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ENGINEERING MECHANICS

UNIT-I Chapter – 2

COPLANAR CONCURRENT FORCE SYSTEM

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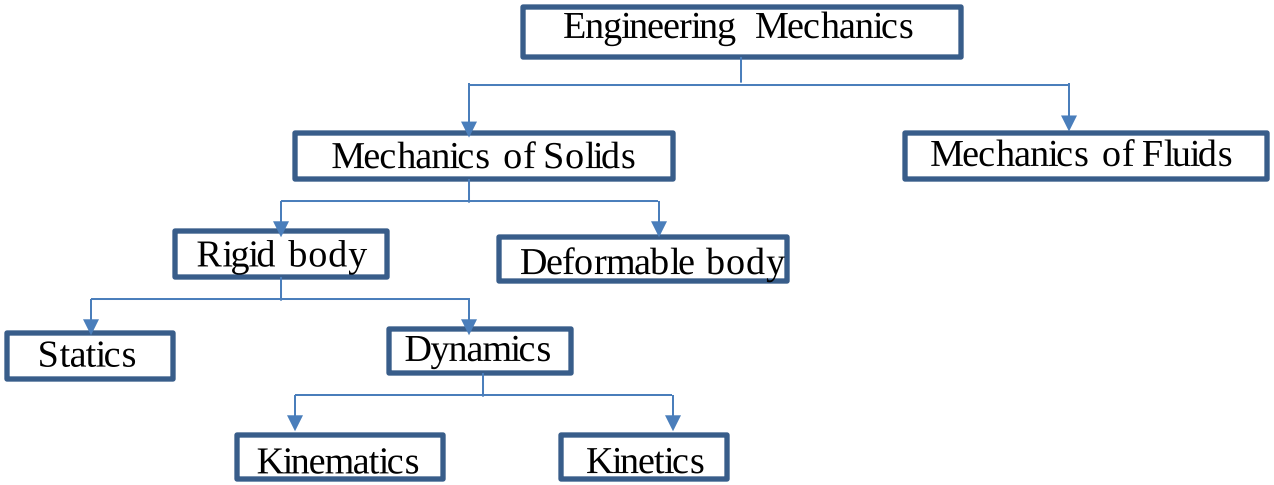
Chapter Content:

- Introduction to Engineering Mechanics and Definition of Engineering Mechanics, Basic idealizations-particle, continuum, body, rigid body, deformable body and force.
- Elements of force, classification of force, force systems.
- Laws of Mechanics – Parallelogram law, Principle of transmissibility of force, law of superposition, Newton’s laws of motion.
- Definitions – Resultant force, composition of forces, Resolution of forces. Analytical formulae for resultant of forces and components of a force using parallelogram law.
- Numerical problems on composition of coplanar concurrent force system.
- Definitions - Equilibrium and equilibrant. Conditions of static equilibrium. Free body Diagram. Lami’s theorem.
- Numerical problems on equilibrium of coplanar concurrent force system.

Engineering Mechanics: "It's a science which describes and predicts the state of rest or a motion of a body under the application of external forces".

Here two important elements are a **body** and a **force.** We are going to study how a force acting on the body changes its state.

Engineering Mechanics is classified as shown in the chart.



Some important definitions

Particle: When the dimensions or size of the body is negligible during its analysis is called as a particle.

Ex: 1) A cricket ball resting on the ground. 2) An aeroplane flying in the sky seen from the ground.

Body: It's a combination of large number of particles held together with a internal cohesion. It has a specific dimensions**.**

Rigid body: A body which is the combination of large number of particles which are occupying fixed positions with respect to each other.

In such bodies the distance between any two arbitrary points is invariant or the relative deformation between its parts is assumed to be negligible during its analysis.

Note: "No body on this earth is perfectly a rigid body".

Deformable body: If the size or dimensions of the body are going to change during its analysis is called as deformable body..

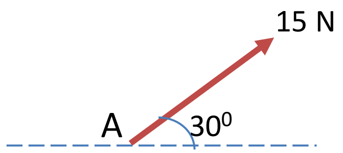
Ex: 1) A beam deflecting due to applied load. 2) A liquid flowing in any direction.

Continuum: When any body is assumed to be consists of continuous distribution of matter throughout its entire volume then such a body is called as continuum.

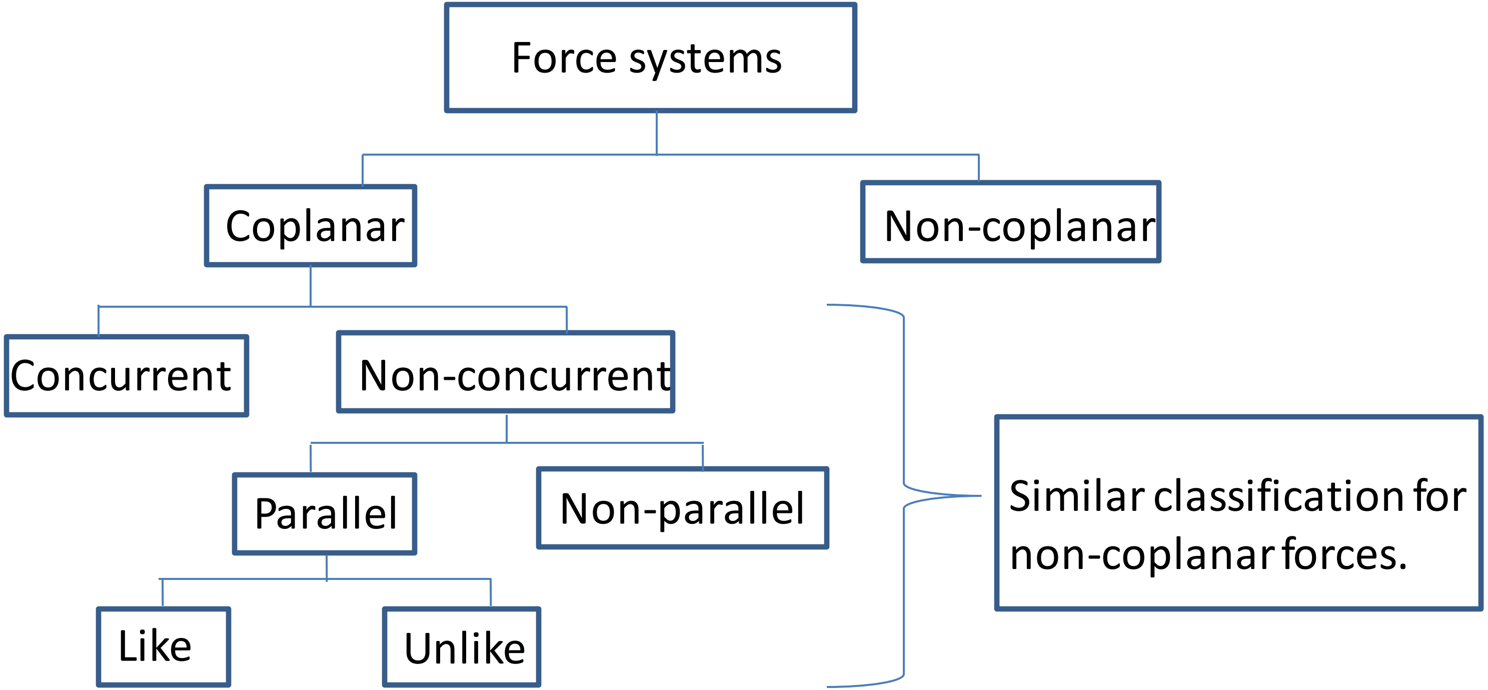
Force: It is the one which tries to change the state of rest or state of motion of a body when it acts on it.

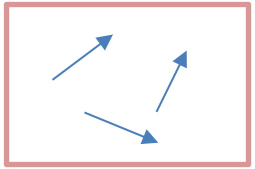
Following are the characteristics of a force:

1) Magnitude
2) Direction (Sense)
3) Point of application.

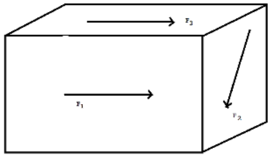


Classification of forces:

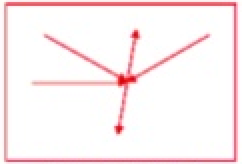




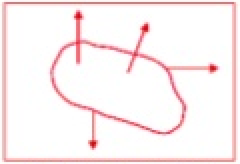
Coplanar forces



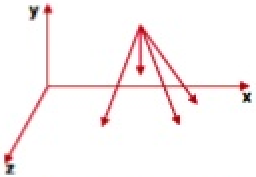
Non-coplanar forces



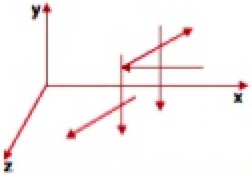
Coplanar concurrent



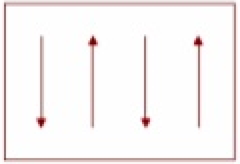
Coplanar non-concurrent



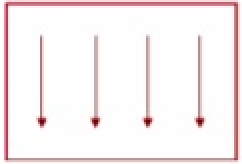
Non-coplanar concurrent



Non-coplanar non-concurrent



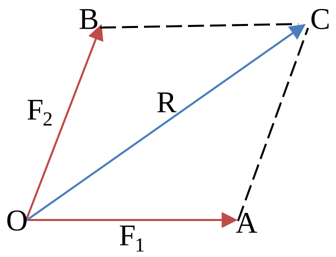
Coplanar parallel



Coplanar like parallel

Laws of Mechanics:

Parallelogram law of forces: " If two forces acting simultaneously on a body at a point can be represented in magnitude and direction by the two adjacent sides of the parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces".



