VLSI Devices
Lecture 2

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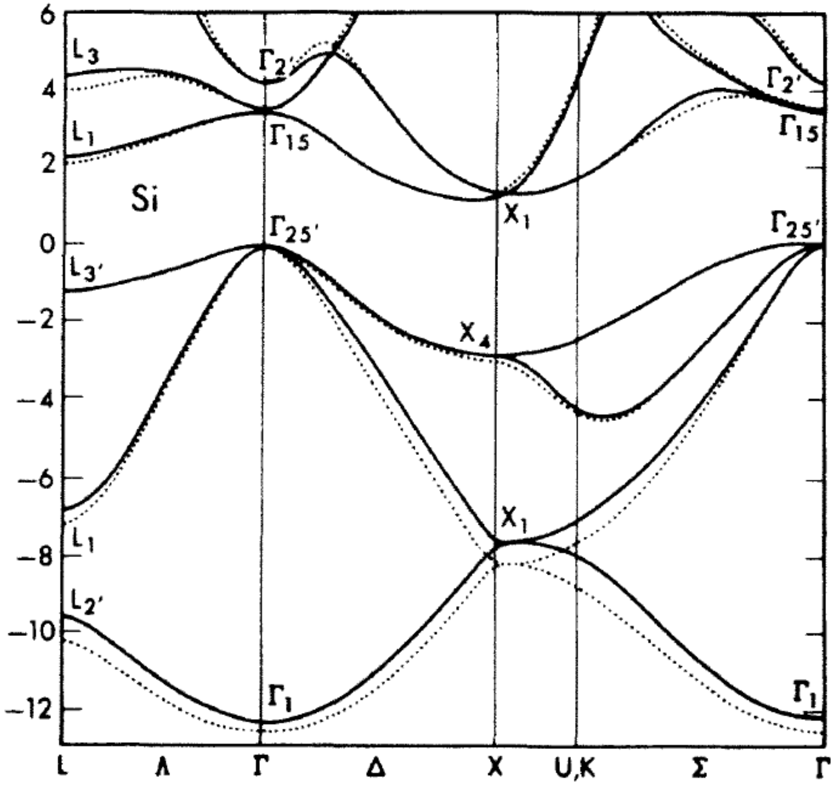
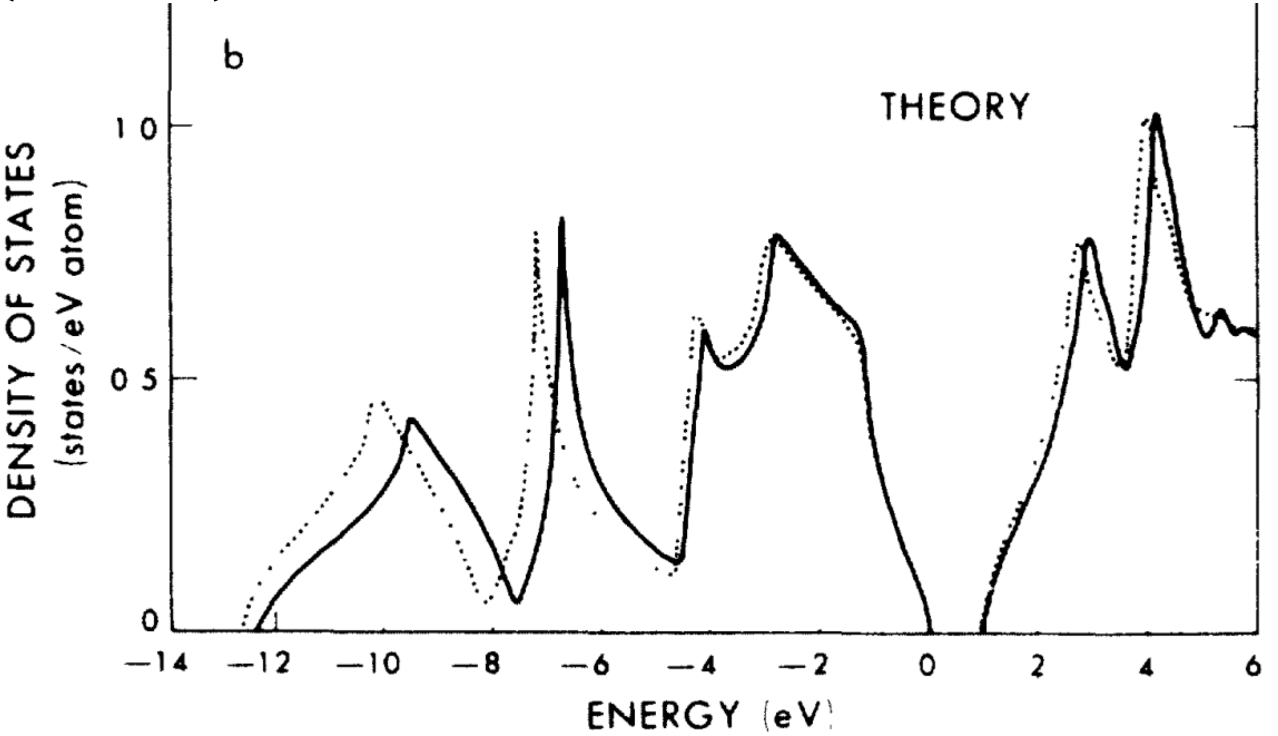
Density-of-states (DOS)

