Chapter-1

MATHEMATICAL MODELING

There are many ways in which models can be described.

- Words
- Drawings or sketches
- Physical models
- Computer programs
- **Mathematical Models**.

What is Mathematical modeling?

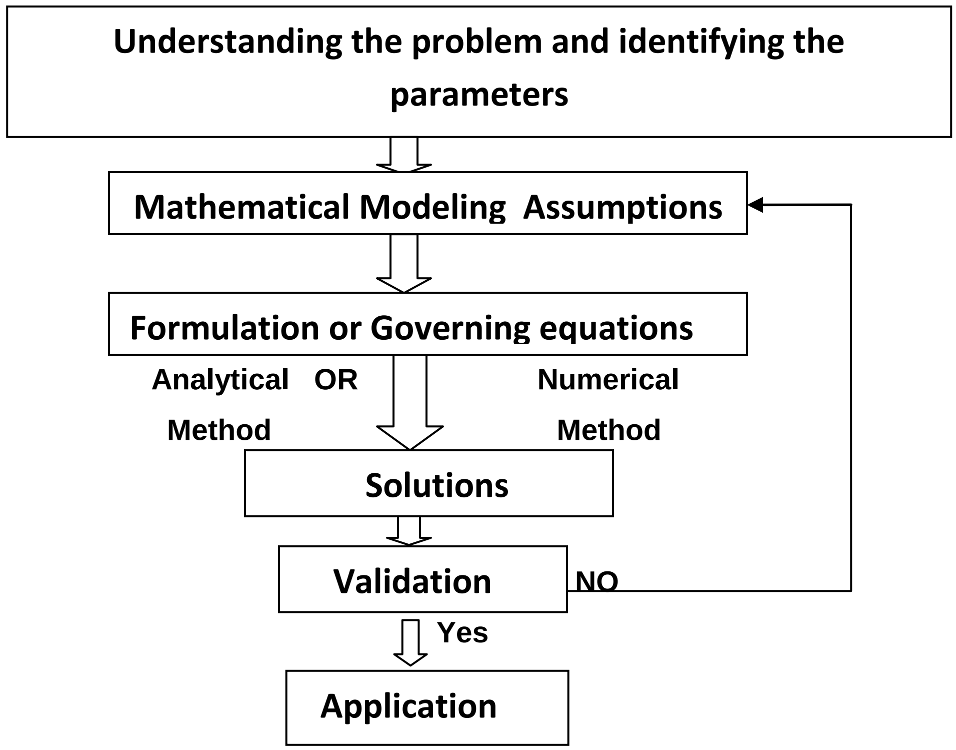
Mathematical Modeling is a process that uses mathematics

- To represent
- Analyze
- Make Prediction
- Provide insight into real-world phenomena.

Types of Mathematical Model:

1) **Theoretical Models :** Theoretical models are models that are based on first principles
and physical laws. Can be extrapolated to a wide variety of situations.
2) **Empirical Models :** Empirical models are models based on experimental data. These models are applicable in the areas with identical conditions as those in which the relationship was formulated.

Process of Mathematical Modeling



- The first step towards mathematical modeling is about understanding the problem and also identifying the parameters.

  (**In this step we analyze the problem and see which parameters have a major influence on the solution to the problem.)**

- The next step is to construct the basic framework of the model by making certain assumptions.

  (**In this step we state those parameters which are not essential and can be neglected**)

- If the assumptions are sufficiently precise, they may lead directly to the formulation or governing equation.

  "***In some cases the formulation itself is the solution***."

- If the formulation is not the solution then we apply analytical method to solve the equation.
- When analytical methods are unproductive we can use numerical methods to obtain the solution.

➢ After obtaining the solution we start testing the validity of the model by comparing the theoretical and practical results.

- If the model is valid then we move towards the application.
- If not we recheck our assumptions and repeat the steps until we get a valid model.

Example 1:

Build mathematical model for free falling parachutist of mass m kg jumps out of a stationary hot air balloon where drag co-efficient is c kg/s

Soln:

Understanding the problem: Find the velocity prior to opening the parachute.

Identifying the parameters: Forces acting on the body and mass.

Mathematical Modeling Assumptions : No horizontal force is acting on the body and mass of the parachute is negligible

Formulation or Governing equations:

By the Newton's second law : $F = ma$

F = net force acting on the body

m = mass of the object (kg)

a = its acceleration (m/s^2)

Since two forces acting on the body, upward force(F_u) and downward force(F_d)

Net force acting on the body is: $F = F_d + F_u$-------1

F_d = Force due to gravity = *mg*

F_u= Force due to air resistance = *-cv* (*-c = drag coefficient in opposite direction*)

Substituting in equation 1 we get

$$ma = mg - cv$$

$$m\frac{dv}{dt} = mg - cv$$

$$\boxed{\boldsymbol{\frac{dv}{dt} = g - \left(\frac{c}{m}\right) v}}$$

is a model for acceleration.

$$\frac{dv}{dt} + \left(\frac{c}{m}\right) v = g$$

Is a linear differential equation, solving by analytical method

$$I.F = e\char`^ \int \frac{c}{m} dt = e^{\left(\frac{c}{m}\right)t}$$

General solution is $v \times I.F = \int I.F \times g \, dt$

$$v \times e^{\left(\frac{c}{m}\right)t} = \int e^{\left(\frac{c}{m}\right)t} \times g \, dt$$

$v \times e^{\left(\frac{c}{m}\right)t} = g \frac{e^{\left(\frac{c}{m}\right)t}}{\frac{c}{m}} + k$ --------2 (Where k is integral constant)

