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1) If $f(x,y,z) = x\sin yz$

a) find grad of f

b) find D.D. of f at $(1,3,0)$ in the direction of

c) $\vec{v} = i + 2j - k$

2) For the foll. f^ns

(i) Find grad of f

(ii) Evaluate the grad at point P.

(iii) Find the rate of change of f at P. in the direction of vector u.

a) $f(x,y) = \sin(2x+3y)$, $P(-6,4)$, $\vec{u} = \frac{1}{2}(\sqrt{3}i - j)$

b) $f(x,y,z) = x^2yz - xyz^3$, $P(2,-1,1)$, $\vec{u} = \langle 0, \frac{4}{5}, \frac{-3}{5} \rangle$

1). $f(x,y,z) = x\sin yz$

a) $\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$

$= \sin yz\, i + \cancel{x\sin yz} + \frac{xz\cos yz}{z} + \frac{kx\cos yz\, z\, x\, y}{y}$.

$\nabla f = \cancel{f}\sin yz +$

$\nabla f = i\sin yz + xz\cos yz\, j + xy\cos yz\, k$

$\nabla f_{(1,3,0)} = 0 + 0 + 3k$

$= \underline{3k}$

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b) $\nabla f = (\sin yz)i + (xz\cos yz)j + (xy\cos yz)k$

D.D. $= \frac{\nabla f \cdot \vec{a}}{|\vec{a}|}$

$= (\sin yz)i + (xz\cos yz)j + (xy\cos yz)_{(1,3,0)} \cdot \frac{(i+2j-k)}{\sqrt{6}}$

$= 3k \cdot \frac{(i+2j-k)}{\sqrt{6}}$

$= \frac{-3}{\sqrt{6}} = -\frac{\sqrt{3}\times\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}\times\sqrt{2}} = \underline{-\sqrt{\frac{3}{2}}}$.

2)(i) $\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$

$= 2\cos(2x+3y)i + 3\cos(2x+3y)j + 0$.

$\nabla f_{(-6,4)} = 2\cos(-12+12)i + 3\cos(-12+12)j$

$= 2i + 3j$

D.D. $= \frac{2i+3j \cdot \left(\frac{\sqrt{3}}{2}i - \frac{1}{2}j\right)}{1} = \sqrt{3} \mp \frac{3}{2}$

D.D. $= \sqrt{3} \mp \frac{3}{2} = \sqrt{3} - \frac{3}{2} = \underline{\frac{2\sqrt{3}-3}{2}}$.

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(ii) $f(x,y,z) = x^2yz - xyz^3$

$\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$

$= \cancel{(2xyz)i} +$

$= (2xyz - yz^3)i + (x^2z - xz^3)j + (x^2y - 3xyz^2)k$

$\nabla f_{(2,-1,1)} = (-4+1)i + (4-2)j + (-4+6)k$

$= -3i + 2j + 2k$

$D.D. = (-3i+2j+2k).\frac{(4/5 j - 3/5 k)}{1}$

$= 0 + \frac{8}{5} - \frac{6}{5}$

$= \frac{2}{5}$

Find DO of the f^n $f(x,y) = x^2y^3 - 4y$, $(2,-1)$ in the direction of the vector $\vec{V} = 2i + 5j$

$\Rightarrow$ $\nabla f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$

$= (2xy^3)i + (3x^2y^2 - 4)j + (0)k$

$\nabla f_{(2,-1)} = -4i + 8j$

$D.D. = -4i + 8j \frac{(2i+5j)}{\sqrt{29}} = \frac{-8+40}{\sqrt{29}}$

$= \frac{+32}{\sqrt{29}} = \frac{32}{\sqrt{29}}$

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Find D.D. or $D_u f(x,y)$ if $f(x,y) = x^3 - 3xy + 4y^2$ and u is a unit vector given by $\theta = \frac{\pi}{6}$. What is

