VLSI Devices
Lecture 25

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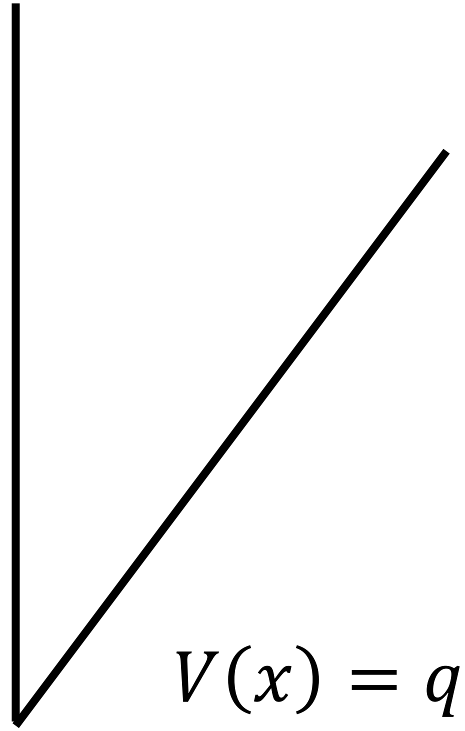
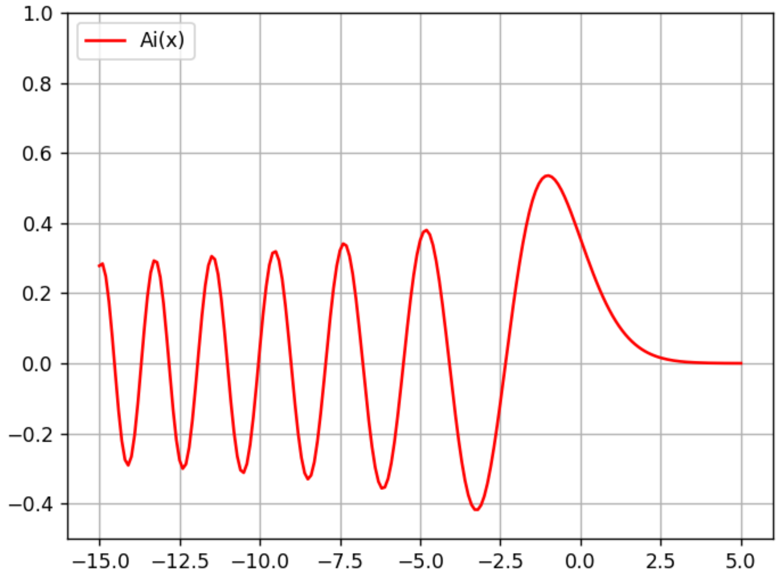
Triangular potential approximation

- Parabolic potential profile
  - However, it is further approximated as a linear potential. ➔ Triangular potential well
  - Then, the Schrödinger equation reads

$$\left[-\frac{\hbar^2}{2m_{xx}}\frac{d^2}{dx^2}+q\mathcal{E}_s x\right]\psi(x)=E\psi(x)$$



Its solution



- Airy function

$$Ai(x) = \frac{1}{\pi}\int_0^{\infty} \cos\left(\frac{t^3}{3} + xt\right) dt$$

  – Its seond derivative is

$$\frac{d^2}{dx^2}Ai(x) = -\frac{1}{\pi}\int_0^{\infty} t^2 \cos\left(\frac{t^3}{3} + xt\right) dt$$

  – Note that $\frac{d}{dt}\sin\left(\frac{t^3}{3} + xt\right) = (t^2 + x)\cos\left(\frac{t^3}{3} + xt\right)$.