$At \quad t = 0, v = 0 \implies k = \frac{-gm}{c}$ Substitute in equation in 2

And simplify we get velocity of the parachutist is the solution of the model

$$\boxed{\boldsymbol{v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)}}$$

Validation:

Consider parachutist of mass m=68.1 kg jumps out of a stationary hot air ballon. Find the velocity prior to opening the chute. The drag coefficient is c=12.5 kg/s

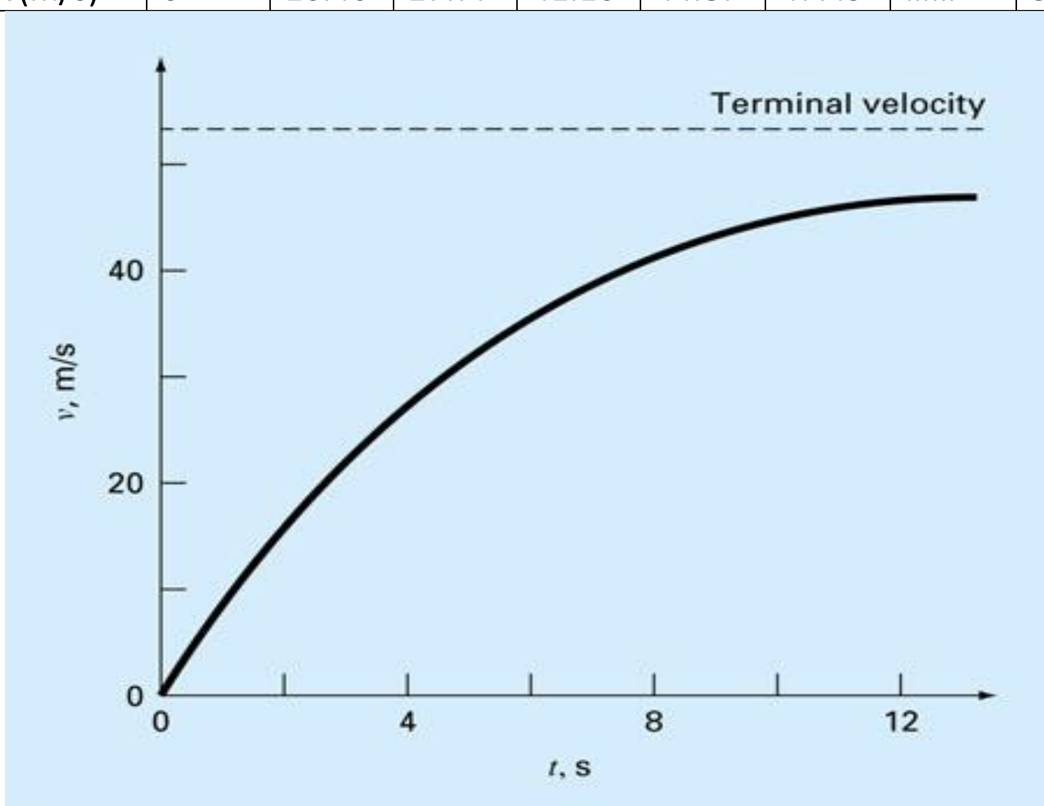
Soln: velocity is $v(t) = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right)$

By substituting $g = 9.8\ m/s^2$ $c = 12.5\ kg/s$ $m = 68.1\ kg$

We get velocity at any time t $\boxed{v(t) = 53.39(1 - e^{-0.18355t})}$

for different values of t velocities are calculated is shown in the table and same is depicted in the graph.

t(sec)	0	2	4	8	10	12	……	∞
v(m/s)	0	16.40	27.77	41.10	44.87	47.49	……	53.39



Conclusion: Table and graph tells us that after long time velocity remains constant that is 53.39 m/s is the terminal velocity.

Example 2:

Empirical model:

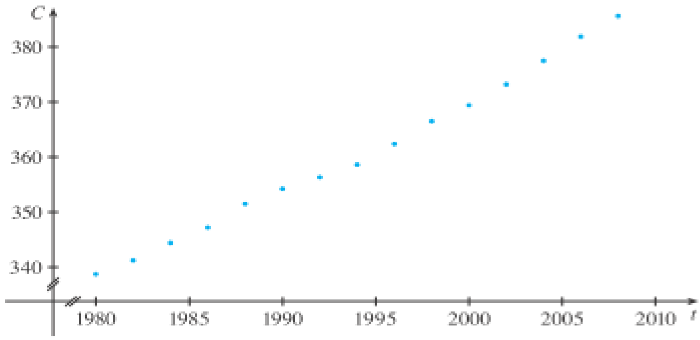
Table lists the average carbon dioxide level in the atmosphere, measured in parts per million at Mauna Loa Observatory from 1980 to 2008. Use the data in Table to find a model for the carbon dioxide level.

Year	CO_2 level (in ppm)	Year	CO_2 level (in ppm)
1980	338.7	1996	362.4
1982	341.2	1998	366.5
1984	344.4	2000	369.4
1986	347.2	2002	373.2
1988	351.5	2004	377.5
1990	354.2	2006	381.9
1992	356.3	2008	385.6
1994	358.6		

Soln:

Understanding the problem: from the given set of data we have to find the carbon dioxide level in the atmosphere

Identifying the parameters: Identify the relation between the two variables year and CO2 by putting scatter plot



Mathematical Modeling Assumptions: Assumptions will not come in the empirical model.

Formulation or Governing equations: As the data points appear to lie close to a straight line, so it's natural to choose a linear model in this case.

$$CO_2 = a(year) + b$$

Find slope "a" and CO2 intercept "b" using initial and final values we obtain

$$\boxed{CO_2 = 1.677(year) - 2977.8}$$

Is the model for CO2