KLE Technological University
Creating Value
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DEPARTMENT OF MATHEMATICS

FMTH0301/Rev.5.3

Course Plan

Semester: **II (All divisions)** Year: 2021-22

Course Title: **Multivariable Calculus**	*Course Code*: 18EMAB102
Total Contact Hours:50	*Duration of ESA*: 3 hrs
ESA Marks: 50	*ISA Marks*: 50
Lesson Plan Author: Dr.Shaila Chougala, Dr. D. A. Patil	*Date*: 20-04-2022
Checked By: Dr. Uma Neeli	*Date*: 20-04-2022

Prerequisites

This course requires the student to know about Single variable calculus and vectors algebra.

Course Outcomes-(CO's)

At the end of the course student will be able to:

i. Solve problems on directional derivatives, gradient, extreme values, errors and approximations for functions of several variables using partial derivatives.
ii. Evaluate the area and the volume of the oriented surfaces using double and triple integrals.
iii. Solve the problems on fundamental theorems of vector calculus viz. Green's, Stokes and divergence.
iv. Solve engineering problems using higher order differential equations.

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Course Articulation Matrix: Mapping of Course Outcomes (CO) with Program Outcomes

Course Title: **Multivariable Calculus**	Semester:**02**
Course Code: **18EMAB102**	Year: 2021-22

Course Outcomes (CO) / Program Outcomes (PO)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1. Solve problems on directional derivatives, gradient, extreme values, errors and approximations for functions of several variables using partial derivatives.	**H**											
2. Evaluate the area and the volume of the oriented surfaces using double and triple integrals	**H**											
3. Define Solve the problems on fundamental theorems of vector calculus viz.,. Green's, Stokes and divergence.	**H**											
4. Solve engineering problems using higher order differential equations.	**H**											

Degree of compliance **L**: Low **M**: Medium **H**: High

Competency addressed in the Course and corresponding Performance Indicators

Competency	**Performance Indicators**
1.1 - Demonstrate the competence in mathematical modeling.	1.1.1 – Apply Mathematical Techniques to solve problems.
	1.1.2- Apply discipline specific Advanced Mathematical Techniques to modeling and problem solution.
	1.1.3- Apply fundamentals of Mathematics to solve problems.

Course Content

Course Code: **18EMAB102**	Course Title: **Multivariable Calculus**	
L-T-P: 4-1-0	Credits: 5	Contact Hrs: 70
ISA Marks: 50	ESA Marks: 50	Total Marks: 100
Teaching Hrs: 50		Exam Duration: 3 hrs

Content	**Hrs**
Unit – 1	
Chapter No.1: Partial differentiation Function of several variables, Partial derivatives, Level curves, Chain rule, Errors and Approximations. Extreme value problems. Lagrange's multipliers. **Chapter No.2: Double Integrals** Double integrals - Rectangular and polar coordinates, Change the order of integration. Change of variables, Jacobian, Applications of double integrals. **Matlab:** optimization problems, application of double integrals.	12 hrs 08 hrs
Unit - 2	
Chapter No.3: Triple integrals Triple integrals, Cartesian, Cylindrical and Spherical coordinate Application of triple integrals. **Chapter No.4: Calculus of Vector Fields** Vector fields, Gradient and directional derivatives. Line and Surface integrals. Independence of path and potential functions. Green's theorem, Divergence of vector field, Divergence theorem, Curl of vector field. Stokes theorem. **Matlab:** Application of triple integrals and Vector calculus problems**.**	07 hrs 13 hrs
Unit - 3	
Chapter No.5: Differential Equations of higher order a) Linear differential equations of second and higher order with constant coefficients, Method of Variation of parameters, Initial and boundary value problems. b) Applications of second order differential equations-Newton's 2^{nd} law, electrical circuits, Simple Harmonic motion. Series solution of differential equations. Validity of Series Solution of differential equations. **Matlab:** Application of differential equations.	5 hrs 5 hrs

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Text Book: Early Transcendental Calculus- James Stewart (INDIA EDITION)

References:

1. Calculus Single and Multivariable, Hughues-Hallett Gleason, Wiley India Ed, 4ed, 2009.
2. Thomas calculus, George B Thomas, Pearson India, 12ed, 2010

Evaluation Scheme

ISA Scheme

Assessment	Weightage in Marks
ISA- 1	15
ISA- 2	15
Post test	10
Matlab Test	10
Total	**50**

Course Unitization for Minor Exams and End Semester Assessment

Topics / Chapters	Teaching hours	No.of Questions in ISA-1	No.of Questions in ISA-2	No.of Questions in Post test	No.of Questions in Matlab test	No.of Questions in ESA
Unit I						
1.Partial differentiations	12	6	--	10	2	3
2.Double integrals	08	3	--	10		
Unit II						
3.Triple integrals	07	--	3	10	2	3
4. Calculus of vector fields	13	--	6	10		
Unit III						
5. Differential equations of higher orders	10	--	--	10	1	2

Note* *Each Question carries 20 marks and may consist of sub-questions.*

- Mixing of sub-questions from different chapters within a unit (***only for Unit I and Unit II***) is allowed in ISA-1, ISA-2 and ESA.
- Answer 5 full questions of 20 marks each (***two full questions from Unit I, Unit II, and one full question from Unit III***) out of 8 in ESA

Course Assessment Plan

Course Title: **Multivariable Calculus**		Code: **18EMAB102**				
Course outcomes (COs)	Weightage in assessment	Assessment Methods				
		ISA-1	ISA-2	Post Test	Matlab test	ESA
1. Solve problems on directional derivatives, gradient, extreme values, errors and approximations for functions of several variables using partial derivatives.	25%	✓		✓	✓	✓
2. Evaluate the area and the volume of the oriented surfaces using double and triple integrals	30%	✓	✓	✓	✓	✓
3. Define Solve the problems on fundamental theorems of vector calculus viz. Green's, Stokes and divergence.	25%		✓	✓	✓	✓
4. Solve engineering problems using higher order differential equations.	20%			✓	✓	✓
Weightage		15%	15%	10%	10%	50%

Date: 20 -04-2022

Head of Department

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Chapter wise Plan

*Course Code and Title: **18EMAB102*** / **Multivariable Calculus**	
Chapter Number and Title: **01. Partial Differentiation**	*Planned Hours:* **12**

Learning Outcomes:

At the end of the topic the student should be able to:

Sr.No	TLO's	CO's	B L	CA Code
1	Represent and interpret the function of two variables graphically using contour maps.	CO1	L2	1.1
2	Find and interpret the partial derivatives for functions of two variables.	CO1	L3	1.1
3	Extend the concept of differentiability to a function of two variables and use a differentiable as an approximation.	CO1	L3	1.1
4	Determine extreme values for the functions of two variables.	CO1	L3	1.1
5	Find extrema of function of two or more variables with constraints by using the Lagrange multiplier method.	CO1	L3	1.1

Lesson Schedule

Class No. Portion covered per hour

1. Introduction to functions of several variables
2. Partial derivatives
3. Level curves
4. Implicit functions, differentials and examples
5. Examples
6. Composite functions, Chain rule
7. Examples
8. Errors and approximations
9. Extreme value problems
10. Examples
11. Lagrange's multipliers
12. Examples

