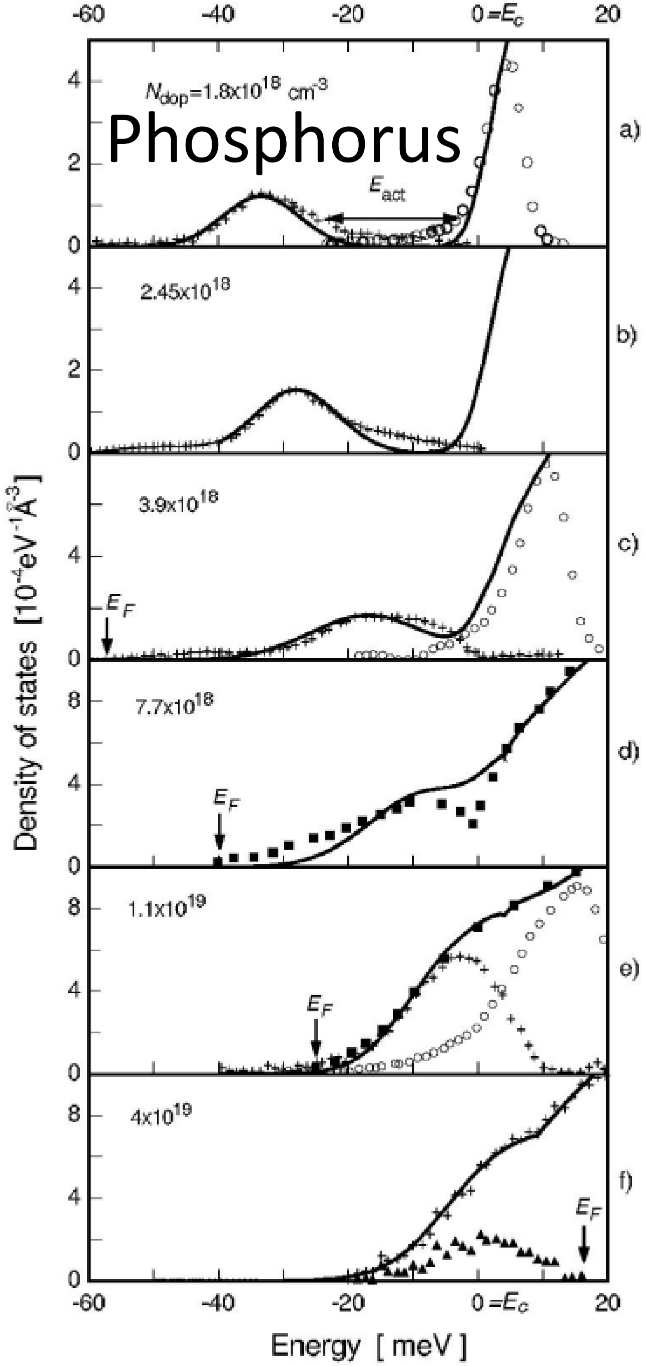


Incomplete dopant ionization

- @ high dopant densities

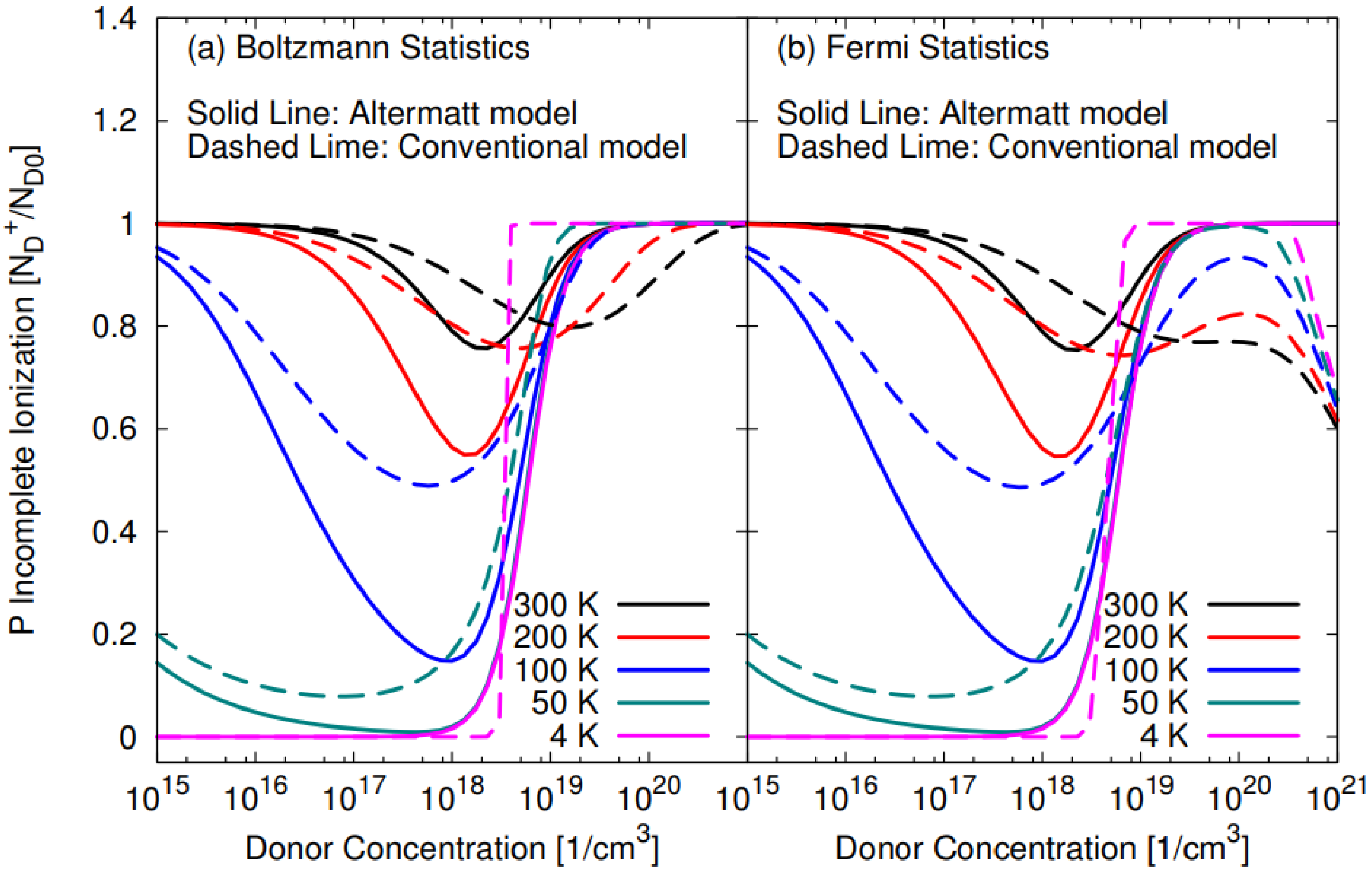


Ionization ratio for P, B, and As
(Schenk et al., SISPAD 2006)



