Lecture14:
MOSFET, small-signal model

Sung-Min Hong (smhong@gist.ac.kr)

Semiconductor Device Simulation Lab.
School of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology

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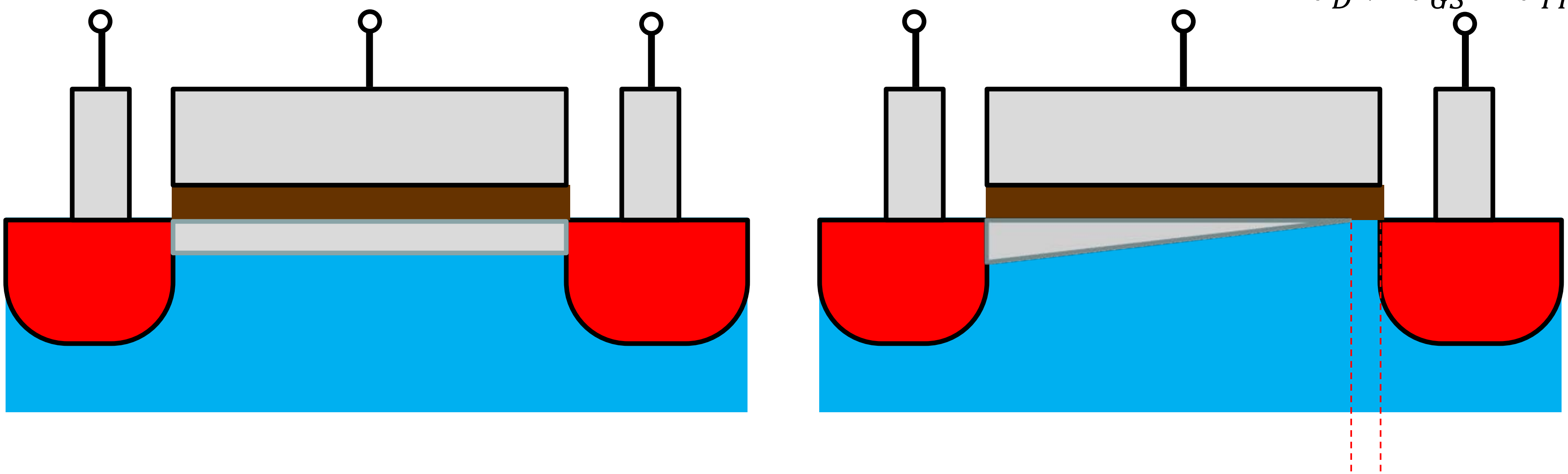
Review of previous lecture

