Electric Potential

Electric Potential and Potential Difference

When a charge q is placed in an electric field $\vec{E}$, created by some source charge distribution, the particle in a field model tells us that there is an electric force q$\vec{E}$ acting on the charge. This force is conservative because the force between charges described by Coulomb's law is conservative. Let us identify the charge and the field as a system. If the charge is free to move, it will do so in response to the electric force. Therefore, the electric field will be doing work on the charge. This work is internal to the system. This situation is similar to that in a gravitational system: When an object is released near the surface of the Earth, the gravitational force does work on the object. This work is internal to the object–Earth system.

When analyzing electric and magnetic fields, it is common practice to use the notation $\overrightarrow{ds}$ to represent an infinitesimal displacement vector that is oriented tangent to a path through space. This path may be straight or curved, and an integral performed along this path is called either a *path integral* or a *line integral* (the two terms are synonymous).

For an infinitesimal displacement $\overrightarrow{ds}$ of a point charge *q* immersed in an electric field, the work done within the charge field system by the electric field on the charge is

$$W_{\text{int}} = \overrightarrow{F_e} \cdot \overrightarrow{ds} = q\vec{E} \cdot \overrightarrow{ds} .$$

Internal work done in a system is equal to the negative of the change in the potential energy of the system: $W_{\text{int}} = -\Delta U$. Therefore, as the charge *q* is displaced, the electric potential energy of the charge field system is changed by an amount $\Delta U = -W_{int} = -q\vec{E} \cdot \overrightarrow{ds}$. For a finite displacement of the charge from some point A in space to some other point B, the change in electric potential energy of the system is

$$\Delta U = -q\int_A^B \vec{E} \cdot \overrightarrow{ds} \text{ ----------------(1)}$$

The integration is performed along the path that *q* follows as it moves from A to B. Because the force $q\vec{E}$ is conservative, this line integral does not depend on the path taken from A to B.

For a given position of the charge in the field, the charge–field system has a potential energy U relative to the configuration of the system that is defined as U = 0. Dividing the potential energy by the charge gives a physical quantity that depends only on the source charge distribution and has a value at every point in an electric field. This quantity is called the **electric potential** (or simply the **potential**) V:

$$V = \frac{U}{q} \text{ ------------(2)}$$

Because potential energy is a scalar quantity, electric potential also is a scalar quantity. The **potential difference** $\Delta V = V_B - V_A$ between two points A and B in an electric field is defined as the change in electric potential energy of the system when a charge q is moved between the points (Eq. 1) divided by the charge:

$$V = \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot \vec{ds} \qquad \text{---------------(3)}$$

In this definition, the infinitesimal displacement $\vec{ds}$ is interpreted as the displacement between two points in space rather than the displacement of a point charge as in Equation 1.

Just as with potential energy, only *differences* in electric potential are meaningful. We often take the value of the electric potential to be zero at some convenient point in an electric field.

Potential difference should not be confused with difference in potential energy. The potential *difference* between A and B exists solely because of a source charge and depends on the source charge distribution (consider points A and B in the discussion above *without* the presence of the charge q). For a potential *energy* to exist, we must have a system of two or more charges. The potential energy belongs to the system and changes only if a charge is moved relative to the rest of the system. This situation is similar to that for the electric field. An electric *field* exists solely because of a source charge. An electric *force* requires two charges: the source charge to set up the field and another charge placed within that field.

Let's now consider the situation in which an external agent moves the charge in the field. If the agent moves the charge from A to B without changing the kinetic energy of the charge, the agent performs work that changes the potential energy of the system: $W = \Delta U$. From Equation 3, the work done by an external agent in moving a charge q through an electric field at constant velocity is

$$W = q\Delta V \text{ -------------(4)}$$

Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a **volt** (V):

$$1 \text{ V} = 1 \text{ J/C}$$

That is, as we can see from Equation 4, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

Equation 3 shows that potential difference also has units of electric field times distance. It follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1 \text{ N/C} = 1 \text{ V/m}$$

Therefore, we can state a new interpretation of the electric field:

The electric field is a measure of the rate of change of the electric potential with respect to position.

A unit of energy commonly used in atomic and nuclear physics is the **electron volt** (eV), which is defined as the energy a charge–field system gains or loses when a charge of magnitude *e* (that is, an electron or a proton) is moved through a potential difference of 1 V. Because 1 V = 1 J/C and the fundamental charge is equal to 1.60×10^{-19} C, the electron volt is related to the joule as follows:

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ C. V} = 1.60 \times 10^{-19} \text{ J} \quad \text{----------------(5)}$$

For instance, an electron in the beam of a typical dental x-ray machine may have a speed of 1.4×10^{8} m/s. This speed corresponds to a kinetic energy 1.1×10^{-14} J, which is equivalent to 6.7×10^{4} eV. Such an electron has to be accelerated from rest through a potential difference of 67 kV to reach this speed.

Potential Difference in a Uniform Electric Field

Equations 1 and 3 hold in all electric fields, whether uniform or varying, but they can be simplified for the special case of a uniform field. First, consider a uniform electric field directed along the negative *y* axis as shown in Figure 1a. Let's calculate the potential difference between two points A and B separated by a distance *d*, where the displacement S points from A toward B and is parallel to the field lines. Equation 3 gives

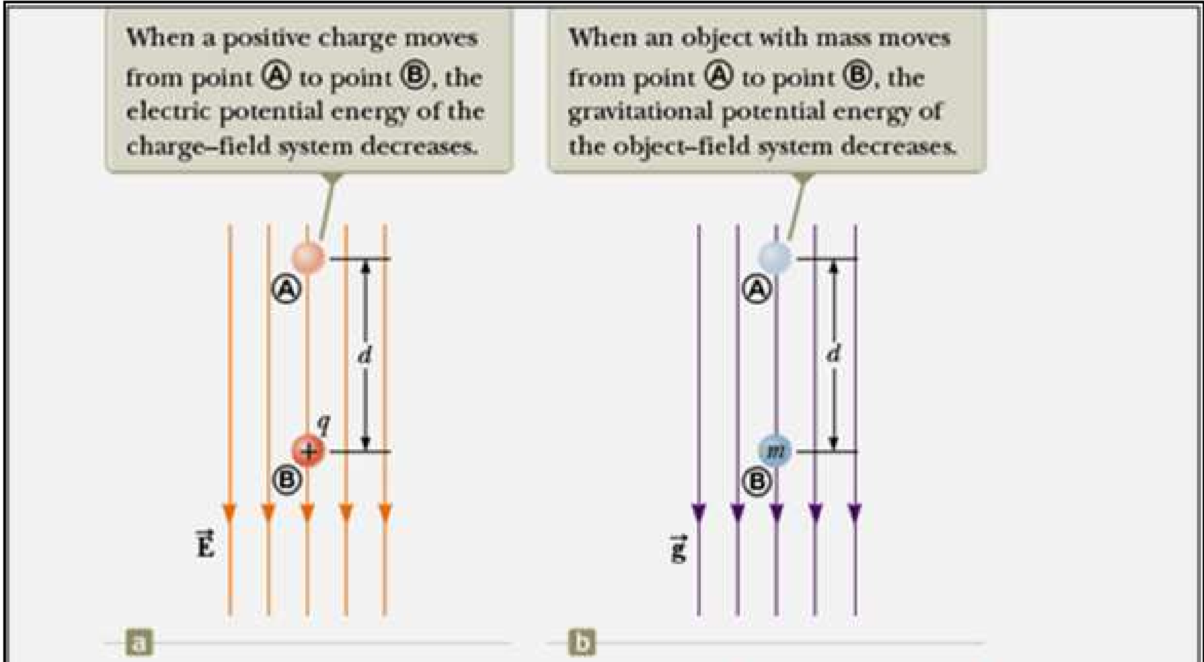


Figure 1. *(a) When the electric field is directed downward, point B is at a lower electric potential than point A. (b) A gravitational analogue to the situation* in (a).

$$V_B - V_A = \Delta V = -\int_A^B \vec{E}\cdot\vec{ds} = -\int_A^B E\cdot ds(\cos 0^0) = -\int_A^B E\cdot ds$$

Because E is constant, it can be removed from the integral sign, which gives

$$\Delta V = -E\int_A^B ds$$

$$\boxed{\Delta V = -Ed} \text{-------------------(6)}$$

The negative sign indicates that the electric potential at point B is lower than at point A; that is, $V_B < V_A$. Electric field lines *always* point in the direction of decreasing electric potential as shown in Figure 1a. Now suppose a charge q moves from A to B. We can calculate the change in the potential energy of the charge–field system from Equations 3 and 6:

$$\Delta U = q\Delta V = -qEd \text{ ----------------(7)}$$

This result shows that if q is positive, then ΔU is negative. Therefore, in a system consisting of a positive charge and an electric field, the electric potential energy of the system decreases when the charge moves in the direction of the field. If a positive charge is released from rest in this electric field, it experiences an electric force $q\vec{E}$ in the direction of $\vec{E}$ (downward in Fig. 1a).

Therefore, it accelerates downward, gaining kinetic energy. As the charged particle gains kinetic energy, the electric potential energy of the charge field system decreases by an equal amount. This equivalence should not be surprising; it is simply conservation of mechanical energy in an isolated system.

Figure 1b shows an analogous situation with a gravitational field. When a particle with mass *m* is released in a gravitational field, it accelerates downward, gaining kinetic energy. At the same time, the gravitational potential energy of the object–field system decreases.

The comparison between a system of a positive charge residing in an electrical field and an object with mass residing in a gravitational field in Figure 1 is useful for conceptualizing electrical behaviour. The electrical situation, however, has one feature that the gravitational situation does not: the charge can be negative. If q is negative, then ΔU in Equation 7 is positive and the situation is reversed.

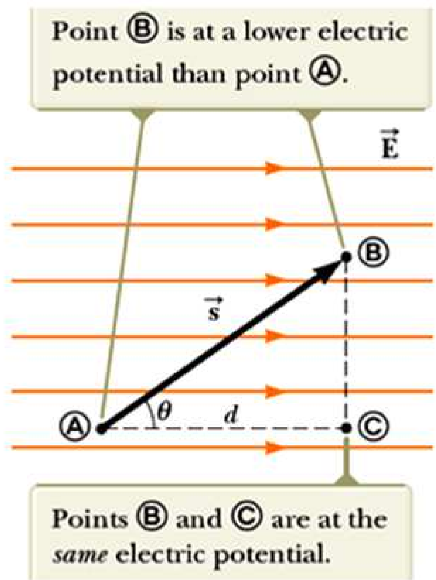


Figure 2. *A uniform electric field directed along the positive x axis. Three points in the electric field are labelled.*

A system consisting of a negative charge and an electric field *gains* electric potential energy when the charge moves in the direction of the field. If a negative charge is released from rest in an electric field, it accelerates in a direction *opposite* the direction of the field. For the negative charge to move in the direction of the field, an external agent must apply a force and do positive work on the charge. Now consider the more general case of a charged particle that moves between A and B in a uniform electric field such that the vector $\vec{S}$ is *not* parallel to the field lines as shown in Figure 2. In this case, Equation 3 gives

$$\Delta V = -\int_A^B \vec{E} \cdot \vec{ds} = -\vec{E} \cdot \int_A^B \vec{ds} = -\vec{E} \cdot \vec{s} \text{------------}(8)$$

where again $\vec{E}$ was removed from the integral because it is constant. The change in potential energy of the charge field system is

$$\Delta U = q\Delta V = -q\vec{E} \cdot \vec{s} \text{------------------}(9)$$

Finally, we conclude from Equation 8 that all points in a plane perpendicular to a uniform electric field are at the same electric potential. We can see that in Figure 2, where the potential difference V_B - V_A is equal to the potential difference V_C - V_A. Therefore, $V_B = V_C$. The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.

