VLSI Devices
Lecture 8

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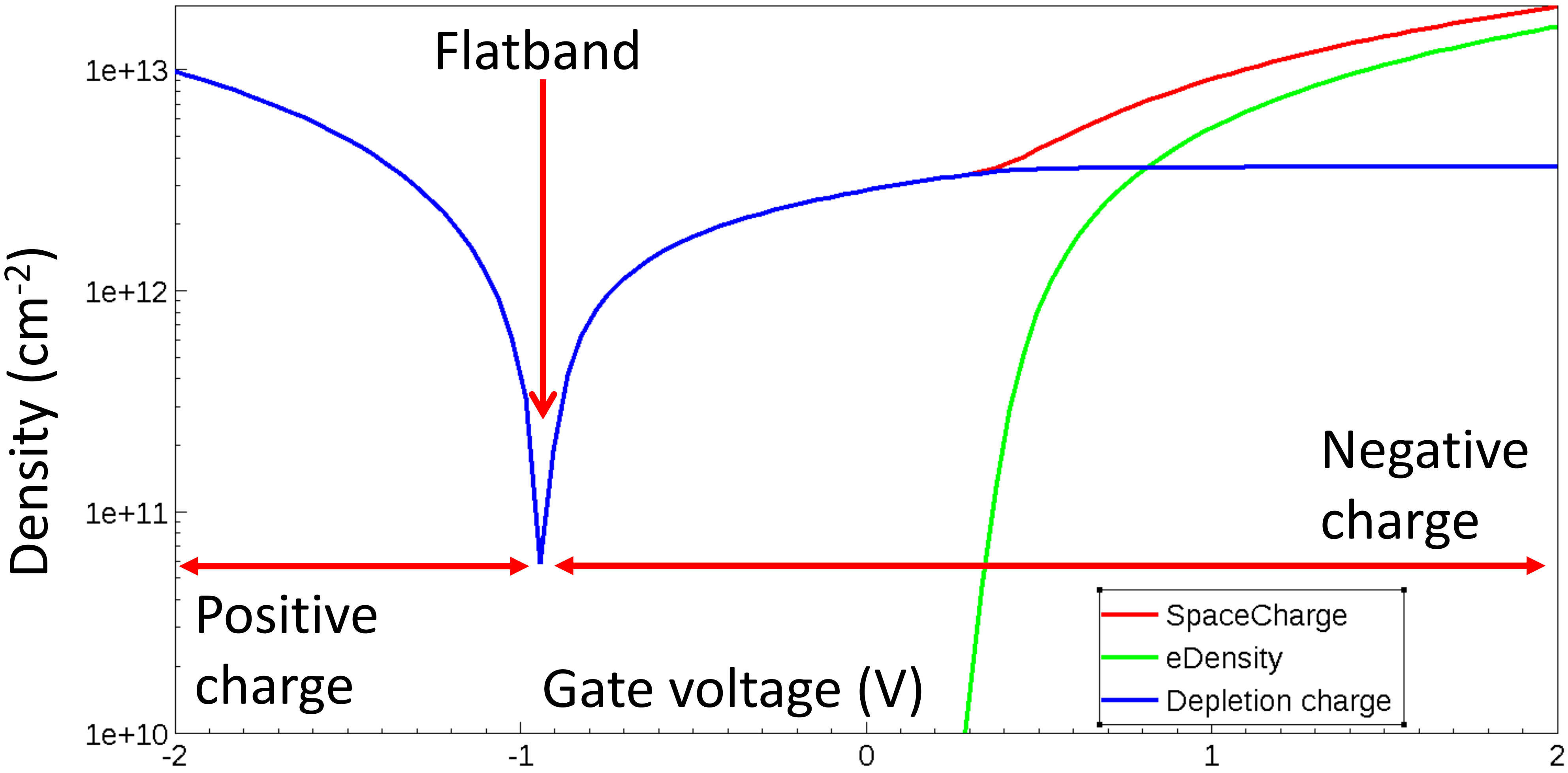
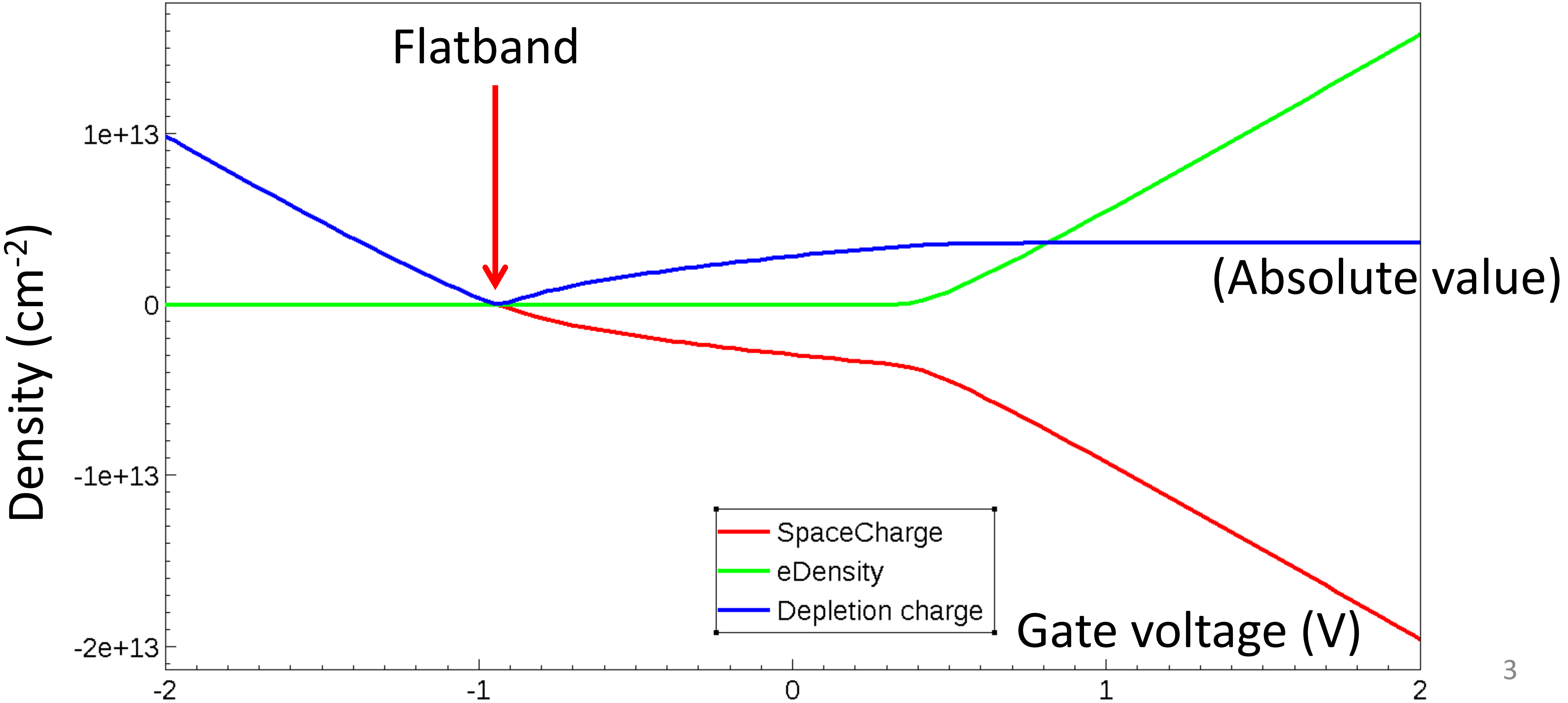
Threshold voltage (1)

- Draw quantities as functions of the gate voltage.



