[Digital IC]
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# HW#5 report

Elmore delay is simple, so it's well used in digital delay estimation. However, it has some drawback about accuracy, so it's important to discuss about the mathematical accuracy of Elmore delay calculation. The purpose of this paper is to prove the property of Elmore delay which is that Elmore delay can be the upper and lower bound of 50% actual delay value of RC tree. Moreover this paper says Elmore delay can develop in any type of input response and as the transition time of input signal is increasing, the 50% delay of the output response will be near to Elmore delay. To prove these properties, the author used statistical method and PBR(Penfield and Rubinstein's Bounds) model.

## 1. Introduction to Elmore Delay concept with impulse response

- The author used RC tree model(**Fig. 1**) to analyze Elmore delay. For example, we can easily calculate the Elmore delay at $C_1$ node. $T_D = 80 \times (1 + 1.2 + 0.5 + 1 + 1 + 1 + 1.2) \times 10^{-12} = 0.552\text{ns}$.

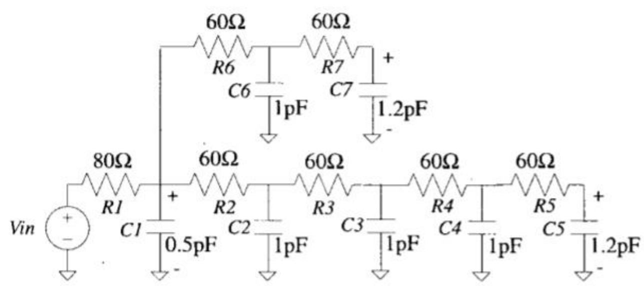


**Figure 1. RC tree**

- Before we get proving, we have to know the meaning of Elmore delay and 50% delay in step response which is simple response to deal. First, 50% delay can be defined at the point $\tau$ which is developed from integrating the unit impulse response, $h(t)$.

$$\int_{o}^{\tau} h(t)\, dt = 0.5$$

Elmore proposed a method to estimate $\tau$ as a mean value of $h(t)$ distribution. Thus, if the impulse response is ideal which mean symmetric like **Fig 2.**, then 50% delay is matched with this estimation. However, the real impulse is skewed asymmetrically like **Fig 3**. Thus, we have to show that the mean value is bounded.

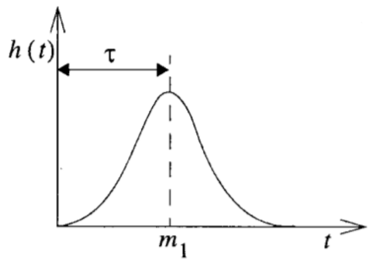


Figure 2. ideal impulse response

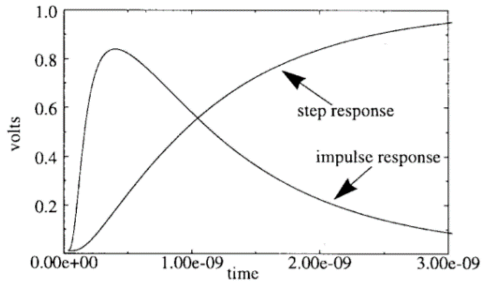


Figure 3. real impulse response

Table 1. Delay values for each method

| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|
| Node | Actual delay | Elmore delay, $T_D$ | Lower bound, $T_D - \sigma$ | Single pole approx. $T_D.\ln(2)$ | PRH upper bound, $t_{max}$ | PRH lower bound, $t_{min}$ |
| C1 | 0.196 ns | 0.55 ns | 0 ns | 0.383 ns | 0.55 ns | 0 ns |
| C5 | 0.919 ns | 1.2 ns | 0.2 ns | 0.83 ns | 1.32 ns | 0.51 ns |
| C7 | 0.45 ns | 0.75 ns | 0 ns | 0.524 ns | 1.02 ns | 0.054 ns |

- Elmore delay can also be applied to estimate the dominant time constant by using moment approximation. This lead us that any single pole response delay can be calculated by below equation.

$$v(t) = 1 = e^{-p_a t}$$

With this approximation, we can estimate 50% point delay to $T_D \ln(2)$. However this estimation is not valid some node. For example, from **Fig 1.**, at $C_5$ node, the approximation is valid, but $C_1$ node it's not valid(from **Table 1.** Column (1) with (3)). Because at node $C_1$, the response is excessive skew as seen from **Fig 4.**, so the approximation is not effective. This means that we can't explain Elmore delay with this method for some type of response. Thus, the author applied another method, Penfield and Rubinstein's Bounds(PRB).(We will not deal the process of Penfield and Rubinstein's Bounds mathematically, we'll just focus to how this method is used to prove the Elmore delay bound)

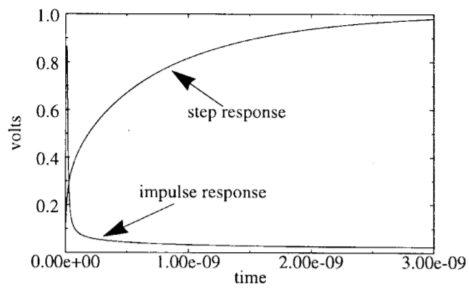


Figure 4. Highly skewed impulse response

By using PRB method, the author can estimate the upper($t_{max}$) and lower($t_{min}$) bound of 50% point delay(from **Table 1.** notation). Then, the author can get the result that $T_D \le t_{max}$. This result implies that the possibility of the Elmore delay may be able to estimate the upper bound of 50% point delay. To make sure the hypothesis, the author develop this idea more mathematically.

