

Special Topics on Basic EECS I

VLSI Devices

Lecture 4

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A special case

- Sometimes, $f(\mathbf{k})$ depends on only the energy, $f(E)$.
 - In such a case, the electron density can be written as

$$n = \frac{1}{(2\pi)^3} \iiint_{\substack{\text{Entire} \\ \mathbf{k} \text{ space}}} f(\mathbf{k}) dk_x dk_y dk_z = \int_{E_c}^{\infty} N(E) f(E) dE$$

~ Taur, Eq. (2.7)

- When do we have $f(E)$, instead of $f(\mathbf{k})$?
 - The equilibrium state is a typical example.

Fermi-Dirac distribution

- At equilibrium, the Fermi-Dirac distribution holds

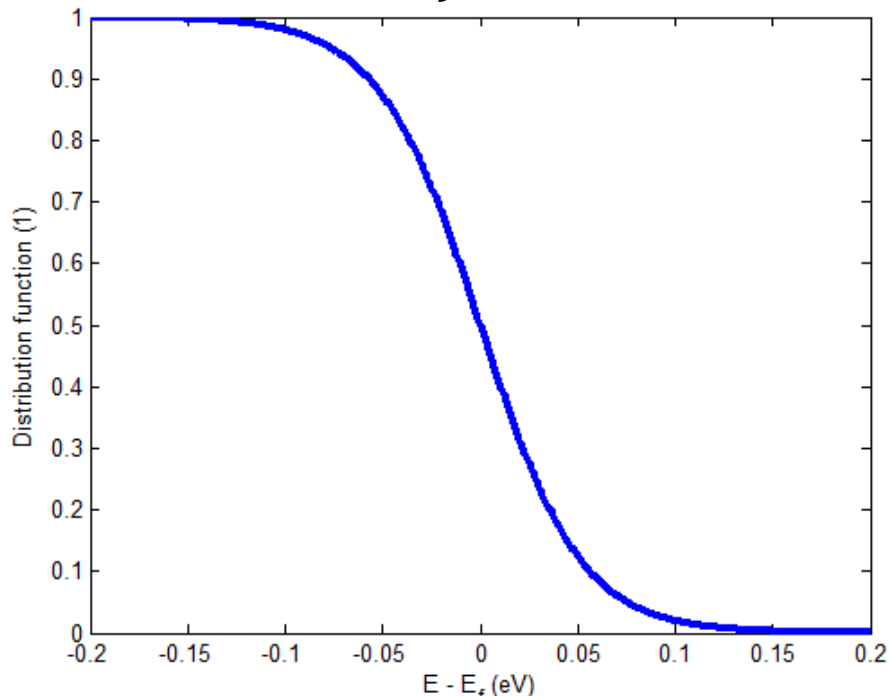
$$f_D(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{k_B T}\right)}$$

Taur, Eq. (2.4)

