VLSI Devices
Lecture 7

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General relation beyond depletion approx. (2)

- Following Taur's notation,

For a while, $\phi(\infty) = -\phi_B$ is used as the reference value. Therefore,

$$n(x) = n_i \exp\left(\frac{q\phi(x)}{k_B T}\right) \rightarrow n(x) = n(\infty) \exp\left(\frac{q\phi(x)}{k_B T}\right)$$ Taur, Eq. (2.178)

$$p(x) = n_i \exp\left(-\frac{q\phi(x)}{k_B T}\right) \rightarrow p(x) = p(\infty) \exp\left(-\frac{q\phi(x)}{k_B T}\right)$$ Taur, Eq. (2.177)

- The Poisson equation

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}\left[N_a\left(\exp\left(-\frac{q\phi}{k_B T}\right) - 1\right) - \frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_B T}\right) - 1\right)\right]$$

Taur, Eq. (2.179)

General relation beyond depletion approx. (3)

- Multiplying $\frac{d\phi}{dx}dx$,
  - The Poisson equation
  $$\frac{d\phi}{dx}d\left(\frac{d\phi}{dx}\right)$$
  $$= -\frac{q}{\epsilon_{si}}\left[N_a\left(\exp\left(-\frac{q\phi}{k_BT}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_BT}\right)-1\right)\right]d\phi$$
  - Integrate the above equation.
  $$\int_0^{-E_x(x)}\frac{d\phi}{dx}d\left(\frac{d\phi}{dx}\right)$$
  $$= -\frac{q}{\epsilon_{si}}\int_0^{\phi(x)}\left[N_a\left(\exp\left(-\frac{q\phi}{k_BT}\right)-1\right)-\frac{n_i^2}{N_a}\left(\exp\left(\frac{q\phi}{k_BT}\right)-1\right)\right]d\phi$$

Taur, Eq. (2.180)

General relation beyond depletion approx. (4)

- (Square of) Electric field
  - From $\frac{1}{2}E_x^2(x) = -\frac{q}{\epsilon_{si}}\left[-N_a \frac{k_B T}{q} \exp\left(-\frac{q\phi}{k_B T}\right) - N_a\phi + N_a \frac{k_B T}{q} - \frac{n_i^2}{N_a}\frac{k_B T}{q}\exp\left(\frac{q\phi}{k_B T}\right) + \frac{n_i^2}{N_a}\phi + \frac{n_i^2}{N_a}\frac{k_B T}{q}\right]$, we get

$$E_x^2(x) = \frac{2k_B T N_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi}{k_B T}\right) + \frac{q\phi}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi}{k_B T}\right) - \frac{q\phi}{k_B T} - 1\right)\right]$$

Taur, Eq. (2.181)

General relation beyond depletion approx. (5)

- At $x = 0$, we have $\phi(0) = \phi_s$.
  - Then,

$$E_s^2 = \frac{2k_B T N_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]$$

General relation beyond depletion approx. (6)

- At $x = 0$, we have $\phi(0) = \phi_s$.
  - From $Q_s = -\epsilon_{si} E_s$,

$$Q_s = \pm\sqrt{2\epsilon_{si} k_B T N_a}\left[\left(\exp\left(-\frac{q\phi_s}{k_B T}\right) + \frac{q\phi_s}{k_B T} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_B T}\right) - \frac{q\phi_s}{k_B T} - 1\right)\right]^{1/2}$$

Taur, Eq. (2.182)

Homework (You don't have to submit it.)