Threshold voltage (2)

- The same graph, in the linear scale,



Threshold voltage (3)

- A criterion for the onset of strong inversion
  - The surface potential reaches $2\phi_B$.

$$\phi_s = 2\phi_B = 2\frac{k_B T}{q}\log\left(\frac{N_a}{n_i}\right)$$

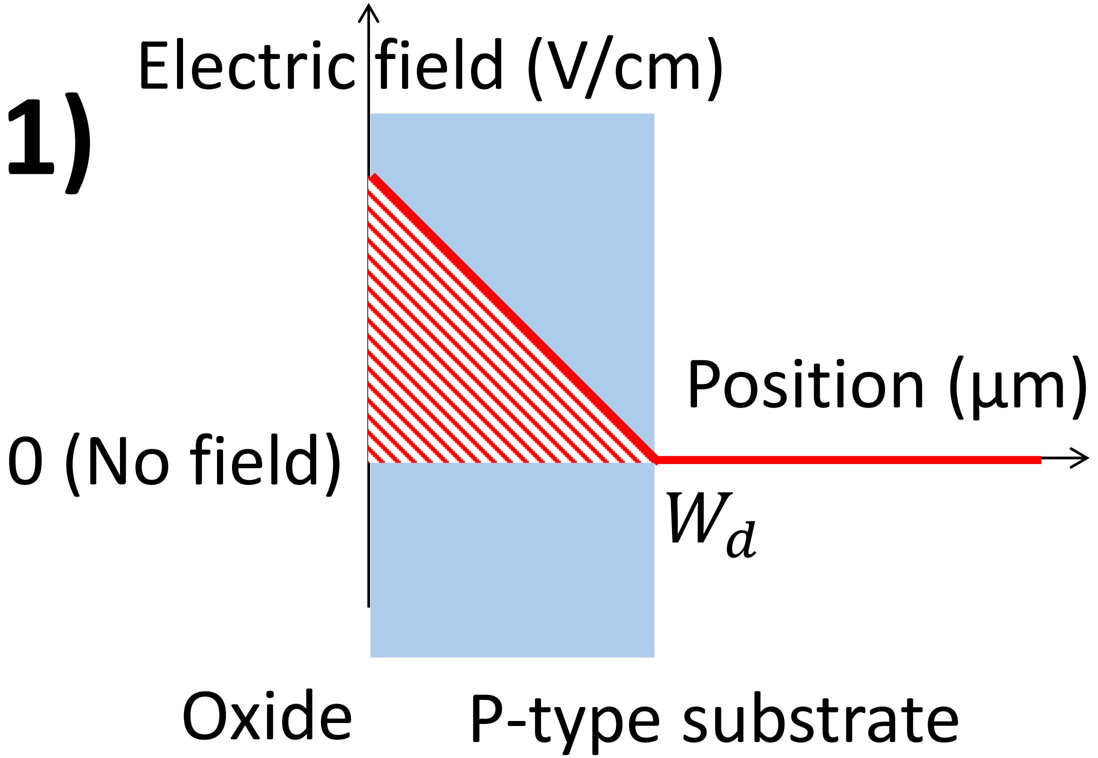
Taur, Eq. (2.183)

  - Remember that $n(x) = n(\infty)\exp\left(\frac{q\phi(x)}{k_B T}\right)$.
  - It means that

$$n(x=0) = n(\infty)\exp\left(\frac{2q\phi_B}{k_B T}\right) = p(\infty)$$

(Of course, it is difficult to measure $n(x=0)$.)

Depletion approximation (1)



- Consider a depleted MOS structure.
  - With the depletion width, W_d,

$$\phi_s = \frac{1}{2} W_d \left(q \frac{N_a}{\epsilon_{si}} W_d \right)$$

  - Then,

$$W_d = \sqrt{\frac{2\epsilon_{si}\phi_s}{qN_a}}$$ Taur, Eq. (2.188)

  - Total depletion charge in silicon, Q_d, is

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si}qN_a\phi_s}$$ Taur, Eq. (2.189)

Depletion approximation (2)

- Another derivation
  - When $V_g > V_{fb}$, $\frac{d\phi}{dx} = -\sqrt{\frac{2k_BTN_a}{\epsilon_{si}}}\left[\left(\exp\left(-\frac{q\phi_s}{k_BT}\right) + \frac{q\phi_s}{k_BT} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_BT}\right) - \frac{q\phi_s}{k_BT} - 1\right)\right]^{1/2}$
  - In the depletion region, $2\phi_B > \phi_s > \frac{k_BT}{q}$, $\frac{d\phi}{dx}$ is well approximated as

$$-\sqrt{\frac{2k_BTN_a}{\epsilon_{si}}}\left[\left(\exp\left(-\frac{q\phi_s}{k_BT}\right) + \frac{q\phi_s}{k_BT} - 1\right) + \frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_BT}\right) - \frac{q\phi_s}{k_BT} - 1\right)\right]^{1/2}$$

Depletion approximation (3)

- A good approximation in the depletion region

$$\frac{d\phi}{dx} = -\sqrt{\frac{2qN_a\phi}{\epsilon_{si}}}$$

  - Rearranged as

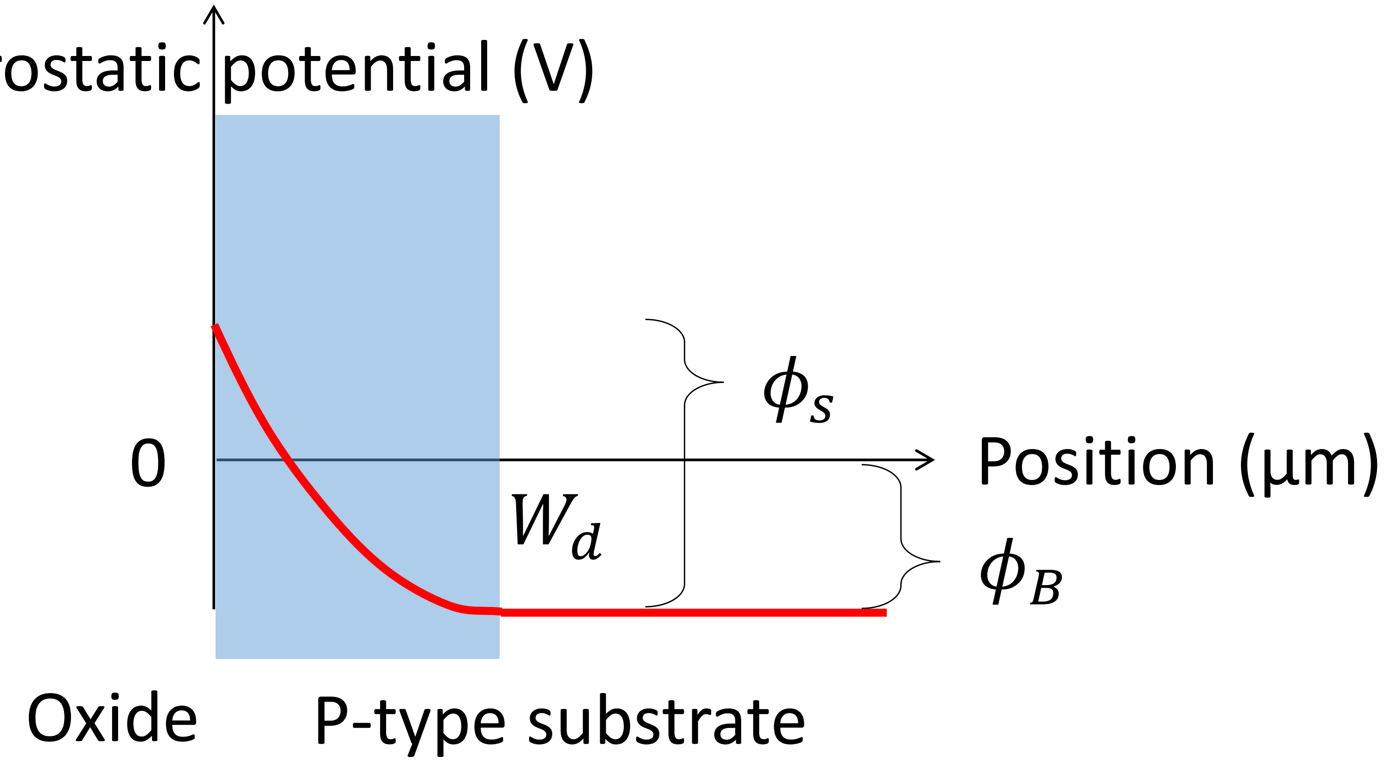
$$\frac{1}{\sqrt{\phi}}d\phi = -\sqrt{\frac{2qN_a}{\epsilon_{si}}}dx$$