– At $E = E_f$ (Fermi level),

$$f_D(E_f) = \frac{1}{2}$$

~ 0.02585 meV
@ 300 K

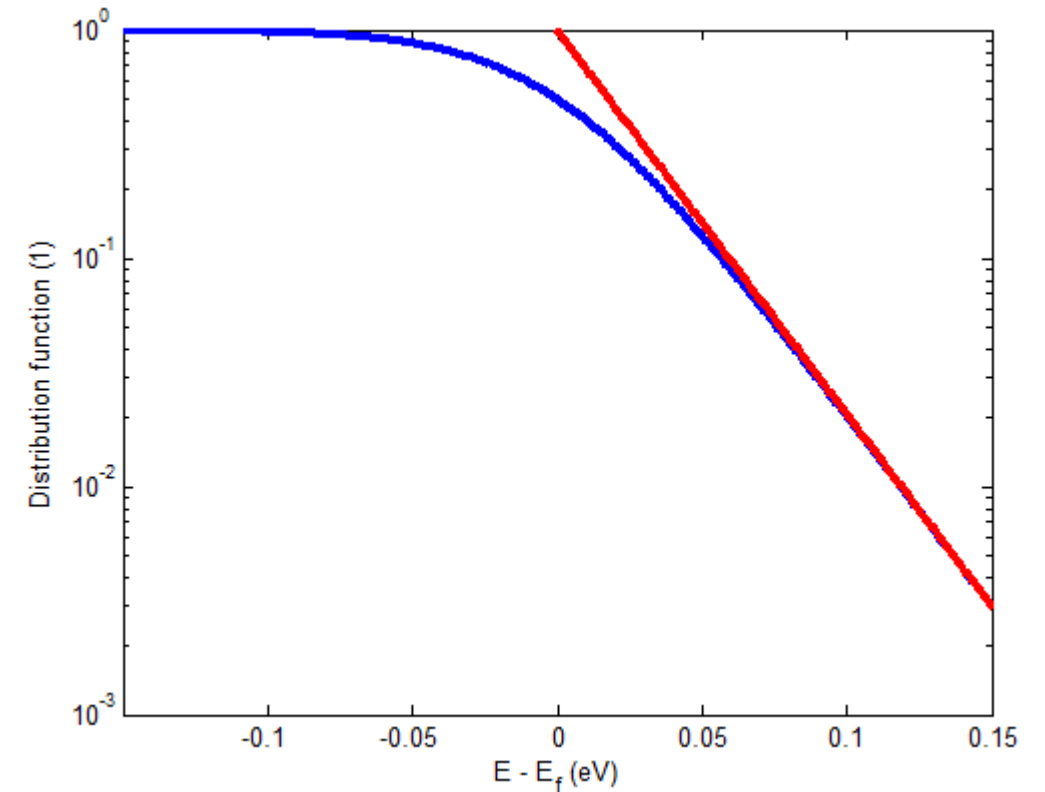
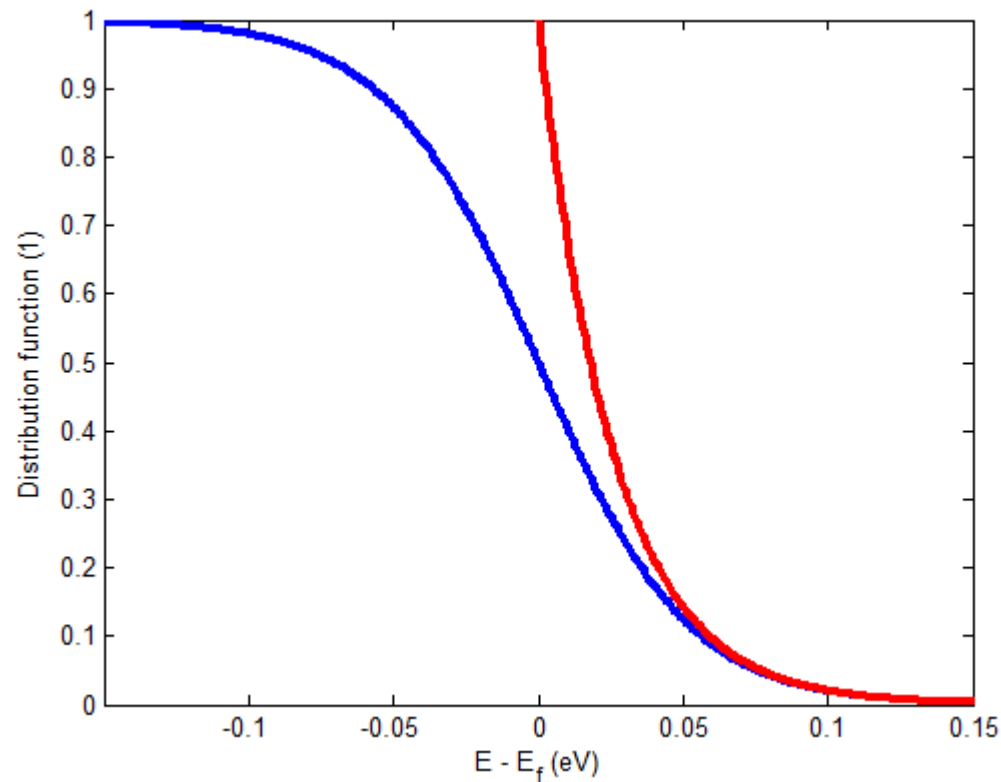


Boltzmann limit

- When $E > E_f$,

$$f_D(E) \approx \exp\left(-\frac{E - E_f}{k_B T}\right)$$

Taur, Eq. (2.5)