Principle of transmissibility: “The state of rest or motion of the rigid body is unaltered if a force acting on the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force ”.

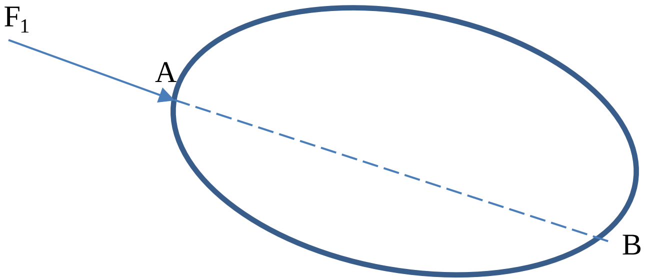


Fig. a

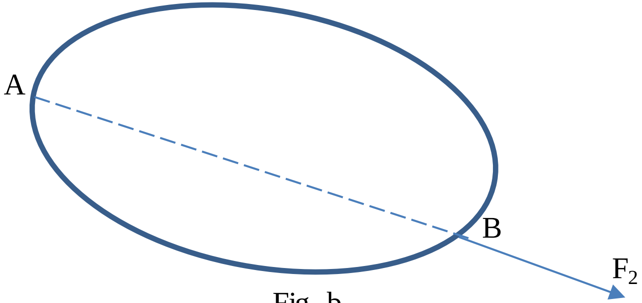


Fig. b

In the above figures magnitudes of F_1 and F_2 are same.

Note: Principle of transmissibility is only applicable to rigid bodies.

Law of superposition: “The action of a given system of forces on a rigid body is not changed by adding or subtracting another system of forces in equilibrium”.

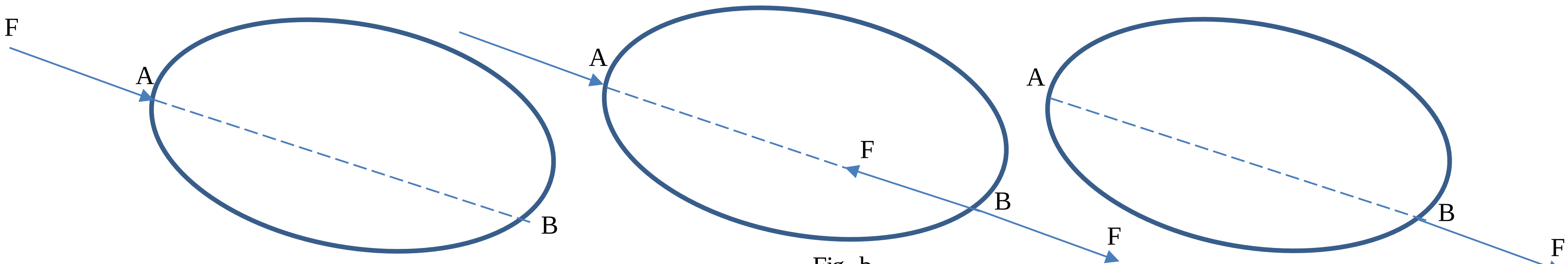


Fig. a Fig. b Fig. c

Newton’s first law: “ Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by an external agency acting on it”.

Newton’s second law: “ The rate of change of momentum of a body is directly proportional to the applied force and it takes place in the direction of force acting on it”.

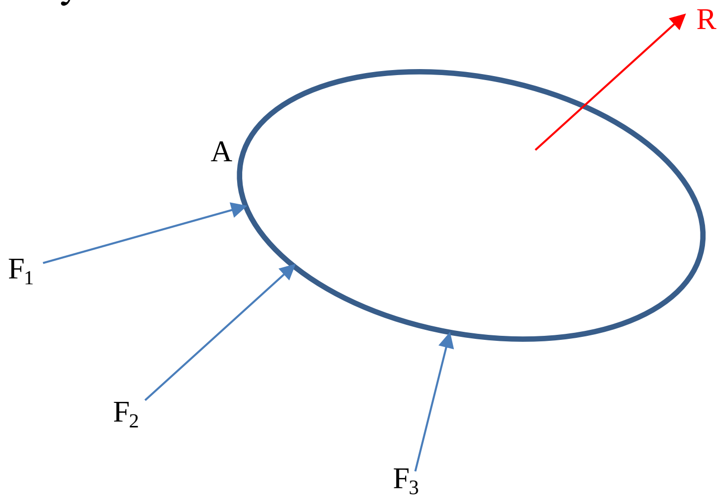
i.e

$$F = m \times a$$

Newton’s third law: “ For every action there is an equal and opposite reaction”.

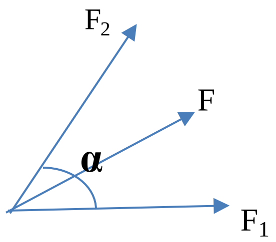
This third law is important as far as Engineering Mechanics syllabus is concerned.

Resultant force: “It is a single force which will have the same effect as that of a number of forces acting on the body”.

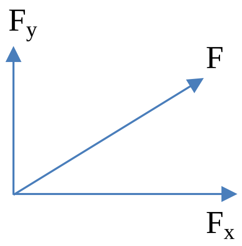


Composition of forces: “The process of finding the resultant force of several forces is known as composition of forces”.

Resolution of a force: "It is the process of finding the components of a force without changing its effect on the body".



Force F is not resolved along two mutually perpendicular directions.



Force F is resolved along two mutually perpendicular directions F_x and F_y.

Resultant of two concurrent forces:

Consider the two forces F_1 and F_2 acting on a particle as shown in Fig. a. Let the angle between the two forces be ‘θ’. If the parallelogram ABCD is drawn as shown in Fig. b, with AB representing F_1 and AD representing F_2 to same scale, according to parallelogram law of forces AC represents the resultant R. Drop perpendicular CE to AB.

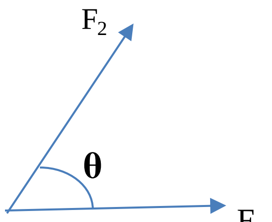


Fig. a

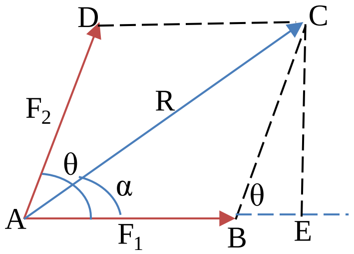


Fig. b

The resultant R of F_1 and F_2 is given by:

From triangle ACE $\quad R = AC = \sqrt{AE^2 + CE^2}$

$$R = AC = \sqrt{(AB + BE)^2 + CE^2}$$

But $$AB = F_1$$

$$BE = BC\cos\theta = F_2\cos\theta$$

$$CE = BC\sin\theta = F_2\sin\theta$$

$$R = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

$$R = \sqrt{F_1^2 + F_2^2\cos\theta^2 + 2F_1F_2\cos\theta + F_2^2\sin\theta^2}$$

$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$ since $\sin^2\theta + \cos^2\theta = 1$

The inclination of the resultant to the direction of the force F_1 is given by α, when

$$\tan\alpha = \frac{CE}{AE} = \frac{CE}{AB + BE} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Hence

$$\boxed{\alpha = \tan^{-1}\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}}$$

Particular cases:

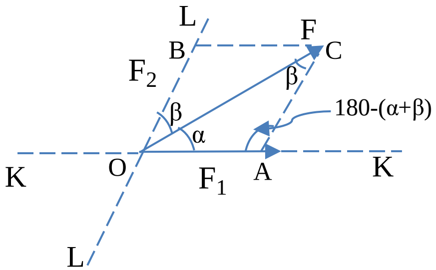
1. When $\theta = 90^0$ $\quad R = \sqrt{F_1{}^2 + F_2{}^2}$ $\quad \alpha = \tan^{-1}\frac{F_2}{F_1}$

2. When $\theta = 0^0$ $\quad R = F_1 + F_2$

3. When $\theta = 180^0$ $\quad R = F_1 - F_2$

Resolution of a force along two arbitrary directions:

Consider a force F acting at point 'O' which is to be resolved along two arbitrary directions KK and LL as shown in the below figure. Let F_1 be the component of F along KK and F_2 be the component of F along LL. KK makes an angle α with OC and LL makes an angle β with OC.

