

VLSI Devices

Lecture 9

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MOS equation

- Up to now, $Q_s(\phi_s)$ is found. We can control only V_g .

Total silicon
charge per
unit area

- Relation between V_g and ϕ_s

$$V_g - V_{fb} = V_{ox} + \phi_s = -\frac{Q_s}{C_{ox}} + \phi_s \quad \text{Taur, Eq. (2.195)}$$

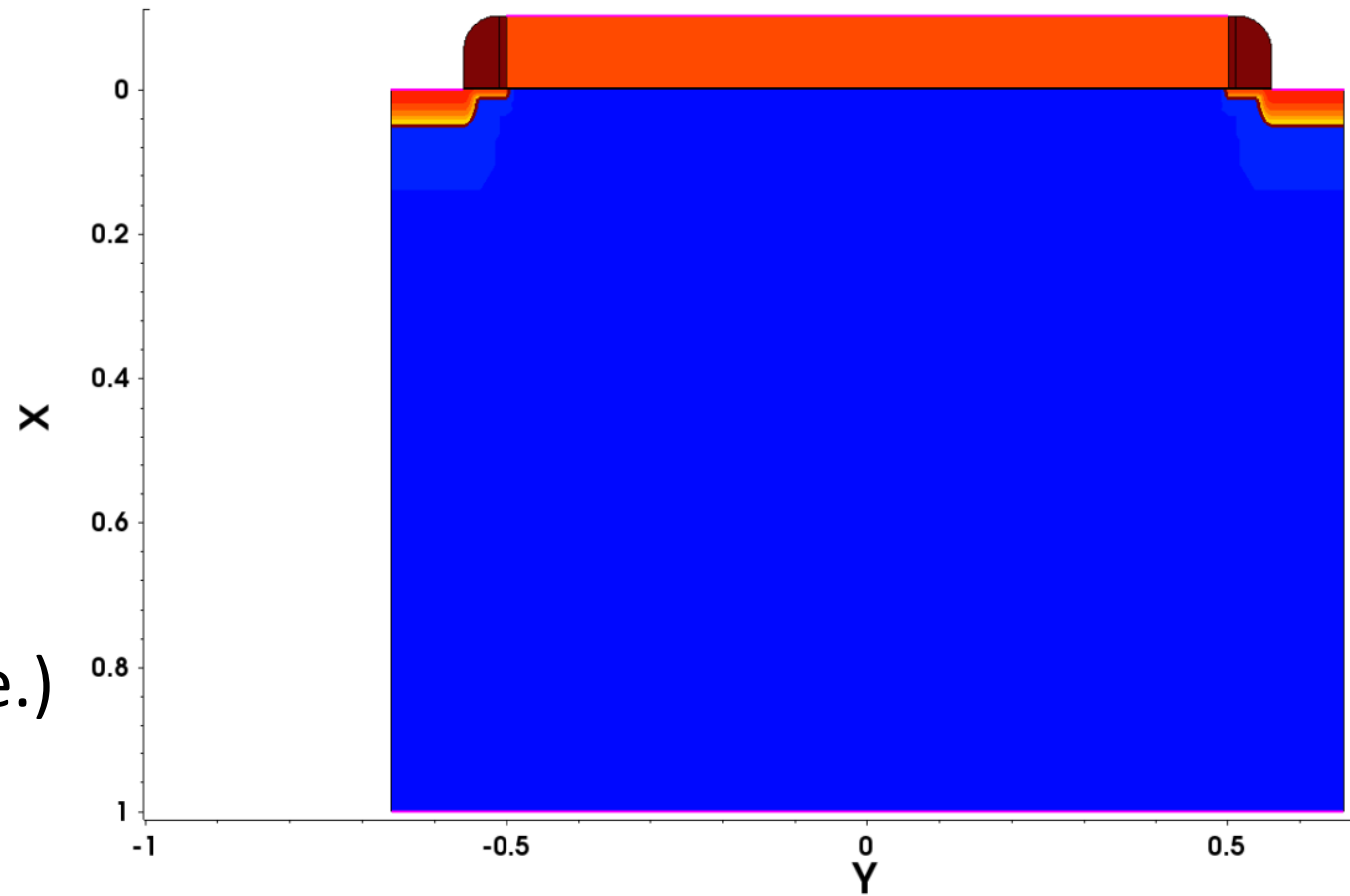
$\frac{\epsilon_{ox}}{t_{ox}}$, oxide capacitance per unit area

- In general, $Q_s(\phi_s)$ is known. We can solve the above equation.

