VLSI Devices
Lecture 11

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Drain current

$$\mathbf{J}_n = -q\mu_n n \nabla\phi_n$$

- Electron current density at a point (x, y)

$$J_n(x,y) = -q\mu_n n(x,y)\frac{dV}{dy}$$ Taur, Eq. (3.5)

  - (It includes both the drift and diffusion currents.)
  - When integrated from $x = 0$ to x_i, (and from $z = 0$ to W)

$$I_d(y) = qW\int_0^{x_i} \mu_n n(x,y)\frac{dV}{dy}dx$$ Taur, Eq. (3.6)

Sign change due to convention of terminal current

z-directional width

Further simplification

- Electron current density at a point (x, y)

$$I_d(y) = qW \int_0^{x_i} \mu_n n(x,y) \frac{dV}{dy} dx = -\mu_{eff} W \frac{dV}{dy} \left(-q \int_0^{x_i} n(x,y) dx \right)$$

$$= -\mu_{eff} W \frac{dV}{dy} Q_i(y)$$

Taur, Eq. (3.8)

  - We introduce an effective mobility, μ_{eff}.
  - Since V is a function of y only, V is interchangeable with y.

$$Q_i(y) = Q_i(V)$$

  - Then,

$$I_d(y) dy = \mu_{eff} W [-Q_i(V)] dV$$

$I_d(y)$ is actually a constant.

- When integrated from $y = 0$ to L, (from $V = 0$ to V_{ds})

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV$$ Taur, Eq. (3.10)

  - Then, how can we find $Q_i(V)$? (We must perform the x-directional integration.)

$$Q_i = -q \int_0^{x_i} n(x,y) dx = -q \int_{\phi_s}^{\delta} n(\phi, V) \frac{dx}{d\phi} d\phi$$

It's small, but not zero.

$$= -q \int_{\delta}^{\phi_s} \frac{(n_i^2/N_a) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$ Taur, Eq. (3.12)

Pao-Sah double integral

- Finally, the expression for I_d reads

$$I_d = q\mu_{eff}\frac{W}{L}\int_0^{V_{ds}}\left[\int_{\delta}^{\phi_s}\frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_BT}(\phi - V)\right)}{E_x(\phi,V)}d\phi\right]dV$$

Taur, Eq. (3.13)

  - It is the Pao-Sah double integral.
  - Rigorous within the GCA, but it is difficult to evaluate.

How can we determine ϕ_s?

- For given V_{gs} and V, we can solve the MOS equation.

$$V_{gs} = V_{fb} + \phi_s - \frac{Q_s}{C_{ox}}$$

$$= V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T}(\phi_s - V)\right)\right]^{1/2}$$

Taur, Eq. (3.14)

Only two important terms are kept.

 - We can numerically solve the above equation to obtain ϕ_s. (Newton method)

Write a code. (1) 🡰 These are optional.

- Calculate I_d at V_{gs} = 1.1 V and V_{ds} = 0.5 V.
  - We follow the Riemann sum approach.
  - Then, let's introduce 101 V values, from 0 V to 0.5 V. (5 mV spacing)

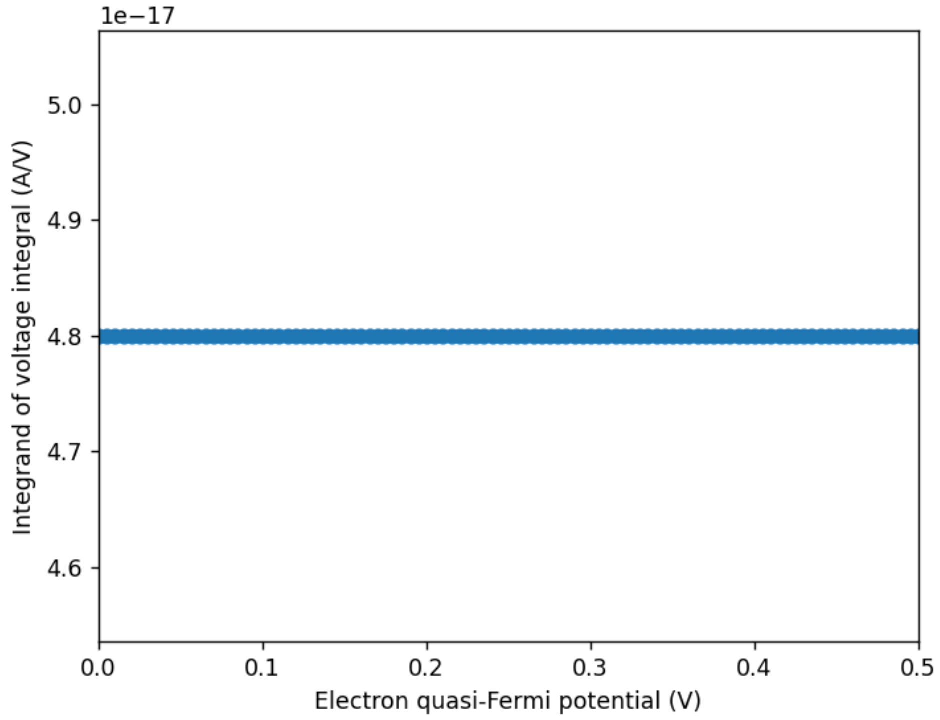
$$I_d \approx q\mu_{eff}\frac{W}{L}\sum_{j=0}^{100}\left[\int_{\delta}^{\phi_s}\frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_B T}\left(\phi - V_j\right)\right)}{E_x\left(\phi, V_j\right)}d\phi\right]\Delta V_j$$