$D_u f(1,2) = ?$

$\Rightarrow$ $\nabla f = (3x^2 - 3y)i + (-3x + 8y)j + (0)k$

$\nabla f_{(1,2)} = (3-6)i + (-3+16)j$

$= -3i + 13j$

$D.D. = |-3i + 13j| \cos\frac{\pi}{6}$

$= 13.34 \times 0.866$

$= 11.55$

$\vec{u} = \langle \cos\theta, \sin\theta \rangle$

$\theta = \frac{\pi}{6}$

$\vec{u} = \left(\frac{\sqrt{3}}{2} i + \frac{1}{2} j\right)$

$D.D. = \nabla f . \frac{\vec{u}}{|\vec{u}|}$

$= (-3i + 13j).\frac{(\sqrt{3}/2\, i + 1/2\, j)}{1}$

$= -3\sqrt{3}/2 + 13/2 = \frac{13 - 3\sqrt{3}}{2}$

Find DO of the foll. f^{ns} at a given point in the direction of vector $\vec{V}$.

(i) $f(x,y) = \frac{x}{x^2+y^2}$, $P(1,2)$, $\vec{v} = <3,5>$

$$\nabla f = \left[\frac{(x^2+y^2) \cdot - (2x)x}{(x^2+y^2)^2}\right] i - \frac{2xy}{(x^2+y^2)^2} j$$

$$\nabla f_{(1,2)} = \left(\frac{5-2}{25}\right) i - \frac{4}{25} j$$

$$\nabla f_{(1,2)} = \frac{3}{25} i - \frac{4}{25} j$$

$$D.D. = \nabla f \cdot \frac{\vec{v}}{|\vec{v}|}$$

$$= \frac{3}{25} i - \frac{20}{25} j \; \frac{(3i + 5j)}{\sqrt{34}}$$

$$= \frac{\frac{9}{25} - \frac{\cancel{5}20}{25}}{\sqrt{34}} = \frac{4-11}{25\sqrt{34}} = \frac{-11}{25\sqrt{34}}$$

(ii) $f(x,y,z) = \sqrt{xyz}$, $P(3,2,6)$, $\vec{v} = <-1,-2,2>$

$$\nabla f = \frac{yz}{2\sqrt{xyz}} i + \frac{xz}{2\sqrt{xyz}} j + \frac{xy}{2\sqrt{xyz}} k$$

$$\nabla f_{(3,2,6)} = \frac{\cancel{12}}{\cancel{2}\sqrt{36}} i + \frac{\cancel{18}^{9}}{\cancel{2}\sqrt{36}} j + \frac{\cancel{6}^{3}}{\cancel{2}\sqrt{36}} k$$

$$= \frac{6i}{6} + \frac{9}{6} j + \frac{3}{6} k$$

$$D.D. = \nabla f \cdot \frac{\vec{v}}{|\vec{v}|}$$

$$= \frac{6}{6} i + \frac{9}{6} j + \frac{3}{6} k \left(\frac{-1i - 2j + 2k}{3}\right)$$

$$= \frac{-\frac{6}{6} - \frac{18}{6} + \frac{6}{6}}{3} = \frac{-\cancel{1} - 3 + \cancel{1}}{3} = -1.$$

Find the max. rate of change of f at a point in the direction in which it occurs.

⟹ (i) $f(x,y) = 4y\sqrt{x}$ at $(4,1)$

(ii) $f(x,y) = \sin xy$ at $(1,0)$

(iii) $f(x,y,z) = \frac{x+y}{z}$ at $(1,1,-1)$

(i) $\nabla f = \frac{4y}{2\sqrt{x}} i + 4\sqrt{x} j$

$$\nabla f_{(4,1)} = \frac{4}{2 \times 2} i + 4 \times 2 j = \frac{4i + 8j}{4} = 1i + 8j$$

Max D.D. $= |\nabla f| = \sqrt{1+64} = 8.06 = \sqrt{65}$.

(ii) $\nabla f = \cancel{y \sin xy i} \cdot y \cos xy i + x \cos xy j$

$\nabla f_{(1,0)} = 0 + 1j = 1j$

Max D.D. $= |\nabla f| = \sqrt{1} = 1$.