Dopant freeze-out

- @ low temperatures



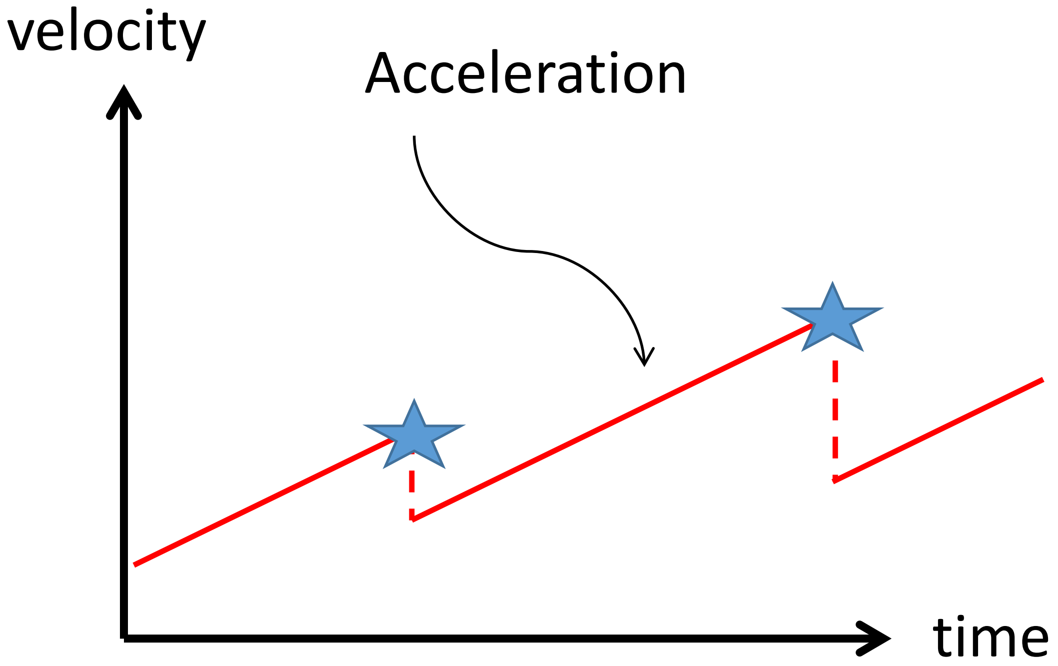
Comparison of the incomplete ionization models at various temperatures (Jin et al., SISPAD 2021)