  – Therefore,

$$-xAi(x) + \frac{d^2}{dx^2}Ai(x) = -\frac{1}{\pi}\int_0^{\infty} \frac{d}{dt}\sin\left(\frac{t^3}{3} + xt\right) dt = 0$$

Simple manipulation

$$\frac{2m_{xx}}{\hbar^2} q\mathcal{E}_s$$

- The Schrödinger equation is written as

$$\left[\frac{d^2}{dx^2} - \frac{2m_{xx}}{\hbar^2}(q\mathcal{E}_s x - E)\right]\psi = \left[\frac{d^2}{dx^2} - \alpha^3\left(x - \frac{1}{q\mathcal{E}_s}E\right)\right]\psi = 0$$

  - With a new variable, $\xi = \alpha\left(x - \frac{1}{q\mathcal{E}_s}E\right)$, it becomes

$$\left[\frac{d^2}{d\xi^2} - \xi\right]\psi = 0$$

  - The solution is $\psi(x) \sim Ai(\xi) = Ai\left(\alpha\left(x - \frac{1}{q\mathcal{E}_s}E\right)\right)$.
  - At $x = 0$, the wavefunction must vanish:

$$-\alpha\frac{1}{q\mathcal{E}_s}E_j = a_j$$

Zeros of the Airy function

$$a_0 \approx -2.3381$$
$$a_1 \approx -4.0879$$

Eigenenergy

- Zeros are well approximated as $a_j \approx -\left[\frac{3\pi}{2}\left(j+\frac{3}{4}\right)\right]^{2/3}$.
  - Then, the eigenenergy becomes

$$E_j = \frac{q\mathcal{E}_s}{\alpha}\left[\frac{3\pi}{2}\left(j+\frac{3}{4}\right)\right]^{2/3} = \left[\frac{3hq\mathcal{E}_s}{4\sqrt{2m_{xx}}}\left(j+\frac{3}{4}\right)\right]^{2/3}$$

Taur, Eq. (4.46)

  - There are two different m_{xx} values: $0.91m_0$ (degeneracy of 2, g=2) and $0.19m_0$ (degeneracy of 4 , g'=4)

Total inversion charge per unit area

- For a subband,
  - The number of electrosn per unit area

$$n = \frac{4\pi k_B T}{h^2} g\sqrt{m_y m_z} \ln\left[1 + \exp\frac{E_f - E_{min}}{k_B T}\right]$$

Taur, Eq. (A12.5)

  - Summation over subbands

$$Q_i^{QM} = -\frac{4\pi q k_B T}{h^2}\left(g m_t \sum_j \ln\left(1 + \exp\frac{E_f - E_c' - E_j}{k_B T}\right) + g'\sqrt{m_l m_t}\sum_j \ln\left(1 + \exp\frac{E_f - E_c' - E_{j'}}{k_B T}\right)\right)$$

Bottom of the conduction energy at the interface, $E_c' = E_c(\infty) - q\phi_s$

Taur, Eq. (4.49)

Subthreshold region

- In this case,
  - It is well approximated as

$$Q_i^{QM} \approx -\frac{4\pi q k_B T}{h^2}\left(g m_t \sum_j \exp\frac{-E_j}{k_B T} + g'\sqrt{m_l m_t}\sum_j \exp\frac{-E_{j'}}{k_B T}\right)\exp\frac{E_f - E_c'}{k_B T}$$

