Special Topics on Basic EECS I
VLSI Devices
Lecture 5

Sung-Min Hong (smhong@gist.ac.kr)
Semiconductor Device Simulation Laboratory
School of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology

Effective DOS

	N_c (cm^{-3})	N_v (cm^{-3})
Silicon	2.8x10^{19}	1.04x10^{19}
Gallium arsenide	4.7x10^{17}	7.0x10^{18}
Germanium	1.04x10^{19}	6.0x10^{18}

N_c and N_v @ 300 K (Hu's book)

- Now we know that

$$n = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5}(m_l m_t^2)^{0.5}\exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

  - With the effective DOS,

Dimension?

$$N_c = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5}(m_l m_t^2)^{0.5}$$ Taur, Eq. (2.10)

  - The electron density can be simply written as

$$n = N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right)$$ Taur, Eq. (2.9)

  - Following a similar derivation, $p = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$ Taur, Eq. (2.11)

Intrinsic carrier concentration

- In this case, $n = p$. Then, what is E_f?

$$N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$$

  - From the above equation,

$$E_f = \frac{E_c + E_v}{2} - \frac{k_B T}{2} \ln \frac{N_c}{N_v}$$

Taur, Eq. (2.12)

  - This energy level is called the intrinsic Fermi level, E_i.
  - In this case,

$$n = p = n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_c - E_v}{k_B T}\right)$$

Taur, Eq. (2.13)

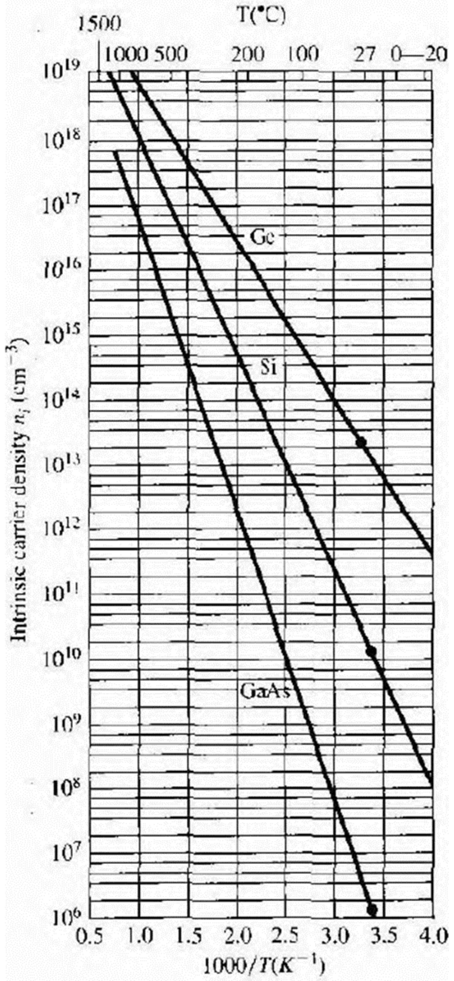
Its temperature dependence

- Recall that $N_c = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5}(m_l m_t^2)^{0.5}$. ($N_v$ has a similar form.)
  - Therefore,

$$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_c - E_v}{k_B T}\right)$$

It is dominant.

$T^{1.5}$, but it is not dominant.



Intrinsic carrier density (Neamen's book)

Using the intrinsic carrier density,

- Carrier densities are expressed as

$$n = n_i \exp\left(-\frac{E_i - E_f}{k_B T}\right)$$ Taur, Eq. (2.14)

$$p = n_i \exp\left(\frac{E_i - E_f}{k_B T}\right)$$ Taur, Eq. (2.15)

  – A useful, general relationship is that the product

$$np = n_i^2$$ Taur, Eq. (2.16)

  in equilibrium is a constant, independent of the Fermi level position.

