Special Topics on Basic EECS I
VLSI Devices
Lecture 16

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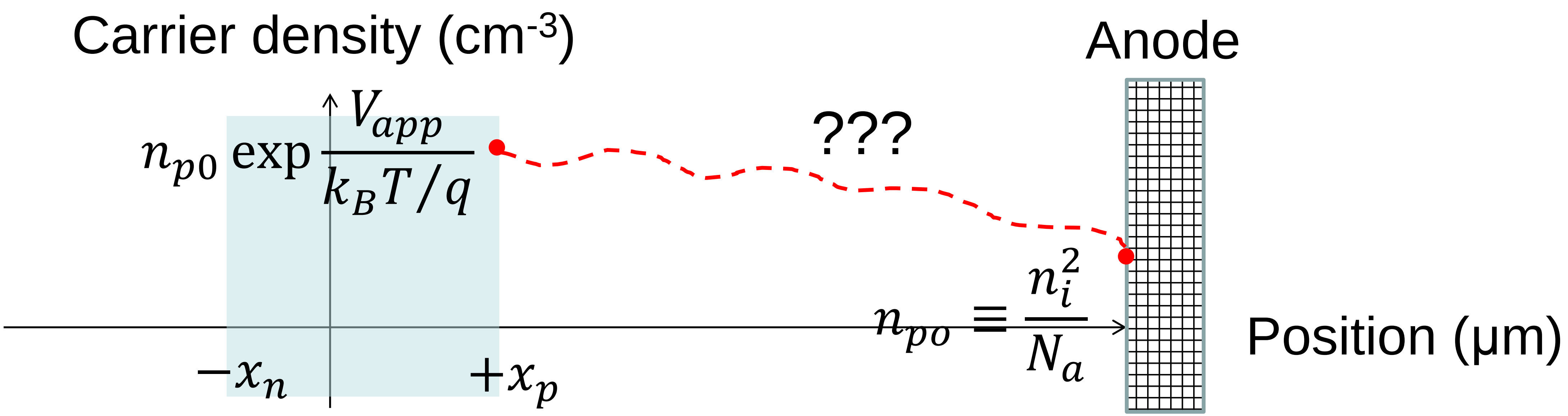
Problem simplified

- With the law of junction, we have a simplified problem.
  - Boundary densities are fixed.
  - Electron density profile in the p-type region?



Electron continuity in p-type

- We must solve the electron continuity equation.

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}J_n - R_n + G_n \qquad \text{Taur, Eq. (2.109)}$$

  - In the p-type region, the electric field is weak.

$$\frac{\partial n}{\partial t} = D_n \frac{\partial^2}{\partial x^2}n - \frac{n - n_{p0}}{\tau_n} \qquad \text{Taur, Eq. (2.113)}$$

  - At steady state,

$$\frac{d^2}{dx^2}n = \frac{n - n_{p0}}{D_n \tau_n} = \frac{n - n_{p0}}{L_n^2} \qquad \text{Taur, Eq. (2.115)}$$

Electron diffusion length

Solution of diffusion equation

- Boundary values

$$n(x_p) = n_{p0} \exp \frac{V_{app}}{k_B T/q}$$

$$n(\infty) = n_{p0}$$

  – The solution is obtained as

$$n(x) = n_{p0}\left(\exp\frac{V_{app}}{V_T} - 1\right)\exp\left(-\frac{x - x_p}{L_n}\right) + n_{p0}$$

(Our textbook considers a finite thickness of the quasineutral region.)

Finite thickness, W_p

- In our textbook (Taur), the quasineutral region starts at $x = 0$.