Electric Potential and Potential Energy Due to Point Charges

An isolated positive point charge q produces an electric field directed radially outward from the charge. To find the electric potential at a point located a distance r from the charge, let's begin with the general expression for potential difference, Equation 3,

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{ds}$$

where A and B are the two arbitrary points shown in Figure 3. At any point in space, the electric field due to the point charge is $\vec{E} = \left(k_e q / r^2 \right) \hat{r}$ (Eq. 9), where $\hat{r}$ is a unit vector directed radially outward from the charge. Therefore, the quantity $\vec{E} \cdot \vec{ds}$ can be expressed as

$$\vec{E} \cdot \vec{ds} = k_e \frac{q}{r^2} \hat{r} \cdot \vec{ds}$$

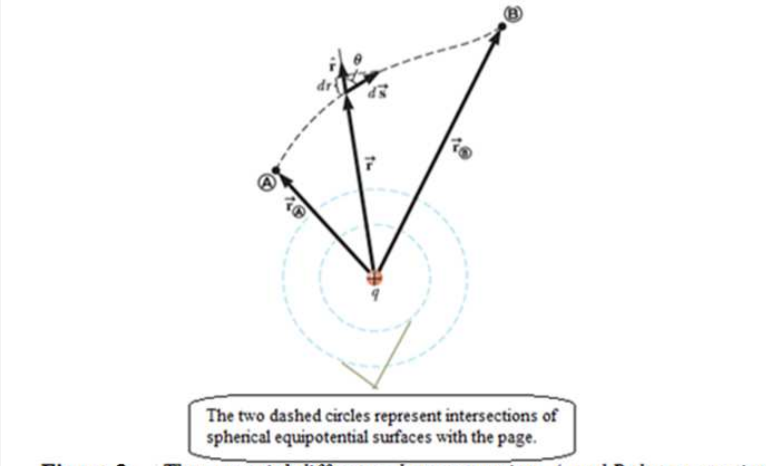


Figure 3. *The potential difference between points A and B due to a point charge q depends only on the initial and final radial coordinates r_A and r_B.*

Because the magnitude of $\hat{r}$ is 1, the dot product $\hat{r}\cdot\vec{ds} = ds\cos\theta$, where θ is the angle between $\hat{r}$ and $\vec{ds}$. Furthermore, $ds\cos\theta$ is the projection of $\vec{ds}$ onto $\hat{r}$; therefore, $ds\cos\theta = dr$. That is, any displacement $\vec{ds}$ along the path from point A to point B produces a change dr in the magnitude of $\vec{r}$, the position vector of the point relative to the charge creating the field. Making these substitutions, we find that $\vec{E}\cdot\vec{ds} = \left(k_e q / r^2\right)\vec{dr}$; hence, the expression for the potential difference becomes

$$V_B - V_A = -k_e q\int_{r_A}^{r_B}\frac{dr}{r^2} = k_e \frac{q}{r}\Big]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \text{----------------(10)}$$

Equation 10 shows us that the integral of $\vec{E}\cdot\vec{ds}$ is *independent* of the path between points A and B. Multiplying by a charge q_0 that moves between points A and B, we see that the integral of $q_0\vec{E}\cdot\vec{ds}$ is also independent of path. This latter integral, which is the work done by the electric force on the charge q_0, shows that the electric force is conservative. We define a field that is related to a conservative force as a **conservative field.** Therefore, Equation 10 tells us that the electric field of a fixed point charge q is conservative. Furthermore, Equation 10 expresses the important result that the potential difference between any two points A and B in a field created by a point charge depends only on the radial coordinates r_A and r_B. It is customary to choose the reference of electric potential for a point charge to be V =0 at r_A =x. With this reference choice, the electric potential due to a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \text{------------------(11)}$$

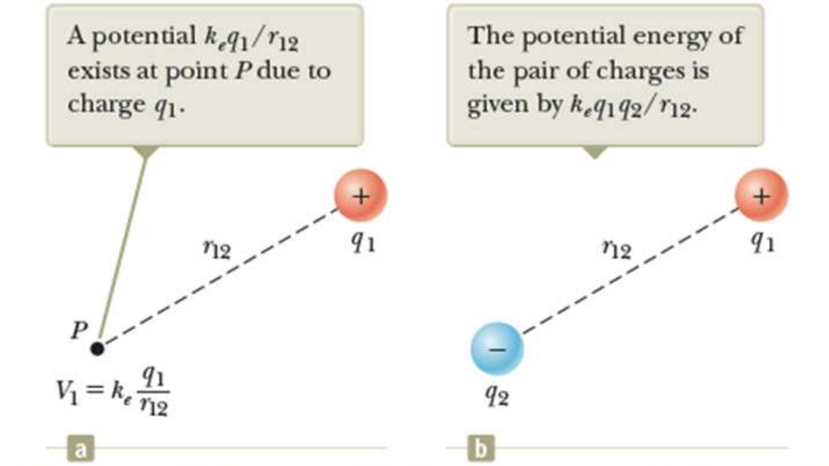


Figure 4. *(a) Charge q_1 establishes an electric potential V_1 at point P.*
(b) Charge q_2 is brought from infinity to point P.

We obtain the electric potential resulting from two or more point charges by applying the superposition principle. That is, the total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at P as

$$V = k_e \sum_i \frac{q_i}{r_i} \text{ --------------(12)}$$

Figure 4a shows a charge q_1, which sets up an electric field throughout space. The charge also establishes an electric potential at all points, including point P, where the electric potential is V_1. Now imagine that an external agent brings a charge q_2 from infinity to point P. The work that must be done to do this is given by Equation 4, $W = q_2 \Delta V$. This work represents a transfer of energy across the boundary of the two-charge system, and the energy appears in the system as potential energy U when the particles are separated by a distance r_{12} as in Figure 4b. From Equation 4, we have $W = \Delta U$. Therefore, the **electric potential energy** of a pair of point charges can be found as follows:

$$\Delta U = W = q_2 \Delta V \qquad \rightarrow \quad U - 0 = q_2 \left(k_e \frac{q_1}{r_{12}} - 0 \right)$$

$$U = k_e \frac{q_1 q_2}{r_{12}} \qquad \text{-----------------(13)}$$

If the charges are of the same sign, then U is positive. Positive work must be done by an external agent on the system to bring the two charges near each other (because charges of the same sign repel). If the charges are of opposite sign, as in Figure 4b, then U is negative. Negative work is done by an external agent against the attractive force between the charges of opposite sign as they are brought near each other; a force must be applied opposite the displacement to prevent q_2 from accelerating toward q_1.

If the system consists of more than two charged particles, we can obtain the total potential energy of the system by calculating U for every *pair* of charges and summing the terms algebraically. For example, the total potential energy of the system of three charges shown in Figure 5 is

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \text{----------------(14)}$$

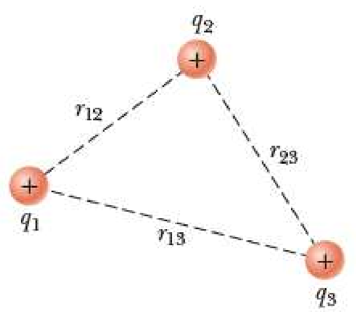


Figure 5 Three point charges are fixed at the positions shown.

Physically, this result can be interpreted as follows. Imagine q_1 is fixed at the position shown in Figure 5 but q_2 and q_3 are at infinity. The work an external agent must do to bring q_2 from infinity to its position near q_1 is $U = k_e \left(\frac{q_1 q_2}{r_{12}} \right)$, which is the first term in Equation 14. The last two terms represent the work required to bring q_3 from infinity to its position near q_1 and q_2. (The result is independent of the order in which the charges are transported.)

Obtaining the Value of the Electric Field from the Electric Potential

The electric field $\vec{E}$ and the electric potential *V* are related as shown in Equation 3, which tells us how to find ΔV if the electric field $\vec{E}$ is known. What if the situation is reversed? How do we calculate the value of the electric field if the electric potential is known in a certain region?

From Equation 3, the potential difference *dV* between two points a distance *ds* apart can be expressed as

$$\Delta V = -\vec{E}\cdot\vec{ds} \text{-----------------(15)}$$

If the electric field has only one component E_x, then $\vec{E}\cdot\vec{ds} = \vec{E_x}\cdot\vec{dx}$. Therefore, Equation 15 becomes $dV = \vec{E_x}\cdot\vec{dx}$, or

$$\vec{E_x} = -\frac{dV}{dx}\cdot \text{ --------------(16)}$$

That is, the *x* component of the electric field is equal to the negative of the derivative of the electric potential with respect to *x*. Similar statements can be made about the *y* and *z* components. Equation 16 is the mathematical statement of the electric field being a measure of the rate of change with position of the electric potential.

Experimentally, electric potential and position can be measured easily with a voltmeter (a device for measuring potential difference) and a meter stick. Consequently, an electric field can be determined by measuring the electric potential at several positions in the field and making a graph of the results. According to Equation 16, the slope of a graph of *V* versus *x* at a given point provides the magnitude of the electric field at that point.

Imagine starting at a point and then moving through a displacement *d* **s**S along an equipotential surface. For this motion, *dV* = 0 because the potential is constant along an equipotential surface. From Equation 15, we see that $dV = \vec{E}\cdot\vec{ds} = 0$; therefore, because the dot product is zero, $\vec{E}$ must be perpendicular to the displacement along the equipotential surface. This result shows that the equipotential surfaces must always be perpendicular to the electric field lines passing through them. The equipotential surfaces associated with a uniform electric field consist of a family of planes perpendicular to the field lines. Figure 6a shows some representative equipotential surfaces for this situation.

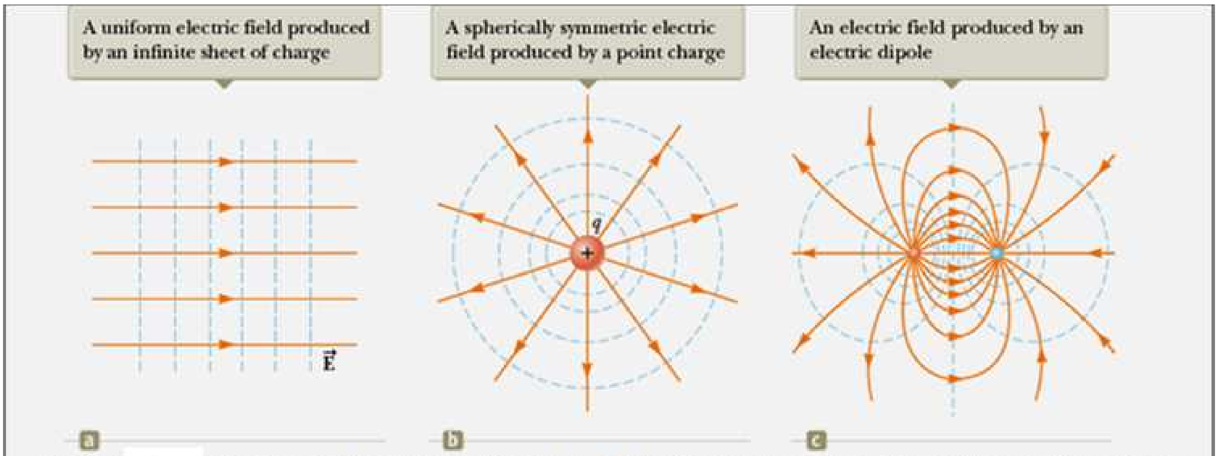


Figure 6. *Equipotential surfaces (the dashed blue lines are intersections of these surfaces with the page) and electric field lines. In all cases, the equipotential surfaces are perpendicular to the electric field lines at every point.*

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance r, the electric field is radial. In this case, $\vec{E}\cdot\vec{ds} = E_r dr$, and we can express dV as $dV = -E_r \cdot dr$. Therefore,

$$E_r = -\frac{dV}{dr} \quad \text{---------------------(17)}$$

For example, the electric potential of a point charge is $V = k_e \, q/r$. Because V is a function of r only, the potential function has spherical symmetry. Applying Equation 17, we find that the magnitude of the electric field due to the point charge is $E_r = k_e \, q/r^2$, a familiar result. Notice that the potential changes only in the radial direction, not in any direction perpendicular to r. Therefore, V (like E_r) is a function only of r, which is again consistent with the idea that equipotential surfaces are perpendicular to field lines. In this case, the equipotential surfaces are a family of spheres concentric with the spherically symmetric charge distribution (Fig. 6b).