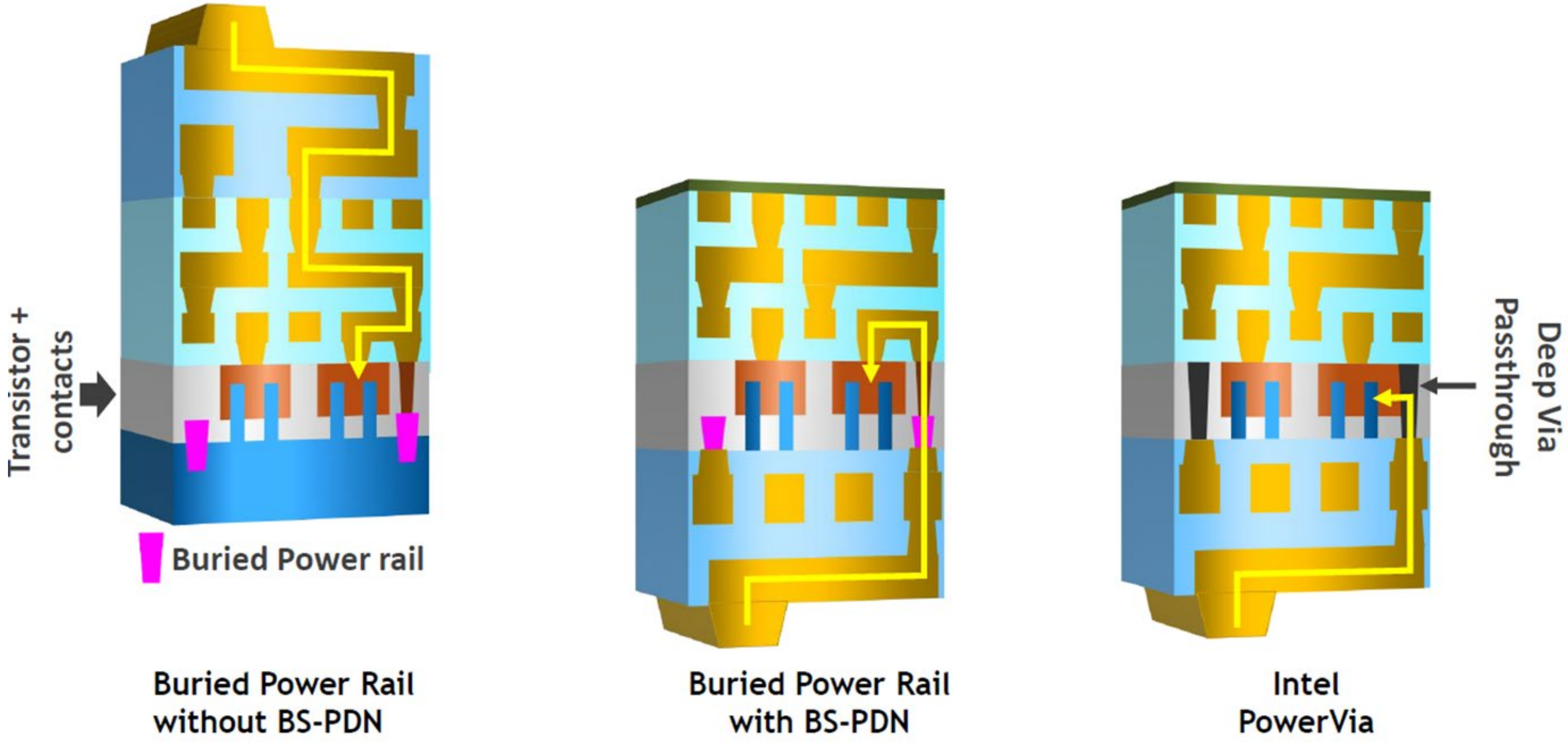
Recall that

- We have 7×10^{23} electrons/cm^3 in Si.
  - At 300 K, only ~ 1.4×10^{10} electrons/cm^3 can be found in the conduction band. Only a single elelctron among 5×10^{13} electrons occupies the conduction band.
  - There are 4 moonwalkers among 8.1×10^9 people.