  - Integration yields

$$2\left(\sqrt{0} - \sqrt{\phi_S}\right) = -\sqrt{\frac{2qN_a}{\epsilon_{si}}}W_d \qquad \text{Taur, Eq. (2.188)}$$

Potential profile

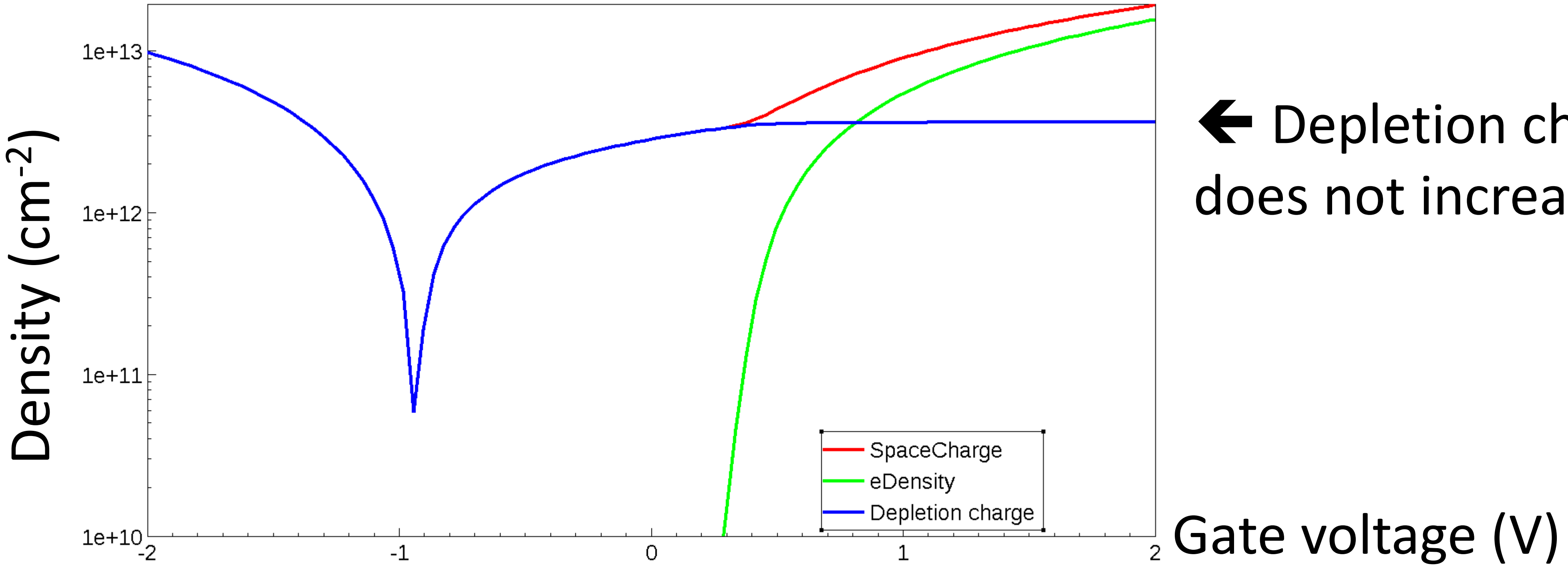
- A parabolic potential profile
  - The depletion region cannot grow indefinitely.
  - When $\phi_s = 2\phi_B$, $n(0) = p(\infty)$
  - The surface behaves like n-type material. (Inversion)
  - ϕ_s is approximately pinned.



Maximum depletion width

- Therefore, maximum depletion width becomes

$$W_d = \sqrt{\frac{4\epsilon_{si}\phi_B}{qN_a}} = \sqrt{\frac{4\epsilon_{si}k_B T\ \ln(N_a/n_i)}{q^2 N_a}}$$ Taur, Eq. (2.190)



🡸 Depletion charge does not increase.

Beyond threshold voltage

It's not perfectly fixed.

- The surface potential is *almost* fixed. (Surface potential pinning)
  - Small additional change in ϕ_s induces an exponential increase of the electron density.
  - Remember that $n = n_i \exp\left(\frac{q\phi}{k_B T}\right)$.
  - When $\phi_s = 2\phi_B$, (in other words, $\phi(0) = \phi_B$)

$$n(0) = n_i \exp\left(\frac{q\phi_B}{k_B T}\right) = p(\infty)$$

  - Additional potential ($\Delta\phi$) yields

$$n(0) = p(\infty) \exp\left(\frac{q\Delta\phi}{k_B T}\right)$$

It's a high density.