Taur, Eq. (2.182)

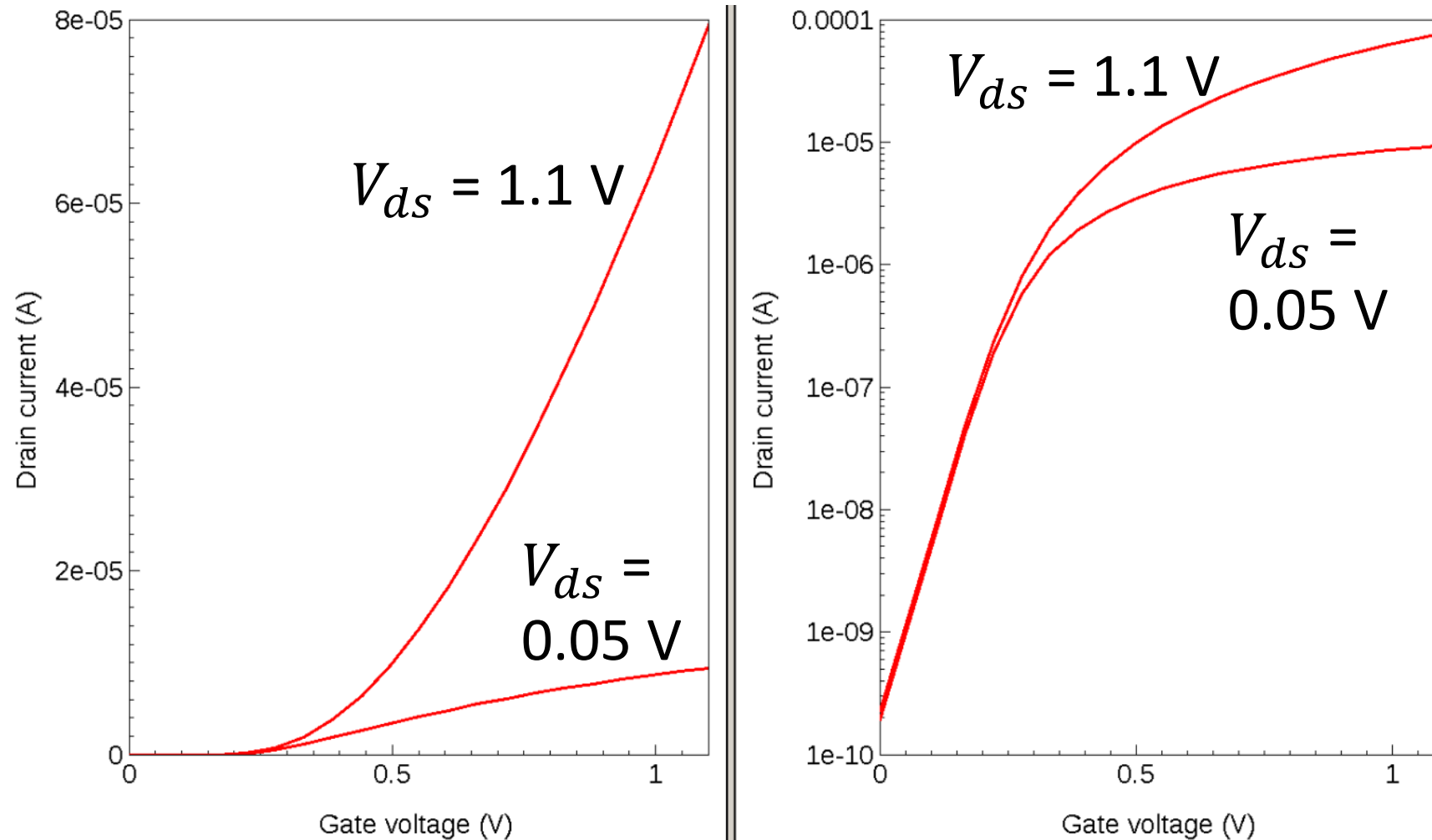
TCAD simulation of a long-channel MOSFET

- Channel length, 1 μm
 - V_{fb} is -1.08 V.
 - t_{ox} is 1.2 nm.
 - C_{ox} is 2.88×10^{-6} F/cm².
 - V_{DD} of 1.1 V
 - (Estimate its technology node.)



