VLSI Devices
Lecture 16

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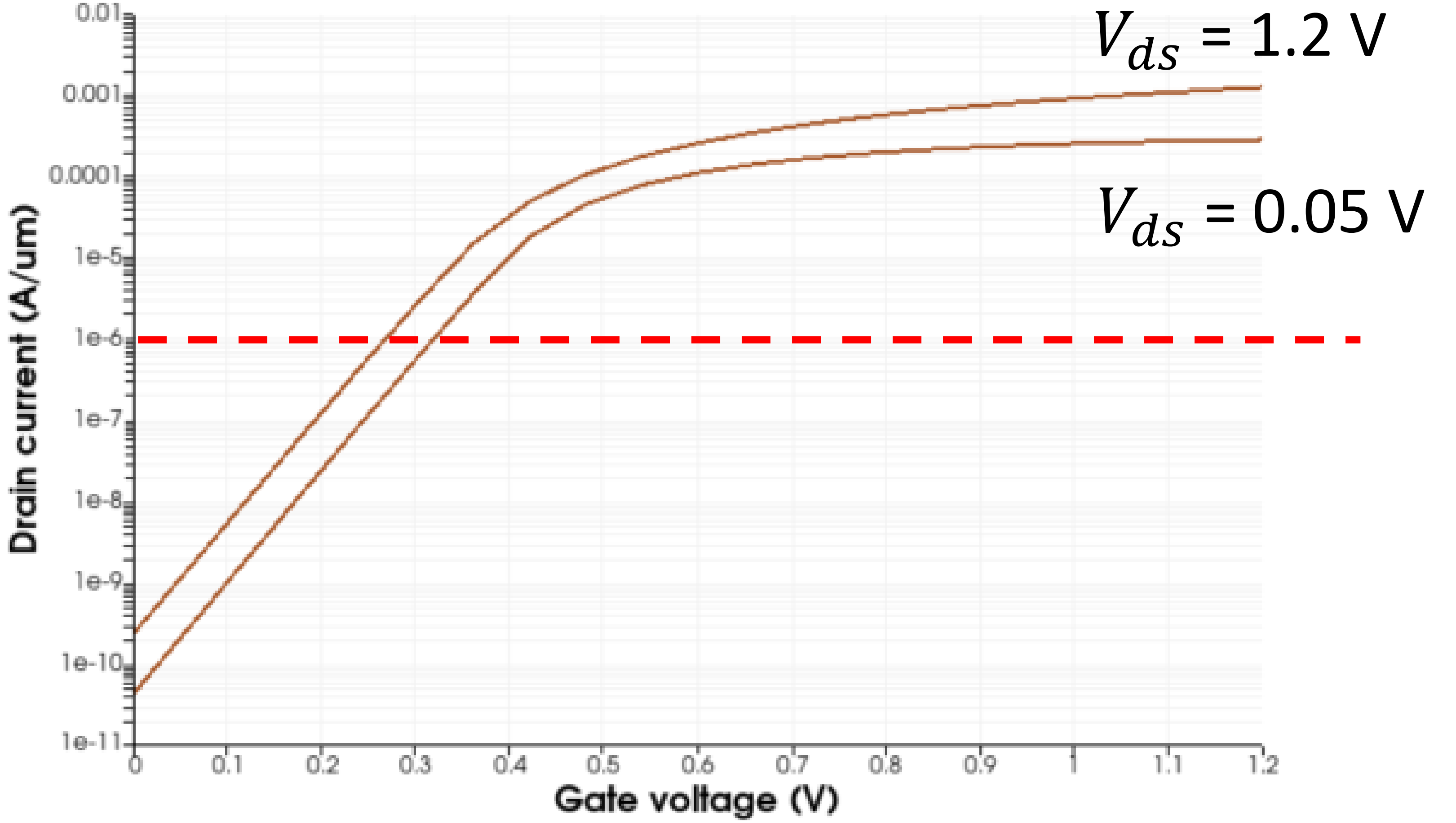
Coverage

