# VLSI Devices
# Lecture 3

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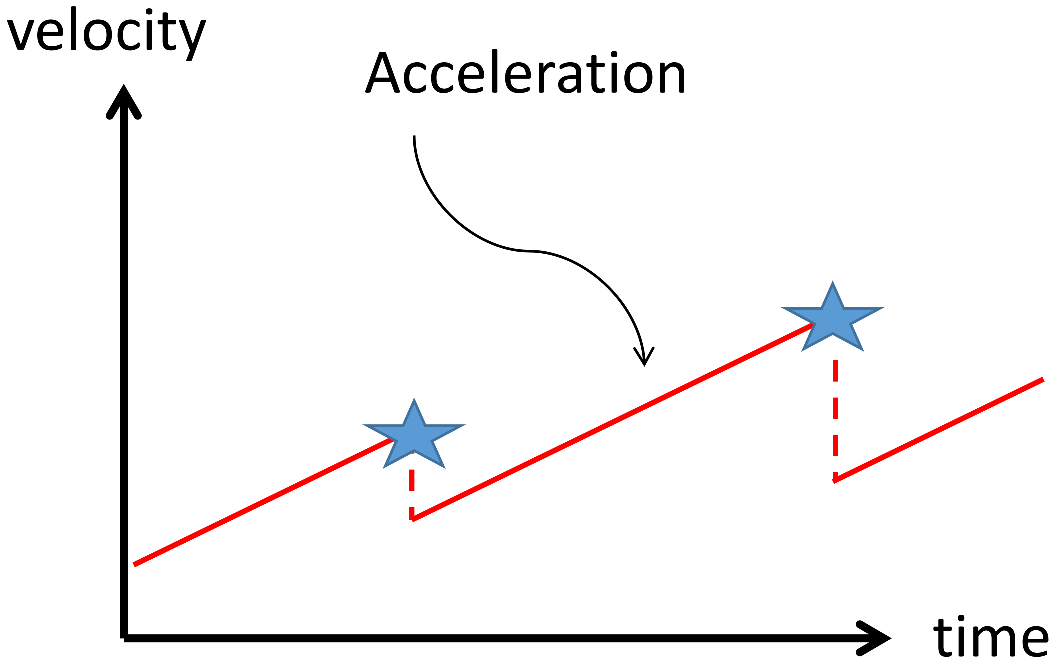
# Drift

- Net movement of charge due to an electric field
  - Since electrons/holes are charged particles, they are accelerated by an electric field. (V/cm)
  - For electrons, $\mathbf{F} = -q\mathbf{E}$ (For holes, $\mathbf{F} = +q\mathbf{E}$ .)
  - According to Newton's 2nd law, the velocity satisfies $\frac{d\mathbf{v}}{dt} = -\frac{q\mathbf{E}}{m_n}$
  - Here, $m_n$ is the conductivity effective mass of electrons.
  - Then, $\mathbf{v}(t) = \mathbf{v}(0) - \frac{q\mathbf{E}}{m_n}t$ (*Right?*)

# Scattering

- The velocity of the carriers…
  - Does not increase indefinitely under the field acceleration. Why?
  - They are scattered frequently and lose the momentum.




Velocity of an electron as a function of time, when scattering events are considered.

# Average velocity

- The velocity of each carrier
  - An indivisual electron exhibits sharp transitions.
- The average velocity
  - However, the average velocity follows a much smoother trajectory.
  - Therefore, it would be better to write

$$\frac{d\langle \mathbf{v} \rangle}{dt} = -\frac{q\mathbf{E}}{m_n} - \frac{\langle \mathbf{v} \rangle}{\tau_n}$$

  - Here, $\tau_n$ is the mean time between collisions.

# Mobility

| | $\mu_n$ | $\mu_p$ |
|---|---|---|
| Si | 1350 | 480 |
| GaAs | 8500 | 400 |
| Ge | 3900 | 1900 |

- Mobility
  - Conduction currents are the result of the drift motion of charge carriers under the influence of an applied electric field.
  - Average drift velocity is directly proportional to the electric field intensity:

$$\langle \mathbf{v} \rangle = -\frac{q\tau_n}{m_n}\mathbf{E} = -\mu_n \mathbf{E}$$

Taur, Eq. (2.26)

Negative sign due to polarity

  - $\mu_n$: Electron mobility in ($cm^2\ V^{-1}\ sec^{-1}$)
  - When the above relation is used, the drift current density becomes

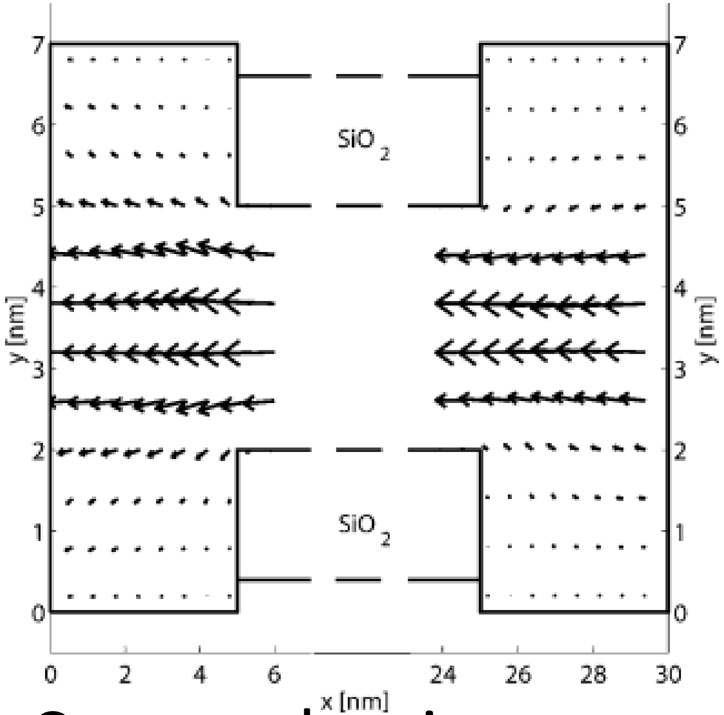
$$\mathbf{J} = q\left(\mu_n n + \mu_p p\right)\mathbf{E} = \sigma \mathbf{E}$$

Taur, Eq. (2.28) and Eq. (2.30)