Drift

- Net movement of charge due to an electric field
  - Since electrons/holes are charged particles, they are accelerated by an electric field. (V/cm)
  - For electrons, $\mathbf{F} = -q\mathbf{E}$ (For holes, $\mathbf{F} = +q\mathbf{E}$.)
  - According to Newton's 2nd law, the velocity satisfies $\frac{d\mathbf{v}}{dt} = -\frac{q\mathbf{E}}{m_n}$
  - Here, m_n is the conductivity effective mass of electrons.
  - Then, $\mathbf{v}(t) = \mathbf{v}(0) - \frac{q\mathbf{E}}{m_n}t$ (*Right?*)

Scattering

- The velocity of the carriers…
  - Does not increase indefinitely under the field acceleration. Why?
  - They are scattered frequently and lose the momentum.



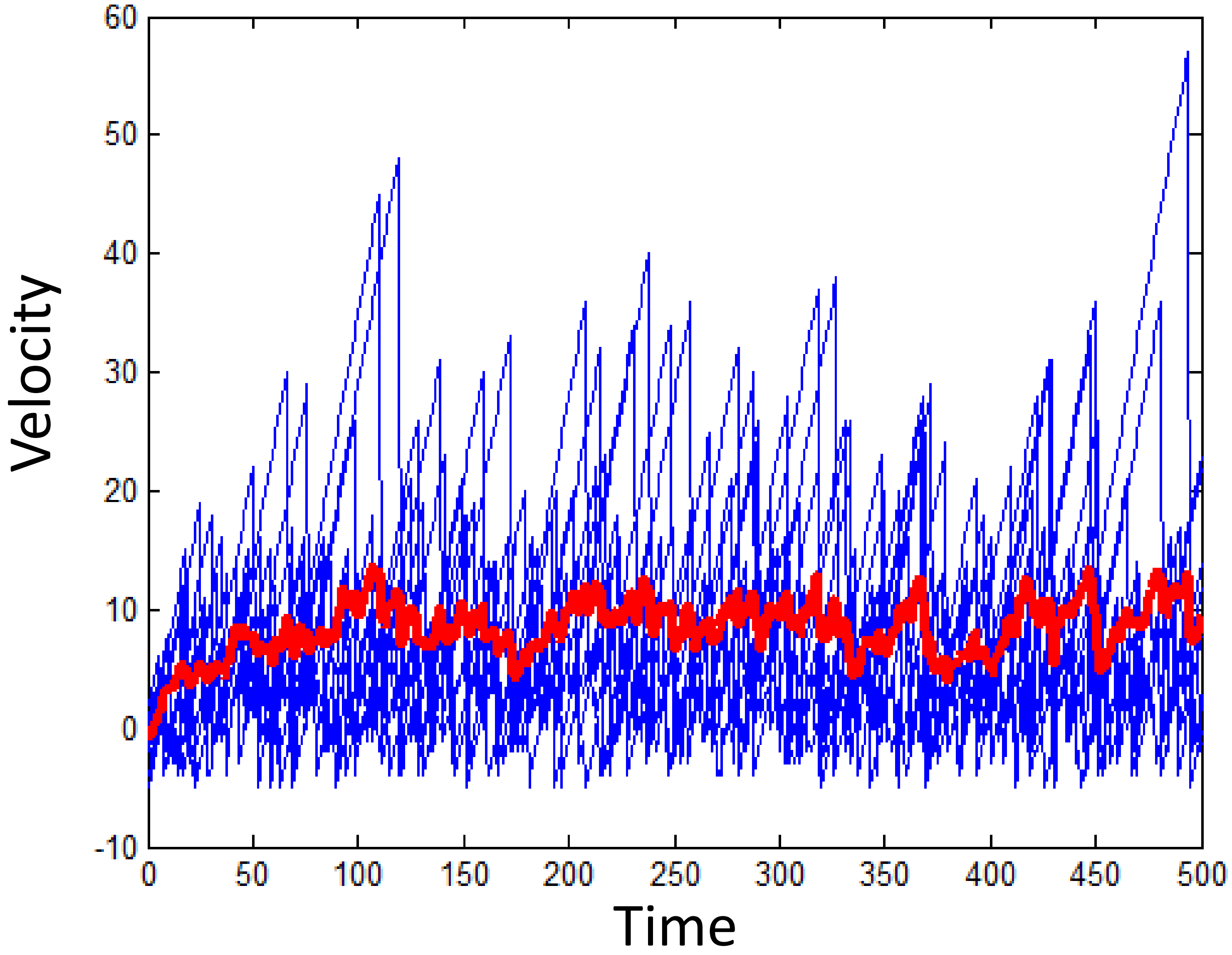
Velocity of an electron as a function of time.
When scattering is considered.