- Two YouTube lectures reserved for advanced topics
  - L14: ~~Substrate bias~~, ~~channel mobility~~
  - L15: 3.2.1
  - L16: 3.2.1 (Continued)
  - L17: Velocity saturation (3.2.2)
  - L18: Channel length modulation and so on (3.2.3, 3.2.4, 3.2.5)
  - L19: MOSFET scaling
  - L20: MOSFET scaling (Continued)
  - L21: Quantum effect (4.2.4)
  - L22: Double-gate MOSFETs (10.3)
  - L23: FinFETs
  - L24: CFETs

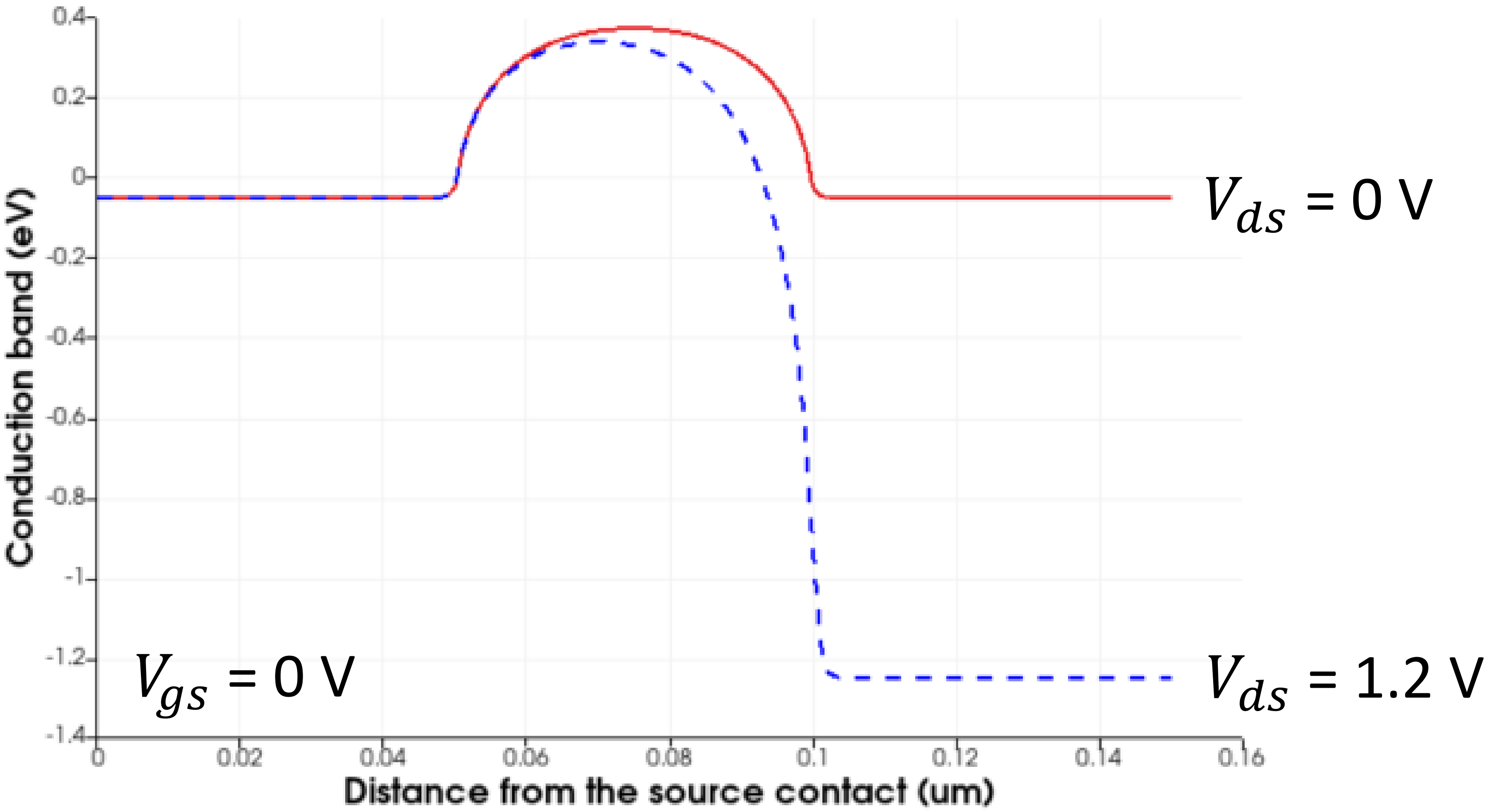
Drain-induced barrier lowering (DIBL)

