Lecture2: Circuit theory

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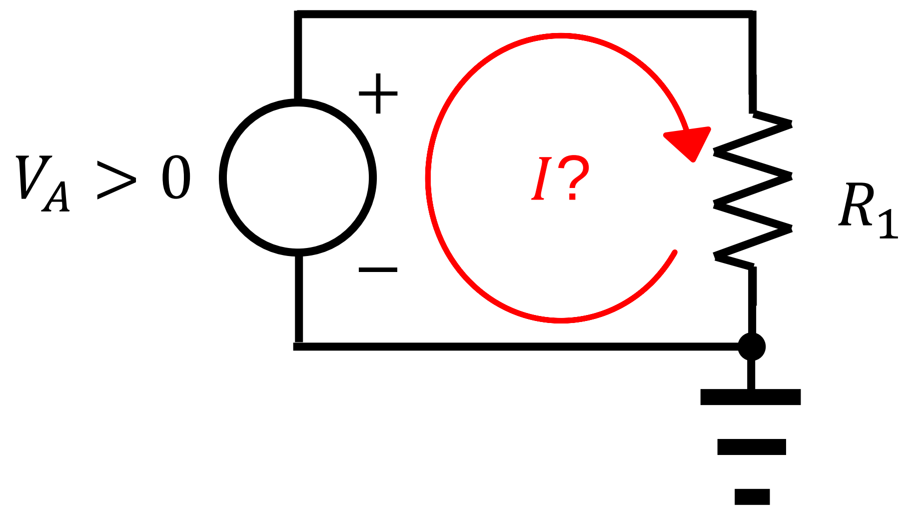
A simple problem

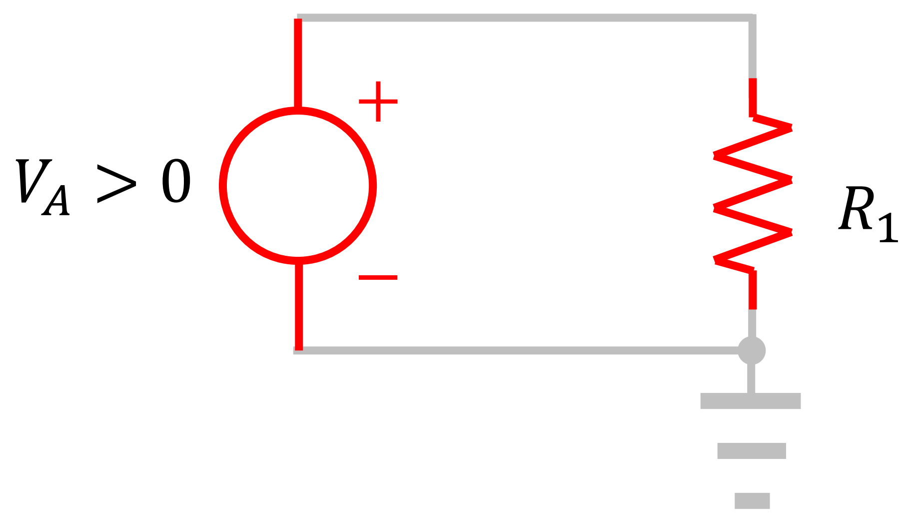
- Solve the problem.
  - What is the loop current?



  - It is an easy problem.

