classmate

Date

Page

① $y'' - 3y' + 2y = 0$, $y(0) = 1$, $y(3) = 0$

→ Auxiliary equation is.

$$D^2 - 3D + 2 = 0$$

$$D^2 - 2D - D + 2 = 0$$

$$D(D-2) - 1(D-2) = 0$$

$$\therefore D = 2, 1$$

The roots are real and distinct

$$C.F. = c_1 e^{2x} + c_2 e^{x} = y \rightarrow ①$$

Put $x=0$, $y=1$ in eq. ①

$$1 = c_1 e^0 + c_2 e^0$$

$$\therefore c_1 + c_2 = 1 \rightarrow ②$$

Put $x=3$, $y=0$ in eq. ①

$$0 = c_1 e^6 + c_2 e^3$$

$$\therefore c_2 = -\frac{c_1 e^6}{e^3} = -c_1 e^3 \rightarrow ③$$

From ② & ③

$$c_1 + c_2 = 1$$

$$c_1 - c_1 e^3 = 1$$

$$c_1(1 - e^3) = 1$$

$$\therefore c_1 = \frac{1}{1-e^3} \qquad \therefore c_2 = \frac{-e^3}{1-e^3}$$

$\therefore$ soln is

$$y = \frac{e^{2x}}{1-e^3} - \frac{e^{3+x}}{1-e^3} \Rightarrow \frac{e^{2x} - e^{3+x}}{1-e^3}$$

classmate

Date
Page

② $y'' + 2y' = 0 \quad (y(0)) = 1, \; y(1) = 2$

$\Rightarrow$ A.E. is

$$D^2 + 2D = 0$$

$$D(D+2) = 0$$

$$\therefore D = 0, -2$$

The roots are real and distinct

$$C.F. = y = c_1 e^{0x} + c_2 e^{-2x} = c_2 e^{-2x} + c_1 \rightarrow ①$$

Put $x = 0$, $y = 1$ in ①

$$1 = c_2 + c_1 \rightarrow ②$$

Put $x = 1$, $y = 2$ in ①

$$2 = c_1 + c_2 e^{-2}$$

$$\therefore c_2 = \frac{2 - c_1}{e^{-2}} = e^2(2 - c_1) \rightarrow ③$$

From ② & ③

$$c_1 + c_2 = 1$$

$$c_1 + e^2(2 - c_1) = 1$$

$$c_1(1 - e^2) = 1 - 2e^2$$

$$\therefore c_1 = \frac{1 - 2e^2}{1 - e^2}$$

$$c_2 = e^2 \; \frac{2 - \cancel{2e^2} - 1 + \cancel{2e^2}}{1 - e^2} = \frac{e^2}{1 - e^2}$$

$\therefore$ soln is

$$y = \frac{1 - 2e^2}{1 - e^2} + \frac{e^{-2x+2}}{1 - e^2}$$

classmate

Date
Page

③ $9y'' - 18y' + 10y = 0, \; y(0) = 0, \; y(\pi) = 1$

$\Rightarrow$ A.E. is

$$9D^2 - 18D + 10 = 0$$

$$D = 1 \pm \frac{1}{3} i$$

The roots are imaginary

$$C.F. = y = e^x \left(c_1 \cos(x/3) + c_2 \sin(x/3) \right) \rightarrow ①$$

Put $x = 0$, $y = 0$ in ①

$$0 = 1(c_1 + 0)$$

$$\therefore c_1 = 0$$

Put $x = \pi$, $y = 1$ in ①

$$1 = [c_1 \cos(\pi/3) + c_2 \sin(\pi/3)] e^\pi$$

$$c_2 \left(\frac{\sqrt{3}}{2} \right) = \frac{1}{e^\pi}$$

$$\therefore c_2 = \frac{2}{e^\pi \sqrt{3}}$$

$\therefore$ The soln is

$$y = e^x \left[\frac{2}{e^\pi \sqrt{3}} \sin\left(\frac{x}{3}\right) \right]$$

(4) $y'' + y' - 2y = 0$, $y(0) = 0$, $y'(0) = 3$

$\Rightarrow$ A.E. is

$$D^2 + D - 2 = 0$$

$$\therefore D = -2, 1$$

The roots are real and distinct

$\therefore$ C.F. is

$$y = c_1 e^{-2x} + c_2 e^{x} \rightarrow (1)$$

Put $x = 0$, $y = 0$ in (1)