  - When j = 0 or j = 100, ΔV_j is 2.5 mV.
  - Otherwise, ΔV_j is 5 mV.
  - Use $q\mu_{eff}\frac{W}{L}$ = 4.8 X 10^{-17} A cm^2 V^{-1}. (A rough estimation for n_i, 10^{10} cm^{-3}, is used.)

Write a code. (2)

- Test the integral.
  - Assume that $\left[\int_{\delta}^{\phi_s} \frac{(n_i^2/N_a)\exp\left(\frac{q}{k_B T}(\phi - V_j)\right)}{E_x(\phi, V_j)} d\phi\right]$ = 1 cm^{-2}.
  - Then, the integral shoud be 2.4 X 10^{-17} A.
  - Check your code.

2.39999999999999955e-17



Write a code. (3)

- Now, we must perform the ϕ-integral.

$$\int_{\delta}^{\phi_s} \frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_B T}\left(\phi - V_j\right)\right)}{E_x\left(\phi, V_j\right)} d\phi$$

  - We must know two bounds, δ and ϕ_s.
  - We can set a small value for δ. (It doesn't matter. $\delta = V_j$ + 1 mV, for example)
  - However, ϕ_s is not easy to evaluate. (We must adopt the Newton method.)
  - Instead, an approximate value for ϕ_s will be used.

Write a code. (4)

- The ϕ-integral is approximated as

$$\int_{\delta}^{\phi_s} \frac{(n_i^2/N_a)\exp\left(\frac{q}{k_B T}(\phi - V_j)\right)}{E_x(\phi, V_j)} d\phi$$

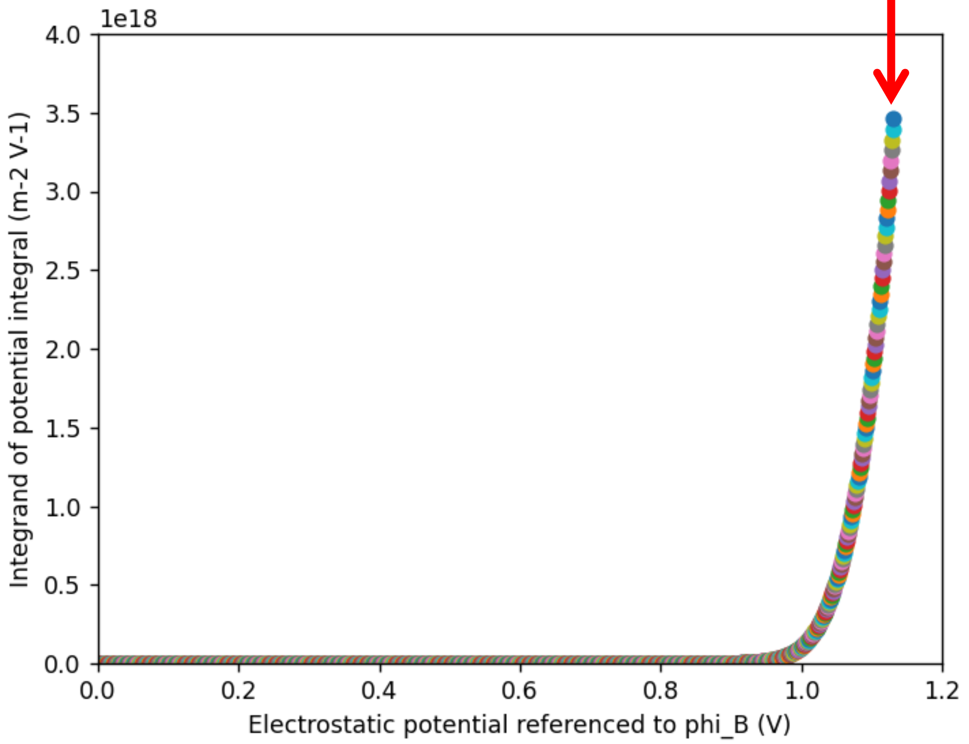
$$\approx \sum_{k=1}^{k_s} \frac{(n_i^2/N_a)\exp\left(\frac{q}{k_B T}(k\ \mathrm{mV})\right)}{E_x(V_j + k\ \mathrm{mV}, V_j)} \times 1\mathrm{mV}$$