Review Questions

Sr. No	Questions	TLO	B L	PI Code
1.	Define level curves and draw the contours of $f(x,y) = (x+y)^2$ for the values 1, 2, 3, and 4. Give a description of the surface defined by $f(x,y)$.	TLO1	L2	1.1.1
2.	Draw the contours of $f(x,y) = x^2 + y^2$ for the values 1, 2, 3, and 4. Relate the contour diagram to the graph off(x,y). From the graph, determine whether the value of $f(x,y)$ at (0, 0) is a local minimum or maximum?	TLO1	L2	1.1.1
3.	Find the first order partial derivatives / rate of change / slope of (i) $f(x,y) = x^5 + 3x^3y^2 + 3xy^4$ (ii) $f(s,t) = \frac{st^2}{(s^2+t^2)}$ (iii) $f(x,y) = \sqrt{x^2+y^2}$ at the point (3,4) in the x-direction. (iv) $f(x,y,z) = \frac{x}{y+z}$ at the point (3,2,1) in y-direction	TLO2	L3	1.1.1
4.	Find all second order partial derivatives (i) $u = e^{-s}\sin t$ (ii) $z = y + \tan 2x$	TLO2	L3	1.1.1
5.	(i) If $u = \sin(x-at) + \ln(x+at)$ then show that $u_{tt} = a^2 u_{xx}$.	TLO2	L3	1.1.1

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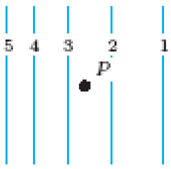
<table>
<tr><td></td><td>(ii) If $u = e^{-x}\cos y - e^{-y}\cos x$ then show that $u_{xx} + u_{yy} = 0$
(iii) If $u = \ln\sqrt{x^2 + y^2}$ then show that $u_{xy} = u_{yx}$
(iv) Verify Clairaut's theorem for $u = xye^y$
(v) Show that u=$\frac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of the three dimensional Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$.</td><td></td><td></td><td></td></tr>
<tr><td>6.</td><td>Suppose that the right circular cylinder is some sort of container for which the height can vary, such as the interior of a piston. What is the instantaneous rate of change of the volume, with respect to height, when the height is 0.3 meters, if the radius is held constant at 0.1 meters?</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>7.</td><td>Use the level curves of the function $z = f(x, y)$ to decide the sign (positive, negative, or zero) of each of the following partial derivatives at the point P. Assume the x- and y-axes are in the usual positions.
(a) f_x (b) f_y (c) f_{xx} (d) f_{xy} (e) f_{yy}


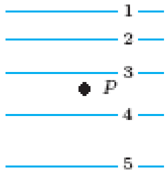
Fig.a


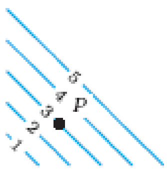
Fig.b


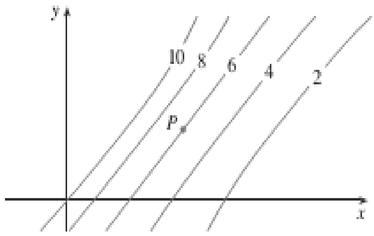
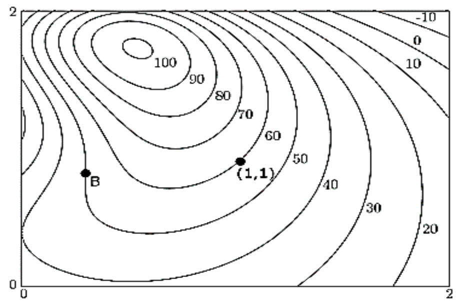
Fig.c


Fig. d</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>8.</td><td>For the contour diagram shown below, representing the function h(x, y).


(i) Determine $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ at (1,1) as (a) positive (b) negative (c) zero
(ii) Determine $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ at the point B as (a) positive (b) negative (c) zero</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>9.</td><td>Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$,
(i) $x^2 + y^2 + z^2 = 3xyz$ (ii) $\sin(xyz) = x + 2y + 2z$.</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>10.</td><td>Use Chain rule to find $\frac{dz}{dt}$ for $z = x^2y + xy^2$; $x = 2 + t^2$; $y = 1 - t^3$</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
<tr><td>11.</td><td>Use Chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for $z = x^2 + xy + y^2$; $x = s + t$; $y = st$</td><td>TLO2</td><td>L3</td><td>1.1.1</td></tr>
</table>

12.	Use Chain rule to find the indicated partial derivatives $z = x^2 + xy^3; \quad x = uv^2 + w^3, \ y = u + ve^w; \ \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}, \frac{\partial z}{\partial w}$ when $u = 2, \ v = 1, \ w = 0$	TLO2	L3	1.1.1
13.	If $u = f(x, y)$, where $x = e^s \cos t$ and $y = e^s \sin t$ then show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right]$	TLO2	L3	1.1.1
14.	The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's Law, to find the current I is changing at the moment when $R = 400\Omega$, $I = 0.08A$, $\frac{dv}{dt} = -0.01v/sec$ and $\frac{dR}{dt} = 0.03\Omega/sec$.	TLO2	L3	1.1.1
15.	Find the differential of the following functions: i) $u = e^t \sin\theta$ (ii) $u = \ln\sqrt{x^2 + y^2 + z^2}$	TLO3	L3	1.1.1
16.	If R is the total resistance of three resistors, connected in parallel, with resistances R_1, R_2, R_3 then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. If the resistances are measured in ohms as $R_1 = 25\,\Omega$, $R_2 = 40\,\Omega$ and $R_3 = 50\,\Omega$ with a possible error of 0.1% in each case, estimate the maximum relative error in the calculated value of R.	TLO3	L3	1.1.1
17.	Find the local maximum, minimum values and saddle points of the function (i) $f(x, y) = x^2 + y^2 + x^2y + 4$ (ii) $f(x, y) = x^3 - 12xy + 8y^3$	TLO4	L3	1.1.1
18.	Use Lagrange multipliers to find maximum and minimum values of the function $f(x, y) = 2x + 6y + 10z$; subject to given condition $x^2 + y^2 + z^2 = 35$	TLO5	L3	1.1.1
19.	Find the shortest distance from the point (1, 0, -2) to plane $x + 2y + z = 4$.	TLO5	L3	1.1.1
20.	Find the dimensions of the box with volume 1000cm^3 that has minimal surface area.	TLO5	L3	1.1.1
21.	A rectangular box without a lid is to be made from 12m^2 of cardboard. Find the maximum volume of such a box.	TLO5	L3	1.1.1

Practice Problems (Text book: Calculus by James Stewart, India edition)

Section: 14.1 Problems 32, 45.

Section: 14.3 Problems 39 to 42, 45 to 48, 51 to 60, 72 and 74.

Section: 14.4 Problems 33 to 38.

