Differential eqn of

An eqn involving an unknown function y (dependent variable) and 1 or more of its derivatives y' y'' y''' is known as diff eqn.

Suppose a diff eqn involves only unknown function & its 1st order derivative is called 1st order diff eqn. If the eqn contains 2nd order derivative of unknown fun y then it is called 2nd order diff eqn.

Ex: $\frac{dx}{dt} + t = \sin t$ — 1st order

$\sin x \frac{d^2y}{dx^2} + e^1 y = 0$ — 2nd order

In general an n^{th} order linear diff eqn is of the form $P_0 \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots + P_n y = q(x)$

where P_0, P_1 ~~and~~ ... & $q(x)$ are continuous real valued fun on some interval & $P_0 \neq 0$.

In particular P_0, P_1 & .... P_n are constants $q(x)$ is a function of x then the eqn is called linear diff eqn with constant coefficients.

The theoretical structure and methods of soln that we develop for 2nd order linear diff eqn extend directly to linear diff eqn of 3rd & higher order diff eqns.

Second order linear diff eqns with const coeffs and its sols.

A general form of linear diff eqn with const coeff is in the form $a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = x$

Ex 2 $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin x$

3 $\frac{d^2y}{dx^2} + 3y = e^x$ $\quad \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

In eq ① if RHS is 0 then eqn is called Homogeneous LDE otherwise it is called non Homogeneous LDE.

The general sol of such eqn will depend on 2 constants.

An initial value problem for the second order eqn consist of finding soln of 2nd order eqn that satisfies the condition $y(x_0) = y_0$ $y'(x_0) = y_1$

Def: A boundary value problem for 2nd order diff eqn consists of finding soln of 2nd order diff eqn that satisfies condition $y(x_0) = y_0$ $y(x_1) = y_1$

The method to find general sol of the Homogeneous 2nd order LDE.

→ The general 2nd order ~~ho~~ linear diff eqn with const coeff is given by.

$a_0\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = 0 \qquad a_0 \neq 0$

$y(x) = c_1 y_1(x) + c_2 y_2(x)$

$y = c_1y_1 + c_2y_2$

Step 1: Write auxillary eqn (AE) for ①

i.e, $a_0m^2 + a_1m + a_2 = 0$

Step 2: Find roots of auxillary eqn then general soln is given by

case i): If the roots of auxilary eqn are real and diff m_1 & m_2 st $m_1 \neq m_2$

e.g. $y(x) = c_1e^{m_1x} + c_2e^{m_2x}$

case ii): If roots are real & repeated m_1 & m_2 st $m_1 = m_2$ then $y(x) = (c_1 + c_2x)e^{m_1x}$

$= c_1e^{m_1x} + c_2e^{m_2x}$

If the roots of eqn are pair of complex roots $m_1, m_2 = \alpha \pm i\beta$ $\quad y(x) = (c_1\cos\beta x + c_2\sin\beta x)e^{\alpha x}$

Q. Solve the following diff eqns

i) $9y'' - 12y' + 4y = 0$

Given diff eqn is homogeneous 2nd order LDE.

∴ Its general sol is $y(x) = c_1y_1(x) + c_2y_2(x)$

1. Write auxilary eqn

$9m^2 - 12m + 4 = 0$

$m = \frac{2}{3}$ (2 times)

$y = (c_1 + c_2x)e^{\frac{2}{3}x}$

$b^2 - 4ac > 0$ - real dist

$b^2 - 4ac = 0$ - real & repea

$b^2 - 4ac < 0$ - imaginary

ii) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = 0$

$y(x) = c_1y_1(x) + c_2y_2(x)$

$m^2 - 6m + 8 = 0$

$m = 4 \quad m = 2$ real & distinct.

$y = (c_1e^{2t} + c_2e^{4t})$

iii) $y'' - 4y' + 4y = 0$

$m^2 - 4m + 4 = 0$

$m = 2$ (2 times)

$y = (c_1 + c_2x)e^{2x}$

iv) $y'' - 2y' + 2y = 0$

$m^2 - 2m + 2 = 0$

$m = (1+i)\ (1-i)$

$y(x) = (c_1\cos x + c_2\sin x)e^x$

v) $\frac{d^2x}{dt^2} + 4x = 0$

$m^2 + 4 = 0$

$2i \quad -2i$

$x(t) = (c_1\cos 2t + c_2\sin 2t)e^{t}$

$8\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 5y = 0$

$8m^2 + 12m + 5 = 0$ $\quad y(t) = (C_1 \cos\frac{1}{4}t + C_2 \sin\frac{1}{4}t)e^x$

$m = -\frac{3}{4} + \frac{1}{4}i$

$m = -\frac{3}{4} - \frac{1}{4}i$

Q. Solve following DE

1. $4y'' - 4y' + y = 0$ $\quad y(0) = 1 \quad y'(0) = 1.5$

$4m^2 - 4m + 1 = 0 \quad m = \frac{1}{2}$ (2 times)

$y = C_1 e^{\frac{1}{2}x} + C_2 e^{\frac{1}{2}x} = e^{\frac{1}{2}x}(C_1 + C_2)$

G.S is $y = (C_1 + C_2 x)e^{\frac{1}{2}x}$

Given $y(0) = 1$ i.e at $x=0$ $y=1$

$1 = (C_1 + C_2(0))e^0 \Rightarrow C_1 = 1$

$y'(0) = 1.5$ i.e, $x=0$ $y' = 1$

$y' = C_1 e^{\frac{1}{2}x} \times \frac{1}{2} + C_2\left(x e^{\frac{1}{2}x} \times \frac{1}{2} + e^{\frac{1}{2}x}\right) = 1.5$

$= \frac{C_1}{2} + C_2(1) = 1.5 \quad \cancel{C_2 = \frac{1}{4}} \quad C_2 = 1$

Substituting these values in eq ①. req soln is $y = (1+x)e^{\frac{x}{2}}$

2. $y'' - 3y' + 2y = 0$ $\quad y(0) = 1$ & $y(3) = 0$.