Moonwalkers (NASA)

Copper (A good conductor)

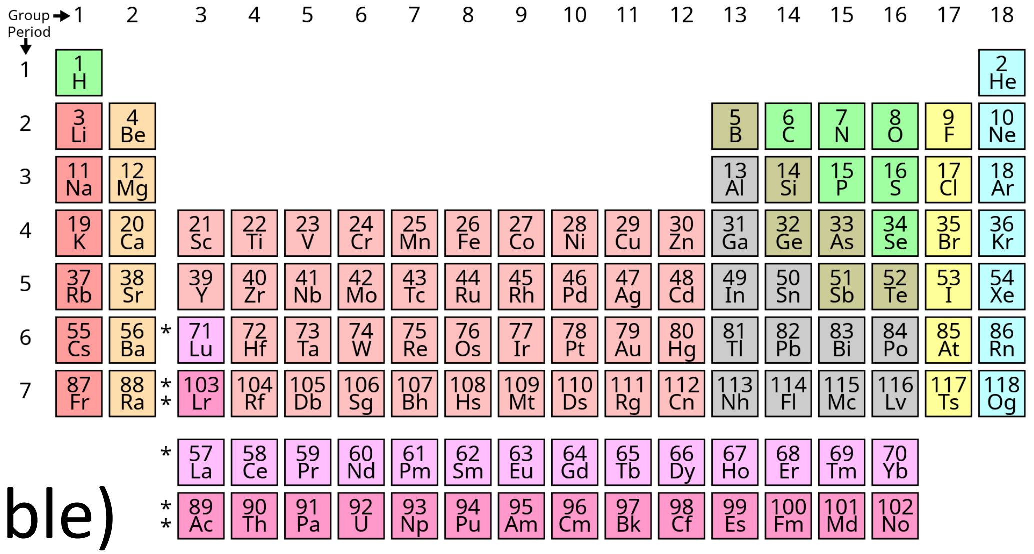
- How many conduction electrons in 1 cm^3?
  - Cu: ~ 8.5×10^{22} cm^{-3}
  - Best for interconnect

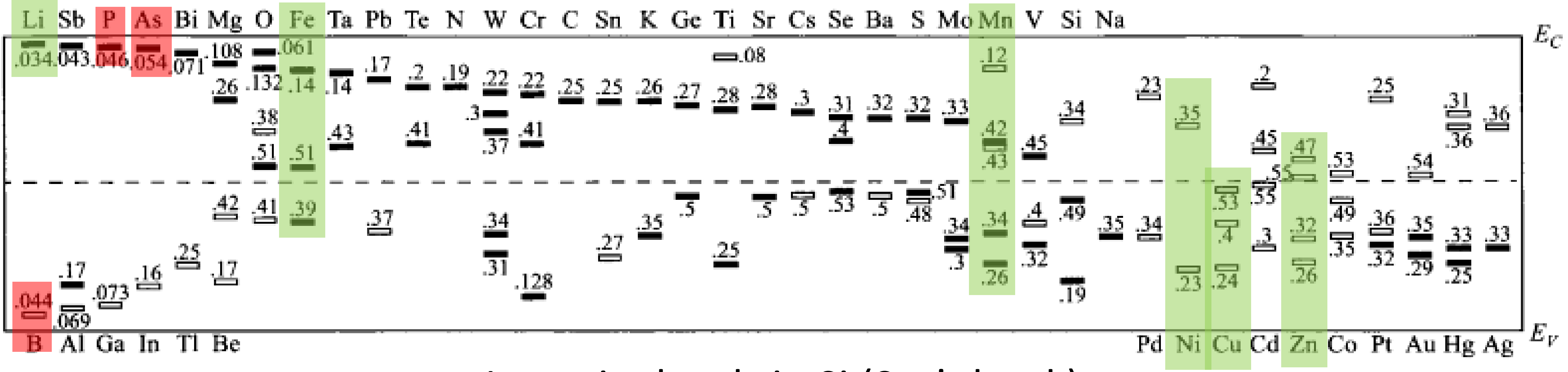


Various ways to supply the power to transistors (Intel, VLSI 2023)

Dopants

- 5 X 10^{20} impurities / cm^3 is 1 % of Si.
  - Find As, P, and B.
  - Find Fe, Cu, Li, Zn, Mn, and Ni. (Undesirable)





Impurity levels in Si (Sze's book)

