VLSI Devices
Lecture 17

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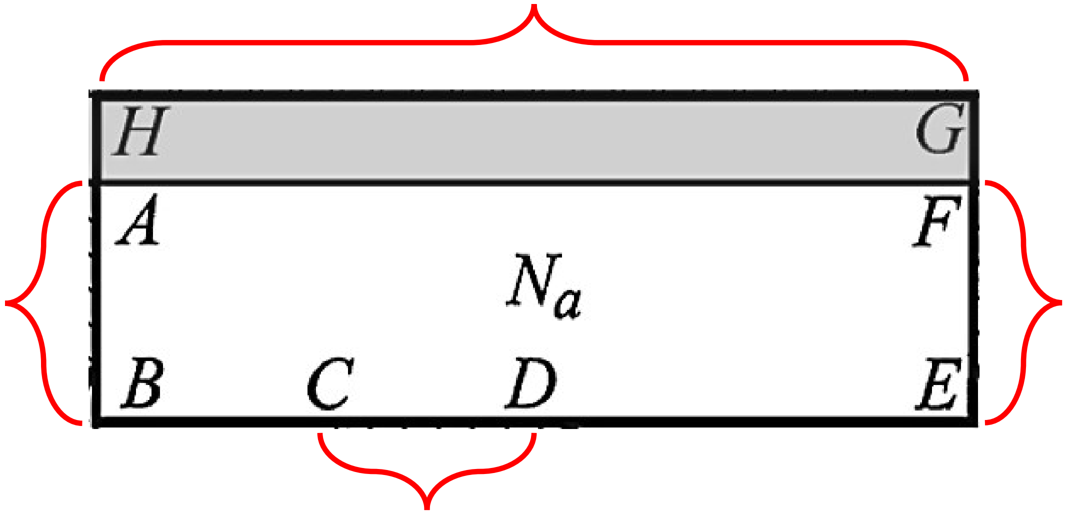
Coverage

- Two YouTube lectures reserved for advanced topics
  - L14: ~~Substrate bias~~, ~~channel mobility~~
  - L15: ~~3.2.1~~
  - L16: 3.2.1 (Continued)
  - L17: Velocity saturation (3.2.2)
  - L18: Channel length modulation and so on (3.2.3, 3.2.4, 3.2.5)
  - L19: MOSFET scaling
  - L20: MOSFET scaling (Continued)
  - L21: Quantum effect (4.2.4)
  - L22: Double-gate MOSFETs (10.3)
  - L23: FinFETs
  - L24: CFETs

Boundary conditions

- Potential reference is $\phi(\infty, y) = -\phi_B$.
  - Along GH: $\phi = V_{gs} - V_{fb}$ Taur, Eq. (A9.3)
  - Along AB: $\phi = \phi_{bi} \approx \frac{E_g}{2q} + \phi_B$ Taur, Eq. (A9.4)
  - Along EF: $\phi = \phi_{bi} + V_{ds}$ Taur, Eq. (A9.5)
  - Along CD: $\phi = 0$ Taur, Eq. (A9.6)
  - $(3t_{ox}$ for uniform permittivity)



Solution

- Poisson equation with boundary conditions
  - Try the following function for the electrostatic potential:

$$\phi(x,y) = v(x,y) + u_L(x,y) + u_R(x,y) + u_B(x,y)$$ Taur, Eq. (A9.9)

Poisson equation with upper b.c. (Long-channel)

Laplace equation to match left b.c.

Laplace equation to match right b.c.

Laplace equation to match bottom b.c.

Solution, $v(x, y)$

- Actually, it is $v(x)$.
  - For the oxide region ($-3t_{ox} \leq x \leq 0$),

$$v(x, y) = \phi_s - \frac{V_{gs} - V_{fb} - \phi_s}{3t_{ox}} x \qquad \text{Taur, Eq. (A9.10)}$$

  - For the silicon region ($0 \leq x \leq W_d$),

$$v(x, y) = \phi_s \left(1 - \frac{x}{W_d}\right)^2 \qquad \text{Taur, Eq. (A9.11)}$$