$m^2 - 3m + 2 = 0 \quad m = 2, m = 1$

$y = C_1 e^{2x} + C_2 e^x$

$y(0) = 1 \quad 1 = C_1 + C_2 \quad y(3) = 0. \quad 0 = C_1 e^6 + C_2 e^3$

$C_2 = C_1 e^2$

$1 = C_1 + C_1 e^2$

$= C_1(1 + e^2)$

$C_1 = \frac{+1}{1 + e^2} =$

C_1

$C_2 = 0.00523$

Q. $y'' - 2y' + 5y = 0$ $\quad y(\pi) = 0 \quad y'(\pi) = 2$

$m^2 - 2m + 5 = 0 \quad m_1 = 1 + 2i \quad m_2 = 1 - 2i$

$y = (C_1 \cos 2x + C_2 \sin 2x)e^{x}$

$y(\pi) = (C_1)e^x = 0. \quad C_1 = 0$

$y'(\pi) = 2 \quad y' = C_1(e^x(-\sin 2x \cdot 2) + e^x \cos 2x)$

$+ C_2(e^x \sin 2x + \cos 2x \cdot 2e^x) = 2$

$C_2 \cdot (e^\pi \times 0 + \cos 2\pi \cdot 2e^\pi) = 2$

$C_2 = \frac{2}{2e^\pi} = e^{-\pi}$

$y = (e^{-\pi} \sin 2x)e^x$

Q. $y'' - 6y' + 9y = 0$ $\quad y(0) = 1$ & $y(1) = 0$.

$m^2 - 6m + 9 = 0. \quad m = 3$ (2 times) $y = C_1 e^{3x} + C_2 e^{3x}$

$y(0) = 1 \Rightarrow 1 = e^0(C_1 + C_2) \Rightarrow C_1 + C_2 = 1$

$y(1) = 0 \Rightarrow 0 = e^3(C_1 + C_2) \Rightarrow C_1 + C_2 = 0$

$C_1 = 1 \quad C_2 = -1$

$y = e^{3x} - e^{3x}$

Q. $y'' - 6y' + 8y = 0$ $\quad y(0) = 2 \quad y'(0) = 2$

$m^2 - 6m + 8 = 0 \quad m_1 = 4 \quad m_2 = 2 \quad y = C_1 e^{4x} + C_2 e^{2x}$

$C_1 = 2 - C_2$

$y(0) = 2 \Rightarrow 2 = C_1 + C_2$

$y'(0) = 2 \Rightarrow C_1 e^{4x} \cdot 4 + C_2 e^{2x} \cdot 2 = 2$

$4C_1 + 2C_2 = 2$

$4(2 - C_2) + 2C_2 = 2$

$8 - 4C_2 + 2C_2 = 2$

$8 - 2 = 2C_2 \quad C_2 = 3$

$C_1 = 2 - 3 = -1$

$y = -e^{4x} + 3e^{2x}$

$y'' + 4y = 0$. $y(0) = 3$ ~~$y(0) =$~~ $y(\pi/4) = -2$.

$m^2 + 4 = 0$

~~$m = \pm 2$~~ ~~$y = C_1 e^{2x} + C_2 e^{-2x}$~~

~~$y(0) = 3$ $3 = C$~~

$m = 2i$ $m = -2i$

$y = (C_1 \times \cos 2x - C_2 \sin 2x) e^{0}$

$3 = C_1$

$-2 = (C_1 \times 0 - C_2(1))$ $C_2 = 2$

$y = (3\cos 2x - C_2\, 2 \sin 2x) e^{0x}$

$y = (3 \cos 2x - 2 \sin 2x)$

Method of variation of Parameters

consider a non-homogeneous LDE with const coeff.

$$a_0 \frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = X \rightarrow (1)$$

where a_0, a_1, a_2 are consts & X is function of. The general soln of (1) is

y = complementary fun + Particular integral
= CF + PI.

where CF is g.s of homogeneous eqn

i.e, $f(D) y = 0$

Methods of finding particular integral.

1. Method of variation of parameters
This is a general method to find the particular solution for given non homogeneous

2. Method to find particular integral
Find general soln of homo diff eqn & denote it as CF i.e, $CF = C_1 y_1(x) + C_2 y_2(x)$

2. Particular integral is obtained by replacing constants C_1 & C_2 by A & B then

$PI = A y_1(x) + B y_2(x)$ A & B are functions of x & are given by the formula

$$A = \int \frac{-y_2}{W} \times dx$$

$$B = \int \frac{y_1}{W} \times dx$$

y_1 & y_2 are solns in complementary functions X is RHS of given eqn & W is Wronskain of solns Y_1 & Y_2 and is given by $W = \begin{bmatrix} y_1 & y_2 \\ y_1' & y_2' \end{bmatrix}$

Substituting these values in PI we get particular soln for the given non-homogenous eqn. Required gen sol is y = complementary f + PI

Q. Solve $y'' + y = \tan x$ $0 \leq x < \pi/2$

→ This is non homogeneous diff eqn $X = \tan x$

∴ G.S is y = comp F + PI

To find CF

consider $y'' + y = 0$

AE $m^2 + 1 = 0$ $m = \pm i = \alpha \pm i\beta$

∴ $y = C_1 \cos x + C_2 \sin x$

To find particular integral

$PI = A \cos x + B \sin x = A y_1 + B y_2$

$A = \int \frac{-y_2}{W} \times dx$; $B = \int \frac{y_1}{W} \times dx$

now $A = \int -\frac{\sin x}{W} \times dx$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$A = -\int \frac{\sin x}{1} \times dx = -\int \sin x \tan x \, dx$$

$$= -\int \frac{\sin^2 x}{\cos x} \, dx$$

$$= -\int \frac{1-\cos^2 x}{\cos x} \, dx$$

$$= -\int \sec x - \cos x \, dx = -\cancel{\sec x \tan x} + \sin x.$$

$$= -\log|\sec x + \tan x| + \sin x$$

$$B = \int \frac{y_1}{W} \times dx = \int \cos x \tan x \, dx$$

$$= \int \sin x \, dx = -\cos x$$

$\therefore$ $PI = A\cos x + B\sin x.$

$$= (-\log|\sec x + \tan x| + \sin x)\cos x - \sin x \cos x$$

$$= -\log|\sec x + \tan x| \cdot \cos x$$

$$y = CF + PI$$

$$= C_1\cos x + C_2 \sin x - \log|\sec x + \tan x| \cdot \cos x$$

5 ii) $y'' - 3y' + 2y = \dfrac{1}{1+e^{-x}}$

It is a non-hom d.e. $\therefore$ it g.s is

$y = CF + PI$

$CF = m^2 - 3m + 2 = 0 \quad m = 2 \quad m = 1$

$y = C_1 e^{2x} + C_2 e^{x} \qquad Ae^{2x} + Be^{x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \begin{vmatrix} e^{2x} & e^{x} \\ 2e^{2x} & e^{x} \end{vmatrix}$$