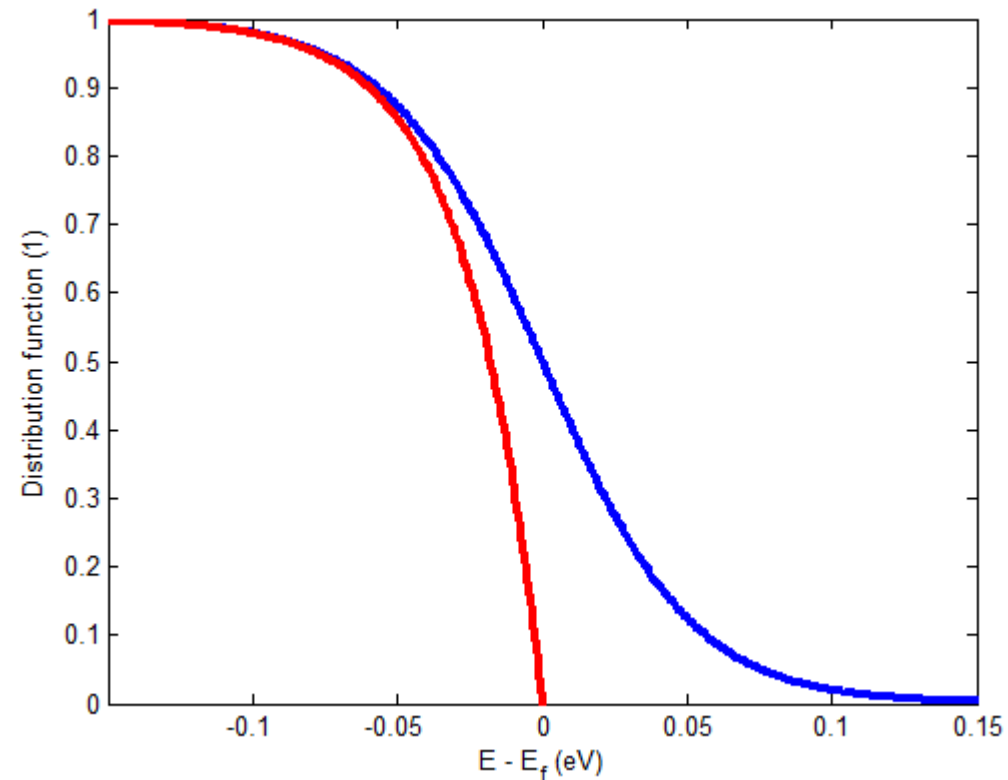


Another Boltzmann limit

- When $E < E_f$,

$$f_D(E) \approx 1 - \exp\left(\frac{E - E_f}{k_B T}\right)$$

Taur, Eq. (2.6)



Carrier concentration (Electron)

- Recall that

Spin and valley
degeneracy

$$n = \int_{E_c}^{\infty} N(E) f(E) dE$$
$$N(E) = 2g \frac{4\pi}{h^3} (2m_l m_t^2)^{0.5} (E - E_c)^{0.5}$$
$$f_D(E) = \exp\left(-\frac{E - E_f}{k_B T}\right)$$

– Collecting them all,

$$n = \frac{8\pi g}{h^3} (2m_l m_t^2)^{0.5} \int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_f}{k_B T}\right) dE$$

Taur, Eq. (2.8)

Manipulation

- It is found that

$$\begin{aligned} n &= \frac{8\pi g}{h_{\infty}^3} (2m_l m_t^2)^{0.5} \exp\left(-\frac{E_c - E_f}{k_B T}\right) \\ &\times \int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_c}{k_B T}\right) dE \end{aligned}$$

– Integral can be evaluated as

$$\begin{aligned} \int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_c}{k_B T}\right) dE &= (k_B T)^{1.5} \int_0^{\infty} z^{0.5} \exp(-z) dz \\ &= (k_B T)^{1.5} \frac{\sqrt{\pi}}{2} \end{aligned}$$

Effective DOS

	$N_c \text{ (cm}^{-3}\text{)}$	$N_v \text{ (cm}^{-3}\text{)}$
Silicon	2.8×10^{19}	1.04×10^{19}
Gallium arsenide	4.7×10^{17}	7.0×10^{18}
Germanium	1.04×10^{19}	6.0×10^{18}

- Now we know that

$$n = 2g \left(\frac{2\pi k_B T}{h^2} \right)^{1.5} (m_l m_t^2)^{0.5} \exp \left(-\frac{E_c - E_f}{k_B T} \right)$$

N_c and N_v
@ 300 K
(Hu's book)