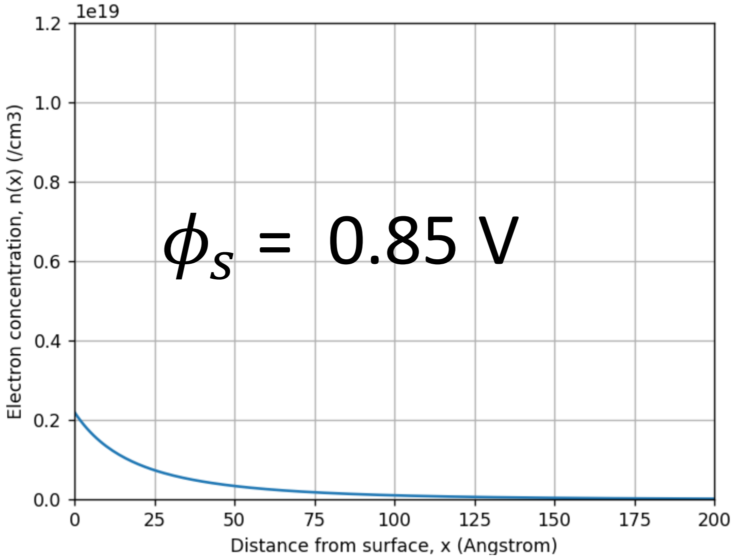
Strong inversion

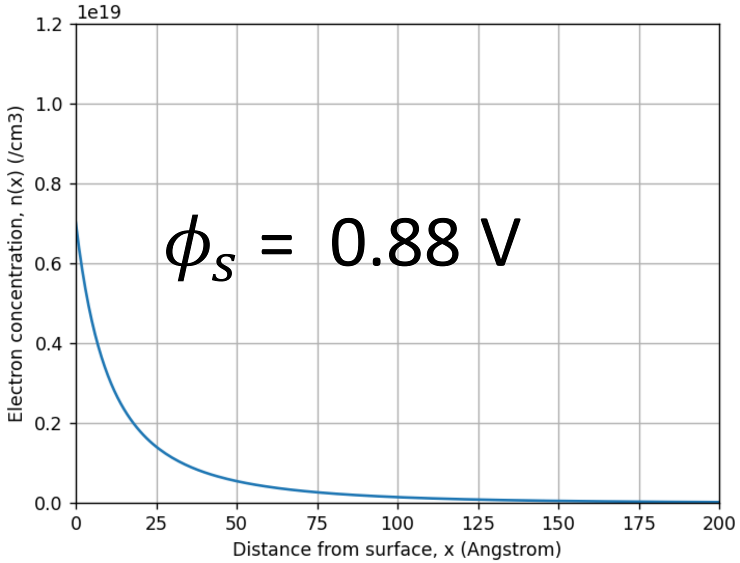
- Beyond strong inversion,

$$\frac{d\phi}{dx} \approx -\sqrt{\frac{2k_B T N_a}{\epsilon_{si}}\left(\frac{q\phi}{k_B T} + \frac{n_i^2}{N_a^2}\exp\left(\frac{q\phi}{k_B T}\right)\right)}$$

Taur, Eq. (2.191)

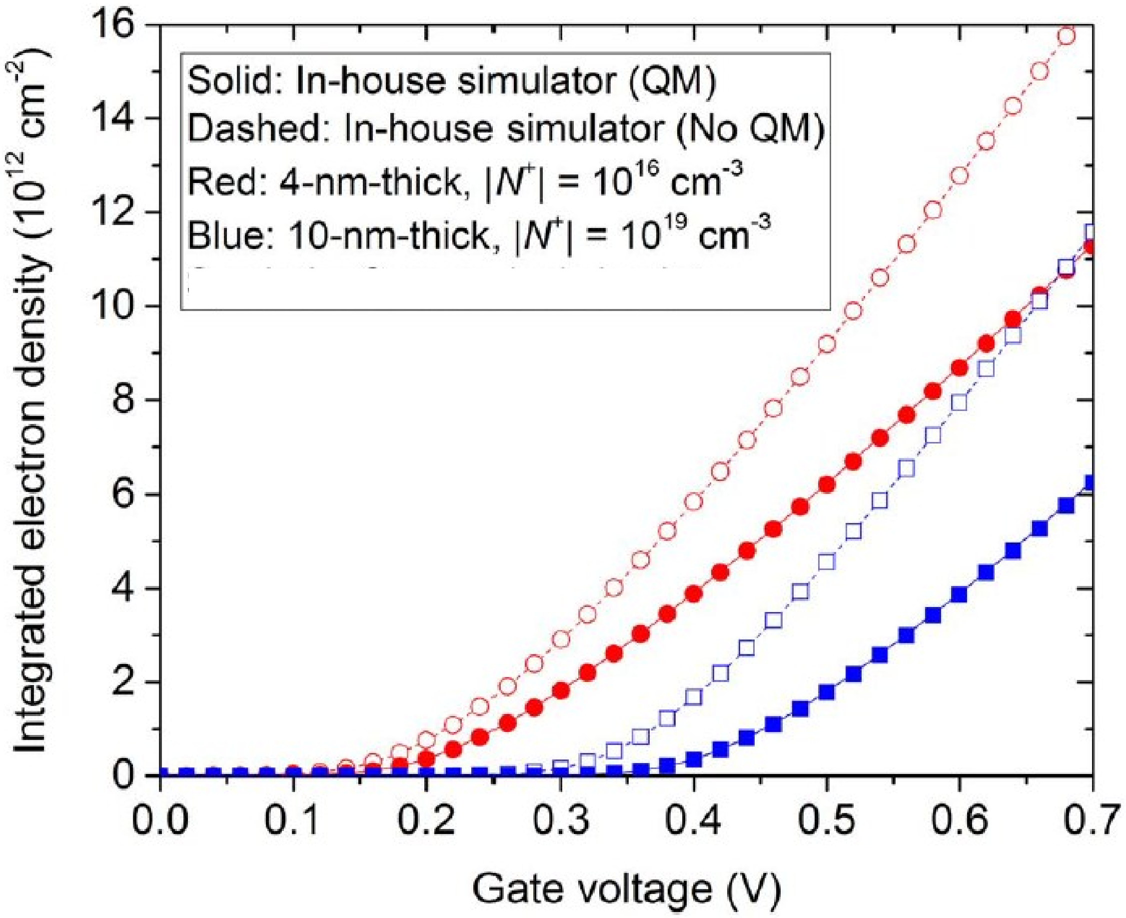
  – The electrons are distributed extremely close to the surface with an inversion-layer width less than 50 Å.





Strong inversion

- Quantum confinement effect
  - A peak distribution 10~20 Å away from the surface



MOS equation

Total silicon charge per unit area

- Up to now, $Q_s(\phi_s)$ is found. We can control only V_g.
  - Relation between V_g and ϕ_s

$$V_g - V_{fb} = V_{ox} + \phi_s = -\frac{Q_s}{C_{ox}} + \phi_s$$

Taur, Eq. (2.195)

