VLSI Devices
Lecture 6

Sung-Min Hong (smhong@gist.ac.kr)
Semiconductor Device Simulation Laboratory
Department of Electrical Engineering and Computer Science
Gwangju Institute of Science and Technology (GIST)

Surface potential, $\boldsymbol{\phi_s \equiv \phi(0) - \phi(\infty)}$

- A downward bending of bands in the p-type silicon near the surface
  - It is important to note that

$$V_g - V_{fb} = \phi_s + V_{ox} \qquad \text{Taur, Eq. (2.172)}$$

  - At the silicon-oxide interface,

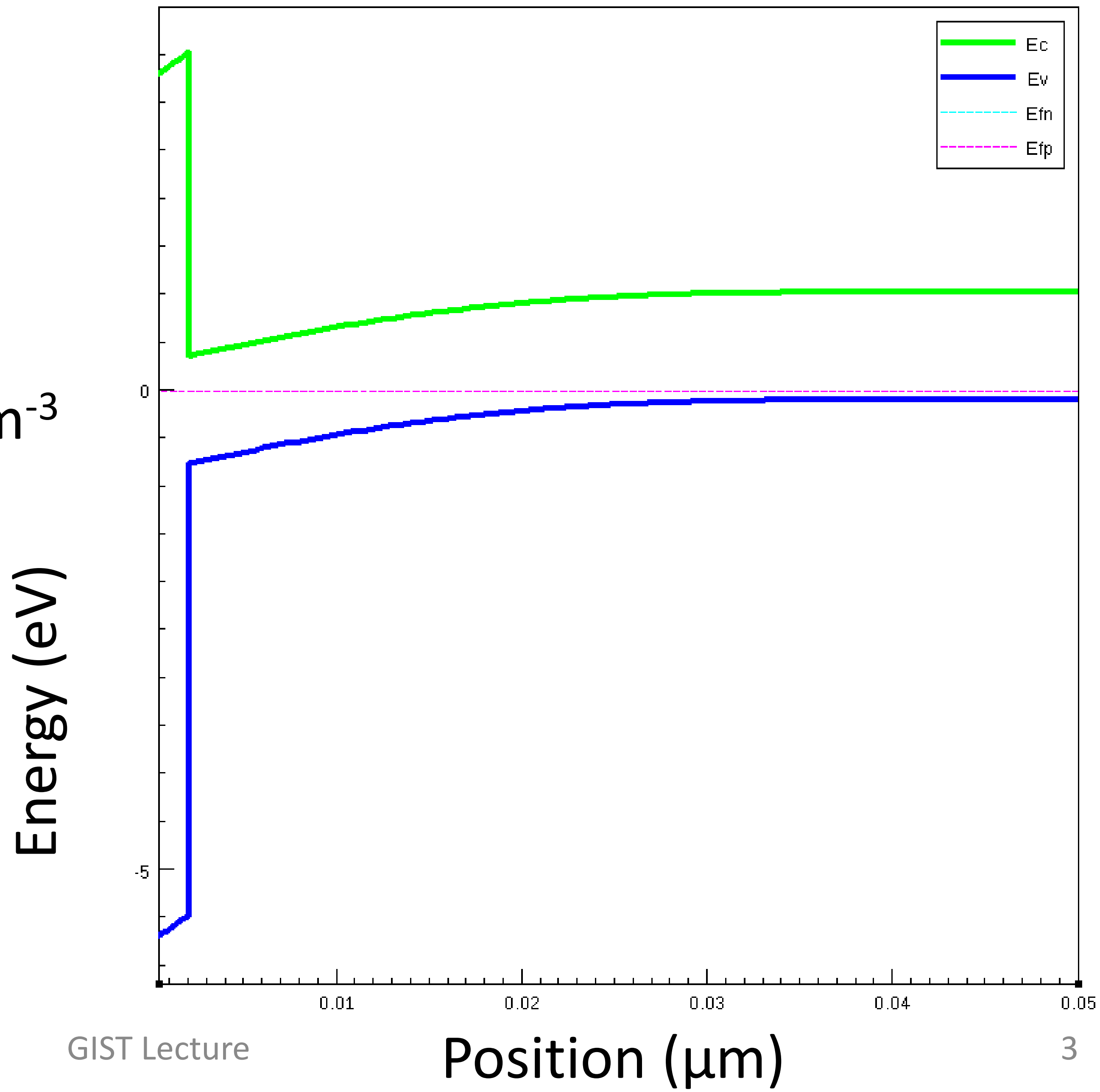
$$\epsilon_{ox}|\mathbf{E}_{ox}| = \epsilon_{si}|\mathbf{E}_{si}| \qquad \text{Taur, Eq. (2.173)}$$

