Special Topics on Basic EECS I
VLSI Devices
Lecture 3

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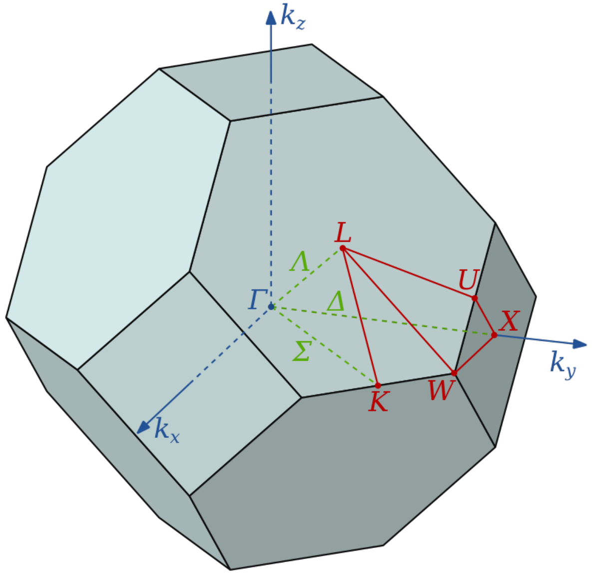
Velocity and inverse mass



- Velocity

$$\mathbf{v}(\mathbf{k}) = \frac{1}{\hbar} \nabla_k E(\mathbf{k})$$

- Inverse mass (its ij component)

$$m_{ij}^{-1} = \frac{1}{\hbar^2} \frac{\partial}{\partial k_i} \frac{\partial}{\partial k_j} E(\mathbf{k})$$

- Example) Silicon conduction band

$$E(\mathbf{k}) - E_c = \frac{\hbar^2}{2} \left(\frac{1}{m_{xx}} k_x^2 + \frac{1}{m_{yy}} k_y^2 + \frac{1}{m_{zz}} k_z^2 \right)$$ ~ Taur, Eq. (2.2)

  – Among three masses, one is m_l and the other two are m_t.

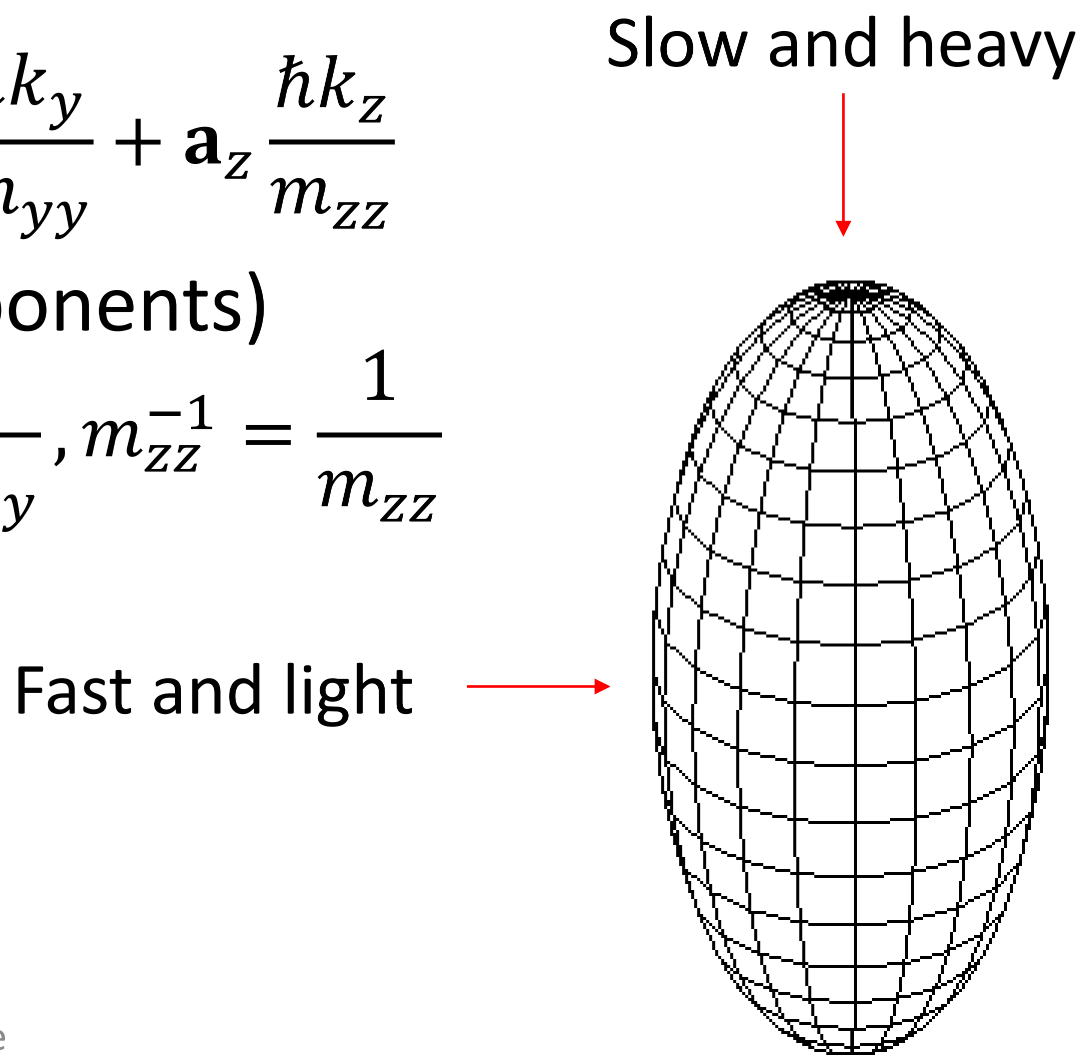
$\mathbf{v}$ and m^{-1} of an ellipsoidal valley