Fermi level in extrinsic silicon

- Charge neutrality
  - For an n-type bulk material at equilibrium,

$$p - n + N_d - N_d f_D(E_d) = 0 \qquad \text{Taur, Eq. (2.17)}$$

  - It is known that

$$f(E_d) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_f}{k_B T}\right)} \qquad \text{Taur, Eq. (2.18)}$$

Due to the spin degeneracy

Discussion

- Let me try to explain the reason.
  - We must start from the Fermi-Dirac distribution…

Equation for the Fermi level

- Assume N_d and E_d are given. Then,

$$N_v \exp\left(\frac{E_v - E_f}{k_B T}\right) - N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) + \frac{N_d}{1 + 2\exp\left(-\frac{E_d - E_f}{k_B T}\right)} = 0$$

Taur, Eq. (2.19)

  - For shallow donor impurities,

$$-N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) + N_d = 0$$

$$E_c - E_f = k_B T \ln \frac{N_c}{N_d}$$

Taur, Eq. (2.20)

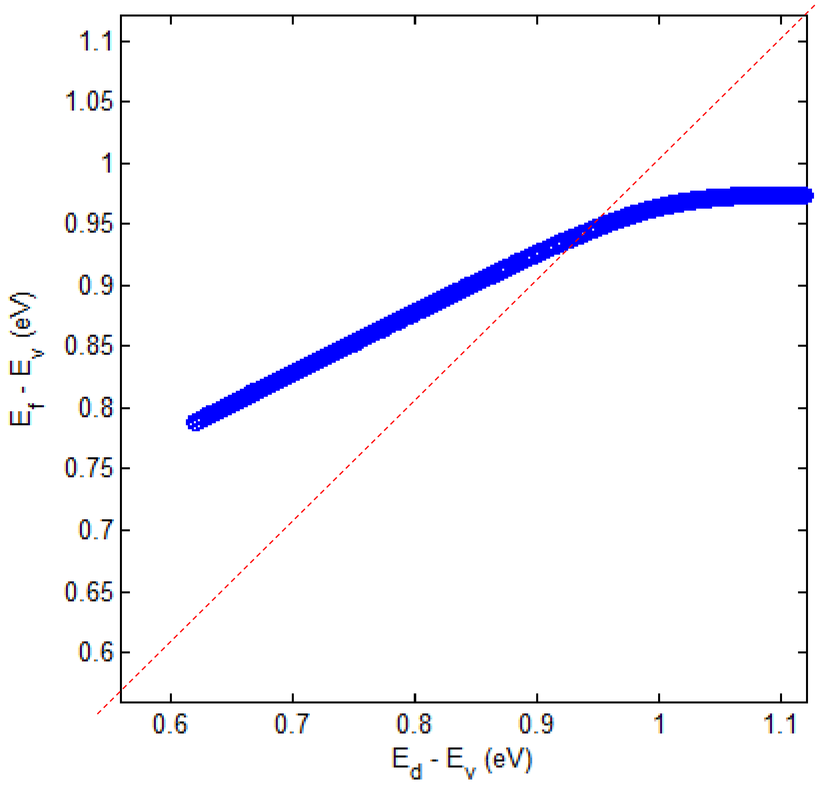
  - Hole density, $p = \frac{n_i^2}{N_d}$