- DOS, $N(E)$, (per spin, per valley)

$$N(E)dE = \frac{1}{(2\pi)^3} \iiint_{E(\mathbf{k})\in(E,E+dE)} dk_x dk_y dk_z$$

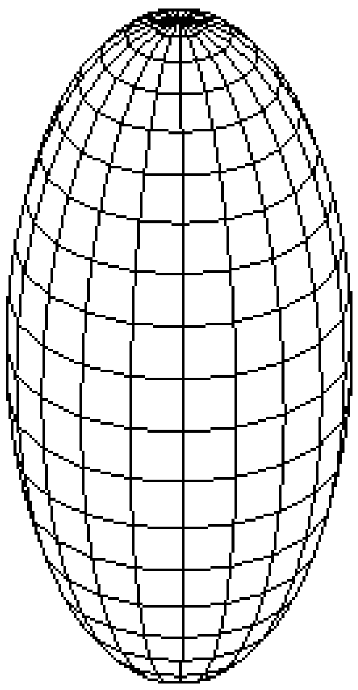
~ Taur, Eq. (2.1)





(J. R. Chelikowsky and M. L. Cohen, PRB, 1974)

Density-of-states (DOS) of an ellipsoidal valley



- Volume in the $\mathbf{k}$-space
  - With $m^* = \left(m_{xx}m_{yy}m_{zz}\right)^{\frac{1}{3}}$, the volume of an ellipsolid is

$$\frac{4\pi}{3}\left(\frac{1}{\hbar}\right)^3 (2m^*)^{1.5}(E-E_c)^{1.5}$$

  - For a shell within a range between $E-E_c$ and $E-E_c+dE$,

$$4\pi\left(\frac{1}{\hbar}\right)^3 \left(2m_{xx}m_{yy}m_{zz}\right)^{0.5}(E-E_c)^{0.5}dE$$

- DOS of silicon conduction band (per spin, per valley)

$$N(E)dE = \frac{4\pi}{h^3}(2m_l m_t^2)^{0.5}(E-E_c)^{0.5}dE$$

~ Taur, Eq. (2.3)

Electron density

0, when empty
1, when occupied

- Number of electrons

$$\# = \sum_{\substack{all\ \color{red}{occupied} \\ \mathbf{k}\ states}} 1 = \sum_{all\ \mathbf{k}\ states} f(\mathbf{k})$$

  - Instead of a sum,

$$\# = \sum_{all\ \mathbf{k}\ states} f(\mathbf{k}) \approx \frac{SystemVolume}{(2\pi)^3} \iiint_{\substack{Entire \\ \mathbf{k}\ space}} f(\mathbf{k}) dk_x dk_y dk_z$$

  - Electron density (per spin, per valley)

$$n = \frac{\#}{SystemVolume} = \frac{1}{(2\pi)^3} \iiint_{\substack{Entire \\ \mathbf{k}\ space}} f(\mathbf{k}) dk_x dk_y dk_z$$