  - It is noted that

$$\phi_s = \frac{qN_a W_d^2}{2\epsilon_{si}} \qquad \text{Taur, Eq. (A9.12)}$$

A mode for $u_R(x, y)$

- For three boundaries, it should vanish.
  - At only one side, it has non-zero values.
  - We can try ($\lambda \equiv W_d + 3t_{ox}$)

$$u_{R,n}(x, y) = \sinh\left(\frac{n\pi y}{\lambda}\right) \sin\left(\frac{n\pi(x + 3_{tox})}{\lambda}\right)$$

$u_R = 0$

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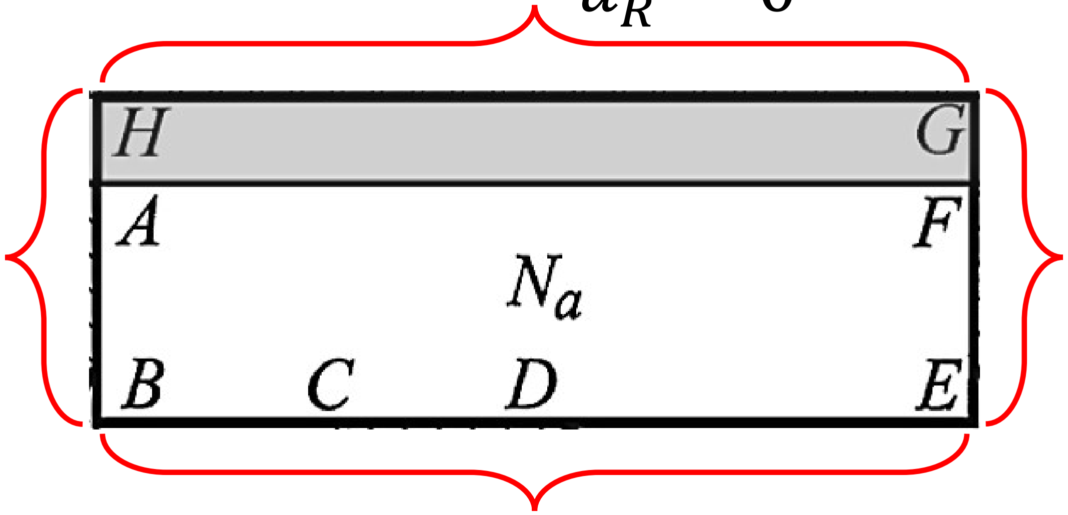


$u_R = \phi(x, L) - v(x, L)$

$u_R = 0$

Series expansion of $\boldsymbol{u_R(x,y)}$

- $u_R(x,L) = \phi(x,L) - v(x,L)$ can be expanded with coefficients c_n's.
  - Therefore,

$$u_R(x,y) = \sum_{n=1}^{\infty} c_n \frac{\sinh\left(\frac{n\pi y}{\lambda}\right)}{\sinh\left(\frac{n\pi L}{\lambda}\right)} \sin\left(\frac{n\pi(x+3_{tox})}{\lambda}\right)$$

Taur, Eq. (A9.15)

- Similar solutions are found for $u_L(x,y)$ and $u_B(x,y)$.

$$u_L(x,y) = \sum_{n=1}^{\infty} b_n \frac{\sinh\left(\frac{n\pi(L-y)}{\lambda}\right)}{\sinh\left(\frac{n\pi L}{\lambda}\right)} \sin\left(\frac{n\pi(x+3_{tox})}{\lambda}\right)$$

Taur, Eq. (A9.14)

First-order expansion

- Keep only b_1 and c_1. (u_B is neglected.)
  - Then,

$$\phi(x,y) = \phi_s\left(1-\frac{x}{W_d}\right)^2 + \frac{b_1\sinh\left(\frac{\pi(L-y)}{\lambda}\right)+c_1\sinh\left(\frac{\pi y}{\lambda}\right)}{\sinh\left(\frac{\pi L}{\lambda}\right)}\sin\left(\frac{\pi(x+3_{tox})}{\lambda}\right)$$

Taur, Eq. (A9.22)

  - Approximate values for b_1 and c_1 are $\frac{4}{\pi}(\phi_{bi}-a\phi_s)$ and $\frac{4}{\pi}(\phi_{bi}+V_{ds}-a\phi_s)$, respectively. $a\approx 0.4$.