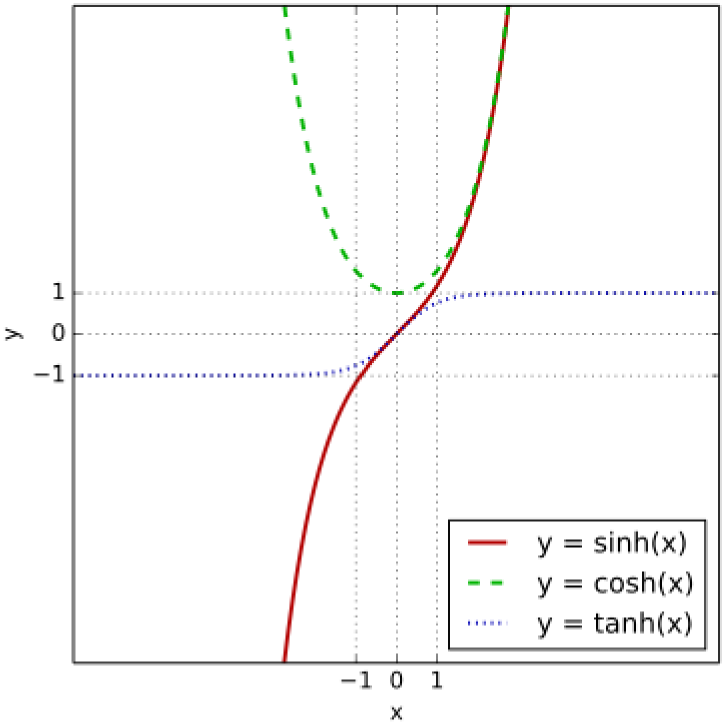
$$n(0) = n_{p0} \exp \frac{V_{app}}{k_B T/q}$$

$$n(W_p) = n_{p0}$$

  - The solution is obtained as

$$n(x) = n_{p0} \left(\exp \frac{V_{app}}{V_T} - 1 \right) \frac{\sinh\left(\frac{W_p - x}{L_n}\right)}{\sinh(W_p/L_n)} + n_{p0}$$

Taur, Eq. (2.119)



Hyperbolic functions (Wikipedia)

Electron current density for finite thickness

- The electron current density at x, $J_n(x)$, is found as

$$J_n(x) = qD_n\frac{dn}{dx} = -q\frac{D_n}{L_n}n_{p0}\left(\exp\frac{V_{app}}{k_BT/q} - 1\right)\frac{\cosh\left(\frac{W_p - x}{L_n}\right)}{\sinh\left(W_p/L_n\right)}$$

  – Then, at $x = 0$,

$$J_n(0) = -q\frac{D_n}{L_n}n_{p0}\left(\exp\frac{V_{app}}{k_BT/q} - 1\right)\frac{1}{\tanh\left(W_p/L_n\right)}$$

Taur, Eq. (2.120)

  – When $W_p \to \infty$,

$$J_n(0) = -q\frac{D_n}{L_n}n_{p0}\left(\exp\frac{V_{app}}{k_BT/q} - 1\right)$$

Electron flux

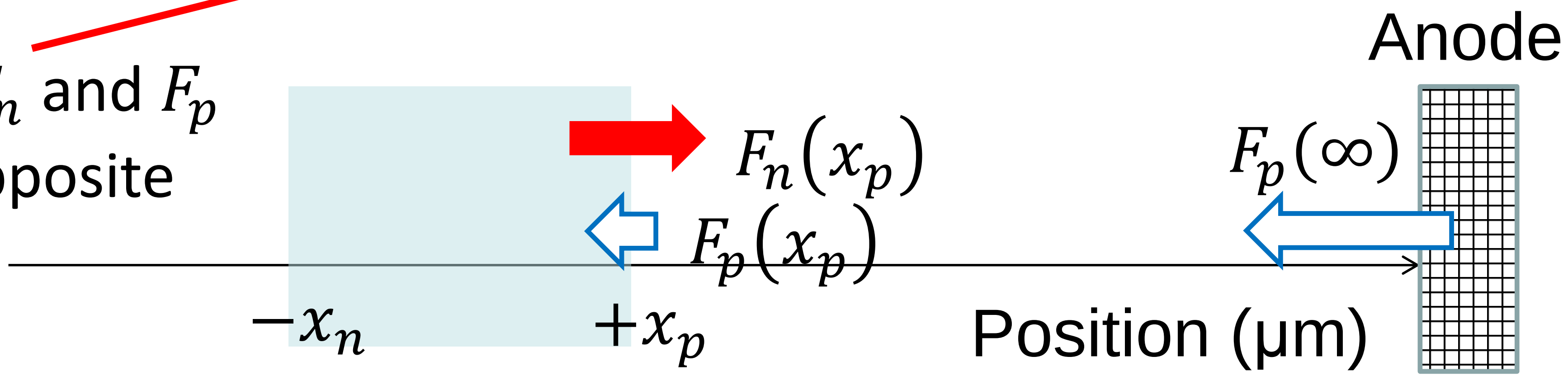
- The electron flux, $F_n(x)$, is found as