A simple game (1)

- We have ten players.
  - Initially, each of them has an integer velocity, [-5, 5]. (Uniform distribution)
  - After one time step, 1 is added to the velocity.
  - Randomly select a player. Then, the player’s velocity is again randomly distributed over [-5,5].

A simple game (2)

- A realization
  - Blue: Ten trajectories
  - Red: Average



Average velocity

- The velocity of each carrier
  - An indivisual electron exhibits sharp transitions.
- The average velocity
  - However, the average velocity follows a much smoother trajectory.
  - Therefore, it would be better to write

$$\frac{d\langle \mathbf{v} \rangle}{dt} = -\frac{q\mathbf{E}}{m_n} - \frac{\langle \mathbf{v} \rangle}{\tau_n}$$

  - Here, τ_n is the mean time between collisions.

Mobility

	μ_n	μ_p
Si	1350	480
GaAs	8500	400
Ge	3900	1900

- Mobility
  - Conduction currents are the result of the drift motion of charge carriers under the influence of an applied electric field.
  - Average drift velocity is directly proportional to the electric field intensity:

$$\langle \mathbf{v} \rangle = -\frac{q\tau_n}{m_n}\mathbf{E} = -\mu_n \mathbf{E}$$

Taur, Eq. (2.26)

Negative sign due to polarity

  - μ_n: Electron mobility in (cm^2/V/sec)
  - When the above relation is used, the drift current density becomes

$$\mathbf{J} = q\left(\mu_n n + \mu_p p\right)\mathbf{E} = \sigma\mathbf{E}$$

Taur, Eq. (2.28) and Eq. (2.30)