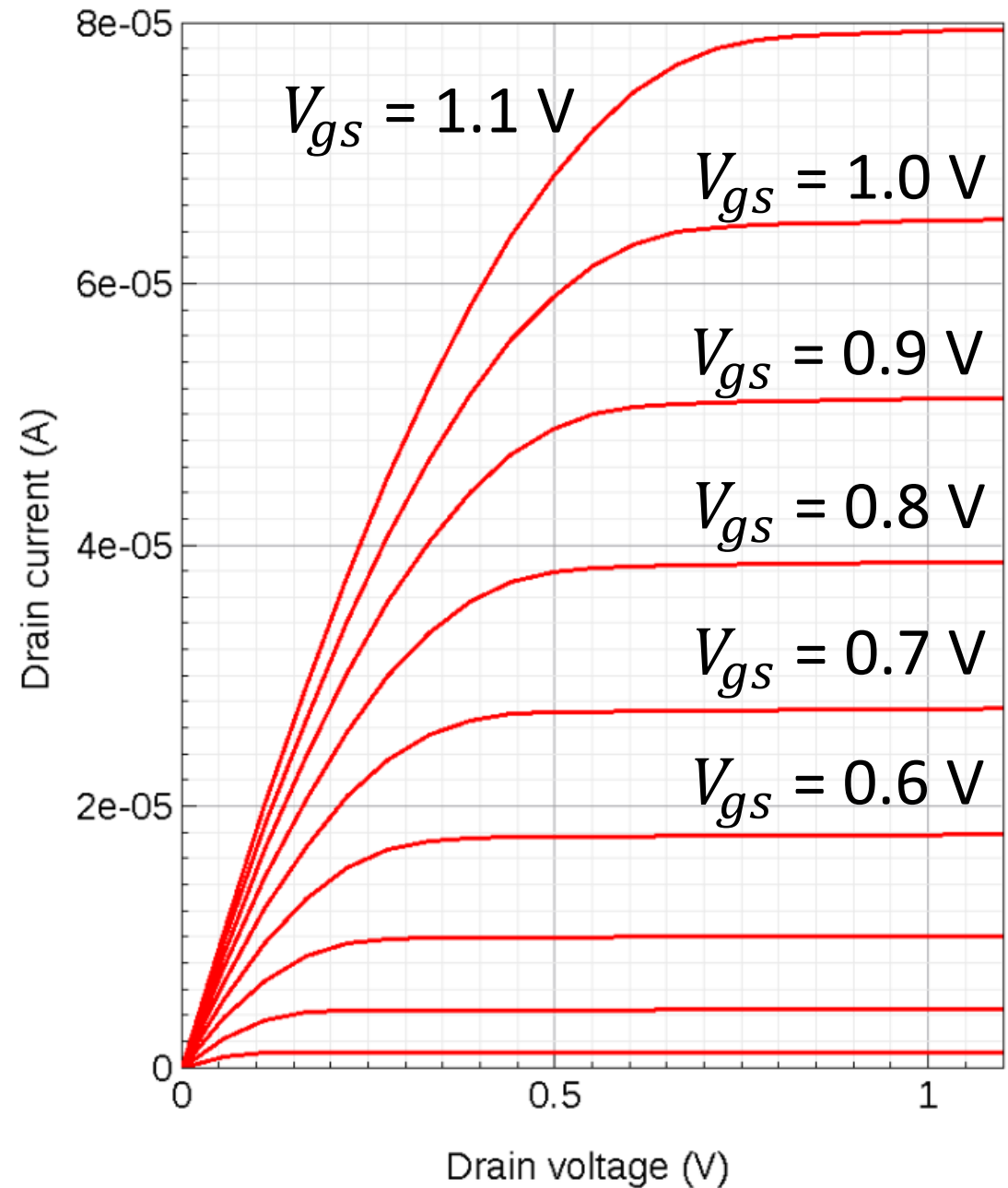
I_d versus V_{gs}

- Input characteristics



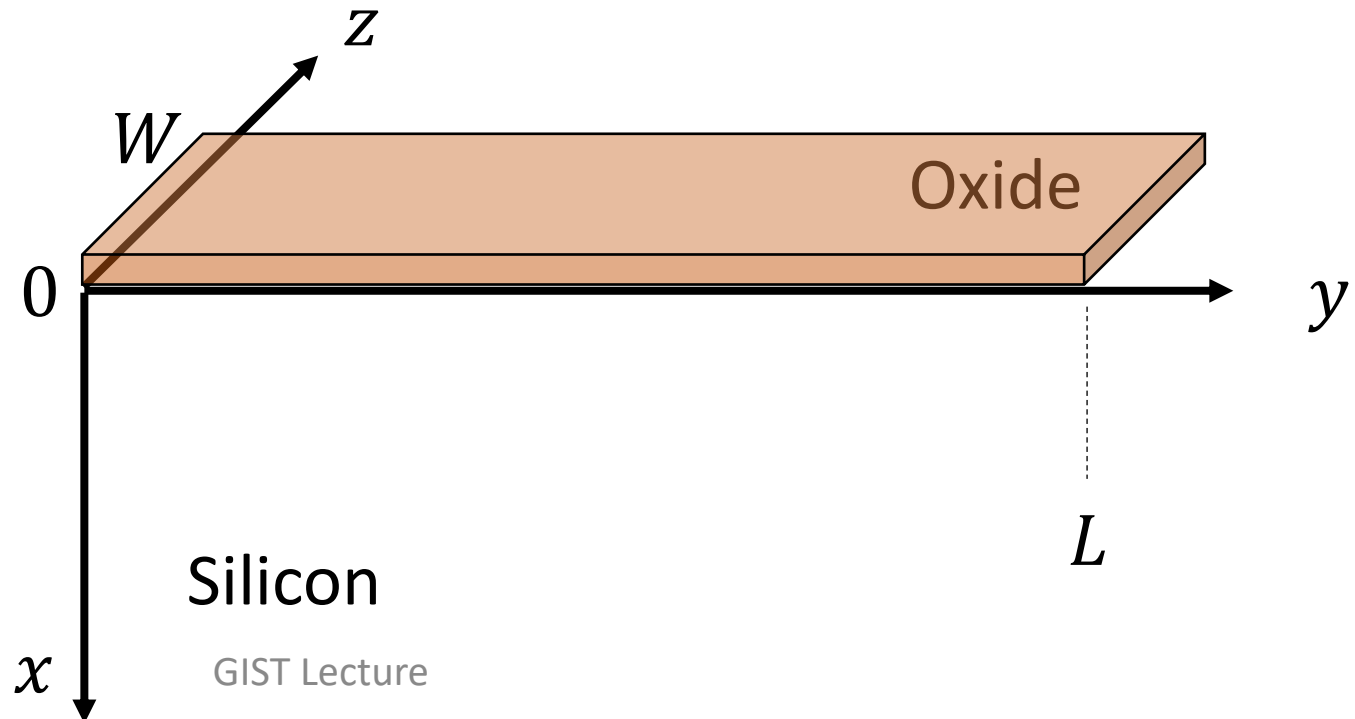
I_d versus V_{ds}

- Output characteristics
 - Triode mode
 - Saturation mode



Schematic

- $x = 0$ at silicon surface
 - $y = 0$ at the source and $y = L$ at the drain
 - Source and substrate are grounded.
 - Uniform p-type substrate



Gradual channel approximation

- Variation of the electric field in the y -direction is much less than the corresponding variation in the x -direction.

$$\left| \frac{\partial^2 \phi}{\partial x^2} \right| \gg \left| \frac{\partial^2 \phi}{\partial y^2} \right|$$

– Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\frac{q}{\epsilon_{si}} [p(x, y) - n(x, y) - N_a]$$

– Poisson equation under the GCA

$$\frac{d^2 \phi}{dx^2} = -\frac{q}{\epsilon_{si}} [p(x, y) - n(x, y) - N_a] \quad \text{Taur, Eq. (2.175)}$$

Electron quasi-Fermi potential, $V(y)$

It was written as ϕ_n in Taur, Eq. (2.61).

- It is assumed that V is independent of x .
 - *Why?* MOSFET current flows predominantly along the y -direction.
 - Since $\mathbf{J}_n = -q\mu_n n \nabla \phi_n$, V varies mainly along the y -direction.
 - Boundary conditions:

$$\begin{aligned} V(y = 0) &= V_s = 0 \\ V(y = L) &= V_d = V_{ds} \end{aligned}$$

- Electron density, $n(x, y)$

$$n(x, y) = \frac{n_i^2}{N_a} \exp \left(\frac{q}{k_B T} (\phi - V) \right)$$

Taur, Eq. (3.1)

Still, ϕ_B is the reference value.