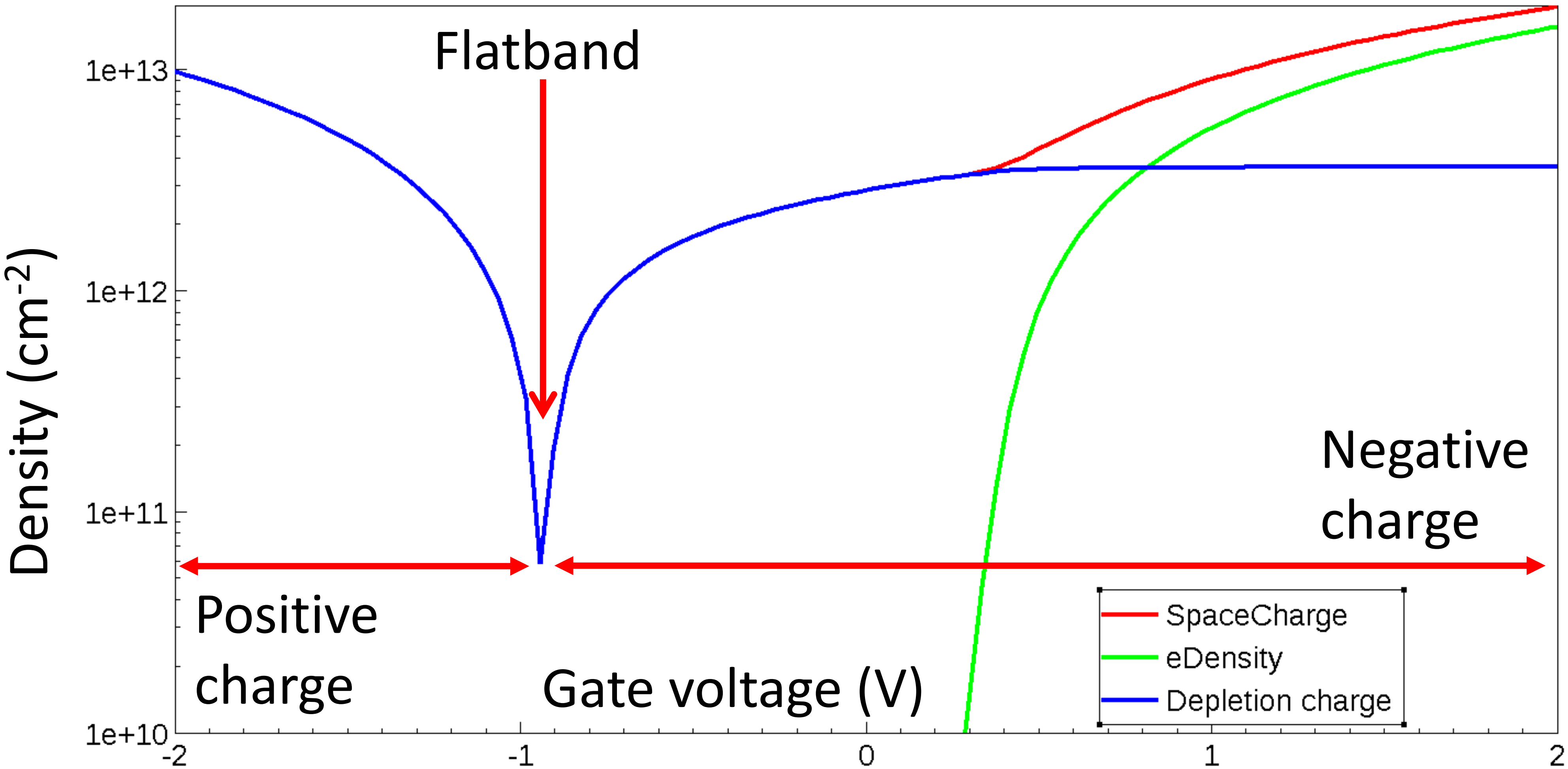
- You may calculate Q_s (or E_s) as a function of ϕ_s.
  - Then, from E_s, you can also calculate $V_{ox} = t_{ox}E_{ox} = t_{ox}\frac{\epsilon_{si}}{\epsilon_{ox}}E_s$.
  - Remember that

$$V_g - V_{fb} = \phi_s + V_{ox}$$

  - Now, you can draw Q_s as a function of V_g.
- Use the parameters for our example.