Section: 14.5 Problems 1 to 12, 21 to 34, 38 ,40, 41, 45 to 50

Section: 14.7 Problems 5,6,9,39,40,41,48

Section: 14.8 Problems 3 to 10

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Chapter wise Plan

Course Code and Title: ***18EMAB102*** / **Multivariable Calculus**	
Chapter Number and Title: **2. Double Integrals**	*Planned Hours:***08**

Learning Outcomes:

At the end of the topic the student should be able to:

Sr.No	TLO's	CO's	B L	CA Code
1	Evaluate a double integral – Rectangular and Polar coordinates.	CO 2	L3	1.1
2	Evaluate double integrals by changing the order of integration and by change of variables.	CO 2	L3	1.1
3	Find area of plane region using double integrals.	CO 2	L3	1.1

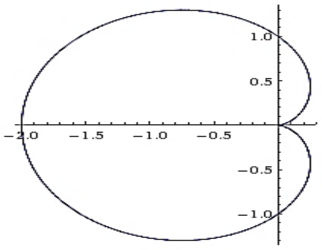
Lesson Schedule

Class No. Portion covered per hour

1. Concept of iterated integrals
2. Evaluation of double integrals over the regions
3. Examples
4. Evaluation of double integrals by change of order of integration
5. Examples
6. Jacobian and evaluation of double integrals by change of variables,
7. Examples
8. Application of double integrals to area

Review Questions

Sr. No	Questions	TLO	B L	PI Code
1.	Evaluate the double integrals (i) $\int_0^1 \int_x^{2-x} (x^2 - y)\,dxdy$ (ii) $\int_0^{\pi} \int_0^2 r\sin\theta\, dr d\theta$ (iii) $\iint_R \frac{xy^2}{x^2+1} dA,\ R = \{(x,y)\|0 \le x \le 1,\ \ -3 \le y \le 3\}$	TLO 1	L3	1.1.1
2.	Evaluate the double integrals (i) $\iint_D (x+y)dA$, D is bounded by $y = \sqrt{x}$ and $y = x^2$ (ii) $\iint_D y^3 dA$, D is triangular region with vertices (0, 2), (1, 1) and (3, 2)	TLO 1	L3	1.1.1
3.	The population density of a certain city is described by the function $f(x,y) = 10{,}000e^{-0.2x-0.1y}$ Where the origin (0, 0) gives the location of the city hall. What is the population inside the rectangular area described by $R = \{(x,y) - 10 \le x \le 10, -5 \le y \le 5\}$.	TLO 1	L3	1.1.1
4.	A thin plate covers the triangular region bounded by x-axis and the lines x=1 and y = 2x in the first quadrant. The plate's density at the point (x, y) is $\rho(x,y) = 6(x+y+1)$. Find the plates mass about the coordinate's axis.	TLO 1	L3	1.1.1
5.	Evaluate the integral by reversing the order of integration i) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dxdy$ ii) $\int_0^1 \int_x^1 e^{x/y}\,dydx$ iii) $\int_0^1 \int_{x^2}^{2-x} xy\,dydx$ iv) $\int_0^1 \int_{\sqrt{y}}^{2-y} xy\,dydx$	TLO 2	L3	1.1.1

6.	Evaluate the integral by converting to polar coordinates i) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}dydx$ ii) $\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)dxdy$ (iii)$\iint_D (x+y)dA$, D is the region that lies to the left of the y-axis between the circles $x^2+y^2=1$ and $x^2+y^2=4$.	TLO 2	L3	1.1.1
7.	Write a double integral $\iint_R f(x,y)dA$ which gives the volume of the top half of a solid ball of radius 5. Hence evaluate.	TLO 2	L3	1.1.1
8.	Use double integral to find area of the region (i) area bounded by the parabola $y=x^2$ and the line $y=2x+3$. (ii) smaller of the areas bounded by the circle $x^2+y^2=9$ and line $x+y=3$ (iii) The region inside the circle $r=4\sin\theta$ and outside the circle $r=2$. iv) One loop of the rose r=cos3θ	TLO3	L3	1.1.1
9.	Paul is trying to impress his girlfriend and is planning on planting flowers in her front yard in the shape of a heart. He has drawn his plan out on paper and is trying to figure out the area that he will cover with flowers. Here is the drawing with units in meters. What is the area used by Paul. 	TLO 3	L3	1.1.1

Practice Problems (Text book: Calculus by James Stewart, India edition)
Section 15.2: Problems 3 to 6, 10, 12, 15 to 22 Section 15.3: Problems 1 to 18, 39 to 45 Section 15.4: Problems 1 to 10, 16 to 18, 29, 30, 32.

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Chapter wise Plan

Course Code and Title: ***18EMAB102*** */* **Multivariable Calculus**	
Chapter Number and Title: **3 Triple Integrals**	*Planned Hours:* **07**

Learning Outcomes:

At the end of the topic the student should be able to:

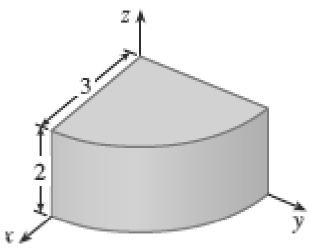
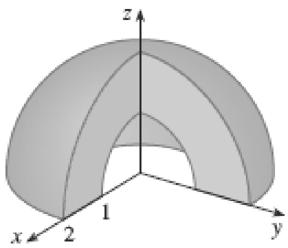
Sr.No	TLO's	CO's	B L	CA Code
1	Evaluation of triple integrals in Rectangular, Cylindrical and Spherical coordinates.	CO 2	L3	1.1
2	Find the volume using triple integrals.	CO 2	L3	1.1

Lesson Schedule

Class No. Portion covered per hour

1. Evaluation of triple integrals
2. Examples.
3. Evaluation of triple integrals : Cartesian coordinates
4. Evaluation of triple integrals : Cylindrical coordinates
5. Evaluation of triple integrals: Spherical coordinates.
6. Find the volume using triple integrals
7. Application problems using multiple integrals

Review Questions

Sr. No	Questions	TLO	B L	PI Code
1.	Evaluate the triple integral (i) $\int_0^1 \int_0^z \int_0^y ze^{-y^2} dxdydz$ (ii) $\int_0^3 \int_0^1 \int_0^{\sqrt{1-x^2}} ze^y dzdxdy$ (iii) $\iiint_E xydV$ where E is the solid tetrahedron with vertices (0,0,0), (1,0,0), (0,2,0) and (0,0,3)	TLO1	L3	1.1.1
2.	Set up the triple integral of an arbitrary continuous function $f(x,y,z)$ in cylindrical or spherical coordinates over the solid shown. Hence evaluate when $f(x,y,z) = 1$.  	TLO1	L2	1.1.1
3.	Find the mass (in kg) of a ball, which has a radius of 2m and density $\delta(x,y,z) = 2kg/m^2$	TLO1	L3	1.1.1
4.	A solid fills the region between two concentric spheres of radii a and b, 0 < a < b. The density at each point is inversely proportional to its square distance from the origin. Find the total mass.	TLO1	L3	1.1.1