  - Here, $V_j + k_s$ mV is a good approximation of ϕ_s.
  - Parameters: V_{fb} is -1.08 V. C_{ox} is 2.88X10^{-6} F/cm^2. (t_{ox} is 1.2 nm.) N_a is 10^{18} cm^{-3}.

Write a code. (5)

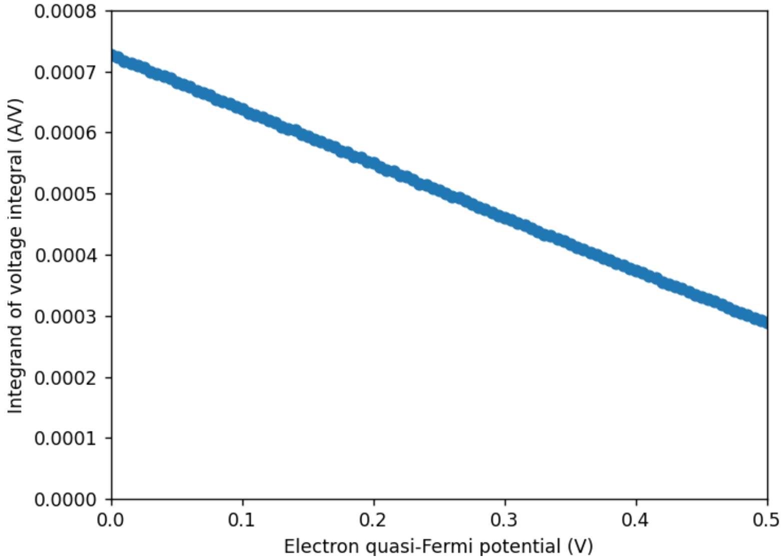
- A specific example, when V_j is 0 V.
  - Draw $\dfrac{(n_i^2/N_a)\exp\left(\frac{q}{k_B T}(\phi - V_j)\right)}{E_x(\phi, V_j)}$ as a function of ϕ.
  - Stopped when the MOS equation changes its sign. (Approximate ϕ_s)
  - Integrated electron density is about 1.517 X 10^{13} cm^{-2}.