Fermi-Dirac distribution

- At equilibrium, $f(\mathbf{k})$ follows the Fermi-Dirac distribution

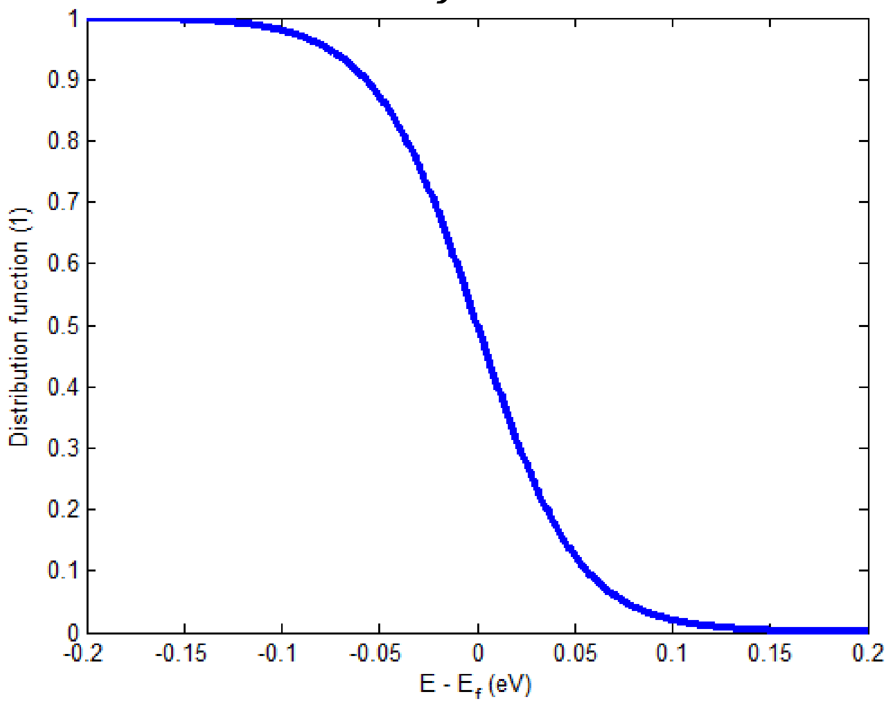
$$f_D(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{k_B T}\right)}$$

Taur, Eq. (2.4)