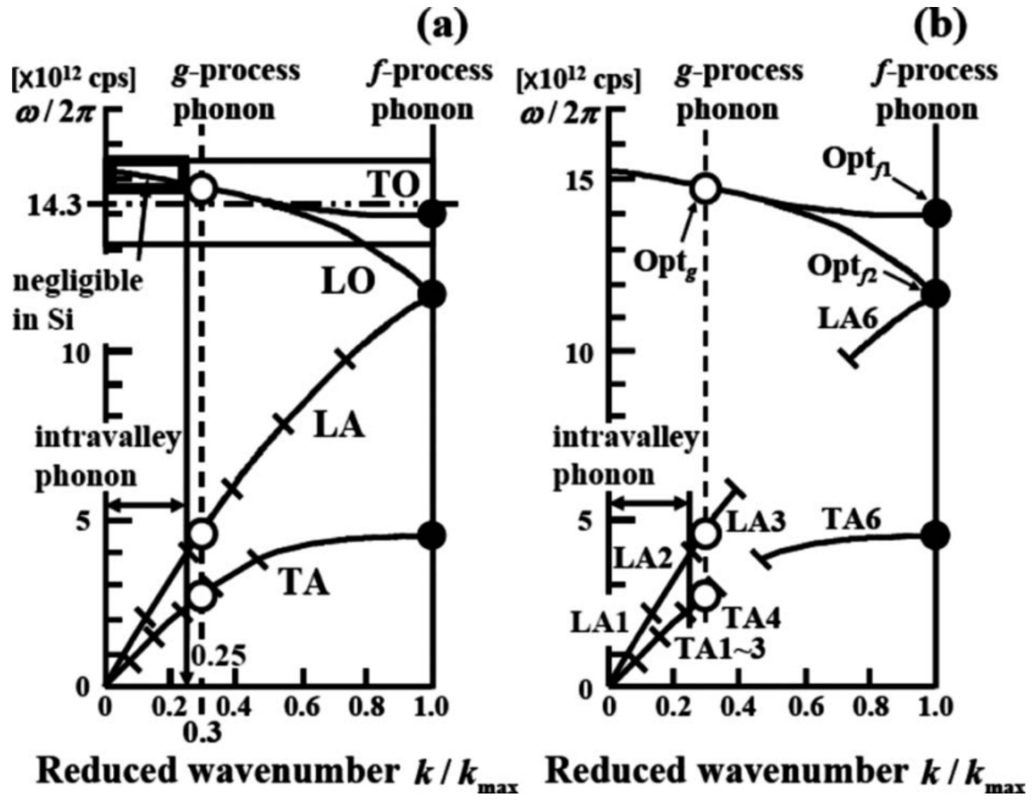
Example 2-2 of Hu's book

- Hole mobility, $\mu_p = 470\ \mathrm{cm^2V^{-1}s^{-1}}$
  - When the electric field is $10^3\ \mathrm{V\ cm^{-1}}$, the drift velocity is $4.7 \times 10^5\ \mathrm{cm\ s^{-1}}$.
  - Momentum relaxation time (with $m_p = 0.39\ m_0$) is 0.1 psec.

Phonon scattering

- Various phonon modes
  - Acoustic phonon : Low energy
  - Optical phonon : High energy, often treated as dispersion-less

~60 meV



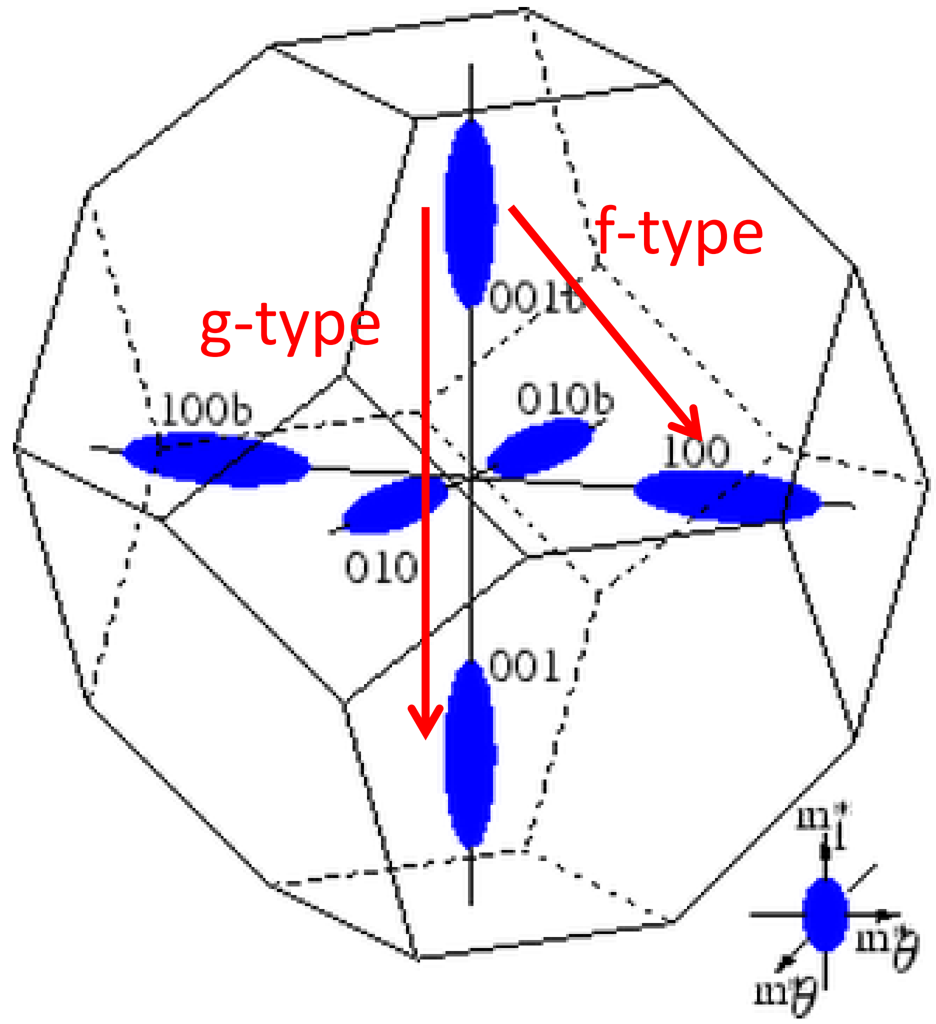
"Selection rule" matters.
Intravalley / f-process / g-process