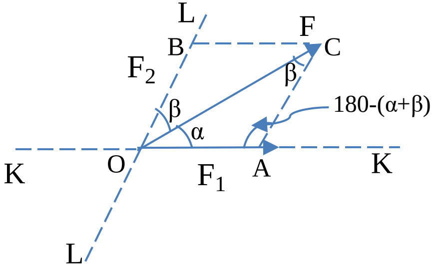


From triangle OCA it is clear that angle OCA is also β and angle OAC is 180-(α+ β)

Applying sine rule for triangle OCA we get:

$$\frac{OC}{\sin[180-(\alpha+\beta)]}=\frac{OA}{sin\beta}=\frac{AC}{sin\alpha}$$

$$\frac{OC}{\sin[180-(\alpha+\beta)]} = \frac{OA}{sin\beta} = \frac{AC}{sin\alpha}$$



$$\frac{F}{\sin(\alpha+\beta)} = \frac{F_1}{sin\beta} = \frac{F_2}{sin\alpha}$$

$$\frac{F_1}{sin\beta} = \frac{F}{\sin(\alpha+\beta)}$$

$$\frac{F_2}{sin\alpha} = \frac{F}{\sin(\alpha+\beta)}$$

$$\boxed{F_1 = \frac{Fsin\beta}{\sin(\alpha+\beta)}}$$

$$\boxed{F_2 = \frac{Fsin\alpha}{\sin(\alpha+\beta)}}$$

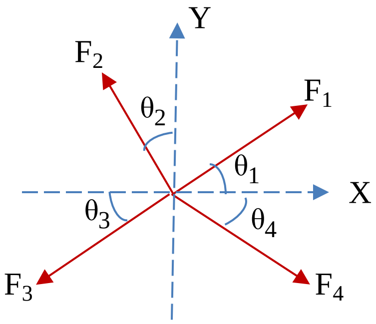
When $(\alpha+\beta) = 90^0$ then we get components along two mutually perpendicular directions. When α is known we have β = 90 – α

$\boxed{F_1 = F\cos\alpha}$ & $\boxed{F_2 = F\sin\alpha}$

Similarly when β is known we get $\boxed{F_1 = F\sin\beta}$ & $\boxed{F_2 = F\cos\beta}$

Resultant of several coplanar concurrent forces:

Now we will study hoe to find the magnitude and direction of resultant of several concurrent force system step by step. Consider a force system shown below.



Step 1: Resolve all the forces along horizontal and vertical directions.

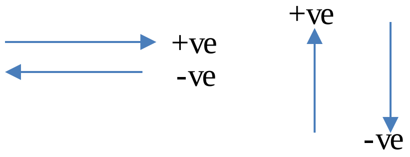
Step 2: Find algebraic sum of all the horizontal components i.e. $\sum F_x$

Step 3: Find algebraic sum of all the vertical components i.e. $\sum F_y$

Step 4: Find the magnitude of resultant using R = $\sqrt{\sum {F_x}^2 + \sum {F_y}^2}$

Step 5: Find the direction of resultant using θ = tan^{-1} $[\frac{\sum F_y}{\sum F_x}]$

Note:

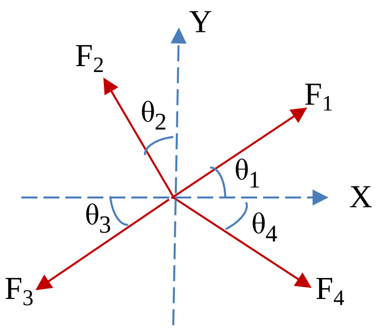


$$\sum F_x = F_1 cos\theta_1 - F_2 sin\theta_2 \ - F_3 cos\theta_3 + F_4 cos\theta_4$$

$$\sum F_y = F_1 sin\theta_1 + F_2 cos\theta_2 \ - F_3 sin\theta_3 - F_4 sin\theta_4$$

$$R = \sqrt{\sum {F_x}^2 + \sum {F_y}^2}$$

$$\theta = \tan^{-1} [\frac{\sum F_y}{\sum F_x}]$$



Problems:

1) A machine foundation bolt as shown in Fig, is subjected to four force system. The bolt can withstand a resultant force of 205 N. Check for safety of bolt. if, F_1 = 80 N, F_2 = 150 N, F_3 = 100 N, F_4 = 110 N, $\theta_1 = 20°$, $\theta_2 = 30°$, $\theta_3 = 15°$.

Solution: Resolving the forces along x direction and adding them

$$\sum F_x = -80\sin 20^0 + 150\cos 30^0 + 100\cos 15^0$$

$$= 199.13 \text{ N}$$

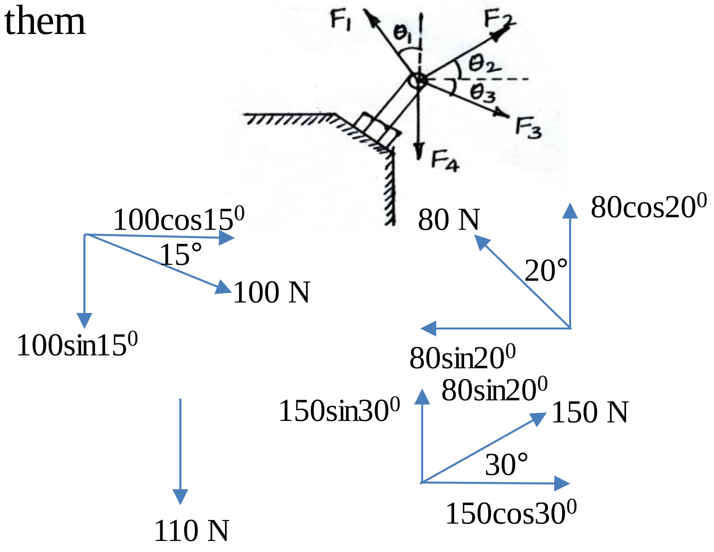
$$\sum F_y = 80\cos 20^0 + 150\sin 30^0 - 100\sin 15^0 - 110$$

$$= 14.29 \text{ N}$$

Magnitude of the resultant:

$$R = \sqrt{\sum F_x^{\,2} + \sum F_y^{\,2}}$$

$$R = \sqrt{199.13^2 + 14.29^2}$$



$$R = \sqrt{199.13^2 + 14.29^2}$$

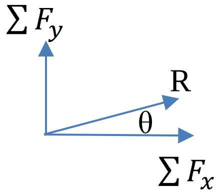
Magnitude $R = 199.64\ N$

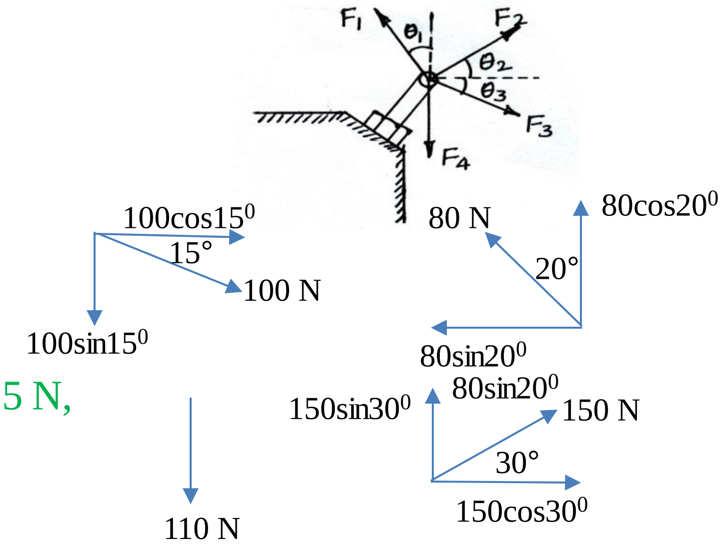
Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

$$\theta = \tan^{-1}\left[\frac{14.29}{199.13}\right]$$

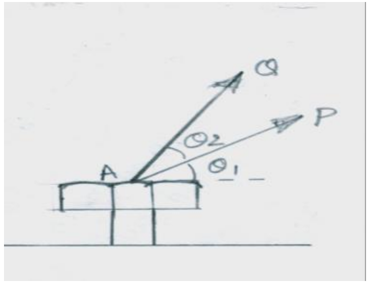
$$\theta = 4.10^0$$

Since the obtained magnitude of the resultant is less than 205 N, the bolt is safe to withstand the applied forces.





2) Two forces P&Q act on bolt as shown in the fig. Determine the resultant if P=40 N, Q=60 N, θ_1=20°, θ_2=25°.