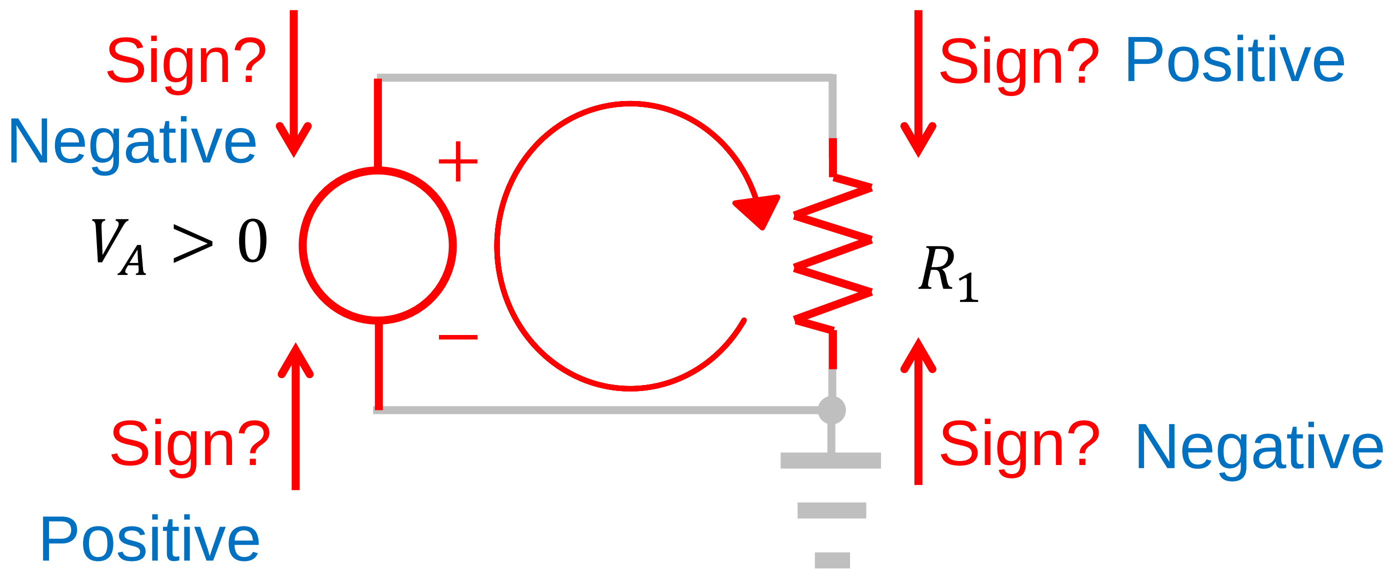
Elements

- Resistors, capacitors, etc
  - They can have multiple terminals.
  - A resistor has two terminals.
  - A diode has two terminals.
  - A MOSFET has three (or four) terminals.



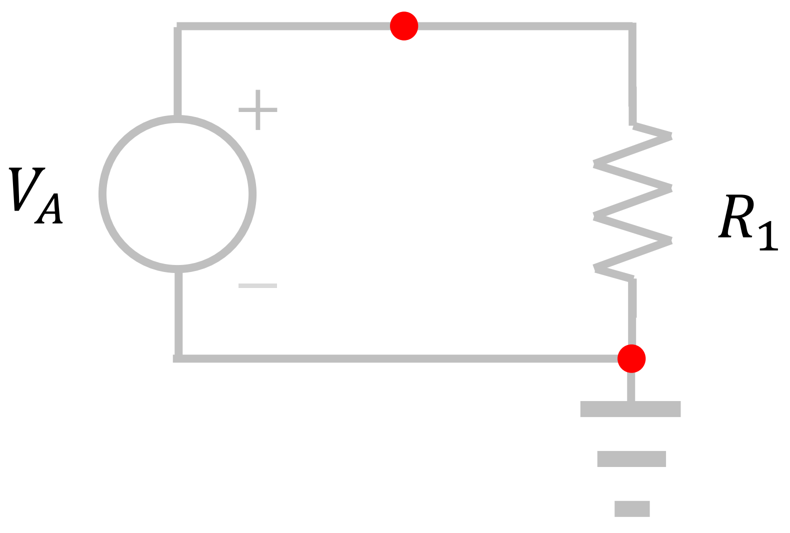
Convention for current

- Terminal current
  - When the current flows into the element, the terminal current is positive.