~ 25.85 meV
@ 300 K

  - At $E = E_f$ (Fermi level),

$$f_D(E_f) = \frac{1}{2}$$

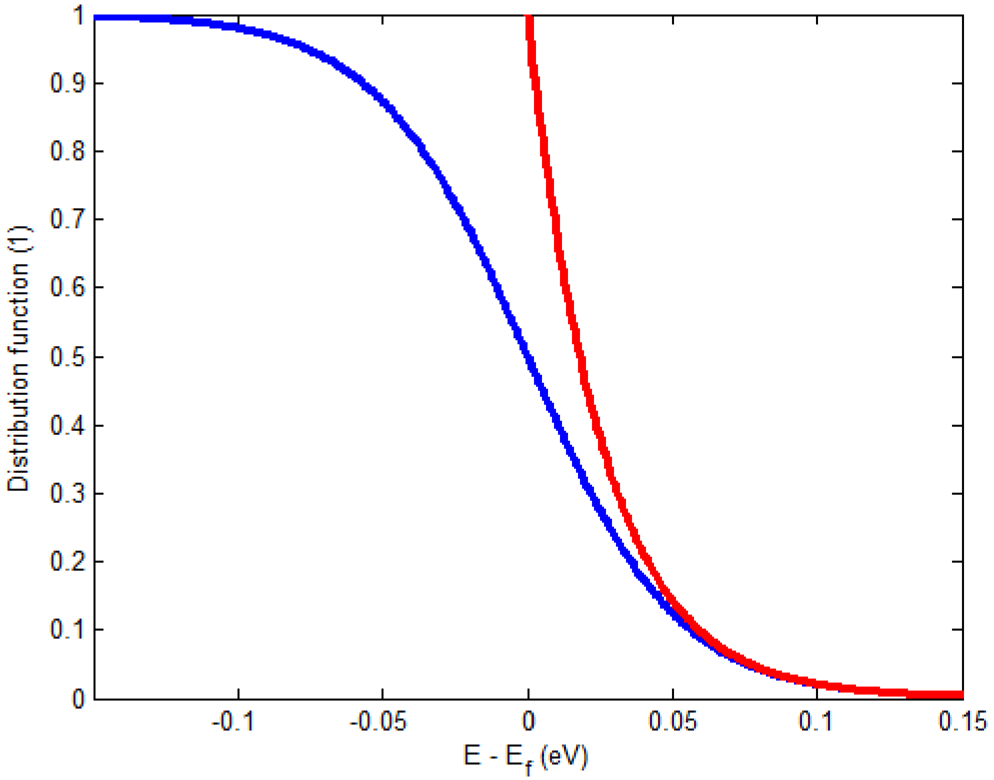


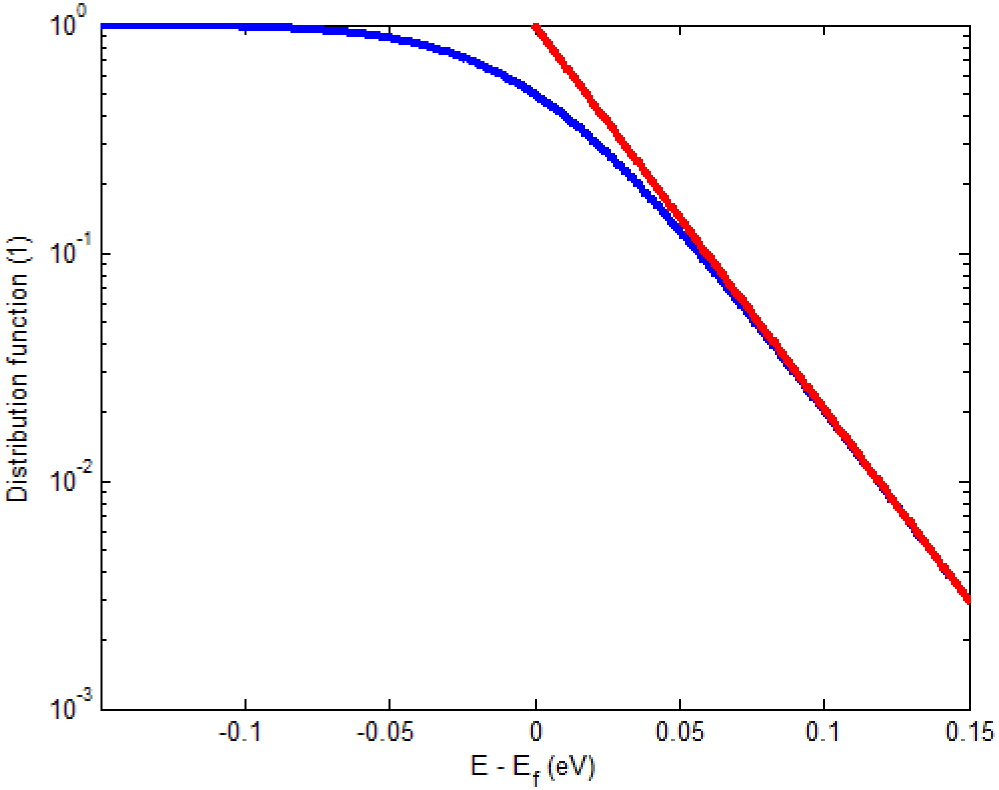
Boltzmann limit

- When $E > E_f$,

$$f_D(E) \approx \exp\left(-\frac{E - E_f}{k_B T}\right)$$

Taur, Eq. (2.5)