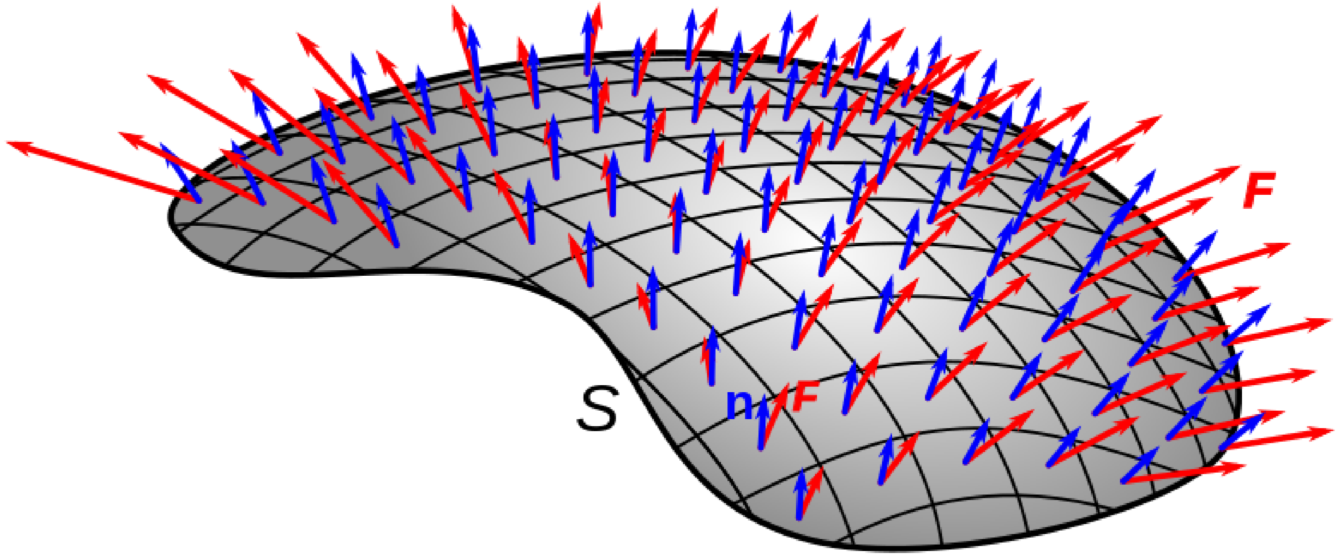
# Terminal current vs. current density

- **J**, current per area (defined everywhere)

$$I_{terminal} = -\int_{terminal} \mathbf{J} \cdot d\mathbf{a}$$




Current density vector (Luisier et al., JAP, vol. 100, p. 043713, 2006)




Surface integral of a vector field (Wikipedia)

# Resistivity & sheet resistivity

- Drift current density

$$\mathbf{J}_{n,drift} = q\mu_n n\mathbf{E}$$ Taur, Eq. (2.28)

$$\mathbf{J}_{p,drift} = q\mu_p p\mathbf{E}$$ Taur, Eq. (2.30)

  – Then, the total resistivity is

Ω cm $\rightarrow$ $$\rho = \frac{1}{q\mu_n n + q\mu_p p}$$ Taur, Eq. (2.32)

- Sheet resistivity
  – The resistance of a conductor of length, $L$, and width, $W$,

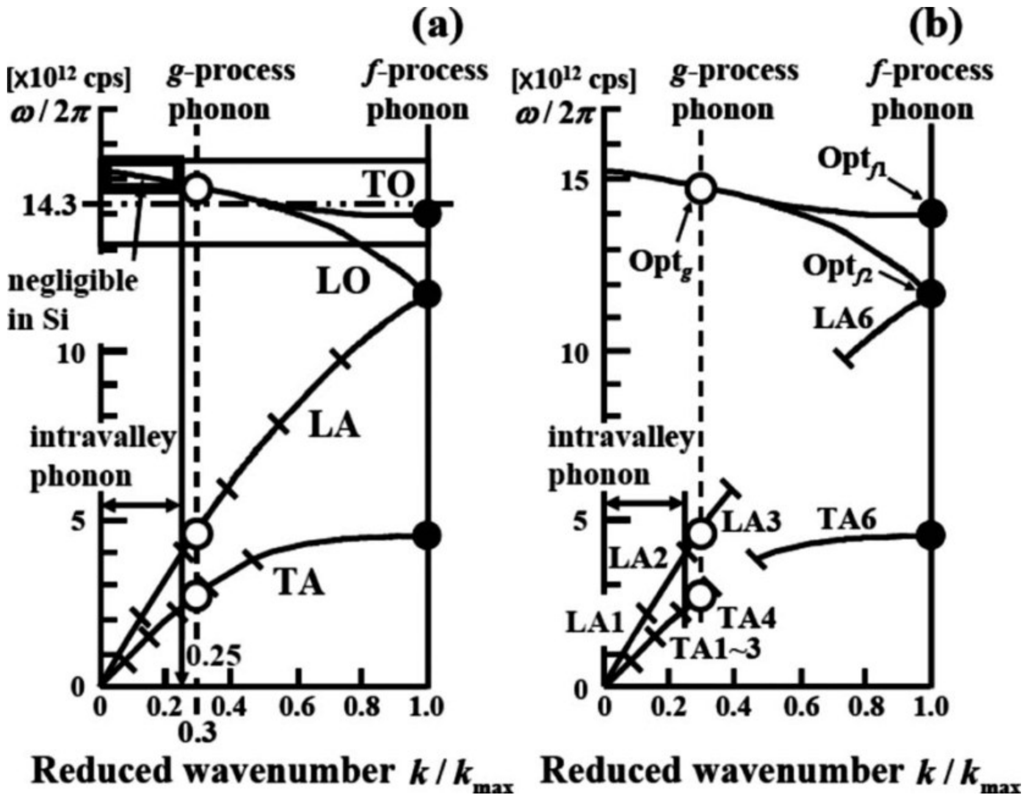
$$R = \frac{L}{W}\rho_{sh}$$ Taur, Eq. (2.35)

Ω/□

# Phonon scattering

- Various phonon modes
  - Acoustic phonon : Low energy
  - Optical phonon : High energy, often treated as dispersion-less