  - Since $\epsilon_{ox} = 3.9\epsilon_0$ and $\epsilon_{si} = 11.7\epsilon_0$,

$$|\mathbf{E}_{ox}| \approx 3|\mathbf{E}_{si}|$$

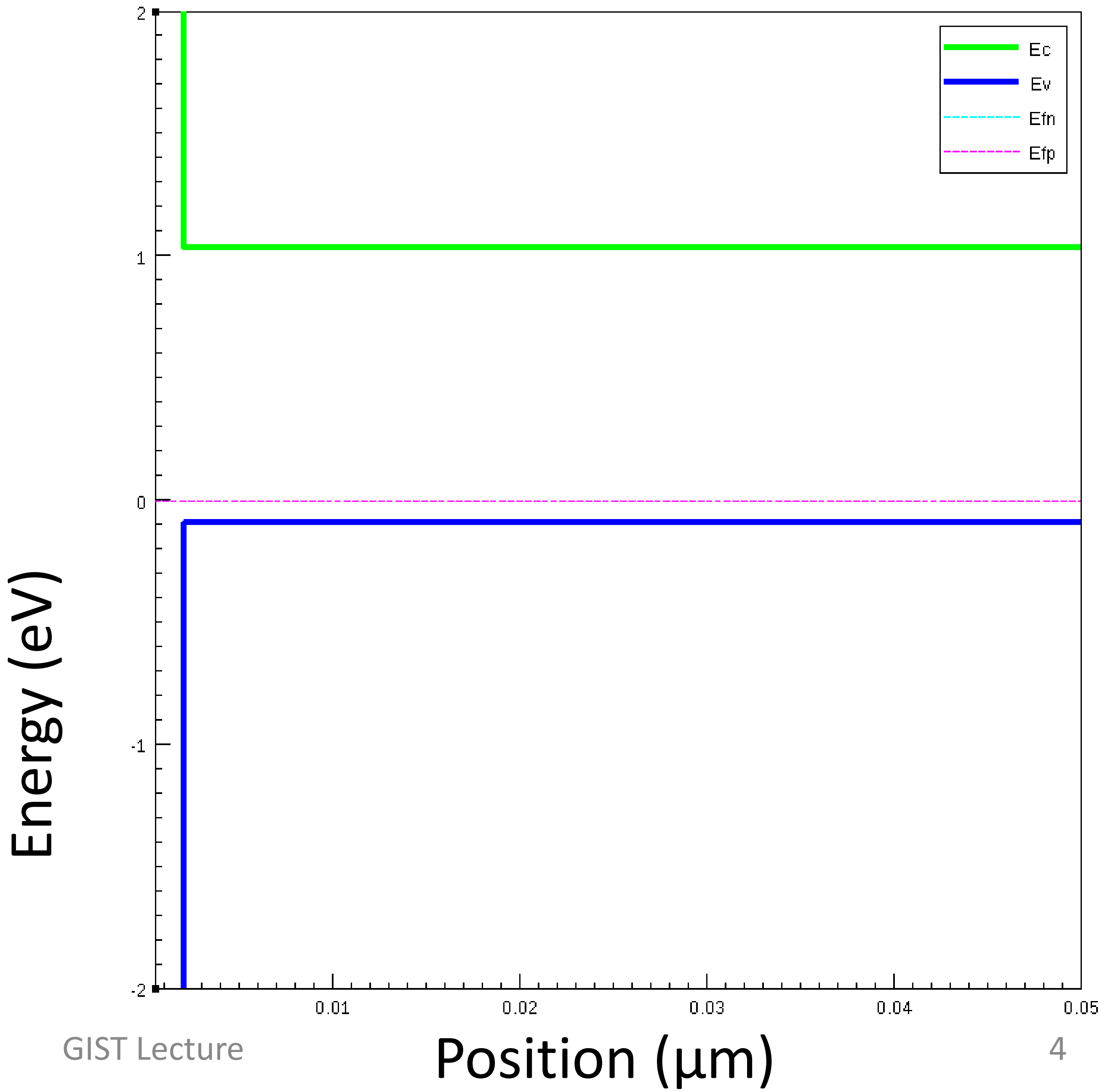
TCAD simulation

- Model parameter
  - Workfunction of 4.17 eV
  - Oxide thickness of 20 Å
  - P-type doping of $1X10^{18}$ cm^{-3}