- Much worse than the long-channel device
  - 45 mV/V @ I_D = 10^{-6} A/µm



Conduction band, again

- At a high V_{ds}, the energy barrier is further reduced.



Simplified geometry for an analytic solution

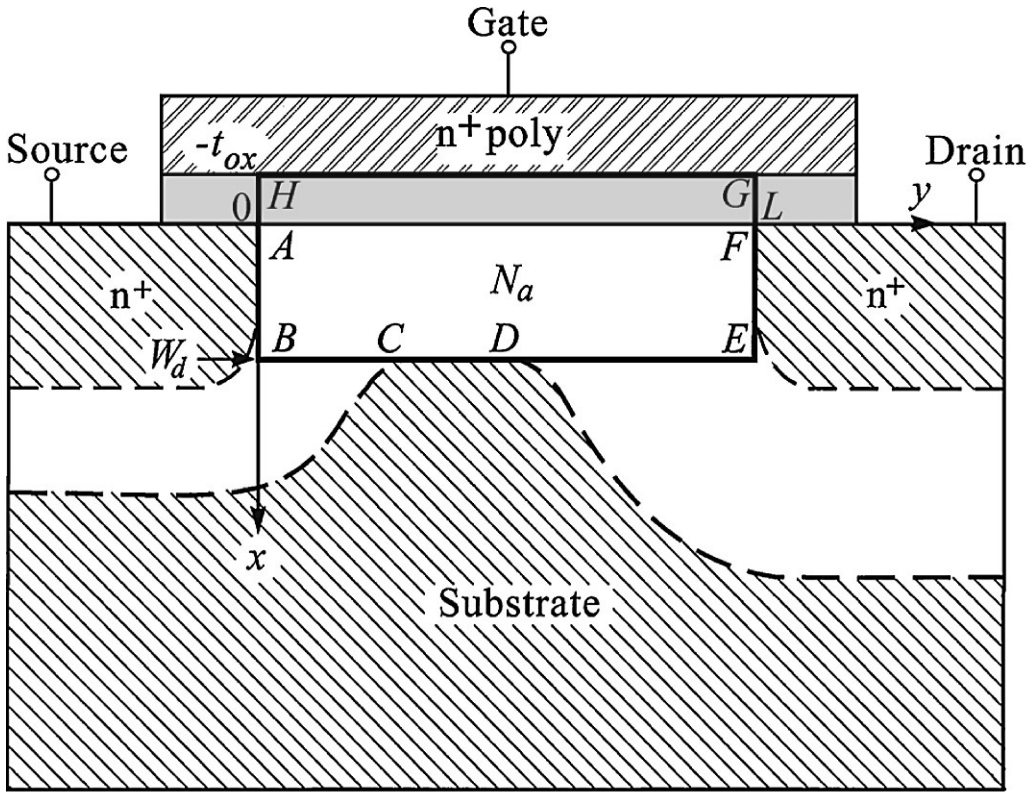
- Poisson equation
  - For AFGH

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$ Taur, Eq. (A9.1)

  - For ABEF (Depleted)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{qN_a}{\epsilon_{si}}$$