Surface potential

- At $x = 0$,

$$\phi(0,y) = \phi_s + \frac{b_1 \sinh\left(\frac{\pi(L-y)}{\lambda}\right) + c_1 \sinh\left(\frac{\pi y}{\lambda}\right)}{\sinh\left(\frac{\pi L}{\lambda}\right)} \sin\left(\frac{\pi 3 t_{ox}}{\lambda}\right)$$

Taur, Eq. (A9.22)

  – Let's find the minimum potential.

$$\frac{d}{dy}\phi(0,y) = \frac{\pi}{\lambda} \frac{-b_1 \cosh\left(\frac{\pi(L-y)}{\lambda}\right) + c_1 \cosh\left(\frac{\pi y}{\lambda}\right)}{\sinh\left(\frac{\pi L}{\lambda}\right)} \sin\left(\frac{\pi 3 t_{ox}}{\lambda}\right) = 0$$

$$b_1 \cosh\left(\frac{\pi(L-y)}{\lambda}\right) = c_1 \cosh\left(\frac{\pi y}{\lambda}\right)$$

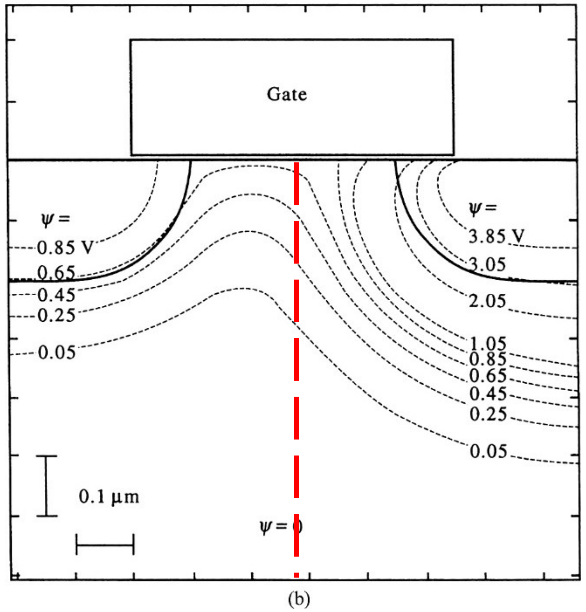
Position for minimum potential, $y = y_c$

- For a positive z, $\cosh z \approx \frac{\exp z}{2}$.

$$\exp \frac{\pi(L - 2y_c)}{\lambda} = \frac{c_1}{b_1}$$

$$y_c = \frac{L}{2} - \frac{\lambda}{2\pi} \ln \frac{c_1}{b_1} \approx \frac{L}{2} - \frac{W_d + 3t_{ox}}{2\pi} \ln\left(1 + \frac{V_{ds}}{\phi_{bi} - a\phi_s}\right)$$

Taur, Eq. (A9.23)



Potential profile (Taur, Fig. 3.20(b))

Minimum potential at $y = y_c$

- Using some approximations,

$$\phi(0, y_c) = \phi_s + 2\sqrt{b_1 c_1}\exp\left(-\frac{\pi L}{2\lambda}\right)\sin\left(\frac{\pi 3 t_{ox}}{\lambda}\right)$$

$$\approx \phi_s + \left(\frac{6\pi t_{ox}}{\lambda}\right)\left(\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})} - \frac{2\phi_{bi} + V_{ds}}{2\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})}} a\phi_s\right)\exp\left(-\frac{\pi L}{2\lambda}\right)$$

~ Taur, Eq. (A9.24)

  – Threshold voltage lowering, ΔV_t,

$$\Delta V_t = \left(\frac{24 t_{ox}}{W_{dm}}\right)\left(\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})} - a(2\phi_B)\right)\exp\left(-\frac{\pi L}{2(W_{dm} + 3t_{ox})}\right)$$

Taur, Eq. (A9.25)

Typical values in the textbook

- Following Taur, Eq. (3.67),

$$\Delta V_t = \left(\frac{24t_{ox}}{W_{dm}}\right)\left(\sqrt{\phi_{bi}(\phi_{bi} + V_{ds})} - a(2\phi_B)\right)\exp\left(-\frac{\pi L}{2(W_{dm} + 3t_{ox})}\right)$$