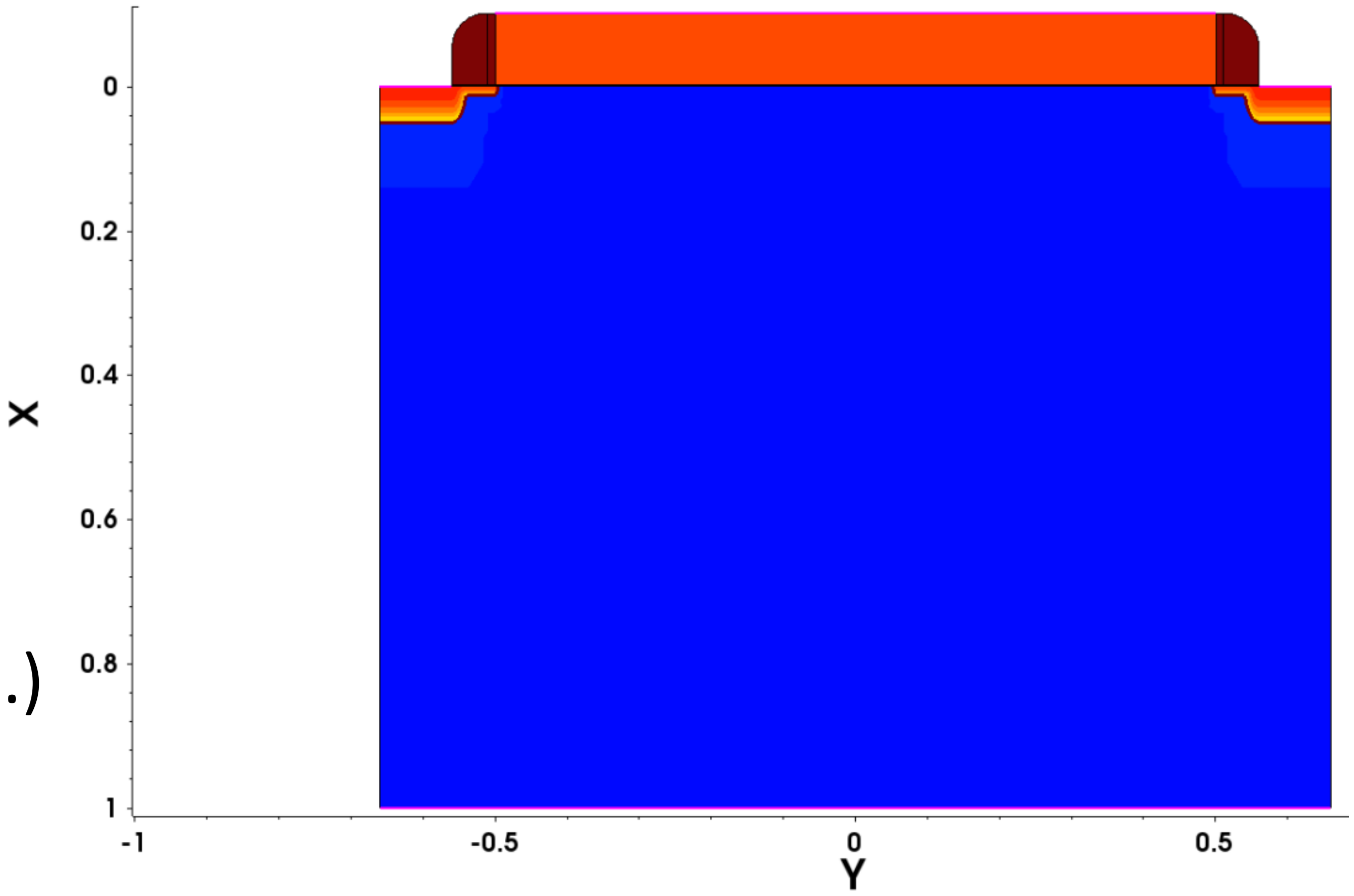
$\frac{\epsilon_{ox}}{t_{ox}}$, oxide capacitance per unit area

  - In general, $Q_s(\phi_s)$ is known. We can solve the above equation.

Taur, Eq. (2.182)

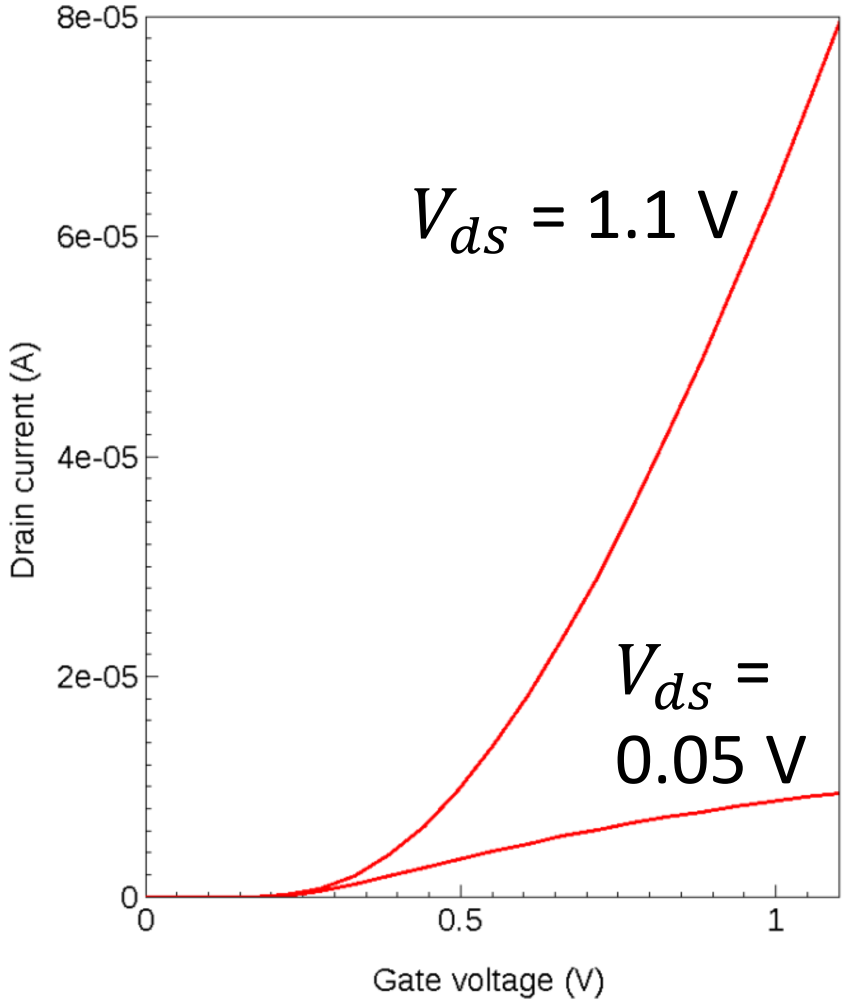
TCAD simulation of a long-channel MOSFET

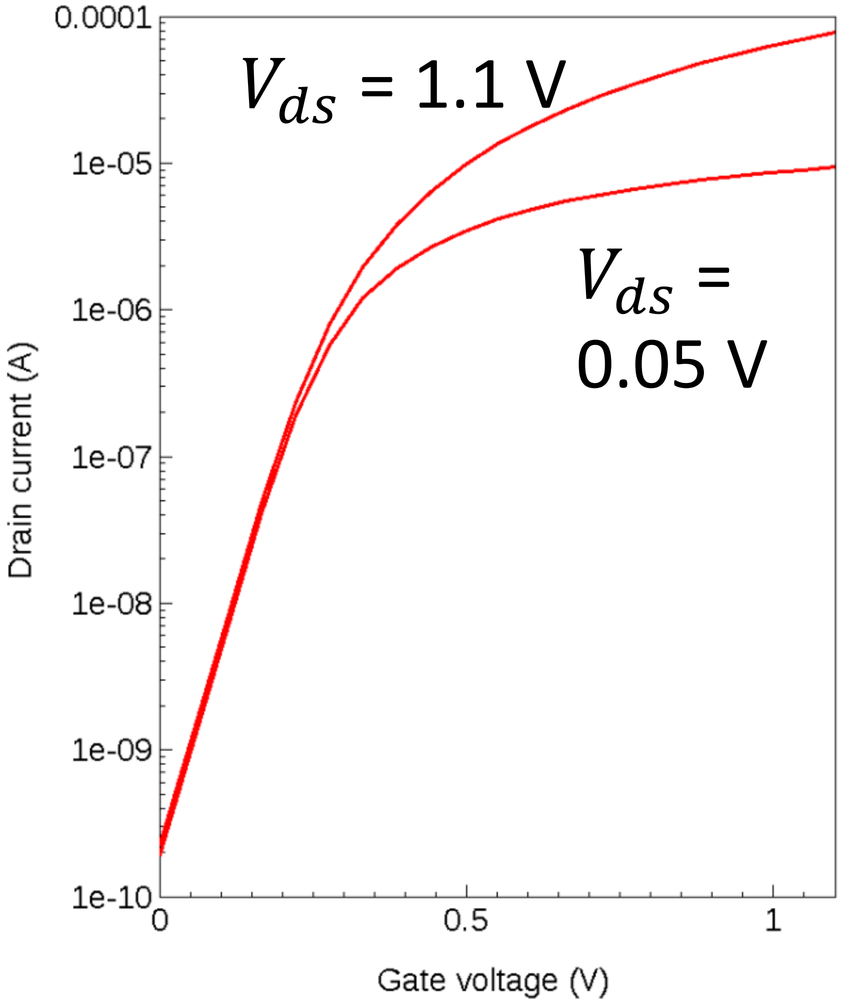
- Channel length, 1 μm
  - V_{fb} is -1.08 V.
  - t_{ox} is 1.2 nm.
  - C_{ox} is 2.88X10^{-6} F/cm^2.
  - V_{DD} of 1.1 V
  - (Estimate its technology node.)