5.	Evaluate the following integrals by changing the variables i. $\iiint xyz\,dxdydz$, over the positive octant of the sphere $x^2+y^2+z^2=a^2$. ii. $\iiint_V (x^2+y^2)dxdydz$, taken over the region V bounded by the paraboloid $z=9-x^2-y^2$ and the plane z = 0. iii. $\iiint x^2+y^2+z^2 dxdydz$ taken over the region $0\le z\le x^2+y^2\le 1$.	TLO1	L3	1.1.1
6.	Consider the cube of side 1, then express the integral $\int_V f\,dV$ (where f is a function of x, y, z) as a triple integral and hence evaluate $\int_V (y^2+z^2)dV$.	TLO1	L3	1.1.1
7.	If V is the tetrahedron bounded by planes x=0, y=0, z=0 and $x+y+z=4$ then express $\int_V f\,dV$ (where f is a function of x, y, z) as a triple integral and hence evaluate $\int_V x\,dV$.	TLO1	L3	1.1.1
8.	Use triple integral to find the volume of the given solid (i) The tetrahedron enclosed by the coordinate planes and the plane $2x+y+z=4$ (ii) The solid enclosed by the paraboloid $z^2+y^2=x$ and plane $x=16$. (iii) The solid enclosed by the cylinder $x^2+y^2=9$ and the planes $y+z=5$ and $z=1$.	TLO2	L3	1.1.1

Practice Problems (Text book: Calculus by James Stewart, India edition)
Section 15.6: Problems 3 to 9, 12,15,16,19,22.

Chapter wise Plan

Course Code and Title: ***18EMAB102*** / **Multivariable Calculus**	
Chapter Number and Title: **4. Calculus of Vector Fields**	*Planned Hours:* **13**

Learning Outcomes:

At the end of the topic the student should be able to:

Sr.No	TLO's	CO's	B L	CA Code
1	Find gradient, curl and divergence of a vector function and directional derivative of scalar function.	CO3	L3	1.1
2	Evaluate the line and surface integrals	CO3	L3	1.1
3	Apply Green's theorem, Stoke's theorem, Gauss-divergence theorem to evaluate the integrals.	CO3	L3	1.1

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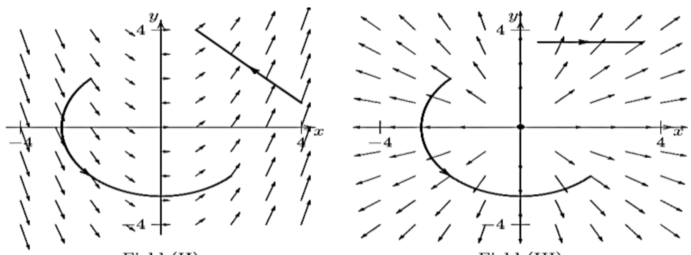
Lesson Schedule

Class No. Portion covered per hour

1. Vector and scalar functions, Gradient
2. Directional derivatives , examples
3. Line and Surface integrals
4. Examples
5. Independence of path and potential functions
6. Green's theorem
7. Examples
8. Divergence of vector field and Examples
9. Divergence theorem
10. Examples
11. Curl of vector field
12. Stokes theorem
13. Examples.

Review Questions

Sr. No	Questions	TLO	B L	PI Code
1.	Suppose the temperature T at each point (x, y, z) in a region of space is given by $T = 100 - x^2 - y^2 - z^2$ and F (x, y, z) is defined to be gradient of T. Find the vector field.	TLO1	L3	1.1.1
2.	Plot gradient vector field together with contour map of f and how they are related to each other? (i) $f(x,y) = x^2 + y^2$ (ii) $f(x,y) = x^2 - y^2$ (iii) $f(x,y) = xy$ (iv) $f(x,y) = \sqrt{x^2 + y^2}$	TLO1	L3	1.1.1
3.	Calculate the angle between the normal to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3)	TLO1	L3	1.1.1
4.	Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1,-2,1).	TLO1	L3	1.1.1
5.	Find the directional derivative of the function at the given point in the direction of vector u where $f(x,y) = 1 + 2x\sqrt{y}$; $P(3,4)$; $u = \langle 4, -3 \rangle$	TLO1	L3	1.1.1
6.	Find the directional derivative of $f(x,y,z) = x^2 + y^2 + z^2$ at P(2,1,3) in the direction of origin.	TLO1	L3	1.1.1
7.	Find the maximum rate of change of $f(x,y,z) = \ln(xy^2z^3)$ at a point (1,-2,-3) and the direction in which it occurs.	TLO1	L3	1.1.1
8.	Suppose that over a certain region of space the electrical potential V is given by $V(x,y,z) = 5x^2 - 3xy + xyz$ (i) Find the rate of change of the potential at P(3,4,5) in the direction of the vector $a = i + j - k$(ii) In which direction does V change most rapidly at P? (iii) What is the maximum rate of change at P?	TLO1	L3	1.1.1

9.	A fly is flying around a room in which the temperature is given by $T(x,y,z) = x^2 + y^4 + 2z^2$.The fly is at the point (1, 1, 1) and realizes that he's cold. In what direction should he fly to warm up most quickly? If he flies in this direction, what will be the instantaneous rate of change of his temperature?	TLO1	L3	1.1.1
10.	You're hiking a mountain which is the graph of $f(x,y) = 15 - x^2 - 2xy - 3y^2$. You're standing at (1, 1, 9). You wish to head in a direction which will maintain your elevation (so you want the instantaneous change in your elevation to be 0). How many possible directions are there for you to head? What are they?	TLO1	L3	1.1.1
11.	Evaluate $\int_c y^2\,dx + x\,dy$, where (a) C=C_1 is the line segment from (-5,-3) to (0,2) and (b) C=C_2 is the arc of the parabola $x = 4 - y^2$ from (-5,-3) to (0,2)	TLO2	L3	1.1.1
12.	Here are two vector fields. For each curve C drawn on the vector field, determine if possible whether $\int_C \mathrm{F.dr}$ would be positive, negative, or zero. 	TLO2	L3	1.1.1
13.	Find the work done by the force field $F(x,y) = x^2 i + xyj$ on a particle that moves once around the circle $x^2 + y^2 = 4$ oriented in the counterclockwise direction. Interpret the answer.	TLO2	L3	1.1.1
14.	Find the work done by the force field $F(x,y,z) =< x - y^2, y - z^2, z - x^2 >$ on the particle that moves along the line segment from (0,0,1) to (2,1,0).	TLO2	L3	1.1.1
15.	Find the work done by the force field $F(x,y,z) = \langle y+z, x+z, x+y \rangle$ on a particle that moves along the line segment from (1,0,0) to (3,4,2)	TLO2	L3	1.1.1
16.	Use Greens theorem to evaluate the line integral along the given positively oriented curve $\int_c (y + e^{\sqrt{x}})dx + (2x + \cos y^2)dy;$ C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$	TLO3	L3	1.1.1
17.	Employ Green's theorem to prove that the area of a simple closed curve C is $\frac{1}{2}\int_C xdy - ydx$Hence find the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and deduce the area bounded by the circle $x^2 + y^2 = a^2$	TLO3	L3	1.1.1
18.	Using Green's theorem, find the area of the region in the first quadrant bounded by the curves $y = x, y = \frac{1}{x}, y = \frac{x}{4}$.	TLO3	L3	1.1.1
19.	Evaluate $\oint_c y^2\,dx + 3xy\,dy$, where C is the boundary of the semiannular region D in the upper half–plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.	TLO3	L3	1.1.1
20.	Define curl and divergence of a vector point function and find curl and divergence of $F(x,y,z) = i + (x + yz)j + (xy - \sqrt{z})k$	TLO1	L2	1.1.1
21.	Show that the vector $V = (x + 3y)i + (y - 3z)j + (x - 2z)k$ is solenoidal.	TLO1	L3	1.1.1