(iii) $f(x,y,z) = \frac{x+y}{z}$

$$\nabla f = \frac{1}{z} i + \frac{1}{z} j - \frac{(x+y)}{z^2} k$$

$$\nabla f_{(1,1,-1)} = -1i - 1j - 2k$$

$$\text{Max D.D.} = \sqrt{1+1+4} = \sqrt{6} = \underline{2.449}.$$

Find the divergence and curl of

(i) $F(x,y,z) = xzi + xyzj - y^2k$

$$\text{div} = \frac{\partial f_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= z + xz + (0)$$

$$= z + xz$$

$$\text{curl} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & xyz & -y^2 \end{vmatrix}$$

$$= i(-2y - xy) - j(0 - x) + k(yz - 0)$$

$$= (-2y - xy)i + xj + yz\,k$$

(ii) $F(x,y,z) = xe^{-y} i + xzj + ze^{y} k$

$$\text{div } \vec{F} = e^{-y} + 0 + e^{y}$$

$$= e^{-y} + e^{y}$$

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xe^{-y} & xz & ze^{y} \end{vmatrix}$$

$$= i(ze^{y} - x) - j(0 - 0) + k(z + xe^{-y})$$

$$= (ze^{y} - x)i + (z + xe^{-y})k$$

(iii) $F(x,y,z) = \ln x\, i + \ln(xy)\, j + \ln(xyz) k$

$$\text{div } \vec{F} = \frac{1}{x} + \frac{x}{xy} + \frac{xy}{xyz}$$

$$= \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

$$\text{Curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \ln x & \ln xy & \ln xyz \end{vmatrix}$$

$$= i\left[\frac{xz}{xyz} - 0\right] - j\left[\frac{yz}{xyz} - 0\right] + k\left[\frac{y}{xy} - 0\right]$$

$$= \frac{1}{y} i - \frac{1}{x} j + \frac{1}{x} k$$

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(iv) $F(x,y,z) = \frac{x}{x^2+y^2+z^2}\, i + \frac{y}{x^2+y^2+z^2}\, j + \frac{z}{x^2+y^2+z^2}\, k$

$$\text{div } \vec{F} = \frac{(x^2+y^2+z^2) - x(2x) + (x^2+y^2+z^2) - y(2y) - z(2z)}{(x^2+y^2+z^2)^2}$$

$$= \frac{3(x^2+y^2+z^2) - 2x^2 - 2y^2 - 2z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^2}$$

$$\therefore \text{div } \vec{F} = \frac{1}{x^2+y^2+z^2}$$

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{x^2+y^2+z^2} & \frac{y}{x^2+y^2+z^2} & \frac{z}{x^2+y^2+z^2} \end{vmatrix}$$

$$= i\left[\frac{-2yz}{(x^2+y^2+z^2)^2} + \frac{2yz}{(x^2+y^2+z^2)^2}\right] - j\left[\frac{-2xz}{(x^2+y^2+z^2)^2} + \frac{2xz}{(x^2+y^2+z^2)^2}\right]$$
$$+ k\left[\frac{-2xy}{(x^2+y^2+z^2)^2} + \frac{2xy}{(x^2+y^2+z^2)^2}\right]$$

$$\therefore \text{curl } \vec{F} = \underline{\underline{0}}$$

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Determine whether $\vec{V} = (-x^2+yz)i + (4y-z^2x)j + (2xz-4z)k$ is solenoidal or not.

$\Rightarrow$ $\text{div } \vec{V} = -2x + 4 + 2x - 4$
$= 0$

$\therefore \vec{V}$ is solenoidal

Determine whether or not F is a conservative vector field if it is find 'f' such that $\vec{F} = \nabla f$

(i) $F(x,y,z) = (\ln y + 2xy^3)i + (3y^2x^2 + x/y)j$

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \ln y + 2xy^3 & 3y^2x^2 + x/y & 0 \end{vmatrix}$$

$$= i\left[0 - \left[6yx^2 - \frac{x}{y^2}\right]\right] - j\left[0 - \left(\frac{1}{y} + 6xy^2\right)\right]$$
$$+ k\left[6y^2x + \frac{1}{y} - \left[\frac{1}{y} + 6xy^2\right]\right]$$