- How to analyze a circuit with a MOSFET and a resistor
- Transconductance

Channel length modulation

- Channel length modulation

$$V_D > V_{GS} - V_{TH}$$

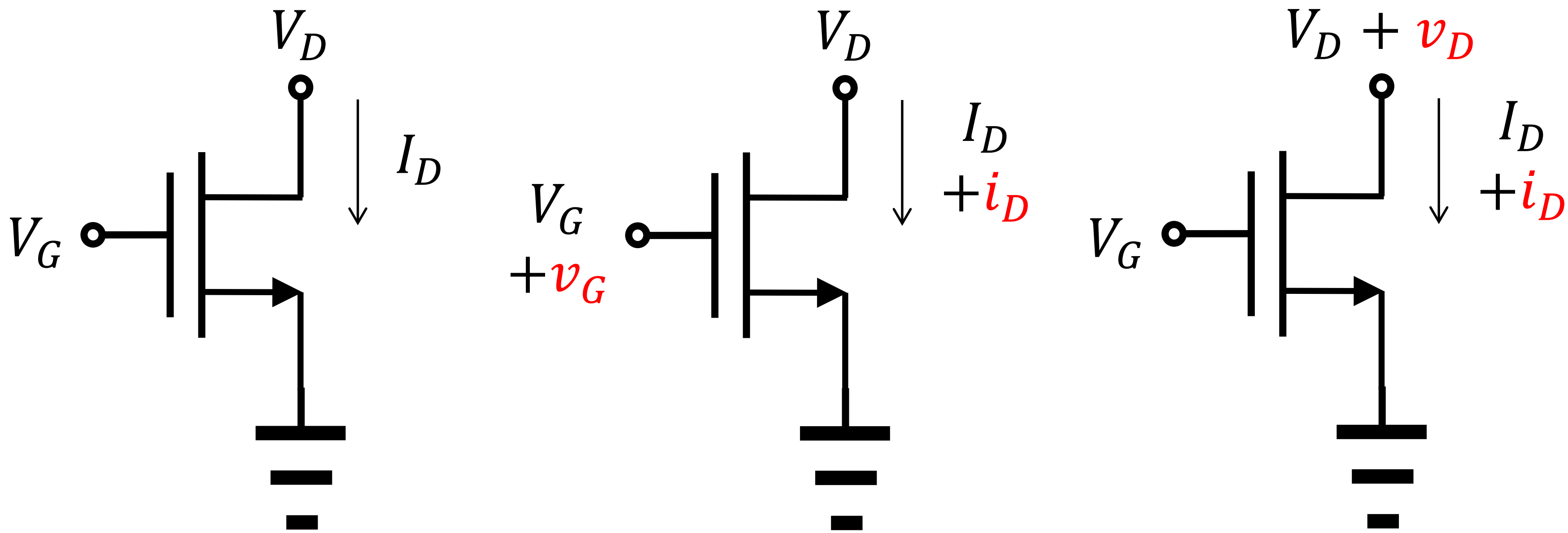


- Output resistance?

$$r_O = \frac{\Delta V_{DS}}{\Delta I_D}$$

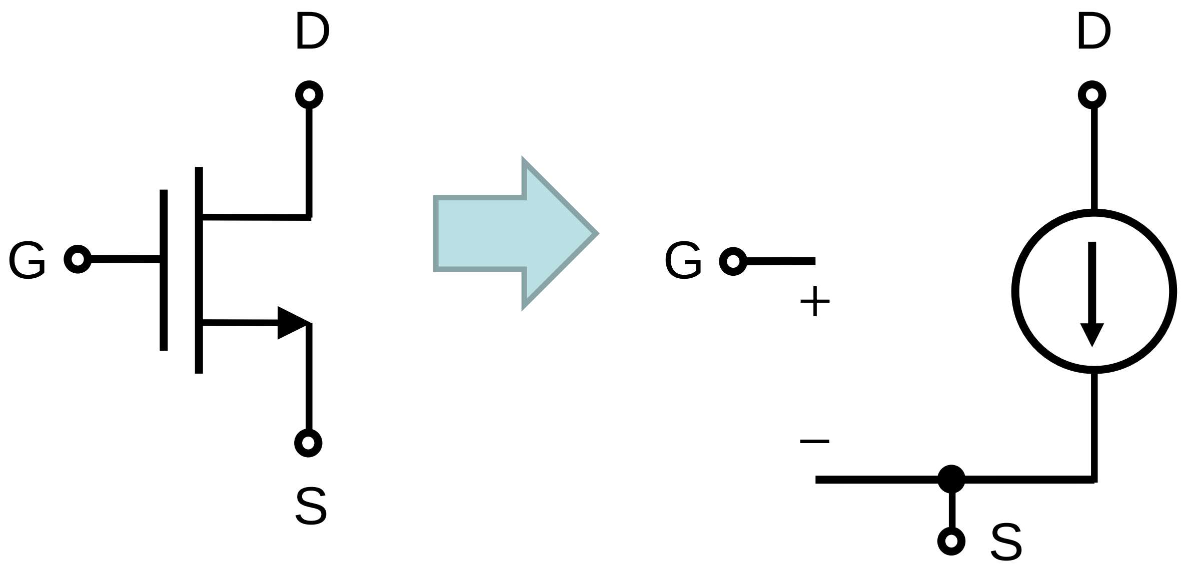
Derivatives

- We can define two derivatives.
  - Transconductance, g_m. Output resistance, r_o.