$$= e^{3x} - 2e^{3x} = e^{3x}.$$

$$A = -\int \frac{y_1}{W} \times dx = -\int \frac{e^{2x}}{e^{3x}} \times \frac{1}{1+e^{-x}} \, dx$$

$$= -\int \frac{e^{-x}}{1+e^{-x}} \, dx$$

Put $1 + e^{-x} = t$

$-e^{-x} dx = dt$

$A = \int + \frac{dt}{t} = +\ln t \qquad A = +\ln(1+e^{-x})$

$$B = -\int \frac{e^{x}}{e^{3x}} \times \frac{1}{1+e^{-x}} \, dx$$

$$= \int \frac{e^{-2x}}{1+e^{-x}} \, dx \qquad 1 + e^{-x} = t, \quad -e^{-x} dx = dt$$

$$= -\int \frac{e^{-x}}{t} \, dt = -\int \frac{t-1}{t} \, dt$$

$$= \int \frac{1}{t} - 1 = \ln t - t$$

$$B = \ln(1+e^{-x}) - (1+e^{-x})$$

$$PI = \ln(1+e^{-x}) \cdot e^{2x} + \ln(1+e^{-x}) \cdot e^{2x} - (1+e^{-x})e^{x}$$

$$= \ln(1+e^{-x})(e^{x} + e^{2x}) - (1+e^{-x})e^{2x}$$

$y = CF + PI$

$= C_1 e^{2x} + C_2 e^{x} + \ln(1+e^{-x})(e^{x} + e^{2x}) - (1+e^{-x})e^{2x}$

iii) $y'' - 2y' + y = e^{x} \log x \qquad X = e^{x} \log x$

$m^2 - 2m + 1 = 0. \quad m = 1$ (2 times)

$y = (C_1 + C_2 x)e^{x} \qquad Ay_1(x) + By_2(x)$

$Ae^{x} + Be^{x}x.$

$$W \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \cancel{\begin{vmatrix} e^{x} & e^{x} \\ e^{x} & e^{x} \end{vmatrix}} \begin{vmatrix} e^{x} & xe^{x} \\ e^{x} & xe^{x} + e^{x} \end{vmatrix}$$

$$e^{x}(xe^{x} + e^{x}) - e^{x}(xe^{x}) = e^{2x}$$

$$= e^{2x}$$

$A = -\int \frac{y_2}{W} \times dx = -\int \frac{e^x \cdot x}{e^{2x}} dx \times e^x \log x$

$= -\int e^{-x} \cdot x \, dx$

$= \cancel{x(-e^{-x}) - \int (-e^{-x}) dx}$

$= \cancel{x e^{-x} - e^{-x} dx}$

$\cancel{A = x e^{-x} + e^{-x}}$

$\cancel{B = \int \frac{y_1}{W}}$

$A = -\int x \log x \, dx$

$= -\left(\log x \cdot \frac{x^2}{2} - \int \left(\frac{1}{x} \times \frac{x^2}{2}\right)\right)$

$= -\log x \cdot \frac{x^2}{2} + \frac{x^2}{4}$

$B = \int \frac{e^x}{e^{2x}} e^x \log x \, dx$

$= \int \log x \, dx = x \log x - x$

$PI = -\log x \cdot \frac{x^2}{2} \cdot e^x + \frac{x^2}{4} e^x + e^{2x} (x \log x - x)$

9. $y'' - 2y' + 2y = e^x \tan x$

$m^2 - 2m + 2 = 0 \quad m_1 = 1+i \quad m_2 = 1-i$

$CF = y = (C_1 \cos x + C_2 \sin x) e^x$

$PI = A y_1(x) + B y_2(x) = A \cos x e^x + B \sin x e^x$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x e^x & \sin x e^x \\ -e^x \sin x + \cos x e^x & e^x \cos x + \sin x e^x \end{vmatrix}$

$= e^{2x} \cos^2 x + \sin x \cos x e^{2x} + e^{2x} \sin^2 x - \sin x \cos x e^{2x}$

$= e^{2x} + \cancel{e^{2x} \sin x \cos x} = e^{2x} (1 + \cancel{\sin x \cos x})$

$A = -\int \frac{y_2}{W} \times dx = \int \frac{\sin x e^x}{e^{2x}(1+\sin 2x)} e^x \tan x \, dx$

$= \int \frac{\sin x \tan x}{1 + \sin 2x} dx$

$\int \frac{\sin^2 x}{\cos x} dx = \int \frac{1 - \cos^2 x}{\cos x} dx$

$= \int \sec x - \cos x \, dx$

$= -\log |\sec x + \tan x| + \sin x$

$B = \int \frac{y_1}{W} \times dx = \int \frac{\cos x e^x}{e^{2x}} e^x \tan x \, dx$

$= -\cos x$

$PI = (-\log |\sec x + \tan x| + \sin x) \cos x e^x$

$- \cos x (\sin x) e^x = \log |\sec x + \tan x|$

$y = CF + PI$

$= (C_1 \cos x + C_2 \sin x) e^x - \log |\sec x + \tan x|$

10. $y'' - 2y' + y = \frac{e^x}{1+x^2} \quad X = \frac{e^x}{1+x^2}$

$m^2 - 2m + 1 = 0 \quad m = 1 \text{ (2 times)}$

$y = C_1 e^x + C_2 x e^x \qquad PI = A y_1(x) + B y_2(x)$

$W = \begin{vmatrix} e^x & x e^x \\ e^x & x e^x + e^x \end{vmatrix}$

$W = x e^{2x} + e^{2x} - x e^{2x} = e^{2x}$

$A = -\int \frac{x e^x}{e^{2x}} \cdot \frac{e^x}{1+x^2} dx = \int \frac{x}{1+x^2} dx$

$1 + x^2 = t \quad 2x \, dx = dt$

$= -\int \frac{1}{2} \times \frac{1}{t} dt = -\frac{\log |t|}{2}$

$B = \int \frac{e^x}{e^{2x}} \cdot \frac{e^x}{1+x^2} dx = \int \frac{1}{1+x^2} dx = \tan^{-1} x$

$\cancel{A}$ PI $-\frac{\log(1+x^2)}{2} e^x + \tan^{-1}x \cdot xe^x$.