Nodes

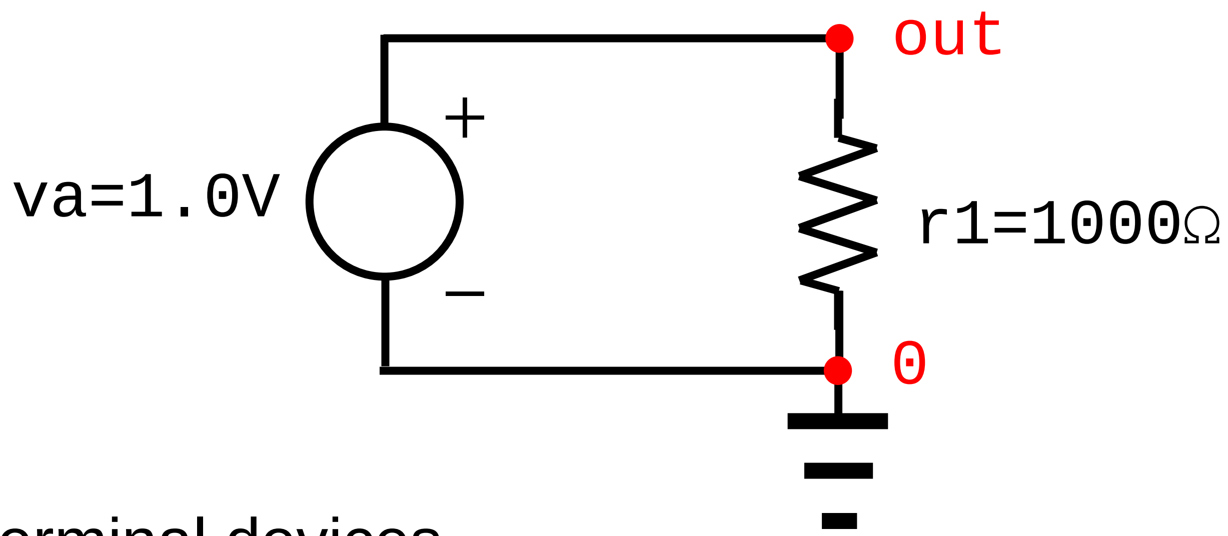
- A point to which multiple terminals are tied.
  - Usually, a dot is used to represent a node.
  - There is a special node, GND.



How to describe a circuit

- Of course, we can draw a circuit schematic. What else?
- A netlist for this circuit looks like:

```
va out 0 1.0
r1 out 0 1000
```