The equipotential surfaces for an electric dipole are sketched in Figure 6c. In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the Cartesian coordinates, the electric field components E_x, E_y, and E_z can readily be found from $V(x, y, z)$ as the partial derivatives

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z} \quad \text{------------------(18)}$$

Electric Potential Due to Continuous Charge Distributions

we found how to determine the electric potential due to a small number of charges. What if we wish to find the potential due to a continuous distribution of charge? The electric potential in this situation can be calculated using two different methods. The first method is as follows. If the charge distribution is known, we consider the potential due to a small charge element *dq*, treating this element as a point charge (Fig. 7). From Equation 11, the electric potential *dV* at some point *P* due to the charge element *dq* is

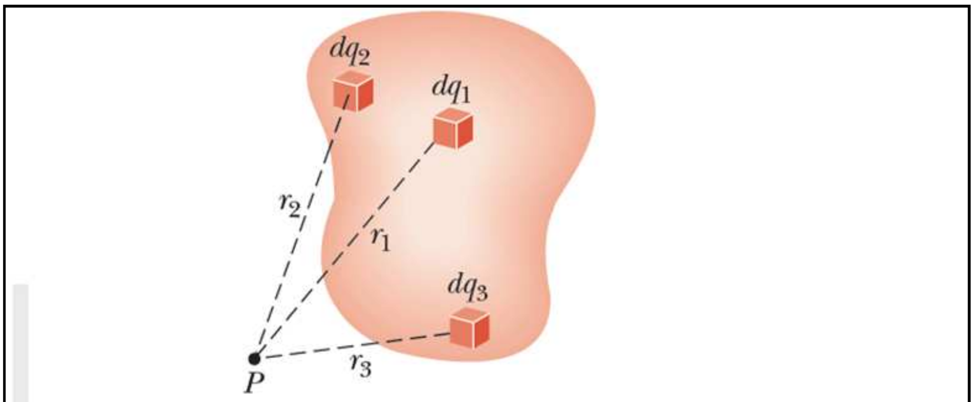


Figure 7. *The electric potential at point P due to a continuous charge distribution can be calculated by dividing the charge distribution into elements of charge dq and summing the electric potential contributions over all elements. Three sample elements of charge are shown.*

$$dV = k_e \frac{dq}{r} \text{ --------------(19)}$$

where *r* is the distance from the charge element to point *P*. To obtain the total potential at point *P*, we integrate Equation 19 to include contributions from all elements of the charge distribution. Because each element is, in general, a different distance from point *P* and k_e is constant, we can express *V* as

$$dV = k_e \int \frac{dq}{r} \text{ -------------(20)}$$

In effect, we have replaced the sum in Equation 12 with an integral. In this expression for *V*, the electric potential is taken to be zero when point *P* is infinitely far from the charge distribution.

The second method for calculating the electric potential is used if the electric field is already known from other considerations such as Gauss's law. If the charge distribution has sufficient symmetry, we first evaluate $\vec{E}$ using Gauss's law and then substitute the value obtained into Equation 3 to determine the potential difference ΔV between any two points. We then choose the electric potential *V* to be zero at some convenient point.

The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in Figure 8. The dipole is along the x axis and is centered at the origin.

(A) Calculate the electric potential at point P on the y axis.

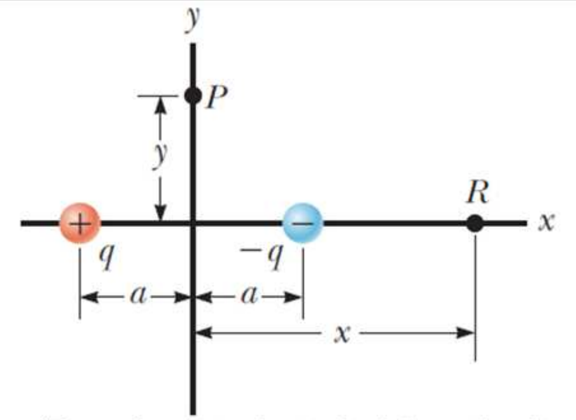


Figure 8. *(Example .1) An electric dipole located on the x axis.*

Solution:

Conceptualize Compare this situation to that in part (B) of Example 1. It is the same situation, but here we are seeking the electric potential rather than the electric field.

Categorize We categorize the problem as one in which we have a small number of particles rather than a continuous distribution of charge. The electric potential can be evaluated by summing the potentials due to the individual charges.

Analyze Use Equation 12 to find the electric potential at P due to the two charges:

$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{q}{\sqrt{a^2 + y^2}} - \frac{-q}{\sqrt{a^2 + y^2}} \right) = 0$$

(B) Calculate the electric potential at point R on the positive x axis.

Solution:

Use Equation 12 to find the electric potential at R due to the two charges:

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e \left(\frac{-q}{\sqrt{a^2 + y^2}} - \frac{q}{\sqrt{a^2 + y^2}} \right) = \frac{2k_e qa}{x^2 - a^2}$$

(C) Calculate *V* and *Ex* at a point on the *x* axis far from the dipole.

Solution:

For point *R* far from the dipole such that $x >> a$, neglect a^2 in the denominator of the answer to part (B) and write *V* in this limit:

$$V_R = \lim_{x \rangle\rangle a} \left(-\frac{2k_e qa}{x^2 - a^2} \right) \approx -\frac{2k_e qa}{x^2} \qquad (x \rangle\rangle a)$$

Use Equation 16 and this result to calculate the *x* component of the electric field at a point on the *x* axis far from the dipole:

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2} \right)$$

$$= 2k_e qa \frac{d}{dx}\left(\frac{1}{x^2} \right) = -\frac{4k_e qa}{x^3} \qquad (x \rangle\rangle a)$$

Finalize The potentials in parts (B) and (C) are negative because points on the positive *x* axis are closer to the negative charge than to the positive charge. For the same reason, the *x* component of the electric field is negative. Notice that we have a $1/r^3$ falloff of the electric field with distance far from the dipole, similar to the behavior of the electric field on the *y* axis in Example 1.

What If?

Suppose you want to find the electric field at a point *P* on the *y* axis. In part (A), the electric potential was found to be zero for all values of *y*. Is the electric field zero at all points on the *y* axis?

Answer No. That there is no change in the potential along the *y* axis tells us only that the *y* component of the electric field is zero. Look back at Figure 8 in Example 1. We showed there that the electric field of a dipole on the *y* axis has only an *x* component. We could not find the *x* component in the current example because we do not have an expression for the potential near the *y* axis as a function of *x*.

Electric Potential Due to a Uniformly Charged Ring

(A) Find an expression for the electric potential at a point *P* located on the perpendicular central axis of a uniformly charged ring of radius *a* and total charge *Q*.

(B) Find an expression for the magnitude of the electric field at point *P*.

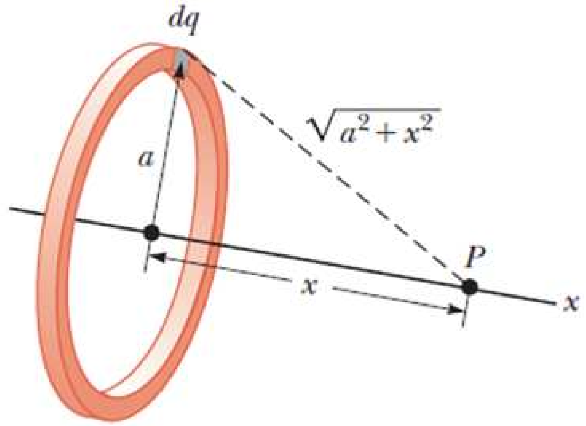


Figure 9 ***(Example 2) A uniformly charged ring of radius a lies in a plane perpendicular to the x axis. All elements dq of the ring are the same distance from a point P lying on the x axis.***

Conceptualize: Study Figure 9, in which the ring is oriented so that its plane is perpendicular to the *x* axis and its center is at the origin. Notice that the symmetry of the situation means that all the charges on the ring are the same distance from point *P*. Notice that no vector considerations are necessary here because electric potential is a scalar.

Categorize: Because the ring consists of a continuous distribution of charge rather than a set of discrete charges, we must use the integration technique represented by Equation 20 in this example.

Analyze: We take point *P* to be at a distance *x* from the center of the ring as shown in Figure 9. Use Equation 20 to express *V* in terms of the geometry:

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{a^2 + x^2}}$$

Noting that *a* and *x* do not vary for an integration over the ring, bring $\sqrt{a^2 + x^2}$ in front of the integral sign and integrate over the ring:

$$V = \frac{k_e}{\sqrt{a^2 + x^2}} \int dq = \frac{k_e Q}{\sqrt{a^2 + x^2}} \quad \text{----------------(21)}$$

(B)Find an expression for the magnitude of the electric field at point P.

From symmetry, notice that along the *x* axis $\vec{E}$ can have only an *x* component. Therefore, apply Equation 16 to Equation 21:

$$E_x = -\frac{dV}{dx} = -k_e Q \frac{d}{dx}\left(a^2 + x^2\right)^{-1/2}$$

$$= -k_e Q\left(-\frac{1}{2}\right)\left(a^2 + x^2\right)^{-3/2}(2x)$$

$$E_x = -\frac{k_e x}{\left(a^2 + x^2\right)^{-3/2}} Q \quad \text{---------------------------(22)}$$

Finalize The only variable in the expressions for *V* and E_x is *x*. That is not surprising because our calculation is valid only for points along the *x* axis, where *y* and *z* are both zero. This result for the electric field agrees with that obtained by direct integration. For practice, use the result of part (B) in Equation to verify that the potential is given by the expression in part (A).

Applications of Electrostatics

The practical application of electrostatics is represented by such devices as lightning rods and electrostatic precipitators and by such processes as xerography and the painting of automobiles. Scientific devices based on the principles of electrostatics include electrostatic generators, the field-ion microscope, and ion-drive rocket engines. Details of two devices are given below.