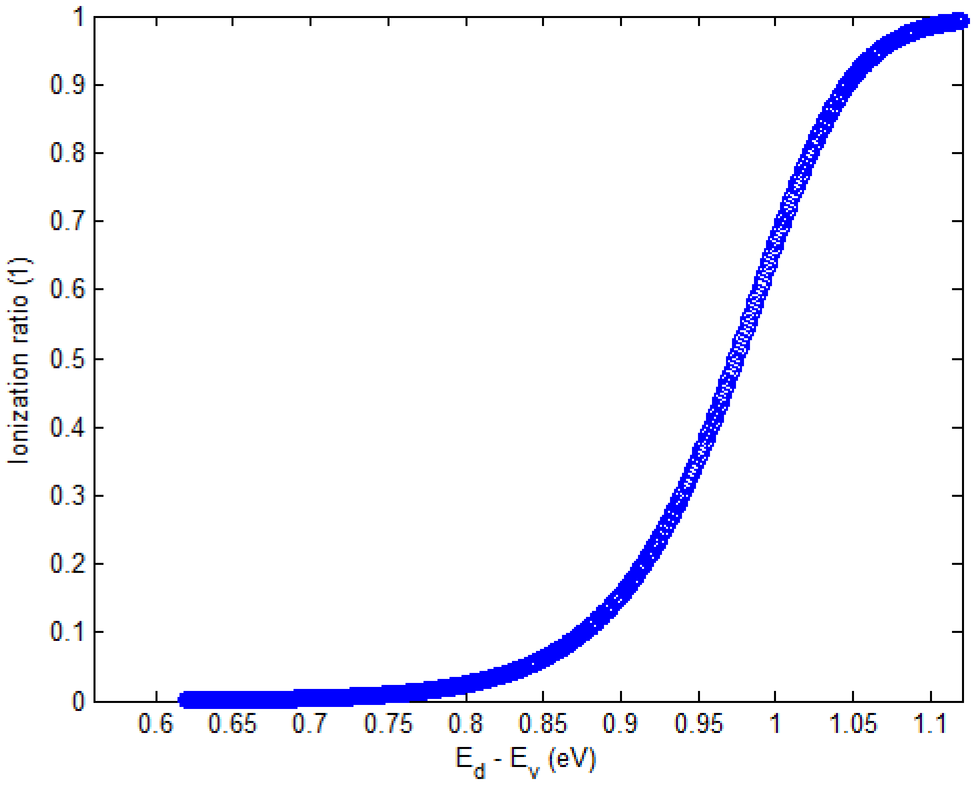
Consider a deep donor state.

	N_c (cm^{-3})	N_v (cm^{-3})
Silicon	2.8×10^{19}	1.04×10^{19}

- Assume N_d is 10^{17} cm^{-3} and T is 300 K.
  - Now, draw E_f and $1 - f_D(E_d)$ as a function of E_d.