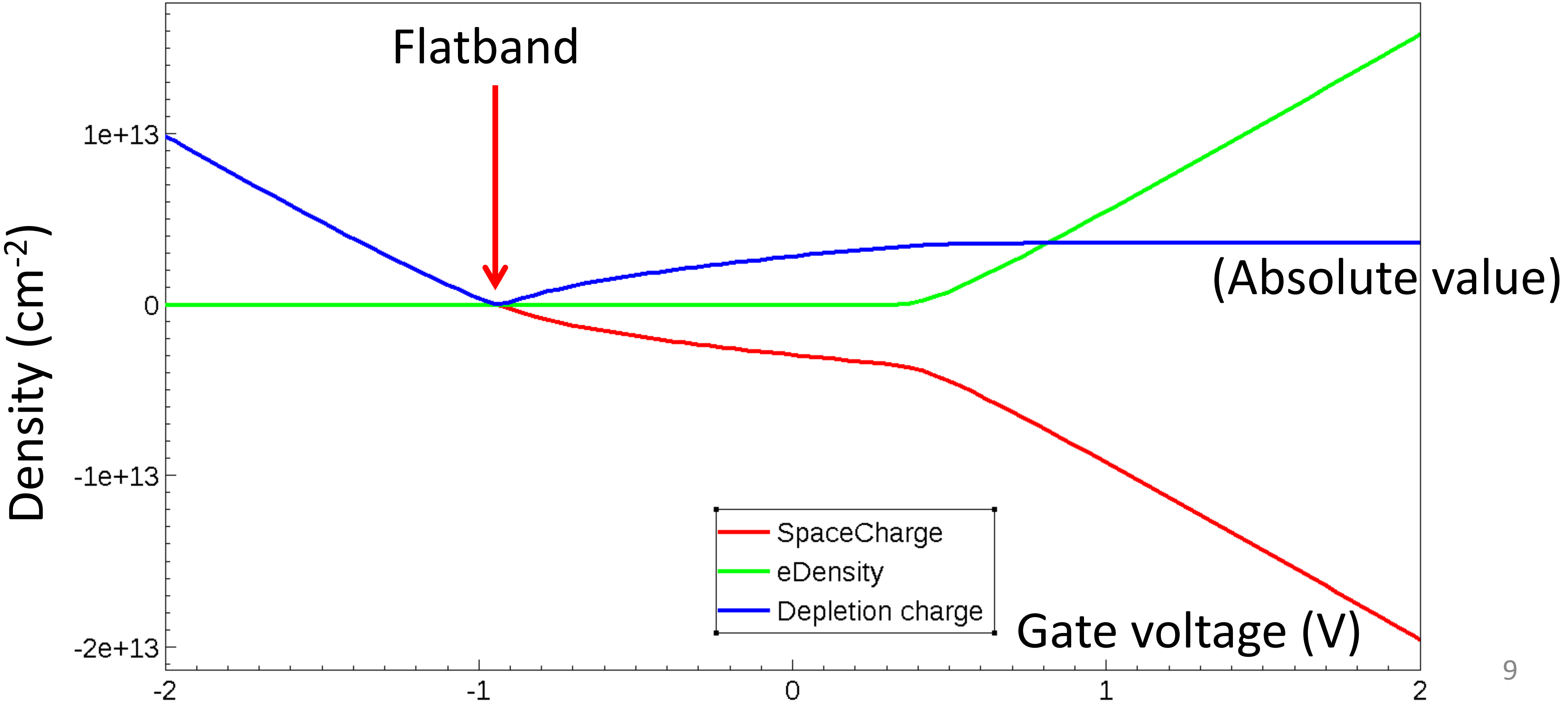
Threshold voltage (1)

- Draw quantities as functions of the gate voltage.



Threshold voltage (2)

- The same graph, in the linear scale,



Threshold voltage (3)

- A criterion for the onset of strong inversion
  - The surface potential reaches $2\phi_B$.

$$\phi_s = 2\phi_B = 2\frac{k_B T}{q}\log\left(\frac{N_a}{n_i}\right)$$

Taur, Eq. (2.183)

  - Remember that $n(x) = n(\infty)\exp\left(\frac{q\phi(x)}{k_B T}\right)$.
  - It means that

$$n(x=0) = n(\infty)\exp\left(\frac{2q\phi_B}{k_B T}\right) = p(\infty)$$

(Of course, it is difficult to measure $n(x=0)$.)