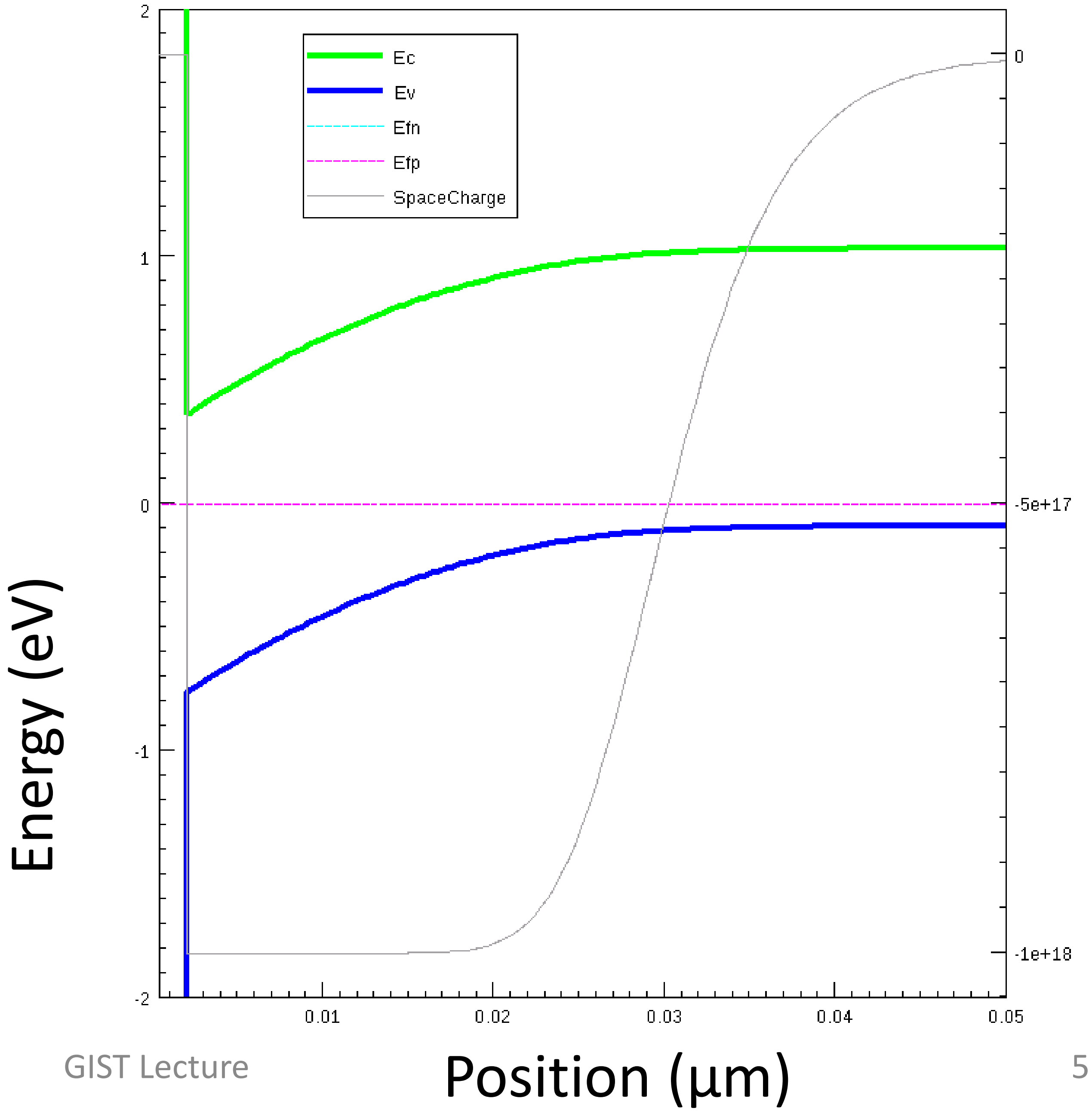
Case 1

- V_g = -0.94 V
  - Flatband condition



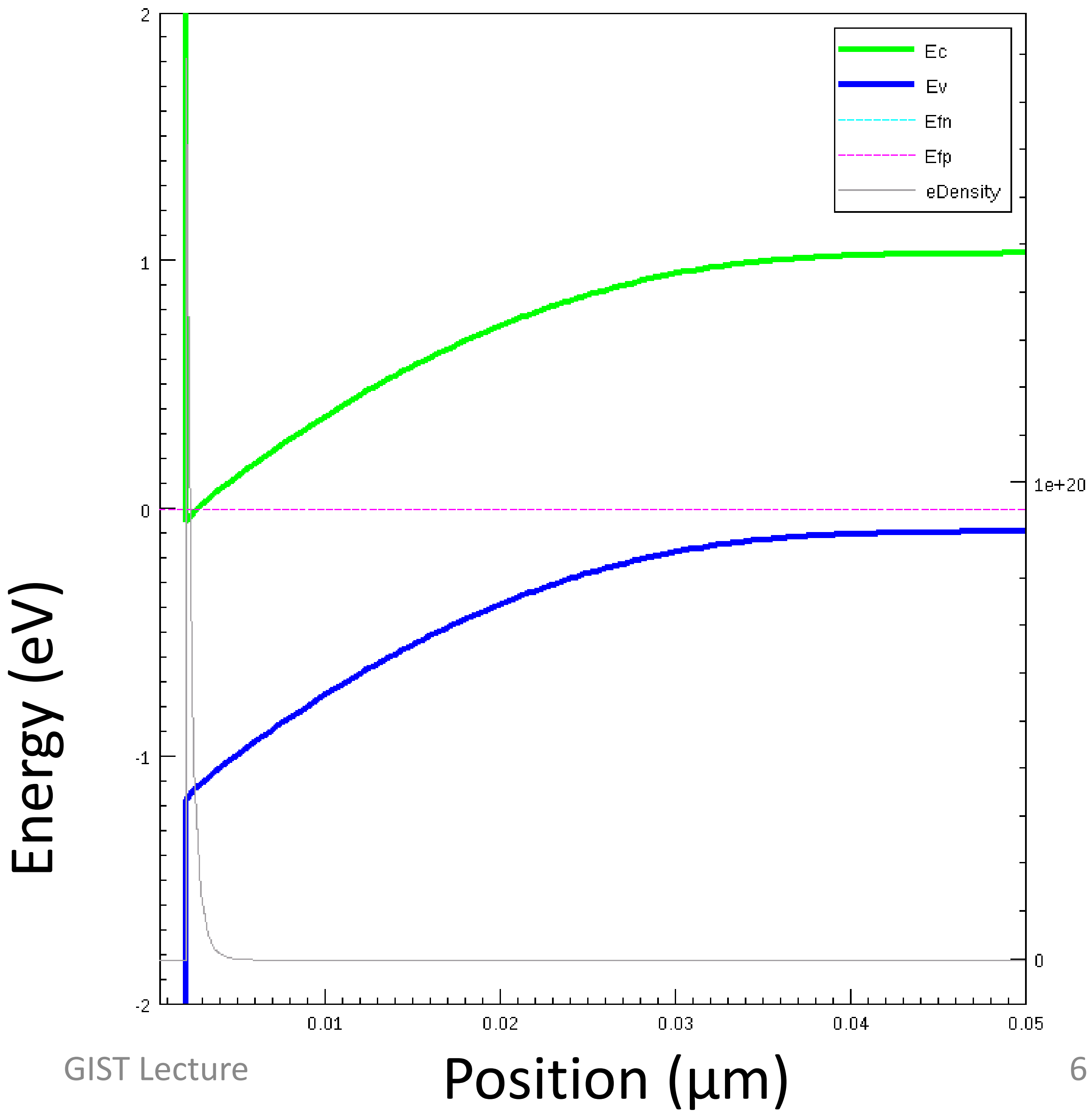
Case 2

- V_g = 0.0 V
  - Depletion
- Space charge



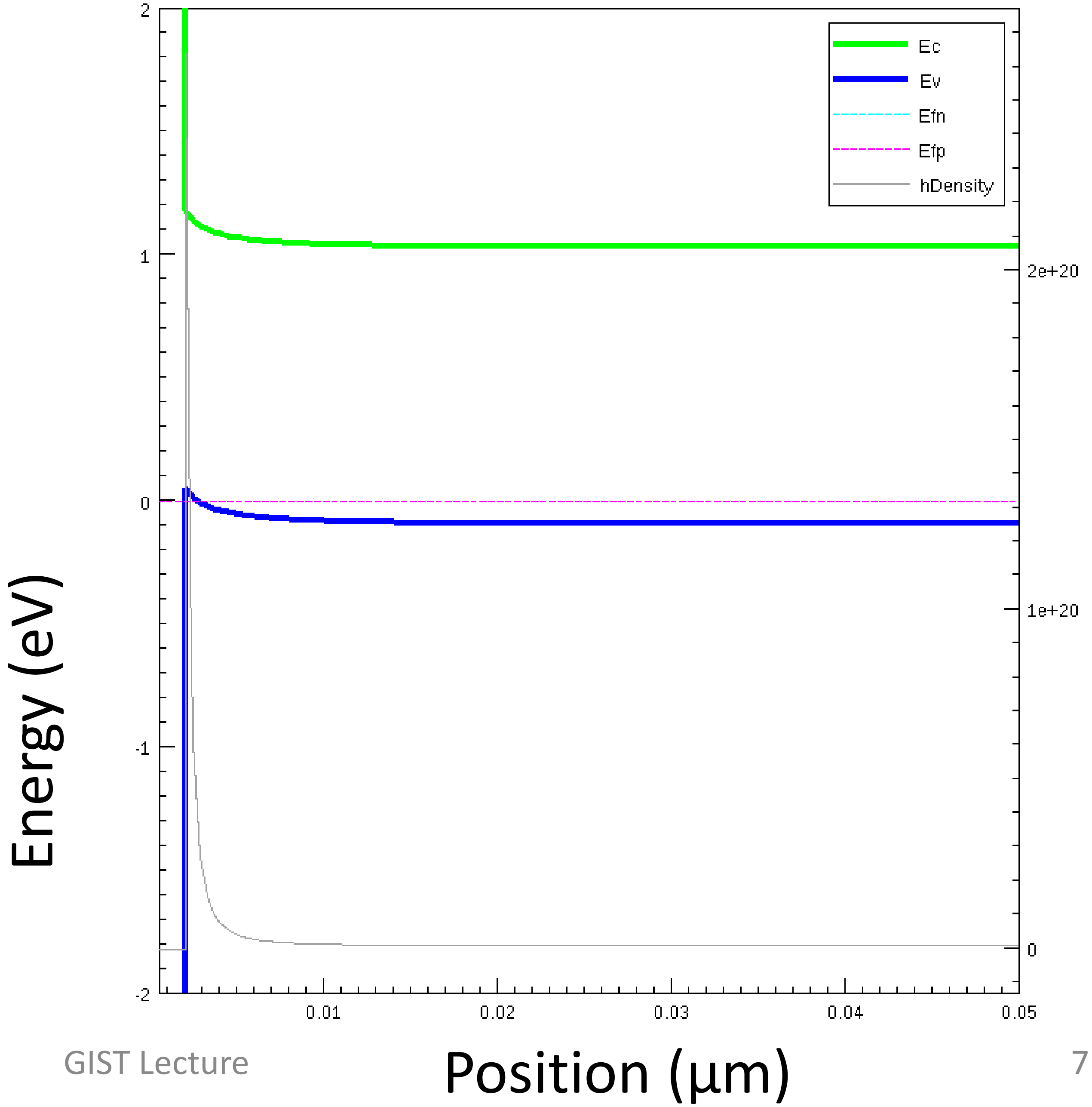
Case 3

- V_g = 1.0 V
  - Inversion
- Electron density



Case 4

- V_g = -2.0 V
  - Accumulation
- Hole density



General relation beyond depletion approx. (1)

- With the depletion approximation, we obtained

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si}qN_a\phi_s}$$

- We can do much better!
  - A generation relation for $Q_s = Q_d + Q_i$
  - The Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}[p(x) - n(x) - N_a]$$

Taur, Eq. (2.175)

Inversion charge

General relation beyond depletion approx. (2)

- Following Taur's notation,

For a while, $\phi(\infty) = -\phi_B$ is used as the reference value. Therefore,

$$n(x) = n_i \exp\left(\frac{q\phi(x)}{k_B T}\right) \rightarrow n(x) = n(\infty) \exp\left(\frac{q\phi(x)}{k_B T}\right)$$ Taur, Eq. (2.178)

$$p(x) = n_i \exp\left(-\frac{q\phi(x)}{k_B T}\right) \rightarrow p(x) = p(\infty) \exp\left(-\frac{q\phi(x)}{k_B T}\right)$$ Taur, Eq. (2.177)