Our previous expressions (1)

- They are modified by V . (Terms related with the electron density)
 - Electric field

$$\begin{aligned} E_x^2(x, y) &= \left(\frac{d\phi}{dx} \right)^2 \\ &= \frac{2k_B T N_a}{\epsilon_{si}} \left[\left(\exp \left(-\frac{q\phi}{k_B T} \right) + \frac{q\phi}{k_B T} - 1 \right) \right. \\ &\quad \left. + \frac{n_i^2}{N_a^2} \left(\exp \left(-\frac{qV}{k_B T} \right) \left(\exp \left(\frac{q\phi}{k_B T} \right) - 1 \right) - \frac{q\phi}{k_B T} \right) \right] \quad \text{Taur, Eq. (3.2)} \end{aligned}$$

Our previous expressions (2)

- They are modified by V . (Terms related with the electron density)

- Surface inversion

$$\phi(0, y) = V(y) + 2\phi_B \quad \text{Taur, Eq. (3.3)}$$

- Maximum depletion layer width

$$W_{dm} = \sqrt{\frac{2\epsilon_{si}[V(y) + 2\phi_B]}{qN_a}} \quad \text{Taur, Eq. (3.4)}$$

- Summary: With the GCA, our MOS expressions are re-used only with modification by V .

Drain current

- Electron current density at a point (x, y)

$$J_n(x, y) = -q\mu_n n(x, y) \frac{dV}{dy}$$

Taur, Eq. (3.5)

- (It includes both the drift and diffusion currents.)
- When integrated from $x = 0$ to x_i ,

$$I_d(y) = qW \int_0^{x_i} \mu_n n(x, y) \frac{dV}{dy} dx$$

Taur, Eq. (3.6)

Sign change due to
convention of
terminal current

z-directional
width

Further simplification

- Electron current density at a point (x, y)

$$\begin{aligned} I_d(y) &= qW \int_0^{x_i} \mu_n n(x, y) \frac{dV}{dy} dx = -\mu_{eff} W \frac{dV}{dy} \left(-q \int_0^{x_i} n(x, y) dx \right) \\ &= -\mu_{eff} W \frac{dV}{dy} Q_i(y) \end{aligned} \quad \text{Taur, Eq. (3.8)}$$

- We introduce an effective mobility, μ_{eff} .
- Since V is a function of y only, V is interchangeable with y .

$$Q_i(y) = Q_i(V)$$

- Then,

$$I_d(y) dy = \mu_{eff} W [-Q_i(V)] dV$$

$I_d(y)$ is actually a constant.

- When integrated from $y = 0$ to L , (from $V = 0$ to V_{ds})

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV \quad \text{Taur, Eq. (3.10)}$$

- Then, how can we find $Q_i(V)$? (We must perform the x -directional integration.)

$$Q_i = -q \int_0^{x_i} n(x, y) dx = -q \int_{\phi_s}^{\delta} n(\phi, V) \frac{dx}{d\phi} d\phi$$

It's small, but not zero.


$$= -q \int_{\delta}^{\phi_s} \frac{(n_i^2 / N_a) \exp\left(\frac{q}{k_B T} (\phi - V)\right)}{E_x(\phi, V)} d\phi \quad \text{Taur, Eq. (3.12)}$$

Then, how can we determine ϕ_s ?

- For given V_{gs} and V , we can solve the MOS equation.

$$\begin{aligned} V_{gs} &= V_{fb} + \phi_s - \frac{Q_s}{C_{ox}} \\ &= V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si}k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T}(\phi_s - V)\right) \right]^{1/2} \end{aligned}$$

Taur, Eq. (3.14)



Only two important terms are kept.

- We can numerically solve the above equation to obtain ϕ_s .

$V(\phi_s)$?