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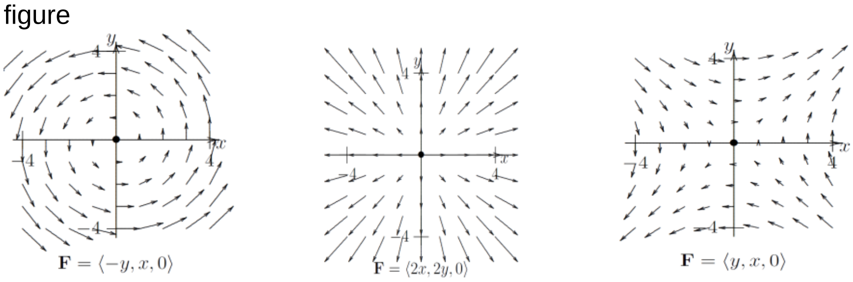
22.	Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $F = \nabla f$ (i) $F(x, y) = (3y^2 - 2y^2)i + (4xy + 3)j$ (ii) $F(x, y) = (ye^x + \sin y)i + (e^x + x\cos y)j$ (iii) $F(x, y, z) = y\cos xyi + x\cos xyj - \sin zk$	TLO1	L3	1.1.1
23.	Find the constants a, b, c so that F is irrotational. $F(x, y, z) = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$	TLO1	L3	1.1.1
24.	Here are sketches of a few vector fields $F = \langle P(x, y), Q(x, y), 0\rangle$ can you tell which one has zero curl? Zero divergence? Explain with reference to the figure  	TLO1	L2	1.1.1
25.	Evaluate $\iint_S \vec{F}.\hat{n}dS$ where $\vec{F} = 18zi - 12j + 3yk$ and S is part of the plane $2x + 3y + 6z = 12$ which is located in the first octant.	TLO2	L3	1.1.1
26.	Find the outward flux of the field $\vec{F} = zi + xj - 3y^2zk$ across the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$	TLO2	L3	1.1.1
27.	Apply Stokes theorem to evaluate $\int_C F.dr$ where C is oriented counter clockwise direction $F(x, y, z) = yzi + 2xzj + e^{xy}k$ C is the circle $x^2 + y^2 = 16, z = 5$	TLO3	L3	1.1.1
28.	Apply Stokes theorem to evaluate $\iint_S curl\vec{F}.\hat{n}dS$ $F(x, y, z) = xi + yj + z^4k$ S is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane z = 1 with downward direction.	TLO3	L3	1.1.1
29.	Find the outward flux of the field $\vec{F} = 6zi + (2x + y)j - xk$ across the entire surface S of the region bounded by the cylinder $x^2 + z^2 = 9$, $x = 0$, $y = 0, z = 0$ and $y = 8$.	TLO3	L3	1.1.1
30.	Use Divergence theorem to calculate the surface integral $\iint_S F.dS; F(x, y, z) = x^2yi + xy^2j + 2xyzk$, where S is the surface of the tetrahedron bounded by planes $x = 0, y = 0, z = 0$ and $x + 2y + z = 2$	TLO3	L3	1.1.1

Practice Problems (Text book: Calculus by James Stewart, India edition)
Section 16.4: Problems 5 to 10 Section 16.5: Problems 1 to 8 and 13 to 18 Section 16.8: Problems 2 to 10 Section 16.9: Problems 5 to 7 and 10 to 12

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Chapter wise Plan

Course Code and Title: ***18EMAB102*** / **Multivariable Calculus**	
Chapter Number and Title: ***5*** **Differential Equations of higher orders**	*Planned Hours:* **10**

Learning Outcomes:

At the end of the topic the student should be able to:

Sr.No	TLO's	CO's	B L	CA Code
1	Solve second order linear differential equation.	CO4	L3	1.1
2	Apply concept of second order linear differential equation to solve problems concerning the vibrations of springs, SHM and electrical circuits.	CO4	L3	1.1
3	Solve differential equation using power series.	CO4	L3	1.1

Lesson Schedule

Class No. Portion covered per hour

1. Introduction to differential equations of second order & higher order with constant Coefficients and solution of homogeneous linear differential equation with examples.
2. Initial and boundary value problems.
3. Solutions of non- homogeneous L.D.E & methods to find P.I of the type $X = e^{ax}$.
4. $X = \sin(ax)$ or $X = \cos(ax)$ and $X = x^m$
5. $X = e^{ax}V$ and $x^m V$
6. Methods of variations of parameters.
7. Applications problems: Vibrating springs
8. Applications problems: Electrical circuits.
9. Power series solutions for differential equation.
10. Examples on solutions for differential equation using power series.

Review Questions

Sr.No	Questions	TLO	B L	PI Code
1.	Solve the homogeneous differential equation. . (i) $9y'' - 12y' + 4y = 0$ (ii) $8\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 5y = 0$	TLO1	L3	1.1.1
2.	Solve the initial values problems (i) $4y'' - 4y' + y = 0$; $y(0) = 1, y'(0) = -1.5$ (ii) $y'' - 2y' + 5y = 0$; $y(\pi) = 0, y'(\pi) = 2$ (iii) $y'' + 2y' + 2y = 0$; $y(0) = 2, y'(0) = 1$	TLO1	L3	1.1.1
3.	Solve the boundary values problems, if possible : (i) $4y'' + y = 0;\ y(0) = 3,\ y(\pi) = -4$ (ii) $y'' - 3y' + 2y = 0;\ y(0) = 1,\ y(3) = 0$ (iii) $y'' - 6y' + 9y = 0;\ y(0) = 1, y(1) = 0$	TLO1	L3	1.1.1
4.	Solve the non-homogeneous differential equation (i) $y'' - 4y' + 5y = e^{-x}$ (ii) $y'' + 9y = e^{3x}$ (iii) $y'' - 2y' + y = e^{2x}$ (iv) $4y'' + y = \cos x$ (v) $y'' - 2y' = \sin 4x$ (vi) $y'' + 3y' + 2y = x^2$ (vii) $y'' + 6y' + 9y = 1 + x$ (viii) $y'' + 2y' + y = xe^{-x}$ (ix) $y'' + 4y = e^{3x} + x\sin 2x$ (x) $y'' - 4y = e^x \cos x$; $y(0) = 1, y'(0) = 2$ (xi) $y'' - y' = xe^x,\ y(0) = 2,\ y'(0) = 1$ (xii) $y'' + 9y = 1 + xe^{9x}$ (xiii) $y'' + y' - 2y = x + \sin 2x,\ y(0) = 1, y'(0) = 0$	TLO1	L3	1.1.1