Large-signal model (1/2)

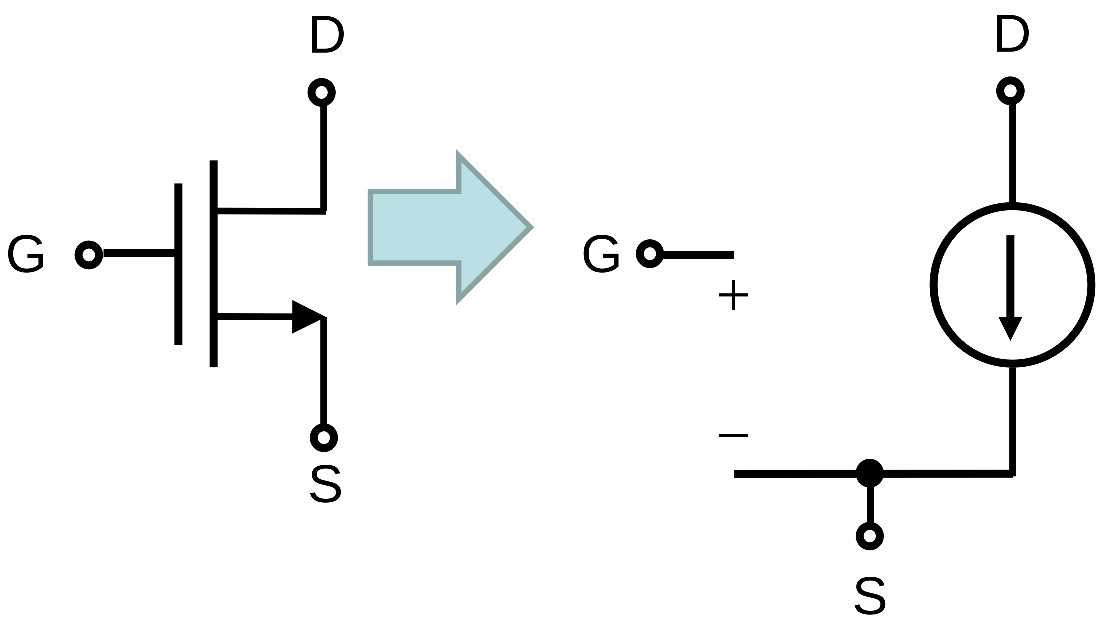
- Saturation region
  - Drain current is determined by gate voltage. (*voltage-controlled current source*)



$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$$

Large-signal model (2/2)