Depletion approximation (1)

- Consider a depleted MOS structure.
  - With the depletion width, W_d,

$$\phi_s = \frac{1}{2} W_d \left(q \frac{N_a}{\epsilon_{si}} W_d \right)$$

  - Then,

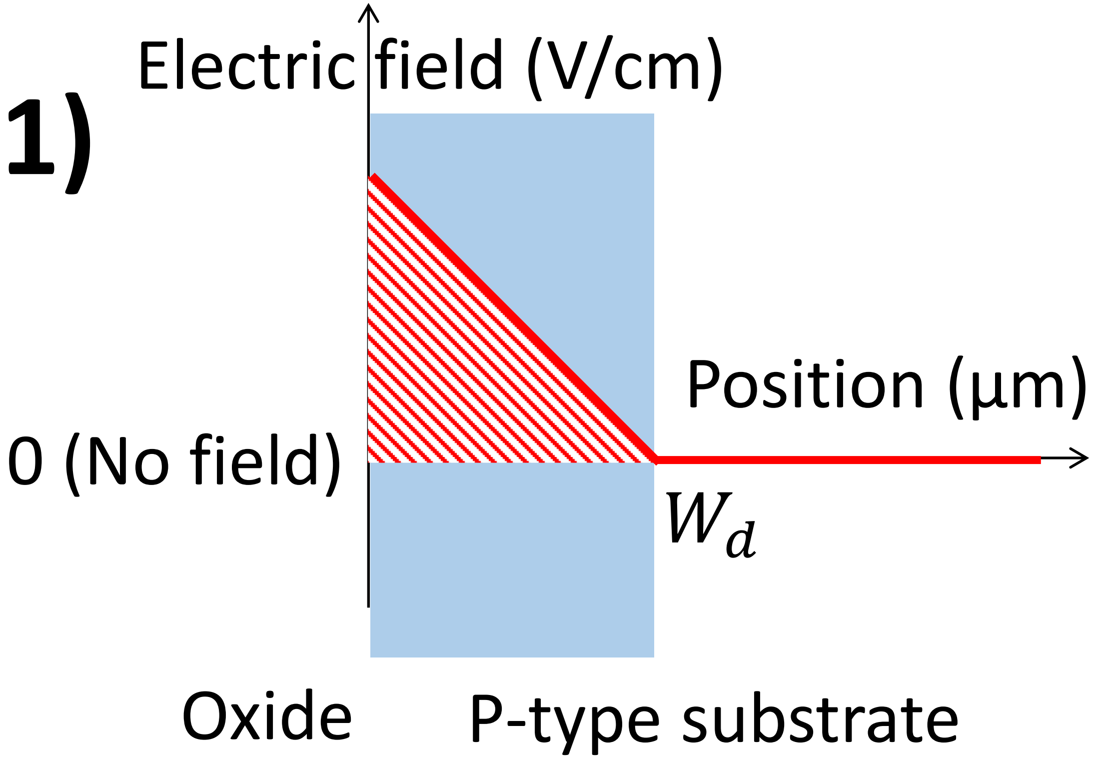
$$W_d = \sqrt{\frac{2\epsilon_{si}\phi_s}{qN_a}}$$

Taur, Eq. (2.188)

  - Total depletion charge in silicon, Q_d, is

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si}qN_a\phi_s}$$

Taur, Eq. (2.189)



Depletion approximation (2)