~60 meV



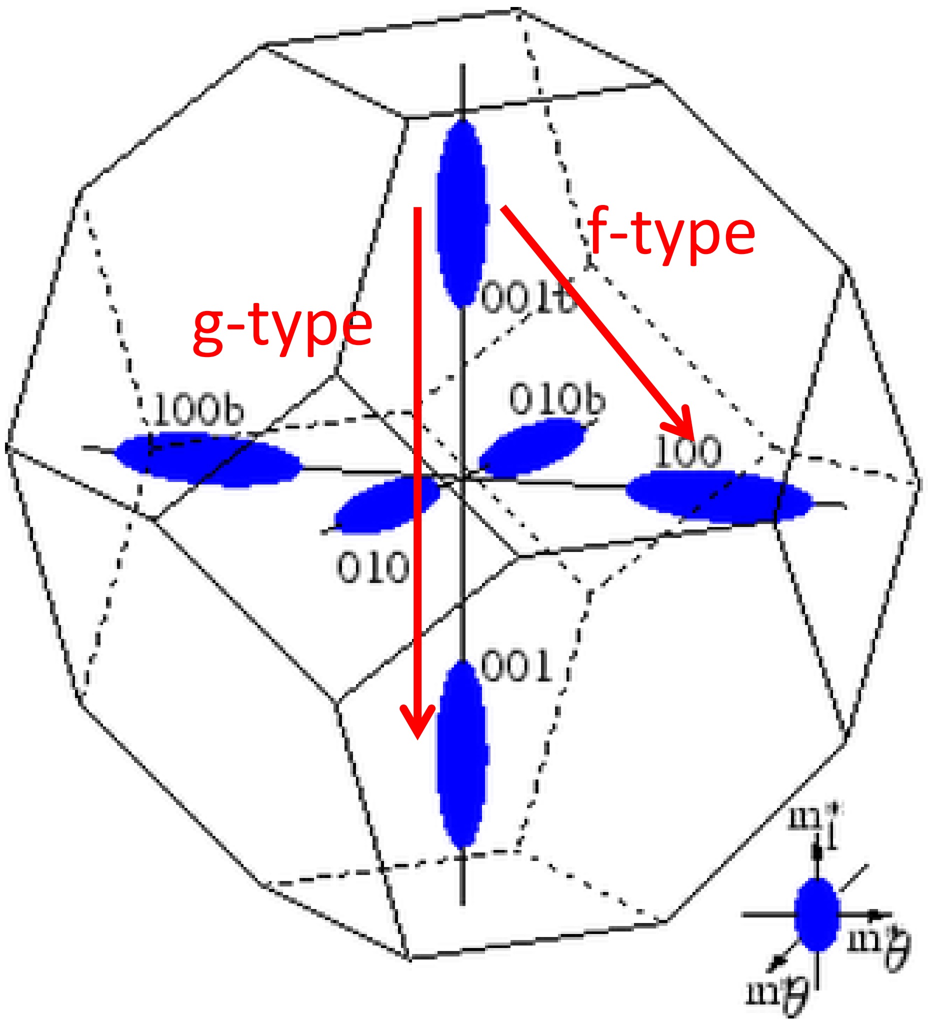

"Selection rule" matters. Intravalley / f-process / g-process

# Typical parameters

- Various phonon modes
  - Acoustic phonon : Low energy

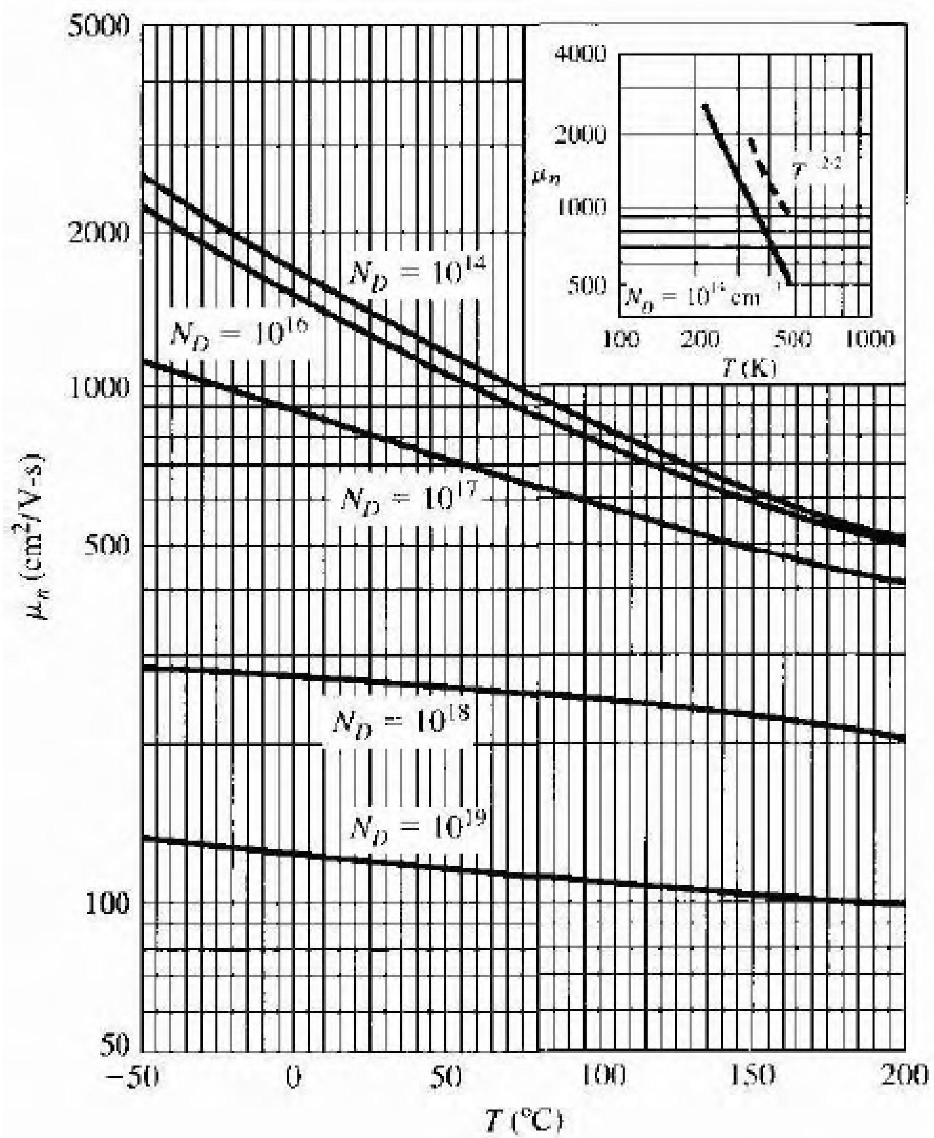
| Mode | $D_t K$ ($10^8$ eV/cm) | $\hbar\omega$ (meV) | Type |
|---|---|---|---|
| TA | 0.470 | 12.1 | g-type |
| LA | 0.740 | 18.5 | g-type |
| LO | 10.23 | 62.0 | g-type |
| TA | 0.280 | 19.0 | f-type |
| LA | 1.860 | 47.4 | f-type |
| LO | 1.860 | 58.6 | f-type |

