$$\frac{\partial z}{\partial s} = e^{(s^2/t + 2s)} \cdot \left[\frac{2s+2t}{t}\right]$$

$$\frac{\partial z}{\partial t} = e^{(s^2/t+2s)} \cdot \left(\frac{s}{t}\right) + e^{(s^2/t+2s)} \cdot (s+2t) \cdot \left(\frac{-s}{t^2}\right)$$

$$= e^{(s^2/t+2s)} \cdot \left(\frac{s}{t}\right)\left[1 - \frac{s}{t} + 2\right]$$

$$= e^{(s^2/t+2s)} \left(\frac{s}{t}\right)\left(\frac{t-s+2t}{t}\right)$$

$$= e^{(s^2/t+2s)} \left(\frac{s}{t}\right) \cdot \left(\frac{3t-s}{t}\right)$$

(iii) $e^{r}\cos\theta = z, \quad r = st, \quad \theta = \sqrt{s^2+t^2}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial s}$$

$$= e^{r}\cos\theta . (t) + (-e^{r}\sin\theta) \frac{1}{\cancel{2}\sqrt{s^2+t^2}} \cdot \cancel{2}s$$

$$= e^{st}\cos(\sqrt{s^2+t^2}).(t) - \frac{e^{st}\sin(\sqrt{s^2+t^2}) \cdot s}{\sqrt{s^2+t^2}}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r} \cdot \frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta} \cdot \frac{\partial \theta}{\partial t}$$

$$= e^{r}\cos\theta . (s) + (-e^{r}\sin\theta) . \frac{1}{\cancel{2}\sqrt{s^2+t^2}} \cdot \cancel{2}t$$

$$= e^{st}\cos(\sqrt{s^2+t^2}) (s) - \frac{e^{st}\sin(\sqrt{s^2+t^2}) \cdot t}{\sqrt{s^2+t^2}}$$

4. Verify Clairaut's theorem:

(i) $u = x\sin(x+2y)$

$$\frac{\partial u}{\partial x} = x\cos(x+2y) + \sin(x+2y)$$

$$\frac{\partial^2 u}{\partial x.\partial y} = -x\,\cancel{co}\sin(x+2y).2 + \cos(x+2y).2$$

$$= 2\cos(x+2y) - 2x\sin(x+2y)$$

$$\frac{\partial u}{\partial y} = x\cos(x+2y).2$$

$$\frac{\partial^2 u}{\partial y.\partial x} = 2[-x\sin(x+2y) + \cos(x+2y)]$$

$$= 2\cos(x+2y) - 2x\sin(x+2y)$$

$$\therefore \frac{\partial^2 u}{\partial x.\partial y} = \frac{\partial^2 u}{\partial y.\partial x}$$

(ii) $u = x^4y^2 - 2xy^5$

$$\frac{\partial u}{\partial x} = 4x^3y^2 - 2y^5 \qquad \frac{\partial^2 u}{\partial x.\partial y} = 8x^3y - 10y^4$$

$$\frac{\partial u}{\partial y} = 2x^4y - 10xy^4 \qquad \frac{\partial^2 u}{\partial y.\partial x} = 8x^3y - 10y^4$$

$$\therefore \frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y.\partial x}$$

classmate

Date

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Use chain rule to find the indicated partial derivative.

$r = 2+1 = 3$ $\quad s = 1+0 = 1$

(i) $u = \sqrt{r^2+s^2}$, $r = y + x\cos t$, $s = x + y\sin t$;