- Another derivation
  - When $V_g > V_{fb}$, $\frac{d\phi}{dx} = -\sqrt{\frac{2k_BTN_a}{\epsilon_{si}}}\left[\left(\exp\left(-\frac{q\phi_s}{k_BT}\right) + \frac{q\phi_s}{k_BT} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_BT}\right) - \frac{q\phi_s}{k_BT} - 1\right)\right]^{1/2}$
  - In the depletion region, $2\phi_B > \phi_s > \frac{k_BT}{q}$, $\frac{d\phi}{dx}$ is well approximated as

$$-\sqrt{\frac{2k_BTN_a}{\epsilon_{si}}}\left[\left(\cancel{\exp\left(-\frac{q\phi_s}{k_BT}\right)} + \frac{q\phi_s}{k_BT} \cancel{- 1}\right) \cancel{+ \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_BT}\right) - \frac{q\phi_s}{k_BT} - 1\right)}\right]^{1/2}$$

Depletion approximation (3)

- A good approximation in the depletion region

$$\frac{d\phi}{dx} = -\sqrt{\frac{2qN_a\phi}{\epsilon_{si}}}$$

  - Rearranged as

$$\frac{1}{\sqrt{\phi}}d\phi = -\sqrt{\frac{2qN_a}{\epsilon_{si}}}dx$$

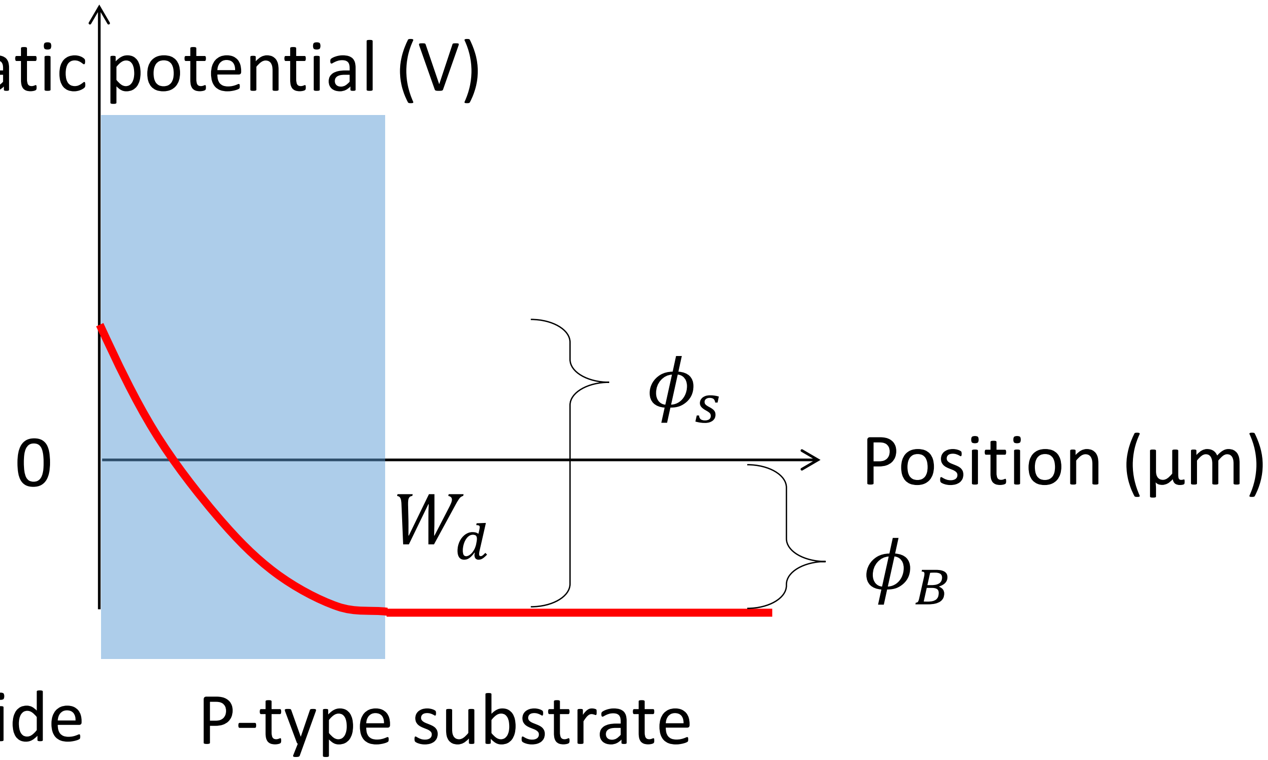
  - Integration yields

$$2\left(\sqrt{0} - \sqrt{\phi_S}\right) = -\sqrt{\frac{2qN_a}{\epsilon_{si}}}W_d$$

