VLSI Devices
Lecture 13

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Drain current

- Using the previous approximation,
  - We can obtain the following expression:

$$I_d = \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left(V_{gs} - V_{fb} + \frac{k_B T}{q} \right) \phi_s - \frac{1}{2} C_{ox} \phi_s^2 - \frac{2}{3} \sqrt{2\epsilon_{si} q N_a} \phi_s^{1.5} + \frac{k_B T}{q} \sqrt{2\epsilon_{si} q N_a \phi_s} \right\} \Bigg|_{\phi_{s,s}}^{\phi_{s,d}}$$

Taur, Eq. (3.21)

  - Only with $\phi_{s,s}$ and $\phi_{s,d}$, we can evaluate the drain current.

Regional approximations

- After the onset of inversion but ***before saturation***,
  - The surface potential, $\phi_s(y)$, can be approximated by

$$\phi(0,y) = V(y) + 2\phi_B$$

Taur, Eq. (3.3)

  - It means that

$$\phi_{s,s} = 2\phi_B$$
$$\phi_{s,d} = 2\phi_B + V_{ds}$$

  - In this case, $\frac{dV}{d\phi_s} = 1$. We must calculate the following term for $\phi_{s,d}$:

$$C_{ox}\left(V_{gs} - V_{fb}\right)(2\phi_B + V_{ds}) - \frac{1}{2}C_{ox}(2\phi_B + V_{ds})^2 - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}(2\phi_B + V_{ds})^{1.5}$$

A simpler form of I_d

- By taking the difference, we can find a simpler form:

$$I_d = \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs} - V_{fb} - 2\phi_B - \frac{1}{2}V_{ds}\right)V_{ds} - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}\left[(2\phi_B + V_{ds})^{1.5} - (2\phi_B)^{1.5}\right]\right\}$$

Taur, Eq. (3.22)

  - For a given V_{gs}, I_d first increases linearly with V_{ds}, then gradually levels off to a saturated value.

Linear (triode) region

- When V_{ds} is small, we may keep only up to the first order.

$$I_d = \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs} - V_{fb} - 2\phi_B\right)V_{ds} - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}\left[\frac{3}{2}(2\phi_B)^{0.5}V_{ds}\right]\right\}$$

$$= \mu_{eff}\frac{W}{L}C_{ox}\left(V_{gs} - V_{fb} - 2\phi_B - \frac{\sqrt{4\epsilon_{si}qN_a\phi_B}}{C_{ox}}\right)V_{ds}$$

$$= \mu_{eff}\frac{W}{L}C_{ox}\left(V_{gs} - V_t\right)V_{ds}$$ Taur, Eq. (3.23)

  - The threshold voltage, V_t, is given by

$$V_t = V_{fb} + 2\phi_B + \frac{\sqrt{4\epsilon_{si}qN_a\phi_B}}{C_{ox}}$$ Taur, Eq. (3.24)

  - It is the gate voltage when the surface potential reaches at $2\phi_B$.

Parabolic region

- We must keep up to the second order.

$$
\begin{aligned}
I_d &= \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs}-V_{fb}-2\phi_B-\frac{1}{2}V_{ds}\right)V_{ds}\right. \\
&\quad \left.-\frac{2}{3}\sqrt{2\epsilon_{si}qN_a}\left[\frac{3}{2}(2\phi_B)^{0.5}V_{ds}+\frac{3}{8}(2\phi_B)^{-0.5}V_{ds}^2\right]\right\} \\
&= \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs}-V_t-\frac{1}{2}V_{ds}\right)V_{ds}-\frac{1}{4}\sqrt{2\epsilon_{si}qN_a}\left[(2\phi_B)^{-0.5}V_{ds}^2\right]\right\} \\
&= \mu_{eff}\frac{W}{L}\left\{C_{ox}(V_{gs}-V_t)V_{ds}-\frac{1}{2}C_{ox}\left[1+\frac{\sqrt{\epsilon_{si}qN_a/(4\phi_B)}}{C_{ox}}\right]V_{ds}^2\right\}
\end{aligned}
$$

Taur, Eq. (3.25)

Let's introduce a factor, $\boldsymbol{m}$.