$$F_n(x) = -D_n \frac{dn}{dx} = \frac{D_n}{L_n} n_{p0} \left(\exp \frac{V_{app}}{k_B T/q} - 1 \right) \exp\left(-\frac{x - x_p}{L_n} \right)$$

  - $F_n(x)$ is non-uniform. The electron flux decreases as x increases.
  - $F_n(\infty)$ vanishes. All injected electrons are recombined.

$$F_p(\infty) = F_n(x_p) + F_p(x_p)$$

Note that F_n and F_p have the opposite directions.



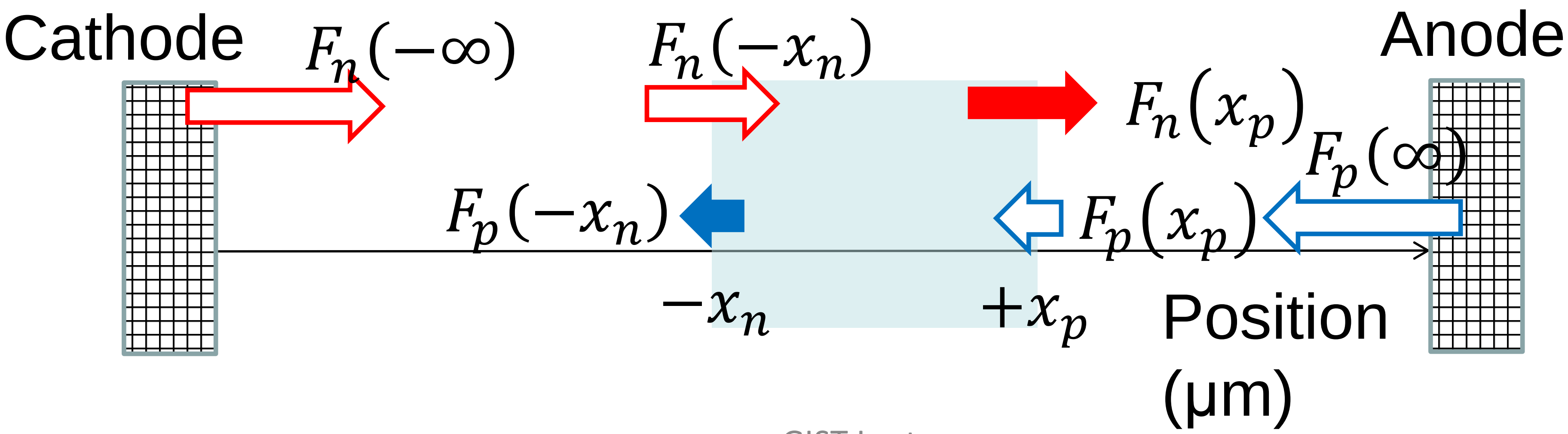
Whole structure

- Similarly, the hole flux can be treated.

$$F_p(\infty) = F_n(x_p) + F_p(x_p)$$
$$F_n(-\infty) = F_n(-x_n) + F_p(-x_n)$$

  - Let us ***assume*** that $F_n(-x_n) = F_n(x_p)$ and $F_p(x_p) = F_p(-x_n)$. Then,

$$F_p(\infty) = F_n(-\infty) = F_n(x_p) + F_p(-x_n)$$



Under such an assumption,

- The total current flowing through pn diode is the sum of the electron current on the p-side and the hole current on the n-side.
  - The diode current density is

$$J_{diode} = -\left[\frac{qD_n n_{p0}}{L_n \tanh\left(W_p/L_n\right)} + \frac{qD_p p_{n0}}{L_p \tanh\left(W_n/L_p\right)}\right]\left(\exp\frac{V_{app}}{k_B T/q} - 1\right)$$