- Format for two-terminal devices

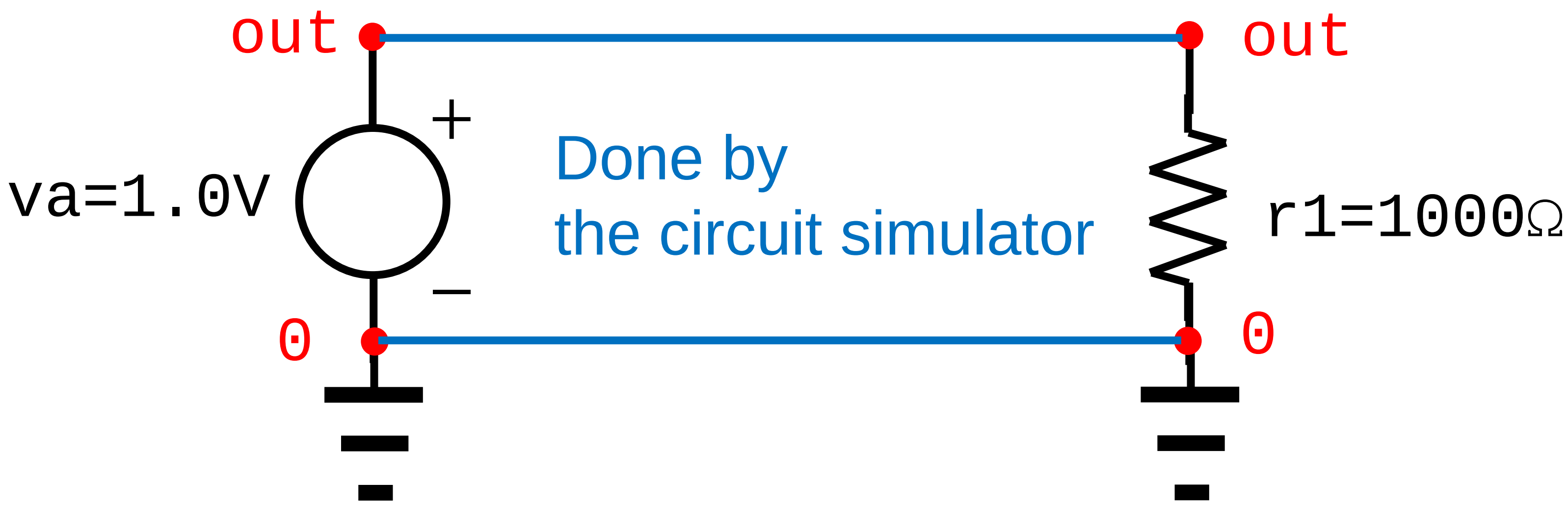
```
elementlabel node1 node2 value
```

From netlist to schematic

- Assume that we have only a netlist.

```
va out 0 1.0
r1 out 0 1000
```

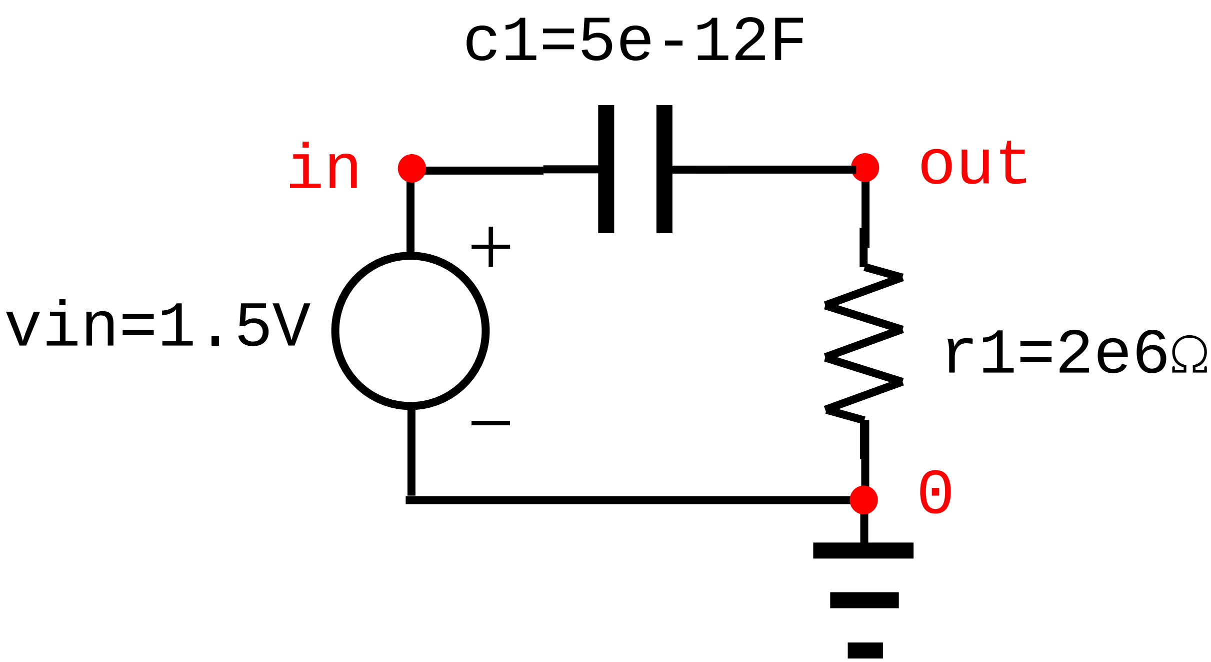
- Let's draw the schematic.
  - The first line gives us a voltage source.
  - The second a resistor.



RC filter

- A netlist for this circuit looks like:

```
c1 in out 5e-12
r1 out 0 2e6
vin in 0 1.5
```



Two-terminal elements

- Consider a two-terminal element.
  - Then, we want to know I_1, I_2, V_1, and V_2.
  - We have four unknowns, therefore, we need four equations.
  - Three equations are obvious:
  - (Current for the terminal 1) + (Current for the terminal 2) = 0

$$I_1 + I_2 = 0$$

  - (Voltage for the terminal 1) – (Connected node voltage) = 0
  - (Voltage for the terminal 2) – (Connected node voltage) = 0
- One remaining equation is element-specific.

V, I , R, C, and L

- Voltage source

$$V_1 - V_2 = V_{source}$$

- Current source

$$I_1 = I_{source}$$

- Resistor

$$I_1 = \frac{V_1 - V_2}{R}$$

- Capacitor

$$I_1 = C\frac{d(V_1 - V_2)}{dt}$$

- Inductor

$$V_1 - V_2 = L\frac{dI_1}{dt}$$

Remaining task

- Four unknowns, four equations
  - The numbers are matched.
  - However, we must know the node voltages.