- It is given as

$$m = 1 + \frac{\sqrt{\epsilon_{si} q N_a / (4\phi_B)}}{C_{ox}} \qquad \text{Taur, Eq. (3.26)}$$

  - From the maximum depletion width,

$$W_{dm} = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_a}} \qquad \text{Taur, Eq. (2.190)}$$

  - An alternative form is available,

$$m = 1 + \frac{C_{dm}}{C_{ox}} = 1 + \frac{3t_{ox}}{W_{dm}} \qquad \text{Taur, Eq. (3.27)}$$

Its physical meaning

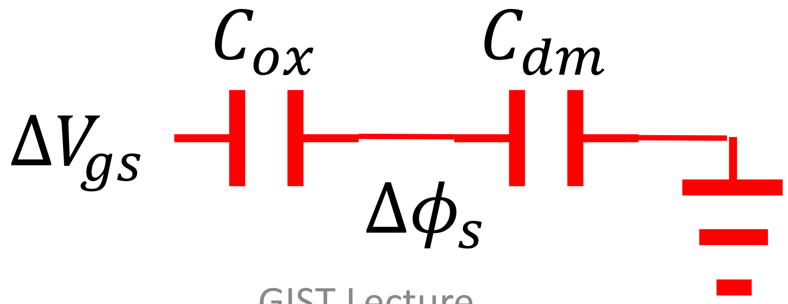
- Serial capacitors give $\frac{C_{ox}C_{dm}}{C_{ox}+C_{dm}}$.
  - Charge across the oxide capacitor:

$$\frac{C_{ox}C_{dm}}{C_{ox}+C_{dm}}\Delta V_{gs} = C_{dm}\Delta\phi_s$$

  - Therefore,

$$m = \frac{C_{ox}+C_{dm}}{C_{ox}} = \frac{\Delta V_{gs}}{\Delta\phi_s}$$

  - m should be kept close to one.



Saturation current

- Maximum value of I_d at a given V_{gs}
  - Recall that

$$I_d = \mu_{eff} \frac{W}{L} \left\{ C_{ox} \left(V_{gs} - V_t \right) V_{ds} - \frac{m}{2} C_{ox} V_{ds}^2 \right\} \quad \text{Taur, Eq. (3.25)}$$

  - When $V_{ds} = V_{dsat} = \frac{V_{gs} - V_t}{m}$,

$$I_d = I_{dsat} = \mu_{eff} C_{ox} \frac{W}{L} \frac{\left(V_{gs} - V_t \right)^2}{2m} \qquad \text{Taur, Eq. (3.28)}$$

Pinch-off

- For $\phi_s = 2\phi_B + V$, the Tylor expansion gives

$$Q_i = -C_{ox}\left(V_{gs} - V_{fb} - 2\phi_B - V\right) + \sqrt{4\epsilon_{si} q N_a \phi_B} + \sqrt{\frac{\epsilon_{si} q N_a}{4\phi_B}} V$$

$$Q_i = -C_{ox}\left(V_{gs} - V_t - V - \frac{1}{C_{ox}}\sqrt{\frac{\epsilon_{si} q N_a}{4\phi_B}} V\right) = C_{ox}\left(V_{gs} - V_t - mV\right)$$

Taur, Eq. (3.29)

  - At $V_{ds} = \frac{V_{gs} - V_t}{m}$ (on-set of saturation), the surface channel vanishes at the drain end of the channel.

Subthreshold current (1)

- Subthreshold region where $V_{gs} < V_t$
  - Recall that

$$-Q_s = \sqrt{2\epsilon_{si} k_B T N_a}\left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2}\exp\left(\frac{q}{k_B T}(\phi_s - V)\right)\right]^{1/2}$$

Taur, Eq. (3.35)

Small in the subthreshold region

  - Its Taylor expansion

$$-Q_s \approx \sqrt{2\epsilon_{si} k_B T N_a}\left[\sqrt{\frac{q\phi_s}{k_B T}} + \frac{1}{2}\sqrt{\frac{k_B T}{q\phi_s}}\frac{n_i^2}{N_a^2}\exp\left(\frac{q}{k_B T}(\phi_s - V)\right)\right]$$

Subthreshold current (2)

- The second term for the inversion charge

$$-Q_i \approx \sqrt{\frac{\epsilon_{si} q N_a}{2\phi_s} \frac{k_B T}{q} \frac{n_i^2}{N_a^2}} \exp\left(\frac{q}{k_B T}(\phi_s - V)\right) \quad \text{Taur, Eq. (3.36)}$$

  - In this case, ϕ_s is a function of V_{gs} only.
  - Recall

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV \qquad \text{Taur, Eq. (3.10)}$$

  - Then, we have

$$I_d = \mu_{eff} \frac{W}{L} \sqrt{\frac{\epsilon_{si} q N_a}{4\phi_B}} \left(\frac{k_B T}{q}\right)^2 \left(\frac{n_i}{N_a}\right)^2 \exp\left(\frac{q\phi_s}{k_B T}\right) \left(1 - \exp\left(-\frac{qV_{ds}}{k_B T}\right)\right)$$

Taur, Eq. (3.37)