- Velocity

$$\mathbf{v}(\mathbf{k}) = \mathbf{a}_x \frac{\hbar k_x}{m_{xx}} + \mathbf{a}_y \frac{\hbar k_y}{m_{yy}} + \mathbf{a}_z \frac{\hbar k_z}{m_{zz}}$$

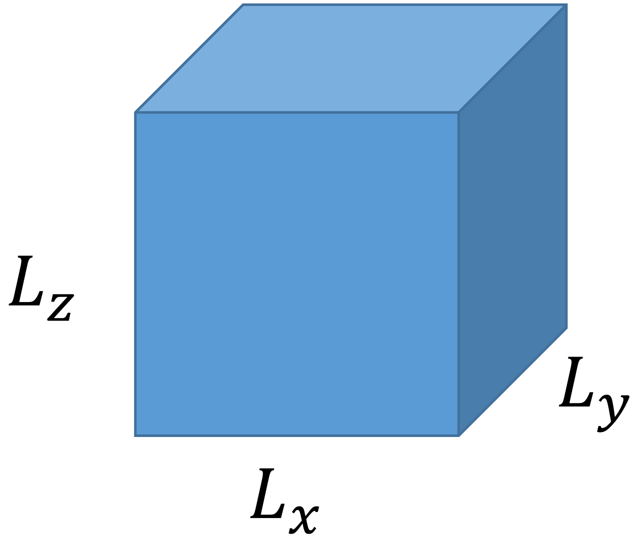
- Inverse mass (non-vanishing components)