$y = CF + PI$

$= C_1e^x + C_2xe^x - \frac{\log(1+x^2)}{2}e^x + \tan^{-1}x \cdot xe^x$.

vi) $y'' + 3y' + 2y = \sin(e^x)$ $\quad X = \sin(e^x)$

$m^2 + 3m + 2 = 0 \quad m = -1 \quad m = -2$

$y = C_1 e^{-x} + C_2 e^{-2x}$

$\cancel{PI}$ $W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$.

$W = -2e^{-3x} + e^{-3x} = -e^{-3x}$

$A = -\int \frac{e^{-2x}}{-e^{-3x}} \sin e^x \, dx = \int e^x \sin e^x \, dx$.

$e^x = t \quad e^x dx = dt$

$\int \sin t \, dt = -\cos e^x$

$B = \int \frac{e^{-x}}{-e^{-3x}} \cancel{dt} \sin e^x \, dx = -\int e^{+2x} \sin e^x dx$

$e^x = t \quad e^x dx = dt$

$\int t \sin t \, dt = -\cos t \cdot t - \int(-\cos t)$

$= -\cos t \cdot t + \sin t$

$= -\sin e^x + \cos e^x \cdot e^x$

$PI = -\cos e^x \times e^{-x} + (\sin e^x + \cos e^x \cdot e^x)e^{-2x}$

$= -\sin e^x \cdot e^{-2x} = -\sin(e^{+x}) \cdot e^{-2x}$.

$y = CF + PI$

$= C_1e^{-x} + C_2e^{-2x} \cancel{+} - \sin(e^x) \cdot e^{-2x}$.

vii) $y'' + y = \frac{1}{1+\sin x} \Rightarrow X = \frac{1}{1+\sin x}$

$m^2 + 1 = 0. \quad m = i \quad m = -i$

$y = (C_1 \cos x + C_2 \sin x)$

$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$

$A = -\int \frac{\sin x}{1} \times \frac{1}{1+\sin x} dx = -\int \frac{\sin x}{1+\sin x} dx$

$= -\int \frac{1+\sin x}{1+\sin x} - \frac{1}{1+\sin x} dx$

$= -\int 1 \, dx + \int \frac{1}{1+\sin x} dx$

$= -x + \int \frac{1}{\cos^2 x/2 + \sin^2 x/2 + 2\sin x/2 \cos x/2} dx$

$= -x + \int \frac{1}{(\cos x/2 + \sin x/2)^2} dx$.

$= -x + \int \frac{dx}{\cos^2 x/2 (1 + \tan x/2)^2}$.

$= -x + \int \frac{\sec^2 x/2}{(1+\tan x/2)^2} dx$

$= \cancel{-x + 2\ln(1+\tan\frac{x}{2})} = -x +$

$B = \int \frac{\cos x}{1+\sin x} dx$

$= -x + \int \frac{1}{t^2} dt$

$= -x - \frac{2}{(1+\tan x/2)}$

$B = \int \frac{\cos x}{1+\sin x}\, dx$

put $1+\sin x = t$

$B = \int \frac{1}{t}\, dt = \ln(1+\sin x)$

$PI =$

Q. $y'' + y = \log(\cos x)$ $\quad X = \log(\cos x)$

$m^2 + 1 = 0 \quad m = i \quad m = -i$

$y = C_1 \cos x + C_2 \sin x$

$A = \int \quad W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$

$A = -\int \frac{\sin x}{1} \log(\cos x)\, dx$

$\cos x = t \quad -\sin x\, dx = dt$

$A = -\int -\log t\, dt = \frac{t}{t} \quad t\log t - t$

$A = \cos x \log(\cos x) - \cos x$

$B = \int \cos x \cdot \log(\cos x)\, dt$

$= \log(\cos x) \cdot \sin x - \int \left(\frac{1}{\cos x} \cdot -\sin x \cdot \sin x\right) dx$

$= \log(\cos x) \cdot \sin x + \int \frac{1-\cos^2 x}{\cos x}\, dx$

$= \log(\cos x) \cdot \sin x + \log|\sec x + \tan x| - \sin x.$

$y = C_1 \cos x + C_2 \sin x$

Method of finding PI to particular function of X.

In shortcut method particular integral is always given by the formula

$PI = \frac{1}{f(D)} X \quad X \to$ RHS of eqn

$f(D) = a_0 D^2 + a_1 D + a_2$

case i) When $X = e^{ax}$

$PI = \frac{1}{f(a)} e^{ax}$ provided $f(a) \neq 0$.

if $f(a) = 0$, it is called case of failure

then $PI = x\left[\frac{1}{f'(a)} e^{ax}\right] \quad [f'(a) \neq 0]$

If $f'(a) = 0$ then $PI = x^2 \left[\frac{1}{f''(a)} e^{ax}\right] \quad [f''(a) \neq 0]$

this process is continued till we have non zero denominator.

complete solution is

$y = CF + PI$

for eg:

Solve the following DE.

i) $y'' - 4y' + 5y = e^{-x}$

This is non homogeneous DE with $X = e^{-x}$

Its soln is $y = CF + PI$

To find CF

$m^2 - 4m + 5 = 0 \quad m = 2+i \quad m = 2-i$

$y = (C_1 \cos x + C_2 \sin x)e^{2x}$

To find PI

Here $X = e^{-x} = e^{ax}$ (We use shortcut method)

Here $a = -1$

$PI = \frac{1}{f(D)} X = \frac{1}{D^2 - 4D + 5} e^{-x} = \frac{1}{(-1)^2 - 4(-1) + 5} e^{-x}$

$= \frac{1}{10} e^{-x}$

Required soln is CF + PI

$(C_1 \cos x + C_2 \sin x)e^{2x} + \frac{1}{10}e^{-x}$

Q. $y'' + 9y = e^{3x}$

$m^2 + 9 = 0$. $m = 3i$ $m = -3i$

$e^{ax} = e^{3x}$ $a = 3$

$y = C_1 \cos 3x + C_2 \sin 3x$

$PI = \frac{1}{f(D)} = \frac{1}{(3)^2+9} e^{3x} = \frac{1}{18} e^{3x}$

$y = C_1 \cos 3x + C_2 \sin 3x + \frac{1}{18} e^{3x}$

Q. $y'' - 2y' + y = e^{2x}$

$m^2 - 2m + 1 = 0$. ~~$m = 1+\sqrt{2}$ $m = 1-\sqrt{2}$~~. $m = 1$ (2 times)

$y = C_1 e^x + C_2 e^x . x$

$PI = \frac{1}{f(D)} = \frac{1}{4-4+1} e^{2x} = \frac{e^{2x}}{1}$

$y = C_1 e^x + C_1 e^x . x + e^{2x}$

Q. $(D^2 - 4D + 4)y = e^{2x}$

$D = 2$ (2 times)

$CF = (C_1 + C_2 x)e^{2x}$

$= \frac{1}{f(D)} x = \frac{1}{(D^2-4D+4)} e^{2x} = \frac{1}{4-8+4} e^{2x}$

$= \frac{1}{0} e^{2x}$

$= x \left[\frac{1}{f'(D)} e^{2x}\right]_{D \to 2} = x\left[\frac{1}{4-4} e^{2x}\right]$ case of failure.