Typical parameters

- Various phonon modes
  - Acoustic phonon : Low energy

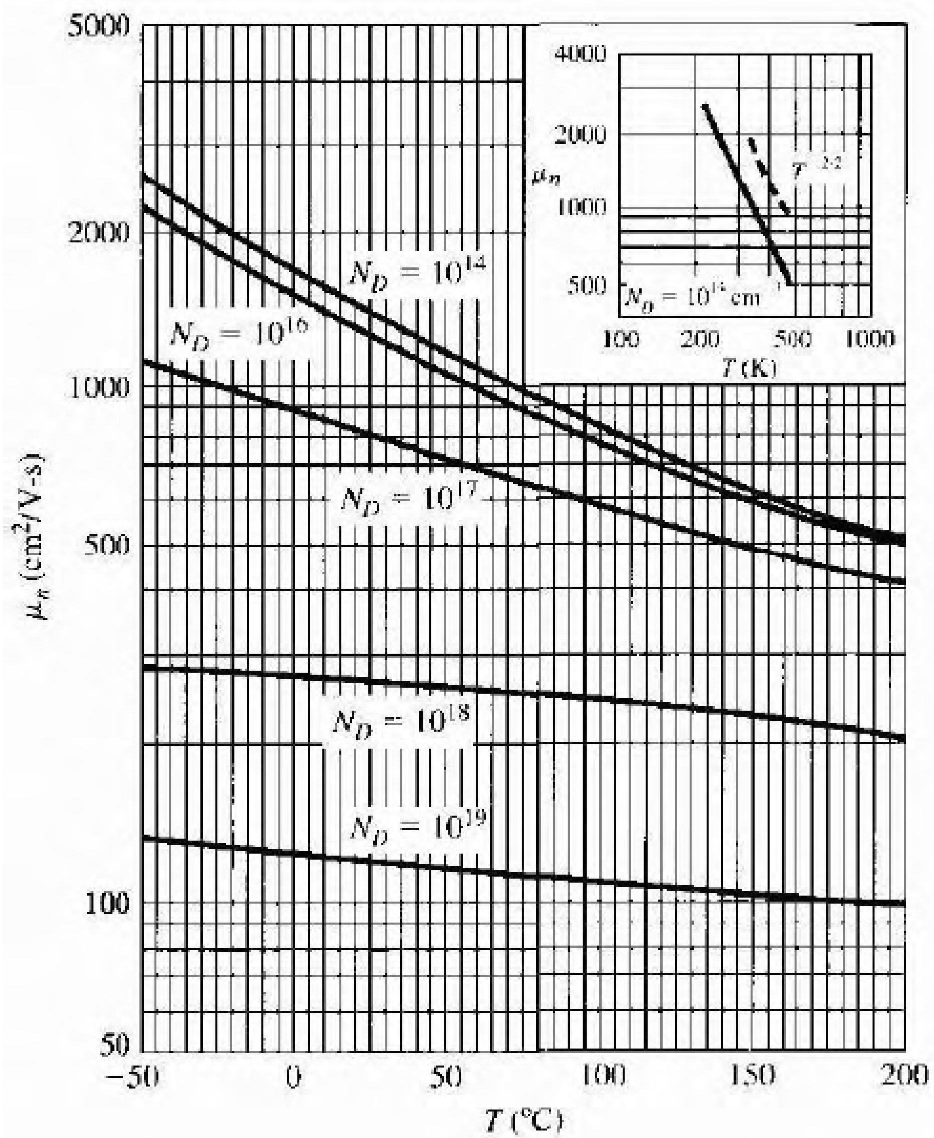
Mode	$D_t K$ (10^8 eV/cm)	$\hbar\omega$ (meV)	Type
TA	0.470	12.1	g-type
LA	0.740	18.5	g-type
LO	10.23	62.0	g-type
TA	0.280	19.0	f-type
LA	1.860	47.4	f-type
LO	1.860	58.6	f-type