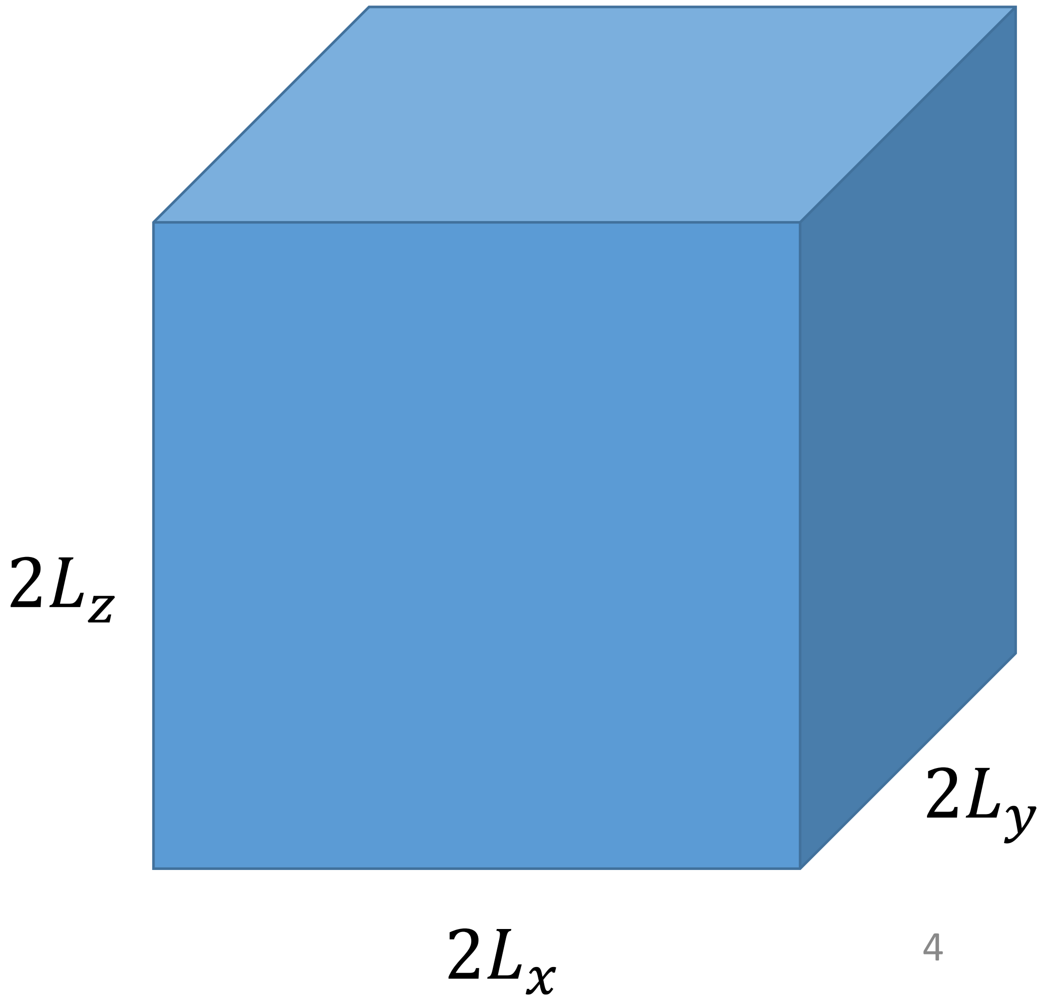
$$m_{xx}^{-1} = \frac{1}{m_{xx}}, m_{yy}^{-1} = \frac{1}{m_{yy}}, m_{zz}^{-1} = \frac{1}{m_{zz}}$$



Volume of a state in the $\mathbf{k}$-space

- A (discrete) $\mathbf{k}$ point corresponds to an electronic state.
  - One state within $\frac{(2\pi)^3}{L_x L_y L_z}$ (Left)
  - One state within $\frac{(2\pi)^3}{8 L_x L_y L_z}$ (Right)
  - In general, one state within $\frac{(2\pi)^3}{Volume}$





Number of states inside $dk_x dk_y dk_z$

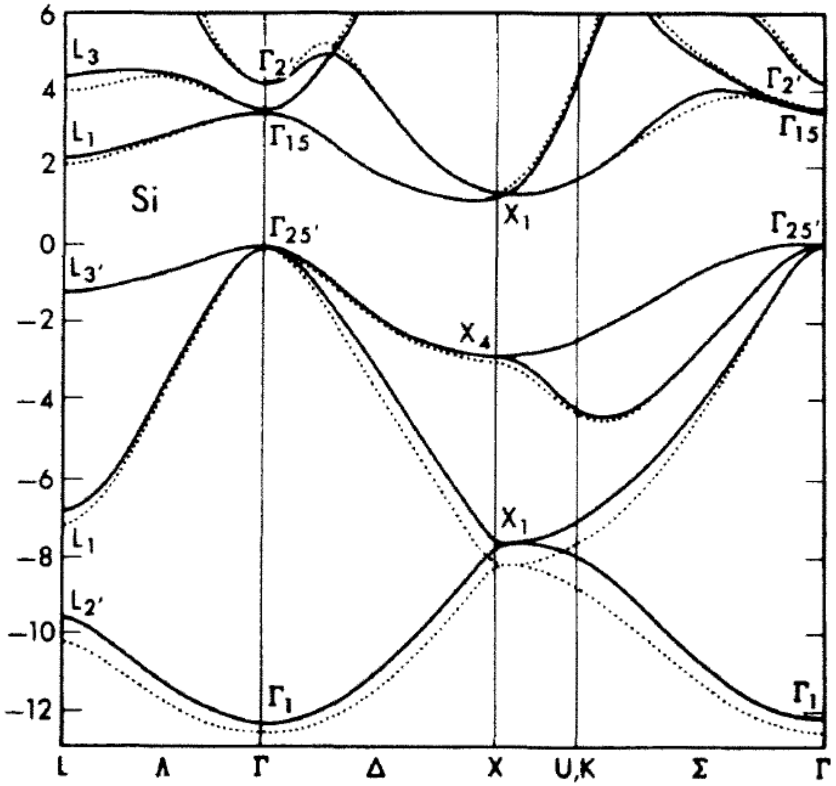
- Since a state takes $\frac{(2\pi)^3}{Volume}$,
  - Number of states inside $dk_x dk_y dk_z$ is
$$\frac{Volume}{(2\pi)^3} dk_x dk_y dk_z$$
  - Number of states inside a range of $[E, E + dE]$,
$$\frac{Volume}{(2\pi)^3} \iiint_{E(\mathbf{k}) \in [E, E+dE]} dk_x dk_y dk_z$$

