Lecture15: Impedance

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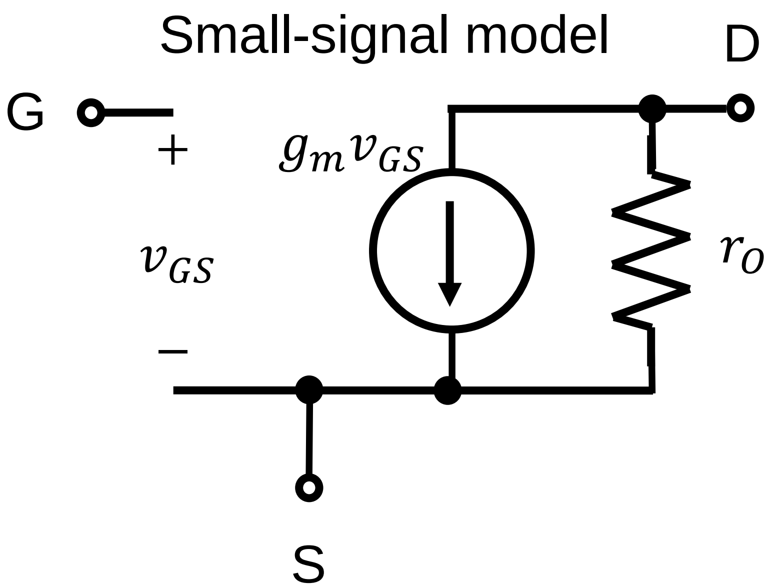
Review of previous lecture

- Small-signal model



Example 6.14 (Razavi)

- Assume that $\mu_n C_{ox} = 100\ \mu\mathrm{A\,V^{-2}}$ and $\frac{W}{L} = 10.$
  - When the drain bias current is 0.5 mA, the gate overdrive voltage, $V_{GS} - V_{TH}$, is 1 V.
  - Then,

$$g_m = 1\ \mathrm{S}$$

- Channel-length modulation
  - With the channel-length modulation coefficient, λ,

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$$

  - Then,

$$r_o = 20\ \mathrm{k\Omega}$$

Simple math

- Following relations are useful.
  - Sine and cosine functions can be expanded with $e^{+j\omega t}$ and $e^{-j\omega t}$.

$$\sin \omega t = -\frac{j}{2} e^{+j\omega t} + \frac{j}{2} e^{-j\omega t}$$

$$\cos \omega t = \frac{1}{2} e^{+j\omega t} + \frac{1}{2} e^{-j\omega t}$$

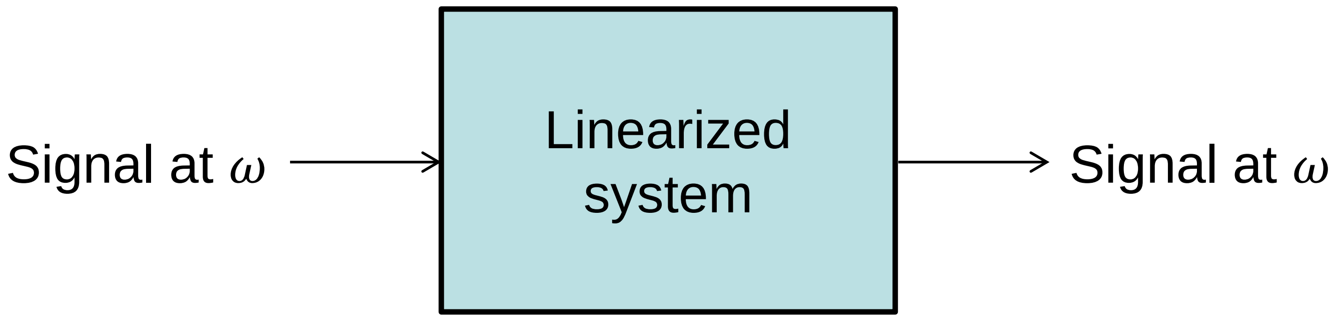
  - Therefore, for a function of $f(t) = f_s \sin \omega t + f_c \cos \omega t$, the expansion is

$$f(t) = \left(-j\frac{f_s}{2} + \frac{f_c}{2}\right) e^{+j\omega t} + \left(+j\frac{f_s}{2} + \frac{f_c}{2}\right) e^{-j\omega t}$$

  - A single complex number, $-j\frac{f_s}{2} + \frac{f_c}{2}$, is enough to represent $f(t)$.