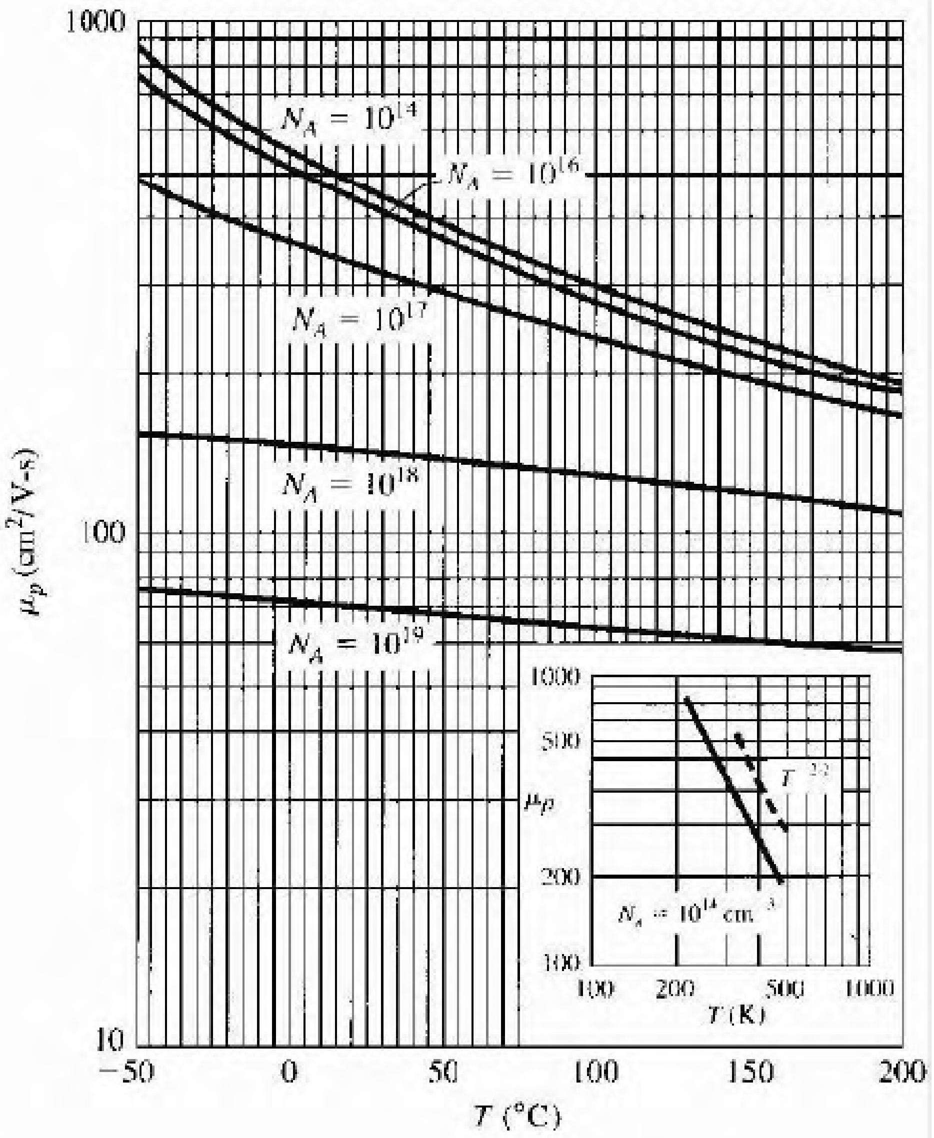
Parameters for inelastic phonon scatterings in the Si conduction band