5.	Solve the differential equation using the method of variation of parameters. (i) $y'' + y = \tan x, 0 < x < \frac{\pi}{2}$ (ii) $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$ (iii) $y'' - 2y' + y = e^x \log x$ (iv) $y'' - 2y' + 2y = e^x \tan x$ (v)$y'' - 2y' + y = \frac{e^x}{1+x^2}$ (vi)$y'' + 3y' + 2y = \sin(e^x)$ (vii) $y'' + y = \frac{1}{1+\sin x}$	TLO1	L3	1.1.1
6.	A series circuit consists of a resister with $R = 40$ Ohms an inductor with $L = 2H$, a capacitor with $C = 0.0025$ F and a 12V battery. If the initial charge is $Q = 0.01C$ and initial current is 0, find charge and current at time.	TLO2	L3	1.1.1
7.	A series circuit contains a resistor with $R = 20$ Ohms an inductor with $L = 1H$, a capacitor with $C = 0.002$ F and a generator producing a voltage of E(t)=12 sin10t. The initial charge is $Q = 0.001C$ and the initial current is 0.Find the charge t at time t.	TLO2	L3	1.1.1
8.	A series circuit consists of a resistor with $R = 40$ Ohms, an inductor with $L = 1H$, a capacitor with $C = 16 \times 10^{-4}, E(t) = 100\cos(10t)$.If the initial charge and current are both 0,find the charge and current at any time.	TLO2	L3	1.1.1
9.	Find the charge on the capacitor in RLC-series circuit where $L = \frac{5}{3}H$, $R = 100\ \Omega$, $c = \frac{1}{30}F$ and $E(t) = 300V$. Assume the initial charge on the capacitor is 0C and the initial current is 9A. What happens to the charge on the capacitor over the time?	TLO2	L3	1.1.1
10.	Show that the frequency of free vibrations in a closed electrical circuit with inductance L and capacitance C in series is $\frac{30}{\pi\sqrt{(LC)}}$ per minute.	TLO2	L3	1.1.1
11.	A particle is executing simple harmonic motion with amplitude 5 meters and time 4 seconds. Find the time required by the particle in passing between points which are at distances 4 and 2 meters from the centre of force and are on the same side of it.	TLO2	L3	1.1.1
12.	A particle moving in a straight line with simple harmonic motion has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2 respectively. Show that the period of motion is $2\pi\sqrt{\frac{x_1^2-x_2^2}{v_2^2-v_1^2}}$	TLO2	L3	1.1.1
13.	A particle of mass of 4gm executing SHM has velocities 8cm/sec and 6cm/sec respectively when it is at distance 3cm and 4cm from the center of its path. Find its period and amplitude. Find also the force acting on the particle when is at a distance of 1cm from the center.	TLO2	L3	1.1.1
14.	A spring with a mass of 2kg has natural length 0.5 m. A force of 25.6N is required to maintain it stretched to a length of 0.7m. If the spring is stretched to a length of 0.7m and then released with initial velocity 0, find the position of the mass at time t.	TLO2	L3	1.1.1
15.	Use power series to solve (i) $y'' + xy' + y = 0$ (ii) $y'' = y$ (iii) $y'' + x^2y = 0,\ y(0) = 1,\ y'(0) = 0$ (iv)$y'' + x^2y + xy = 0,\ y(0) = 0,\ y'(0) = 1$	TLO3	L3	1.1.1

Practice Problems (Text book: Calculus by James Stewart, India edition)
Section 17.2: Problems 1 to 10 and 13 to 18

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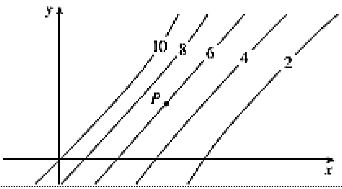
Model Question Paper for ISA-1	
Course Code: 18EMAB102	**Course Title: Multivariable Calculus**
Duration:	**75 minutes**
Max. Marks:	**40**
Note: Answer any TWO full questions	

Q.No	Questions	Marks	CO	PI Code	B L
1 a	Level curves are shown for a function. Determine whether the following partial derivatives are positive or negative at the point. (a)f_x (b) f_y (c) f_{xx} (d) f_{xy} (e) f_{yy} 	6	CO1	1.1.1	L3
b	The length l, width w and height h of a box change with time. At a certain instant the dimensions are l=1m and w=h=2m and l and w are increasing at a rate of 2m/s while h is decreasing at a rate of 3m/s. At that instant find the rates at which the following quantities are changing. (i) The volume (ii) The surface area (iii) The length of a diagonal.	7	CO1	1.1.1	L3
c	Use double integral to find area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	7	CO2	1.1.1	L3
2 a	Evaluate the double integrals (i) $\int_1^3\int_0^1(1+4xy)dxdy$ (ii)$\int_0^{\pi/2}\int_0^{2\cos\theta} e^{\sin\theta}\,dr d\theta$	6	CO2	1.1.1	L3
b	Evaluate the integral by reversing the order of integration $\int_0^1\int_{\sqrt{y}}^1\sqrt{x^3+1}dxdy$	7	CO2	1.1.1	L3
c	If $u=f(x,y)$, where $x=e^s\cos t$ and $y=e^s\sin t$ then show that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2+\left(\frac{\partial u}{\partial t}\right)^2\right]$	7	CO1	1.1.1	L2
3 a	Show that $u=\frac{1}{\sqrt{x^2+y^2+z^2}}$ is a solution of the three dimensional Laplace equation $u_{xx}+u_{yy}+u_{zz}=0$.	6	CO2	1.1.1	L3
b	If x, y, z are the angles of a triangle , show that the maximum value of $\cos x\cos y\cos z=1/8$.	7	CO1	1.1.1	L3
c	Evaluate$\int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}dydx$ by converting to polar coordinates	7	CO2	1.1.1	L3

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Model Question Paper for ISA-2	
Course Code: 18EMAB102	**Course Title: Multivariable Calculus**
Duration:	**75 minutes**
Max. Marks:	**40**

Note: Answer any TWO full questions

Q.No	Questions	Marks	CO	PI Code	B L
1 a	Find the directional derivative of $f(x,y,z)=x^2+y^2+z^2$ at P(2,1,3) in the direction of origin.	6	CO3	1.1.1	L3
b	Use triple integral to find the volume of the tetrahedron enclosed by the coordinate planes and the plane $2x+y+z=4$	7	CO2	1.1.1	L3
c	Apply Stokes theorem to evaluate $\int_C F\cdot dr$ where $F(x,y,z)=(x+y^2)i+(y+z^2)j+(z+x^2)k$, C is the triangle with vertices (1,0,0), (0,1,0) and (0,0,1)	7	CO3	1.1.1	L3
2 a	Evaluate $\iiint_E (x^3+xy^2)dV$, where E is bounded by the planes z=0 and z=x+y+3 and by the cylinders $x^2+y^2=4$ and $x^2+y^2=9$.	6	CO2	1.1.1	L3
b	Evaluate $\iiint_E yz\cos(x^5)dV$, where $E(x,y,z)=\{(x,y,z):0\le x\le 1, 0\le y\le x, x\le z\le 2x\}$	7	CO3	1.2.1	L3
c	Use Divergence theorem to calculate the surface integral $\iint_S F.dS$ where $F(x,y,z)=x^2yi+xy^2j+2xyzk$; S is the surface of the tetrahedron bounded by planes$x=0,=0,z=0\ \&\ x+2y+z=2$	7	CO3	1.1.1	L3
3 a	Determine whether or not the vector field $F(x,y,z)=2xyi+(x^2+2yz)j+y^2k$ is conservative. If it is conservative, find a function f such that $F=\nabla f$.	6	CO3	1.1.1	L3
b	Find the work done by the force field $F(x,y,z)=<x-y^2,y-z^2,z-x^2>$ on the particle that moves along the line segment from (0, 0, 1) to (2, 1, 0).	7	CO2	1.1.1	L3
c	Plot gradient vector field together with contour map of f and how they are related to each other? $f(x,y)=x^2+y^2$	7	CO3	1.1.1	L3