- With the effective DOS,

Dimension? $\longrightarrow N_c = 2g \left(\frac{2\pi k_B T}{h^2} \right)^{1.5} (m_l m_t^2)^{0.5}$

Taur, Eq. (2.10)

- The electron density can be simply written as

$$n = N_c \exp \left(-\frac{E_c - E_f}{k_B T} \right)$$

Taur, Eq. (2.9)

- Following a similar derivation, $p = N_v \exp \left(\frac{E_v - E_f}{k_B T} \right)$

Taur, Eq. (2.11)

Intrinsic carrier concentration

- In this case, $n = p$. Then, what is E_f ?

$$N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$$

- From the above equation,

$$E_f = \frac{E_c + E_v}{2} - \frac{k_B T}{2} \ln \frac{N_c}{N_v}$$

Taur, Eq. (2.12)

- This energy level is called the intrinsic Fermi level, E_i .

- In this case,

$$n = p = n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_c - E_v}{k_B T}\right)$$

Taur, Eq. (2.13)

Its temperature dependence

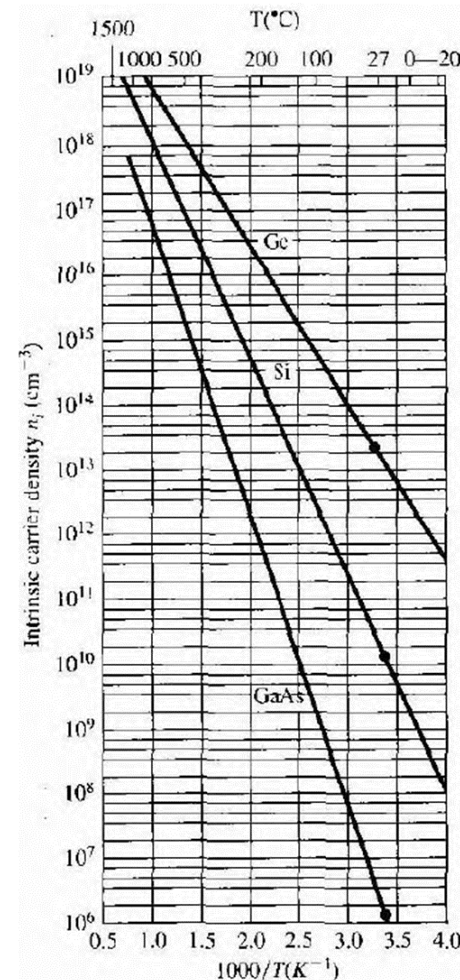
- Recall that $N_c = 2g \left(\frac{2\pi k_B T}{h^2} \right)^{1.5} (m_l m_t^2)^{0.5}$. (N_v has a similar form.)

– Therefore,

$$n_i = \sqrt{N_c N_v} \exp \left(-\frac{E_c - E_v}{k_B T} \right)$$

It is dominant.

$T^{1.5}$, but it is not dominant.



Intrinsic carrier density
(Neamen's book)

Using the intrinsic carrier density,

- Carrier densities are expressed as

$$n = n_i \exp\left(-\frac{E_i - E_f}{k_B T}\right) \quad \text{Taur, Eq. (2.14)}$$

$$p = n_i \exp\left(\frac{E_i - E_f}{k_B T}\right) \quad \text{Taur, Eq. (2.15)}$$

- A useful, general relationship is that the product

$$np = n_i^2 \quad \text{Taur, Eq. (2.16)}$$

in equilibrium is a constant, independent of the Fermi level position.

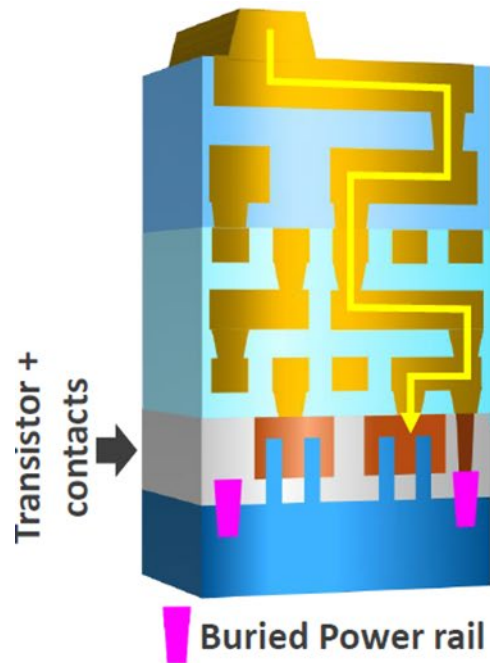
Recall that

- We have 7×10^{23} electrons/cm³ in Si.
 - At 300 K, only $\sim 1.4 \times 10^{10}$ electrons/cm³ can be found in the conduction band. Only a single electron among 5×10^{13} electrons occupies the conduction band.
 - There are 4 moonwalkers among 8.1×10^9 people.