Taur, Eq. (2.188)

Potential profile

- A parabolic potential profile
  - The depletion region cannot grow indefinitely.
  - When $\phi_s = 2\phi_B$, $n(0) = p(\infty)$
  - The surface behaves like n-type material. (Inversion)
  - ϕ_s is approximately pinned.

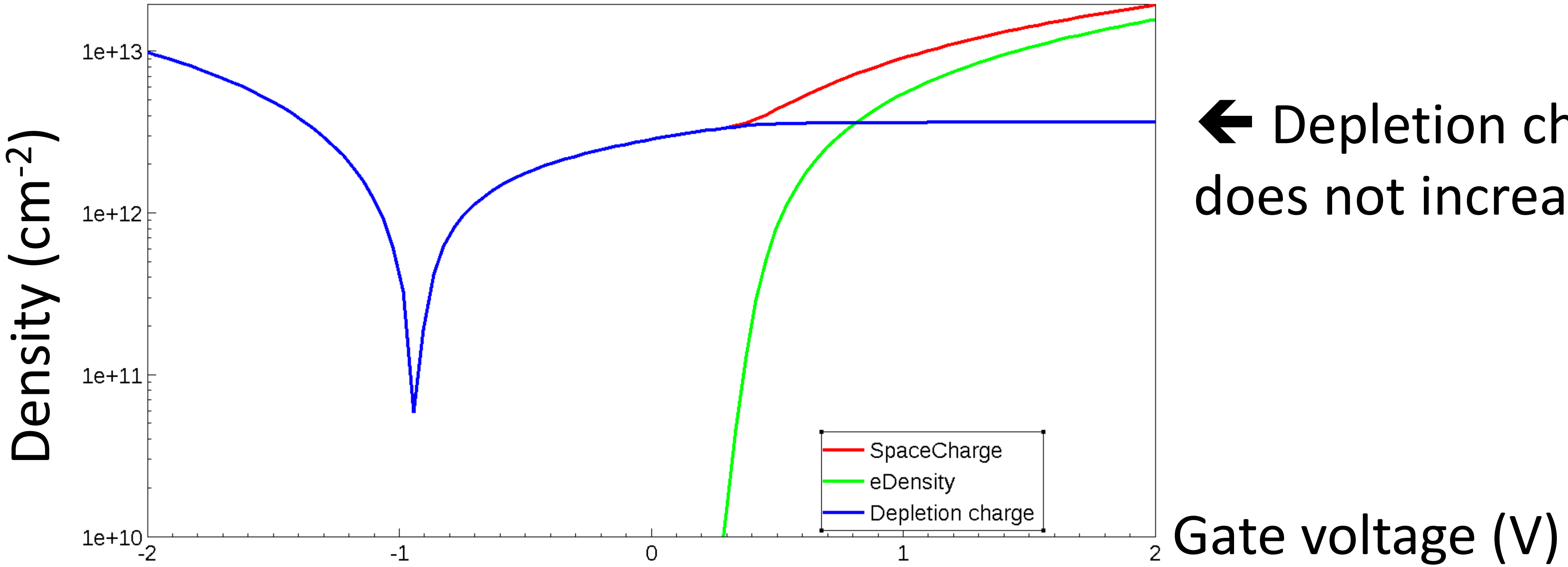


Maximum depletion width

- Therefore, maximum depletion width becomes

$$W_d = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_a}} = \sqrt{\frac{4\epsilon_{si}k_BT\ \ln(N_a/n_i)}{q^2N_a}}$$

Taur, Eq. (2.190)



🡰 Depletion charge does not increase.