Density-of-states (DOS)

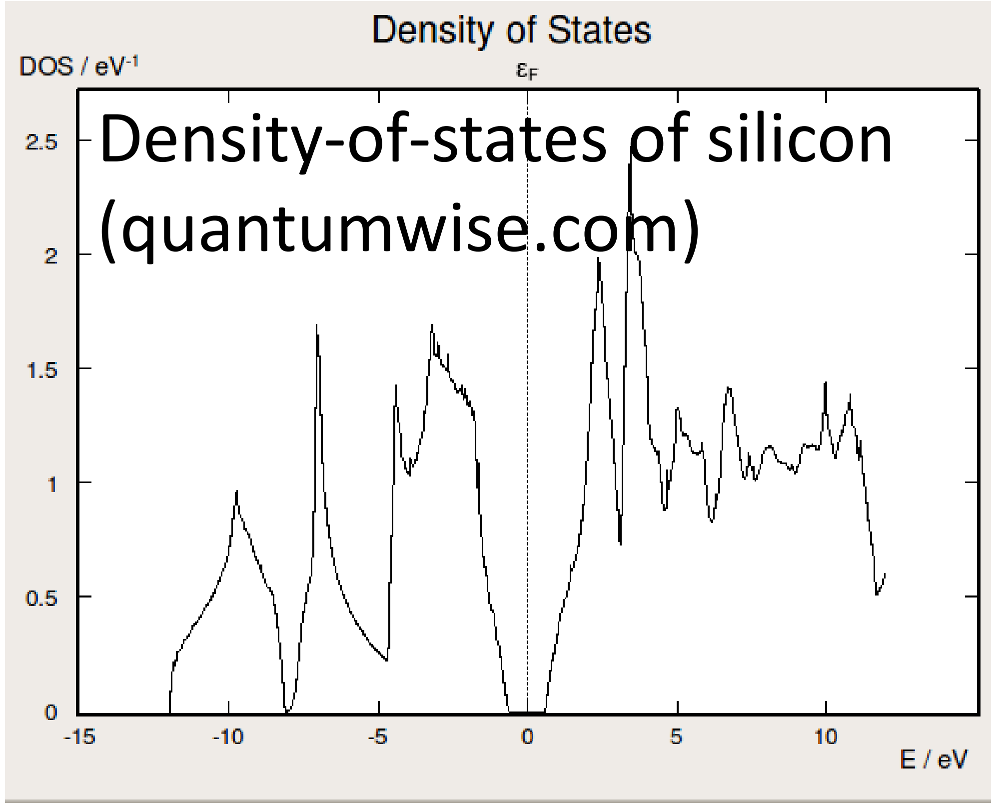
- DOS, $N(E)$, (per spin, per valley)

$$N(E)dE = \frac{1}{(2\pi)^3} \iiint_{E(\mathbf{k})\in(E,E+dE)} dk_x dk_y dk_z$$

~ Taur, Eq. (2.1)



GIST Lecture



Density-of-states of silicon (quantumwise.com)

Density-of-states (DOS) of an ellipsoidal valley

- Volume in the $\mathbf{k}$-space
  - With $m^* = \left(m_{xx}m_{yy}m_{zz}\right)^{\frac{1}{3}}$,

$$\frac{4\pi}{3}\left(\frac{1}{\hbar}\right)^3 (2m^*)^{1.5}(E-E_c)^{1.5}$$

  - Therefore, within a range between $E-E_c$ and $E-E_c+dE$,

$$4\pi\left(\frac{1}{\hbar}\right)^3 \left(2m_{xx}m_{yy}m_{zz}\right)^{0.5}(E-E_c)^{0.5}dE$$

- DOS of silicon conduction band (per spin, per valley)

$$N(E)dE = \frac{4\pi}{h^3}(2m_l m_t^2)^{0.5}(E-E_c)^{0.5}dE$$

~ Taur, Eq. (2.3)

Homework#1

- Non-parabolicity, α
  - Consider an isotropic valley,

$$E(1+\alpha E)=\frac{\hbar^2}{2m^*}k^2$$

  - For this valley, express the velocity, the inverse mass, and the DOS using E.