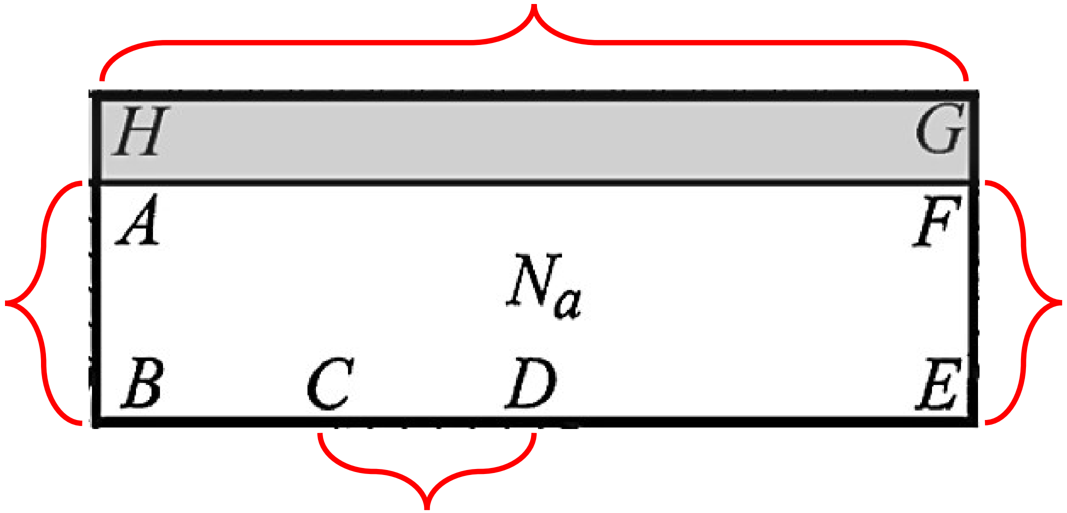
Taur, Eq. (A9.2)



Simplified geometry (Taur, Fig. A9.1)

Boundary conditions

- Potential reference is $\phi(\infty, y) = -\phi_B$.
  - Along GH: $\phi = V_{gs} - V_{fb}$ Taur, Eq. (A9.3)
  - Along AB: $\phi = \phi_{bi} \approx \frac{E_g}{2q} + \phi_B$ Taur, Eq. (A9.4)
  - Along EF: $\phi = \phi_{bi} + V_{ds}$ Taur, Eq. (A9.5)
  - Along CD: $\phi = 0$ Taur, Eq. (A9.6)
  - $(3t_{ox}$ for uniform permittivity)



Solution

- Poisson equation with boundary conditions
  - Try the following function for the electrostatic potential:

$$\phi(x,y) = v(x,y) + u_L(x,y) + u_R(x,y) + u_B(x,y)$$ Taur, Eq. (A9.9)

Poisson equation with upper b.c. (Long-channel)

Laplace equation to match left b.c.

Laplace equation to match right b.c.

Laplace equation to match bottom b.c.

Solution, $\boldsymbol{v(x, y)}$

- Actually, it is $v(x)$.
  - For the oxide region ($-3t_{ox} \le x \le 0$),

$$v(x, y) = \phi_s - \frac{V_{gs} - V_{fb} - \phi_s}{3t_{ox}} x \qquad \text{Taur, Eq. (A9.10)}$$

  - For the silicon region ($0 \le x \le W_d$),

$$v(x, y) = \phi_s \left(1 - \frac{x}{W_d}\right)^2 \qquad \text{Taur, Eq. (A9.11)}$$

