# Digital I.C Assignment

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In the original work of 1948, Elmore attempted to estimate the 50% delay of a monotonic step response by the mean of the impulse response. However, because the median of the impulse response is the exact 50% delay and Elmore is approximating the median by the mean, Penfield and Rubinstein developed best and worst case bounds on the step response waveform. But even with the higher order approximations with accuracy comparable to SPICE, the Elmore delay remains a popular metric merely for its simplicity. In this paper, they prove that the Elmore delay value is an absolute upper bound on the 50% delay for an RC tree.

An RC tree is an RC circuit with capacitors from all nodes to ground, no capacitors between nonground nodes, and no resistors connected to ground, making it a natural model for characterizing digital gate and interconnect delays. Although it is somewhat erroneous for the real impulse response which is skewed asymmetrically, it is the skew which will allow us to bound the delay by the mean in this paper.

The Elmore value for the output at node I is given by $T_{Di} = \sum_{k=1}^{N} R_{ki} C_k$ where $R_{ki}$ is the resistance of the portion of the path between the input and node I, that is common with the unique path between the input and node k, and $C_k$ is the capacitance at node k. The Elmore delay has also been used as a dominant time constant approximation which is given by $\mathrm{H}(s) = \sum_{k=0}^{\infty} \frac{(-1)^k}{q!} s^k \int_0^{\infty} t^k h(t) dt$.

'Mode ≤ Median ≤ Mean' states that the Elmore delay, or the mean of the impulse response, is truly an upper bound on the median or the 50% point delay. Corollary1 : A lower bound on the 50% delay for an RC tree is given by '$\max(\mu - \sigma, 0)$' where $\mu$ is the mean and $\sigma = \sqrt{\mu_2}$. Corollary 2 : For an RC circuit with a monotonically increasing, piecewise-smooth input $v_i(t)$ such that $v'_i(t)$ is a unimodal function, Mode ≤ Median ≤ Mean holds for the output response $v_o(t)$ at any node. Corollary3 : For a finite sized RC circuit with a monotonically increasing piecewise-smooth input $v_i(t)$ such that $v'_i(t)$ is a symmetric function, as the rise time of the input signal, $t_r \to \infty$, the 50% delay of the output response → $T_D$ , i.e., Median → Mean.

The Elmore delay is an extremely popular timing performance metric which is used at all levels of electronic circuit design automation. A lower bound on the delay is also developed as a function of the Elmore delay, which is the first moment of the impulse response, along with the second moment of the impulse response. Improved bounds may be possible with more moments, but moment matching techniques, such as asymptotic waveform evaluation (AWE), are preferable when higher order moments are available