Electron density

0, when empty
1, when occupied

- Number of electrons

$$\# = \sum_{\substack{all\ \color{red}{occupied} \\ \mathbf{k}\ states}} 1 = \sum_{all\ \mathbf{k}\ states} f(\mathbf{k})$$

  – Instead of a sum,

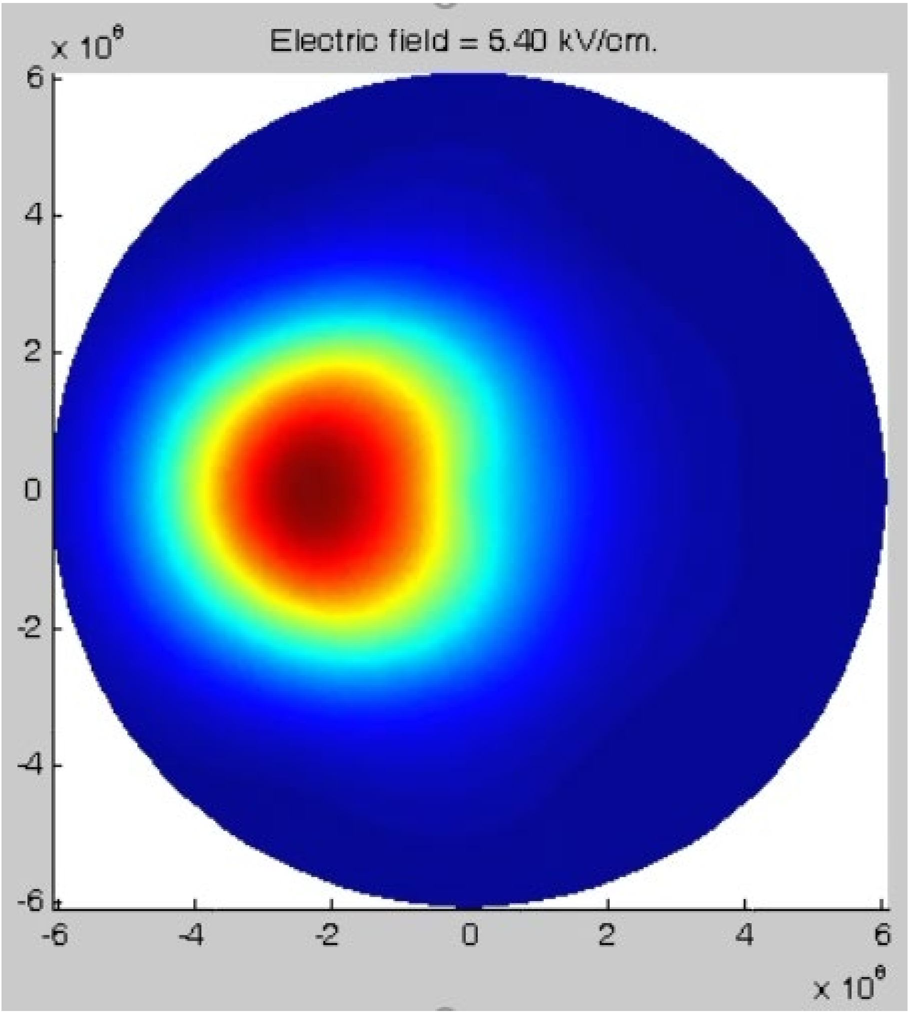
$$\# = \sum_{all\ \mathbf{k}\ states} f(\mathbf{k}) \approx \frac{Volume}{(2\pi)^3} \iiint_{\substack{Entire \\ \mathbf{k}\ space}} f(\mathbf{k}) dk_x dk_y dk_z$$

  – Electron density (per spin, per valley)

$$n = \frac{\#}{Volume} = \frac{1}{(2\pi)^3} \iiint_{\substack{Entire \\ \mathbf{k}\ space}} f(\mathbf{k}) dk_x dk_y dk_z$$

Distribution function

- $f(\mathbf{k})$ is the distribution function.
  - It is 0, when the state is completely empty.
  - It is 1, when the state is fully occupied.
  - It is in a range of [0,1].
  - In general, it is a function of $\mathbf{k}$.



Distribution function of
graphene at a high electric field

Thank you!