I_d versus V_{gs}

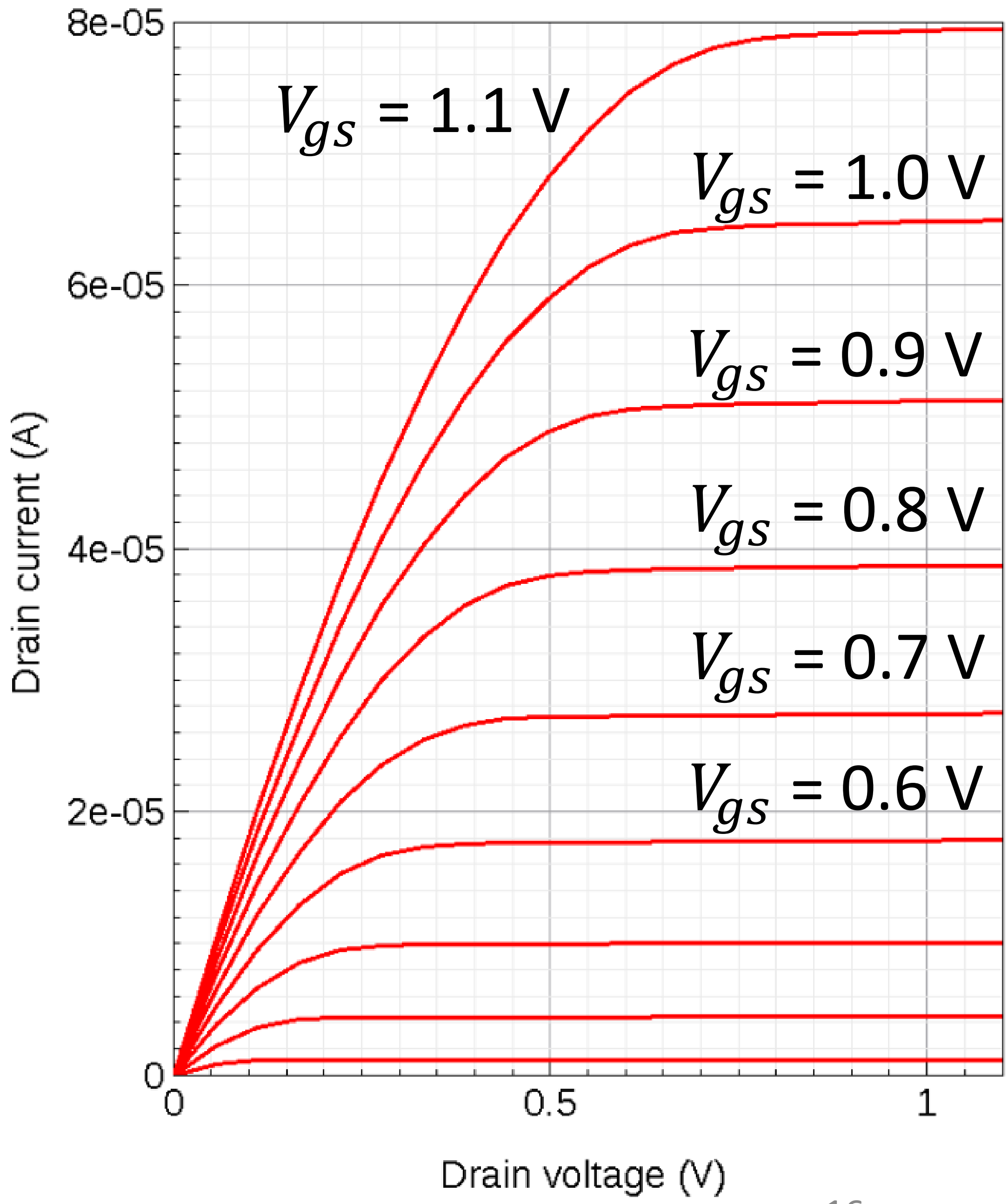
- Input characteristics





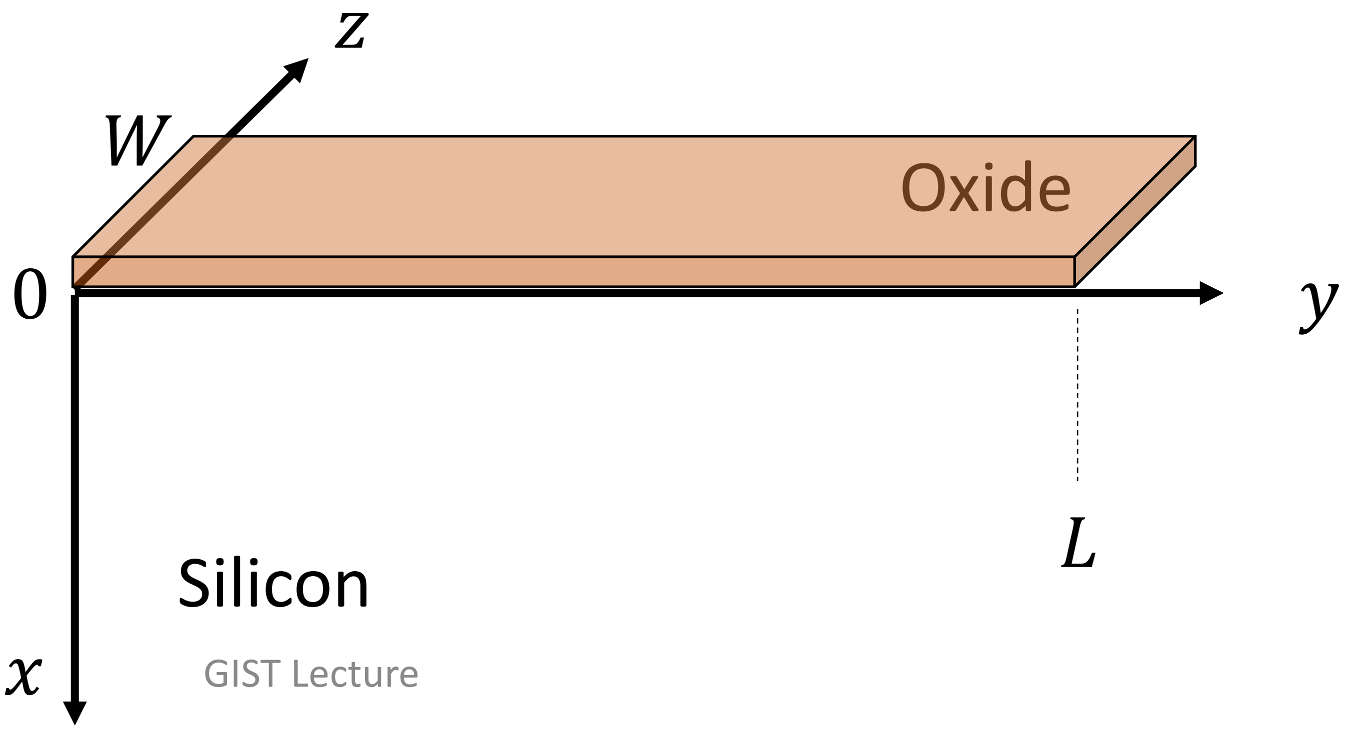
I_d versus V_{ds}

- Output characteristics
  - Triode mode
  - Saturation mode



Schematic

- $x = 0$ at silicon surface
  - $y = 0$ at the source and $y = L$ at the drain
  - Source and substrate are grounded.
  - Uniform p-type substrate



Gradual channel approximation

- Variation of the electric field in the y-direction is much less than the corresponding variation in the x-direction.

$$\left|\frac{\partial^2\phi}{\partial x^2}\right| \gg \left|\frac{\partial^2\phi}{\partial y^2}\right|$$

  - Poisson equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = -\frac{q}{\epsilon_{si}}[p(x,y) - n(x,y) - N_a]$$

  - Poisson equation under the GCA

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}[p(x,y) - n(x,y) - N_a]$$

Taur, Eq. (2.175)

Electron quasi-Fermi potential, $V(y)$

It was written as ϕ_n in Taur, Eq. (2.61).

- It is assumed that V is independent of x.
  - *Why?* MOSFET current flows predominantly along the y-direction.
  - Since $\mathbf{J}_n = -q\mu_n n \nabla \phi_n$, V varies mainly along the y-direction.
  - Boundary conditions:

$$V(y=0) = V_s = 0$$
$$V(y=L) = V_d = V_{ds}$$

- Electron density, $n(x,y)$

$$n(x,y) = \frac{n_i^2}{N_a} \exp\left(\frac{q}{k_B T}(\phi - V)\right)$$

Taur, Eq. (3.1)

Still, ϕ_B is the reference value.

Our previous expressions (1)

- They are modified by V. (Terms related with the electron density)
  - Electric field

$$E_x^2(x,y) = \left(\frac{d\phi}{dx}\right)^2$$
$$= \frac{2k_BTN_a}{\epsilon_{si}}\left[\left(\exp\left(-\frac{q\phi}{k_BT}\right) + \frac{q\phi}{k_BT} - 1\right)\right.$$
$$\left. + \frac{n_i^2}{N_a^2}\left(\exp\left(-\frac{qV}{k_BT}\right)\left(\exp\left(\frac{q\phi}{k_BT}\right) - 1\right) - \frac{q\phi}{k_BT}\right)\right]$$

Taur, Eq. (3.2)

Our previous expressions (2)

- They are modified by V. (Terms related with the electron density)
  - Surface inversion

$$\phi(0, y) = V(y) + 2\phi_B$$

Taur, Eq. (3.3)

  - Maximum depletion layer width

$$W_{dm} = \sqrt{\frac{2\epsilon_{si}[V(y) + 2\phi_B]}{qN_a}}$$

Taur, Eq. (3.4)

- Summary: With the GCA, our MOS expressions are re-used only with modification by V.

Drain current

- Electron current density at a point (x, y)

$$J_n(x,y) = -q\mu_n n(x,y)\frac{dV}{dy}$$ Taur, Eq. (3.5)

  - (It includes both the drift and diffusion currents.)
  - When integrated from $x = 0$ to x_i,