– The Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}\left[N_a\left(\exp\left(-\frac{q\phi}{k_B T}\right) - 1\right) - \frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_B T}\right) - 1\right)\right]$$

Taur, Eq. (2.179)

General relation beyond depletion approx. (3)

- Multiplying $\frac{d\phi}{dx}dx$,
  - The Poisson equation
    $$\frac{d\phi}{dx}d\left(\frac{d\phi}{dx}\right) = -\frac{q}{\epsilon_{si}}\left[N_a\left(\exp\left(-\frac{q\phi}{k_BT}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_BT}\right)-1\right)\right]d\phi$$
  - Integrate the above equation.
    $$\int_0^{-E_x(x)}\frac{d\phi}{dx}d\left(\frac{d\phi}{dx}\right) = -\frac{q}{\epsilon_{si}}\int_0^{\phi(x)}\left[N_a\left(\exp\left(-\frac{q\phi}{k_BT}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_BT}\right)-1\right)\right]d\phi$$

Taur, Eq. (2.180)

General relation beyond depletion approx. (4)

- (Square of) Electric field
  - From $\frac{1}{2}E_x^2(x) = -\frac{q}{\epsilon_{si}}\left[-N_a\frac{k_BT}{q}\exp\left(-\frac{q\phi}{k_BT}\right) - N_a\phi + N_a\frac{k_BT}{q} - \frac{n_i^2}{N_a}\frac{k_BT}{q}\exp\left(\frac{q\phi}{k_BT}\right) + \frac{n_i^2}{N_a}\phi + \frac{n_i^2}{N_a}\frac{k_BT}{q}\right]$, we get

$$E_x^2(x) = \frac{2k_BTN_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi}{k_BT}\right) + \frac{q\phi}{k_BT} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi}{k_BT}\right) - \frac{q\phi}{k_BT} - 1\right)\right]$$

Taur, Eq. (2.181)

General relation beyond depletion approx. (5)

- At $x = 0$, we have $\phi(0) = \phi_s$.
  - Then,

$$E_s^2 = \frac{2k_B T N_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]$$

General relation beyond depletion approx. (6)

- At $x = 0$, we have $\phi(0) = \phi_s$.
  - From $Q_s = -\epsilon_{si} E_s$,

$$Q_s = \pm\sqrt{2\epsilon_{si} k_B T N_a}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]^{1/2}$$

Taur, Eq. (2.182)

Homework (You don't have to submit it.)

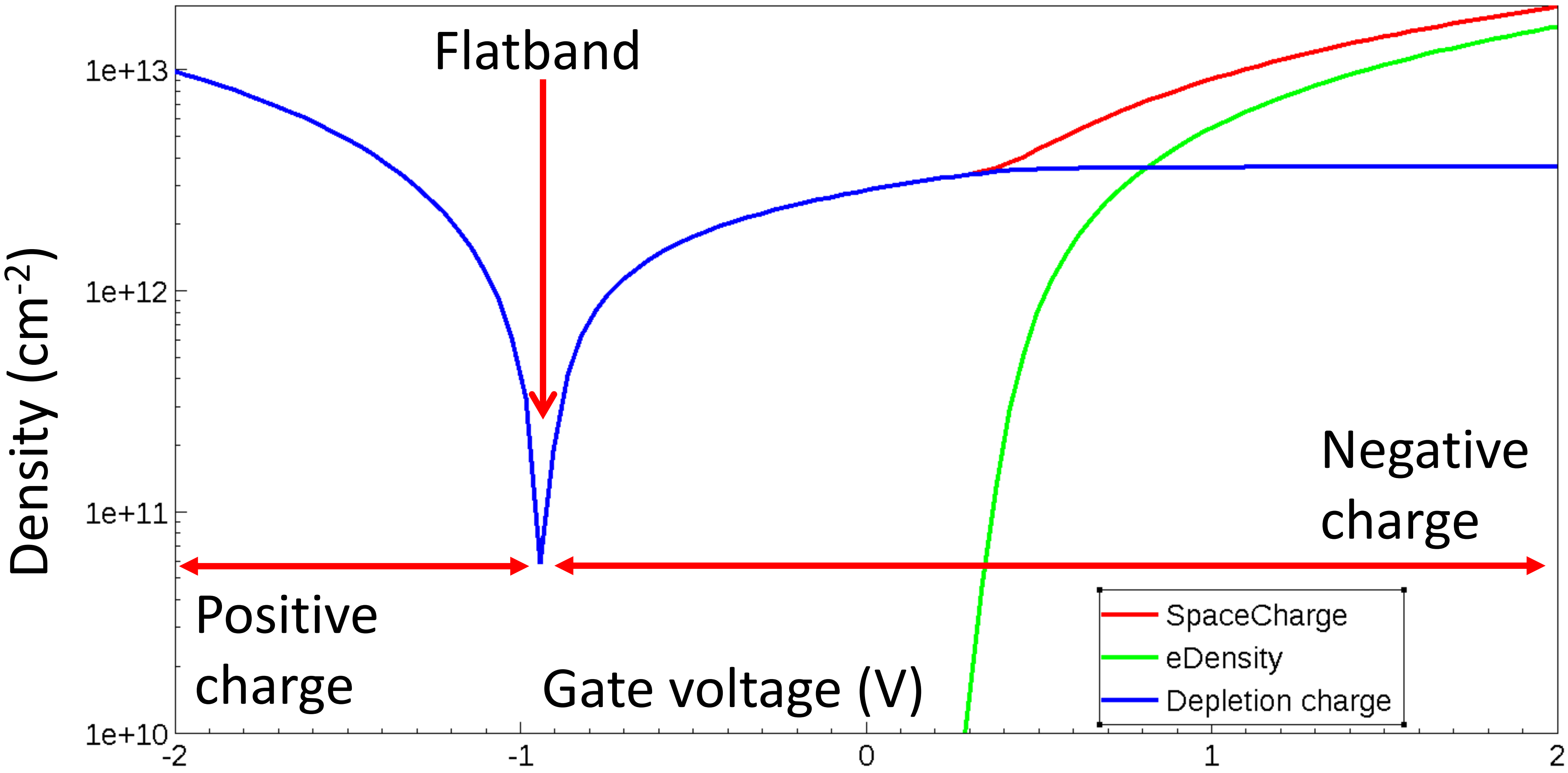
- You may calculate Q_s (or E_s) as a function of ϕ_s.
  - Then, from E_s, you can also calculate $V_{ox} = t_{ox}E_{ox} = t_{ox}\frac{\epsilon_{si}}{\epsilon_{ox}}E_s$.
  - Remember that