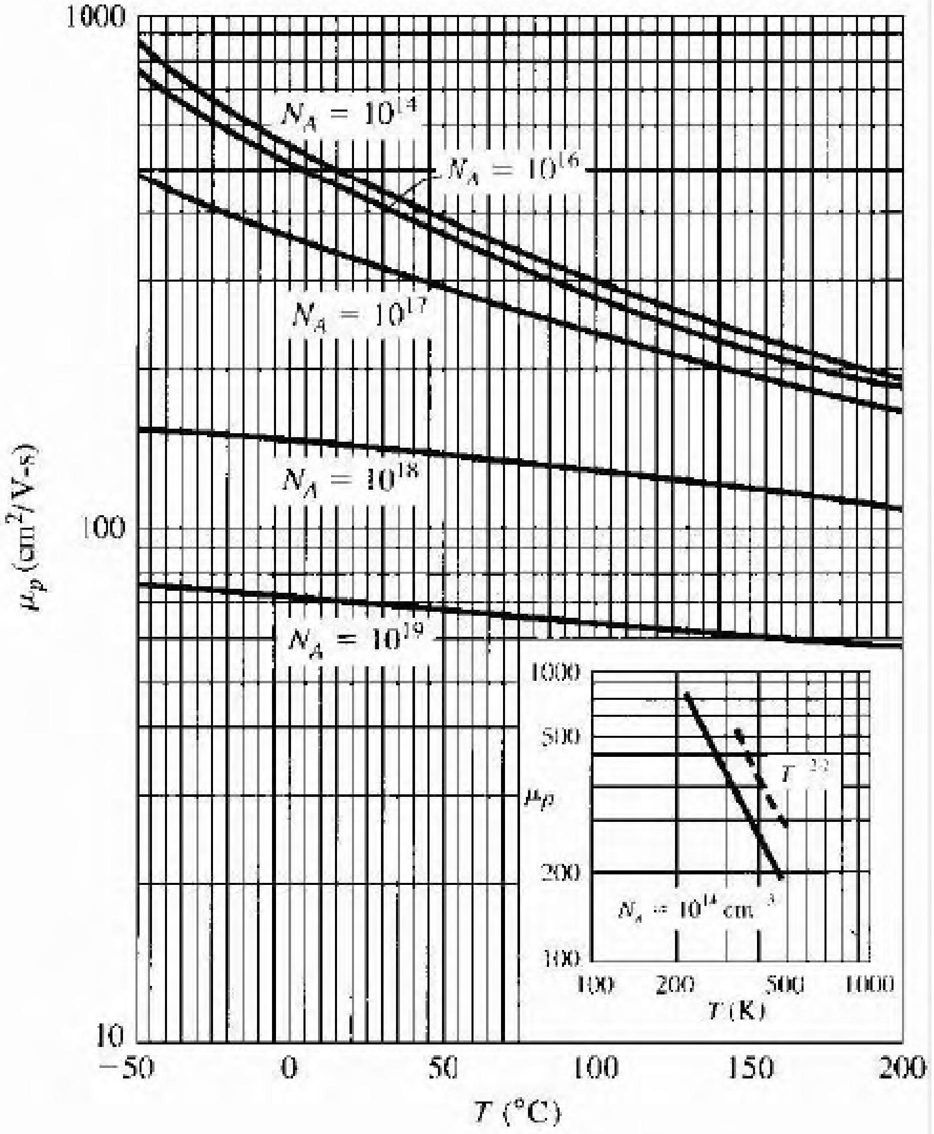
Parameters for inelastic phonon scatterings in the Si conduction band



# Temperature & doping

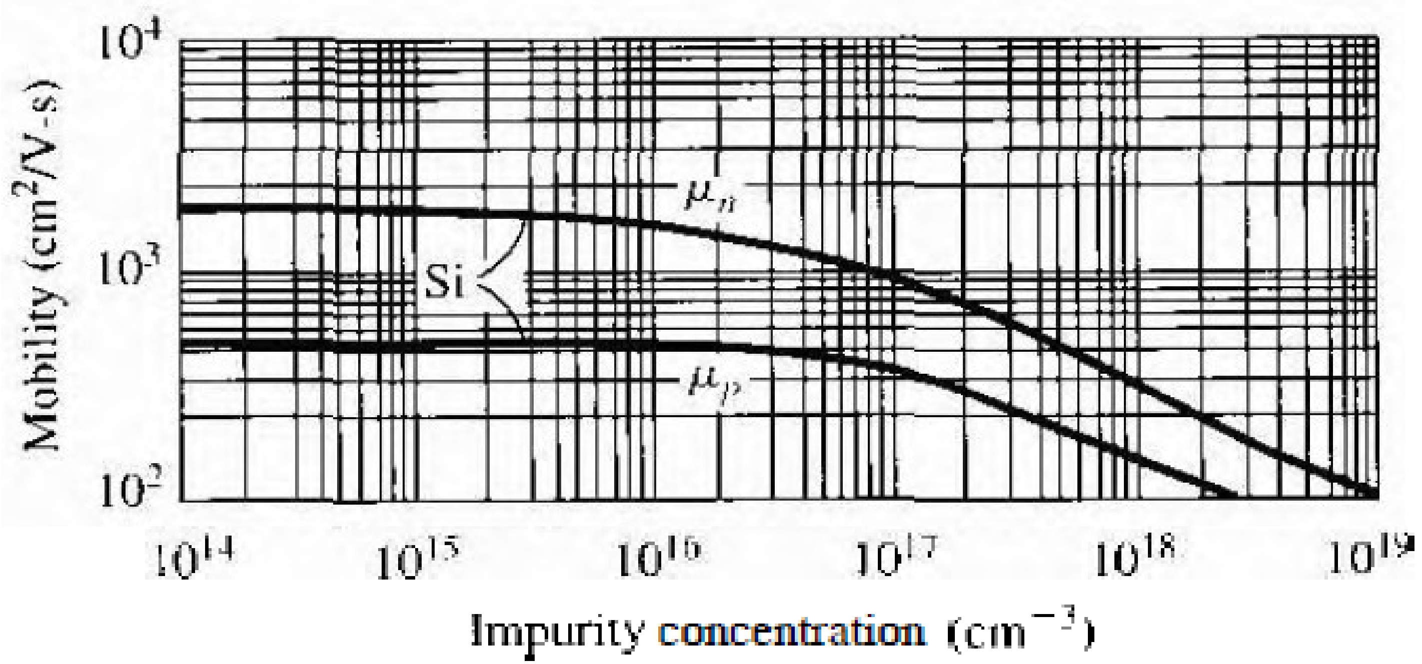
(Neamen's book)



# Impurity concentration

- It is modeled as an elastic scattering process.