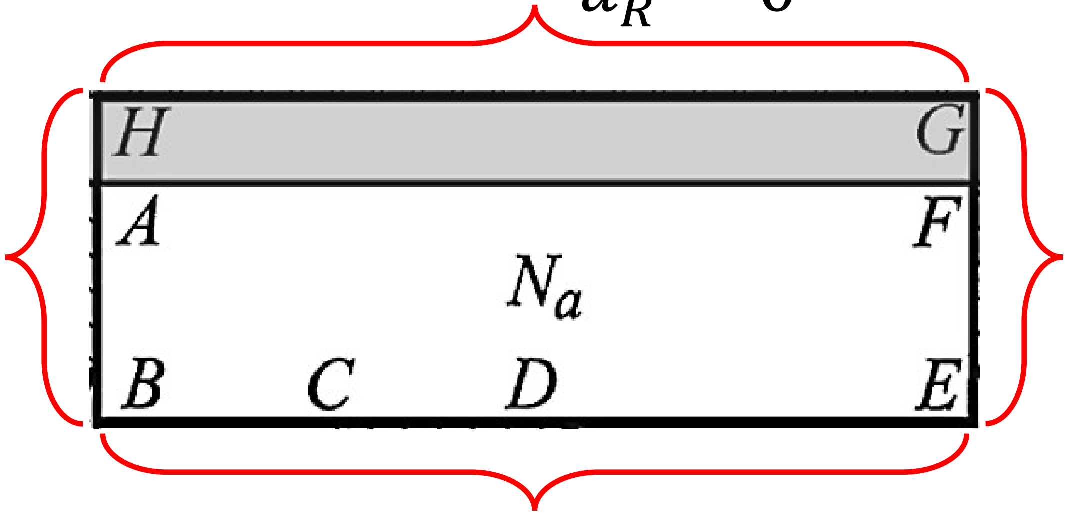
  - It is noted that

$$\phi_s = \frac{qN_a W_d^2}{2\epsilon_{si}} \qquad \text{Taur, Eq. (A9.12)}$$

A mode for $u_R(x, y)$

- For three boundaries, it should vanish.
  - At only one side, it has non-zero values.
  - We can try ($\lambda \equiv W_d + 3t_{ox}$)

$$u_{R,n}(x,y) = \sinh\left(\frac{n\pi y}{\lambda}\right)\sin\left(\frac{n\pi(x+3_{tox})}{\lambda}\right)$$



Series expansion of $\boldsymbol{u_R(x,y)}$

- $u_R(x,L) = \phi(x,L) - v(x,L)$ can be expanded with coefficients c_n's.
  - Therefore,

$$u_R(x,y) = \sum_{n=1}^{\infty} c_n \frac{\sinh\left(\frac{n\pi y}{\lambda}\right)}{\sinh\left(\frac{n\pi L}{\lambda}\right)} \sin\left(\frac{n\pi(x+3_{tox})}{\lambda}\right)$$

Taur, Eq. (A9.15)

- Similar solutions are found for $u_L(x,y)$ and $u_B(x,y)$.

$$u_L(x,y) = \sum_{n=1}^{\infty} b_n \frac{\sinh\left(\frac{n\pi (L-y)}{\lambda}\right)}{\sinh\left(\frac{n\pi L}{\lambda}\right)} \sin\left(\frac{n\pi(x+3_{tox})}{\lambda}\right)$$

Taur, Eq. (A9.14)

First-order expansion

- Keep only b_1 and c_1. (u_B is neglected.)
  - Then,

$$\phi(x,y) = \phi_s\left(1-\frac{x}{W_d}\right)^2 + \frac{b_1\sinh\left(\frac{\pi(L-y)}{\lambda}\right)+c_1\sinh\left(\frac{\pi y}{\lambda}\right)}{\sinh\left(\frac{\pi L}{\lambda}\right)}\sin\left(\frac{\pi(x+3_{tox})}{\lambda}\right)$$

Taur, Eq. (A9.22)

  - Approximate values for b_1 and c_1 are $\frac{4}{\pi}(\phi_{bi}-a\phi_s)$ and $\frac{4}{\pi}(\phi_{bi}+V_{ds}-a\phi_s)$, respectively. $a \approx 0.4$.

