Lecture5:
Small-signal analysis

Sung-Min Hong (smhong@gist.ac.kr)

Semiconductor Device Simulation Lab.

School of Electrical Engineering and Coumputer Science

Gwangju Institute of Science and Technology

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A simple math

- Taylor series expansion
  - Consider a function, $f(x)$.
  - Then, at $x_0 + \Delta x$ (Δx is small.), the function value would be similar to that at x_0:

$$f(x_0 + \Delta x) \approx f(x_0)$$

  - A better approximation?

$$f(x_0 + \Delta x) \approx f(x_0) + \left.\frac{df(x)}{dx}\right|_{x=x_0} \Delta x$$

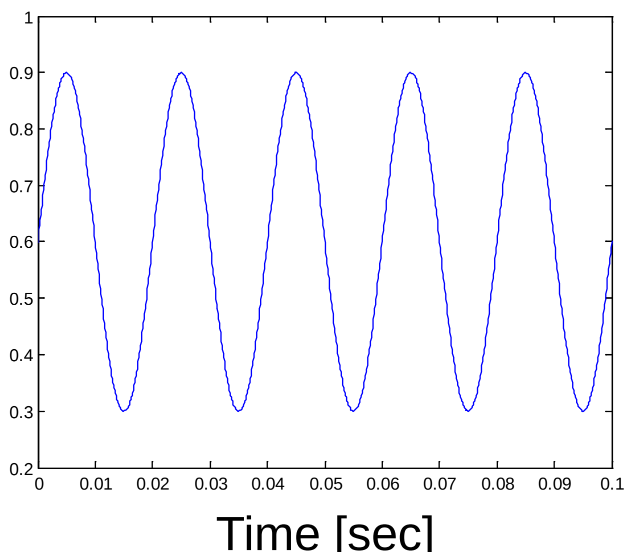
  - Nonlinear function ➔ linearly approximated!

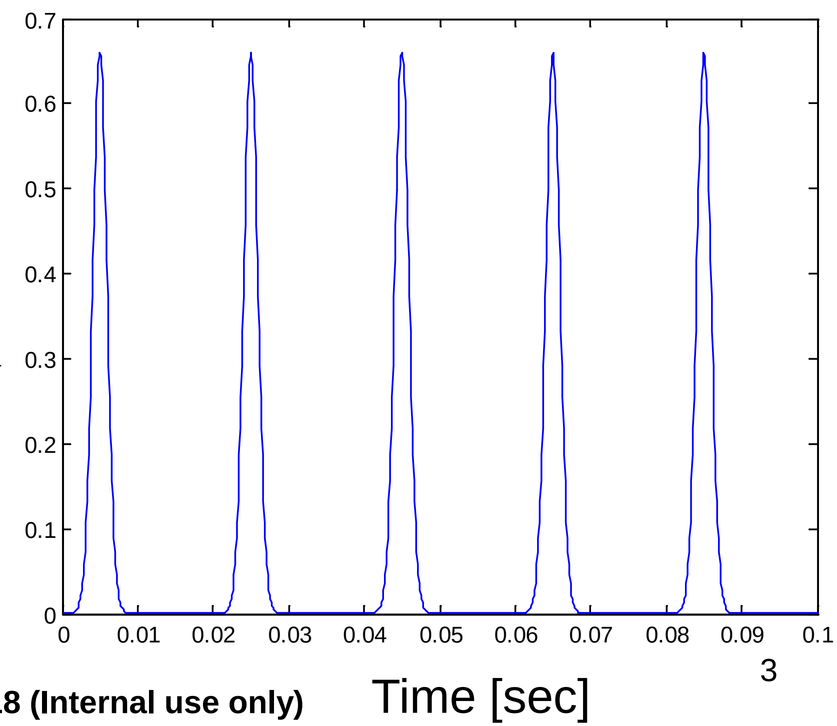
Nonlinear system

- A diode: Input V_D, output I_D

$$V_D \longrightarrow \boxed{I_D = I_S \exp\frac{V_D}{V_T}} \longrightarrow I_D$$