The Van de Graff Generator

Experimental results show that when a charged conductor is placed in contact with the inside of a hollow conductor, all the charge on the charged conductor is transferred to the hollow conductor. In principle, the charge on the hollow conductor and its electric potential can be increased without limit by repetition of the process. In 1929, Robert J. Van de Graff (1901–1967) used this principle to design and build an electrostatic generator, and a schematic representation of it is given in Figure 10. This type of generator was once used extensively in nuclear physics research. Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material. The high-voltage electrode is a hollow metal dome mounted on an insulating column. The belt is charged at point A by means of a corona discharge between comb-like metallic needles and a grounded grid. The needles are maintained at a positive electric potential of typically 10^4 V. The positive charge on the moving belt is transferred to the dome by a second comb of needles at point B. Because the electric field inside the dome is negligible, the positive charge on the belt is easily transferred to the conductor regardless of its potential. In practice, it is possible to increase the electric potential of the dome until electrical discharge occurs through the air. Because the "breakdown" electric field in air is about $3 \text{ X}10^6$ V/m, a sphere 1.00 m in radius can be raised to a maximum potential of $3 \text{ X } 10^6$ V. The potential can be increased further by increasing the dome's radius and placing the entire system in a container filled with high-pressure gas. Van de Graaff generators can produce potential differences as large as 20 million volts. Protons accelerated through such large potential differences receive enough energy to initiate nuclear reactions between themselves and various target nuclei. Smaller generators are often seen in science classrooms and museums. If a person insulated from the ground touches the sphere of a Van de Graaff generator, his or her body can be brought to a high electric potential. The person's hair acquires a net positive charge, and each strand is repelled by all the others as in the photograph.

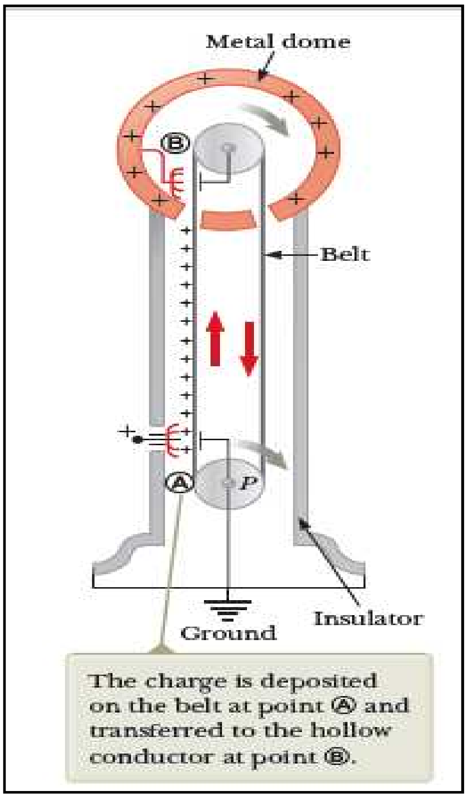


Figure 10. *Schematic diagram of a Van de Graff generator.*
Charge is transferred to the metal dome at the top by means of a moving belt.

Capacitance and dielectrics:

Definition of Capacitance

Consider two conductors as shown in Figure 11. Such a combination of two conductors is called a **capacitor.** The conductors are called *plates.* If the conductors carry charges of equal magnitude and opposite sign, a potential difference ΔV exists between them.

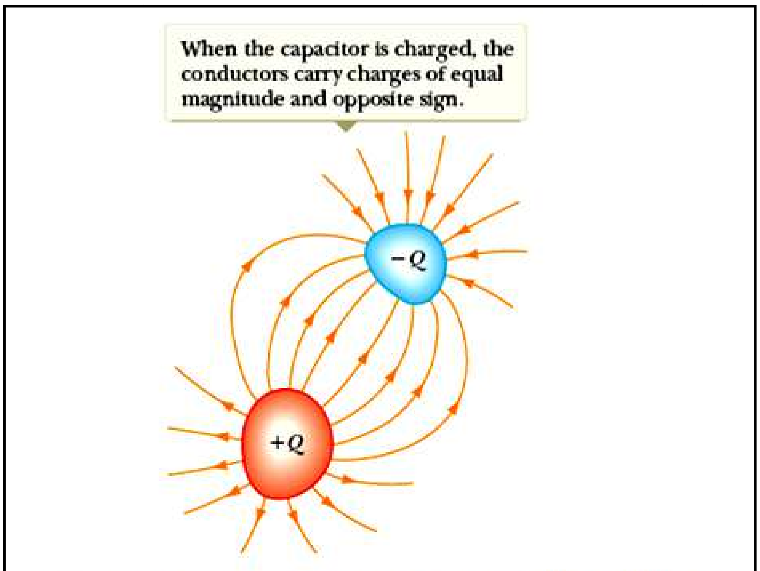


Figure 11. *A capacitor consists of two conductors*

What determines how much charge is on the plates of a capacitor for a given voltage? Experiments show that the quantity of charge Q on a capacitor1 is linearly proportional to the potential difference between the conductors; that is, $Q \propto \Delta V$. The proportionality constant depends on the shape and separation of the conductors.2 This relationship can be written as $Q = C\ \Delta V$ if we define capacitance as follows:

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C = \frac{Q}{\Delta V} \quad \text{----------------(23)}$$

By definition *capacitance is always a positive quantity.* Furthermore, the charge Q and the potential difference DV are always expressed in Equation 23 as positive quantities. From Equation 23, we see that capacitance has SI units of coulombs per volt. Named in honor of Michael Faraday, the SI unit of capacitance is the **farad** (F):

$$1F = 1 C/V$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F). We shall use the symbol mF to represent microfarads. In practice, to avoid the use of Greek letters, physical capacitors are often labeled "mF" for microfarads and "mmF" for micromicrofarads or, equivalently, "pF" for picofarads.

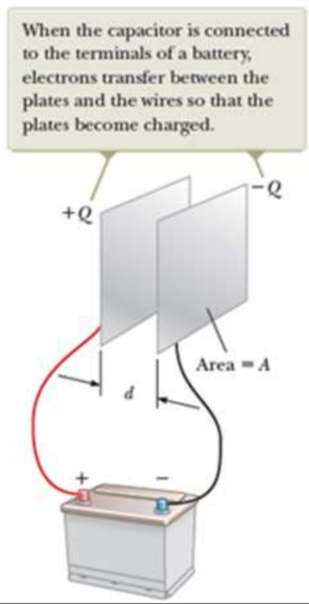


Figure 12. *A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d.*

Let's consider a capacitor formed from a pair of parallel plates as shown in Figure 12. Each plate is connected to one terminal of a battery, which acts as a source of potential difference. If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires when the connections are made. Let's focus on the plate connected to the negative terminal of the battery. The electric field in the wire applies a force on electrons in the wire immediately outside this plate; this force causes the electrons to move onto the plate. The movement continues until the plate, the wire, and the terminal are all at the same electric potential. Once this equilibrium situation is attained, a potential difference no longer exists between the terminal and the plate; as a result, no electric field is present in the wire and the electrons stop moving. The plate now carries a negative charge. A similar process occurs at the other capacitor plate, where electrons move from the plate to the wire, leaving the plate positively charged. In this final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.

Calculating Capacitance

We can derive an expression for the capacitance of a pair of oppositely charged conductors having a charge of magnitude Q in the following manner. First we calculate the potential difference. We then use the expression C = Q / ΔV to evaluate the capacitance. The calculation is relatively easy if the geometry of the capacitor is simple. Although the most common situation is that of two conductors, a single conductor also has a capacitance. For example, imagine a single spherical, charged conductor. The electric field lines around this conductor are exactly the same as if there were a conducting, spherical shell of infinite radius, concentric with the sphere and carrying a charge of the same magnitude but opposite sign. Therefore, we can identify the imaginary shell as the second conductor of a two-conductor capacitor. The electric potential of the sphere of radius a is simply k_eQ /a, and setting V = 0 for the infinitely large shell gives

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e \, Q/a} = \frac{a}{k_e} = 4\pi\varepsilon_0 a \quad \text{---------------(24)}$$

This expression shows that the capacitance of an isolated, charged sphere is proportional to its radius and is independent of both the charge on the sphere and its potential, as is the case with all capacitors. Equation 24 is the general definition of capacitance in terms of electrical parameters, but the capacitance of a given capacitor will depend only on the geometry of the plates. The capacitance of a pair of conductors is illustrated below with three familiar geometries, namely, parallel plates, concentric cylinders, and concentric spheres. In these calculations, we assume the charged conductors are separated by a vacuum.

Parallel-Plate Capacitors

Two parallel, metallic plates of equal area *A* are separated by a distance *d* as shown in Figure 12. One plate carries a charge +*Q* , and the other carries a charge -*Q*. The surface charge density on each plate is $\sigma = Q/A$. If the plates are very close together (in comparison with their length and width), we can assume the electric field is uniform between the plates and zero elsewhere. The value of the electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals *Ed*; therefore,

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A}$$

Substituting this result into Equation 24, we find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\varepsilon_o A}$$

$$C = \frac{\varepsilon_o A}{d} \quad \text{--------(25)}$$

That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation. Let's consider how the geometry of these conductors influences the capacity of the pair of plates to store charge. As a capacitor is being charged by a battery, electrons flow into the negative plate and out of the positive plate. If the capacitor plates are large, the accumulated charges are able to distribute themselves over a substantial area and the amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Therefore, it is reasonable that the capacitance is proportional to the plate area *A* as in Equation 25. Now consider the region that separates the plates. Imagine moving the plates closer together. Consider the situation before any charges have had a chance to move in response to this change. Because no charges have moved, the electric field between the plates has the same value but extends over a shorter distance. Therefore, the magnitude of the potential difference between the plates $\Delta V = Ed$ is smaller. The difference between this new capacitor voltage and the terminal voltage of the battery appears as a potential difference across the wires connecting the battery to the capacitor, resulting in an electric field in the wires that drives more charge onto the plates and increases the potential difference between the plates. When the potential difference between the plates again matches that of the battery, the flow of charge stops. Therefore, moving the plates closer together causes
the charge on the capacitor to increase. If *d* is increased, the charge decreases. As a result, the inverse relationship between *C* and *d* in Equation 25 is reasonable.

The Cylindrical Capacitor

A solid cylindrical conductor of radius *a* and charge *Q* is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$ (Fig. 13a). Find the capacitance of this cylindrical capacitor if its length is *l*.

Solution: Conceptualize Recall that any pair of conductors qualifies as a capacitor, so the system described in this example therefore qualifies. Figure 13b helps visualize the electric field between the conductors. We expect the capacitance to depend only on geometric factors, which, in this case, are *a*, *b*, and *l*.

Categorize Because of the cylindrical symmetry of the system, we can use results from previous studies of cylindrical systems to find the capacitance.

Analyze Assuming *l* is much greater than a and b, we can neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 26.4b).

Write an expression for the potential difference between the two cylinders from Equation 9:

$$V_b - V_a = -\int_a^b \vec{E} \cdot \vec{ds}$$

Apply the above equation for the electric field outside a cylindrically symmetric charge distribution and notice from Figure 13b that $\vec{E}$ is parallel to $\vec{ds}$ along a radial line:

$$V_b - V_a = -\int_a^b \vec{E_r} \cdot \vec{dr} = -2k_e\lambda\int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

Substitute the absolute value of ΔV into Equation 25 and use $\lambda = Q/l$,:

$$C = \frac{Q}{\Delta V} = \frac{Q}{2k_e Q/l \, \ln\left(b/a\right)} = \frac{l}{2k_e \ln\left(b/a\right)} \qquad \text{----------------------(26)}$$

Finalize The capacitance depends on the radii a and b and is proportional to the length of the cylinders. Equation 26 shows that the capacitance per unit length of a combination of concentric cylindrical

An example of this type of geometric arrangement is a coaxial cable, which consists of two concentric cylindrical conductors separated by an insulator. You probably have a coaxial cable attached to your television set if you are a subscriber to cable television. The coaxial cable is especially useful for shielding electrical signals from any possible external influences.