$$0 = c_1 + c_2 \rightarrow (5)$$

Diff. (1) w.r.t. x

$$y' = -2c_1 e^{-2x} + c_2 e^{x} \rightarrow (2)$$

Put $y' = 3$, $x = 0$ in (2)

$$-2c_1 + c_2 = 3 \rightarrow (4)$$

From (3) & (4)

$$\therefore c_1 = -1 \text{ \& } c_2 = 1$$

$\therefore$ The soln is

$$y = -e^{-2x} + e^{x}$$

(5) $y'' - 3y' + 2y = 0$, $y(0) = 3$; $y'(0) = 0$

$\Rightarrow$ A.E. is

$$D^2 - 3D + 2 = 0$$

$$\therefore D = 2, 1$$

The roots are real and distinct

$\therefore$ C.F. is

$$y = c_1 e^{2x} + c_2 e^{x} \rightarrow (1)$$

Put $x = 0$, $y = 3$ in (1)

$$c_1 + c_2 = 3 \rightarrow (2)$$

Diff. (1) w.r.t. x

$$y' = 2c_1 e^{2x} + c_2 e^{x} \rightarrow (3)$$

Put $y' = 0$, $x = 0$ in (3)

$$2c_1 + c_2 = 0 \rightarrow (4)$$

From (2) & (4)

$$c_1 = -3 \quad c_2 = 6$$

$\therefore$ The soln is

$$y = -3e^{2x} + 6e^{x}$$

(e) $y'' + 3y' + 2y = 4e^{-x}$

$\Rightarrow$ A.E. is

$$D^2 + 3D + 2 = 0$$

$$D = -1, -2$$

The roots are real and distinct

$$C.F. = c_1 e^{-2x} + c_2 e^{-x}.$$

$$P.I. = \frac{1}{D^2+3D+2} 4e^{-x} \quad [D=-1]$$

$$= 4 \frac{1}{1-3+2=0} e^{-x}$$

$$= 4 x \frac{1}{2D+3} . e^{-x} \quad [D=-1]$$

$$\therefore PI = 4x . e^{-x}$$

The sol^n is

$$y = C.F. + P.I.$$

$$= (c_1 e^{-2x} + c_2 e^{-x}) + 4x . e^{-x}$$

(f) $y'' - 2y' + y = 3e^{2x}$

$\Rightarrow$ A.E. is

$$D^2 - 2D + 1 = 0$$

$$D = 1, 1$$

The roots are real and equal

$$C.F. = (c_1 + c_2 x) e^{x}$$

$$P.I. = 3 \frac{1}{D^2-2D+1} e^{2x}$$

$$D = 2$$

$$= 3 \frac{1}{4-4+1} e^{2x} = 3e^{2x}$$

$$\therefore PI = 3e^{2x}$$

The sol^n is

$$y = C.F. + P.I.$$

$$= (c_1 + c_2 x) e^{x} + 3e^{2x}$$

$y'' + 6y' + 9y = 1 + x$

$y'' - y' = x e^{x}, \quad y(0) = 2, \; y'(0) = 1$

$\Rightarrow$ A.E. $D^2 - D = 0$

$D(D-1) = 0$

$D = 0, 1$

C.F. $\Rightarrow c_1 e^{0x} + c_2 e^{x} = c_1 + c_2 e^{x}$

P.I. $= \dfrac{1}{D^2 - D} \cdot x \cdot e^{x}$

$= e^{x} \dfrac{1}{(D+1)^2 - (D+1)} \cdot x.$

$= e^{x} \dfrac{1}{D^2 + D} \cdot x$

$= e^{x} \dfrac{1}{D(1+D)} \cdot x$

$= e^{x} \cdot \dfrac{1}{D} (1+D)^{-1} x = e^{x} \cdot \dfrac{1}{D} (1 - D + D^2 - - -) x$

$= e^{x} \cdot \dfrac{1}{D} [x - 1]$

$= e^{x} \left(\dfrac{x^2}{2} - x \right)$

$y'' - 2y' + 2y = e^{x} \tan x$