① $\int_{x=0}^{2}\int_{y=0}^{\pi/2} x \sin y . dx . dy$ $\qquad \frac{\pi^2(1-\cos 2)}{8}$

$= \int_0^2 x\left[-\cos y\right]_0^{\pi/2} . dx$

$= \int_0^2 x . dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} = 2$

② $\int_{y=1}^{2}\int_{x=y}^{2} x . y . dx\, dy$

$= \int_1^2 y . \left[\frac{x^2}{2}\right]_y^2 . dy = \int_1^2 y . \left[2 - \frac{y^2}{2}\right] . dy$

$= \int_1^2 2y - \frac{y^3}{2} . dy$

$= \left[\frac{2y^2}{2} - \frac{y^4}{4 \times 2}\right]_1^2$

$= \left[4 - \frac{\cancel{16}\,2}{\cancel{4 \times 2}}\right] - \left[\frac{\cancel{2}}{\cancel{2}} - \frac{1}{8}\right]$

$= \left[4 - 2\right] - \left[\frac{7}{8}\right]$

$= 2 - \frac{7}{8}$

$= \frac{17}{8} = \frac{9}{8}$

Ans

① $\displaystyle\int_{x=0}^{2}\int_{y=0}^{\pi/2} x \sin y . dx . dy$ $\qquad\qquad \dfrac{\pi^2(1-\cos 2)}{8}$

$= \displaystyle\int_0^2 x\left[-\cos y\right]_0^{\pi/2} . dx$

$= \displaystyle\int_0^2 x . dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} = 2$

② $\displaystyle\int_{y=1}^{2}\int_{x=y}^{2} x . y . dx\, dy$

$= \displaystyle\int_1^2 y . \left[\frac{x^2}{2}\right]_y^2 . dy = \int_1^2 y . \left[2 - \frac{y^2}{2}\right] . dy$

$= \displaystyle\int_1^2 2y - \frac{y^3}{2} . dy$

$= \left[\dfrac{2y^2}{2} - \dfrac{y^4}{4\times 2}\right]_1^2$

$= \left[4 - \dfrac{\cancel{16}}{\cancel{4\times 2}}\right] - \left[\dfrac{\cancel{2}}{\cancel{2}} - \dfrac{1}{8}\right]$

$= \left[4 - 2\right] - \left[\dfrac{7}{8}\right]$

$= 2 - \dfrac{7}{8}$

$= \dfrac{\cancel{17}}{\cancel{8}} = \dfrac{9}{8}$

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③. $\int_{y=2}^{4}\int_{x=1}^{7} (x^2+y^2)\, dx\, dy$

$= \int_{2}^{4}\left[\frac{x^3}{3} + xy^2\right]_1^7 . dy$

$= \int_{2}^{4}\left[\frac{343}{3} + 7y^2\right] - \left[\frac{1}{3} + y^2\right] . dy$

$= \int_{2}^{4}\left[\frac{342}{3} + 6y^2\right] . dy$

$= \left[\frac{\overset{114}{\cancel{342}} y}{\cancel{3}} + \frac{\overset{2}{\cancel{6}} y^3}{\cancel{3}}\right]_2^4$

$= [456 + 128] - [228 + 16]$

$= \underline{\underline{340}}.$

④. $\iint_R x\sin(x+y)\, dA, \quad R = \left[0, \frac{\pi}{6}\right] \times \left[0, \frac{\pi}{3}\right]$

$= \int_0^{\pi/3}\int_0^{\pi/6} x \sin(x+y) . dx . dy$

$= \int_0^{\pi/3} x\left[-\cos(x+y)\right]_0^{\pi/6} . dx$

$= \int_0^{\pi/3} x$

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⑤. $\iint_R x^3y^2.dA \quad R = \{(x,y) \mid 0 \le x \le 2, -x \le y \le x\}$

$$= \int_{x=2}^{10}\int_{y=-x}^{x} x^3.y^2.dx.dy.$$

$$= \int_2^{10} x^3\left[\frac{y^3}{3}\right]_{-x}^{x}.dx$$

$$= \int_2^{10} x^3\left[\frac{x^3}{3}+\frac{x^3}{3}\right].dx$$

$$= \int_2^{10} x^3 \times \frac{2x^3}{3}.dx$$

$$= \frac{2}{3}\left[\frac{x^7}{7}\right]_{0}^{2}$$

$$= \frac{2}{3} \times \frac{128}{7} = \frac{256}{21}.$$

⑥. $\int_2^4\int_{-1}^1 (x^2+y^2).dy.dx$

$$= \int_2^4\left[\frac{x^3}{3}+xy^2\right]_{-1}^{1}.dy.$$

$$= \int_2^4\left[\frac{1}{3}+1y^2\right]-\left[\frac{-1}{3}-y^2\right].dy$$

$$= \int_2^4 \left(\frac{1}{3} + \frac{1}{3} + 2y^2\right).dy$$

$$= \left[\frac{2}{3}y + \frac{2y^3}{3}\right]_2^4$$

$$= \left(\frac{2\times4}{3} + \frac{64\times2}{3}\right) - \left[\frac{4}{3} + \frac{16}{3}\right]$$

