Mid-term exam, VLSI Devices, Spring semester, 2025

April 21, 2025

Some useful relations:

For any vector field, $\mathbf{V}(\mathbf{r})$,

$$\int_{\Omega} \nabla \cdot \mathbf{V}(\mathbf{r}) d^3 r = \int_{\text{Surface of } \Omega} \mathbf{V}(\mathbf{r}) \cdot d\mathbf{a}$$

Net charge density, $\rho(\mathbf{r})$,

$$\rho(\mathbf{r}) = qp(\mathbf{r}) - qn(\mathbf{r}) + qN_{dop}^{+}(\mathbf{r})$$

1. The total current density is given as $\mathbf{J}_{total} = \mathbf{J}_n + \mathbf{J}_p + \frac{\partial}{\partial t}\mathbf{D}$. By using three equations shown below (the Poisson equation, the electron continuity equation, and the hole continuity equation), calculate $\nabla \cdot \mathbf{J}_{total}$.

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{J}_n = q \frac{\partial}{\partial t} n$$

$$\nabla \cdot \mathbf{J}_p = -q \frac{\partial}{\partial t} p$$

2. It is assumed that the surface-normal component of $\mathbf{J}_{total}$ vanishes at non-contact boundaries. On the other hand, for a terminal α, its terminal current, I_α, is given as

$$I_\alpha = - \int_{\text{Surface of } \alpha} \mathbf{J}_{total} \cdot d\mathbf{a}.$$

Using these relations and the answer of Problem #1, calculate $\sum_\alpha I_\alpha$.

3. The electrostatic potential and the electron quasi-Fermi potential are given as $\phi(\mathbf{r})$ and $\phi_n(\mathbf{r})$, respectively. Write down an expression for the electron density, $n(\mathbf{r})$. Temperature is T. You may assume the Boltzmann statistics. When you introduce some quantities, define them. (Of course, in

this problem, consider the conventional meaning of $\phi(\mathbf{r})$. It is not a special one for the MOS system.)

4. The electron current density can be written as

$$J_n = q\mu_n n\mathbf{E} + qD_n\nabla n,$$

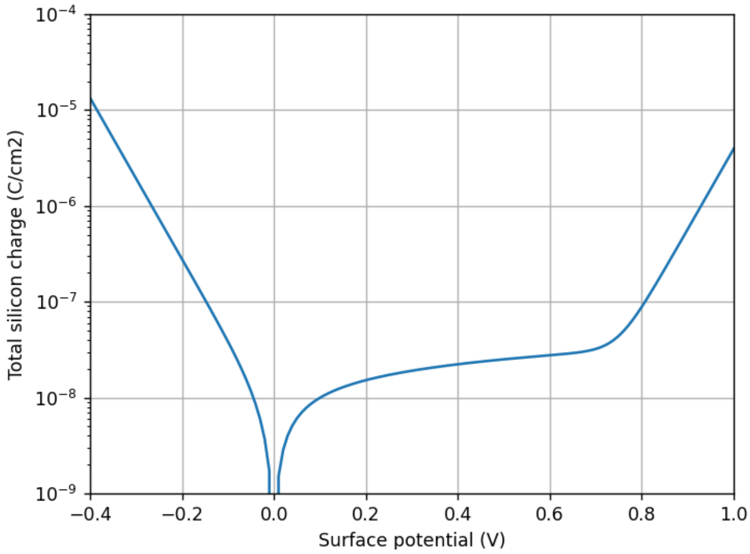
where μ_n and D_n are the electron mobility and the electron diffusion coefficient, respectively. There are two terms, one for the drift mechanism and the other for the diffusion mechanism. Using the Einstein relation and the answer of Problem #3, unify these two terms.

5. In our lectures, we have derived a general relation for the semiconductor charge, Q_s, inside the MOS substrate. When the substrate is infinitely long and grounded, it was given as

$$Q_s = \pm\sqrt{2\epsilon_{si}k_BTN_a}\left[\left(\exp\left(-\frac{q\phi_s}{k_BT}\right)+\frac{q\phi_s}{k_BT}-1\right)+\frac{n_i^2}{N_a^2}\left(\exp\left(\frac{q\phi_s}{k_BT}\right)-\frac{q\phi_s}{k_BT}-1\right)\right]^{1/2}.$$

It is noted that the surface potential, ϕ_s, is referenced to the p-type substrate. Derive this relation.