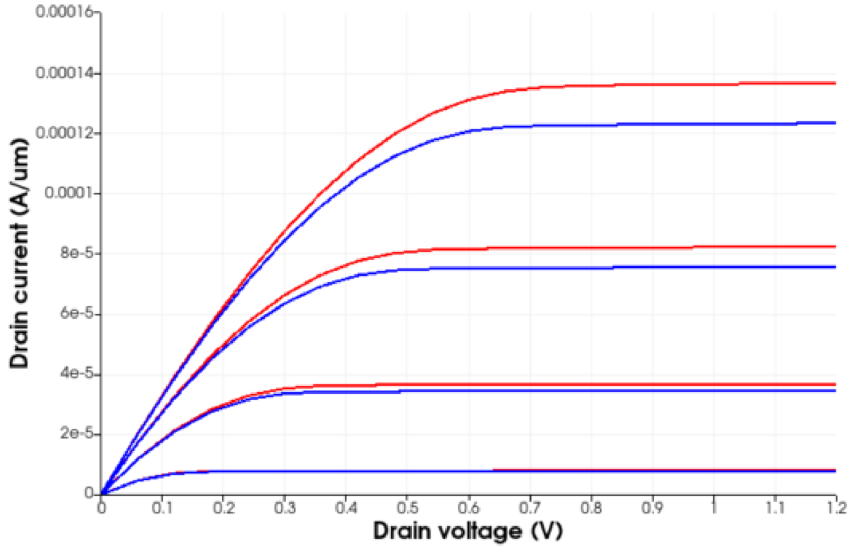
  - Using typical values,

$$0.1 = (2.4)\left(\sqrt{2} - 0.4\right)\exp\left(-\frac{\pi L}{2(1.3W_{dm})}\right)$$

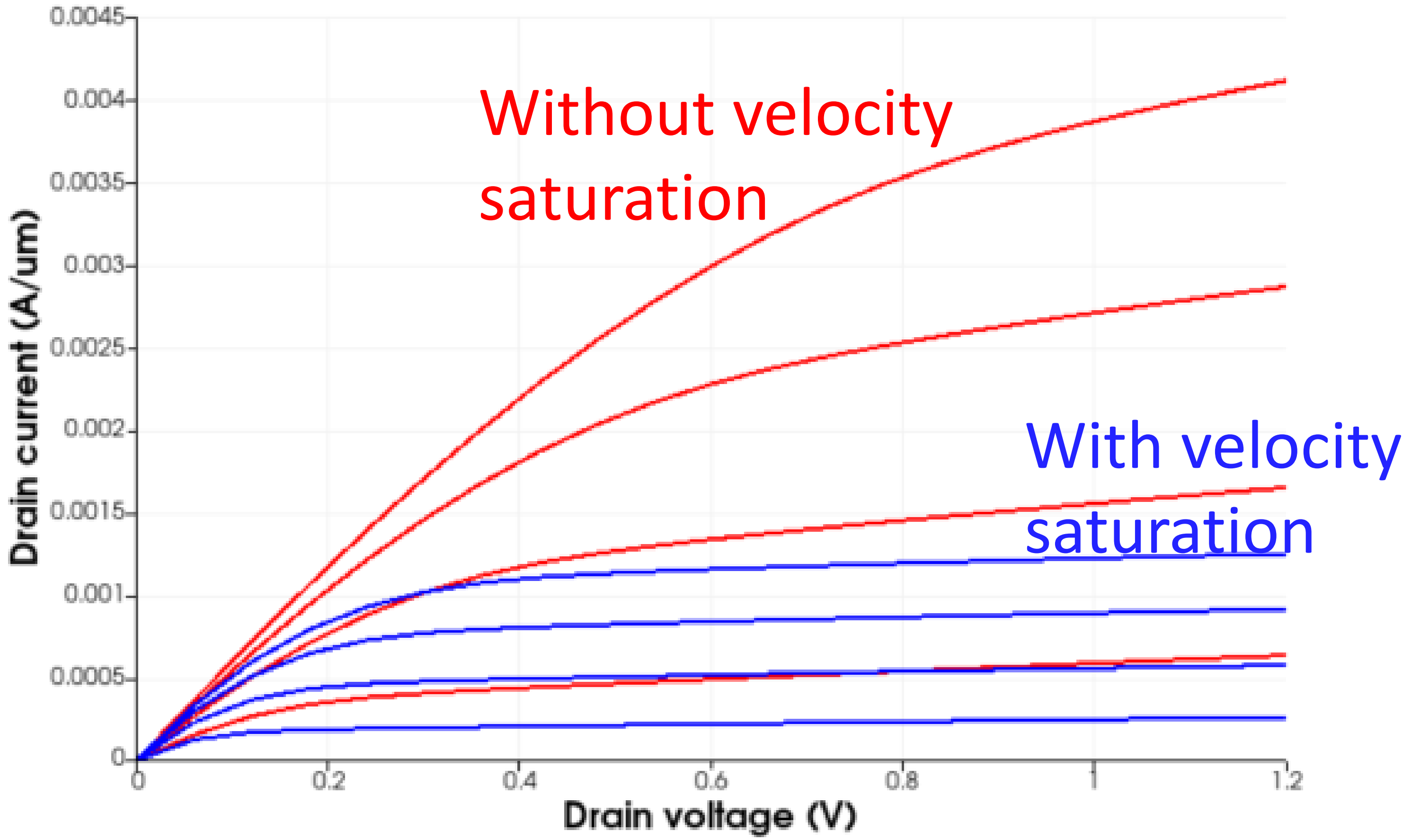
  - We get $L \approx 2.6W_{dm} \approx 2(W_{dm} + 3t_{ox})$.
  - The minimum allowable channel length is $L_{min} \approx 2(W_{dm} + 3t_{ox})$.