$= \underline{0}.$

$\frac{\partial f}{\partial x} = \ln y + 2xy^3$ $\qquad$ $\frac{\partial f}{\partial y} = 3y^2x^2 + \frac{x}{y}$

$f = x\ln y + x^2y^3 + f(y)$ $\qquad$ $f = y^3x^2 + x\ln y + f(x)$

$\therefore f = x\ln y + x^2y^3 + C$

(ii) $F(x,y,z) = y^2 i + (2xy + e^{3z}) j + 3ye^{3z} k$

$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + e^{3z} & 3ye^{3z} \end{vmatrix}$

$= i\left[3e^{3z} - 3e^{3z}\right] - j\left[0 - 0\right] + k\left[2y - 2y\right]$

$= 0$

$\frac{\partial f}{\partial x} = y^2$ $\qquad \frac{\partial f}{\partial y} = 2xy + e^{3z}$

$\therefore f = \cancel{\frac{y^3}{3}}\, xy^2 + f(y,z)$ $\qquad f = xy^2 + ye^{3z} + g(x,z)$

$\frac{\partial f}{\partial z} = 3ye^{3z}$

$\therefore f = ye^{3z} + h(x,z)$

$\therefore f = xy^2 + ye^{3z} + c$

(iii) $F(x,y,z) = yz\, i + xz\, j + (xy + 2z) k$

$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & (xy + 2z) \end{vmatrix}$

$= i(x - x) - j(y - y) + k(z - z)$

$= 0$

$\frac{\partial f}{\partial x} = yz$ $\qquad \frac{\partial f}{\partial y} = xz$ $\qquad \frac{\partial f}{\partial z} = xy + 2z$

$\therefore f = xyz + f(y,z)$ $\quad \therefore f = xyz + f(x,z)$ $\quad \therefore f = xyz + z^2 + f(x,y)$

$\therefore f = xyz + z^2 + c$

(iv) $F(x,y,z) = y^2 z^3 i + 2xyz^3 j + 3x^2 y^2 z^2 k$

$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3x^2y^2z^2 \end{vmatrix}$

$= i(6x^2yz^2 - 6xyz^2) - j(6xy^2z^2 - 3y^2z^2)$
$+ k(2yz^3 - 2yz^3)$

$\neq 0$ so 'f' does not exist

(v) $F(x,y) = e^x \cos y\, i + e^x \sin y\, j$

$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x \cos y & e^x \sin y & 0 \end{vmatrix}$

$= i(0 - 0) - j(0 - 0) + k(e^x \sin y + e^x \sin y)$

$\neq 0$ so 'f' does not exist

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(1). $\vec{F}(x,y,z) = xi - zj + yk$

sphere $x^2+y^2+z^2 = 4$. $R = y$

$P = x \quad Q = -z \quad g = x^2+y^2+z^2-4 = 0$

$\frac{\partial g}{\partial x} = 2x \quad \frac{\partial g}{\partial y} = 2y \quad g = z = \sqrt{4-x^2-y^2}$ ✗

$\frac{\partial g}{\partial x} = \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2-y^2}} \quad \frac{\partial g}{\partial y} = \frac{-\cancel{2}y}{\cancel{2}\sqrt{4-x^2-y^2}}$

Flux of $\vec{F} = \iint_S \vec{F}\cdot\hat{n}\, ds = \iint_D \left[\frac{x^2}{\sqrt{4-x^2-y^2}} + \frac{-yz}{\sqrt{4-x^2-y^2}} + y\right] dA$

$\iint_D \left(\frac{x^2}{\sqrt{4-x^2-y^2}} - \frac{y(\sqrt{4-x^2-y^2})}{\sqrt{4-x^2-y^2}} + y\right) dA$