Taur, Eq. (2.121)

Draw Figure 2.20 of Taur's book

- Long diode with N_d = N_a = 10^{17} cm^{-3}
  - The diode current density is

$$J_{diode} = -\left[\frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}\right]\left(\exp\frac{V_{app}}{k_B T/q} - 1\right)$$

  - Parameters are:

μ_n = 763 cm^2/V sec

μ_p = 474 cm^2/V sec

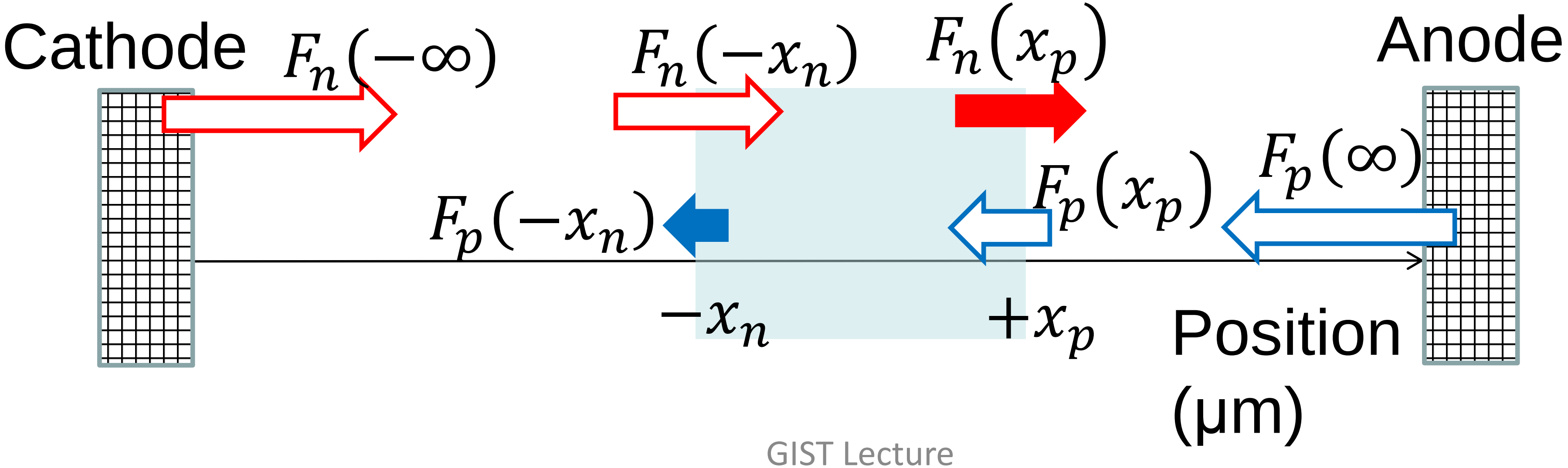
τ_n = 2.89 µsec

τ_p = 12.5 µsec

Recombination

- When the recombination in the depletion region is considered, we can find the following relation:

$$F_p(\infty) = F_n(-\infty) = F_n(x_p) + F_p(-x_n) + \int_{-x_n}^{x_p} [R(x) - G(x)]dx$$



Approximate expression

- We can estimate the upper bound.

$$\int_{-x_n}^{x_p} [R(x) - G(x)]dx < [R(x) - G(x)]_{maximum}\underbrace{(x_p + x_n)}_{W_d}$$

  - Assume that $R - G = CN_t \frac{np-n_i^2}{n+p+2n_i}$ for the SRH centers.
  - Also, in the depletion region, $np = n_i^2 \exp\frac{V_{app}}{k_B T/q}$. (Why?)
  - Then, we have the maximum value

$$[R(x) - G(x)]_{maximum} = CN_t \frac{n_i}{2}\left(\exp\frac{V_{app}}{2\,k_B T/q} - 1\right)$$

$$= \frac{n_i}{2\tau_i}\left(\exp\frac{V_{app}}{2\,k_B T/q} - 1\right)$$

Current

- By using the previous results, the current can be obtained.