$$V_g - V_{fb} = \phi_s + V_{ox}$$

  - Now, you can draw Q_s as a function of V_g.
- Use the parameters for our example.

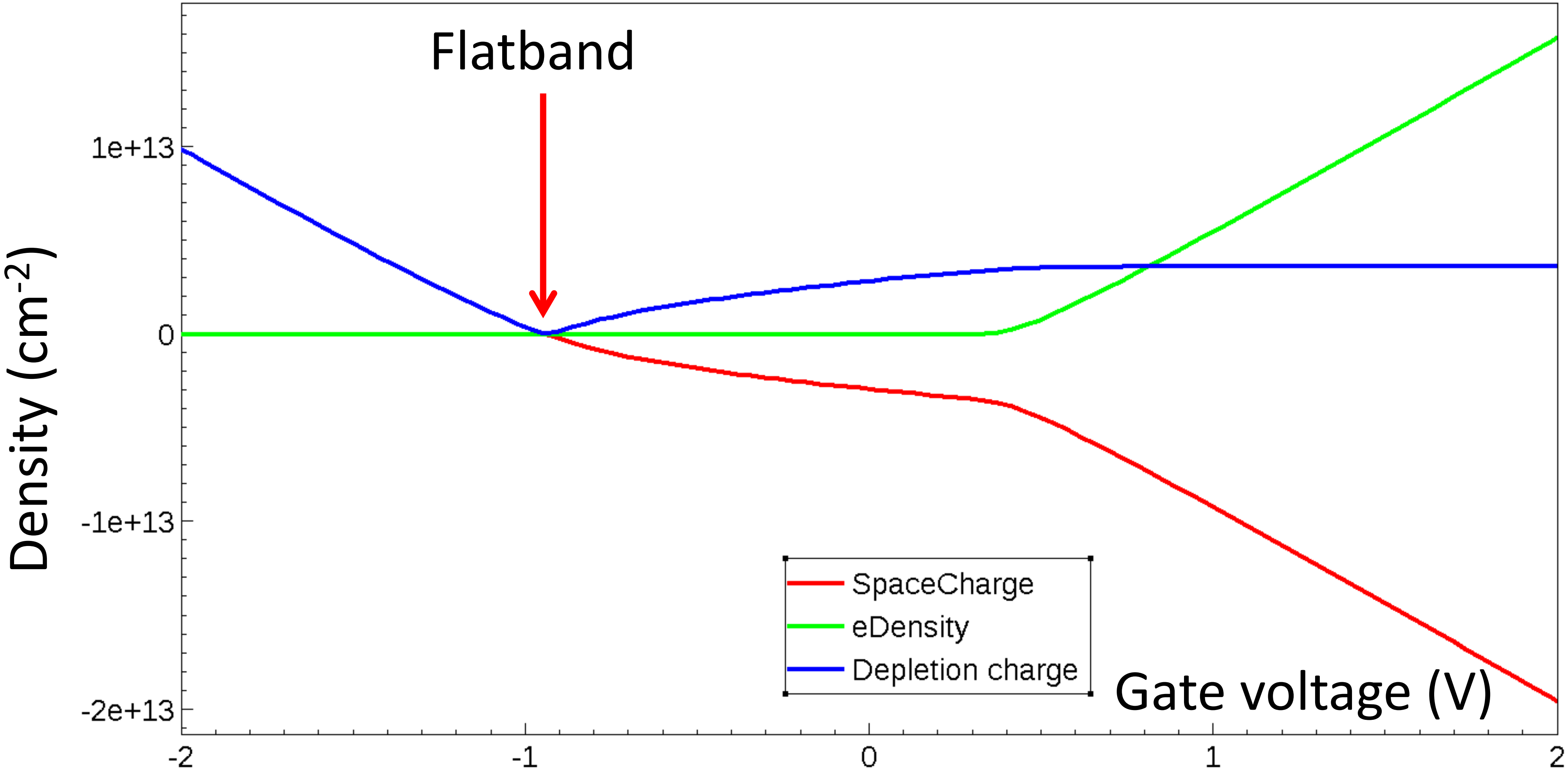
Threshold voltage (1)

- Draw quantities as functions of the gate voltage.