$r \to 0$ to 2

$\theta \to 0$ to $\pi/2$

$x \Rightarrow r\cos\theta$

$y = r\sin\theta$

$= \int_{\theta=0}^{\pi/2}\int_{r=0}^{2} \left(\frac{r^2\cos^2\theta}{\sqrt{4-r^2}}\right) r\,dr\,d\theta$

$= \int_{\theta=0}^{\pi/2}\int_{r=0}^{2} \frac{r^3\cos^2\theta}{\sqrt{4-r^2}}\,dr\,d\theta$

$4 - r^2 = t$

$-2r\,dr = dt$

$\therefore dr = \frac{dt}{-2r}$

$= \int_{\theta=0}^{\pi/2} - \int_{t=4}^{0} \frac{r^3\cos^2\theta}{\sqrt{t}} \cdot d\theta . \frac{dt}{+2r}$

when $r = 0$

$t = 4$

when $r = 2$

$t = 0$.

$= \int_{\theta=0}^{\pi/2}\int_{t=0}^{4} \frac{(4-t)}{\sqrt{t}}\frac{\cos^2\theta}{2}\, d\theta . dt$

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$= \int_{\theta=0}^{\pi/2}\int_{t=0}^{4} \left(\frac{4}{\sqrt{t}} - \frac{t}{\sqrt{t}}\right)\frac{\cos^2\theta}{2} . dt . d\theta$

$= \int_{\theta=0}^{\pi/2} \left[\frac{4 . t^{1/2}}{1/2} - \frac{t^{3/2}}{3/2}\right]_0^4 \frac{\cos^2\theta}{2} . d\theta$

$= \int_{\theta=0}^{\pi/2} \frac{1}{2} \times \frac{32}{3}\left[\frac{1+\cos 2\theta}{2}\right] . d\theta$

$= \frac{16}{3}\left[\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$

$= \frac{\cancel{16}^{4}}{3} \times \frac{\pi}{\cancel{4}} = \underline{\underline{\frac{4\pi}{3}}}$.

(26. $\vec{F} = zi + xj - 3y^2zk$ $\quad x^2+y^2 = 16$ b/w $z = 0$ & $z = 5$

$\Rightarrow$ $P = z \quad Q = x \quad R = -3y^2z \quad g = 16 - x^2 - y^2$ in the first octant.

$\frac{\partial g}{\partial x} = -2x \quad \frac{\partial g}{\partial y} = -2y$

Flux of $\vec{F} = \iint_S \vec{F}\cdot\hat{n}\, ds$ ✗

$\vec{F} = zi + xj - 3y^2zk, \quad x^2+y^2 = 16$

$z=0, \ z=5$

$\Rightarrow P = z, \ Q = x \quad R = -3y^2z$

$S: \ x = \sqrt{16-y^2} = g(y,z)$

$\frac{\partial g}{\partial y} = \frac{1}{2\sqrt{16-y^2}}(-2y) \cdot \quad \frac{\partial g}{\partial z} = 0$

$$\iint \vec{F}.\hat{n}\,ds = \iint_D \left(P - Q\frac{\partial g}{\partial y} - R\frac{\partial g}{\partial z}\right).dA$$

$$= \iint_D \left(z - x\left(\frac{-y}{\sqrt{16-y^2}}\right) - 0\right).dA$$

$$= \iint_D \left(z + y\frac{\cancel{\sqrt{16-y^2}}}{\cancel{\sqrt{16-y^2}}}\right).dA$$

$$= \int_{z=0}^{5}\int_{y=0}^{4} (y+z)\,dy\,dz = \underline{\underline{90}}.$$

xy-plane $\langle P, Q, R\rangle \rightarrow \langle -\frac{\partial g}{\partial x}, -\frac{\partial g}{\partial y}, 1\rangle$

yz-plane $\langle P, Q, R\rangle \rightarrow \langle 1, -\frac{\partial g}{\partial \cancel{x}y}, -\frac{\partial g}{\partial z}\rangle$

xz-plane $\langle P, Q, R\rangle \rightarrow \langle -\frac{\partial g}{\partial x}, 1, -\frac{\partial g}{\partial z}\rangle$

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