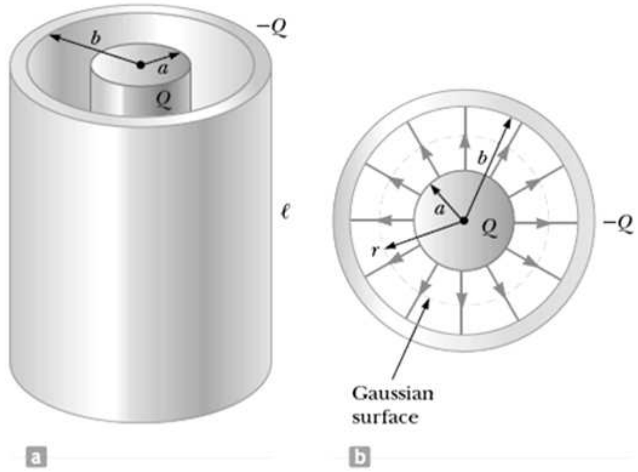


Figure 13 (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius *a* and length ℓ surrounded by a coaxial cylindrical shell of radius *b*. (b) End view. The electric field lines are radial. The dashed line represents the end of a cylindrical gaussian surface of radius *r* and length ℓ.

The Spherical capacitor

A spherical capacitor consists of a spherical conducting shell of radius b and charge -Q concentric with a smaller conducting sphere of radius a and charge Q (Fig. 14). Find the capacitance of this device.

Solution: Conceptualize This system involves a pair of conductors and qualifies as a capacitor. We expect the capacitance to depend on the spherical radii a and b.

Categorize Because of the spherical symmetry of the system, we can use results from previous studies of spherical systems to find the capacitance.

Analyze The direction of the electric field outside a spherically symmetric charge distribution is radial and its magnitude is given by the expression $E = k_eQ/r^2$. In this case, this result applies to the field between the spheres ($a < r < b$).

Write an expression for the potential difference between the two conductors from Equation 3:

$$V_b - V_a = -\int_a^b \vec{E} \cdot \vec{ds}$$

Apply the result of Example .3 for the electric field outside a spherically symmetric charge distribution and note that $\vec{E}$ is parallel to $\vec{ds}$ along a radial line:

$$V_b - V_a = -\int_a^b \overrightarrow{E_r} \cdot \overrightarrow{dr} = -k_e Q \int_a^b \frac{dr}{r^2} = k_e Q \left[\frac{1}{r}\right]_a^b$$

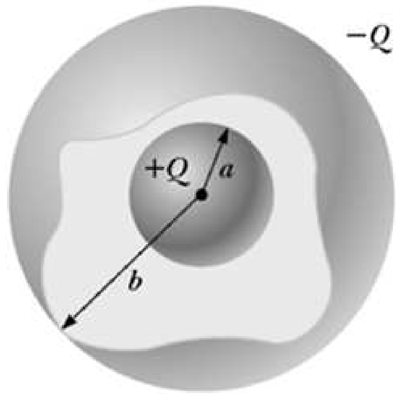
$$(1)\ \ V_b - V_a = k_e Q \left[\frac{1}{b} - \frac{1}{a}\right] = k_e Q \frac{a-b}{ab}$$

Substitute the absolute value of ΔV into 25:

$$C = \frac{Q}{\Delta V} = \frac{Q}{V_b - V_a} = \frac{ab}{k_e (a-b)} \quad \text{------------(27)}$$

Finalize The capacitance depends on a and b as expected. The potential difference between the spheres in Equation (1) is negative because Q is positive and b>a. Therefore, in Equation 27, when we take the absolute value, we change a - b to b- a. The result is a positive number.

Figure 14.
A spherical capacitor consists of an inner sphere of radius *a* surrounded by a concentric spherical shell of radius *b*. The electric field between the spheres is directed radially outward when the inner sphere is positively charged.



Combinations of Capacitors

Two or more capacitors often are combined in electric circuits. We can calculate the equivalent capacitance of certain combinations using methods described in this section. Throughout this section, we assume the capacitors to be combined are initially uncharged. In studying electric circuits, we use a simplified pictorial representation called a **circuit diagram.** Such a diagram uses **circuit symbols** to represent various circuit elements. The circuit symbols are connected by straight lines that represent the wires between the circuit elements. The circuit symbols for capacitors, batteries, and switches as well as the color codes used for them in this text are given in Figure 15. The symbol for the capacitor reflects the geometry of the most common model for a capacitor, a pair of parallel plates. The positive terminal of the battery is at the higher potential and is represented in the circuit symbol by the longer line.

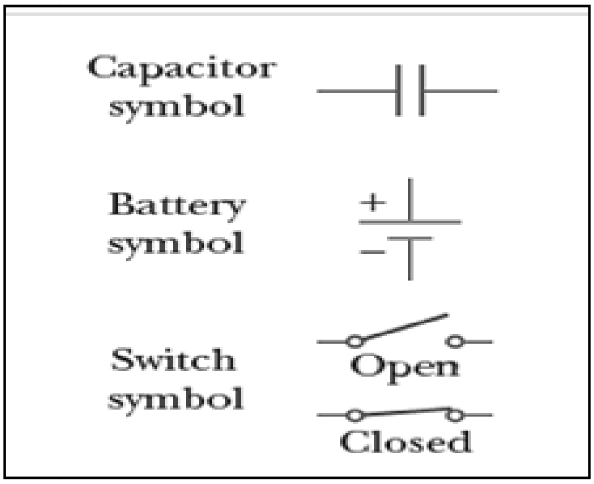


Figure 15 *Circuit symbols for capacitors, batteries, and switches. Notice that capacitors are in blue, batteries are in green, and switches are in red. The closed switch can carry current, whereas the open one cannot.*

Parallel Combination

Two capacitors connected as shown in Figure 16a are known as a **parallel combination** of capacitors. Figure 16b shows a circuit diagram for this combination of capacitors. The left plates of the capacitors are connected to the positive terminal of the battery by a conducting wire and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and so are both at the same potential as the negative terminal. Therefore, the individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination. That is,

$$\Delta V_1 = \Delta V_2 = \Delta V$$

where ΔV is the battery terminal voltage.

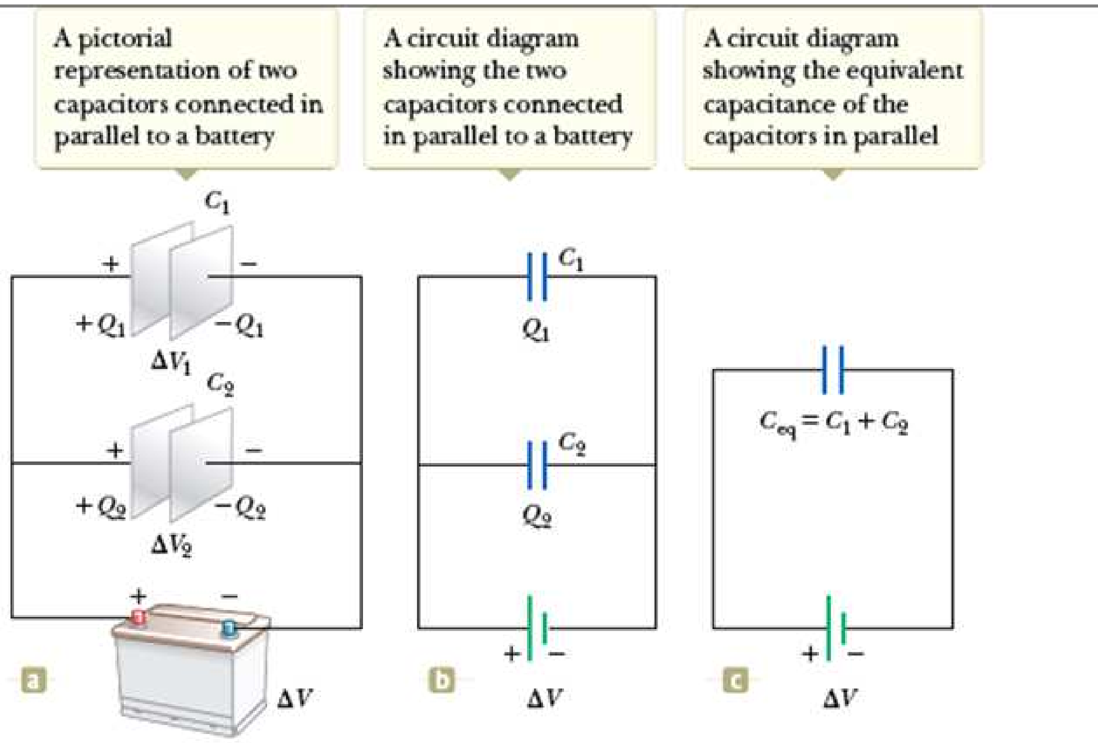


Figure 16. *Two capacitors connected in parallel. All three diagrams are equivalent*

After the battery is attached to the circuit, the capacitors quickly reach their maximum charge. Let's call the maximum charges on the two capacitors Q_1 and Q_2, where $Q_1 = C_1 \Delta V_1$ and $Q_2 = C_2 \Delta V_2$. The *total charge* Q_{tot} stored by the two capacitors is the sum of the charges on the individual capacitors:

$$Q_{tot} = Q_1 + Q_2 = C_1 \Delta V_1 + C_2 \Delta V_2 \quad \text{------------------------(28)}$$

Suppose you wish to replace these two capacitors by one *equivalent capacitor* having a capacitance C_{eq} as in Figure 16c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store charge Q_{tot} when connected to the battery. Figure 16c shows that the voltage across the equivalent capacitor is ΔV because the equivalent capacitor is connected directly across the battery terminals. Therefore, for the equivalent capacitor,

$$Q_{tot} = C_{eq} \Delta V$$

Substituting this result into Equation 28 gives

$$C_{eq} \Delta V = C_1 \Delta V_1 + C_2 \Delta V_2$$

$$C_{eq} = C_1 + C_2 \quad \text{(Parallel combination)}$$

where we have canceled the voltages because they are all the same. If this treatment is extended to three or more capacitors connected in parallel, the **equivalent capacitance** is found to be

$$C_{eq} = C_1 + C_2 + C_3 + \dots\dots\dots\dots \quad \text{(Parallel combination)}\text{--------------}(29)$$

Therefore, the equivalent capacitance of a parallel combination of capacitors is

(1) the algebraic sum of the individual capacitances and

(2) greater than any of the individual capacitances.

Statement (2) makes sense because we are essentially combining the areas of all the capacitor plates when they are connected with conducting wire, and capacitance of parallel plates is proportional to area.