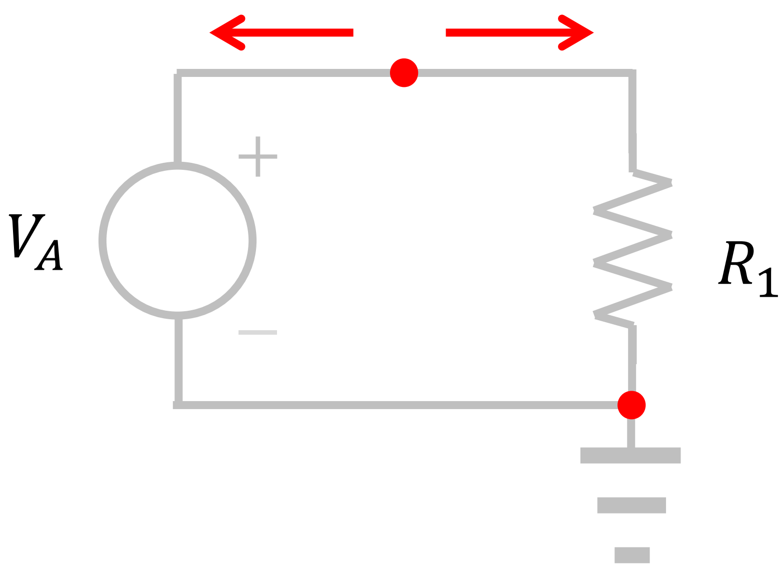
$$V_1 - \text{(Connected node voltage)} = 0$$

$$V_2 - \text{(Connected node voltage)} = 0$$

  - Therefore, we need more equations, whose number is the number of nodes.

KCL

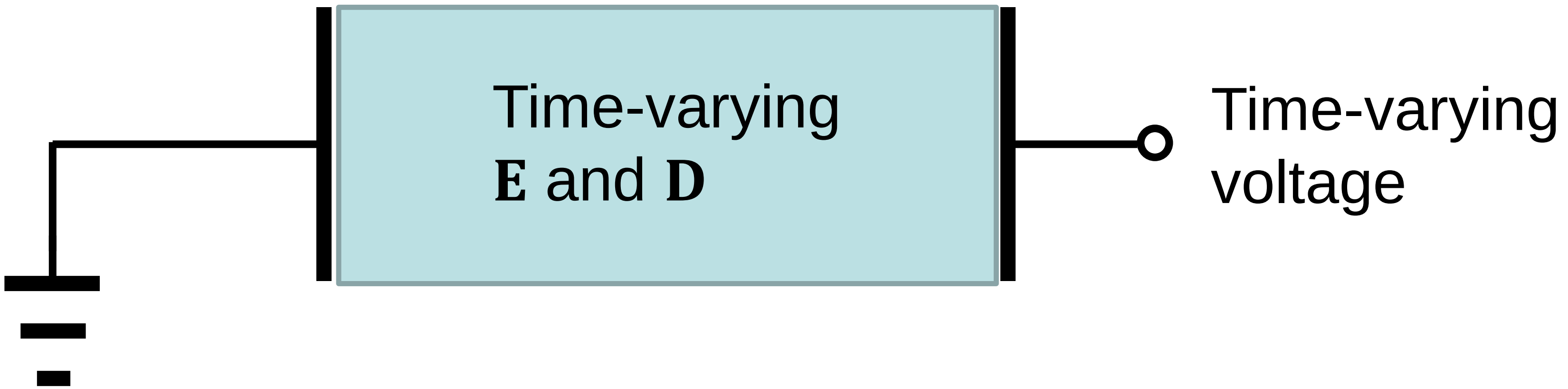
- The basic principle of circuit analysis is…
  - Kirchhoff's current law (KCL)!
  - At any node in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.