$= x^2 \left[\frac{1}{2} e^{2x}\right] = \frac{x^2}{2} e^{2x}$.

Q. $y'' - 6y' + 8y = e^{2x} + e^x$

CF $y = C_1 e^{4x} + C_2 e^{2x}$

$PI = \frac{1}{f(D)} X = \frac{1}{D^2 - 6D + 8} (e^{2x} + e^x)$

$= \frac{1}{4-12+8} e^{2x} + \frac{1}{1-6+8} e^x$

$= \frac{1}{0} e^{2x} + \frac{1}{3} e^x$

$= x\left[\frac{1}{2D-6}\right] e^{2x} + \frac{1}{3}e^x = x\left(\frac{-1}{2}\right) e^{2x} + \frac{1}{3}e^x$

Q. $(D^2 - D - 6)y = e^{-2x} + e^{2x}$ $D = 3$ $D = -2$

$CF = C_1 e^{3x} + C_2 e^{-2x}$

$PI = \frac{1}{9-3-6} e^{3x} + \frac{1}{4+4-6} e^{-2x}$

$= \frac{1}{0} e^{3x} + \frac{1}{2} e^{-2x}$

$= \frac{1}{2D-1} e^{3x} + \frac{1}{2} e^{-2x}$

$= \frac{1}{5} e^{3x} + \frac{1}{2} e^{-2x}$

$y = C_1 e^{3x} + C_2 e^{-2x} + \frac{1}{5} e^{3x} + \frac{1}{2} e^{-2x}$

Q. $y'' - 4y' + 4y = e^{2x} + e^{-x}$ $D = 2$ (2 times)

$D^2 - 4D + 4 = 0$.

$CF = (C_1 + C_2 x) e^{2x}$

$PI = \frac{1}{D^2 - 4D + 4} X = \frac{1}{4-8+4} e^{2x} + \frac{1}{1+4+4} e^{-x}$

$= \frac{1}{0} e^{2x} + \frac{1}{9} e^{-x}$

$y = \frac{x(1)}{2D-4} e^{2x} + \frac{1}{9} e^{-x}$

$= \frac{x^2}{2} e^{2x} + \frac{1}{9} e^{-x}$

Case 2: X = polynomial in x.

$X = \phi(x) = a x^n + a_1 x^{n-1} + \dots + a_n$

$PI = \frac{1}{f(D)} X = \frac{1}{f(D)} \phi(x)$

Here we find PI by division method by writing $\phi(x)$ in descending powers of x.

$f(D)$ is ascending powers of D

when remainder = 0.

PI = quotient of division

for eg:

$(D^2+3D+2)y = 12x^2$

non homogeneous DE

$y = CF + PI$

$CF = c_1 e^{-x} + c_2 e^{-2x}$.

To find PI

$PI = \frac{1}{f(D)} X = \frac{1}{D^2+3D+2} 12x^2$

$2+3D+D^2 \overline{)\, 12x^2}$ $6x^2$

$12x^2 + 36x$

$6x^2$

Q. $(D^2+6D+9)y = 1+x$

$PI = \frac{1}{D^2+6D+9}(1+x)$

$9+6D+D^2 \overline{)\, x+1}$ $\frac{x}{9} + \frac{1}{27}$

$-x \pm 2/3$

$1/3$

$-1/3$

0.

Q. $y'' + 3y' + 2y = x^2$

$PI = D^2+3D+2 \overline{)\, x^2}$ $\frac{x^2}{2} - \frac{3}{2}x + \frac{7}{4}$

$-x^2 \pm 3x \pm 1$

$-3x - 1$

$-3x \mp 9/2$

$7/2$

$-7/2$

0.

Q. $y'' - y + 6y = e^x + 1 + x^2$

$PI = \frac{1}{D^2-D+6} e^x = \frac{1}{6} e^x$.

$\cancel{6-D+D^2} \overline{)\, \cancel{x^2} \; x}$ $\frac{1}{6}$

$6-D+D^2 \overline{)\, x^2+x \quad +1}$ $\frac{x^2}{6} + \frac{x}{18} + \frac{13}{108}$

$-x^2 - \frac{x}{3} \quad \mp \frac{1}{3}$

$-\frac{x}{3} + \frac{2}{3}$

$-\frac{x}{3} + \frac{1}{18}$

$+ \quad -$

$\frac{13}{18}$

$\frac{13}{18}$

0.

case iii) $\sin ax$ or $\cos ax$

1) Replace only D^2 by $-a^2$ in $f(D)$
& D retain as it is
f(

2) Rationalise denominator and then replace D^2 by $-a^2$

3) Operate 'x' on each term of numerator

if $f(D) = 0$.

Here we multiply numerator by x & denominator diff wrt D

$$x = x\left(\frac{1}{f'(D)} \sin ax \;\middle|\; \cos ax\right)$$

Q. Solve $4y'' + y = \cos x$

$$m = \pm \frac{i}{2}$$

$$CF = \left[C_1 \cos \tfrac{1}{2}x + C_2 \sin \tfrac{1}{2}x\right] e^{0x}$$

$$PI = \frac{1}{f(D)} X = \frac{1}{4D^2+D} \cos x \quad \text{Here } a = 1$$

Replace D^2 by $-a^2$, $= -1$

$$= \frac{1}{4-1} \cos x = \frac{1}{3} \cos x$$

$\therefore$ $y = CF + PI$

Q. $y'' - 2y' = \sin 4x$

$$PI = \frac{1}{f(D)} X = \frac{1}{D^2 - 2D} \sin 4x \quad \text{Here } a = 4$$

-16

$$= \frac{1}{(-16)} = \frac{-1}{224} \sin 4x$$

$m^2 - 2m = 0 \quad m = 0, 2$.

