DIC HW#5

Summarizing of "The Elmore Delay as a Bound for RC Trees with Generalized Input Signals"

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Before this paper, the Elmore delay had some drawbacks that are uncertainty as to whether it is an optimistic or a pessimistic estimate, and the restriction to step response delay estimation. Authors proved that the Elmore delay is an absolute upper bound on the 50% delay of an RC tree response. Also this bound can be not only applied to step signals but also other input signals.

I. RC Trees and Their Approximations

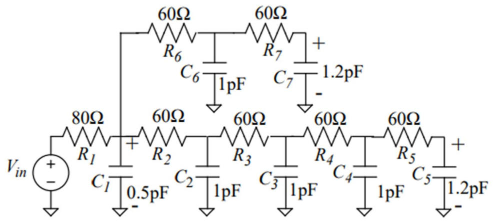

Fig 1. A simple RC tree.

Elmore proposed to approximate τ by the mean of the h(t) distribution at $\int_0^{\tau} h(t)dt = 0.5$. However, the real impulse signal has not symmetric. Therefore the approximation that the mean is equal to median is not valid for real impulse signal.

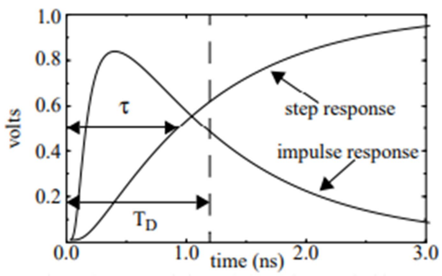

Fig 2. The unit step and the unit impulse response for the voltage across C_5 in Fig. 1.

Fig. 2 shows that the asymmetric of impulse response. However it also shows that we can bound the delay by the mean.

The calculation of the Elmore delay is below.

$$T_{D_i} = \sum_{k=1}^{N} R_{ki} C_k$$

Also calculation of Fig.1. is below.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Node	Actual delay (ns)	Elmore delay, T_D (ns)	Lower bound, T_D-σ (ns)	Single pole, T_D.ln(2) (ns)	RPH upper bound, t_{max} (ns)	RPH lower bound, t_{min} (ns)
C_1	0.196	0.55	0	0.383	0.55	0
C_5	0.919	1.2	0.2	0.83	1.32	0.51
C_7	0.45	0.75	0	0.524	1.02	0.054

Table 1. Delay bounds for circuit in Fig. 1.

The Elmore delay has also been used as a dominant time constant approximation. Also it is then used to fit a single pole approximation.

$$v(t) = 1 - e^{-p_d t}$$

Solving above equation for the 50% delay, the Elmore delay is approximated by ln2. At Table 1, the Single pole approximation can predict bound of Node C_1, but not for Node C_5. One way to explain this is the distributions h(t) of each of them are different. It is difficult to predict distribution behavior of low frequency, therefore Rubinstein and Penfield established bounds for the RC step response delay.

II. The Elmore delay as a bound

There are some definitions and lemmas for showing that

$$H(-\sigma) = \int_{-\infty}^{-\sigma} h\left(\zeta\right) d\zeta \leq \frac{\sigma^2}{\sigma^2 + (-\sigma)^2} = \frac{1}{2}$$

Above equation states that in the new coordinate system, τ = - σ is less than the median. Thus, in the original coordinate system for h(t) we have that

$$\mu - \sigma \leq \text{Median}$$

III. General input signals

Since the digital gate is generally modeled by a saturated ramp, we should extend the Elmore bound to consider a non-zero input signal transition time.

Node	Elmore delay	Rise-time = 1ns		Rise-time = 5ns		Rise-time =10ns	
		Delay	% Error	Delay	% Error	Delay	% Error
A	0.02 ns	0.01 ns	104 %	0.018 ns	11.9 %	0.019 ns	1.54 %
B	1.13 ns	0.72 ns	54.7 %	1.06 ns	6.5 %	1.116 ns	0.86 %
C	1.56 ns	1.2 ns	29.6 %	1.48 ns	4.8 %	1.547 ns	0.64 %

Table2. Delays and relative error at nodes A, B, C along a signal path.

With above table, the Elmore delay is increased toward to leaf of RC Trees or when rise-time increased.