- Recall that

$$V_{gs} = V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si}k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T}(\phi_s - V)\right) \right]^{1/2}$$

Taur, Eq. (3.14)

- We can rearrange

$$\frac{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s)^2}{2\epsilon_{si}k_B T N_a} = \frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T}(\phi_s - V)\right)$$

- Therefore,

$$V = \phi_s - \frac{k_B T}{q} \log \left\{ \frac{N_a^2}{n_i^2} \left[\frac{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s)^2}{2\epsilon_{si}k_B T N_a} - \frac{q\phi_s}{k_B T} \right] \right\}$$

Pao-Sah double integral

- Finally, the expression for I_d reads

$$I_d = q\mu_{eff} \frac{W}{L} \int_0^{V_{ds}} \left[\int_{\delta}^{\phi_s} \frac{(n_i^2 / N_a) \exp\left(\frac{q}{k_B T} (\phi - V)\right)}{E_x(\phi, V)} d\phi \right] dV$$

Taur, Eq. (3.13)

- It is the Pao-Sah double integral.
- Rigorous within the GCA, but it is difficult to evaluate.

Charge-sheet model

- Simpler model with further approximations

- Consider the previous method to calculate Q_i :

$$Q_i = -q \int_{\delta}^{\phi_s} \frac{(n_i^2 / N_a) \exp\left(\frac{q}{k_B T} (\phi - V)\right)}{E_x(\phi, V)} d\phi$$

- A more simple way? Instead, Q_d is approximated as

$$Q_d = -q N_a W_d = -\sqrt{2\epsilon_{si} q N_a \phi_s} \quad \text{Taur, Eq. (3.15)}$$

- Then, Q_i can be approximated as

$$Q_i = Q_s - Q_d = -C_{ox}(V_{gs} - V_{fb} - \phi_s) + \sqrt{2\epsilon_{si} q N_a \phi_s} \quad \text{Taur, Eq. (3.16)}$$


(Of course, it is not exact.)

Change of variable

- Now, Q_i can be a function of ϕ_s .
 - Variable change from V to ϕ_s :

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV = \mu_{eff} \frac{W}{L} \int_{\phi_{s,s}}^{\phi_{s,d}} [-Q_i(\phi_s)] \frac{dV}{d\phi_s} d\phi_s$$

Taur, Eq. (3.17)



Surface potentials at the two ends, $y = 0$ and L . They can be calculated by solving Taur, Eq. (3.14).

$$\frac{dV}{d\phi_s} \text{? (1)}$$

• Recall that

$$V = \phi_s - \frac{k_B T}{q} \log \left\{ \frac{N_a^2}{n_i^2} \left[\frac{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T} \right] \right\}$$

Taur, Eq. (3.18)

– Therefore,

$$\frac{dV}{d\phi_s} = 1 - \frac{k_B T}{q} \frac{-\frac{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s)}{\epsilon_{si} k_B T N_a} - \frac{q}{k_B T}}{\frac{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}}$$

$$\frac{dV}{d\phi_s} \text{? (2)}$$

- Simple rearrange yields

$$\frac{dV}{d\phi_s} = 1 + \frac{2k_B T}{q} \frac{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s) + \epsilon_{si} q N_a}{C_{ox}^2 (V_{gs} - V_{fb} - \phi_s)^2 - 2\epsilon_{si} q N_a \phi_s}$$

Taur, Eq. (3.19)

– It is still very complicated...