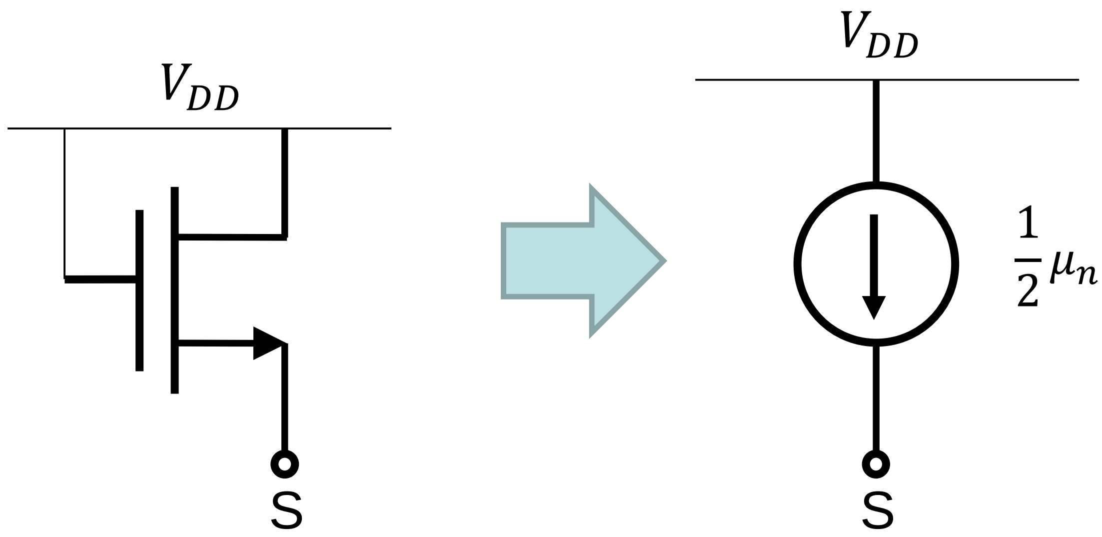
- Triode region
  - Still, it can be described by a *voltage-controlled current source*.



$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS}-V_{DS}^2\right]$$

Example 6.13 (Razavi)

- Always in the saturation region!
  - Any necessary condition?

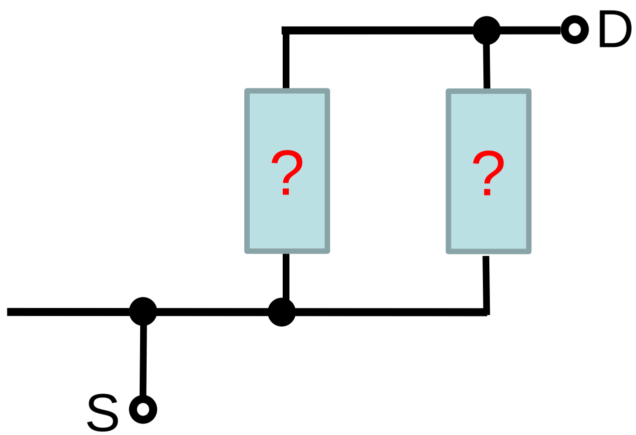


$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{DD} - V_S - V_{TH})^2$$

Gate and drain are tied.
They are connected to V_{DD}.