(Neamen's book)

- Which one is dominant? Phonon or impurity?
  - Matthiessen's rule, $\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$

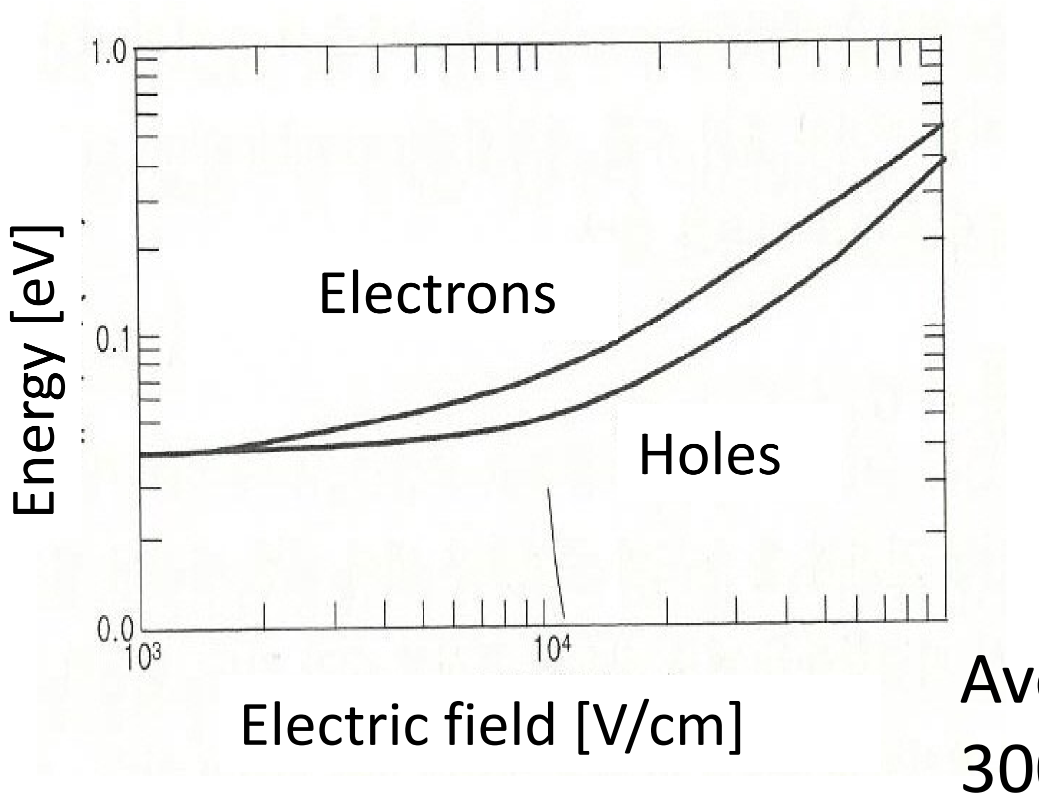
Taur, Eq. (2.27)

# Matthiessen's rule

- When there are multiple contributions to the mobility,
  - (For example, phonon-limited mobility / impurity-limited mobility)
  - The overall collision rate is given by sum of all contributions.
  - $\frac{1}{\tau_m} = \frac{1}{\tau_{mL}} + \frac{1}{\tau_{mI}} + \cdots$
  - (The above relation holds exactly only in the microscopic level.)
  - When recalling $\mu = \frac{q\tau_m}{m_n}$, it means $\frac{1}{\mu_m} = \frac{1}{\mu_{mL}} + \frac{1}{\mu_{mI}} + \cdots$
  - It is very useful.

# Hot electron

- Not only velocity, but also energy…
  - Increases when the electric field increases.
  - Increase of energy is a reason of the velocity saturation. *Why?*



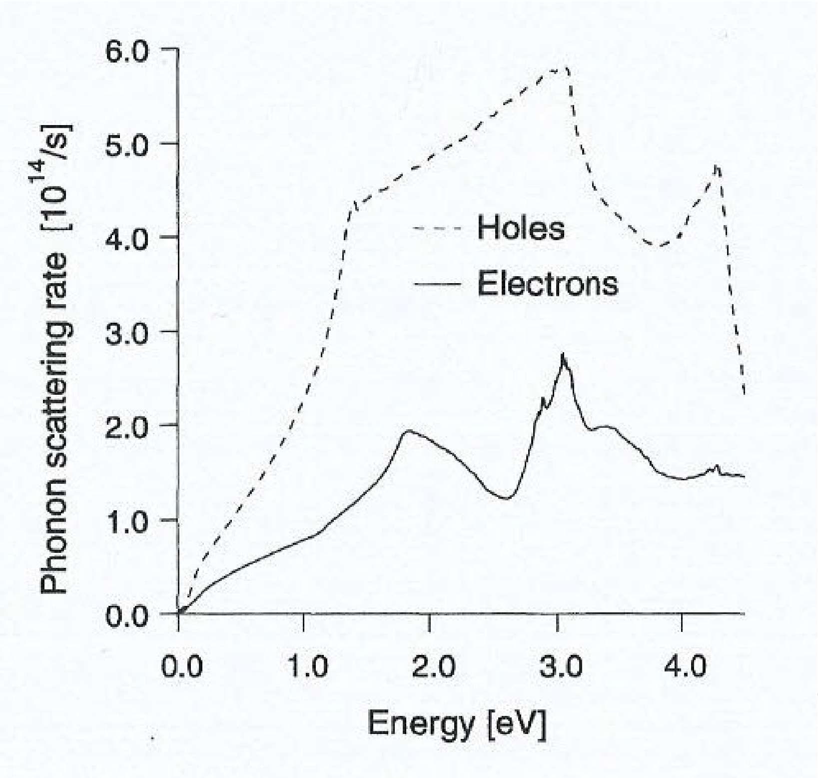

Up to 1 kV/cm, average energy is almost the same with the lattice energy.

Above 10 kV/cm, average energy significantly deviates from the lattice energy.