Integrand

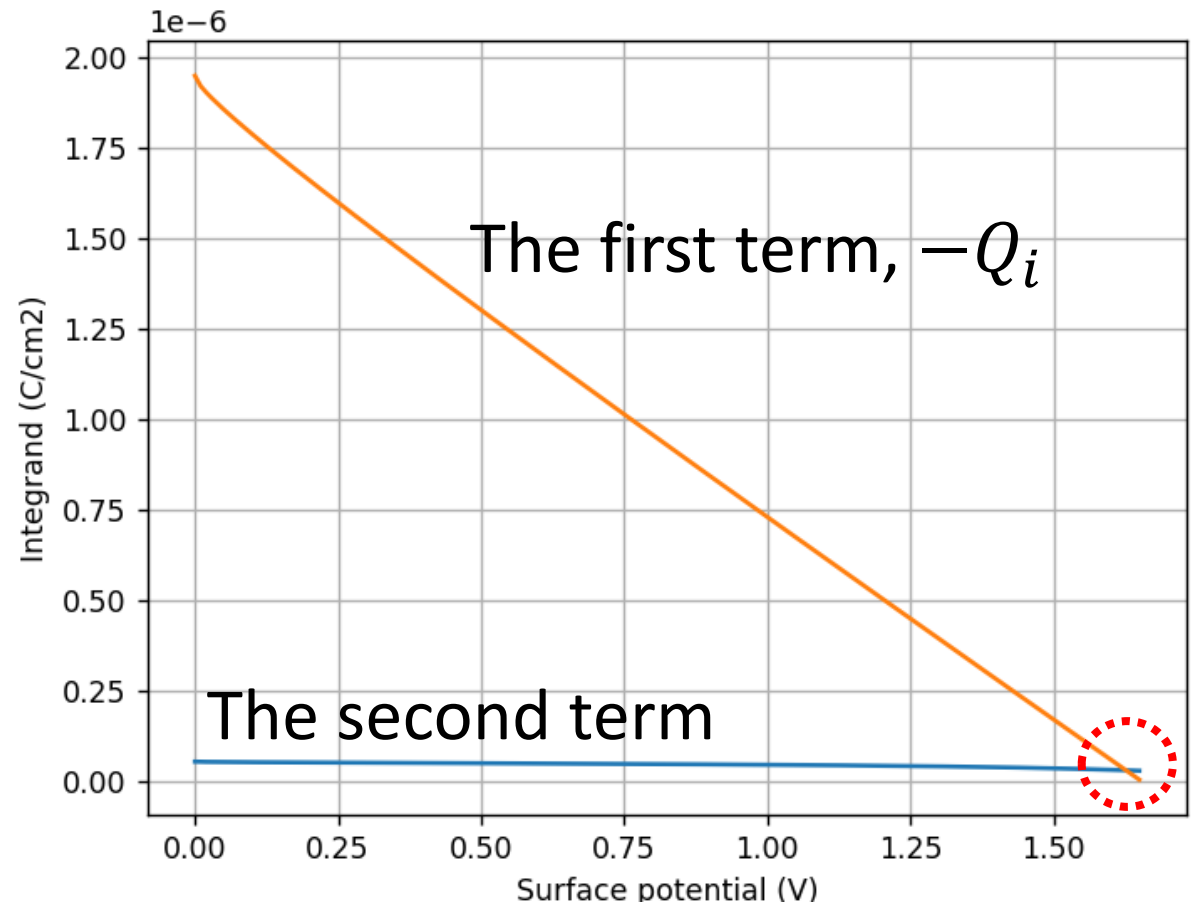
- When multiplied with $-Q_i(\phi_s)$,

$$\begin{aligned} & (-Q_i(\phi_s)) \frac{dV}{d\phi_s} \\ &= -Q_i(\phi_s) + \frac{2k_B T}{q} \frac{C_{ox}^2(V_{gs} - V_{fb} - \phi_s) + \epsilon_{si} q N_a}{C_{ox}(V_{gs} - V_{fb} - \phi_s) + \sqrt{2\epsilon_{si} q N_a \phi_s}} \end{aligned}$$

- The second term is still very complicated...
- *Is it really important?*

Comparison between two terms

- Let's draw two terms.
 - Assume that $N_a = 10^{17} \text{ cm}^{-3}$, $t_{ox} = 10 \text{ nm}$, $V_{gs} = 1.0 \text{ V}$, and $V_{fb} = -0.88 \text{ V}$.
 - The second term is small.
 - It is meaningful only when $Q_i \approx 0$.
 - This is corresponding to $C_{ox}(V_{gs} - V_{fb} - \phi_s)$
 $= \sqrt{2\epsilon_{si}qN_a\phi_s}$.



Integrand, again

- Within this condition,

$$\begin{aligned} (-Q_i(\phi_s)) \frac{dV}{d\phi_s} &\approx -Q_i(\phi_s) + \frac{k_B T}{q} \frac{C_{ox} \sqrt{2\epsilon_{si} q N_a \phi_s} + \epsilon_{si} q N_a}{\sqrt{2\epsilon_{si} q N_a \phi_s}} \\ &= -Q_i(\phi_s) + \frac{k_B T}{q} C_{ox} + \frac{k_B T}{q} \frac{\sqrt{2\epsilon_{si} q N_a}}{2\sqrt{\phi_s}} \end{aligned}$$

Its integration yields $\frac{k_B T}{q} C_{ox} \phi_s$.

Its integration yields $\frac{k_B T}{q} \sqrt{2\epsilon_{si} q N_a \phi_s}$.

Thank you!