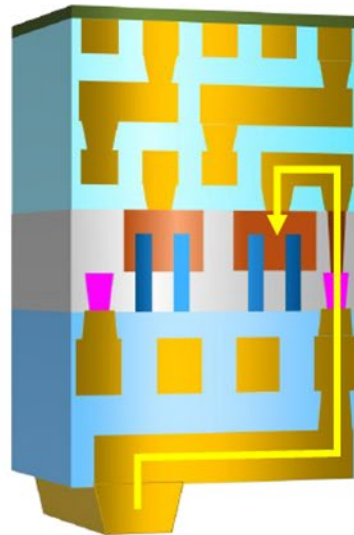


Copper (A good conductor)

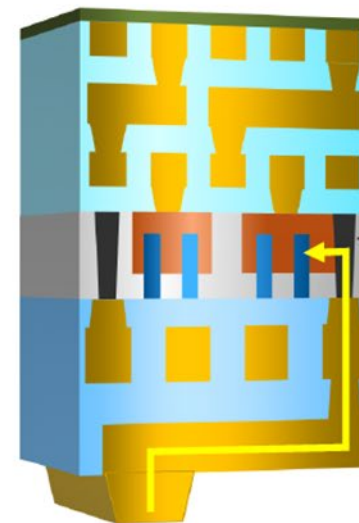
- How many conduction electrons in 1 cm^3 ?
 - Cu: $\sim 8.5 \times 10^{22} \text{ cm}^{-3}$
 - Best for interconnect



Buried Power Rail
without BS-PDN



Buried Power Rail
with BS-PDN



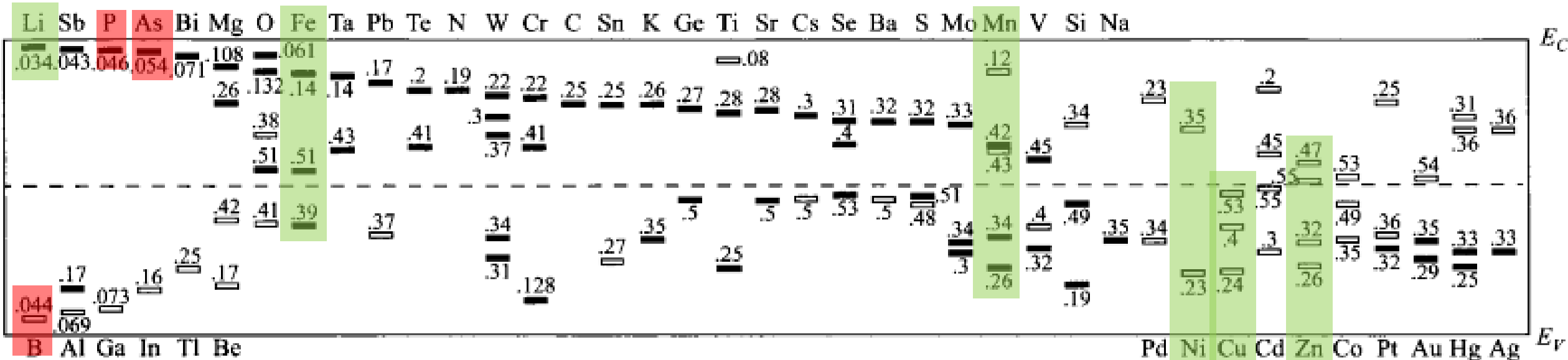
Intel
PowerVia

Various ways to
supply the power
to transistors
(Intel, VLSI 2023)

Dopants

- 5×10^{20} impurities / cm^3 is 1 % of Si.
 - Find As, P, and B.
 - Find Fe, Cu, Li, Zn, Mn, and Ni. (Undesirable)

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	* 71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	* 103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
			* 57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb		
			* 89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No		



Thank you!