Average energy of electrons/holes in Si at 300K (Park's book)

# Velocity saturation

- Electron with higher energy
  - Has a higher chance to be scattered by phonons. (Higher DOS)
  - More frequent scattering : Smaller $\tau_m$

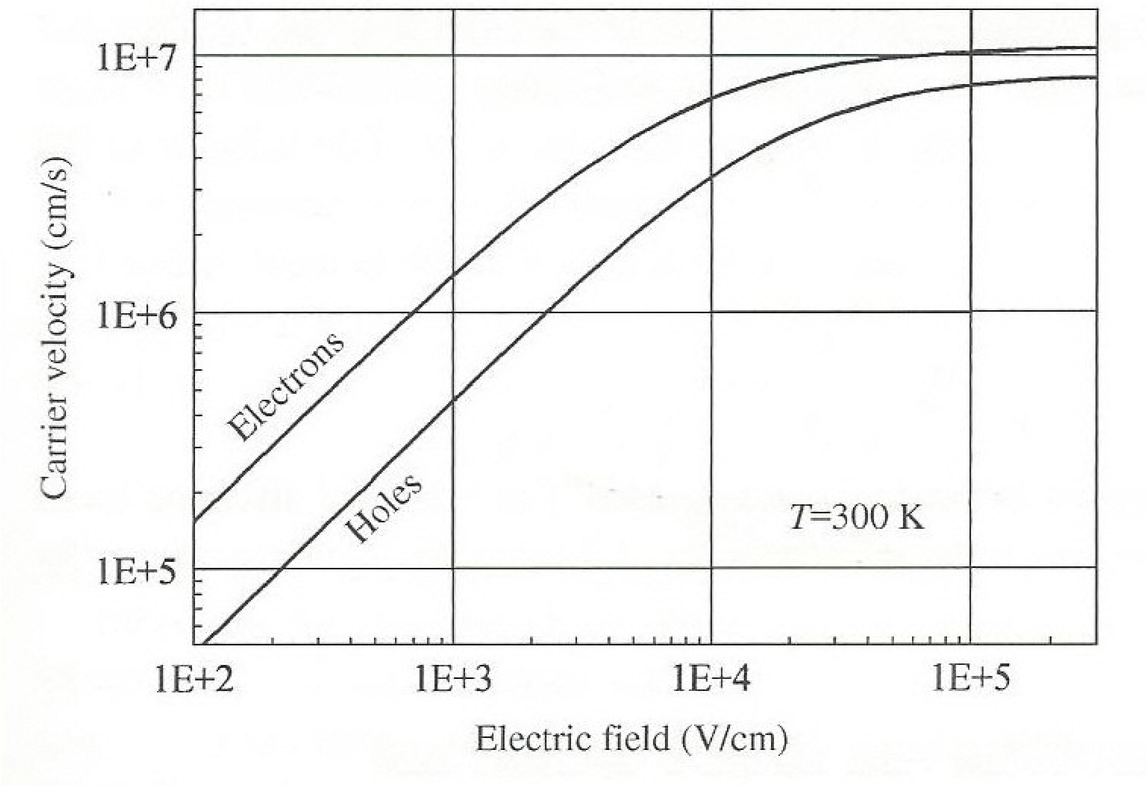



Phonon scattering rate in Si (Jungemann's book)

Phonon scattering rate in Si resembles the Density-Of-States.

# Velocity vs. electric field

- At low electric fields, the linear relationship is valid.
  - At high electric fields, the velocity saturation starts to occur. The saturation velocity of Si is about $10^7$ (cm/sec).




Velocity-field relationship in Si at 300K (Taur's book)

# Caughey-Thomas relation

- For silicon,
  - Electron velocity can be approximated by

$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E}\right)^2\right]^{0.5}}$$

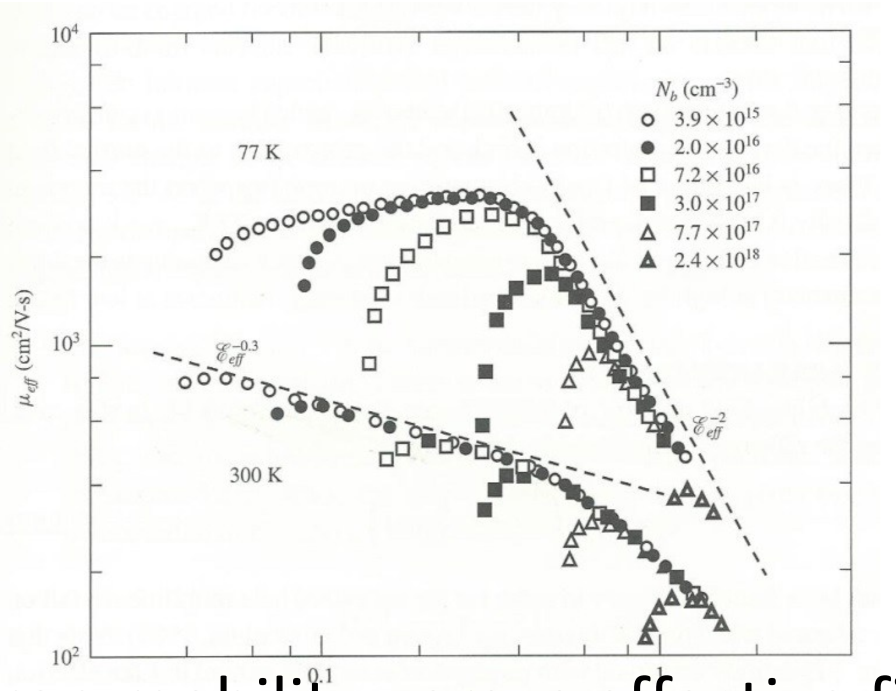
  - Hole velocity

$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E}\right)\right]}$$

  - Why are they different?

# Other scattering mechanisms

- We have discussed about the bulk mobility.
  - Other scattering mechanisms (alloy scattering & impact ionization)
  - Surface scattering severely reduces the inversion mobility.