$$I_{total} = I_{diode} + I_{SC}$$ Taur, Eq. (2.138)

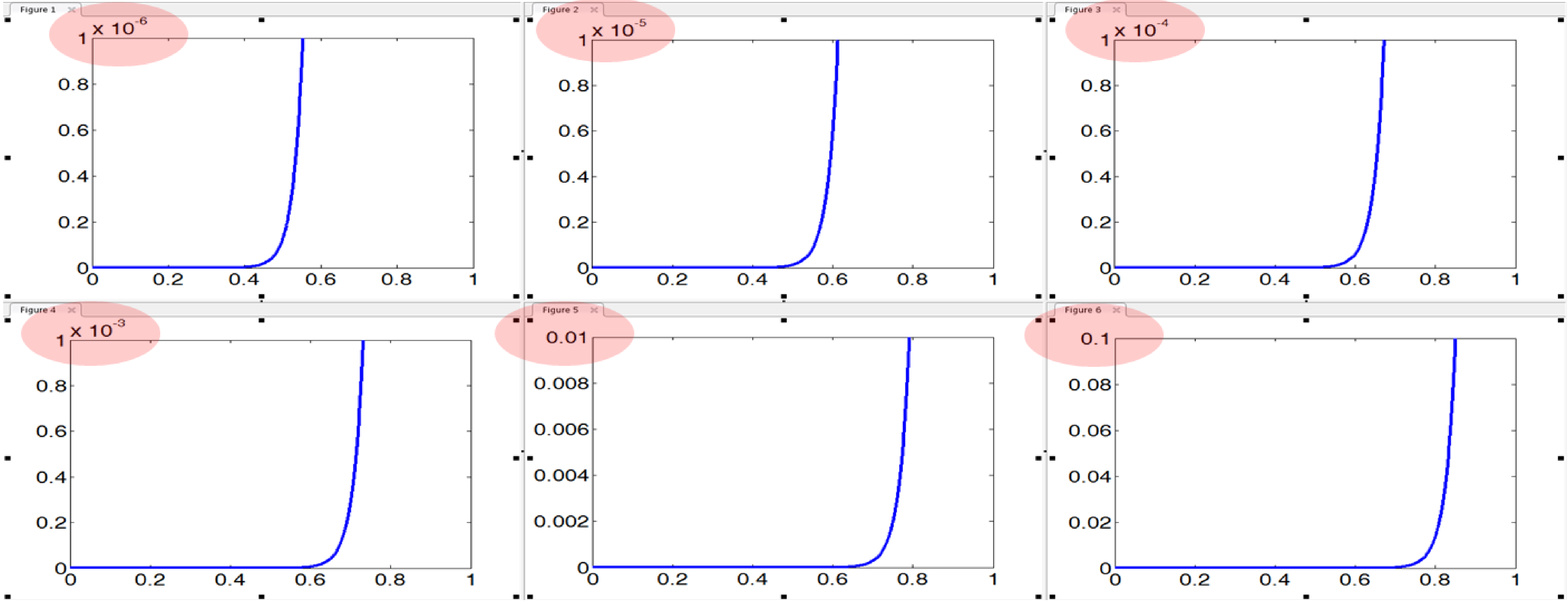
  - With the ideal factor, m, the forward diode current is often expressed in the form

$$I_{total} \sim \exp \frac{qV_{app}}{mk_B T}$$ Taur, Eq. (2.139)

  - When m is unity, the current is considered “ideal.”
  - The nonideality at small forward bias (m ~ 2) is caused by the space-charge-region current.

