VLSI Devices
Lecture 12

Sung-Min Hong (smhong@gist.ac.kr)
Semiconductor Device Simulation Laboratory
Department of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology (GIST)

Charge-sheet model

- Simpler model with further approximations
  - Consider the previous method to calculate Q_i:

$$Q_i = -q \int_{\delta}^{\phi_s} \frac{\left(n_i^2/N_a\right) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$

  - A more simple way? Instead, Q_d is approximated as

$$Q_d = -qN_a W_d = -\sqrt{2\epsilon_{si} q N_a \phi_s}$$

Taur, Eq. (3.15)

  - Then, Q_i can be approximated as

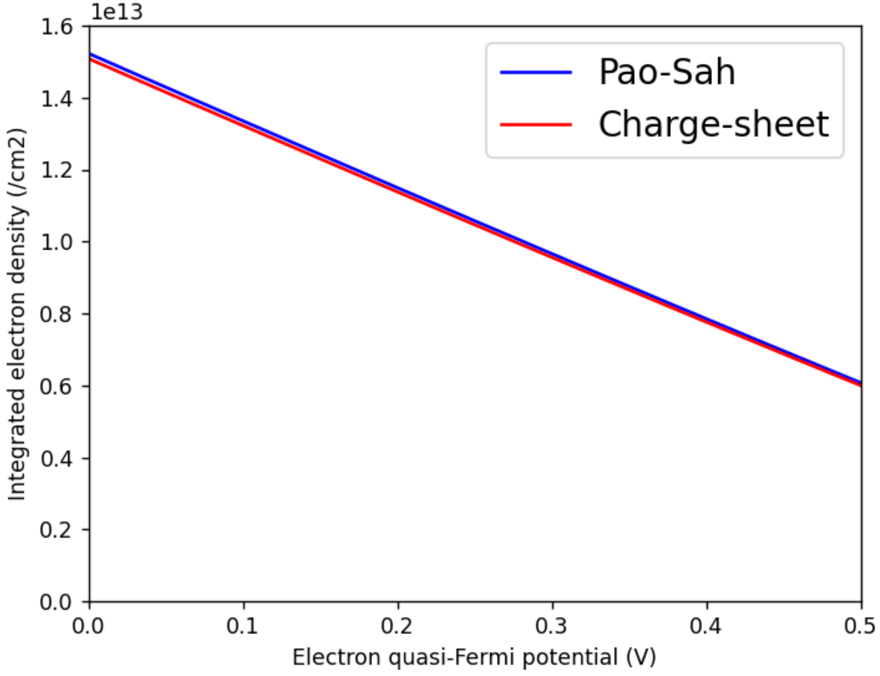
$$Q_i = Q_s - Q_d = -C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si} q N_a \phi_s}$$

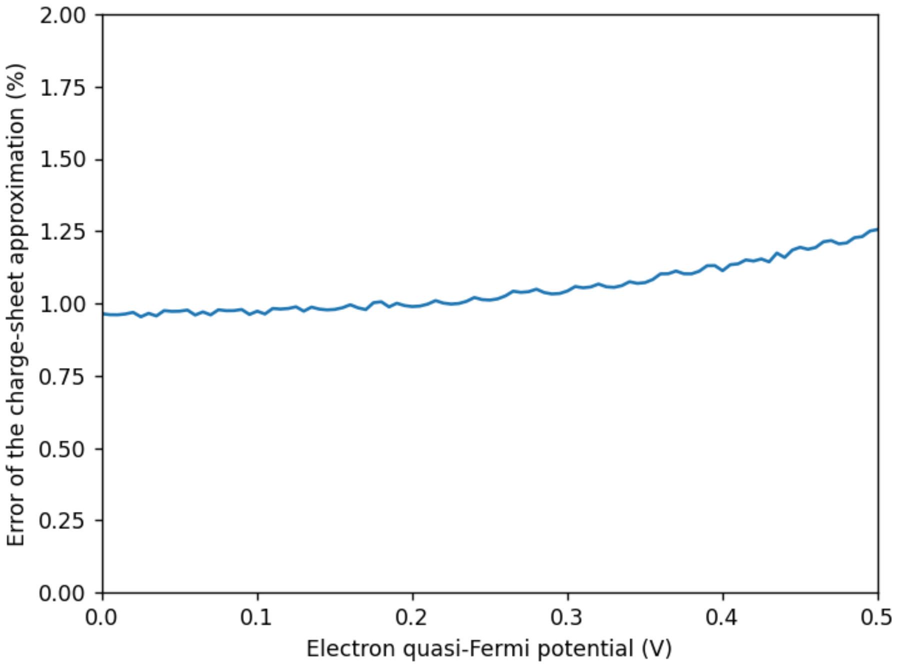
Taur, Eq. (3.16)

(Of course, it is not exact.)

