Final exam, VLSI Devices, Spring semester, 2024

June 10, 2024

1. Consider a PN junction at equilibrium. By using the depletion approximation, derive the depletion width.

2. When a forward bias, V_{app}, is applied to a PN junction, in the neutral p-type region, we must solve the following equation:

$$\frac{d^2}{dx^2}n = \frac{n - n_{p0}}{L_n^2}$$

In the above equation, n is the electron density, n_{p0} is the minority electron density at equilibrium, and L_n is the electron diffusion length. The neutral p-type region starts at $x = 0$ and ends at $x = \infty$. At these boundaries, the boundary values of n are

$$n(x = 0) = n_{p0} \exp \frac{V_{app}}{k_B T/q},$$

$$n(x = \infty) = n_{p0}.$$

Calculate the solution, $n(x)$.

3. Repeat the same problem. However, in this problem, we assume that the neutral p-type region has a finite thickness of W_p. The boundary values of n are

$$n(x = 0) = n_{p0} \exp \frac{V_{app}}{k_B T/q},$$

$$n(x = W_p) = n_{p0}.$$

Calculate the solution, $n(x)$.

4. Consider a long-channel MOSFET operated in the saturation mode. In this case, we can assume that

$$I_d = \mu_{eff} C_{ox} \frac{W}{L} \frac{\left(V_{gs} - V_t\right)^2}{2m},$$

where quantities have the same meanings studied in our lectures. Let us assume that

$$V_{gs}(t) = 1.0 + V_t + 0.5 \cos 2\pi f t,$$

where f is the signal frequency. By calculating I_d explicitly, show that the output current has a signal with a frequency of $2f$.

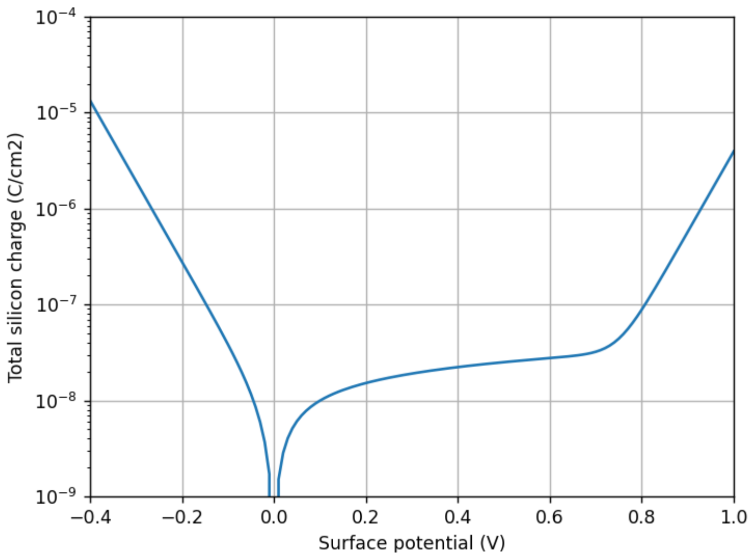
5 (Counted as two problems). In our lectures, we have derived a general relation for the semiconductor charge, Q_s. It was given as

$$Q_s = \pm\sqrt{2\epsilon_{si} k_B T N_a}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]^{1/2}.$$

It is noted that the surface potential, ϕ_s, is referenced to the p-type substrate. Derive this relation.

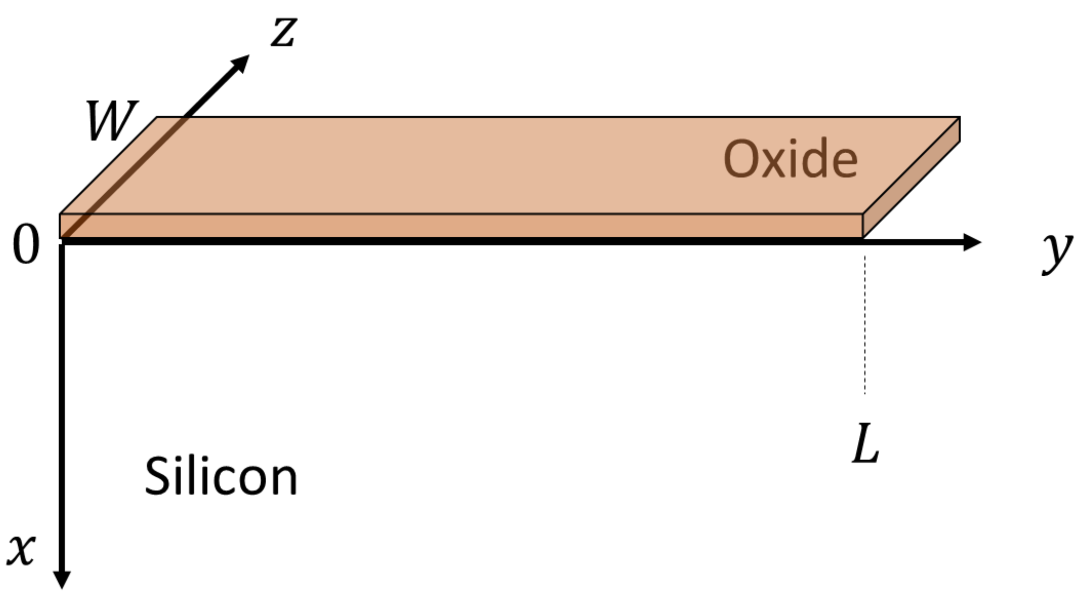
6. The following figure shows $|Q_s|$ as a function of ϕ_s. We can clearly identify three different regions. For each of these three regions, identify the most important term in the square bracket.

