Digital Integrated Circuits Homework 5

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Research Paper

R. Gupta, B. Tutuianu and L. T. Pileggi, "The Elmore delay as a bound for RC trees with generalized input signals," in *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 16, no. 1, pp. 95-104, Jan. 1997.

Introduction

In its first study in 1948, Elmore attempted to estimate a 50% delay in the monotonic step response as the mean of the impulse response. Elmore approximated the median by the mean, so Penfield and Rubinstein developed best and worst case bounds on the step response waveform. Later on, higher order moment matching technique were developed and it can calculate higher order moments with good efficiency.

Despite high-order approximations with the accuracy comparable to SPICE, Elmore delay remains a popular measurement criterion for simplicity because it is the delay metric which is easily measured in terms of net widths and lengths.

The only drawback to this delay metric is the uncertainty of its accuracy and the restriction to it being an estimate only for the step response delay.

The Elmore delay

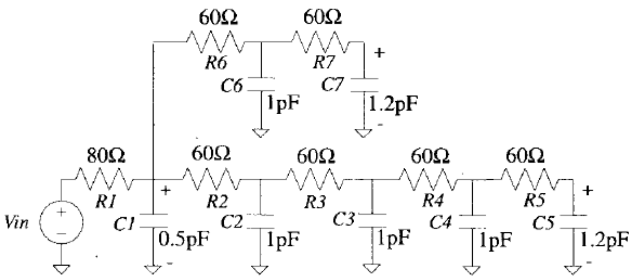

Fig1. A simple RC tree

The mean of this distribution function which is skewed symmetrically is defined by the first moment of the impulse response. Elmore unit step response delay approximation, T_D, is $T_D = m_1 = \int_0^\infty th(t)dt$. However, the real impulse response is skewed asymmetrically, so it can be make error.

Elmore delay can be calculated from two O(N) traversals of the tree, where N is the number of nodes in the tree. The Elmore value for the output at node i is given by $T_{D_i} = \sum_{k=1}^{N} R_{ki}C_k$ where R_{ki} is the resistance of the portion of the path between the input and node i, that is common with the unique path between the input and node k, and C_k is the capacitance at node k.

The Elmore Delay As A Bound

The mean would not coincide with the median in an asymmetrical distribution for the impulse response. For a unimodal skewed distribution function, the mean, median, mode inequality states that these three quantities occur either in alphabetical order or the reverse alphabetical order. (Mode $\leq$ Median $\leq$ Mean or Mean $\leq$ Median $\leq$ Mode) Before getting the bound condition of the median, 2 lemmas and 5 definitions which are in the paper are needed. From lemma 1 and 2, we have that RC each node in an RC tree has a unimodal distribution function for which $\gamma \geq 0$. To get an inequality, Mode $\leq$ Median $\leq$ Mean, let Mean $\leq$ Median $\leq$ Mode hold for any node, α, in an RC tree. The skewness, $\gamma = \frac{Mean-Median}{\sigma}$ should be equal to zero or bigger than zero. However, Mean is equal to Median or smaller than Median, so $\gamma \leq 0$. Namely, the assumption contradicts the condition, $\gamma \geq 0$. Then, Mode $\leq$ Median $\leq$ Mean is correct.

By the corollary 1, the lower bound on the 50% delay for an RC tree is given by $\max(\mu - \sqrt{\mu_2}, 0)$. Also, the output signal transition time, $T_R \propto \sqrt{\mu_2}$.

The above shows the Elmore delay when the input signals are stepwise. However, the signal coming out of the digital gate is generally modeled by a saturated ramp. It is necessary to verify that Elmore delay can act as an upper bound even on general input signals. Corollary 2 indicates that the condition, Mode $\leq$ Median $\leq$ Mean, is satisfied in the piecewise-smooth input. Also, corollary 3 indicates that as the rise time of the input-signal increases without bound, the 50% delay for an RC tree approaches the Elmore delay T_D like below.

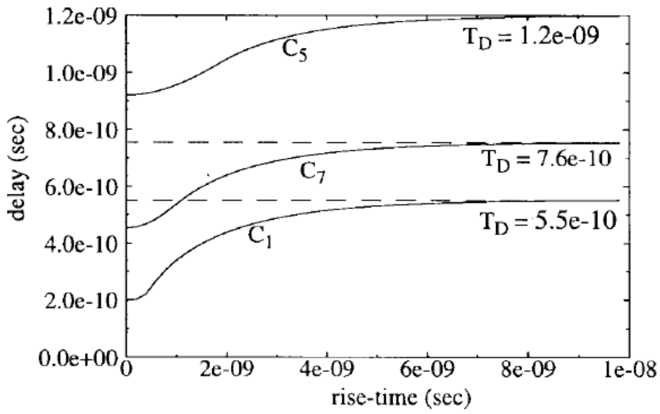

Fig 2. Delay curves show that as the rise time of the input signal increases, the delay approaches T_D.