Total current density

- Why do we have the KCL?
  - The total current density is a sum of the particle current density and the displacement current density:

$$\mathbf{J}_{tot} = \mathbf{J}_{particle} + \frac{\partial}{\partial t}\mathbf{D}$$



  - According to the Maxwell equations,

$$\nabla \cdot \mathbf{J}_{tot} = 0$$

Integration of $\nabla \cdot \mathbf{J}_{tot} = 0$

- Volume integral
  - Integration over a certain volume, Ω, yields

$$\int_{\Omega} (\nabla \cdot \mathbf{J}_{tot}) d\mathbf{r} = \oint_{S} \mathbf{J}_{tot} \cdot d\mathbf{a} = 0$$

  - Here, S is the surface of Ω.
- Branch current
  - By integrating $\mathbf{J}_{tot}$ over a certain surface, we can calculate the current through that surface.

$$\int_{A_i} \mathbf{J}_{tot} \cdot d\mathbf{a} = I_i$$

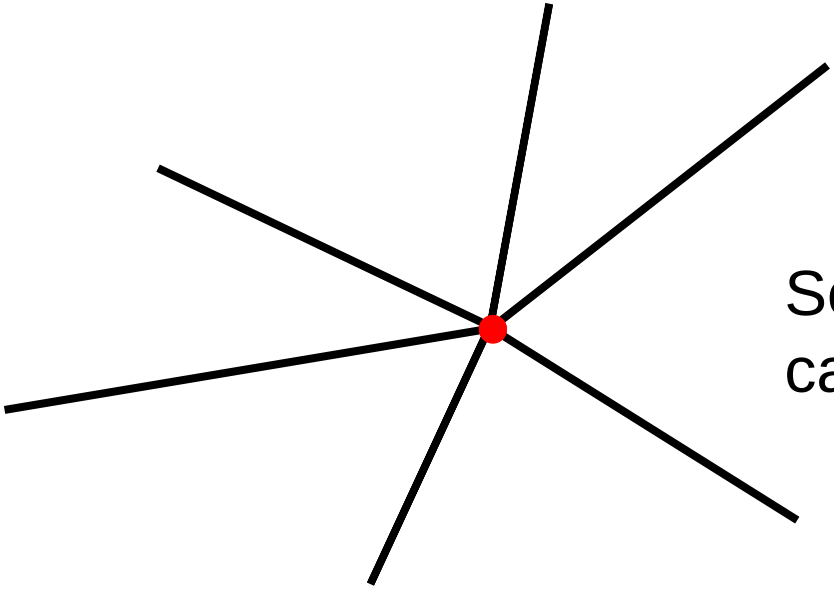
  - Here, A_i is the surface of a certain branch and I_i is a branch current.

Finally,

- By combining the previous relations,
  - We have the KCL.

$$\sum_i I_i = 0$$

  - Its derivation is quite general.



Several branches can be considered.

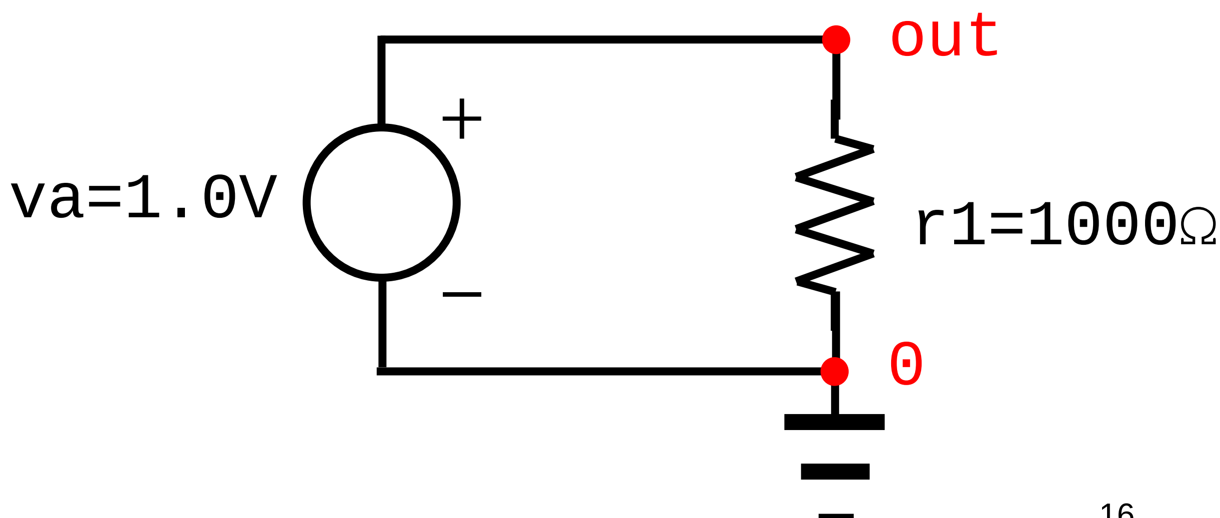
Voltage source + resistor

- Our simple problem
  - Following equations are identified.