$$= \frac{8}{3} + \frac{128}{3} - \frac{4}{3} - \frac{16}{3}$$

$$= -\frac{12}{3} + \frac{128}{3} = \frac{116}{3}.$$

7. $$\int_0^3 \int_1^2 (x^2y).dy.dx$$

$$= \int_0^3 \left[\frac{x^3y}{3}\right]_1^2 .dy$$

$$= \int_0^3 y.\left[\frac{8}{3} - \frac{1}{3}\right].dy$$

$$= \frac{7}{3}\left[\frac{y^2}{2}\right]_0^3 = \frac{7}{\cancel{3}} \times \frac{\cancel{9}^{3}}{2} = \frac{21}{2}$$

Ans $\frac{27}{2}$

$$\int_{\theta=0}^{\pi/2}\int_{r=0}^{4\cos\theta} r.dr.d\theta.$$

$$= \int_0^{\pi/2} \left[\frac{r^2}{2}\right]_0^{4\cos\theta} . d\theta$$

$$= 16\int_0^{\pi/2} \frac{\cos^2\theta}{2} . d\theta$$

$$= \frac{16}{2}\int_0^{\pi/2} \frac{1+\cancel{2}\cos 2\theta}{2} . d\theta$$

$$= \frac{16}{4}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \frac{16}{4}\left[\frac{\pi}{2} + 0\right] = \frac{\pi \times \cancel{16}^{2}}{\cancel{8}} = 2\pi$$

$$\int_0^{\pi/2}\int_0^{\pi/2} \sin x \cos y . dy . dx.$$

$$= \int_0^{\pi/2} \cos y \left[-\cos x\right]_0^{\pi/2} dy$$

$$= \int_0^{\pi/2} \cos y \left[1\right] . dy$$

$$= \left[\sin y\right]_0^{\pi/2} = 1$$

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$$\iint_R x\sin(x+y)\,dA, \quad R=\left[0,\frac{\pi}{6}\right]\times\left[0,\frac{\pi}{3}\right]$$

$$= \int_0^{\pi/6}\int_0^{\pi/3} x\sin(x+y).dx.dy$$

$$\int_0^{\pi/2}\int_{r=0}^{\cos\theta} e^{\sin\theta}.dr.d\theta$$

$$= \int_0^{\pi/2} e^{\sin\theta}.\left[r\right]_0^{\cos\theta}.d\theta.$$

$$= \int_0^{\pi/2} e^{\sin\theta}.\cos\theta.d\theta \qquad \int f'(x)\,e^{f(x)}.dx$$

$$= \left[e^{\sin\theta}\right]_0^{\pi/2} \qquad = e^{f(x)}+C.$$

$$= e^{(\sin\pi/2)} - e^{(\sin 0)} \qquad = e^{1}-1 = e-1$$

$$= e-1.$$

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(4). $\int_0^{\pi/3}\int_0^{\pi/6} x\sin(x+y)\,dx.dy.$

$= \int_0^{\pi/3} \left[x(-\cos(x+y)) - 1(-\sin(x+y))\right]_0^{\pi/6}.dy.$

$= \int_0^{\pi/3} \left[-\frac{\pi}{6}\cos\left(y+\frac{\pi}{6}\right) + \sin\left(y+\frac{\pi}{6}\right)\right] - \sin y$

$\left(= -\frac{\pi}{6}\sin\left(y+\frac{\pi}{6}\right) + \left(-\cos\left(y+\frac{\pi}{6}\right)\right)\right)_0^{\pi/3}$

$= \left[-\frac{\pi}{6}(1-0) - \left(\frac{-\pi}{6}\right)\right) \times$

$= \int_0^{\pi/3} \left[\frac{-\pi}{6}\cos\left(y+\frac{\pi}{6}\right) + \sin\left(y+\frac{\pi}{6}\right) - \sin y\right]_0^{\pi/} dy$

$= -\frac{\pi}{6}(1) + \frac{1}{2} - \left(\frac{-\pi}{6}\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2} + 1\right)$

$= \frac{-\pi}{12} + \left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}$