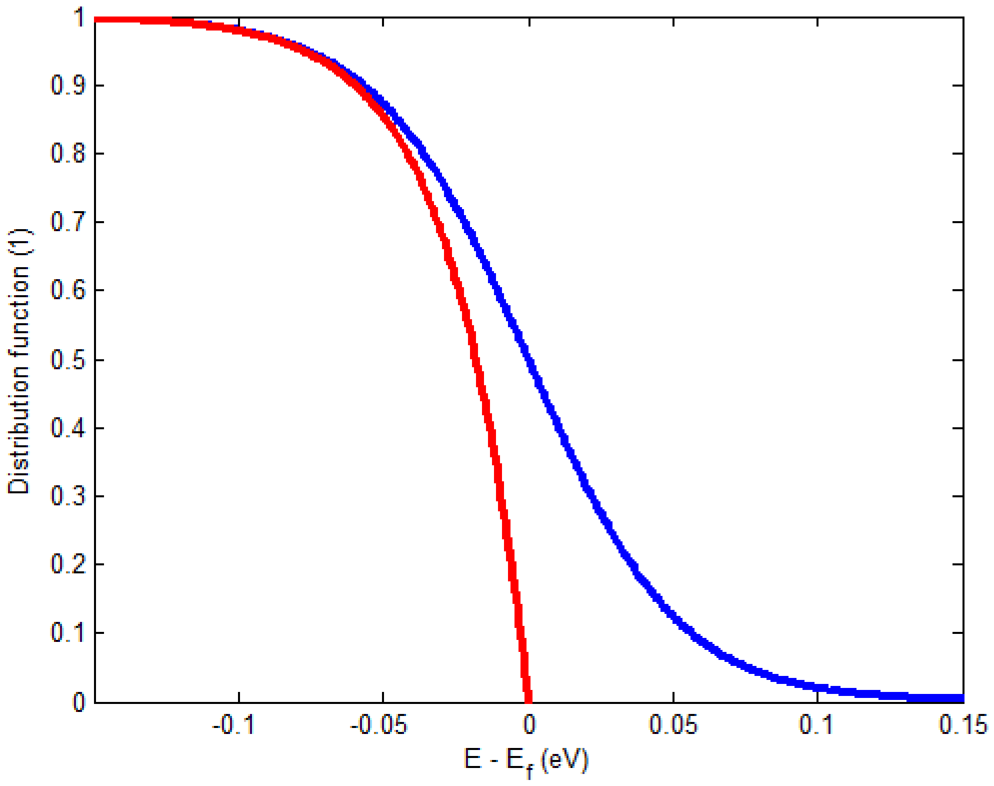


Another Boltzmann limit

- When $E < E_f$,

$$f_D(E) \approx 1 - \exp\left(\frac{E - E_f}{k_B T}\right)$$

Taur, Eq. (2.6)



Carrier concentration (Electron)

- Recall that

Spin and valley degeneracy

$$n = \int_{E_c}^{\infty} N(E) f(E) dE$$

$$N(E) = 2g \frac{4\pi}{h^3} (2m_l m_t^2)^{0.5} (E - E_c)^{0.5}$$

$$f_D(E) = \exp\left(-\frac{E - E_f}{k_B T}\right)$$

– Collecting them all,

$$n = \frac{8\pi g}{h^3} (2m_l m_t^2)^{0.5} \int_{E_c}^{\infty} (E - E_c)^{0.5} \exp\left(-\frac{E - E_f}{k_B T}\right) dE$$

Taur, Eq. (2.8)

Manipulation

- It is found that

$$n = \frac{8\pi g}{h^3}(2m_l m_t^2)^{0.5}\exp\left(-\frac{E_c - E_f}{k_B T}\right) \times \int_{E_c}^{\infty}(E - E_c)^{0.5}\exp\left(-\frac{E - E_c}{k_B T}\right)dE$$

  – Integral can be evaluated as

$$\int_{E_c}^{\infty}(E - E_c)^{0.5}\exp\left(-\frac{E - E_c}{k_B T}\right)dE = (k_B T)^{1.5}\int_{0}^{\infty} z^{0.5}\exp(-z)\,dz = (k_B T)^{1.5}\frac{\sqrt{\pi}}{2}$$

Effective DOS

	N_c (cm^{-3})	N_v (cm^{-3})
Silicon	2.8×10^{19}	1.04×10^{19}
Gallium arsenide	4.7×10^{17}	7.0×10^{18}
Germanium	1.04×10^{19}	6.0×10^{18}

N_c and N_v @ 300 K (Hu's book)

- Now we know that

$$n = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5} (m_l m_t^2)^{0.5} \exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

  - With the effective DOS,

Dimension?

$$N_c = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5} (m_l m_t^2)^{0.5}$$

Taur, Eq. (2.10)

  - The electron density can be simply written as

$$n = N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

Taur, Eq. (2.9)

  - Following a similar derivation, $p = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$

Taur, Eq. (2.11)

Intrinsic carrier concentration

- In this case, $n = p$. Then, what is E_f?