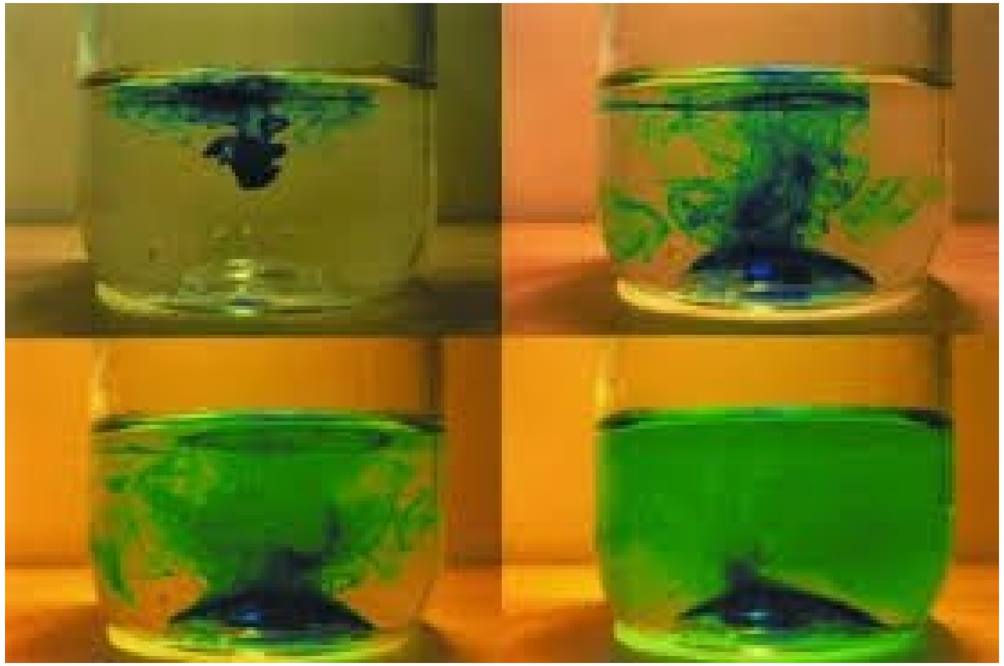

So-called “universal” mobility curve in the Si inversion layer.

Two difference contributions are clearly visible.

Electron mobility versus effective field for several doping concentrations (Takagi’s paper)

# Diffusion

- It is not only for charged particles.
  - For example,



Diffusion of ink
(Google images)

  - Therefore, no polarity is expected.

# Equation

- Flux
  - The electron flux due to the diffusion mechanism is given by

$$\mathbf{F}_n = -D_n \nabla n$$

  where $D_n$ is the electron diffusion coefficient in the unit of (cm$^2$/sec).
  - The diffusion current density is

$$\mathbf{J}_{n,diff} = qD_n \nabla n \qquad \text{Taur, Eq. (2.36)}$$

- How about the hole?
  - The diffusion current density is

$$\mathbf{J}_{p,diff} = -qD_p \nabla p \qquad \text{Taur, Eq. (2.37)}$$

# Revisit the total current density.

- Total current density
  - Electron current density

$$\mathbf{J}_n = q\mu_n n\mathbf{E} + qD_n \nabla n$$

Taur, Eq. (2.54)

  - Hole current density

$$\mathbf{J}_p = q\mu_p p\mathbf{E} - qD_p \nabla p$$

Taur, Eq. (2.55)

  - (Time-dependent) displacement current density

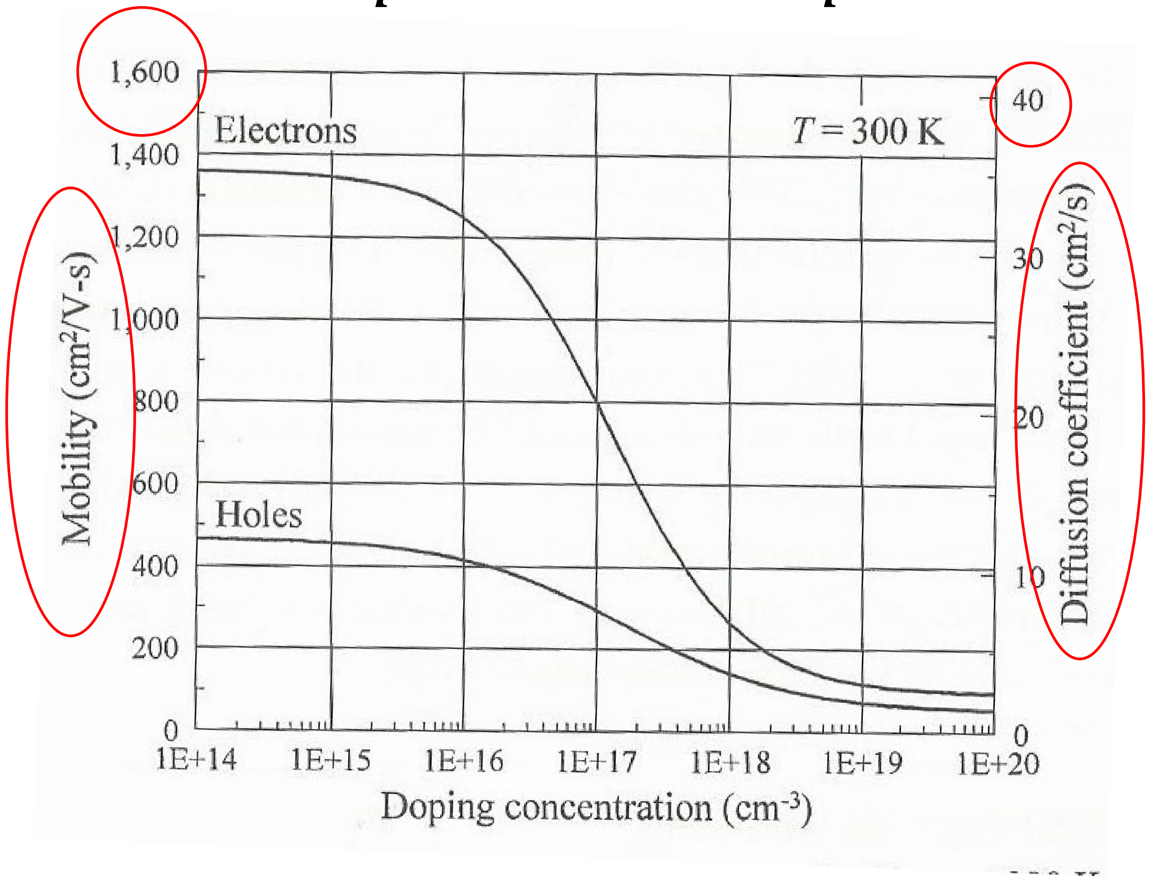
$$\mathbf{J}_{displacement} = \frac{\partial}{\partial t}(\epsilon\mathbf{E})$$