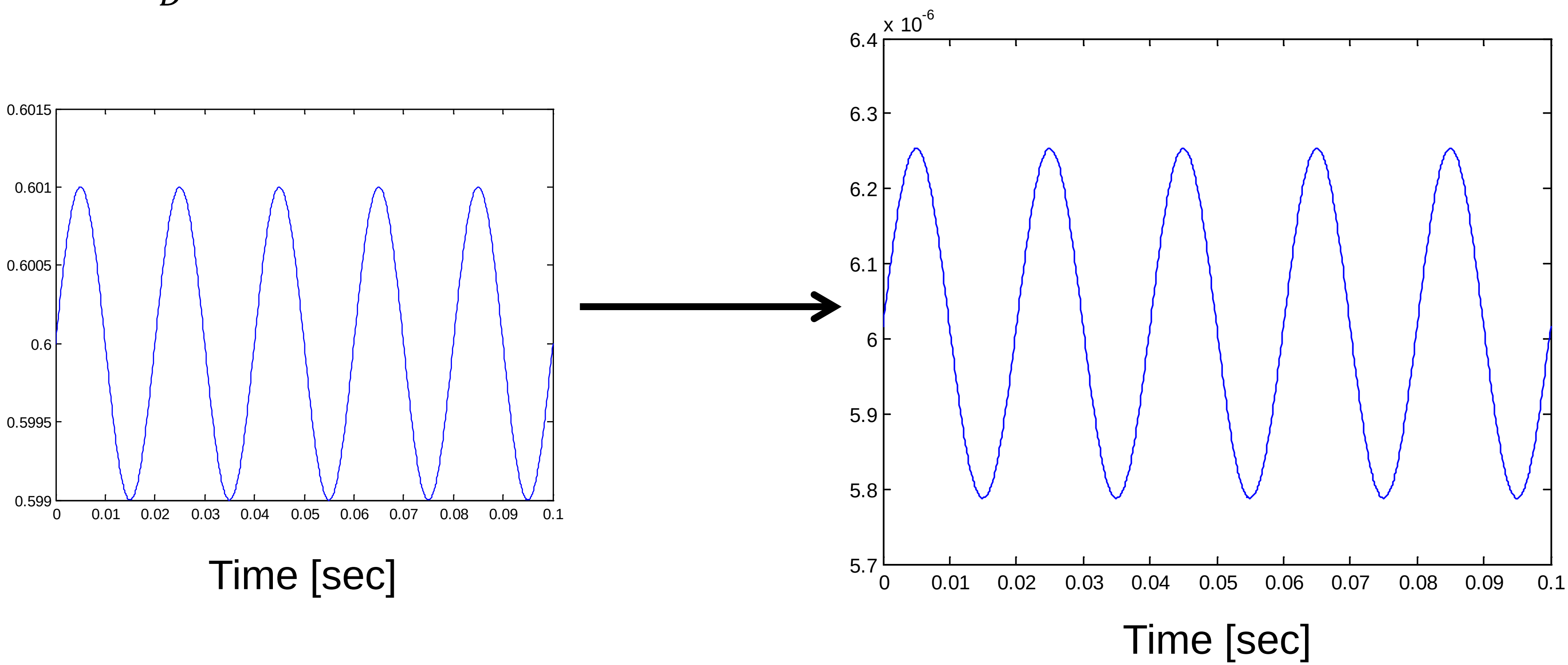
- When $V_{in} = 0.6 + \mathbf{0.3} \sin 2\pi f t$,
  - I_D is not sinusoidal.





Smaller amplitude of input

- When $V_{in} = 0.6 + \mathbf{0.001} \sin 2\pi f t$,
  - I_D is almost sinusoidal.



Small-signal analysis

- General case

$$V_{D,DC} + v_{D,AC}(t) \longrightarrow \boxed{I_D = I_S \exp\frac{V_D}{V_T}} \longrightarrow I_{D,DC} + i_{D,AC}(t)$$

- "Small-signal" case
  - When $v_{D,AC}(t)$ is small, then, the AC current is given by

$$i_{D,AC}(t) \approx I_{D,DC}\,\frac{v_{D,AC}(t)}{V_T}$$

$$v_{D,AC}(t) \longrightarrow \boxed{I_{D,AC}(t) = I_{D,DC}\,\frac{v_{D,AC}(t)}{V_T}} \longrightarrow i_{D,AC}(t)$$

Example3.18 (Razavi)

- A diode is biased at a current of 1 mA.
  - Determine the current change if V_D changes by 1 mV.

$$\Delta I_D = \frac{I_D}{V_T}\Delta V_D \approx 40\mu\mathrm{A}$$

- Small-signal resistance
  - As far as small changes in the diode current and voltage are concerned, the device behaves as a linear resistor.

$$r_d = \frac{V_T}{I_D}$$

Example3.19 (Razavi)

- When the small change in the diode voltage is time-varying,
  - What happens?

$$I_{D2} = I_s \exp \frac{V_{D1} + \Delta V}{V_T} = I_s \exp \frac{V_{D1}}{V_T} \exp \frac{\Delta V}{V_T}$$

$$I_{D2} \approx I_{D1} \left(1 + \frac{\Delta V}{V_T} \right)$$

$$I_{D2} = I_s \exp \frac{V_{D1} + \Delta V \cos \omega t}{V_T} = I_s \exp \frac{V_{D1}}{V_T} \exp \frac{\Delta V \cos \omega t}{V_T}$$

$$I_{D2} \approx I_{D1} \left(1 + \frac{\Delta V \cos \omega t}{V_T} \right)$$

  - The output current has the same frequency!