Beyond threshold voltage

It's not perfectly fixed.

- The surface potential is *almost* fixed. (Surface potential pinning)
  - Small additional change in ϕ_s induces an exponential increase of the electron density.
  - Remember that $n = n_i \exp\left(\frac{q\phi}{k_B T}\right)$.
  - When $\phi_s = 2\phi_B$, (in other words, $\phi(0) = \phi_B$)

$$n(0) = n_i \exp\left(\frac{q\phi_B}{k_B T}\right) = p(\infty)$$

  - Additional potential ($\Delta\phi$) yields

$$n(0) = p(\infty) \exp\left(\frac{q\Delta\phi}{k_B T}\right)$$

It's a high density.

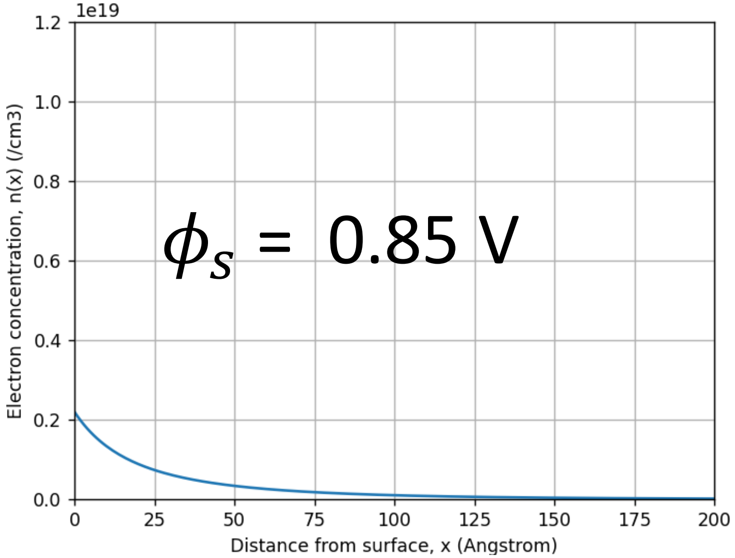
Strong inversion

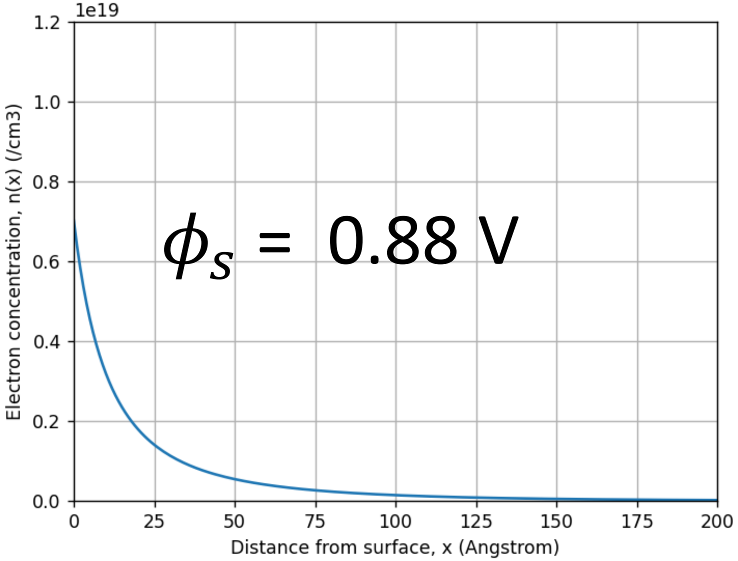
- Beyond strong inversion,

$$\frac{d\phi}{dx} \approx -\sqrt{\frac{2k_BTN_a}{\epsilon_{si}}\left(\frac{q\phi}{k_BT}+\frac{n_i^2}{N_a^2}\exp\left(\frac{q\phi}{k_BT}\right)\right)}$$

Taur, Eq. (2.191)