Temperature & doping

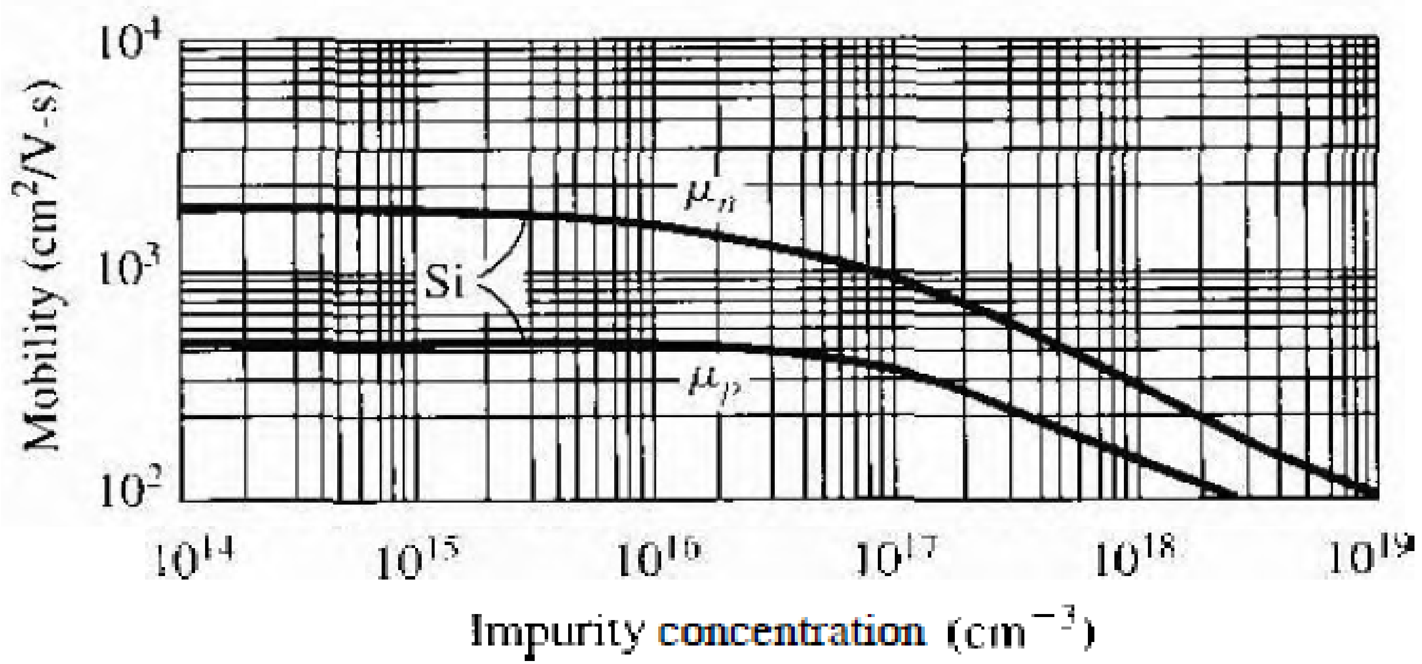
(Neamen's book)





Impurity concentration

- It is modeled as an elastic scattering process.



(Neamen's book)

- Which one is dominant? Phonon or impurity?
  - Matthiessen's rule, $\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$ Taur, Eq. (2.27)

Matthiessen's rule

- When there are multiple contributions to the mobility,
  - (For example, phonon-limited mobility / impurity-limited mobility)
  - The overall collision rate is given by sum of all contributions.
  - $\frac{1}{\tau_m} = \frac{1}{\tau_{mL}} + \frac{1}{\tau_{mI}} + \cdots$
  - (The above relation holds exactly only in the microscopic level.)
  - When recalling $\mu = \frac{q\tau_m}{m_n}$, it means $\frac{1}{\mu_m} = \frac{1}{\mu_{mL}} + \frac{1}{\mu_{mI}} + \cdots$
  - It is very useful.

Thank you!