  - Using $E_c' = E_c(\infty) - q\phi_s$,

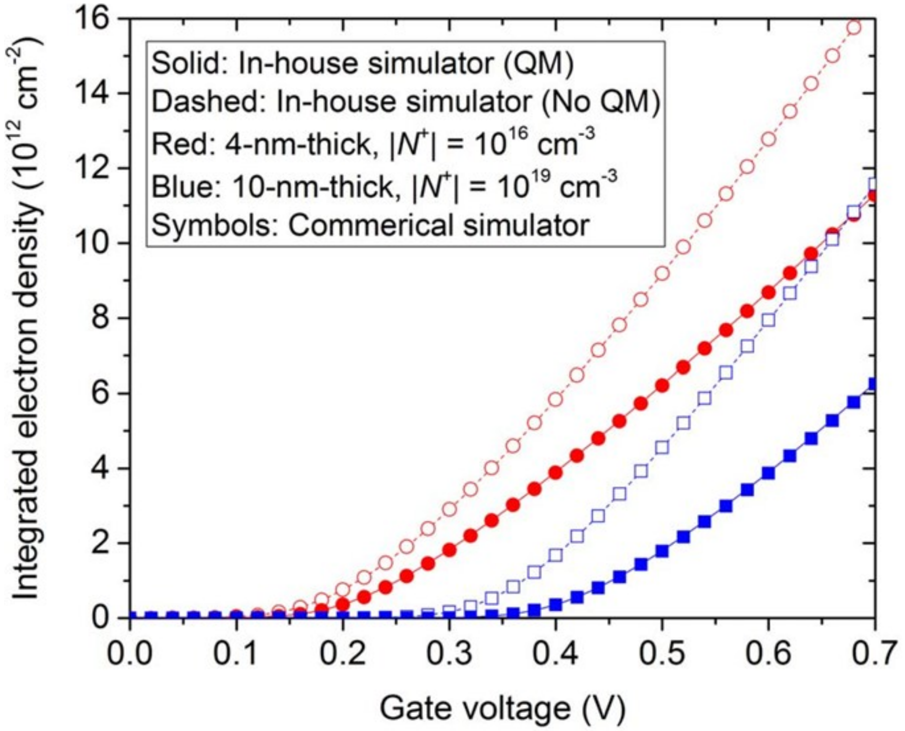
$$Q_i^{QM} \approx -\frac{4\pi q k_B T n_i^2}{h^2 N_c N_a}\left(g m_t \sum_j \exp\frac{-E_j}{k_B T} + g'\sqrt{m_l m_t}\sum_j \exp\frac{-E_{j'}}{k_B T}\right)\exp\frac{q\phi_s}{k_B T}$$

  - (Note that $\exp\frac{E_f - E_c(\infty)}{k_B T} = \frac{n_i^2}{N_c N_a}$.)

Taur, Eq. (4.50)

Shift of threshold voltage

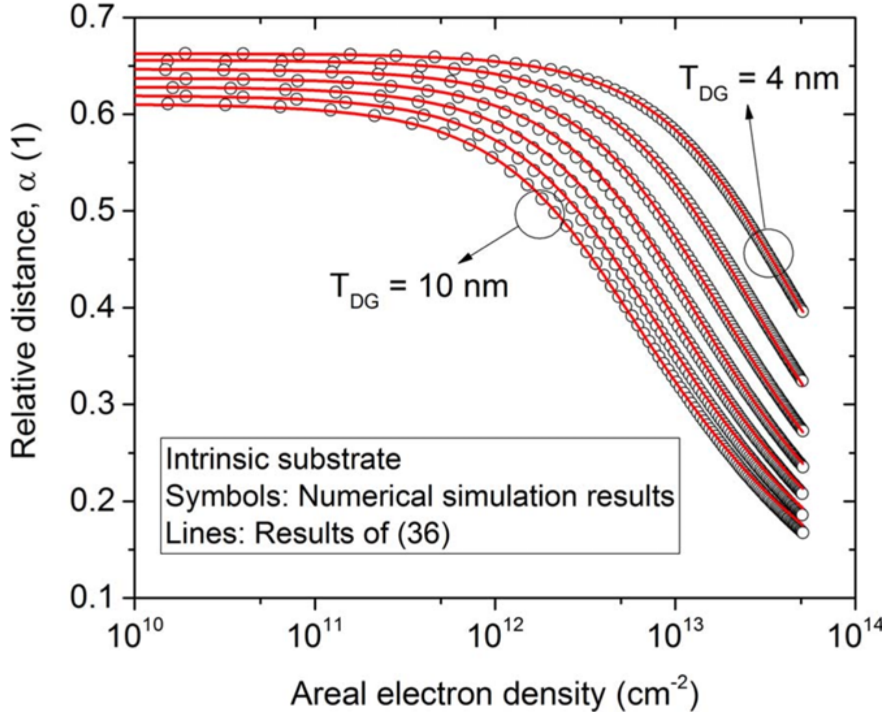
- Q_i^{QM} is smaller than its classical counterpart.
  - Additional band bending is required to achieve the same inversion charge per unit area
  - Example taken from our textbook:
  - When N_a is 3 X 10^{18} cm^{-3}, $\Delta\phi_s^{QM}$ (additional surface potential to match the classical density) is 0.13 V.