$$CF = C_1 e^{0x} + C_2 e^{2x}$$
$$= C_1 + C_2 e^{2x}$$

$$= \frac{1}{-16-2D} \times \frac{(-16+2D)}{(-16+2D)} \times \sin 4x$$

$$= \frac{-16+2D}{(256 - 4D^2)} \times \sin 4x$$

$$= \frac{-16 + 2D \times \sin 4x}{320}$$

$$= \frac{-16 \sin 4x + 20 \sin 4x}{320} \rightarrow 2 \frac{d}{dt}(\sin 4x)$$

$$= \frac{1}{320}(-16 \sin 4x + 8 \cos 4x)$$

Q. $y'' + 4y = e^{3x} + \sin 2x$

$D^2 + 4 = 0$.

$$= \frac{1}{D^2+4} e^{3x} + \frac{1}{D^2+4} \sin 2x$$

$$= \frac{1}{13} e^{3x} + \frac{1}{D^2+4} \sin 2x \quad (D^2 \rightarrow -a^2)$$

$$= \frac{1}{13} e^{3x} + \frac{1}{-4+4} \sin 2x$$

$$= \frac{1}{13} e^{3x} + x\left[\frac{1}{2D}\right] \sin 2x$$

$$= \frac{1}{13} e^{3x} + x\left[\frac{-D}{-2D^2}\right] \sin 2x$$

$$= \frac{e^x}{13} + x\left[\frac{-D}{8}\right] \sin 2x \qquad -\frac{x}{8} \frac{d}{dx}(\sin 2x)$$

$$-\frac{x}{8} \cos 2x \cdot 2$$

$$= \frac{e^x}{13} - \frac{x}{4} \cos 2x \qquad -\frac{x}{4} \cos 2x$$

Q. $y'' + y' - 2y = x + \sin 2x \quad y(0) = 1 \quad y'(0) = 0$

$PI = \frac{1}{D^2+D-2} x + \frac{1}{D^2+D-2} \sin 2x$

$D^2+D-2 \,)\, x \,(\, -\frac{x}{2} - \frac{1}{4}$

$\quad -x - \frac{1}{2}$

$\quad \frac{1}{2}$

$\quad \frac{1}{2}$

$\quad 0$

$= -\frac{x}{2} - \frac{1}{4} + \frac{1}{D-6} \sin 2x$

$= -\frac{x}{2} - \frac{1}{4} + \frac{D-6}{D^2-6^2} \sin 2x$

$= -\frac{x}{2} - \frac{1}{4} + \frac{D-6}{-40} \sin 2x$

$= -\frac{x}{2} - \frac{1}{4} - \frac{1}{40}(\cos 2x . 2 - 6 \sin 2x)$

$= -\frac{x}{2} - \frac{1}{4} - \frac{1}{20} (\cos 2x - 3 \sin 2x)$

case 4: when $X = e^{ax} V$ where V is any function of x
V - polynomial or / trigonometric ($\sin ax$ or $\cos ax$)
then Particular integral $= \frac{1}{f(D)} . X = \frac{1}{f(D)} e^{ax} V$

$= e^{ax} \left(\frac{1}{f(D)}\right)_{D \to D+a} V = e^{ax} \left[\frac{1}{f(D+a)} V\right]$

Q. Solve the following D.E.

i) $y'' + 2y' + y = x e^{-x}$

This is a non homogenous DE with $X = x e^{-x}$ (i.e, $e^{ax} V$ where $a = -1$ & $V = x$)

$\therefore$ Its solution is $y = CF + PI$

To find CF

AE : $m^2 + 2m + 1 = 0$. ($m = -1$ (2 times))

$CF = (C_1 + C_2 x) e^{-x}$

To find $PI = \frac{1}{f(D)} X = \frac{1}{D^2+2D+1} e^{-x} x$

$= e^{-x} \left[\frac{1}{D^2+2D+1} x\right]_{D \to D-1}$

$\int \frac{1}{D} x = \int x \, dx$

$\int \frac{1}{D^2} x = \iint x \, dx \, dx$

$= e^{-x} \left[\frac{1}{(D-1)^2 + 2(D-1) + 1} x\right]$

$= e^{-x} \left[\frac{1}{D^2} x\right]$

$= e^{-x} \iint x \, dx \, dx$

$= e^{-x} \int \frac{x^2}{2} dx$

$= e^{-x} \frac{x^3}{6}$

ii) Solve $y'' - 4y = e^x \cos x$: $y(0) = 1$ & $y'(0)$

$m^2 - 4 = 0$. $m = 2, -2$

$CF = C_1 e^{2x} + C_2 e^{-2x}$.

To find PI :

$PI = \frac{1}{f(D)} X = \frac{1}{D^2-4} e^x \cos x \quad (a=1 \quad V=\cos x)$

$= e^x \left(\frac{1}{D^2-4} \cos x\right]_{D \to D+1}$

$= e^x \left[\frac{1}{(D+1)^2-4} \cos x\right] = e^x \left[\frac{1}{D^2+1+2D-4} \cos x\right]$

$= e^x \left[\frac{\cos x}{D^2+2D-3}\right]$ Replace D^2 by $-a^2$

$= e^x \left[\frac{\cos x}{-1+2D-3}\right] = e^x \left[\frac{\cos x}{+2D-4}\right]$

$= \frac{e^x}{2} \left[\frac{D+2}{D^2-4} \cos x\right] = \frac{e^x}{2} \left[\frac{D+2}{-5} \cos x\right]$

$= \frac{e^x}{-10} (-\sin x + 2\cos x)$

$y = CF + PI = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{10} e^x (2\cos x - \sin x)$

$y(0) = c_1 + c_2 - \frac{1}{10}(2) \Rightarrow c_1 + c_2 - \frac{1}{5} = 1$

$c_1 + c_2 = \frac{6}{5}$

$y'(0) = c_1 e^{2x}.2 - c_2 e^{-2x}.2 - \frac{1}{10}\left(e^x 2\sin x - 2\cos x - 2e^x \cos x - e^x \sin x\right)$

$= c_1 e^{2x}.2 - c_2 e^{-2x}.2 - \frac{1}{5}(-e^{-x}\sin x - \cos x e^x - e^x \cos x - e^x \sin x)$

$= 2c_1 - 2c_2 - \frac{1}{5}(-1-1)$

$= 2c_1 - 2c_2 + \frac{2}{5} = 0.$

$c_1 - c_2 = \frac{21}{20}$ $\qquad 2c_1 = \frac{9}{4}$

$c_1 + c_2 = \frac{6}{5}$ $\qquad c_1 = \frac{9}{8}$

$c_2 = \frac{3}{40}$

substituting c_1 & c_2 in eqn a.