## 2. Elmore delay bound issue

- The fundamental idea is to prove that the distribution of real impulse response in RC tree follows a unimodal and positively skew at any node of RC tree and. In statistics, a unimodal and positively skew distribution satisfies the below property. Mode means the most frequently observed value, median is the half point of total frequency, and mean is defined as we know.

$$\text{Mode} \le \text{Median} \le \text{Mean} \qquad [1]$$

The author proved that the impulse response at any node of RC tree follows a unimodal and positive function(*Lemma1*) and they always have nonnegative skewness $\gamma$ which means positively skew(*Lemma 2*). Thus, for impulse response input, we can prove that the distributions are unimodal and positively skew in RC tree, so the median value(50% point delay) is bounded by mean value(Elmore delay). The lower bound issue is also proved with mathematically. The bound value is $\mu - \sigma$(mean-standard deviation) which are all obtained from impulse distribution.

## 3. General signal response – saturated ramp

- To apply Elmore delay estimation for real world delay calculation, the discussion about only impulse response is not enough. Because signals from digital gates are usually modeled as a saturated ramp which has nonzero transition time.

Then, how we could derive the delay of saturated ramp? First, we'll only focus to output response and establish the factor of delay term. The author prove that the relationship [1] holds for the output response at any node(*Corollary 2*) and 50% delay of output response approaches to Elmore delay as the input rise time increase(*Corollary 3*). Additionally, the author proved that the input and output response delays are equal to Elmore delay, so we don't need to prove about input response.

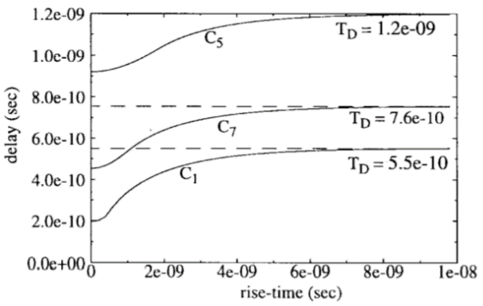


**Figure 5. Delay time for each rise time input signals**

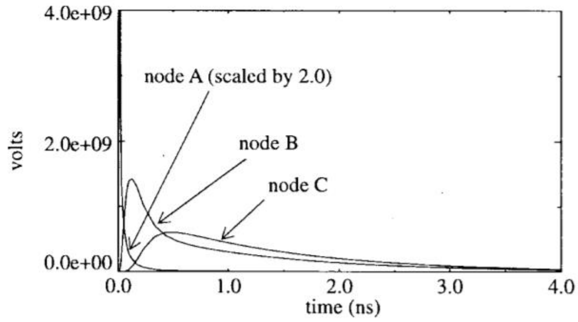


**Figure 6. Impulse responses for distance from driver**

Thus, the author knew that the estimation of 50% delay by using Elmore delay can be derived by the terms of rise time of input signal. As the rise time increase, the 50% delay approaches to Elmore delay which are seen from **Fig 5.** In this process, we can also know that as the effect of the distance from the node and driver. As the distance increase, the response is more asymmetric, so the Elmore delay can be a tighter bound of 50% delay. From **Fig 6.**, farther node from the driver, node C has more skewed impulse response. There are another data to confirm these two effect in **Table 2.** This shows the relative error (Delay-Elmore)/Delay values for each distance node and each inputs that have other rise time. We can know that as the distance from the driver and the rise time are increase, the mismatch can be reduced.

**Table 2. Relative error for each nodes in other input signals**

| Node | Elmore delay | Rise-time = 1ns | | Rise-time = 5ns | | Rise-time =10ns | |
|---|---|---|---|---|---|---|---|
| | | Delay | % Error | Delay | % Error | Delay | % Error |
| A | 0.02 ns | 0.01 ns | 104% | 18.0 ps | 11.9% | 19.0 ps | 1.54% |
| B | 1.13 ns | 0.72 ns | 54.7% | 1.06 ns | 6.5% | 1.116 ns | 0.86% |
| C | 1.56 ns | 1.2 ns | 29.6% | 1.48 ns | 4.8% | 1.547 ns | 0.64% |

## 4. Conclusion

- Until now, we can confirm that the Elmore delay estimation is a valid approximation to estimate the real delay time. Additionally, we also know the another effects about the distance from the driver and the rise time of input. Thus we can use Elmore delay method more appropriate for each situation.