Subthreshold current (3)

- Since $m = \frac{\Delta V_{gs}}{\Delta \phi_s}$, $V_{gs} - V_t = m(\phi_s - 2\phi_B)$.
  - Then,

$$\exp\left(\frac{q\phi_s}{k_B T}\right) = \exp\left(\frac{q\left(V_{gs} - V_t\right)}{m k_B T}\right) \exp\left(2\frac{q\phi_B}{k_B T}\right)$$

  - From the above expression,

$$I_d = \mu_{eff} \frac{W}{L} \sqrt{\frac{\epsilon_{si} q N_a}{4\phi_B}} \left(\frac{k_B T}{q}\right)^2 \exp\left(\frac{q\left(V_{gs} - V_t\right)}{m k_B T}\right) \left(1 - \exp\left(-\frac{q V_{ds}}{k_B T}\right)\right)$$

Taur, Eq. (3.39)

Subthreshold slope (1)

- I_d is independent of V_{ds}, when $V_{ds} \gg k_B T/q$.

$$I_d = \mu_{eff} \frac{W}{L} \sqrt{\frac{\epsilon_{si} q N_a}{2\phi_s}} \left(\frac{k_B T}{q}\right)^2 \exp\left(\frac{q(V_{gs} - V_t)}{m k_B T}\right)$$

  – Its gate voltage dependence is very important.

$$\log_{10} I_d = (a\ constant) + \frac{q(V_{gs} - V_t)}{m k_B T} \log_{10} e$$

$$\frac{d(\log_{10} I_d)}{dV_{gs}} = \frac{q}{m k_B T} \log_{10} e$$

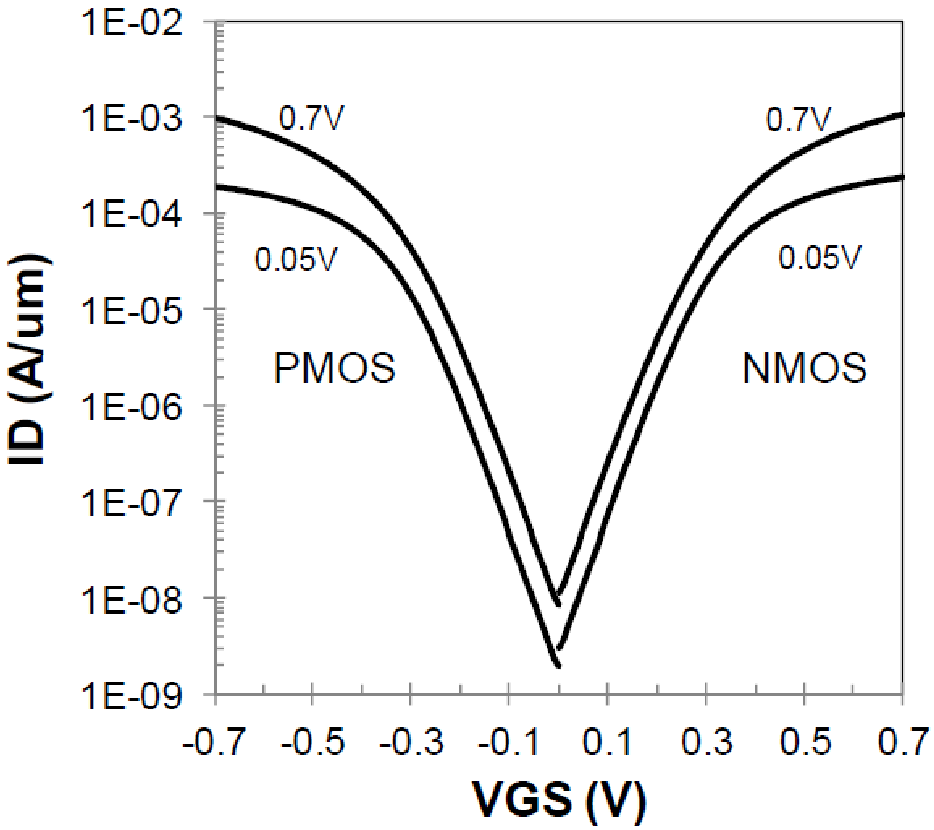
Subthreshold slope

$$S = \left(\frac{d(\log_{10} I_d)}{dV_{gs}}\right)^{-1} = \frac{m k_B T}{q} \ln 10$$

Taur, Eq. (3.41)

Subthreshold slope (2)

- At 300 K, $\frac{k_B T}{q} \ln 10$ is 60 mV/dec.
  - Note that m is larger than 1.



Subthreshold behavior
(Natarajan, IEDM 2024)

Thank you!