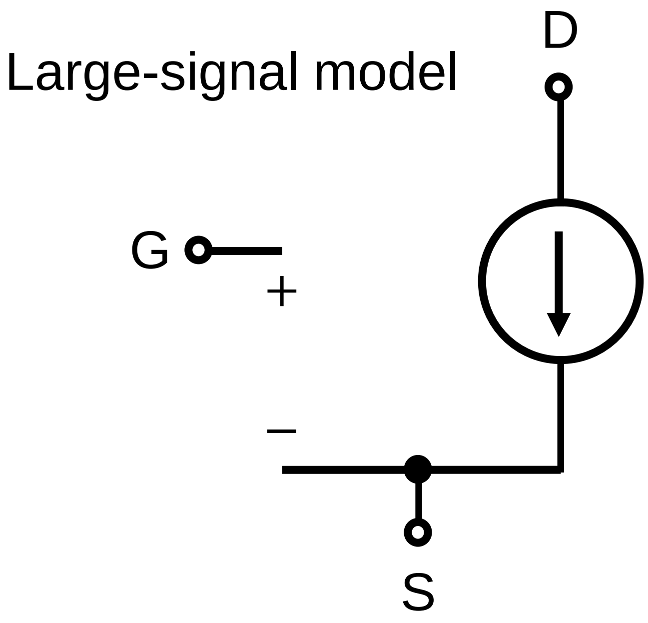
Small-signal current

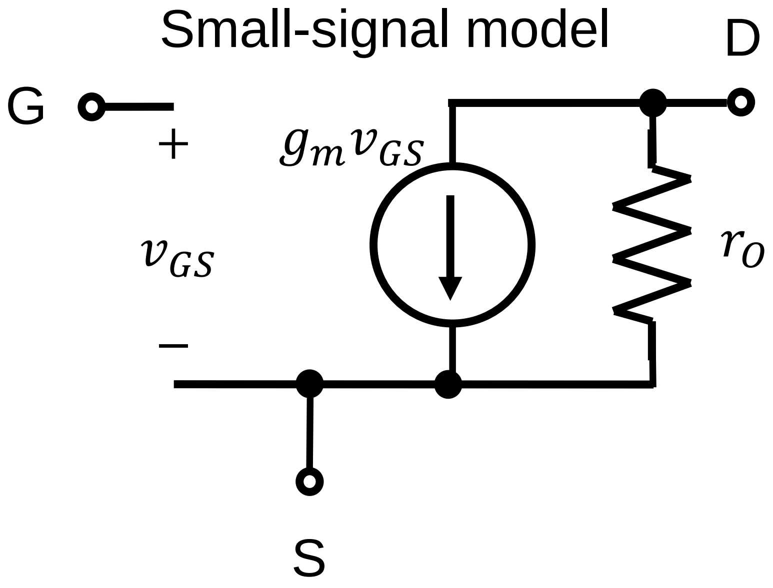
- Using the transconductance (g_m) and the output resistance (r_O),
  - The small-signal drain current is given as $i_D = g_m v_G + \frac{v_D}{r_O}$.
  - When we build a small-signal model, two contributions must be separately considered.



Small-signal model

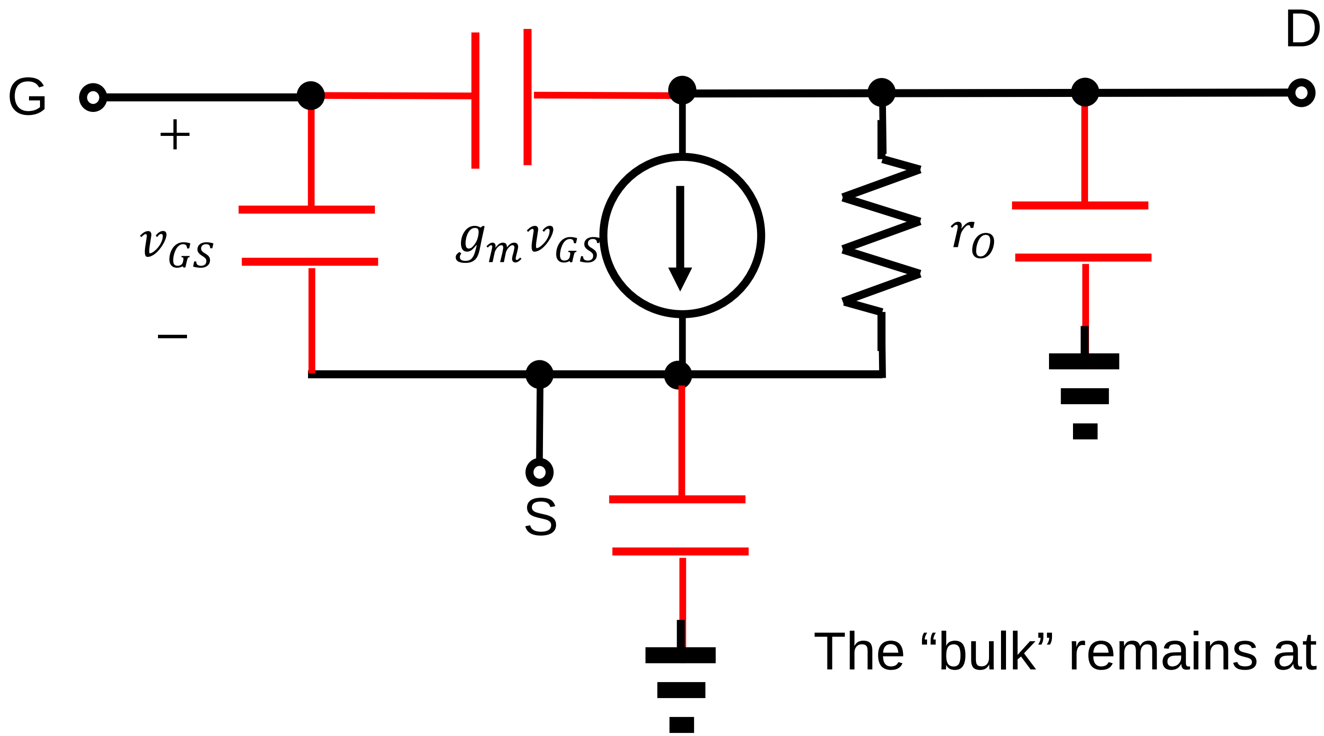
- For small-signal analysis, a small-signal model for the MOSFET is introduced.