$$\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]^{1/2}$$



7. Now, we can express Q_s as a function of ϕ_s. However, the surface potential cannot be directly changed. Instead, by changing V_g, we can indirectly control the surface potential. Write down the relation between V_g and ϕ_s. Of course, $Q_s(\phi_s)$ should appear in your answer.

8. What is the gradual channel approximation? Explain it. (You may use the following coordinate system.)



9. We have assumed that the electron quasi-Fermi potential, V, is independent of the vertical direction, x. Justify this assumption.

10. When V is zero, we have the following results:

$$E_x^2(x,y) = \left(\frac{d\phi}{dx}\right)^2 = \frac{2k_B T N_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi}{k_B T}\right) + \frac{q\phi}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\left(\exp\left(\frac{q\phi}{k_B T}\right) - 1\right) - \frac{q\phi}{k_B T}\right)\right].$$

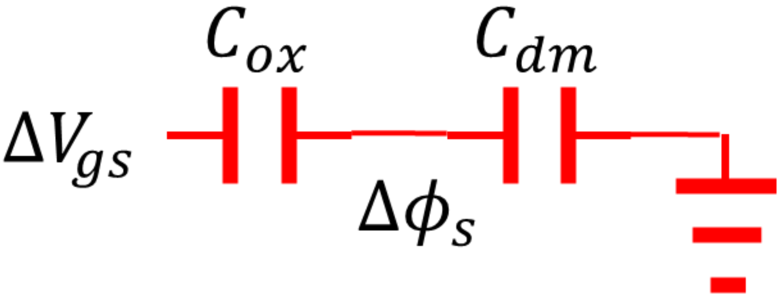
Now, at a certain y, $V(y)$ is non-zero. Modify the above equation to consider this non-zero $V(y)$.

11. After a long derivation procedure, we have found an expression for I_d:

$$I_d = \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs} - V_{fb} + \frac{k_BT}{q}\right)\phi_s - \frac{1}{2}C_{ox}\phi_s^2 - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}\phi_s^{1.5} + \frac{k_BT}{q}\sqrt{2\epsilon_{si}qN_a\phi_s}\right\}\Bigg|_{\phi_{s,s}}^{\phi_{s,d}}$$

Now, assume that $\phi_{s,s}$ and $\phi_{s,d}$ are 1.006 V and 1.206 V, respectively. $\mu_{eff}\frac{W}{L}$ is 250 cm^2 V^{-1} sec^{-1}. The vacuum permittivity is 8.854 X 10^{-12} F m^{-1} and the relative permittivity of silicon is 11.7. The oxide is 10 nm-thick and $V_{gs} - V_{fb}$ is 1.88 V. For the thermal voltage, use 25.85 mV. The substrate doping concentration is 10^{17} cm^{-3}. Of course, q is 1.6X10^{-19} C. By using these numbers, calculate I_d.

12. In our lectures, we introduced a factor, m. Discuss its physical meaning. (You may use the following circuit diagram.)



13. In the subthreshold region, we have an approximate form for the inversion charge,

$$-Q_i \approx \sqrt{\frac{\epsilon_{si}qN_a}{2\phi_s}}\frac{k_BT}{q}\frac{n_i^2}{N_a^2}\exp\left(\frac{q}{k_BT}(\phi_s - V)\right),$$

where ϕ_s can be regarded as a constant. By using a relation,

$$I_d = \mu_{eff}\frac{W}{L}\int_0^{V_{ds}}[-Q_i(V)]dV,$$

derive an expression for I_d.

14. Draw a schematic diagram of a DRAM cell. Describe its READ operation. (The sense amplifier plays an important role.)