Pao-Sah versus charge-sheet

- Consider a bias condition of V_{gs} = 1.1 V and V_{ds} = 0.5 V.
  - Calculate ϕ_s within a maximum error of 0.01 mV. Then, apply the charge-sheet approximation.
  - Only about 1 % error





Change of variable

- Now, Q_i can be a function of ϕ_s.
  - Variable change from V to ϕ_s:

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV = \mu_{eff} \frac{W}{L} \int_{\phi_{s,s}}^{\phi_{s,d}} [-Q_i(\phi_s)] \frac{dV}{d\phi_s} d\phi_s$$

Taur, Eq. (3.17)

Surface potentials at the two ends, $y = 0$ and L. They can be calculated by solving Taur, Eq. (3.14).

$V(\phi_s)$?

- Recall that

$$V_{gs} = V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right) \right]^{1/2}$$

Taur, Eq. (3.14)

  - We can rearrange

$$\frac{C_{ox}^2 \left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} = \frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right)$$

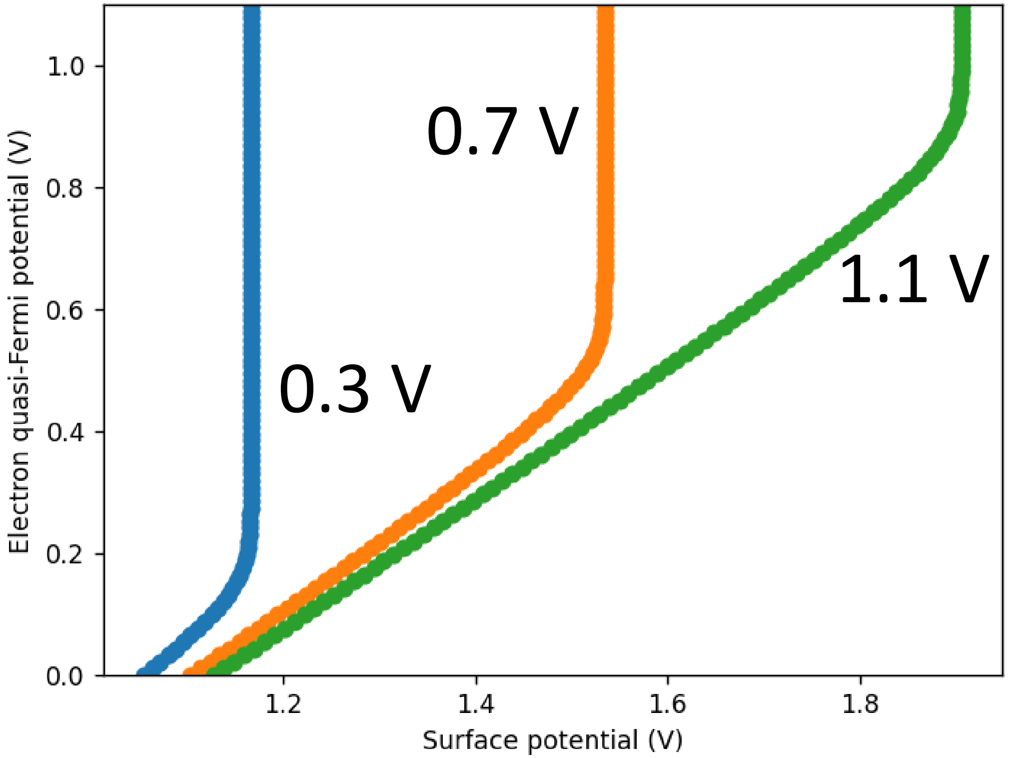
  - Therefore,

$$V = \phi_s - \frac{k_B T}{q} \log\left\{ \frac{N_a^2}{n_i^2} \left[\frac{C_{ox}^2 \left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T} \right] \right\}$$

Taur, Eq. (3.18)

Pao-Sah result

- Consider three gate voltages, V_{gs} = 0.3 V, 0.7 V, and 1.1 V.
  - Draw the electron quasi-Fermi potential (V) as a function of the surface potential (ϕ_s). See Taur, Fig. 3.3.



$\frac{dV}{d\phi_s}$? (1)

- Recall that

$$V = \phi_s - \frac{k_B T}{q} \log\left\{\frac{N_a^2}{n_i^2}\left[\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}\right]\right\}$$

Taur, Eq. (3.18)

  – Therefore,

$$\frac{dV}{d\phi_s} = 1 - \frac{k_B T}{q} \frac{-\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)}{\epsilon_{si} k_B T N_a} - \frac{q}{k_B T}}{\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}}$$

$\frac{dV}{d\phi_s}$? (2)

- Simple rearrange yields

$$\frac{dV}{d\phi_s} = 1 + \frac{2k_BT}{q} \frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right) + \epsilon_{si}qN_a}{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2 - 2\epsilon_{si}qN_a\phi_s}$$

Taur, Eq. (3.19)