Surface potential

- At $x = 0$,

$$\phi(0,y) = \phi_s + \frac{b_1 \sinh\left(\frac{\pi(L-y)}{\lambda}\right) + c_1 \sinh\left(\frac{\pi y}{\lambda}\right)}{\sinh\left(\frac{\pi L}{\lambda}\right)} \sin\left(\frac{\pi 3 t_{ox}}{\lambda}\right)$$

Taur, Eq. (A9.22)

  – Let's find the maximum potential.

$$\frac{d}{dy}\phi(0,y) = \frac{\pi}{\lambda} \frac{-b_1 \cosh\left(\frac{\pi(L-y)}{\lambda}\right) + c_1 \cosh\left(\frac{\pi y}{\lambda}\right)}{\sinh\left(\frac{\pi L}{\lambda}\right)} \sin\left(\frac{\pi 3 t_{ox}}{\lambda}\right) = 0$$

$$b_1 \cosh\left(\frac{\pi(L-y)}{\lambda}\right) = c_1 \cosh\left(\frac{\pi y}{\lambda}\right)$$

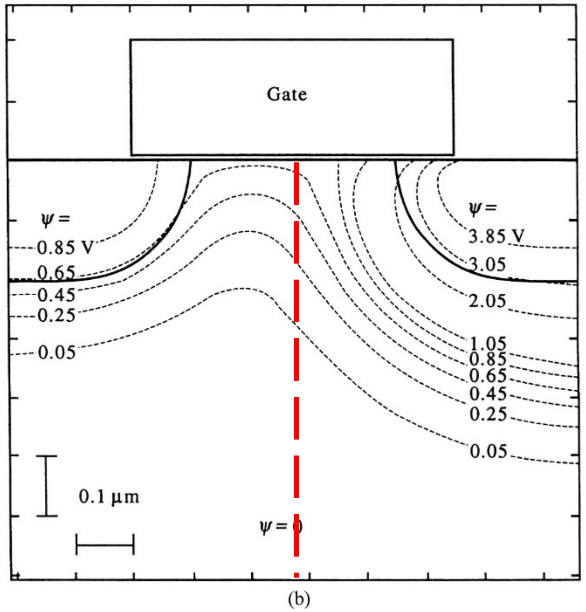
Position for maximum potential, $y = y_c$

- For a positive z, $\cosh z \approx \frac{\exp z}{2}$.

$$\exp \frac{\pi(L - 2y_c)}{\lambda} = \frac{c_1}{b_1}$$

$$y_c = \frac{L}{2} - \frac{\lambda}{2\pi} \ln \frac{c_1}{b_1} \approx \frac{L}{2} - \frac{W_d + 3t_{ox}}{2\pi} \ln \left(1 + \frac{V_{ds}}{\phi_{bi} - a\phi_s} \right)$$

Taur, Eq. (A9.23)



Potential profile (Taur, Fig. 3.20(b))

Maxium potential at $y = y_c$

- Using some approximations,

$$\phi(0, y_c) = \phi_s + 2\sqrt{b_1 c_1}\exp\left(-\frac{\pi L}{2\lambda}\right)\sin\left(\frac{\pi 3 t_{ox}}{\lambda}\right)$$

$$\approx \phi_s + \left(\frac{6\pi t_{ox}}{\lambda}\right)\left(\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})} - \frac{2\phi_{bi} + V_{ds}}{2\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})}} a\phi_s\right)\exp\left(-\frac{\pi L}{2\lambda}\right)$$

~ Taur, Eq. (A9.24)

  – Threshold voltage lowering, ΔV_t,

$$\Delta V_t = \left(\frac{24 t_{ox}}{W_{dm}}\right)\left(\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})} - a(2\phi_B)\right)\exp\left(-\frac{\pi L}{2(W_{dm} + 3t_{ox})}\right)$$

Taur, Eq. (A9.25)

Typical values in the textbook

- Following Taur, Eq. (3.67),

$$\Delta V_t = \left(\frac{24t_{ox}}{W_{dm}}\right)\left(\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})} - a(2\phi_B)\right)\exp\left(-\frac{\pi L}{2(W_{dm} + 3t_{ox})}\right)$$

  - Using typical values,

$$0.1 = (2.4)\left(\sqrt{2} - 0.4\right)\exp\left(-\frac{\pi L}{2(1.3W_{dm})}\right)$$

  - We get $L \approx 2.6W_{dm} \approx 2(W_{dm} + 3t_{ox})$.
  - The minimum allowable channel length is $L_{min} \approx 2(W_{dm} + 3t_{ox})$.

Thank you!