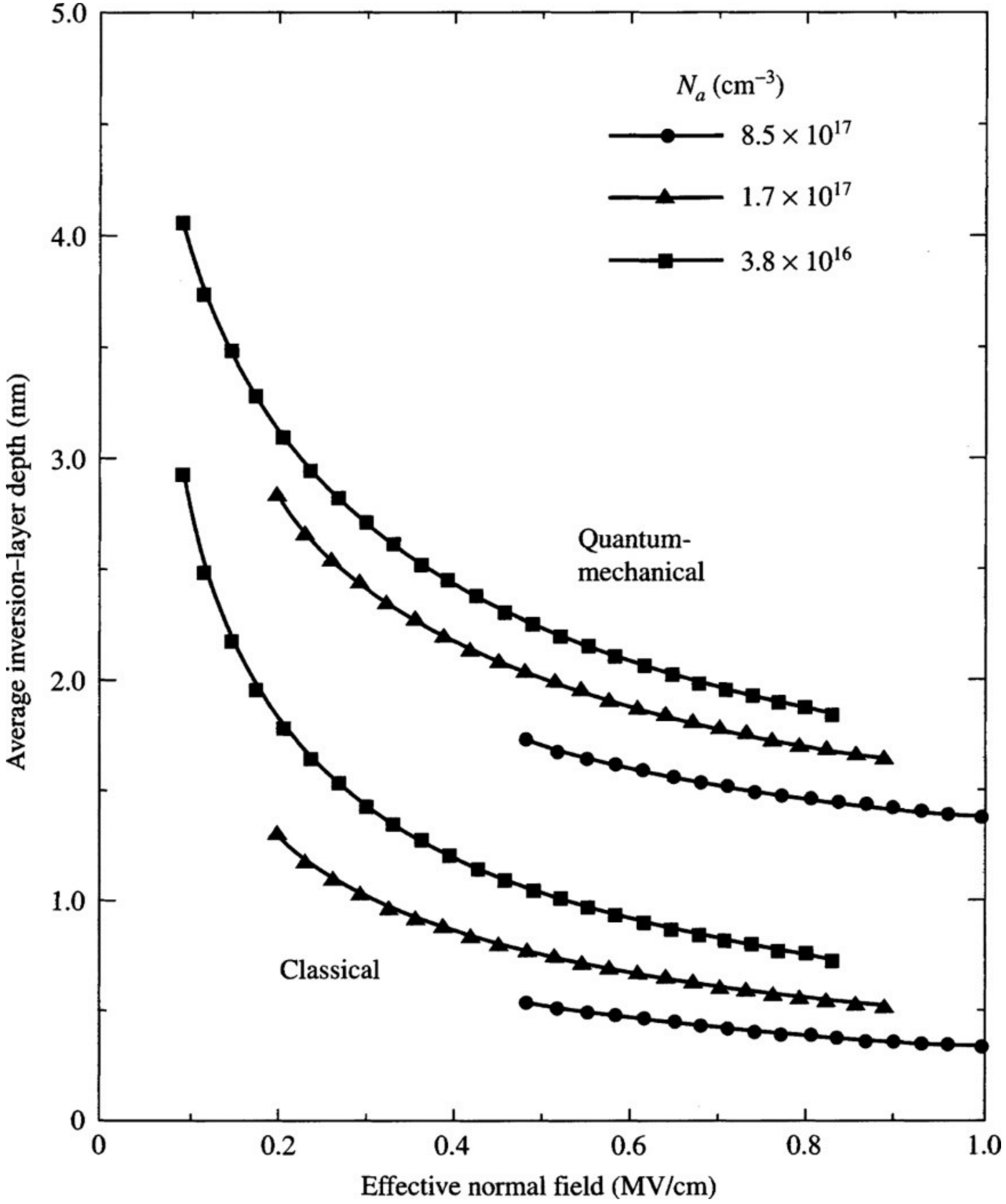
Q-V relations for double-gate MOS structures

Inversion-layer depth

- Average distance
  - It reduces at a high gate voltage.
  - However, QM value is larger than CL one.



Average distance for double-gate MOS structures



Inversion-layer depth (Taur, Fig. 4.21)

Nanosheet

- It is known that the nanowire is ideal in terms of gate controllability.
  - However, nanosheet transistors are area-efficient.