$$I_d(y) = qW\int_0^{x_i} \mu_n n(x,y)\frac{dV}{dy}dx$$ Taur, Eq. (3.6)

Sign change due to convention of terminal current

z-directional width

Further simplification

- Electron current density at a point (x, y)

$$I_d(y) = qW \int_0^{x_i} \mu_n n(x,y) \frac{dV}{dy} dx = -\mu_{eff} W \frac{dV}{dy} \left(-q \int_0^{x_i} n(x,y) dx \right)$$

$$= -\mu_{eff} W \frac{dV}{dy} Q_i(y)$$

Taur, Eq. (3.8)

  - We introduce an effective mobility, μ_{eff}.
  - Since V is a function of y only, V is interchangeable with y.

$$Q_i(y) = Q_i(V)$$

  - Then,

$$I_d(y) dy = \mu_{eff} W [-Q_i(V)] dV$$

$I_d(y)$ is actually a constant.

- When integrated from $y = 0$ to L, (from $V = 0$ to V_{ds})

$$I_d = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} [-Q_i(V)] dV \qquad \text{Taur, Eq. (3.10)}$$

  - Then, how can we find $Q_i(V)$? (We must perform the x-directional integration.)

It's small, but not zero.

$$Q_i = -q \int_0^{x_i} n(x, y) dx = -q \int_{\phi_s}^{\delta} n(\phi, V) \frac{dx}{d\phi} d\phi$$

$$= -q \int_{\delta}^{\phi_s} \frac{(n_i^2/N_a) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi \qquad \text{Taur, Eq. (3.12)}$$

Then, how can we determine ϕ_s?

- For given V_{gs} and V, we can solve the MOS equation.

$$V_{gs} = V_{fb} + \phi_s - \frac{Q_s}{C_{ox}}$$

$$= V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right) \right]^{1/2}$$

Taur, Eq. (3.14)

Only two important terms are kept.

– We can numerically solve the above equation to obtain ϕ_s.

$V(\phi_s)$?

- Recall that

$$V_{gs} = V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si} k_B T N_a}}{C_{ox}} \left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right) \right]^{1/2}$$

Taur, Eq. (3.14)

  - We can rearrange

$$\frac{C_{ox}^2 \left(V_{gs} - V_{fb} - \phi_s \right)^2}{2\epsilon_{si} k_B T N_a} = \frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2} \exp\left(\frac{q}{k_B T} (\phi_s - V) \right)$$

  - Therefore,

$$V = \phi_s - \frac{k_B T}{q} \log\left\{ \frac{N_a^2}{n_i^2} \left[\frac{C_{ox}^2 \left(V_{gs} - V_{fb} - \phi_s \right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T} \right] \right\}$$

Taur, Eq. (3.18)

Pao-Sah double integral

- Finally, the expression for I_d reads

$$I_d = q\mu_{eff}\frac{W}{L}\int_0^{V_{ds}}\left[\int_\delta^{\phi_s}\frac{\left(n_i^2/N_a\right)\exp\left(\frac{q}{k_BT}(\phi - V)\right)}{E_x(\phi,V)}d\phi\right]dV$$

Taur, Eq. (3.13)

  - It is the Pao-Sah double integral.
  - Rigorous within the GCA, but it is difficult to evaluate.