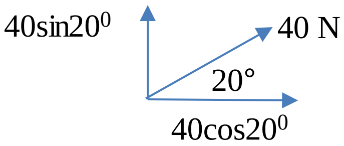
Solution: Resolving the forces along x direction and adding them

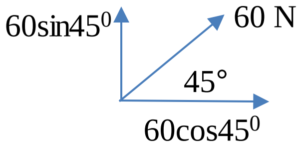
$$\sum F_x = 40\cos 20^0 + 60\cos 45^0$$

$$= 80.01 \text{ N}$$

$$\sum F_y = 40\sin 20^0 + 60\sin 45^0$$

$$= 56.10 \text{ N}$$





Magnitude of the resultant:

$$R = \sqrt{\sum {F_x}^2 + \sum {F_y}^2}$$

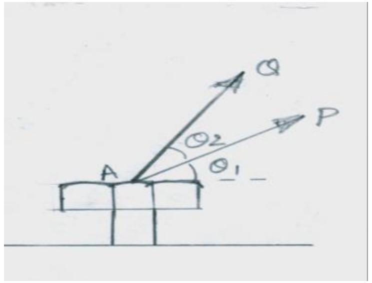
$$R = \sqrt{80.01^2 + 56.10^2}$$

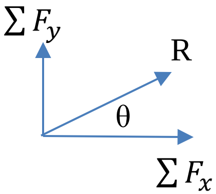
R = 97.71 N

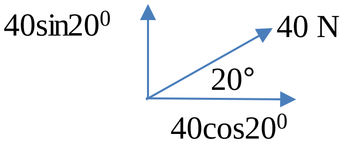
Direction of the resultant $\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]$

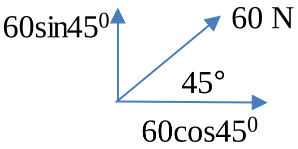
$\theta = \tan^{-1}\left[\frac{56.10}{80.01}\right]$

$\theta = 35.03^0$

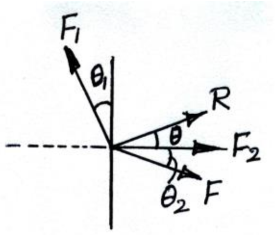








3) Determine the magnitude of the force F for the resultant of the forces shown in figure to be at an angle of θ = 15° with reference to X-axis. Also determine the magnitude of the resultant, if F_1 = 70 N, F_2 = 60 N, θ_1 = 20° θ_2 = 10^0 .

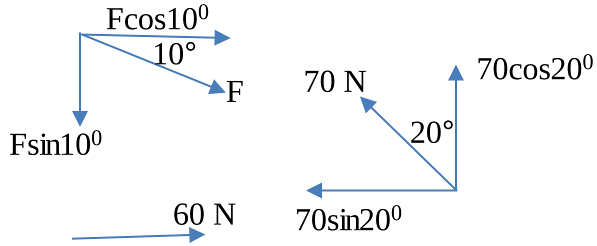


Solution: Resolving the forces along x direction and adding them

$\sum F_x$ = $-70\sin 20^0 + F\cos 10^0 + 60$

$\sum F_y$ = $70\cos 20^0 - F\sin 10^0$

As the direction of the resultant R is known,



$$\tan\theta = \frac{\sum F_y}{\sum F_x}$$

$$\tan 15^0 = \frac{70\cos 20^0 - F\sin 10^0}{-70\sin 20^0 + F\cos 10^0 + 60}$$

0.268(36.058+0.9848F) = 65.78 – 0.174 F

0.4379 = 56.117

F = 128.15 N.

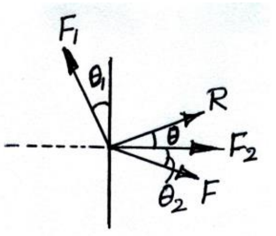
$\sum F_x$ = -70sin20^0+128.15cos10^0+60 = 162.26 N

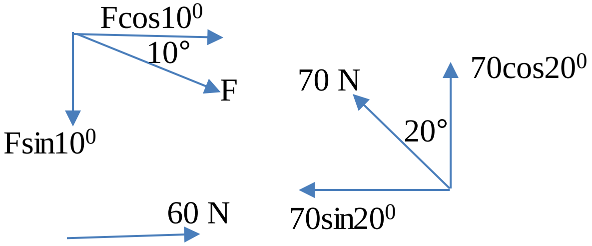
$\sum F_y$ = 70cos20^0-128.15sin10^0 = 43.48 N

$$R = \sqrt{\sum F_x{}^2 + \sum F_y{}^2}$$

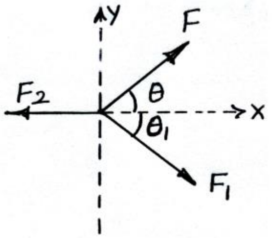
$$R = \sqrt{162.26^2 + 43.48^2}$$

R = 167.98 N





4) The force system shown in figure has a resultant of 250 N pointing vertically upwards along the Y-axis. Calculate the value of F & θ required to give this resultant. F_1 = 250 N, F_2 = 600 N, θ_1 = 30 °

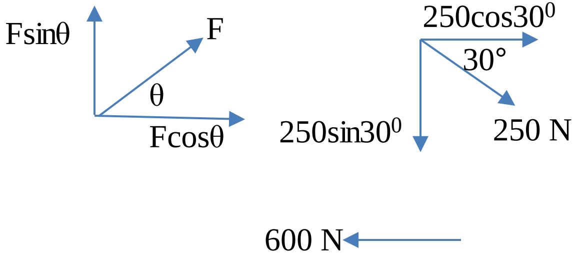


Solution: Resolving the forces along x direction and adding them

$\sum F_x$ = 250cos30^0+Fcosθ-600 -----(i)

$\sum F_y$ = -250cos30^0+Fsinθ ------(ii)

Since the resultant is along y-axis, its component in x-axis is Zero i.e. $\sum F_x$= 0.



Therefore equation (i) becomes 250cos30^0+Fcosθ-600 = 0

Fcosθ = 383.5 ------(iii)

Resultant, R = $\sum F_y$

250 = -250cos30^0+Fsinθ

$250 = -250\cos30^0 + F\sin\theta$

$F\sin\theta = 375$ -------(iv)

Dividing (iv) by (iii)

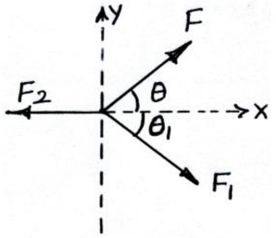
$\tan\theta = \frac{375}{383.5}$

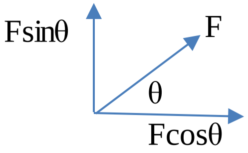
$\theta = 44.36^0$

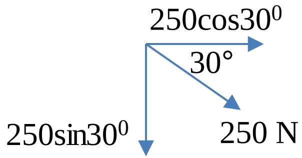
From equation (iv), $F\sin\theta = 375$

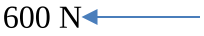
$F = \frac{375}{\sin 44.36^0}$

$F = 536.35\ N$

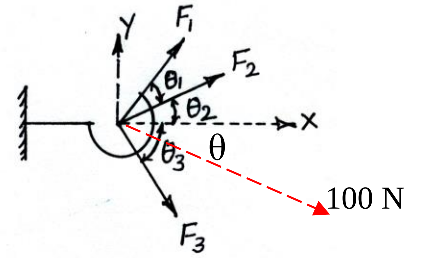








5) Three forces acting on a hook are as shown in figure. Determine the direction of the fourth force 'P' of magnitude 100 N such that the hook is pulled in X –axis only. The hook is safe if resultant is less than 300 N. Take F_1 = 80 N, F_2 = 80 N, F_3 = 200 N, $\theta_1 = 30°$ $\theta_2 = 30°$ $\theta_3 = 60°$. Check the safety of hook.



Solution: Let 100 N force acts as shown making an angle θ with x-axis.

Since the resultant is along x-axis, its component in y-axis is zero.

i.e. $\sum F_y$=0

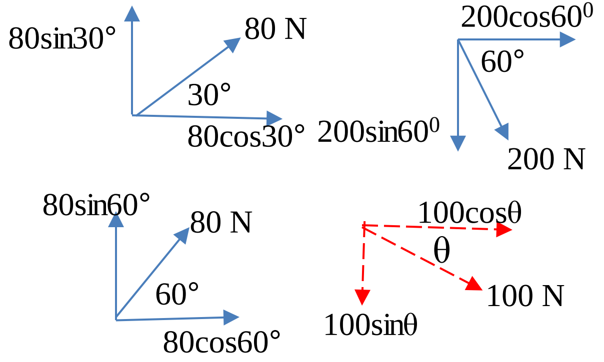
Therefore $\sum F_y$=80sin30^0+80sin60^0-200sin60^0-100sinθ

100sinθ = -63.923

θ = -39.73^0 (i.e. θ is to be measured in anticlockwise direction)

Magnitude of the resultant

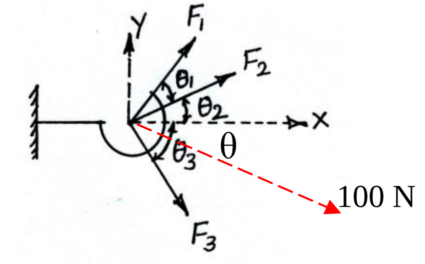
R = $\sum F_x$

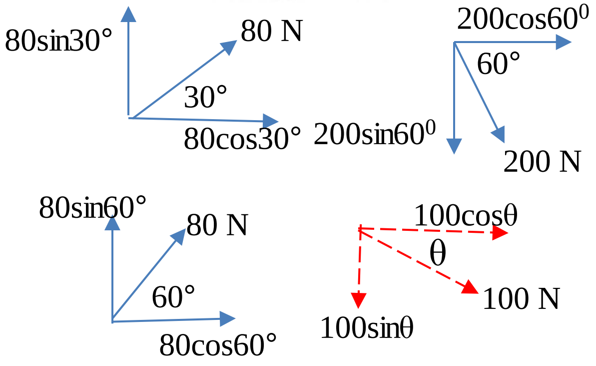


$R = \sum F_x = 80\cos30^0 + 80\cos60^0 + 200\cos60^0 + 100\cos39.73$

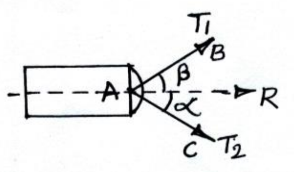
R = 286.19 N

Since the value of resultant is less than 300 N, the hook is safe.