$y = \frac{9}{8} e^{2x} + \frac{3}{40} e^{-2x} - \frac{1}{10} e^x (2\cos x - \sin x)$

Q. $y'' - y' = xe^x \quad y(0) = 2 \quad y'(0) = 1$

$m^2 - m = 0 \qquad m = +1, 0$

CF $y = c_1 e^x + c_2$.

$PI = \frac{1}{f(D)} X = \frac{1}{D^2-D} xe^x = e^x \left[\frac{1}{D^2-D} x\right]_{D \to D+1}$

$= e^x \left(\frac{1}{(D+1)^2-(D+1)} x\right] = e^x \left[\frac{1}{D^2+1+2D-D-1} x\right]$

$= e^x \left[\frac{1}{D^2+D} x\right] = e^x \left[\frac{1}{D^2+D} x\right]$

consider $\frac{1}{D^2+D} x. = \frac{1}{D(D+1)} x = \frac{1}{D}\left[\frac{1}{D+1} x\right]$

$1+D \overline{) \, x \,}$ $x-1$.

$\underline{x + \cancel{x}}$

-1

$\underline{-1}$

0

$\therefore \frac{1}{D}\left[\frac{x}{D+1}\right] = \frac{1}{D}(x-1) = \int (x-1)\,dx$

$= \frac{x^2}{2} - x.$

$\therefore PI = e^x \left[\frac{x^2}{2} - x\right]$

$y = c_1 + c_2 e^x + e^x \left(\frac{x^2}{2} - x\right)$

$y(0) = c_1 + c_2 + [0)$

$c_1 + c_2 = 2$

$y' = c_2 e^x + e^x \frac{x^2}{2} + e^x \frac{2x}{2} - e^x x - e^x$

$= c_2 - 1 = 0 \quad c_2 = 1 \quad c_1 = 2 - 1 =$

$\cancel{c_1+c_2=2}$

$c_1+c_2=2$

Q. $y''+9y = 1+xe^x$

$m^2+9m=0$. $m=\pm 3i$

$CF = c_1\cos 2x + c_2\sin 3x$

$$PI = \left[\frac{1}{D^2+9}(1+xe^x)\right]$$

$$= \frac{1}{D^2+9} + \frac{xe^x}{D^2+9}$$

$$= \frac{1}{D^2+9}e^{0x} + \frac{xe^x}{D^2+9}$$

$$= \frac{1}{9}e^{0x} + e^x\left[\frac{1}{D^2+9}x\right]$$

$$= \frac{1}{9} + e^x\left[\frac{1}{(D+1)^2+9}x\right]$$

$9+D^2$ $\cancel{x}$ $\cancel{x}$ $= \frac{1}{9}+e^x\left[\frac{1}{D^2+2D+1+9}x\right]$

$$= \frac{1}{9}+e^x\left[\frac{1}{D^2+2D+10}x\right]$$

$10+2D+D^2 \,\overline{)\,x\,}\; \frac{x}{10}-\frac{1}{50}$

$-\frac{x}{}+\frac{1}{5}$

$-\frac{1}{5}$

$-\frac{1}{}$

$+\frac{}{5}$

0

$$= \frac{1}{9}+e^x\left[\frac{x}{10}-\frac{1}{50}\right]$$

Q. $y''-2y'+4y = e^x\cos x$

$m^2-2m+4=0$ $m=1+\sqrt{3}i$ $m=1-\sqrt{3}i$

CF $y=(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)e^x$

$$PI = \frac{1}{f(D)}e^x\cos x$$

$$=\left(\frac{1}{D^2-2D+4}e^x\cos x\right) = e^x\left(\frac{1}{(D+1)^2-2D-2+4}\cos x\right)$$

$$=\frac{1}{D^2+1+2D-2D-2+4}\cos x\Big] = e^x\left[\frac{1}{D^2+3}\cos x\right] \quad a=1$$

$$= e^x\left(\frac{1}{-1+3}\cos x\right) = e^x\frac{1}{2}\cos x$$

$$y = CF+PI = (c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)e^x + \frac{e^x}{2}\cos x$$

case v)

when $X = xV$. V is either $\sin ax$ / $\cos ax$

$$PI = x\left(\frac{1}{f(D)}V\right) - \left[\frac{f'(D)}{[f(D)]^2}\right]V$$

$$= x\left(\frac{1}{f(D)}V\right) - f'(D)\left[\frac{1}{(f(D))^2}V\right]$$

Q. Solve $y''+4y = x\sin x$.

$m^2+4=0$ $m=\pm 2i$

$CF = c_1\cos 2x + c_2\sin 2x$

$$PI = \frac{1}{f(D)}\cdot xV = x\left(\frac{1}{f(D)}V\right) - f'(D)\left[\frac{1}{(f(D))^2}V\right]$$

$$= x\left(\frac{1}{D^2+4}\sin x\right) - 2D\left[\frac{1}{(D^2+4)^2}\sin x\right]$$

$$= x\left(\frac{1}{3}\sin x\right) - 2D\left(\frac{1}{9}\sin x\right)$$

$$= x\left(\frac{1}{3}\sin x\right) - \frac{2}{9}\cos x$$

Q. $y'' + 4y = e^{3x} + x\sin 2x$

$m^2 + 4 = 0. \quad m = \pm 2i$

$y = c_1 \cos 2x + c_2 \sin 2x$

$PI_2 = \frac{1}{f(D)} \cdot xV$

$= x\left(\frac{1}{f(D)} V\right) - f'(D)\left(\frac{1}{(f(D))^2} V\right)$

$= x\left(\frac{1}{D^2+4} \sin 2x\right) - \cancel{\frac{2D}{\cos 2x + 2}} \left(\frac{1}{(D^2+4)^2} \cdot \sin 2x\right)$

$= x\left(\frac{1}{0} \sin 2x\right) - 2D\left(\frac{1}{0} \sin 2x\right)$ (case of failure

$= x^2\left(\frac{1}{2D} \sin 2x\right) - 2D\left(\frac{x}{2(D^2+4) \cdot 2D} \sin 2x\right)$