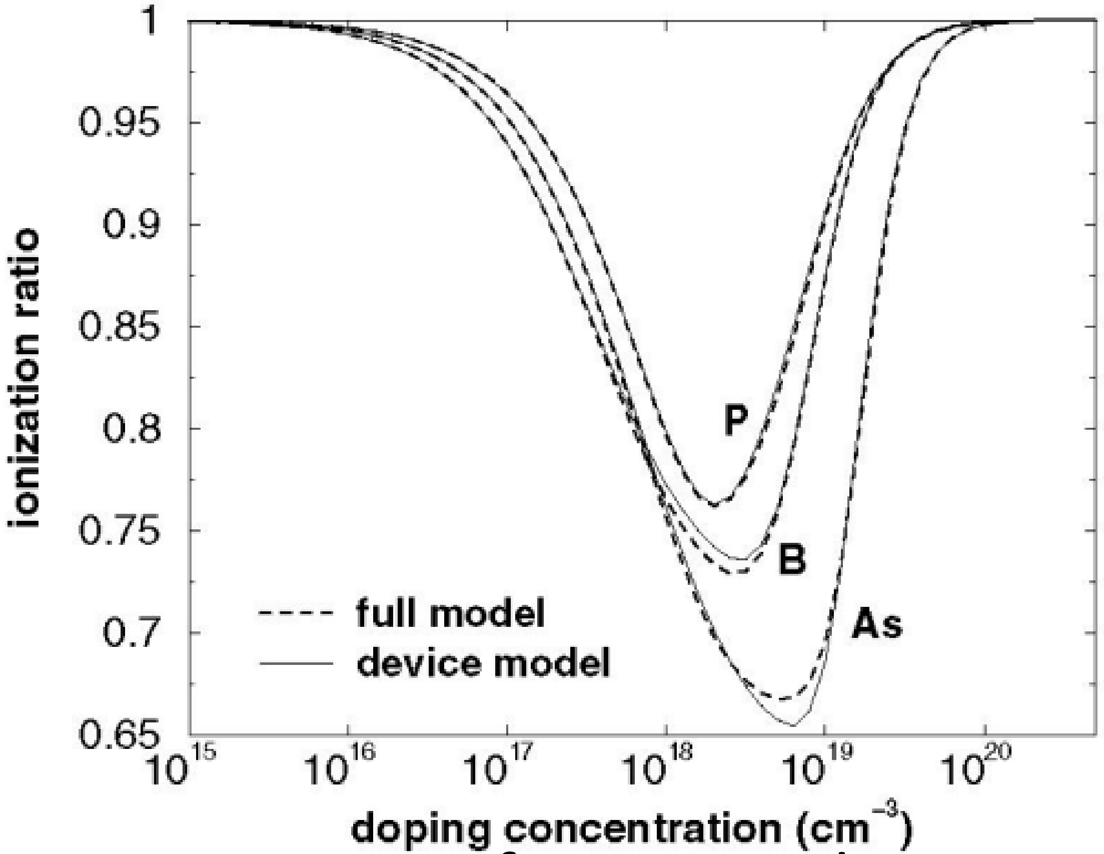
Ionization ratio



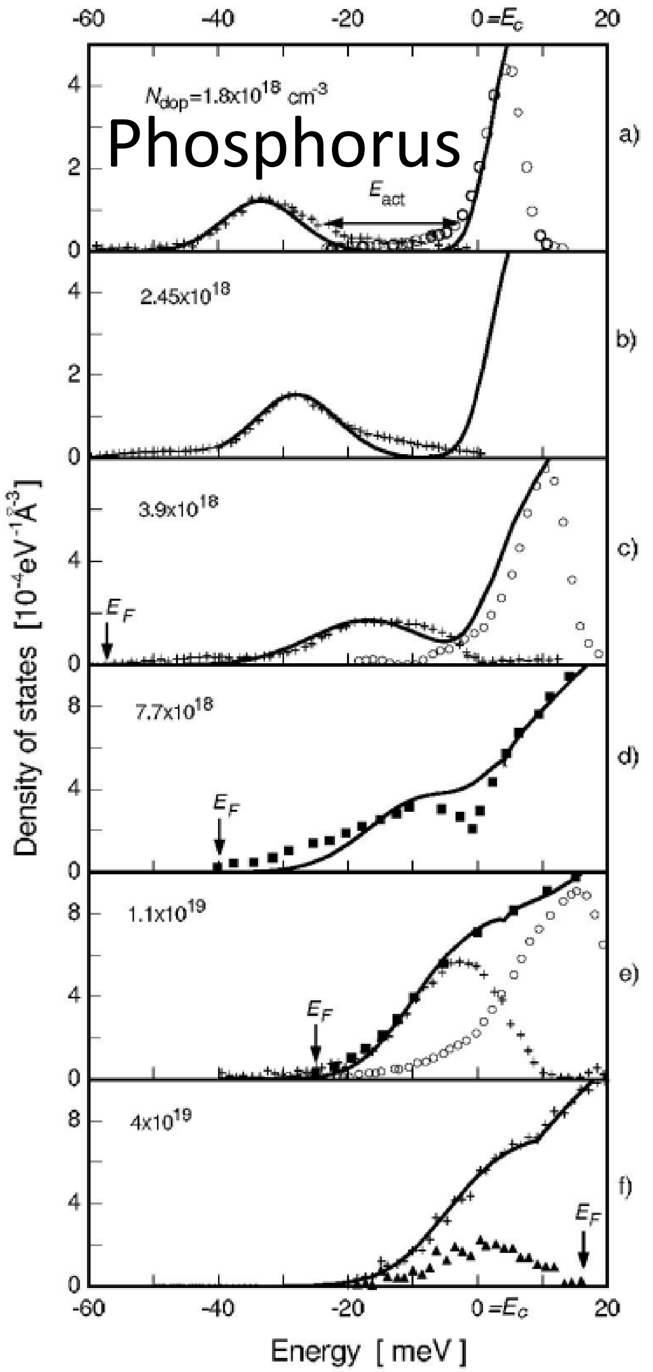


Incomplete dopant ionization

- @ high dopant densities

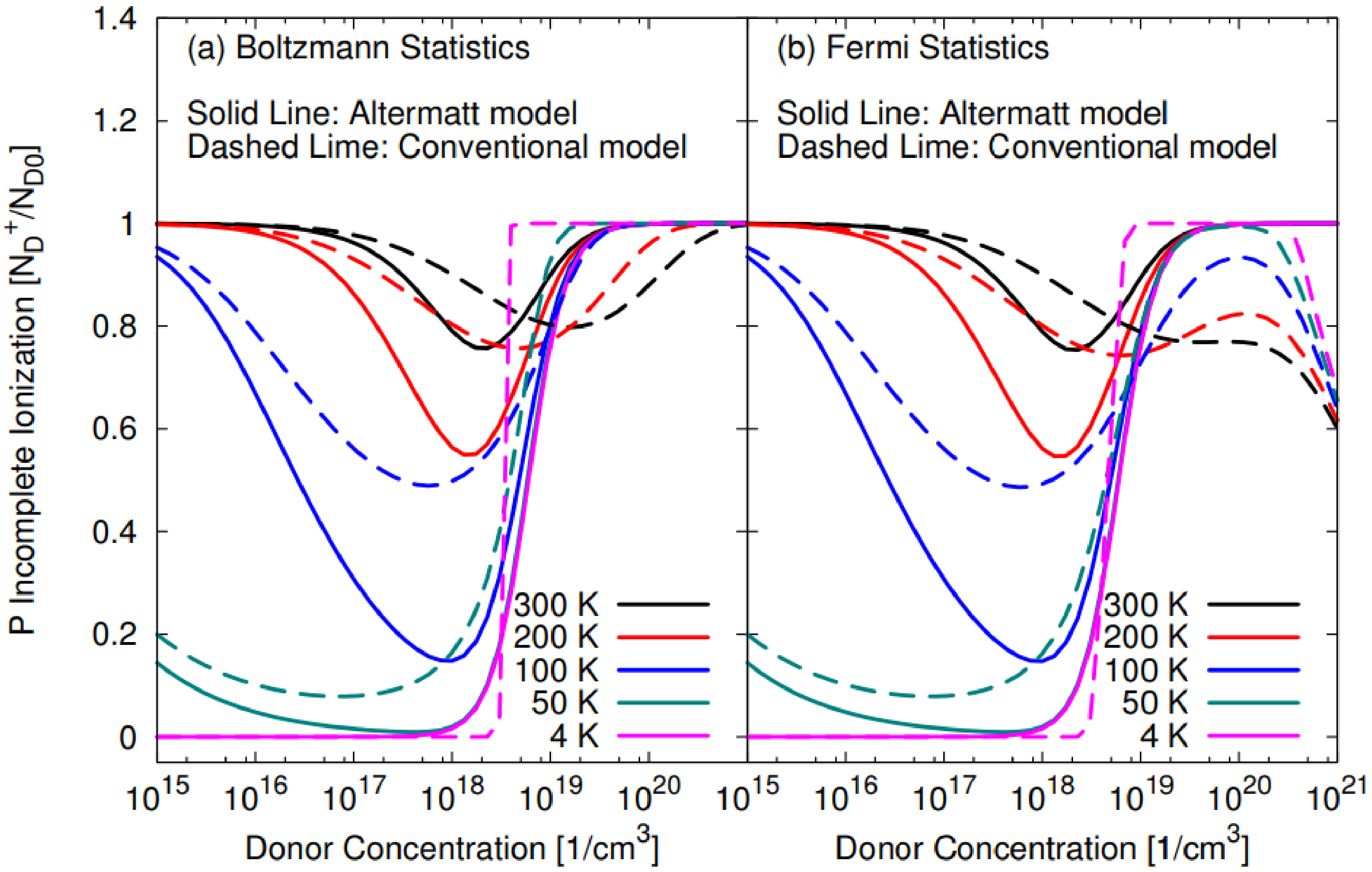


Ionization ratio for P, B, and As
(Schenk et al., SISPAD 2006)



Dopant freeze-out

- @ low temperatures



Comparison of the incomplete ionization models at various temperatures (Jin et al., SISPAD 2021)

Thank you!