Approximate ϕ_s

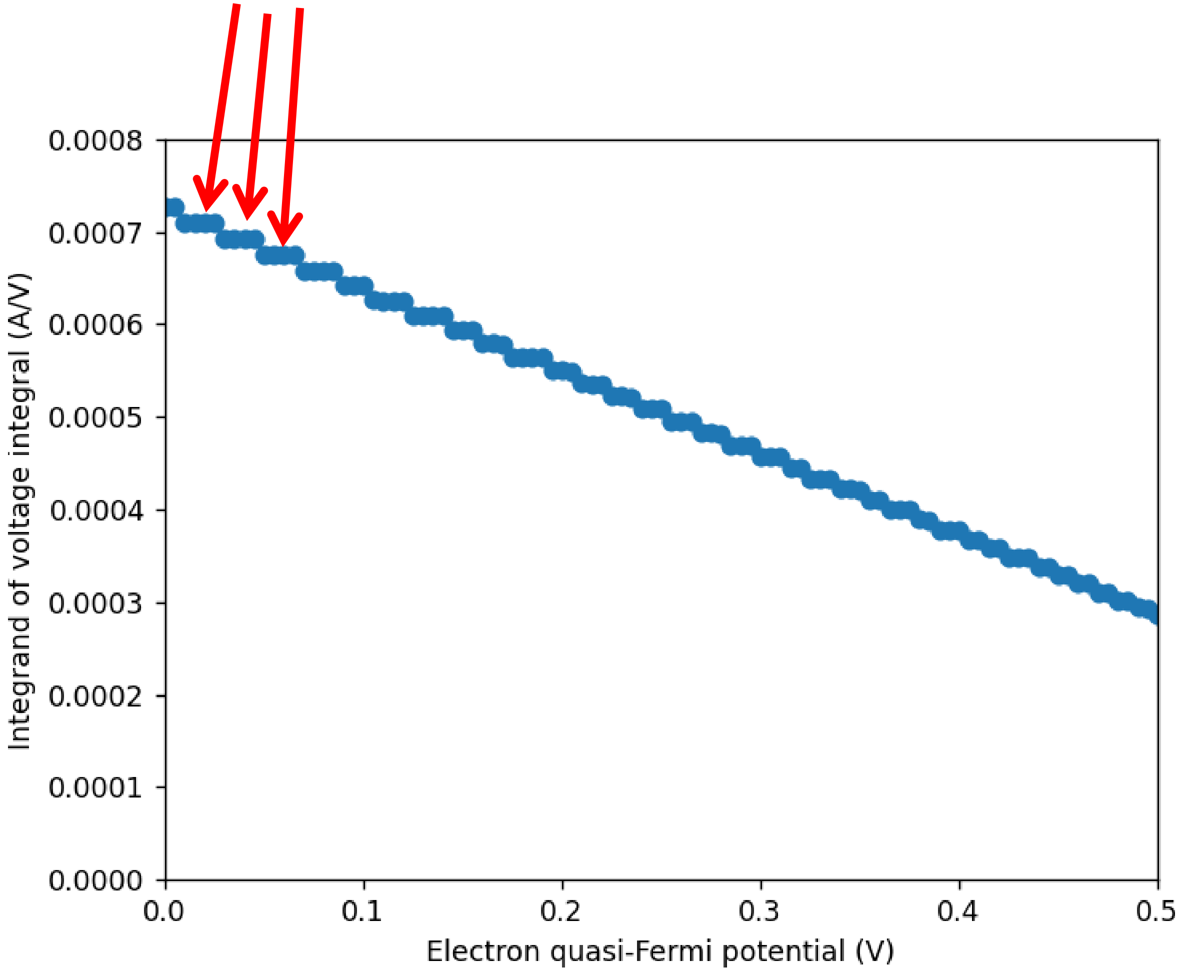


Write a code. (6)

- Integrand of the V-integral
  - By integrating it, we can find I_d = 252.8 μA.
  - Instead of 1 mV, we can try an even finer ϕ spacing.
  - With 0.2 mV, I_d = 252.9 μA.