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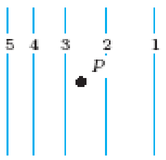
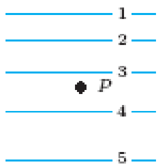
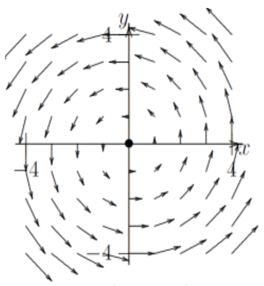
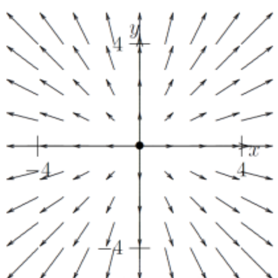
Model Question Paper for End Semester Assessment	
Course Code: 18EMAB102	**Course Title**: **Multivariable Calculus**
Duration:	**3 hrs**
Max. Marks:	**100**

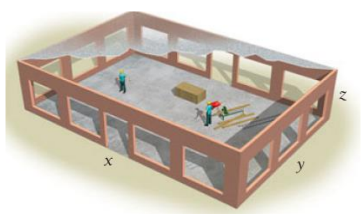
Note: Answer any **five** questions choosing any two full questions from unit I and unit II and any **one** full question from unit III.

Unit-I

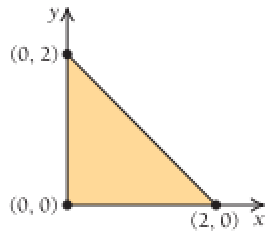
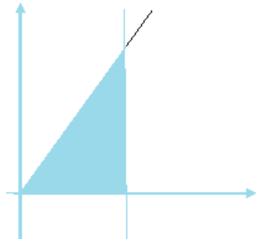
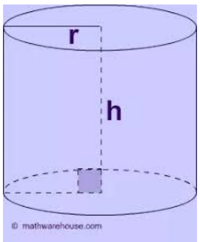
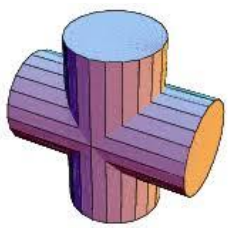
Q.No	Questions	Marks	CO	PI Code	B L
1 a	Use the level curves of the function $z = f(x, y)$ to decide the sign (positive, negative, or zero) of each of the following partial derivatives at the point P. Assume the x- and y-axes are in the usual positions.  Fig.a  Fig.b	6	CO1	1.1.1	L3
b	The pressure, volume and temperature of a mole of an ideal gas are related by the equation $PV = 8.31RT$, where P is measured in kilopascals, V in liters and T in Kelvins. Use differentials to find the approximate change in the pressure if the volume increases from 12L to 12.3L and the temperature decreases from 310K to 305K.	7	CO1	1.1.1	L3
c	Evaluate the integral by reversing the order of integration $\int_0^3 \int_{y^2}^9 y \cos x^2 \, dxdy$	7	CO2	1.1.1	L3
2 a	Find the mass M of a metal plate R bounded by y=x and y=x^2 , with density given by the function $f(x, y) = 1 + xy \ \ kg/\ m^2$	6	CO2	1.1.1	L3
b	Sketch the region in the xy plane bounded by x-axis, y=x, x+y=1 ii)Express the integral of f(x,y) over the region in terms of iterated integrals in two ways iii) Using one of your answers to part (b), evaluate the integral exactly f(x, y)=x.	7	CO2	1.1.1	L3
c	If, $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, find $u_{xx} + u_{yy} + u_{zz}$	7	CO1	1.1.1	L2
3 a	Use double integral to find area of the rose $r = \sin 3\theta$.	6	CO2	1.1.1	L3
b	Find the dimensions of the rectangular box of maximum volume such that the sum of its 12 edges is a constant c.	7	CO1	1.1.1	L3
c	Evaluate by converting to polar coordinates $\iint_R \cos(x^2 + y^2)\, dA$, where R is the region in the first quadrant of circle $x^2 + y^2 = 9$.	7	CO2	1.1.1	L3

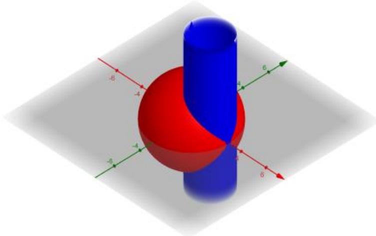
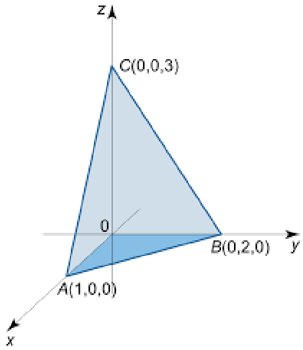
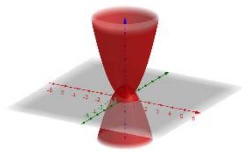
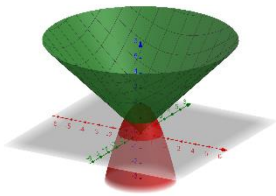
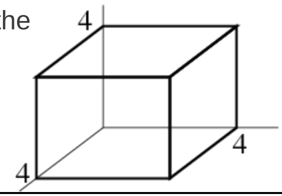
	Unit-II				
4 a	A vector field is given by $\vec{F} = \sin y\, i + x(1+\cos y)j$. Evaluate the line integral over the circular path given by $x^2+y^2=a^2,\ z=0$	6	CO3	1.1.1	L3
b	A solid E lies within the cylinder $x^2+y^2=1$, below the plane z=4 and above the paraboloid $z=1-x^2-y^2$.The density at any point is proportional to its distance from axis of the cylinder. Find the mass of E.	7	CO2	1.1.1	L3
c	Use Divergence theorem to calculate $\iint_S F.dS$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ and$F = x^3 i + y^3 j + z^3 k$	7	CO3	1.1.1	L3
5 a	Use spherical co-ordinates to find the volume of the solid that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$.	6	CO3	1.1.1	L3
b	Evaluate,$\iiint_E z\, dv$ where E is the solid tetrahedron bounded by the four planes and $x=0, y=0, z=0$ and $x+y+z=1$	7	CO2	1.1.1	L3
c	Use Stokes theorem to evaluate $\iint_S \text{curl}\, F.dS$ where $F=(x^2+y-4)i+3xyj+(2xz+z^2)k$ and S is the surface of $x^2+y^2+z^2=16$ above the xy-plane.	7	CO3	1.1.1	L3
6 a	Use greens theorem to find $\int_C (3x-y)dx+(2x+y)dy,\quad \text{where } C: x^2+y^2=a^2$	6	CO3	1.1.1	L3
b	Evaluate $\int_0^x\int_0^y\int_0^z \cos(x+y+z)\, dzdydx$	7	CO2	1.1.1	L3
c	Here are sketches of a few vector fields $F = \langle P(x,y), Q(x,y), 0\rangle$can you tell which one has zero curl? Zero divergence? Explain with reference to the figure.  $F=\langle -y,x,0\rangle$  $F=\langle 2x,2y,0\rangle$	7	CO3	1.1.1	L3