$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial t}$ when $x=1$, $y=2$, $t=0$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial x} \qquad = \frac{1 \cdot 3 + 1}{\sqrt{10}} = \frac{4}{\sqrt{10}}$$

$$= \frac{1}{\cancel{2}\sqrt{r^2+s^2}} \cdot \cancel{2}r \cdot \cos t + \frac{1}{\cancel{2}\sqrt{r^2+s^2}} \cdot \cancel{2}s \cdot (1)$$

$$= \frac{\cos t \cdot r + s}{\sqrt{y^2+x^2\cos^2 t+2xy\cos t+x^2+y^2\sin^2 t+2xy\sin t}}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial y} + \frac{\partial u}{\partial s} \cdot \frac{\partial s}{\partial y}$$

$$= \frac{1}{2\sqrt{r^2+s^2}} \cdot (1) \cdot 2r + \frac{1}{2\sqrt{r^2+s^2}} \cdot (\sin t).$$

$$= \frac{\cancel{2} \times 3}{\cancel{2}\sqrt{10}} + \frac{1}{\cancel{2}\sqrt{10}} \times (0) \cdot \cancel{2} \times 1$$

$$= \frac{3}{\sqrt{10}}.$$

$r = 3 \quad s = 1 \quad t = 0 \qquad x = 1 \quad y = 2$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial r}\cdot\frac{\partial r}{\partial t} + \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial t}$$

$$= \frac{1}{\cancel{2}\sqrt{r^2+s^2}}\cdot \cancel{2}r.(-x\sin t) + \frac{1}{\cancel{2}\sqrt{r^2+s^2}}\cdot \cancel{2}s.(y\cos t)$$

$$= -1\times 0 + \frac{1}{\sqrt{10}}\cdot 1\times 2\times 1$$

$$= \underline{\underline{\frac{2}{\sqrt{10}}}}.$$

$v = 1$

$u = 3$

(ii) $R = \ln(u^2+v^2+w^2)$, $u = x+2y$, $v = 2x-y$

$w = 2xy$, $\frac{\partial R}{\partial x}$, $\frac{\partial R}{\partial y}$, when $x = y = 1$

$w = 2$

$$\Rightarrow \frac{\partial R}{\partial x} = \frac{\partial R}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial R}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial R}{\partial w}\cdot\frac{\partial w}{\partial x}$$

$$= \frac{1}{u^2+v^2+w^2}\cdot 2u\cdot(1) + \frac{1}{u^2+v^2+w^2}\cdot 2v\cdot(2) + \frac{1}{u^2+v^2+w^2}\cdot 2w.2y$$

$$= \frac{1}{9+1+4}\cdot 2\times 3 + \frac{1}{9+1+4}\cdot 2\times 1\cdot\times 2 + \frac{1}{9+1+4}\cdot 2\times 2\times 2\times 1$$

$$= \frac{6+4+8}{14} = \frac{18}{14} = \underline{\underline{\frac{9}{7}}}$$

$$\frac{\partial R}{\partial y} = \frac{\partial R}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial R}{\partial v}\cdot\frac{\partial v}{\partial y} + \frac{\partial R}{\partial w}\cdot\frac{\partial w}{\partial y}$$

$$= \frac{6}{14}\cdot 2 + \frac{2}{14}\cdot(-1) + \frac{4}{14}\cdot 2 = \frac{12-2+8}{14} = \frac{18}{14} = \underline{\underline{\frac{9}{7}}}$$

Find the differential of the following fcs

(i) $z = x^3 \ln y^2$

(ii) $v = y \cos xy$

(iii) $w = xy e^{xy}$

(i) $dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy$

$dz = 3x^2 \ln y^2 \cdot dx + \frac{x^3}{y^2} \cdot 2y \cdot dy$

$dz = 3x^2 \ln y^2 \cdot dx + \frac{2x^3}{y} \cdot dy$

(ii) $dv = \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy$

$= -y^2 \sin xy \cdot dx + [\cos xy + y(-\sin xy) \cdot x] dy$

$= -y^2 \sin xy \cdot dx + \cos xy \cdot dy - xy \sin xy \cdot dy$

(iii) $w = xy e^{xy}$

$dw = \frac{\partial w}{\partial x} \cdot dx + \frac{\partial w}{\partial y} \cdot dy$

$= xy e^{xy} \cdot y + \cancel{xy} e^{xy} \cancel{dx} + [xy \cdot e^{xy} \cdot x + x e^{xy}] \cdot dy$

$= xy^2 \cdot e^{xy} \cdot dx + y e^{xy} \cancel{dx} + x^2 y e^{xy} \cdot dy + x e^{xy} \cdot dy$