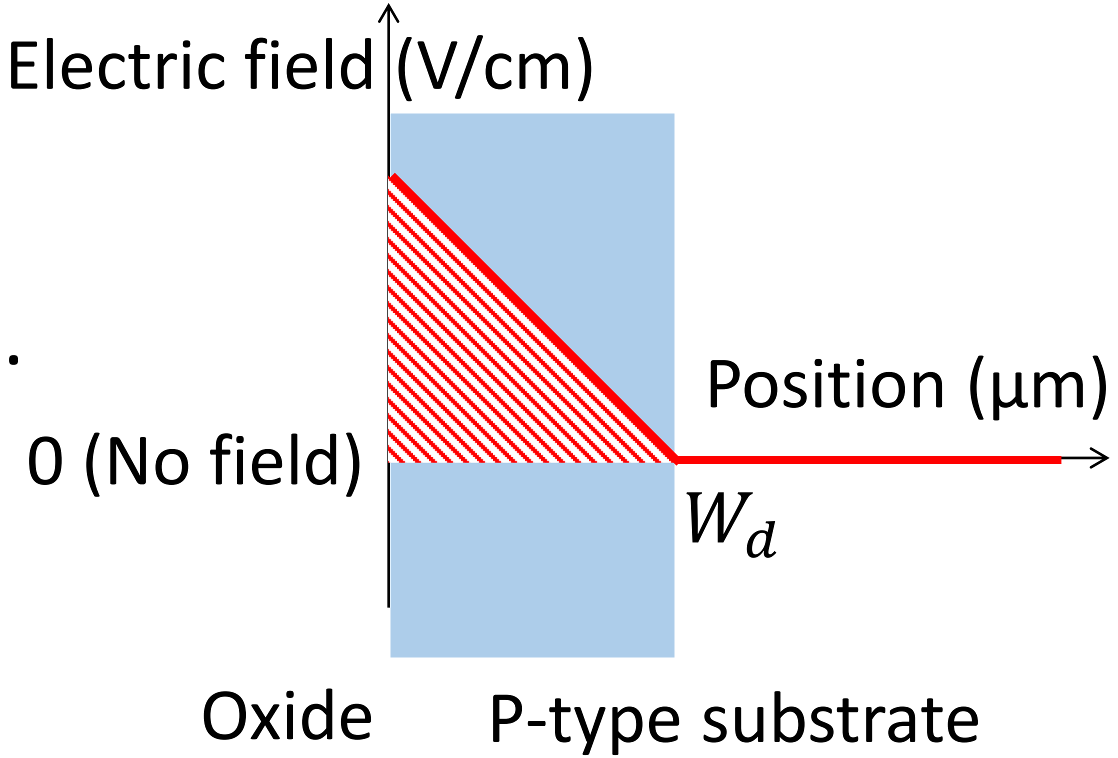


Threshold voltage (2)

- The same graph, in the linear scale,



Depletion approximation



- Consider a depleted MOS structure.
  - With the depletion width, W_d,

$$\phi_s = \frac{1}{2} W_d \left(q \frac{N_a}{\epsilon_{si}} W_d \right)$$

  - Then,

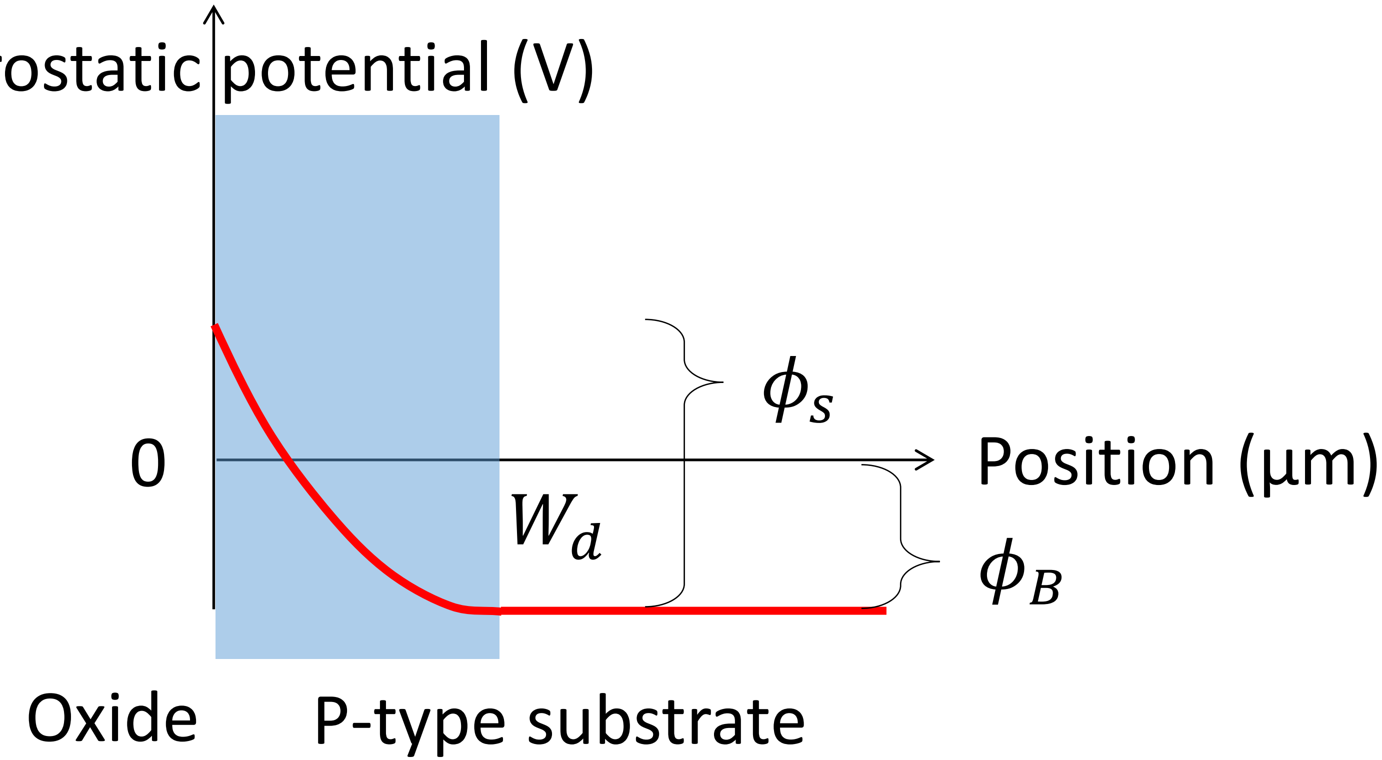
$$W_d = \sqrt{\frac{2\epsilon_{si}\phi_s}{qN_a}}$$ Taur, Eq. (2.188)