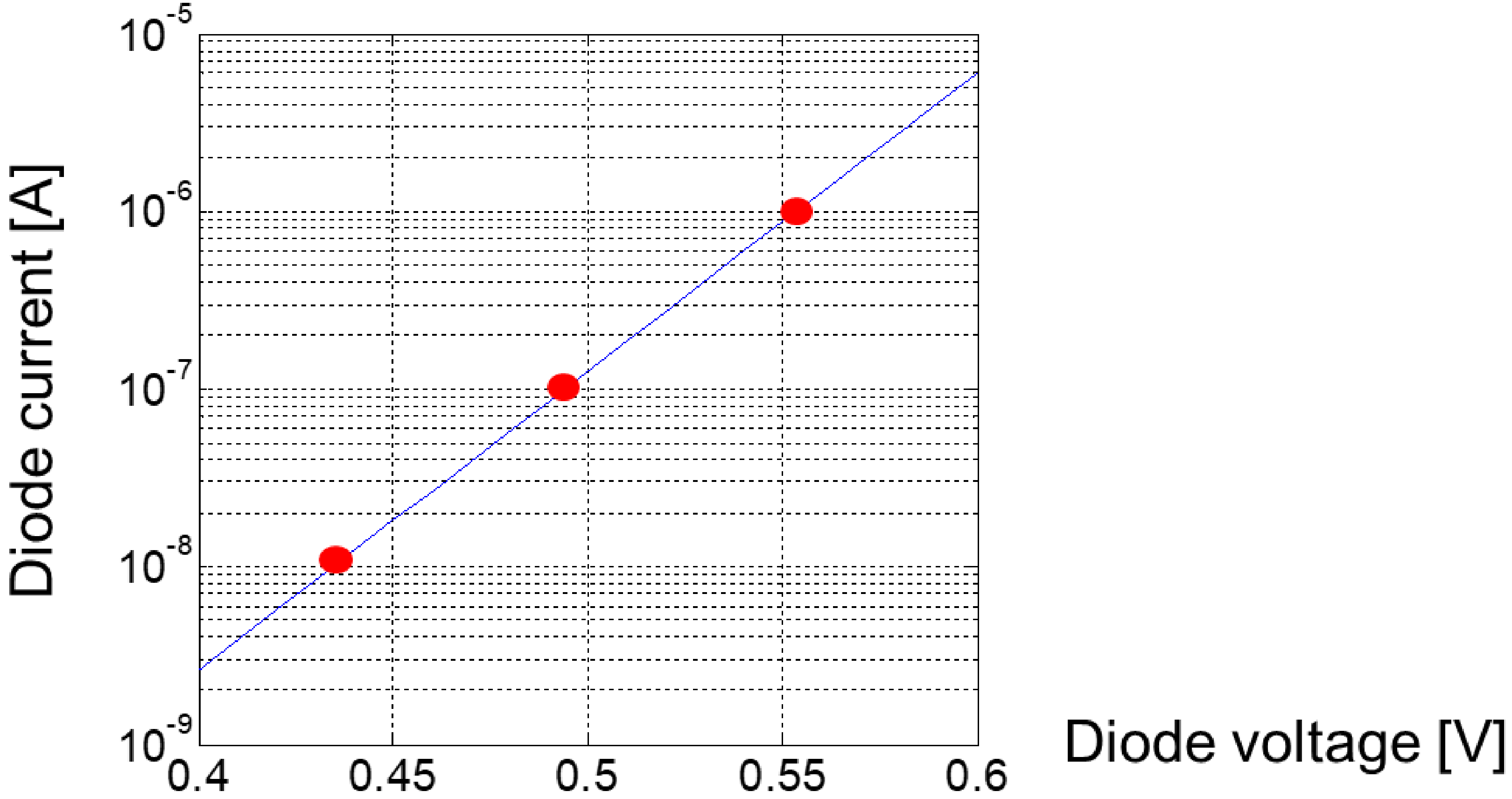
Diode IV curves

- A diode with $I_0 = 5 \times 10^{-16}\mathrm{A}$ (Only different y scales)



Important observation

- In order to obtain 10x higher current,
  - We must apply only 60 mV additionally. (300 K)



Short diode

- Consider a case where p- and n-regions are shorter than the diffusion length.
  - Our previous solution: $n(x) = n_{p0}\left(\exp\frac{V_{app}}{k_B T/q} - 1\right)\exp\left(-\frac{x-x_p}{L_n}\right) + n_{p0}$
  - It assumes infinitely long p- and n-regions.
  - Opposite extreme:

$$n(x) = n_{p0}\left(\exp\frac{V_{app}}{k_B T/q} - 1\right)\left(1 - \frac{x - x_p}{W_p - x_p}\right) + n_{p0}$$

Thank you!