6. Assume that the graph shown below describes $|Q_s|(\phi_s)$ of a substrate. Using this substrate, a MOS capacitor is made. The flatband voltage is -0.9 V and C_{ox} is 2X10^{-7} F cm^{-2}. When the gate voltage is 0.4 V, estimate the total silicon charge.



7. The previous graph shows $|Q_s|(\phi_s)$ of a substrate, whose p-type doping concentration is N_a. Now, draw the same graph for ten times higher doping concentration. Of course, we cannot draw the exact curve. Provide the reason of modification.

8. In the MOS capacitor, let us assume that we can selectively change the electron quasi-Fermi potential to V, while the substrate is still grounded. Then, the expression for Q_s is modified due to V. Write down the modified expression for Q_s.

9. It is assumed that the electrostatic potential, ϕ, is given as

$$\phi(x,y) = \phi_s\left(1-\frac{x}{W_d}\right)^2 + \left(b\sinh\left(\frac{\pi(L-y)}{\lambda}\right) + c\sinh\left(\frac{\pi y}{\lambda}\right)\right)\sin\left(\frac{\pi(x+3t_{ox})}{\lambda}\right),$$

where ϕ_s, W_d, b, L, λ, c, and t_{ox} are given parameters. Specify the gradual channel approximation. For the electrostatic potential introduced above, explicitly derive the condition.

10. We have assumed that the electron quasi-Fermi potential, V, is independent of the vertical direction, x. Justify this assumption.

11. Write down the Pao-Sah double integral.

12. After a long derivation procedure, we have found an expression for I_d:

$$I_d = \mu_{eff}\frac{W}{L}\left\{C_{ox}\left(V_{gs} - V_{fb} + \frac{k_B T}{q}\right)\phi_s - \frac{1}{2}C_{ox}\phi_s^2 - \frac{2}{3}\sqrt{2\epsilon_{si}qN_a}\phi_s^{1.5} + \frac{k_B T}{q}\sqrt{2\epsilon_{si}qN_a\phi_s}\right\}\Bigg|_{\phi_{s,s}}^{\phi_{s,d}}$$

Now, assume that $\phi_{s,s}$ and $\phi_{s,d}$ are 1.132 V and 1.597 V, respectively. $\mu_{eff}\frac{W}{L}$ is 300 cm^2 V^{-1} sec^{-1}. The vacuum permittivity is 8.854 X 10^{-12} F m^{-1} and the relative permittivity of silicon is 11.7. The equivalent oxide thickness is 1.2 nm and $V_{gs} - V_{fb}$ is 2.18 V. For the thermal voltage, use 25.85 mV. The substrate doping concentration is 10^{18} cm^{-3}. Of course, q is 1.6X10^{-19} C. By using these numbers, calculate I_d.

13. The relation between V and ϕ_s is important. Consider the MOS equation with a simplified $Q_s(\phi_s)$ relation:

$$V_{gs} = V_{fb} + \phi_s + \frac{\sqrt{2\epsilon_{si}k_B T N_a}}{C_{ox}}\left[\frac{q\phi_s}{k_B T} + \frac{n_i^2}{N_a^2}\exp\left(\frac{q}{k_B T}(\phi_s - V)\right)\right]^{1/2}.$$

When V_{gs} is fixed, find an explicit expression for $\frac{dV}{d\phi_s}$.

14. In the subthreshold region, we have an approximate form for the inversion charge,

$$-Q_i \approx \sqrt{\frac{\epsilon_{si}qN_a}{2\phi_s}\frac{k_B T}{q}\frac{n_i^2}{N_a^2}}\exp\left(\frac{q}{k_B T}(\phi_s - V)\right),$$

where ϕ_s can be regarded as a constant. By using a relation,

$$I_d = \mu_{eff}\frac{W}{L}\int_0^{V_{ds}}[-Q_i(V)]dV,$$

derive an expression for I_d.

15. We have studied the $Q_s(\phi_s)$ relation of the planar MOS structure. Instead, consider a double-gate structure whose thickness is T_{si}. Using the electrostatic potentials at two points, $x = 0$ (Si/SiO$_2$ interface) and $x = \frac{T_{si}}{2}$ (midpoint), express the silicon charge. You don't have to adopt Prof. Taur's offset potential. Assume that the hole density is negligible.