$$I_{va} + I_{r1} = 0$$ KCL

$$V(out) - 0.0 = 1.0$$ Voltage source

$$I_{r1} = \frac{V(out)}{1000}$$ Resistor



Current source + resistors

- A typical example in this course

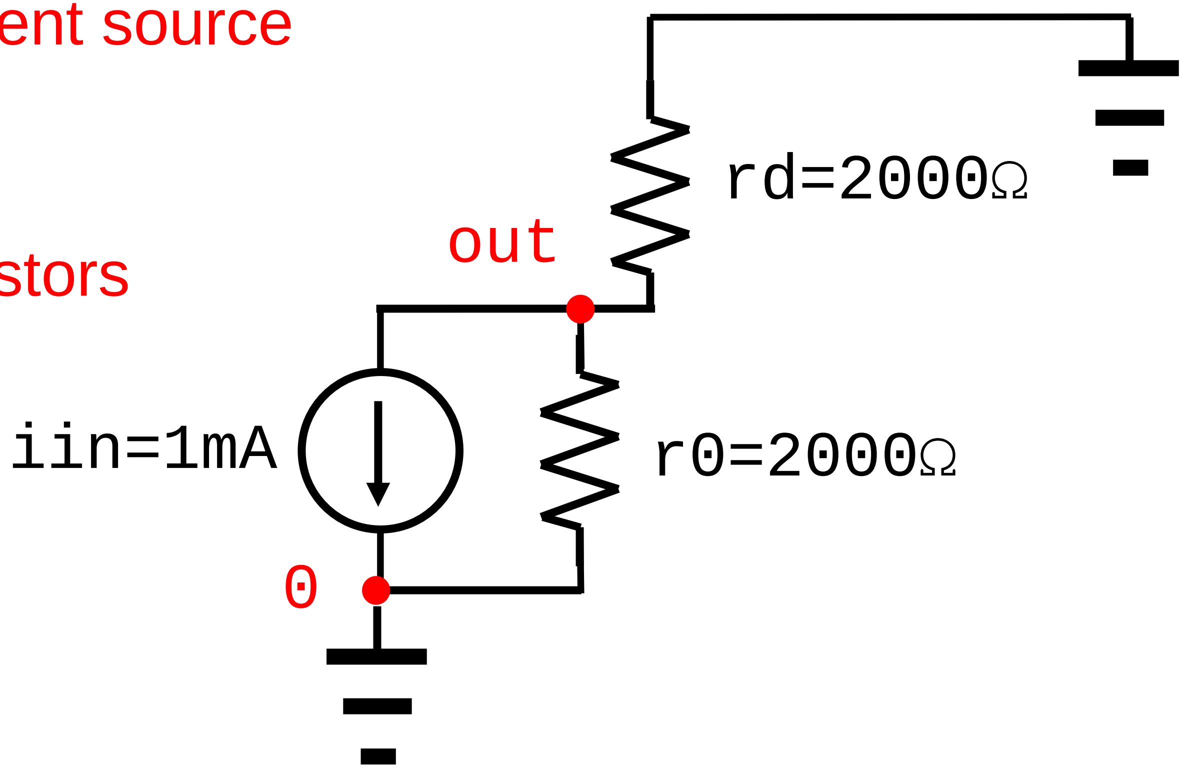
$I_{iin} + I_{r0} + I_{rd} = 0$ KCL

$I_{iin} = 10^{-3}$ Current source

$$I_{r0} = \frac{V(out)}{2000}$$

Resistors

$$I_{rd} = \frac{V(out)}{2000}$$



Source degeneration

- An additional resistor