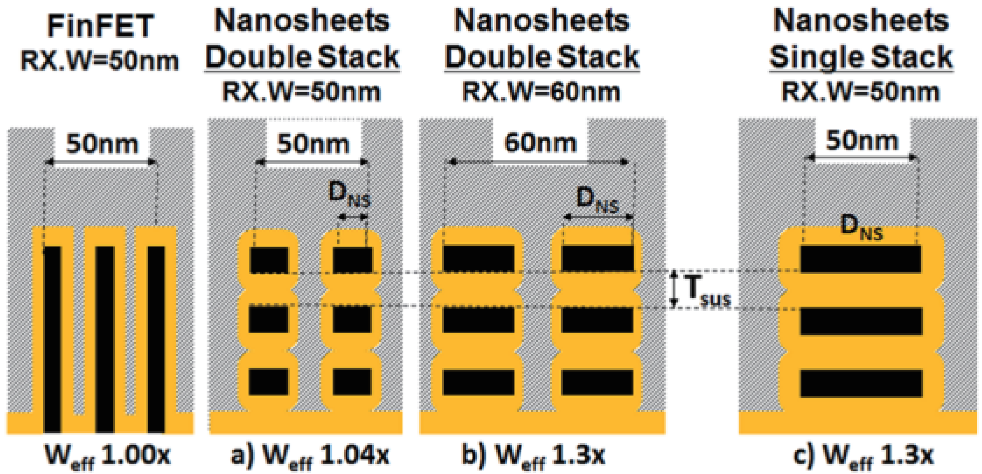


Fig. 2: Increase in W_{eff} going from aggressively scaled FinFET to double and single stack Nanosheets structures. Best improvement is obtained using a single wide Nanosheet stack at constant active width (RX.W).

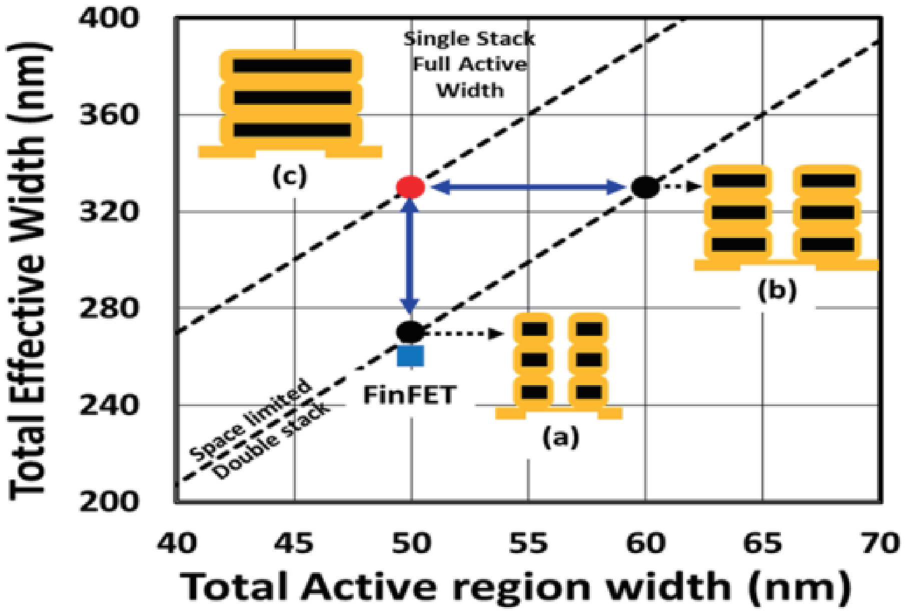


Fig. 3: Improvement in W_{eff} at same footprint going from extremely scaled FinFET to a single wide stack Nanosheet.

Areal efficiency of nanosheet (IBM, VLSI 2017)

Double-gate (DG) MOSFET

- A wide nanosheet can be considered as a double-gate.
  - Two gate contacts
  - The silicon film is usually lightly doped and fully depleted.
  - Performance advantage lies mainly in the ability of DG MOSFETs to scale to a shorter channel length.

Thank you!