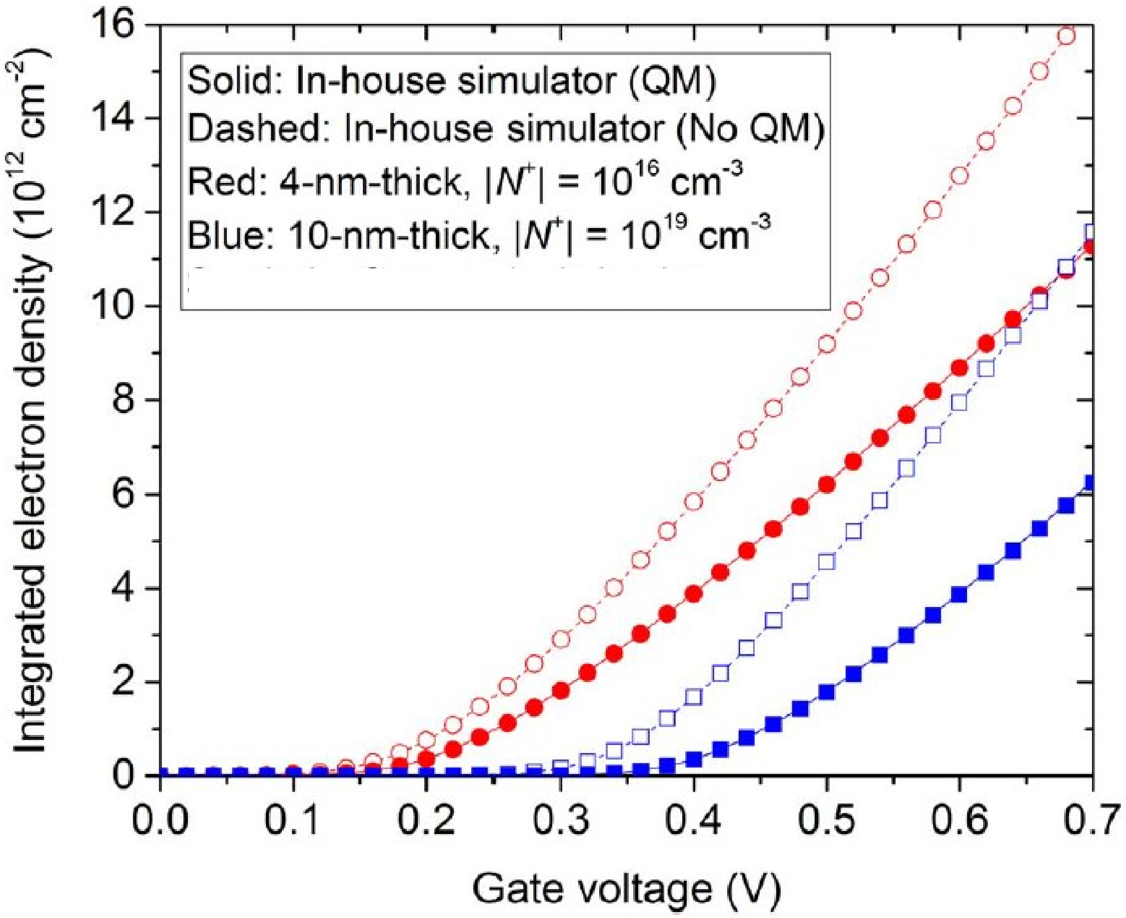
  - The electrons are distributed extremely close to the surface with an inversion-layer width less than 50 Å.





Strong inversion

- Quantum confinement effect
  - A peak distribution 10~20 Å away from the surface



MOS equation

- Up to now, $Q_s(\phi_s)$ is found. We can control only V_g.
  - Relation between V_g and ϕ_s

Total silicon charge per unit area

$$V_g - V_{fb} = V_{ox} + \phi_s = -\frac{Q_s}{C_{ox}} + \phi_s$$ Taur, Eq. (2.195)

$\frac{\epsilon_{ox}}{t_{ox}}$, oxide capacitance per unit area

  - In general, $Q_s(\phi_s)$ is known. We can solve the above equation.

Taur, Eq. (2.182)

Thank you!