6) A disabled automobile is pulled by means of 2 ropes as shown in figure. Knowing that the tension in the rope AB is T_1 = 750 N, determine the tension in rope AC ie T_2 & the value of α so that the resultant force exerted at A, R = 1200 N force directed along the axis of the automobile. β= 30°.



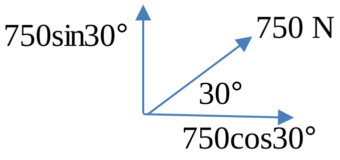
Solution: Resolving the forces along x direction and adding them

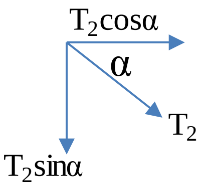
$\sum F_x = 750\cos 30^0 + T_2\cos\alpha$ ------(i)

Resolving the forces along y direction and adding them

$\sum F_y = 750\sin 30^0 - T_2\sin\alpha$ ------(ii)

Given that the resultant is along the axis of automobile i.e. horizontal.





Therefore $\sum F_y = 0$. hence equation (ii) becomes.

$750\sin 30^0 - T_2\sin\alpha = 0$

$T_2\sin\alpha = 375$ -----(a)

Also we know that $R = \sum F_x$

Therefore equation (i) becomes

$750\cos 30^0 + T_2\cos\alpha = 1200$

$T_2\cos\alpha = 550.5$ ------(b)

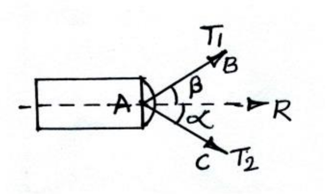
(a)/(b) gives

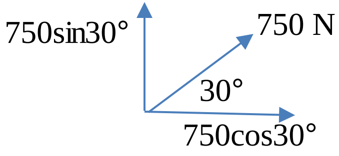
$\tan\alpha = \frac{375}{550.5} = 0.681$

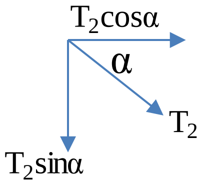
$\alpha = 34.25^0$

From equation (a), $T_2 = \frac{375}{\sin 34.25^0}$

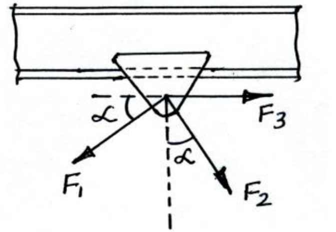
$T_2 = 666.1$ N.







7) A hoist trolley is subjected to three forces as shown in figure. Determine (i) the value of angle α for which the resultant of the three forces is vertical (ii) corresponding magnitude of the resultant. Given, F_1 = 400 N, F_2 = 200 N, F_3 = 250 N.



Solution: Since the resultant is vertical, $\sum F_x = 0$

250+200sinα-400cosα = 0

Or 200sinα - 400cosα + 250 = 0

sinα - 2cosα + 1.25 = 0

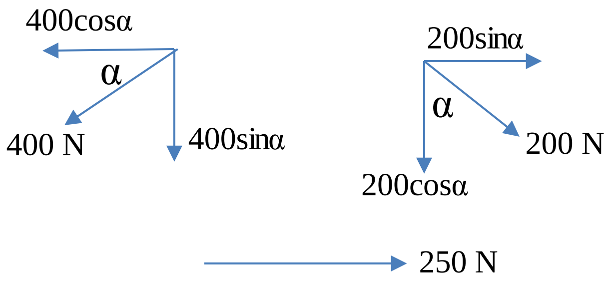
sinα-2$\sqrt{(1 - sin^2\alpha)}$ + 1.25 = 0

sinα + 1.25 = 2$\sqrt{(1 - sin^2\alpha)}$

Squaring on both sides`

$\sin^2\alpha + 2.5 \sin\alpha + 1.25^2 = 4(1 - \sin^2\alpha)$

$5\sin^2\alpha + 2.5\sin\alpha - 2.44 = 0$



$5\sin^2\alpha + 2.5\sin\alpha - 2.44 = 0$

This equation is in quadratic form.

i.e. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Therefore $\sin\alpha = \frac{-2.5 \pm \sqrt{2.5^2 - 4x5x(-2.44)}}{2x5}$

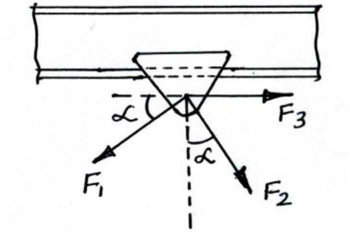
$= (-2.5+7.49)/10$

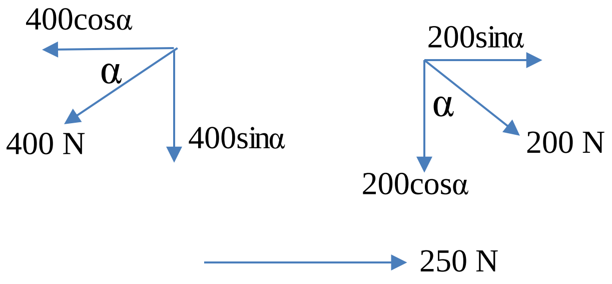
After solving we get

$\alpha = 29.46^0$

Resultant, $R = \sum F_y = -400\sin\alpha - 200\cos\alpha$

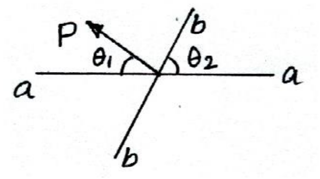
$R = -370.87$ N





8) The force P of magnitude 500 N is to be resolved into two components along lines aa & bb. Determine the angle θ_1 knowing that the component along the line aa is to be 240 N when $\theta_2 = 60°$.

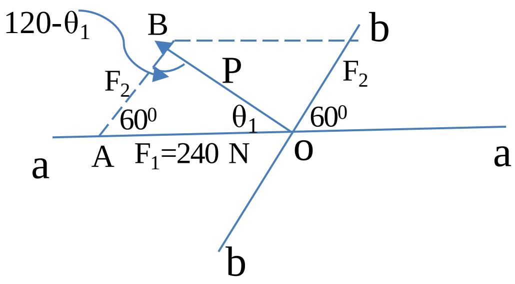
Solution: Considering the triangle OAB and applying sine rule



$$\frac{OB}{\sin 60^0} = \frac{OA}{\sin(120-\theta_1)} = \frac{AB}{sin\theta_1}$$

$$\frac{500}{\sin 60^0} = \frac{240}{\sin(120-\theta_1)} = \frac{F_2}{sin\theta_1}$$

$$\frac{500}{\sin 60^0} = \frac{240}{\sin(120-\theta_1)}$$



After solving we get $\theta 1 = 95.43^0$

$$\frac{500}{\sin 60^0} = \frac{F_2}{sin\theta_1}$$

$F_2 = 574.75$ N this is the component of force along ‘bb’.

Equilibrium of coplanar concurrent forces:

Equilibrium: It is defined as the state of a body at which the resultant of all the forces acting on that body is zero.

For ex: When the two forces F_1 and F_2 acts on a body then to call a body to be in equilibrium following points to be satisfied.

1. The magnitudes of two forces should be equal. i.e. $F_1 = F_2$
2. The directions of two forces should be opposite.
3. The line of action of two forces should be same.

When more than two forces acts on the body then two call a body to be in equilibrium, the resultant of all the forces should be equal to zero.

i.e. $R = 0$

Now the question comes when R will be equal to zero??