  - It is still very complicated…

Integrand

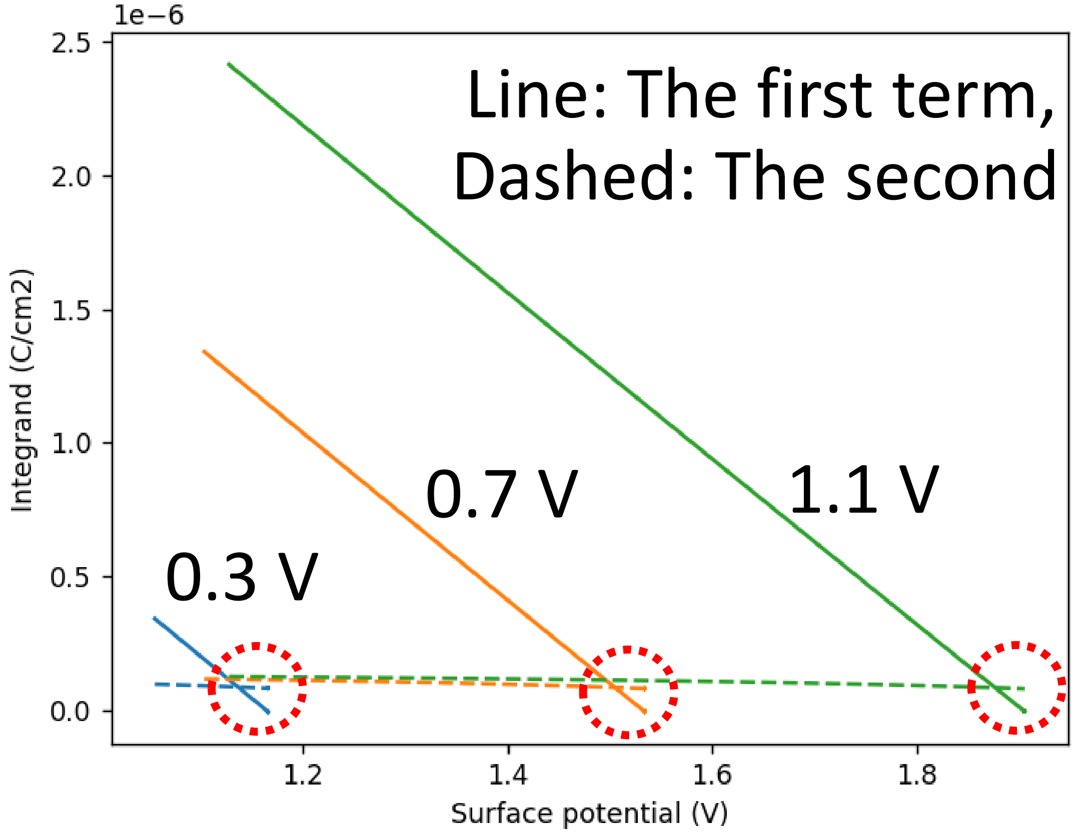
- When multiplied with $-Q_i(\phi_s)$,

$$\left(-Q_i(\phi_s)\right)\frac{dV}{d\phi_s}$$
$$= -Q_i(\phi_s) + \frac{2k_BT}{q}\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right) + \epsilon_{si}qN_a}{C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si}qN_a\phi_s}}$$

  – The second term is still very complicated…
  – *Is it really important?*

Comparison between two terms

- Let's draw two terms.
  - Assume that N_a = 10^{18} cm^{-3}, t_{ox} = 1.2 nm, and V_{fb} = -1.08 V.
  - The second term is small.
  - It is meaningful only when $Q_i \approx 0$.
  - This is corresponding to $C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) = \sqrt{2\epsilon_{si} q N_a \phi_s}$.



Line: The first term, $-Q_i$
Dashed: The second term

Integrand, again

- Within this condition,

$$(-Q_i(\phi_s))\frac{dV}{d\phi_s} \approx -Q_i(\phi_s) + \frac{k_B T}{q}\frac{C_{ox}\sqrt{2\epsilon_{si} q N_a \phi_s} + \epsilon_{si} q N_a}{\sqrt{2\epsilon_{si} q N_a \phi_s}}$$

$$= -Q_i(\phi_s) + \frac{k_B T}{q} C_{ox} + \frac{k_B T}{q}\frac{\sqrt{2\epsilon_{si} q N_a}}{2\sqrt{\phi_s}}$$

Its integration yields $\frac{k_B T}{q} C_{ox}\phi_s$.

Its integration yields $\frac{k_B T}{q}\sqrt{2\epsilon_{si} q N_a \phi_s}$.

Drain current

- Using the previous approximation,
  - We can obtain the following expression:

$$I_d = \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left(V_{gs} - V_{fb} + \frac{k_B T}{q} \right) \phi_s - \frac{1}{2} C_{ox} \phi_s^2 - \frac{2}{3} \sqrt{2\epsilon_{si} q N_a} \phi_s^{1.5} + \frac{k_B T}{q} \sqrt{2\epsilon_{si} q N_a \phi_s} \right\} \Bigg|_{\phi_{s,s}}^{\phi_{s,d}}$$

Taur, Eq. (3.21)

  - Only with $\phi_{s,s}$ and $\phi_{s,d}$, we can evaluate the drain current.

Thank you!