$$N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) = N_v \exp\left(\frac{E_v - E_f}{k_B T}\right)$$

  - From the above equation,

$$E_f = \frac{E_c + E_v}{2} - \frac{k_B T}{2} \ln \frac{N_c}{N_v}$$

Taur, Eq. (2.12)

  - This energy level is called the intrinsic Fermi level, E_i.
  - In this case,

$$n = p = n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_c - E_v}{2 k_B T}\right)$$

Taur, Eq. (2.13)

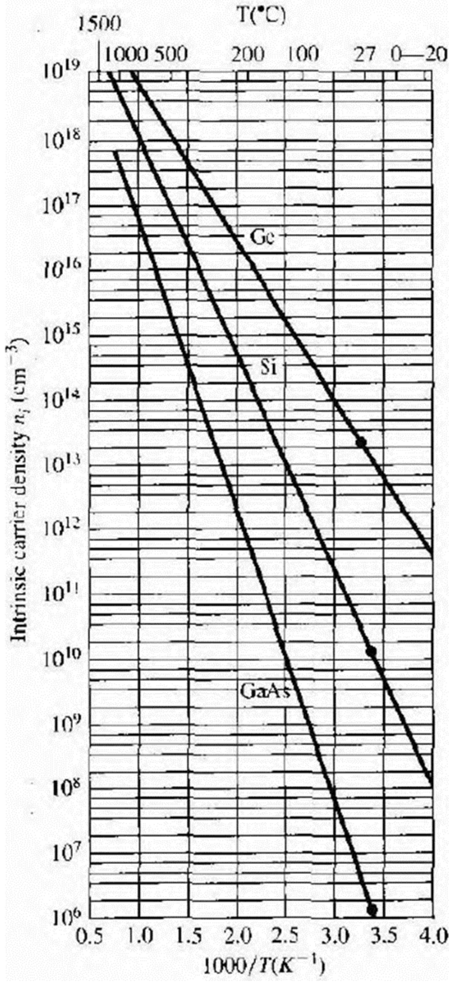
Its temperature dependence

- Recall that $N_c = 2g\left(\frac{2\pi k_B T}{h^2}\right)^{1.5}(m_l m_t^2)^{0.5}$. ($N_v$ has a similar form.)
  - Therefore,

$$n_i = \sqrt{N_c N_v}\exp\left(-\frac{E_c - E_v}{2k_B T}\right)$$

It is dominant.

$T^{1.5}$, but it is not dominant.



Intrinsic carrier density (Neamen's book)

Using the intrinsic carrier density,

- Carrier densities are expressed as

$$n = n_i \exp\left(-\frac{E_i - E_f}{k_B T}\right) \qquad \text{Taur, Eq. (2.14)}$$

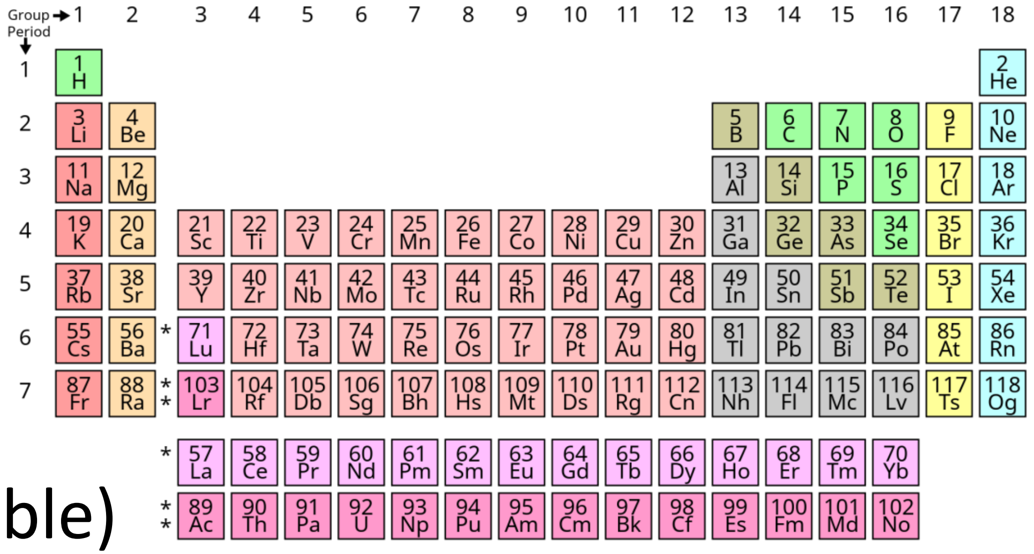
$$p = n_i \exp\left(\frac{E_i - E_f}{k_B T}\right) \qquad \text{Taur, Eq. (2.15)}$$