Series Combination

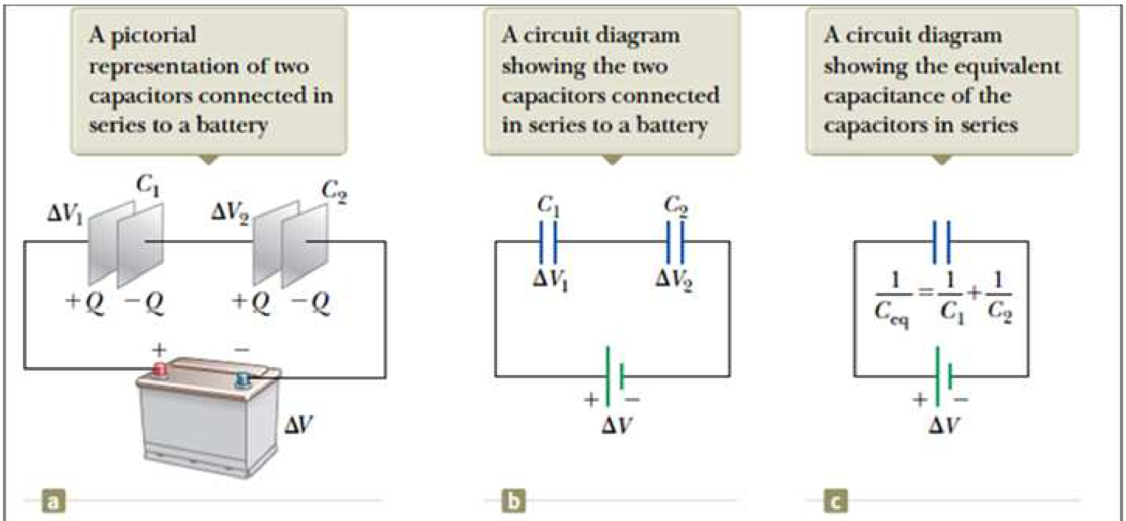


Figure17. *Two capacitors connected in series. All three diagrams are equivalent.*

Two capacitors connected as shown in Figure 17a and the equivalent circuit diagram in Figure 17b are known as a **series combination** of capacitors. The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else; hence, they form an isolated system that is initially uncharged and must continue to have zero net charge. To analyze this combination, let's first consider the uncharged capacitors and then follow what happens immediately after a battery is connected to the circuit. When the battery is connected, electrons are transferred out of the left plate of C_1 and into the right plate of C_2. As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is forced off the left plate of C_2, and this left plate therefore has an excess positive charge. The negative charge leaving the left plate of C_2 causes negative charges to accumulate on the right plate of C_1. As a result, both right plates end up with a charge -Q and both left plates end up with a charge +Q . Therefore, the charges on capacitors connected in series are the same:

$$Q_1 = Q_2 = Q$$

where Q is the charge that moved between a wire and the connected outside plate of one of the capacitors.

Figure 17a shows the individual voltages ΔV_1 and ΔV_2 across the capacitors. These voltages add to give the total voltage ΔV_{tot} across the combination:

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \quad \text{----------------------(30)}$$

In general, the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors. Suppose the equivalent single capacitor in Figure 17c has the same effect on the circuit as the series combination when it is connected to the battery. After it is fully charged, the equivalent capacitor must have a charge of -Q on its right plate and a charge of +Q on its left plate. Applying the definition of capacitance to the circuit in Figure 17c gives

Substituting this result into Equation 30, we have

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Canceling the charges because they are all the same gives

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

When this analysis is applied to three or more capacitors connected in series, the relationship for the **equivalent capacitance** is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ---------- \quad \text{(series combination) (31)}$$

This expression shows that

(1) the inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and

(2) the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.

Energy Stored in a Charged Capacitor

Because positive and negative charges are separated in the system of two conductors in a capacitor, electric potential energy is stored in the system. Many of those who work with electronic equipment have at some time verified that a capacitor can store energy. If the plates of a charged capacitor are connected by a conductor such as a wire, charge moves between each plate and its connecting wire until the capacitor is uncharged. The discharge can often be observed as a visible spark. If you accidentally touch the opposite plates of a charged capacitor, your fingers act as a pathway for discharge and the result is an electric shock. The degree of shock you receive depends on the capacitance and the voltage applied to the capacitor. Such a shock could be dangerous if high voltages are present as in the power supply of a home theater system. Because the charges can be stored in a capacitor even when the system is turned off, unplugging the system does not make it safe to open the case and touch the components inside.

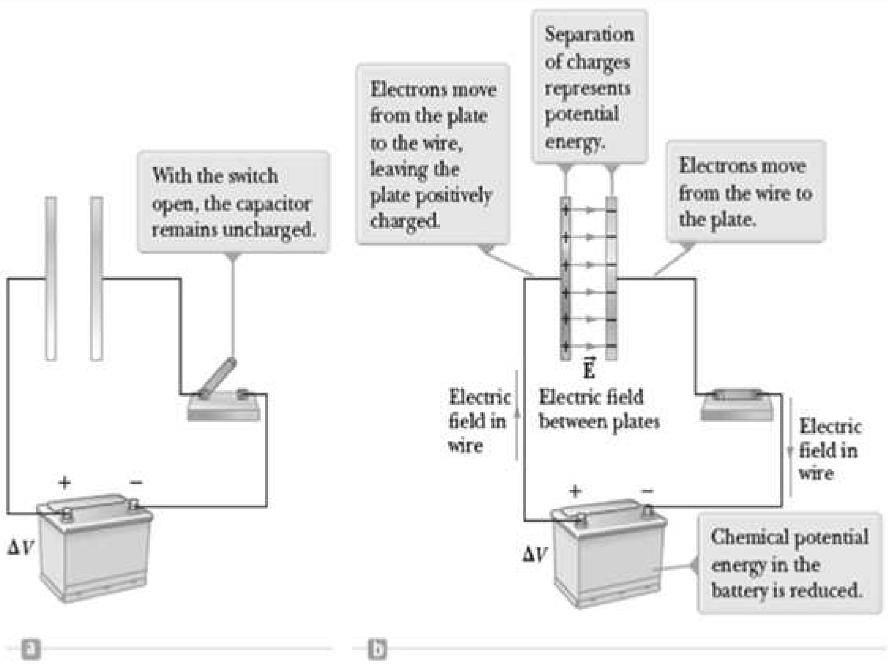


Figure *18.* (a) A circuit consisting of a capacitor, a battery, and a switch. (b) When the switch is closed, the battery establishes an electric field in the wire and the capacitor becomes charged.

Figure 18a shows a battery connected to a single parallel-plate capacitor with a switch in the circuit. Let us identify the circuit as a system. When the switch is closed (Fig. 18b), the battery establishes an electric field in the wires and charges flow between the wires and the capacitor. As that occurs, there is a transformation of energy within the system. Before the switch is closed, energy is stored as chemical potential energy in the battery. This energy is transformed during the chemical reaction that occurs within the battery when it is operating in an electric circuit. When the switch is closed, some of the chemical potential energy in the battery is

transformed to electric potential energy associated with the separation of positive and negative charges on the plates.

To calculate the energy stored in the capacitor, we shall assume a charging process that is different from the actual process described in Section 26.1 but that gives the same final result. This assumption is justified because the energy in the final configuration does not depend on the actual charge-transfer process. Imagine the plates are disconnected from the battery and you transfer the charge mechanically through the space between the plates as follows. You grab a small amount of positive charge on one plate and apply a force that causes this positive charge to move over to the other plate. Therefore, you do work on the charge as it is transferred from one plate to the other. At first, no work is required to transfer a small amount of charge dq from one plate to the other, but once this charge has been transferred, a small potential difference exists between the plates. Therefore, work must be done to move additional charge through this potential difference. As more and more charge is transferred from one plate to the other, the potential difference increases in proportion and more work is required. The overall process is described by the nonisolated system model for energy. The work done on the system by the external agent appears as an increase in electric potential energy in the system. Suppose q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is $\Delta V = q/C$. This relationship is graphed in Figure 19. We know that the work necessary to transfer an increment of charge dq from the plate carrying charge $2q$ to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V \cdot dq = \frac{q}{C} dq$$

The work required to transfer the charge dq is the area of the tan rectangle. Because 1 V = 1 J/C, the unit for the area is the joule. The total work required to charge the capacitor from $q = 0$ to some final charge $q = Q$ is

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q \cdot dq = \frac{Q^2}{2C}$$

The work done in charging the capacitor appears as electric potential energy U_E stored in the capacitor. Using Equation 25, we can express the potential energy stored in a charged capacitor as

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 \qquad \text{-------------(32)}$$

Because the curve in Figure 19 is a straight line, the total area under the curve is that of a triangle of base Q and height ΔV.

Equation 32 applies to any capacitor, regardless of its geometry. For a given capacitance, the stored energy increases as the charge and the potential difference increase. In practice, there is a limit to the maximum energy (or charge) that can be stored because, at a sufficiently large value of ΔV, discharge ultimately occurs between the plates. For this reason, capacitors are usually labeled with a maximum operating voltage.

We can consider the energy in a capacitor to be stored in the electric field created between the plates as the capacitor is charged. This description is reasonable because the electric field is proportional to the charge on the capacitor. For a parallel- plate capacitor, the potential difference is related to the electric field through the relationship $\Delta V = Ed$. Furthermore, its capacitance is $C = \varepsilon_0 A/d$ (Eq. 25). Substituting these expressions into Equation 32 gives Because the volume occupied by the electric field is Ad, the *energy per unit volume* $u_E = U_E/Ad$, known as the *energy density*, is

Although Equation 25 was derived for a parallel-plate capacitor, the expression is generally valid regardless of the source of the electric field. That is, the energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

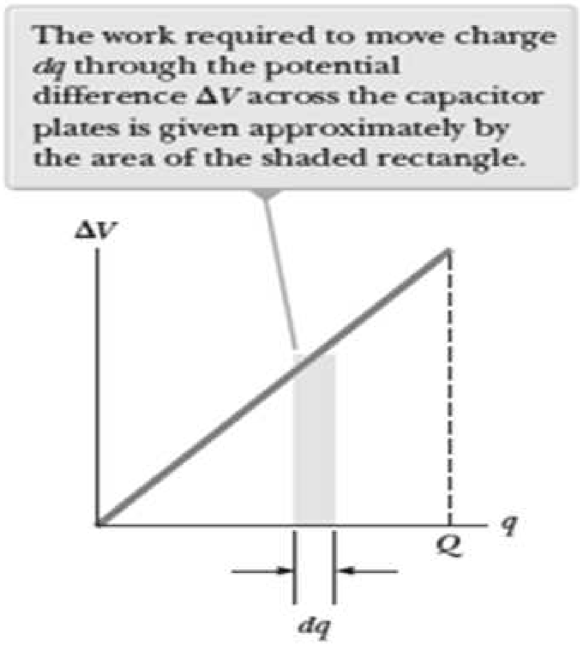


Figure 19. A plot of potential difference versus charge for a capacitor is a straight line having slope $1/C$.

Capacitors with Dielectrics

A **dielectric** is a nonconducting material such as rubber, glass, or waxed paper. We can perform the following experiment to illustrate the effect of a dielectric in a capacitor. Consider a parallel-plate capacitor that without a dielectric has a charge Q_0 and a capacitance C_0. The potential difference across the capacitor is $\Delta V_0 = Q_0/C_0$. Figure 20a illustrates this situation. The potential difference is measured by a device called a *voltmeter*. Notice that no battery is shown in the figure; also, we must assume no charge can flow through an ideal voltmeter. Hence, there is no path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates as in Figure 20b, the voltmeter indicates that the voltage between the plates decreases to a value ΔV. The voltages with and without the dielectric are related by a factor k as follows:

$$\Delta V = \frac{\Delta V_0}{k}$$

Because $\Delta V < \Delta V_0$, we see that k >1. The dimensionless factor k is called the **dielectric constant** of the material. The dielectric constant varies from one material to another. In this section, we analyze this change in capacitance in terms of electrical parameters such as electric charge, electric field, and potential difference; Because the charge Q_0 on the capacitor does not change, the capacitance must change to the value

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0 / k} = k\frac{Q_0}{\Delta V_0}$$

$$\boxed{C = kC_0} \qquad \text{------------------------------(33)}$$

That is, the capacitance *increases* by the factor k when the dielectric completely fills the region between the plates. Because $C_0 = \varepsilon_0 A/d$ (Eq. 26) for a parallel-plate capacitor, we can express the capacitance of a parallel-plate capacitor filled with a dielectric as

$$C = k\frac{\varepsilon_0 A}{d} \qquad \text{----------------(34)}$$

From Equation 34, it would appear that the capacitance could be made very large by inserting a dielectric between the plates and decreasing *d*. In practice, the lowest value of *d* is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** (maximum electric field) of the dielectric. If the magnitude of the electric field in the dielectric exceeds the dielectric strength, the insulating properties break down and the dielectric begins to conduct.