We know that $R = \sqrt{\sum F_x{}^2 + \sum F_y{}^2}$

We know that $R = \sqrt{\sum F_x{}^2 + \sum F_y{}^2}$

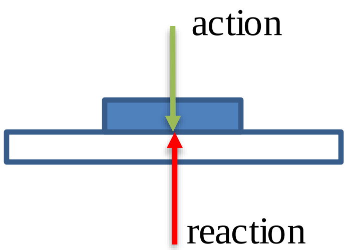
We can achieve $R = 0$, when both $\sum F_x$ & $\sum F_y$ should be simultaneously equal to zero.

Therefore the necessary conditions of equilibrium for several coplanar concurrent forces are

1. $\sum F_x = 0$
2. $\sum F_y = 0$

Action and Reaction: When two bodies are in contact with each other they exert some force on each other, those forces are action and reaction.

For ex: A duster kept on the table



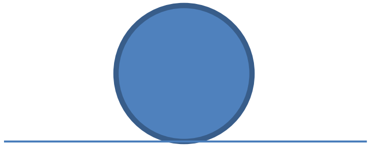
Free body diagram(FBD):

For the analysis of equilibrium condition it is necessary to isolate the body under consideration from the other bodies and draw all the forces acting on the body.

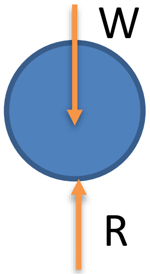
For this first the body is drawn and then all applied forces, self weight and reactions from the other bodies in contact are drawn.

"A body under consideration which is freed from all contact surfaces and is shown with all the forces on it (including self weight, reactions from other contact surfaces) is called free body diagram (FBD)"

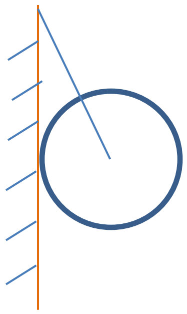
For ex: a ball resting on the ground.



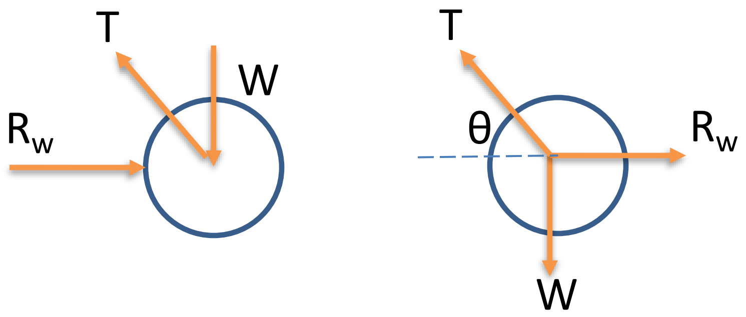
FBD of ball



For ex: a ball which is hanging from a nail in the wall.



FBD of ball



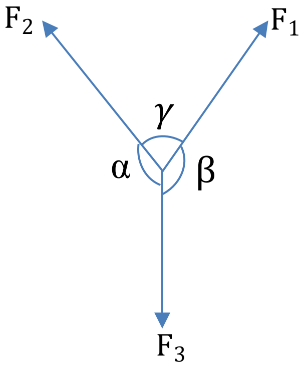
Hence free body diagrams helps to apply conditions of equilibrium to find out required unknowns.

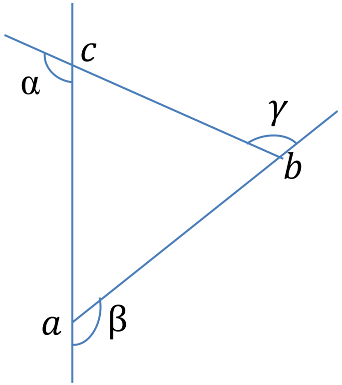
Lami’s theorem:

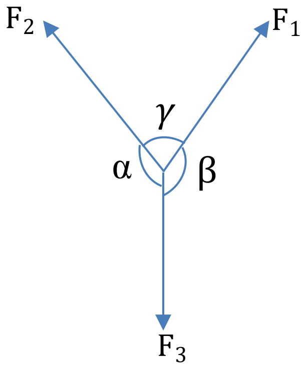
If a body is in equilibrium under the action of only three forces, which are concurrent then this theorem can be used conveniently.

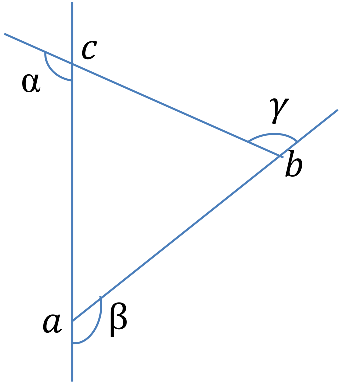
Statement: “If a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the other two forces”

$$\frac{F_1}{sin\alpha}.=\frac{F_2}{sin\beta}=\frac{F_3}{sin\gamma}$$









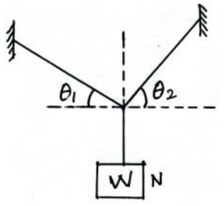
Proof: Draw the three forces F_1, F_2 and F_3 one after the other in direction and magnitude starting from 'a'. Since the body is in equilibrium the resultant should be zero, which means the last point of force diagram should coincide with 'a'. Thus, it results in a triangle of forces abs as shown in above figure. Now the external angles at a, b, and c are equal to β, γ *and* α. Since ab, bc and ca are parallel to F_1, F_2 and F_3 respectively. In the triangle of forces abc,

ab = F_1, bc = F_2 and ca = F_3

Applying sine rule for the triangle abc,

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta)} = \frac{ca}{\sin(180-\gamma)} \longrightarrow \boxed{\frac{F_1}{sin\alpha} = \frac{F_2}{sin\beta} = \frac{F_3}{sin\gamma}}$$

9) A weight of W = 100 N is hung by two wires as shown in figure. Find the tension in two wires. Take θ_1 = 30°,θ_2= 60°.



Solution: since there are only three forces which are concurrent, we can apply Lami's theorem

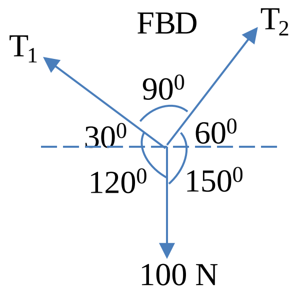
$$\frac{T_1}{sin\,150^0}=\frac{T_2}{sin\,120^0}=\frac{100}{sin\,90^0}$$

$$\frac{T_1}{sin\,150^0}.=\frac{100}{sin\,90^0}$$

T_1 = 50 N

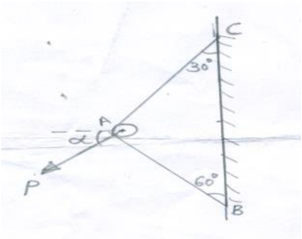
$$\frac{T_2}{sin\,120^0}=\frac{100}{sin 90^0}$$

T_2 = 86.6 N



10) The force P is applied to a small wheel which rolls on the cable ACB as shown in the Fig. Knowing that tension in the cable is 600N. Determine the magnitude and direction of P.

Solution: Considering the equilibrium of point ‘A’, its FBD is as shown.



Using $\sum F_x = 0$

$600\cos 60^0 + 600\cos 30^0 - P\cos\alpha = 0$

$P\cos\alpha = 819.6$ N --------(i)

Using $\sum F_y = 0$

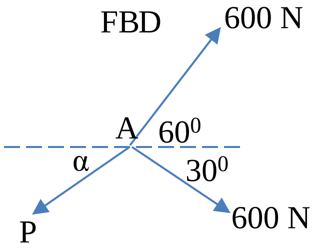
$600\sin 60^0 - 600\sin 30^0 - P\sin\alpha = 0$

$P\sin\alpha = 219.61$ N ---------(ii)

Dividing equation (ii) by (i)

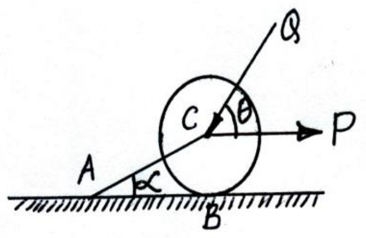
$\tan\alpha = \frac{219.61}{819.60}$ $\quad\quad \alpha = 15^0$

From (ii) $\quad P = \frac{219.61}{\sin 15^0} = 848.5$ N



11) A roller of weight 10 kN rests on a smooth horizontal floor & is tied by a rope AC as shown in figure. Estimate the force in AC & reaction from the floor if the roller is subjected to forces P = 10 kN, Q = 7 kN, α = 30°, θ = 45°.

Solution: Considering the equilibrium of roller. Its FBD is as shown.

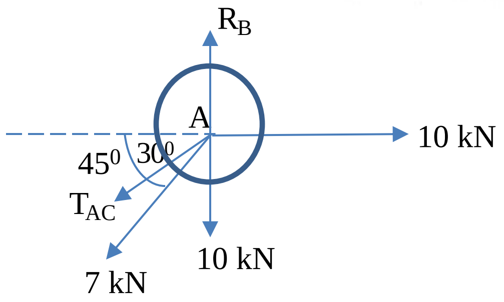


Let T_{AC} be the tension in the rope AC. & R_B be the reaction at floor.