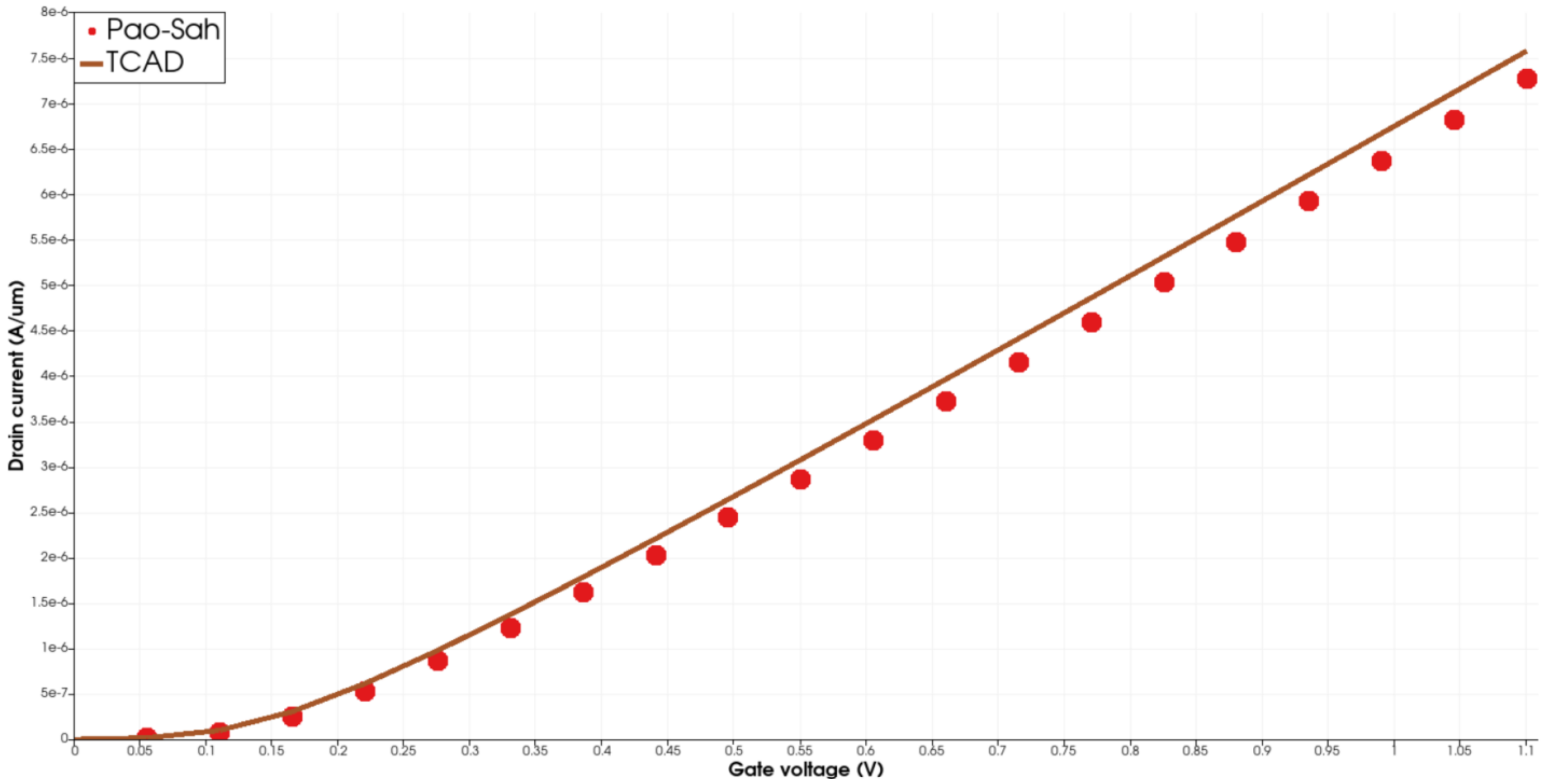


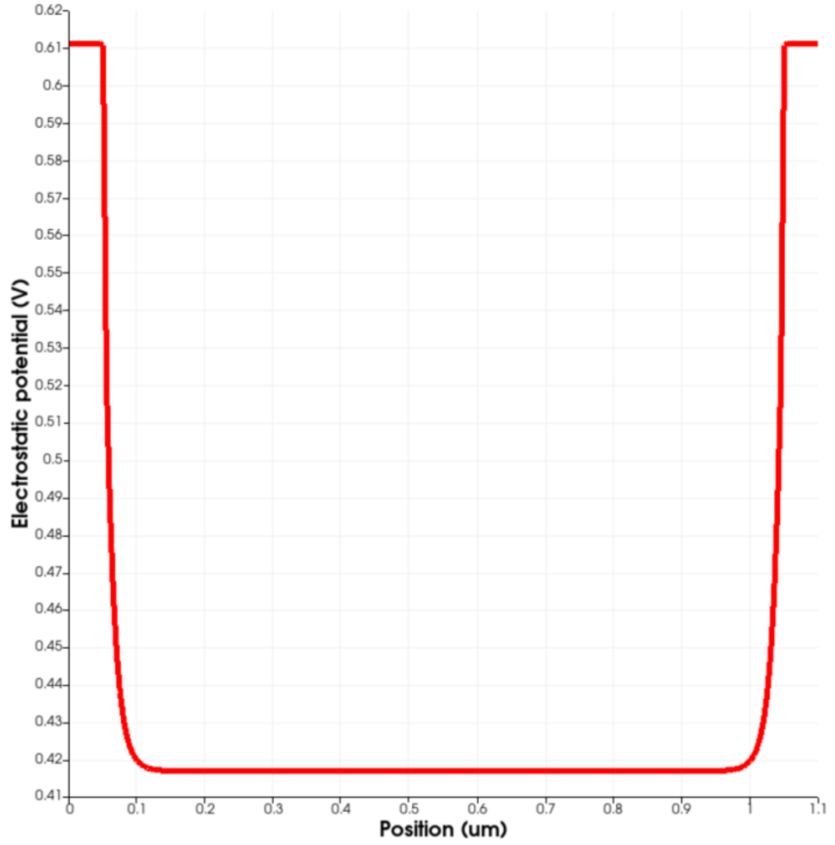
Step-wise???



Verify the result.

- TCAD simulation result (at V_{ds} = 0.01 V)
  - They are similar, but not perfectly matched. *Why?*





Charge-sheet model

- Simpler model with further approximations
  - Consider the previous method to calculate Q_i:

$$Q_i = -q \int_{\delta}^{\phi_s} \frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$

  - A simpler way? Instead, Q_d is approximated as

$$Q_d = -qN_a W_d = -\sqrt{2\epsilon_{si} q N_a \phi_s}$$ Taur, Eq. (3.15)

  - Then, Q_i can be approximated as

$$Q_i = Q_s - Q_d = -C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si} q N_a \phi_s}$$

Taur, Eq. (3.16)

(Of course, it is not exact.)

Thank you!