Time-dependent one?

- In general, capacitive components can be seen.



The “bulk” remains at ac ground.

At low frequencies

- Capacitor current is $I = C\frac{dV}{dt}$.
  - When a sinusoidal dependence, for example $\sin\omega t$, is assumed, the capacitor current is proportional to ω.
  - At low frequencies, ω can be regarded as a small number.
  - In other words, the electric conduction between two nodes becomes rather weak.
  - Therefore, we often neglect the capacitive components in the small-signal model.
  - Of course, at higher frequencies, they become very important.

Simple math

- Following relations are useful.
  - Sine and cosine functions can be expanded with $e^{+j\omega t}$ and $e^{-j\omega t}$.

$$\sin \omega t = -\frac{j}{2}e^{+j\omega t} + \frac{j}{2}e^{-j\omega t}$$
$$\cos \omega t = \frac{1}{2}e^{+j\omega t} + \frac{1}{2}e^{-j\omega t}$$

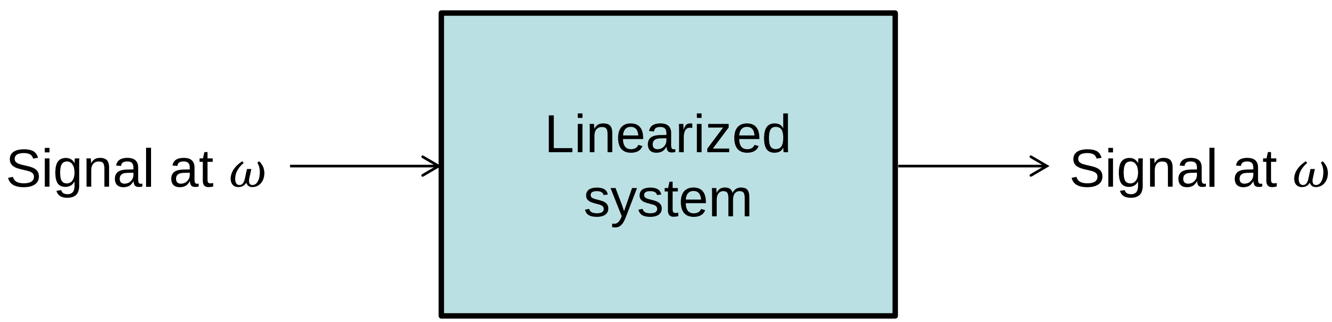
  - Therefore, for a function of $f(t) = f_s \sin \omega t + f_c \cos \omega t$, the expansion is

$$f(t) = \left(-j\frac{f_s}{2} + \frac{f_c}{2}\right)e^{+j\omega t} + \left(+j\frac{f_s}{2} + \frac{f_c}{2}\right)e^{-j\omega t}$$

  - A single complex number, $-j\frac{f_s}{2} + \frac{f_c}{2}$, is enough to represent $f(t)$.