Applying $\sum F_x = 0$

$10 - 7\cos 45^0 - T_{AC}\cos 30^0 = 0$

FBD



Therefore $T_{AC} = \frac{5.05}{cos30^0} = 5.83$ kN

Applying $\sum F_y = 0$

$R_B - 10 - 7\sin 45^0 - T_{AC}\sin 30^0 = 0$

$R_B = 17.865$ kN.

12) A string is subjected to the forces 4 kN and W as shown in the fig. Determine the magnitude of W and the tension induced in various portions of string.

Solution: Consider equilibrium of ‘B’. Its free body diagram is as Shown.

Applying Lami’s theorem to joint ‘B’

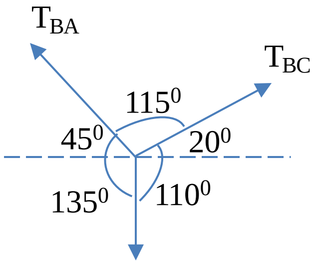
$$\frac{4}{sin\,115^0}.=\frac{T_{BA}}{sin\,110^0} = \frac{T_{BC}}{sin\,135^0}$$

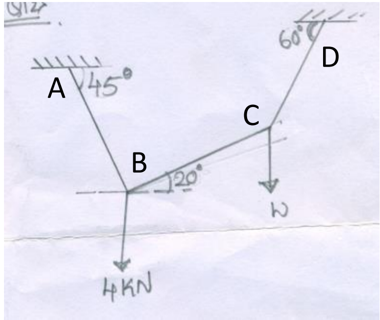
$$\frac{4}{sin\,115^0}.=\frac{T_{BA}}{sin\,110^0}$$

T_{BA} = 4.147 kN

$$\frac{T_{BC}}{sin\,135^0} = \frac{4}{sin\,115^0}.$$

T_{BC} = 3.12 kN





Applying Lami's theorem to joint 'C'

$$\frac{3.12}{sin\,150^0}.=\frac{T_{CD}}{sin\,70^0} = \frac{W}{sin\,140^0}$$

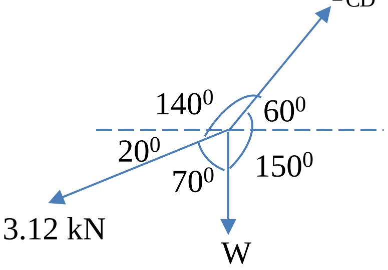
$$\frac{3.12}{sin\,150^0}.=\frac{T_{CD}}{sin\,70^0}$$

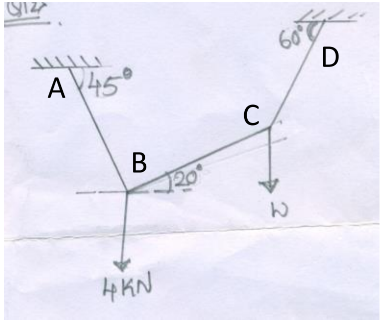
T_{BA} = 5.86 kN

$$\frac{W}{sin\,140^0}=\frac{3.12}{sin\,150^0}$$

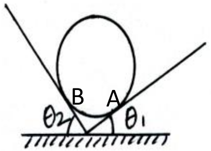
W = 4.01 kN

FBD of C



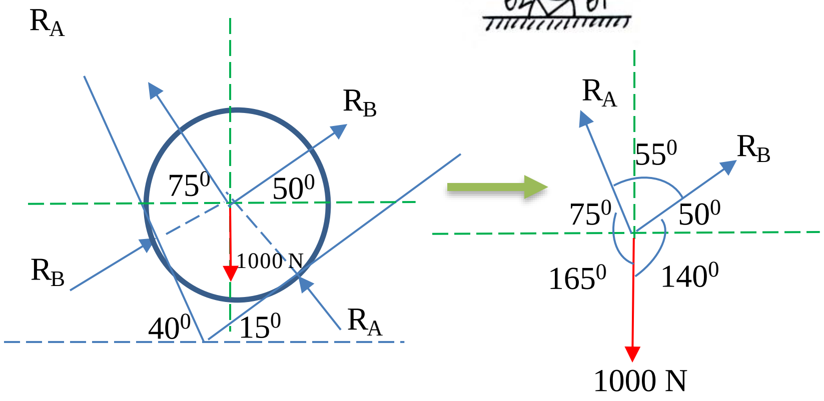


13) A smooth circular cylinder of radius 1.5 m is lying in a triangular groove one side of which makes $\theta 1$ =15° angle and the other $\theta 2$= 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction & the cylinder weighs 1000 N.



Solution: Consider equilibrium of ‘B’. Its free body diagram is as Shown.

FBD



Applying Lami’s theorem to joint ‘B’

$$\frac{1000}{sin55^0}.=\frac{R_B}{sin165^0}=\frac{R_A}{sin140^0}$$

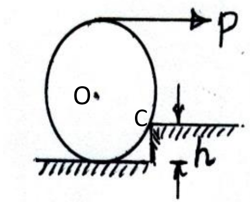
$$\frac{1000}{sin55^0}.=\frac{R_B}{sin165^0}$$

R_B = 315.96 kN

$$\frac{R_A}{sin140^0}=\frac{1000}{sin55^0}$$

R_A = 784.7 kN

14) A roller of radius 12 cm & weight W = 500 N is to be pulled over a kerb of height h = 6 cm by a horizontal force P applied to the end of a string wound around circumference of roller. Find the magnitude of P required to start roller over the kerb.



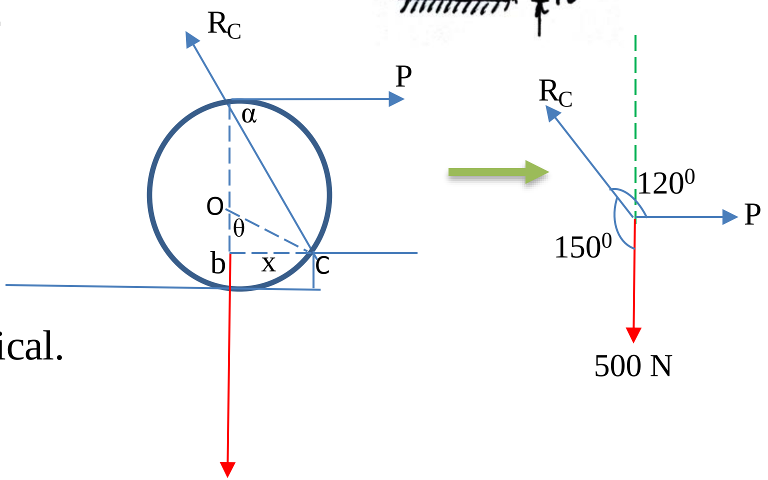
Solution: When the roller is about to turn over the kerb, the contact with the floor is lost and hence, there is no reaction from the Floor. The body is in equilibrium under the action of three forces namely

1. Applied force P, which is horizontal
2. Self weight.
3. Reaction R from the edge of the kerb.

Let θ be the angle made by the line CO with the vertical.

α is angle between R_C and horizontal.

FBD



From figure $\cos\theta = \frac{ob}{oc} = \frac{6}{12}$

Therefore $\theta = 60^0$

also, $bc = x = 12\sin\theta = 12\sin 60^0 = 10.39$ cm

$\tan\alpha = \frac{cf}{ef} = \frac{18}{10.39}$

$\alpha = 60^0$

Applying Lami's theorem:

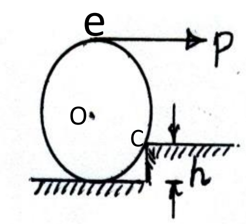
$$\frac{500}{\sin 120^0} = \frac{R_c}{\sin 90^0} = \frac{P}{\sin 150^0}$$

$$\frac{500}{\sin 120^0} = \frac{R_c}{\sin 90^0}$$

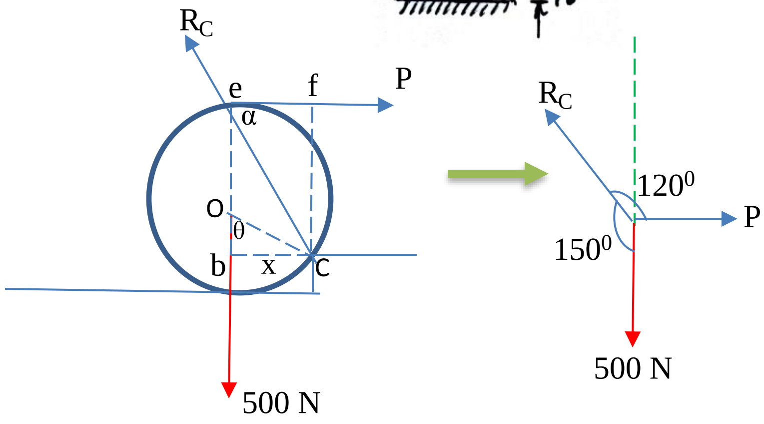
$R_C = 577.35$ N.

$$\frac{500}{\sin 120^0} = \frac{P}{\sin 150^0}$$

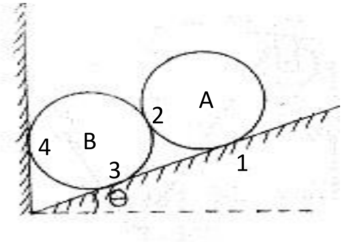
P = 288.67 N



FBD

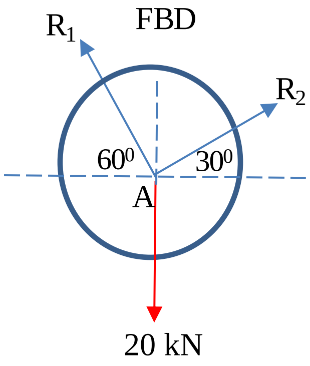


15) Two spheres of equal radius and each of weight 20 KN rest as shown in figure. Determine the reactions at all the points of contact. Take $\theta = 30^0$.