# Einstein relation

- At equilibrium, we have the following relations:

$$D_n = \frac{k_B T}{q}\mu_n, D_p = \frac{k_B T}{q}\mu_p$$

Taur, Eq. (2.38) and Eq. (2.39)




(Park's book)

# Current density with Einstein relation

- Alternative forms
  - Electron current density

$$\mathbf{J}_n = -q\mu_n n\left[\nabla\phi - \frac{k_B T}{q}\frac{1}{n}\nabla n\right] \qquad \text{Taur, Eq. (2.56)}$$

  - Hole current density

$$\mathbf{J}_p = -q\mu_p p\left[\nabla\phi + \frac{k_B T}{q}\frac{1}{p}\nabla p\right] \qquad \text{Taur, Eq. (2.57)}$$

# Revisiting carrier concentrations

$$E_f = -q\phi_f$$

- Carrier densities are expressed as

$$n = n_i \exp\left(-\frac{E_i - E_f}{k_B T}\right) = n_i \exp\left(\frac{\phi - \phi_f}{k_B T/q}\right)$$ Taur, Eq. (2.49)

$$p = n_i \exp\left(\frac{E_i - E_f}{k_B T}\right) = n_i \exp\left(-\frac{\phi - \phi_f}{k_B T/q}\right)$$ Taur, Eq. (2.50)

  – These relations are generally applicable in the presence of net charge and band bending.

# Extension to non-equilibrium cases

- Of course, at non-equilibrium cases, we cannot define $E_f$.
  - However, we *introduce* $\phi_n$ and $\phi_p$ to satisfy:

$$n = n_i \exp\left(\frac{\phi - \phi_n}{k_B T/q}\right)$$

Taur, Eq. (2.61)

$$p = n_i \exp\left(-\frac{\phi - \phi_p}{k_B T/q}\right)$$

Taur, Eq. (2.62)

  - They are called quasi-Fermi potentials. (Electron quasi-Fermi potential and hole quasi-Fermi potential)

# Current density with $\nabla\phi_n$ and $\nabla\phi_p$

- From the expression for the quasi-Fermi potential,
  - It can be written as

$$\phi_n = \phi - \frac{k_B T}{q} \log \frac{n}{n_i}$$

Taur, Eq. (2.65)

  - Taking the gradient,

$$\nabla\phi_n = \nabla\phi - \frac{k_B T}{q} \frac{\nabla n}{n}$$

  - Using this expression,

$$\mathbf{J}_n = -q\mu_n n \left[ \nabla\phi - \frac{k_B T}{q} \frac{1}{n} \nabla n \right] = -q\mu_n n \nabla\phi_n$$

Taur, Eq. (2.63)

$$\mathbf{J}_p = -q\mu_p p \nabla\phi_p$$

Taur, Eq. (2.64)

# Gradient of quasi-Fermi potential

- We have the following relations,

$$\mathbf{J}_n = -\,q\mu_n n \nabla\phi_n$$

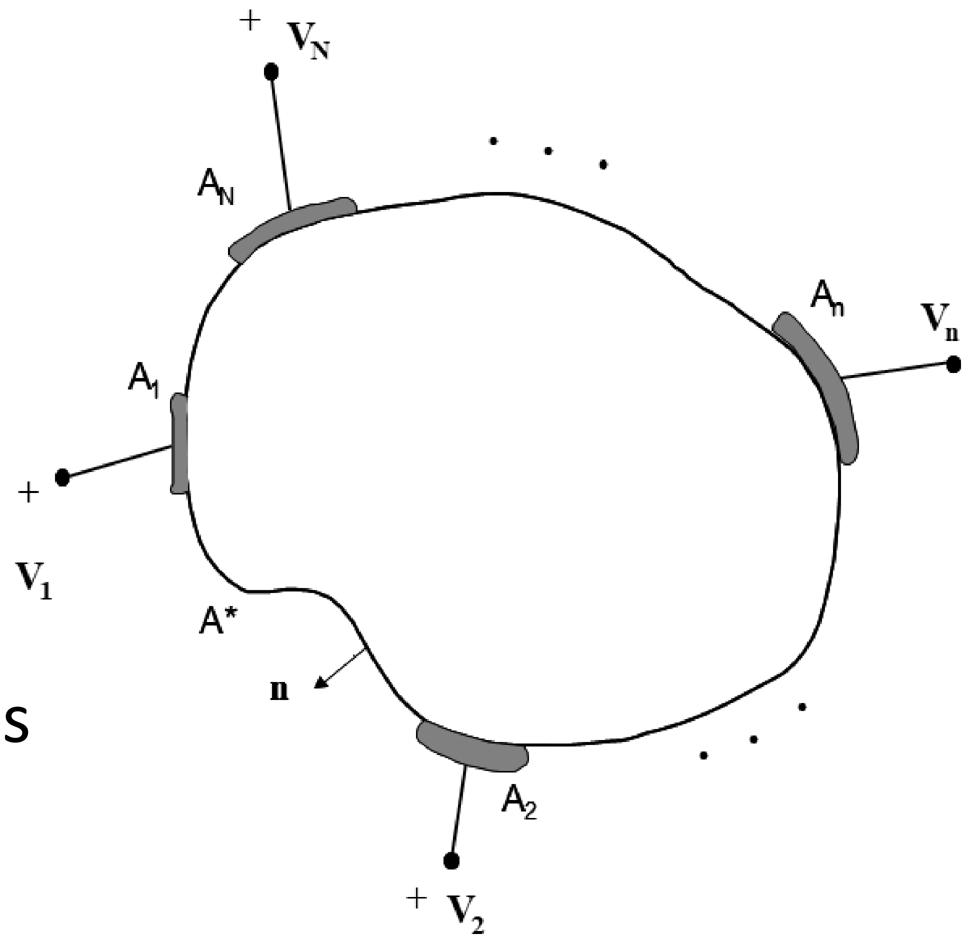
$$\mathbf{J}_p = -q\mu_p p \nabla\phi_p$$

  – The gradient of electron quasi-Fermi potential drives the electron current.
  – The gradient of hole quasi-Fermi potential drives the hole current.

# Boundary conditions for $\phi_n$ and $\phi_p$

- Dirichlet boundary condition
  - At terminals,
    $\phi_n = \phi_p = V_{app}$

- Neumann boundary condition
  - No current through non-contact surfaces
    $\nabla\phi_n \cdot \mathbf{n} = \nabla\phi_p \cdot \mathbf{n} = 0$



# Thank you!