  - Total depletion charge in silicon, Q_d, is

$$Q_d = -qN_a W_d = -\sqrt{2\epsilon_{si} q N_a \phi_s}$$ Taur, Eq. (2.189)

Potential profile

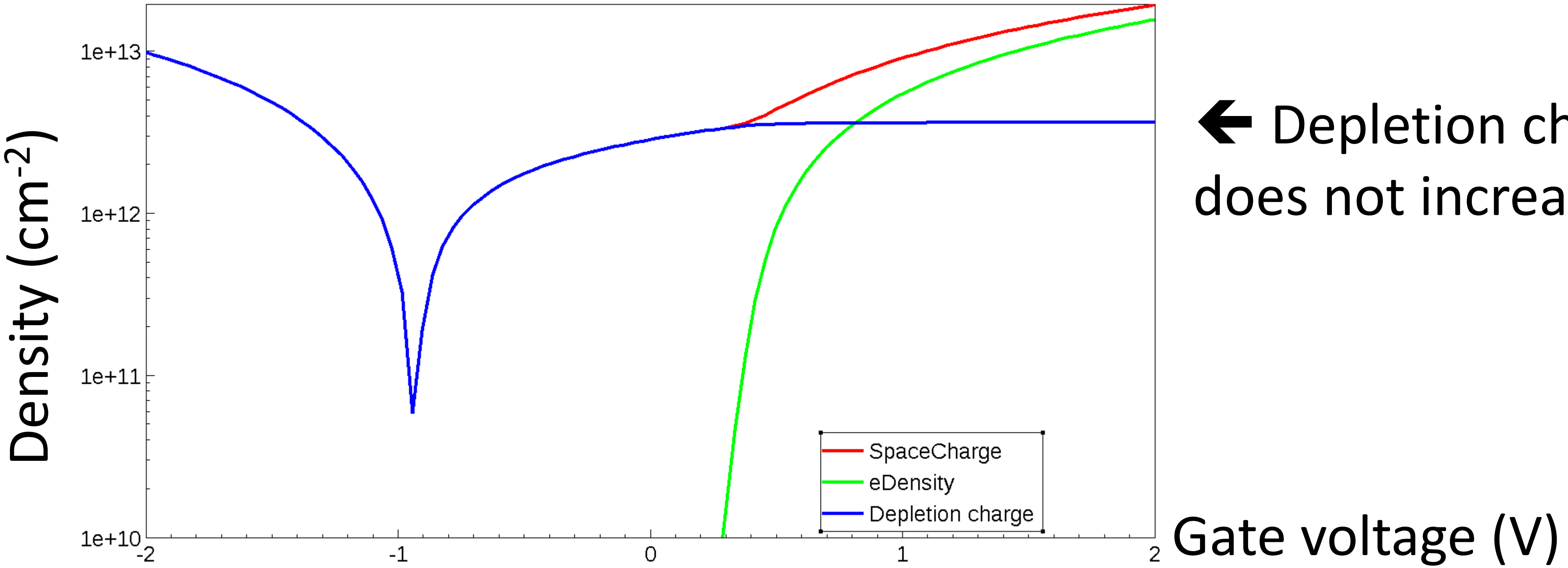
- A parabolic potential profile
  - The depletion region cannot grow indefinitely.
  - When $\phi_s = 2\phi_B$, $n(0) = p(\infty)$
  - The surface behaves like n-type material. (Inversion)
  - ϕ_s is approximately pinned.



Maximum depletion width

- Therefore, maximum depletion width becomes

$$W_d = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_a}} = \sqrt{\frac{4\epsilon_{si}k_BT\ \ln(N_a/n_i)}{q^2N_a}}$$ Taur, Eq. (2.190)



🡸 Depletion charge does not increase.

Beyond threshold voltage

It's not perfectly fixed.

- The surface potential is *almost* fixed. (Surface potential pinning)
  - Small additional change in ϕ_s induces an exponential increase of the electron density.
  - Remember that $n = n_i \exp\left(\frac{q\phi}{k_B T}\right)$.
  - When $\phi_s = 2\phi_B$,

$$n(0) = n_i \exp\left(\frac{q\phi_B}{k_B T}\right) = p(\infty)$$

  - Additional potential ($\Delta\phi$) yields

$$n(0) = p(\infty) \exp\left(\frac{q\Delta\phi}{k_B T}\right)$$

It's a high density.

Thank you!