Solution: Let A & B be the centers of 2 spheres and 1, 2, 3, and 4 be the points of contacts at which reactions are required.

Consider the equilibrium of sphere A, its FBD is as shown.



Applying $\sum F_x = 0$

$R_2\cos30^0 - R_1\cos60^0 = 0$

$R_2 = 0.5773\ R_1$ ----------(i)

$\sum F_y = 0,$

$R_2\sin30^0 + R_1\sin60^0 - 20 = 0$

$0.866 + 0.5773\ R_1 x0.5\ -20 = 0$

$R_1 = 17.32$ kN.

From (i) $R_2 = 10$ kN

Considering equilibrium of sphere ‘B’, its FBD is as shown.

Applying $\sum F_y = 0$,

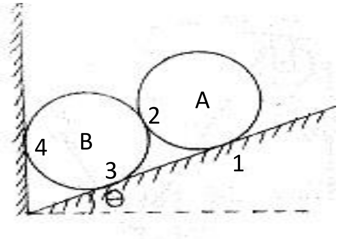
$R_3 \sin 60^0 - 20 - 10 \sin 30^0 = 0$

$R_3 = 28.87$ kN

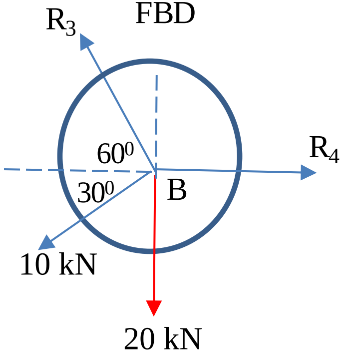
$\sum F_x = 0$,

$R_4 - 10 \cos 30^0 - R_3 \cos 60^0 = 0$

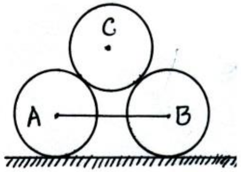
$R_4 = 29.095$ kN



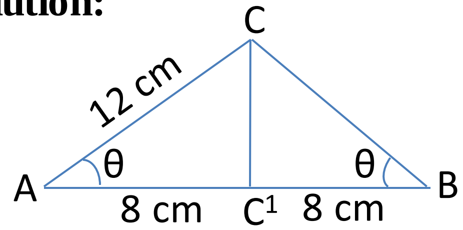
FBD



16) Two smooth circular cylinders each of weight 100 N & radius 6 cm are connected at their centers by a string AB of length 16 cm & rests upon a horizontal plane. Supporting above them a third cylinder of weight 200 N of same radius 6 cm. find the force in the string AB & the reactions at all the points of contact.



Solution:

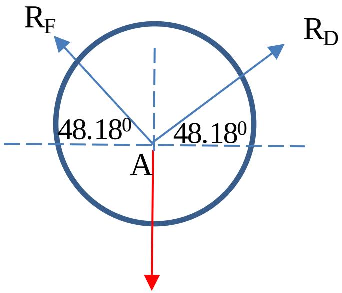


From the triangle ACC[1]

$\cos\theta = \frac{8}{12}$

$\theta = 48.18^0$

FBD of C



Considering the equilibrium of cylinder ‘C’, its FBD is as below.

Using Lami’s theorem

$$\frac{200}{\sin 138.18^0} = \frac{R_F}{\sin 138.18^0} = \frac{R_D}{\sin 83.64^0}$$

$$\frac{200}{\sin 138.18^0} = \frac{R_F}{\sin 138.18^0} = \frac{R_D}{\sin 83.64^0}$$

$$R_F = R_D = \frac{200 \sin 138.18^0}{\sin 83.64^0}$$

$R_F = R_D = 134.18$ N.

Consider cylinder A

Applying $\sum F_x = 0$,

$T_{AB} - 134.18\cos 48.18^0 = 0$

$T_{AB} = 89.47$ N

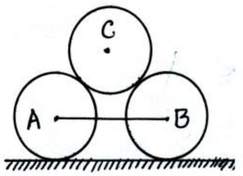
Applying $\sum F_y = 0$,

$R_E - 100 - 134.18\sin 48.18^0 = 0$

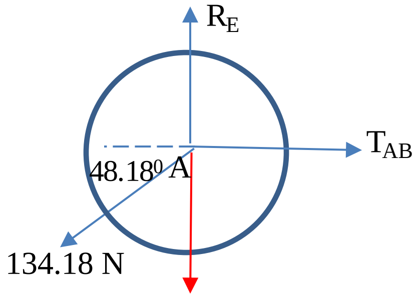
$R_E = 200$ N

Therefore $R_D = R_F = 134.18$ N

$R_E = R_G = 200$ N



FBD of A



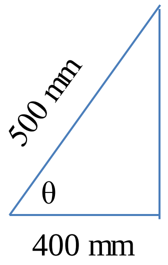
17) Two smooth spheres A & B each of weight W and radius ‘r’ are in equilibrium in a rectangular box as shown. Find the forces exerted at all the points of contact, if r = 250 mm, b = 900 mm and W = 100 N.

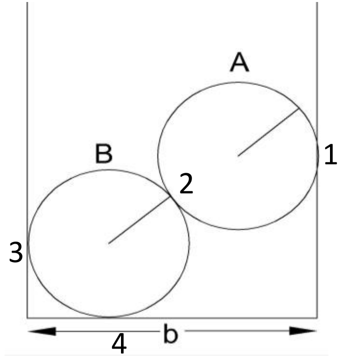
Solution:

From the triangle

$\cos\theta = \frac{400}{500}$

$\theta = 36.86^0$





Considering the equilibrium of sphere ‘A’, its FBD is as below.

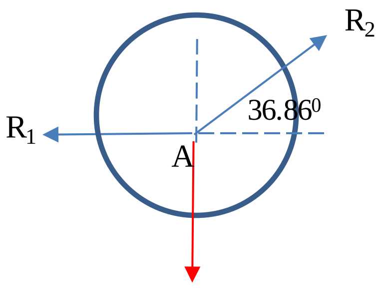
Using Lami’s theorem

$$\frac{100}{\sin 143.14^0} = \frac{R_1}{\sin 126.86^0} = \frac{R_2}{\sin 90^0}$$

$$\frac{100}{\sin 143.14^0} = \frac{R_1}{\sin 126.86^0}$$

$R_1 = 133.38$ N

FBD of C



$$\frac{100}{sin143.14^0}.=\frac{R_2}{sin90^0}$$

R_2= 166.70 N

Considering the equilibrium of sphere ‘B’, its FBD is as below.

Applying $\sum F_x = 0$,

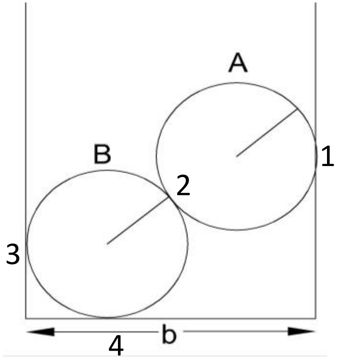
R_3 – 166.70cos36.86^0 = 0

R_3 = 133.37 N

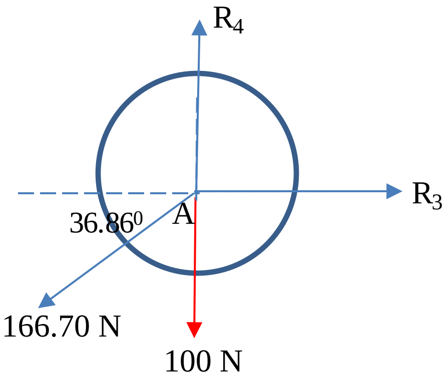
Applying $\sum F_y = 0$,

R_4 – 100 - 166.70sin36.86^0 = 0

R_4 = 200 N



FBD of B



Thank You