$= \frac{x^2}{2}\left(-\frac{\cos 2x}{2}\right) - 2D\left(\frac{x}{4D(D^2+4)} \sin 2x\right) \quad = -2D\left(\frac{\cos 2x}{-4D+4D}\right)$

$\cancel{= \frac{x^2}{4}(-\cos 2x) - 2D\left(\frac{1}{2D} \sin 2x\right)}$

$= -\frac{x^2}{4} \cos 2x - 2D\left(x^2 \frac{1}{4D(2D) + (D^2+4)4} \sin 2x\right)$

$= -\frac{x^2}{4} \cos 2x - 2D\left(x^2 \frac{1}{8D^2 + 4D^2 + 16} \sin 2x\right)$

$= -\frac{x^2}{4} \cos 2x - 2D\left(x^2 \frac{1}{-32 - 16 + 16}\right)$

$= -\frac{x^2}{4} \cos 2x + 2D\left(\frac{x^2}{32} \sin 2x\right)$

$= \frac{x^2}{4} \cos 2x + \frac{2}{32}(2x^2 \cos 2x + \sin 2x \cdot 2x)$

$= -\frac{x^2}{4} \cos 2x + \frac{1}{16}(2x^2 \cos 2x + 2x \sin 2x)$

Q. $2y'' + 2y' + 3y = x^2 + 2x - 1 + \sin x + e^x$

$2m^2 + 2m + 3 = 0 \quad m = -\frac{1}{2} \pm \frac{\sqrt{5}}{2} i,$

$CF = \left(c_1 \cos \frac{\sqrt{5}}{2} x + c_2 \sin \frac{\sqrt{5}}{2} x\right) e^{-\frac{1}{2}x}$

$PI = \frac{1}{f(D)} X = \frac{1}{2D^2 + 2D + 3} (x^2 + 2x - 1 + \sin x + e^x)$

$= \frac{1}{2D^2 + 2D + 3} (x^2 + 2x - 1) + \frac{1}{2D^2 + 2D + 3} \sin x + \frac{1}{2D^2 + 2D + 3} e^x$

$\cancel{1 + 2x + x^2 \;\big)\; 2D^2 + 2D + 3}$

$3 + 2D + 2D^2 \;\big)\; x^2 + 2x - 1 \;\big(\; \frac{x^2}{3} + \frac{2}{9}x - \frac{1}{27}$

$x^2 + 4x/3 + 4/3$

$2/3\, x + 1/3$

$2/3\, x + 4/9$

$-\frac{1}{9}$

$-\frac{1}{27}$

0

$= \frac{x^2}{3} + \frac{2x}{9} - \frac{25}{27} + \frac{1}{2D+1} \sin x + \frac{1}{7} e^x$

$= \frac{x^2}{3} + \frac{2x}{9} - \frac{25}{27} + \frac{2D-1}{-5} \sin x + \frac{1}{7} e^x$

$= \frac{x^2}{3} + \frac{2x}{9} - \frac{25}{27} - \frac{2}{5} \cos x + \frac{1}{5} \sin x + \frac{1}{7} e^x$

Q. $(D^2 + 3D + 2)y = x \sin 2x$

$m = -1, -2$

$CF = c_1 e^{-x} + c_2 e^{-2x}$

$PI = \frac{1}{f(D)} X = \frac{1}{D^2 + 3D + 2}$

$= x\left(\frac{1}{f(D)} V\right) - \left(\frac{f'(D)}{\{f(D)\}^2}\right] V$

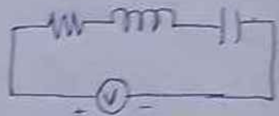
$= x\left(\frac{1}{D^2 + 3D + 2}\right) \sin 2x - (2D+3)\left(\frac{1}{(D^2}\right.$

6. $R = 40\Omega$. $L = 2H$ $C = 0.0025F$ $V = 12V$ $Q = 0.01C$

$q = 0.01C$

Given:

If Q be charge on capacitor at any time 't' then the current in circuit at any time t



$$i = \frac{dq}{dt}$$

$$\therefore \text{KVL total} = L\frac{dI}{dt} + Ri + \frac{q}{c} = E$$

Since $i = \frac{dq}{dt}$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E$$

$L = 2$ $C = 0.0025$ $R = 40$ $E = 12$.

$$2 \times \frac{d^2q}{dt^2} + 40\frac{dq}{dt} + \frac{1}{0.0025} = 12$$

$$2\frac{d^2q}{dt^2} + 40\frac{dq}{dt} + 400q = 12$$

which is LDE with const coff in 'q' which is non homogeneous

$q(t) = 2m^2 + 40m + 400 = 0$.

$m = -10 \pm 10i$

$CF = (C_1 \cos 10t + C_2 \sin 10t)e^{-10t}$

$PI = 2D^2 + 40D + 400 = 12$

$$= \frac{1}{f(D)} x = \frac{1}{2D^2 + 40D + 400} 12$$

$$= 12\left(\frac{1}{2D^2 + 40D + 400} e^{0t}\right)$$

$$= 12\left(\frac{1}{400}\right)$$

$\therefore$ Req soln is

$$q(t) = (C_1 \cos 10t + C_2 \sin 10t)e^{-10t} + \frac{3}{100}$$

This gives charge in circuit at any time t

Given

Initially charge on capacitor $= 0.01C$

$i = 0$.

i.e, at $t = 0$ $q = 0.01$ & $i = 0$.

i.e, $\frac{dq}{dt} = 0$.

applying these conditions

$$0.01 = C_1 + \frac{12}{400}.$$

$C_1 = -0.02$

diff q wrt t we get

$$\frac{dq}{dt} = i = e^{-10t}(-C_1 10 \sin 10t + C_2 10 \cos 10t) + (C_1 \cos 10t + C_2 \sin 10t)(-10e^{-10t})$$

Given at $t=0$ $i=0$

$$0 = 10C_2 + C_1(-10)$$

$$C_2 = 0.02 = C_1$$

Substituting these values in expressions q & i we get charge and current in the circuit as

$$q(t) = (-0.02\cos 10t + -0.02\sin 10t)e^{-10t} + \frac{3}{100}.$$

$$i(t) = e^{-10t}(-0.02 \times 10\sin 10t - 0.2\cos 10t)$$
$$+ (-0.02\cos 10t + -0.02\sin 10t)(-10e^{-10t})$$

7. $R = 20\Omega$ $L = 1H$.