  – A useful, general relationship is that the product

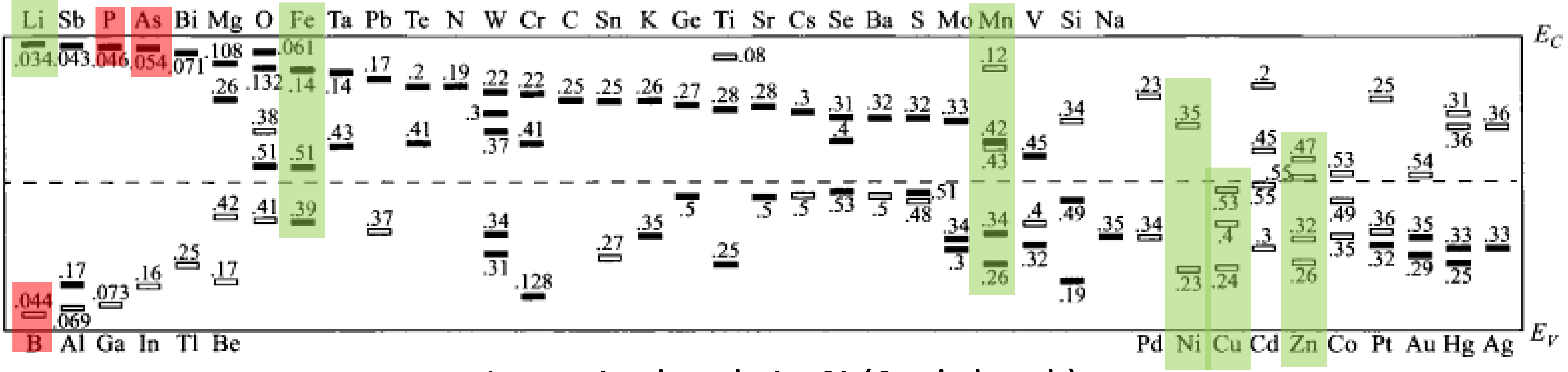
$$np = n_i^2 \qquad \text{Taur, Eq. (2.16)}$$

  in equilibrium is a constant, independent of the Fermi level position.

Dopants



- 5 X 10^{20} impurities / cm^3 is 1 % of Si.
  - Find As, P, and B.
  - Find Fe, Cu, Li, Zn, Mn, and Ni. (Undesirable)



Impurity levels in Si (Sze's book)

Equation for the Fermi level

- Assume N_d and E_d are given. Then,

$$N_v \exp\left(\frac{E_v - E_f}{k_B T}\right) - N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) + \frac{N_d}{1 + 2\exp\left(-\frac{E_d - E_f}{k_B T}\right)} = 0$$

Taur, Eq. (2.19)

- For shallow donor impurities,

$$-N_c \exp\left(-\frac{E_c - E_f}{k_B T}\right) + N_d = 0$$

$$E_c - E_f = k_B T \ln \frac{N_c}{N_d}$$

Taur, Eq. (2.20)

- Hole density, $p = \frac{n_i^2}{N_d}$

Taur, Eq. (2.21)

In terms of n_i and E_i,

- We can write useful expressions:

$$E_c - E_f = k_B T \ln \frac{N_c}{N_d}$$ Taur, Eq. (2.20)

$$\cancel{(E_c - E_i)} + E_i - E_f = k_B T \ln \frac{n_i}{N_d} + \cancel{(E_c - E_i)}$$

$$E_f - E_i = k_B T \ln \frac{N_d}{n_i}$$ Taur, Eq. (2.24)

  - For p-type,

$$E_i - E_f = k_B T \ln \frac{N_a}{n_i}$$ Taur, Eq. (2.25)

Thank you!