Physical capacitors have a specification called by a variety of names, including *working voltage, breakdown voltage,* and *rated voltage.* This parameter represents the largest voltage that can be applied to the capacitor without exceeding the dielectric strength of the dielectric material in the capacitor. Consequently, when selecting a capacitor for a given application, you must consider its capacitance as well as the expected voltage across the capacitor in the circuit, making sure the expected voltage is smaller than the rated voltage of the capacitor.

Insulating materials have values of k greater than unity and dielectric strengths greater than that of air as Table 26.1 indicates. Therefore, a dielectric provides the following advantages:

• An increase in capacitance

• An increase in maximum operating voltage

• Possible mechanical support between the plates, which allows the plates to be
close together without touching, thereby decreasing *d* and increasing *C*

Table 26.1 Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

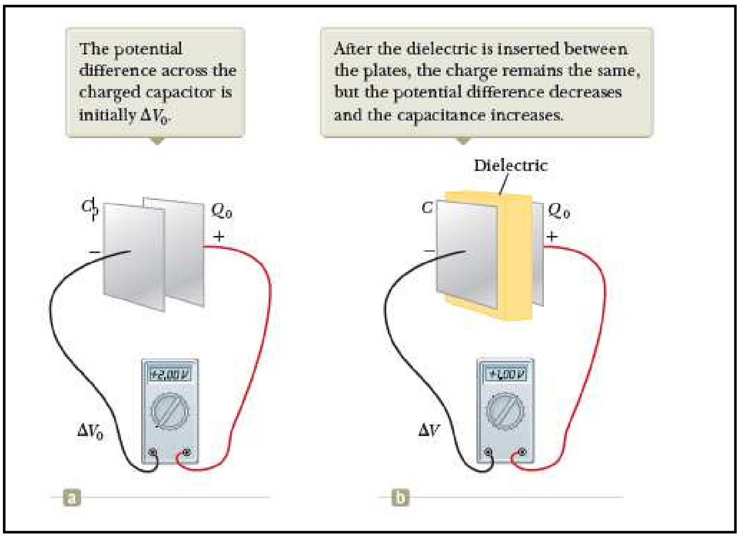


Figure 20 *A charged capacitor (a) before and (b) after insertion of a dielectric between the plates.*

Types of Capacitors

Many capacitors are built into integrated circuit chips, but some electrical devices still use stand-alone capacitors. Commercial capacitors are often made from metallic foil interlaced with thin sheets of either paraffin-impregnated paper or Mylar as the dielectric material. These alternate layers of metallic foil and dielectric are rolled into a cylinder to form a small package (Fig. 21a). High-voltage capacitors commonly consist of a number of interwoven metallic plates immersed in silicone oil (Fig. 21b). Small capacitors are often constructed from ceramic materials.

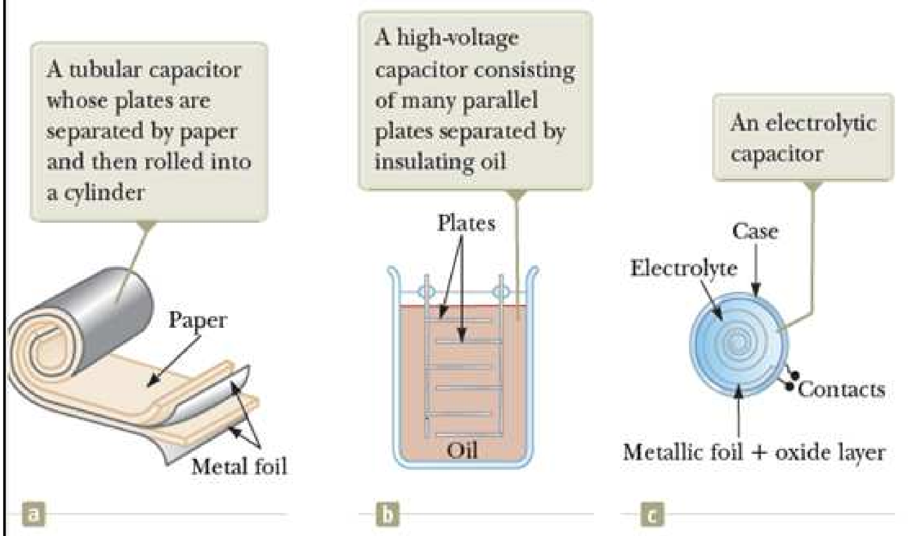


Figure 21. Three commercial capacitor designs.

Often, an *electrolytic capacitor* is used to store large amounts of charge at relatively low voltages. This device, shown in Figure 21c, consists of a metallic foil in contact with an *electrolyte*, a solution that conducts electricity by virtue of the motion of ions contained in the solution. When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide (an insulator) is formed on the foil, and this layer serves as the dielectric. Very large values of capacitance can be obtained in an electrolytic capacitor because the dielectric layer is very thin and therefore the plate separation is very small.

Electrolytic capacitors are not reversible as are many other capacitors. They have a polarity, which is indicated by positive and negative signs marked on the device. When electrolytic capacitors are used in circuits, the polarity must be correct. If the polarity of the applied voltage is the opposite of what is intended, the oxide layer is removed and the capacitor conducts electricity instead of storing charge.

Variable capacitors (typically 10 to 500 pF) usually consist of two interwoven sets of metallic plates, one fixed and the other movable, and contain air as the dielectric (Fig. 22). These types of capacitors are often used in radio tuning circuits.

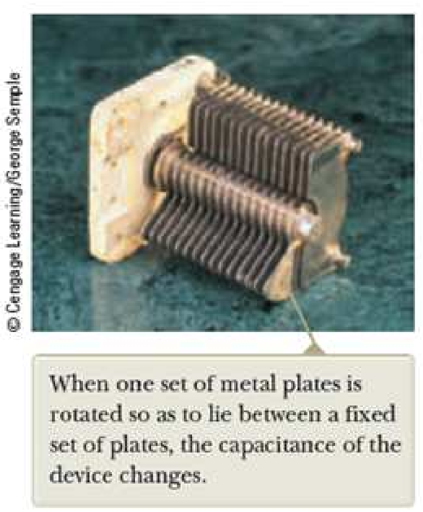


Figure 22. A variable capacitor.

Electric Dipole in an Electric Field

We have discussed the effect on the capacitance of placing a dielectric between the plates of a capacitor. The electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$ as shown in Figure 23. The **electric dipole moment** of this configuration is defined as the vector $\vec{P}$ directed from -q toward +q along the line joining the charges and having magnitude

$$P \equiv 2aq \text{ ----------------------------(34)}$$

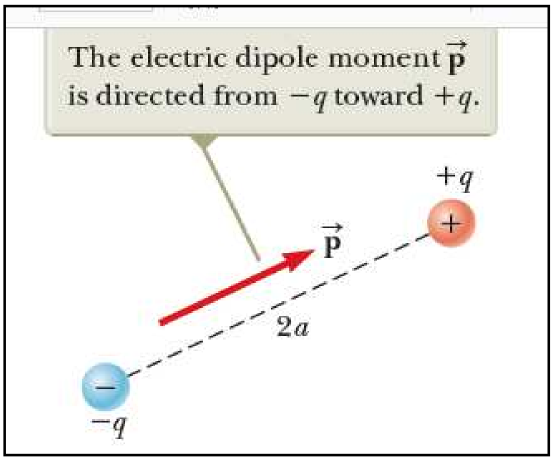


Figure 23 *An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance of 2a.*

Now suppose an electric dipole is placed in a uniform electric field $\vec{E}$ and makes an angle θ with the field as shown in Figure 24. We identify $\vec{E}$ as the field *external* to the dipole, established by some other charge distribution, to distinguish it from the field *due to* the dipole.

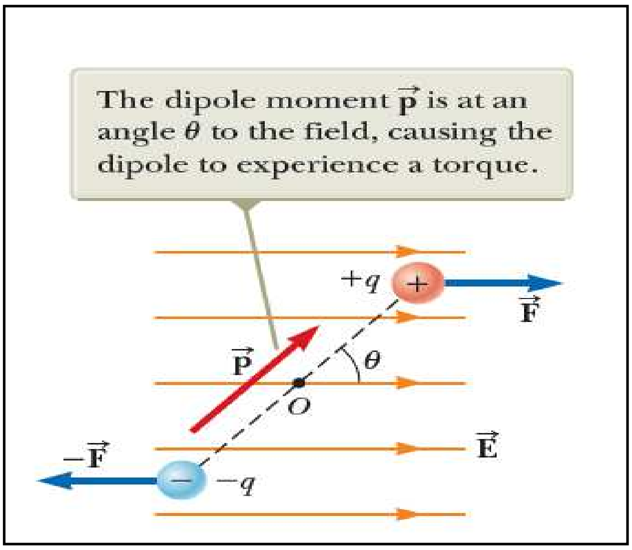


Figure 24. *An electric dipole in a uniform external electric field.*

Each of the charges is modeled as a particle in an electric field. The electric forces acting on the two charges are equal in magnitude ($F = qE$) and opposite in direction as shown in Figure 24. Therefore, the net force on the dipole is zero. The two forces produce a net torque on the dipole, however; the dipole is therefore described by the rigid object under a net torque model. As a result, the dipole rotates in the direction that brings the dipole moment vector into greater alignment with the field. The torque due to the force on the positive charge about an axis through O in Figure 24 has magnitude $Fa\sin\theta$, where a sin u is the moment arm of F about O. This force tends to produce a clockwise rotation. The torque about O on the negative charge is also of magnitude $Fa\sin\theta$; here again, the force tends to produce a clockwise rotation. Therefore, the magnitude of the net torque about O is

$$\tau = 2Fa\sin\theta$$

Because $F = qE$ and $p = 2aq$, we can express τ as

$$\tau = 2aqE\sin\theta = pE\sin\theta \quad \text{------------------------------(35)}$$

Based on this expression, it is convenient to express the torque in vector form as the cross product of the vectors $\vec{p}$ and $\vec{E}$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

We can also model the system of the dipole and the external electric field as an isolated system for energy. Let's determine the potential energy of the system as a function of the dipole's orientation with respect to the field. To do so, recognize that work must be done by an external agent to rotate the dipole through an angle so as to cause the dipole moment vector to become less aligned with the field. The work done is then stored as electric potential energy in the system. Notice that this potential energy is associated with a *rotational* configuration of the system. Previously, we have seen potential energies associated with *translational* configurations: an object with mass was moved in a gravitational field, a charge was moved in an electric field, or a spring was extended. The work dW required to rotate the dipole through an angle is $dW = \tau \cdot d\theta$. Because $\vec{\tau} = \vec{p} \sin\theta$ and the work results in an increase in the electric potential energy U, we find that for a rotation from θ_i to θ_f, the change in potential energy of the system is

$$U_f - U_i = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} pE\sin\theta \cdot d\theta = pE\int_{\theta_i}^{\theta_f} \sin\theta \cdot d\theta$$

$$= pE(-\cos\theta)_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

The term that contains $\cos\theta_i$ is a constant that depends on the initial orientation of the dipole. It is convenient to choose a reference angle of $\theta_i = 90^0$ so that $\cos\theta_i = \cos 90^0 = 0$. Furthermore, let's choose $U_i = 0$ at $\theta_i = 90^0$ as our reference value of potential energy. Hence, we can express a general value of $U_E = U_f$ as

:

$$U_E = -pE\cos\theta \quad \text{----------------(36)}$$

We can write this expression for the potential energy of a dipole in an electric field as the dot product of the vectors $\vec{p}$ and $\vec{E}$

$$U_E = -\vec{p} \cdot \vec{E} \quad \text{----------------(37)}$$

To develop a conceptual understanding of Equation 37, compare it with the expression for the potential energy of the system of an object in the Earth's gravitational field, $U_g = mgy$. First, both expressions contain a parameter of the entity placed in the field: mass for the object, dipole moment for the dipole. Second, both expressions contain the field, *g* for the object, *E* for the dipole. Finally, both expressions contain a configuration description: translational position *y* for the object, rotational position u for the dipole. In both cases, once the configuration is changed, the system tends to return to the original configuration when the object is released: the object of mass *m* falls toward the ground, and the dipole begins to rotate back toward the configuration in which it is aligned with the field.