Charge-sheet model

- Simpler model with further approximations
  - Consider the previous method to calculate Q_i:

$$Q_i = -q \int_{\delta}^{\phi_s} \frac{\left(n_i^2/N_a\right) \exp\left(\frac{q}{k_B T}(\phi - V)\right)}{E_x(\phi, V)} d\phi$$

  - A more simple way? Instead, Q_d is approximated as

$$Q_d = -qN_aW_d = -\sqrt{2\epsilon_{si} q N_a \phi_s}$$ Taur, Eq. (3.15)

  - Then, Q_i can be approximated as

$$Q_i = Q_s - Q_d = -C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si} q N_a \phi_s}$$

Taur, Eq. (3.16)

(Of course, it is not exact.)

Change of variable

- Now, Q_i can be a function of ϕ_s.
  - Variable change from V to ϕ_s:

$$I_d = \mu_{eff}\frac{W}{L}\int_0^{V_{ds}}[-Q_i(V)]dV = \mu_{eff}\frac{W}{L}\int_{\phi_{s,s}}^{\phi_{s,d}}[-Q_i(\phi_s)]\frac{dV}{d\phi_s}d\phi_s$$

Taur, Eq. (3.17)

Surface potentials at the two ends, $y = 0$ and L. They can be calculated by solving Taur, Eq. (3.14).

$\frac{d\boldsymbol{V}}{d\boldsymbol{\phi_s}}$? (1)

- Recall that

$$V = \phi_s - \frac{k_B T}{q} \log\left\{\frac{N_a^2}{n_i^2}\left[\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}\right]\right\}$$

Taur, Eq. (3.18)

  - Therefore,

$$\frac{dV}{d\phi_s} = 1 - \frac{k_B T}{q} \frac{-\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)}{\epsilon_{si} k_B T N_a} - \frac{q}{k_B T}}{\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2}{2\epsilon_{si} k_B T N_a} - \frac{q\phi_s}{k_B T}}$$

$\frac{dV}{d\phi_s}$? (2)

- Simple rearrange yields

$$\frac{dV}{d\phi_s} = 1 + \frac{2k_BT}{q}\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right) + \epsilon_{si}qN_a}{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right)^2 - 2\epsilon_{si}qN_a\phi_s}$$

Taur, Eq. (3.19)

  - It is still very complicated…

Integrand

- When multiplied with $-Q_i(\phi_s)$,

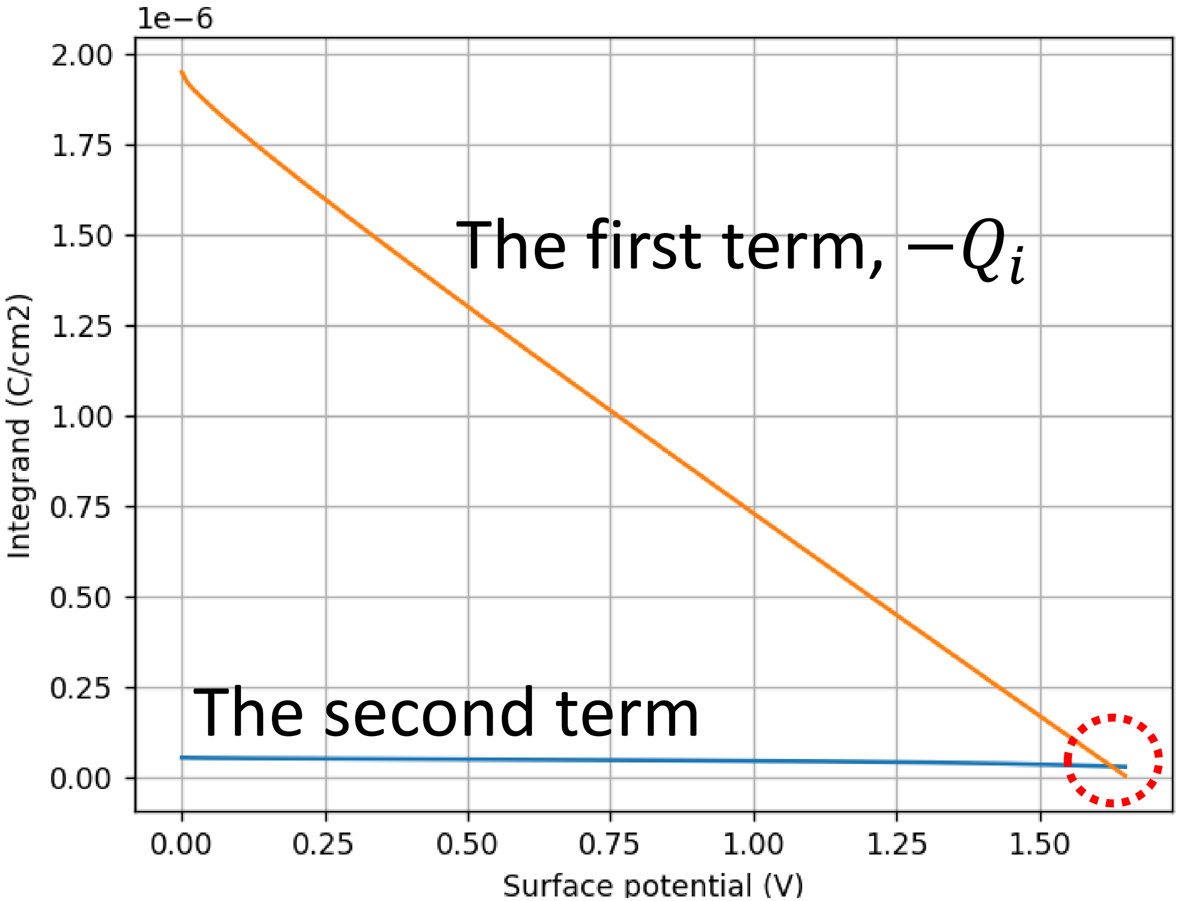
$$\left(-Q_i(\phi_s)\right)\frac{dV}{d\phi_s}$$

$$= -Q_i(\phi_s) + \frac{2k_BT}{q}\frac{C_{ox}^2\left(V_{gs} - V_{fb} - \phi_s\right) + \epsilon_{si}qN_a}{C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) + \sqrt{2\epsilon_{si}qN_a\phi_s}}$$

  - The second term is still very complicated…
  - *Is it really important?*

Comparison between two terms

- Let's draw two terms.
  - Assume that N_a = 10^{17} cm^{-3}, t_{ox} = 10 nm, V_{gs} = 1.0 V, and V_{fb} = -0.88 V.
  - The second term is small.
  - It is meaningful only when $Q_i \approx 0$.
  - This is corresponding to $C_{ox}\left(V_{gs} - V_{fb} - \phi_s\right) = \sqrt{2\epsilon_{si} q N_a \phi_s}$.



Integrand, again

- Within this condition,

$$(-Q_i(\phi_s))\frac{dV}{d\phi_s} \approx -Q_i(\phi_s) + \frac{k_BT}{q}\frac{C_{ox}\sqrt{2\epsilon_{si}qN_a\phi_s} + \epsilon_{si}qN_a}{\sqrt{2\epsilon_{si}qN_a\phi_s}}$$

$$= -Q_i(\phi_s) + \frac{k_BT}{q}C_{ox} + \frac{k_BT}{q}\frac{\sqrt{2\epsilon_{si}qN_a}}{2\sqrt{\phi_s}}$$

Its integration yields $\frac{k_BT}{q}C_{ox}\phi_s$.

Its integration yields $\frac{k_BT}{q}\sqrt{2\epsilon_{si}qN_a\phi_s}$.

Thank you!