Linearized system

- Our circuit is nonlinear in general.
- However, we have *linearized* it.
  - When the input signal has an angluar frequency, ω, the output signal has the same one.
  - It is sufficient to consider the input-output relation at ω.



Impedance

- Resistance, $V(t) = R\, I(t)$
  - It is assumed that $V(t)$ and $I(t)$ are in the same phase.
- Impedance, $V(\omega) = Z(\omega) I(\omega)$
  - Consider $V(t) = V_0 \sin \omega t$ and $I(t) = I_0 \cos \omega t$. (Different phases)
  - We introduce a phasor voltage, $V(\omega)$, and a phasor current, $I(\omega)$.
  - The relation between $V(t)$ and $V(\omega)$ is $V(t) = Re\left[V(\omega) e^{j\omega t}\right]$.
  - When $V(t) = V_0 \sin \omega t$, the phasor voltage is $V(\omega) = -jV_0$.
  - When $I(t) = I_0 \cos \omega t$, the phasor voltage is $I(\omega) = I_0$.
  - In this example, $Z(\omega) = -j\frac{V_0}{I_0}$. A purely imaginary number.

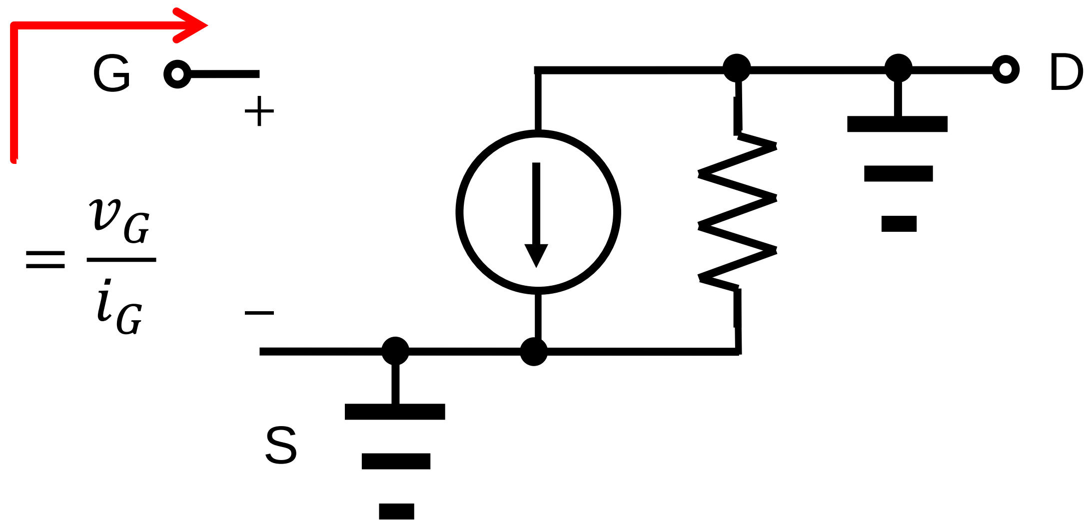
Multi-terminal devices

- When the number of terminals is 3,
  - We can define 9 (= 3 X 3) different impedances.

- Termination condition is important.
  - Depending on the termination condition, the impedance can be heavily changed.
  - In many cases, it is obvious from the problem.

Impedances of MOSFET

- “Looking into the <u>*TERMINAL*</u>,” we see the impedance of the <u>*TERMINAL*</u>.
  - Example) Looking into the gate. The source and drain are ac-grounded.



$$Z_G = \frac{v_G}{i_G}$$

  - Similar for other terminals

Input impedance

- Consider a input signal with a finite internal resistance.
  - Usually, the internal resistance is small, but not zero.
  - The actual small-signal voltage applied to the gate terminal is given by

$$v_G(\omega) = v_{in} \frac{Z_G(\omega)}{r_{int} + Z_G(\omega)}$$