Velocity saturation

- Impact of velocity saturation
  - Saturation occurs at a much lower voltage (than $V_{dsat} = (V_{gs} - V_t)/m$).



Long-channel



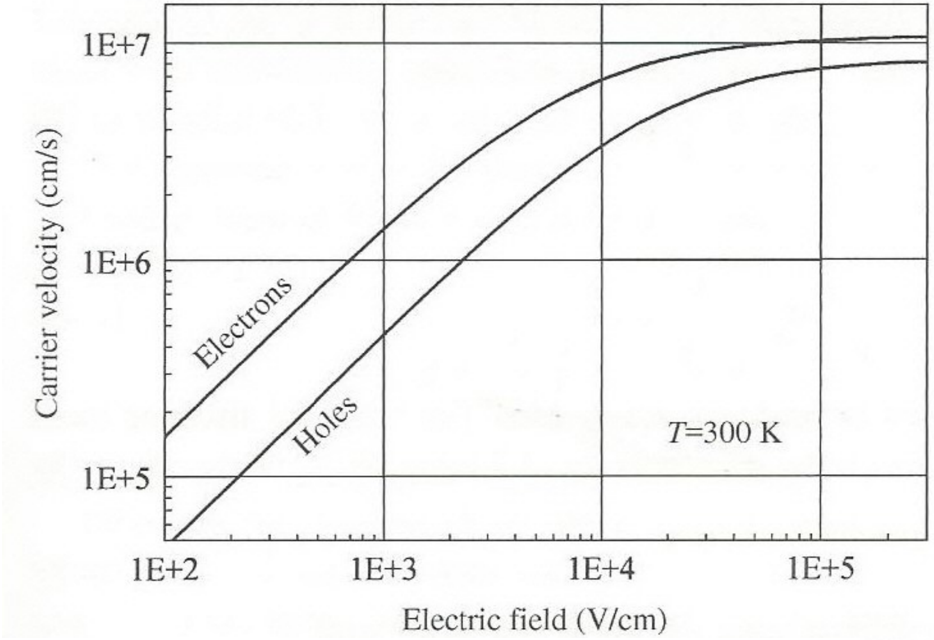
Velocity-field relationship

- Caughey-Thomas
  - Saturation may occur at a much lower voltage (than $V_{dsat} = \frac{V_{gs}-V_t}{m}$).

$$v = \frac{\mu_{eff}\mathcal{E}}{[1 + (\mathcal{E}/\mathcal{E}_c)^n]^{1/n}}$$

Taur, Eq. (3.71)

  - Critical field, $\mathcal{E}_c$
  - For electrons, $n = 2$. For holes, $n = 1$
  - At low fields, $v = \mu_{eff}\mathcal{E}$
  - At high fields ($\mathcal{E} \to \infty$), $v \to \mu_{eff}\mathcal{E}_c = v_{sat}$



Velocity-field relationship (Taur, Fig. 2.10)

Analytic solution for $n = 1$ (1)

- Valid for holes (PMOSFET)

$$I_d = -WQ_i(V)\frac{\mu_{eff}\frac{dV}{dy}}{1+\left(\frac{\mu_{eff}}{v_{sat}}\right)\frac{dV}{dy}}$$