Linearized system

- Our circuit is nonlinear in general.
- However, we have *linearized* it.
  - When the input signal has an angluar frequency, ω, the output signal has the same one.
  - It is sufficient to consider the input-output relation at ω.



Impedance

- Resistance, $V(t) = R\, I(t)$
  - It is assumed that $V(t)$ and $I(t)$ are in the same phase.
- Impedance, $V(\omega) = Z(\omega) I(\omega)$
  - Consider $V(t) = V_0 \sin \omega t$ and $I(t) = I_0 \cos \omega t$. (Different phases)
  - We introduce a phasor voltage, $V(\omega)$, and a phasor current, $I(\omega)$.
  - The relation between $V(t)$ and $V(\omega)$ is $V(t) = Re\left[V(\omega) e^{j\omega t}\right]$.
  - When $V(t) = V_0 \sin \omega t$, the phasor voltage is $V(\omega) = -jV_0$.
  - When $I(t) = I_0 \cos \omega t$, the phasor voltage is $I(\omega) = I_0$.
  - In this example, $Z(\omega) = -j \frac{V_0}{I_0}$. A purely imaginary number.

Multi-terminal devices

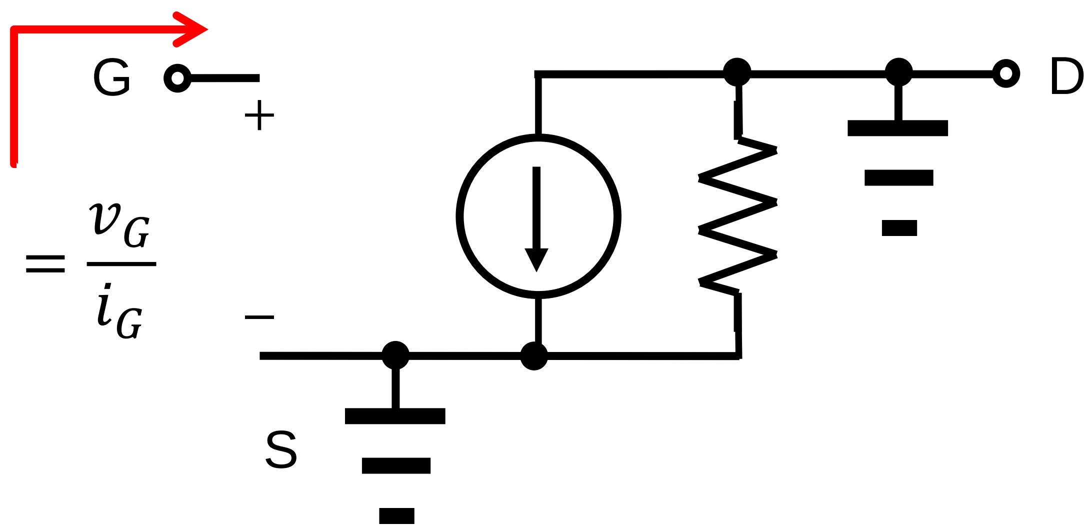
- When the number of terminals is 3,
  - We can define 9 (= 3 X 3) different impedances.

- Termination condition is important.
  - Depending on the termination condition, the impedance can be heavily changed.
  - In many cases, it is obvious from the problem.

Impedances of MOSFET

- "Looking into the *TERMINAL*," we see the impedance of the *TERMINAL*.
  - Example) Looking into the gate. The source and drain are ac-grounded.

$$Z_G = \frac{v_G}{i_G}$$



  - Similar for other terminals

Input impedance

- Consider a input signal with a finite internal resistance.
  - Usually, the internal resistance is small, but not zero.
  - The actual small-signal voltage applied to the gate terminal is given by

$$v_G(\omega) = v_{in} \frac{Z_G(\omega)}{r_{int} + Z_G(\omega)}$$