Molecules are said to be *polarized* when a separation exists between the average position of the negative charges and the average position of the positive charges in the molecule. In some molecules such as water, this condition is always present; such molecules are called **polar molecules.** Molecules that do not possess a permanent polarization are called **nonpolar molecules.**

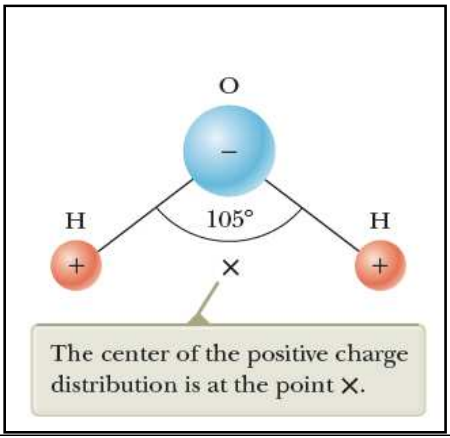


Figure 25. *The water molecule, H_2O, has a permanent polarization resulting from its nonlinear geometry.*

We can understand the permanent polarization of water by inspecting the geometry of the water molecule. The oxygen atom in the water molecule is bonded to the hydrogen atoms such that an angle of 105^0 is formed between the two bonds (Fig. 25). The center of the negative charge distribution is near the oxygen atom, and the center of the positive charge distribution lies at a point midway along the line joining the hydrogen atoms (the point labeled X in Fig. 25). We can model the water molecule and other polar molecules as dipoles because the average positions of the positive and negative charges act as point charges. As a result, we can apply our discussion of dipoles to the behavior of polar molecules.

Washing with soap and water is a household scenario in which the dipole structure of water is exploited. Grease and oil are made up of nonpolar molecules, which are generally not attracted to water. Plain water is not very useful for removing this type of grime. Soap contains long molecules called *surfactants*. In a long molecule, the polarity characteristics of one end of the molecule can be different from those at the other end. In a surfactant molecule, one end acts like a nonpolar molecule and the other acts like a polar molecule. The nonpolar end can attach to a grease or oil molecule, and the polar end can attach to a water molecule. Therefore, the soap serves as a chain, linking the dirt and water molecules together. When the water is rinsed away, the grease and oil go with it.

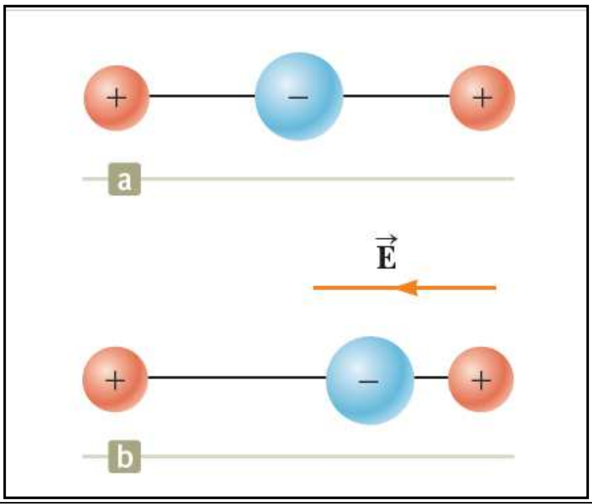


Figure 26 *(a) A linear symmetric molecule has no permanent polarization. (b) An external electric field induces a polarization in the molecule.*

A symmetric molecule (Fig. 26a) has no permanent polarization, but polarization can be induced by placing the molecule in an electric field. A field directed to the left as in Figure 26b causes the center of the negative charge distribution to shift to the right relative to the positive charges. This *induced polarization* is the effect that predominates in most materials used as dielectrics in capacitors.

An Atomic Description of Dielectrics

We found that the potential difference ΔV_0 between the plates of a capacitor is reduced to $\Delta V_0/k$ when a dielectric is introduced. The potential difference is reduced because the magnitude of the electric field decreases between the plates. In particular, if $\vec{E}_0$ is the electric field without the dielectric, the field in the presence of a dielectric is

$$\vec{E} = \frac{\vec{E}_0}{k} \qquad \text{--------------------(38)}$$

First consider a dielectric made up of polar molecules placed in the electric field between the plates of a capacitor. The dipoles (that is, the polar molecules making up the dielectric) are randomly oriented in the absence of an electric field as shown in Figure 27a. When an external field $\vec{E}_0$ due to charges on the capacitor plates is applied, a torque is exerted on the dipoles, causing them to partially align with the field as shown in Figure 27b. The dielectric is now polarized. The degree of alignment of the molecules with the electric field depends on temperature and the magnitude of the field. In general, the alignment increases with decreasing temperature and with increasing electric field.

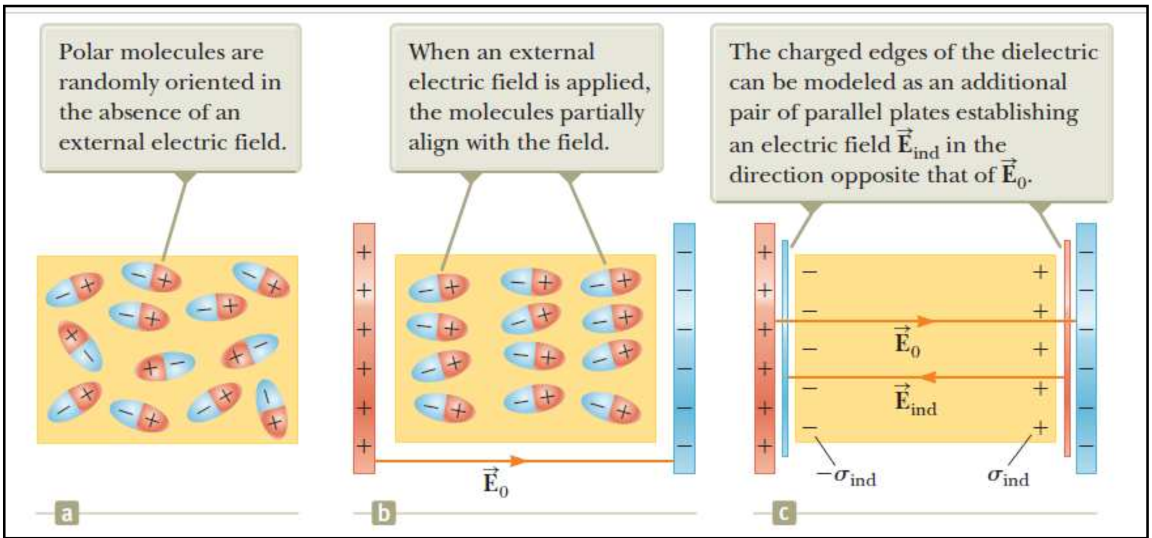


Figure 27 *(a) Polar molecules in a dielectric. (b) An electric field is applied to the dielectric. (c) Details of the electric field inside the dielectric.*

If the molecules of the dielectric are nonpolar, the electric field due to the plates produces an induced polarization in the molecule. These induced dipole moments tend to align with the external field, and the dielectric is polarized. Therefore, a dielectric can be polarized by an external field regardless of whether the molecules in the dielectric are polar or nonpolar.

With these ideas in mind, consider a slab of dielectric material placed between the plates of a capacitor so that it is in a uniform electric field $\vec{E}_0$ as shown in Figure 27b. The electric field due to the plates is directed to the right and polarizes the dielectric. The net effect on the dielectric is the formation of an *induced* positive surface charge density σ_{ind} on the right face and an equal-magnitude negative surface charge density $-\sigma_{ind}$ on the left face as shown in Figure 27c. Because we can model these surface charge distributions as being due to charged parallel plates, the induced surface charges on the dielectric give rise to an induced electric field $\vec{E}_{\text{ind}}$ in the direction opposite the external field $\vec{E}_0$. Therefore, the net electric field $\vec{E}$ in the dielectric has a magnitude

$$E = E_0 - E_{ind} \qquad \text{--------------------(39)}$$

In the parallel-plate capacitor shown in Figure 28, the external field E_0 is related to the charge density σ on the plates through the relationship $E_0 = \sigma/\varepsilon_0$. The induced electric field in the dielectric is related to the induced charge density σ_{ind} through the relationship $E_{\text{ind}} = \sigma_{\text{ind}}/\varepsilon_0$. Because $E = E_0/k = \sigma/k\varepsilon_0$, substitution into Equation 39 gives

$$\frac{\sigma}{k\varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_{ind}}{\varepsilon_0}$$

$$\sigma_{ind} = \left(\frac{k-1}{k}\right)\sigma \qquad \text{----------------------(40)}$$

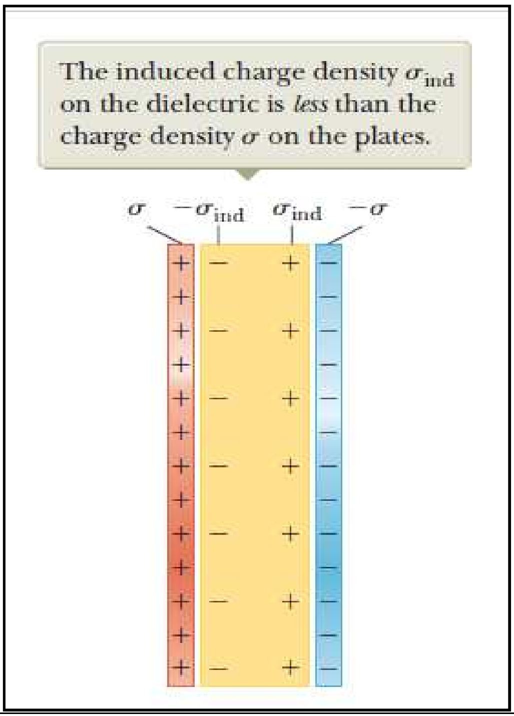


Figure 28. *Induced charge on a dielectric placed between the plates of a charged capacitor.*

Because k >1, this expression shows that the charge density sind induced on the dielectric is less than the charge density s on the plates. For instance, if k = 3, the induced charge density is two-thirds the charge density on the plates. If no dielectric is present, then k = 1 and $\sigma_{ind} = 0$ as expected. If the dielectric is replaced by an electrical conductor for which E = 0, however, Equation 40 indicates that $E_0 = E_{ind}$, which corresponds to $\sigma_{ind} = \sigma$. That is, the surface charge induced on the conductor is equal in magnitude but opposite in sign to that on the plates, resulting in a net electric field of zero in the conductor.