Taur, Eq. (3.73)

  - Rearranging

$$I_d\left[1+\left(\frac{\mu_{eff}}{v_{sat}}\right)\frac{dV}{dy}\right] = -WQ_i(V)\mu_{eff}\frac{dV}{dy}$$

$$I_d = -\left[\mu_{eff}WQ_i(V)+\left(\frac{\mu_{eff}I_d}{v_{sat}}\right)\right]\frac{dV}{dy}$$

Taur, Eq. (3.74)

Analytic solution for $n = 1$ (2)

- Drain current with velocity saturation

$$I_d dy = -\left[\mu_{eff} W Q_i(V) + \left(\frac{\mu_{eff} I_d}{v_{sat}}\right)\right] dV$$

  - Integration from $y = 0$ to L (from $V = 0$ to V_{ds})

$$I_d L = -\mu_{eff} W \int_0^{V_{ds}} Q_i(V) dV - \left(\frac{\mu_{eff} I_d}{v_{sat}}\right) V_{ds}$$

$$I_d L \left(1 + \frac{\mu_{eff} V_{ds}}{v_{sat} L}\right) = -\mu_{eff} W \int_0^{V_{ds}} Q_i(V) dV$$

$$I_d = \frac{-\mu_{eff}(W/L) \int_0^{V_{ds}} Q_i(V) dV}{1 + (\mu_{eff} V_{ds}/v_{sat} L)}$$

Taur, Eq. (3.75)

Analytic solution for $n = 1$ (3)

- Using the chage-sheet model

$$Q_i = -C_{ox}\left(V_{gs} - V_t - mV\right)$$

Taur, Eq. (3.76)

$$I_d = \frac{\mu_{eff} C_{ox} (W/L) \left[\left(V_{gs} - V_t\right)V_{ds} - \frac{m}{2}V_{ds}^2\right]}{1 + \left(\mu_{eff} V_{ds}/v_{sat} L\right)}$$

Taur, Eq. (3.77)

  - By solving $\frac{dI_d}{dV_{ds}} = 0$ at V_{dsat},

$$0 = \frac{\left(V_{gs} - V_t\right) - mV_{dsat}}{1 + \left(\mu_{eff} V_{dsat}/v_{sat} L\right)} - \frac{\left(V_{gs} - V_t\right)V_{dsat} - \frac{m}{2}V_{dsat}^2}{\left[1 + \left(\mu_{eff} V_{dsat}/v_{sat} L\right)\right]^2}\left(\mu_{eff}/v_{sat} L\right)$$

Analytic solution for $n = 1$ (4)

- Manipulation

$$\left[\left(V_{gs} - V_t\right) - mV_{dsat}\right]\left[1 + \left(\mu_{eff} V_{dsat} / v_{sat} L\right)\right]$$
$$= \left[\left(V_{gs} - V_t\right) V_{dsat} - \frac{m}{2} V_{dsat}^2\right]\left(\mu_{eff} / v_{sat} L\right)$$

$$V_{dsat} = \frac{2\left(V_{gs} - V_t\right)/m}{1 + \sqrt{1 + 2\mu_{eff}\left(V_{gs} - V_t\right)/(m v_{sat} L)}} \le \left(V_{gs} - V_t\right)/m$$

Taur, Eq. (3.78)

$-L \to \infty, V_{dsat} = \left(V_{gs} - V_t\right)/m$

$-L \to 0,$

$$V_{dsat} = \sqrt{\frac{2\left(V_{gs} - V_t\right) v_{sat} L}{\mu_{eff} m}}$$

Analytic solution for $n = 1$ (5)

- Two extreme cases
  - $L \to \infty$,

$$I_{dsat} = \mu_{eff} C_{ox} \frac{W}{L} \frac{\left(V_{gs} - V_t\right)^2}{2m} \qquad \text{Taur, Eq. (3.80)}$$

  - $L \to 0$,

$$I_{dsat} = C_{ox} W v_{sat} \left(V_{gs} - V_t\right) \qquad \text{Taur, Eq. (3.81)}$$

  - In this case, I_{dsat} is independent of channel length L and varies linearly with $V_{gs} - V_t$ instead of quadratically as in the long-channel case.

Thank you!