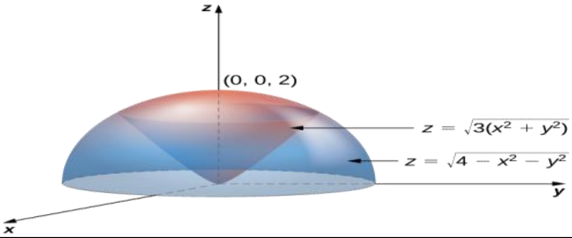
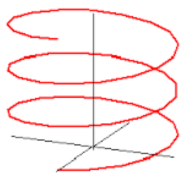
	Unit-III				
7 a	Solve the initial values problem $y'' = 2y' + 5y = 0, \quad y(\pi) = 0, y'(\pi) = 2$	6	CO4	1.1.1	L3
b	Solve the non-homogeneous differential equation $y'' + y = e^x + x^3, y(0) = 2, y'(0) = 0.$	7	CO4	1.1.1	L3
c	Solve $y'' + y = secx, 0 < x < \frac{\pi}{2}$ using the method of variation of parameters.	7	CO4	1.1.1	L3
8 a	At the end of three successive seconds, the distances of a point moving with simple harmonic motion, from its mean position are 1, 5, and 5 respectively. Find the time of a complete oscillation.	4	CO4	1.1.1	L3
b	Use power series to solve $(x^2 + 1)y'' + xy' - y = 0.$	8	CO4	1.1.1	L3
c	A series circuit consists of a resistor with R=20ohm an inductor with L=1 H, a capacitor with C=0.002 F and a 12V battery. If the initial charge and current are both 0, find charge and current at time t.	8	CO4	1.1..1	L3

Sr.No	MATLAB TUTORIAL EXERCISES
	Introduction of Partial derivatives
1	A rectangular box without a lid is to be made from $12\mathrm{m}^2$ of cardboard. Find the maximum volume of such a box.
2	A rectangular box must have volume $500\,\mathrm{in}^3$. Find the shape that has the smallest mailing length (the sum of the three edge lengths).
3	The Beverage-Can Problem. The standard beverage can holds 12 fl. oz, or has a volume of 21.66 what dimensions yield the minimum surface area? Find the minimum surface area. (Assume that the shape of the can is a right circular cylinder 
4	Minimizing construction costs: A company is planning to construct a warehouse whose interior volume is to be 252,000 Construction costs per square foot are estimated to be as follows: Walls: $3.00 Floor: $4.00 Ceiling: 3.00 

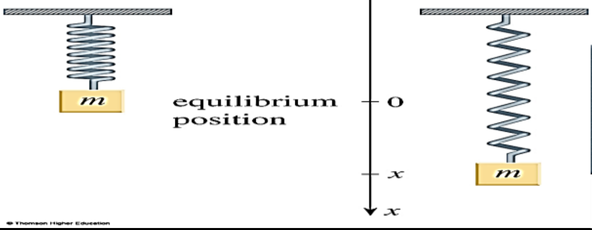
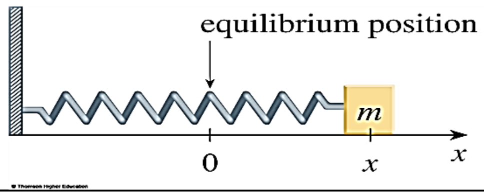
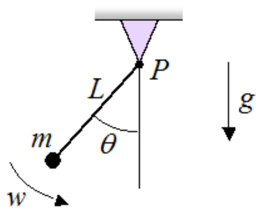
5	Minimizing surface area: An oil drum of standard size has a volume of 200 gal, or 27 ft. What dimensions yield the minimum surface area? Find the minimum surface area. 
6	Minimizing the cost of a container: A trash company is designing an open-top, rectangular container that will have a volume of 320. The cost of making the bottom of the container is \$5 per square foot, and the cost of the sides is \$4 per square foot. Find the dimensions of the container that will minimize total cost. (Hint: Make a substitution using the formula for volume)
7	Evaluate the double integrals I) $\int_1^3\int_0^1(1+4xy)dxdy$ II) $\int_0^1\int_x^{2-x}(x^2-y)dxdy$ III) $\int_0^{\pi/2}\int_0^{\cos\theta}e^{\sin\theta}drd\theta$ IV) $\int_0^2\int_0^{\pi}r\sin\theta drd\theta$ V) $\iint_R \frac{xy^2}{x^2+1}dA,\ R=\{(x,y)\mid 0\le x\le 1,\ \ -3\le y\le 3\}$
8	Integrate the function $\sqrt{x^2+y^2+1}$over the region bounded by the ellipse$3x^2+4y^2=37$
9	Evaluate by converting to polar I) $\int_0^1\int_0^{\sqrt{1-x^2}}e^{x^2+y^2}dydx$ II) $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}dydx$ III) $\int_{y=0}^1\int_{x=\sqrt{y}}^1\sqrt{x^3+1}dxdy$
10	Use double integral to find area of the region (I) area bounded by the parabola $y=x^2$and the line y=2x+3. (II) smaller of the areas bounded by the circle $x^2+y^2=9$ and line x+y=3 (III) The region inside the circle $r=4\sin\theta$ and outside the circle r=2
11	An agricultural sprinkler distributes water in a circular pattern of radius 100ft. It supplies water to a depth of e^{-r}feet per hour at a distance of r feet from the sprinkler. (a) If $0<R<100$, what is the total amount of water supplied per hour to the region between the circle of radius R=5 and R=100, centered at the sprinkler? (b) Determine the average amount of water per hour per square foot supplied to the region inside the circle of radius 5 and 100 feet.
12	The population density of fireflies in a field is given by $f=\frac{1}{100}x^2y$where $0\le x\le 30$ and $0\le y\le 20$, x and y are in feet, and f is the number of fireflies per square foot. Determine the total population of fireflies in the field.
13	The density of students living near a university is modeled by $p(x,y)=9-x^2-y^2$. Where x and y are in miles and p is the number of students per square mile, in hundreds. Assume the university is located at (0, 0). Find the number of students who live in the shaded region shown below.

	
14	The population density of a certain city is described by the function $f(x,y) = 10{,}000e^{-0.2x-0.1y}$ Where the origin (0, 0) gives the location of the city hall. What is the population inside the rectangular area described by $R = \{(x,y) - 10 \le x \le 10,\ -5 \le y \le 5\}$.
15	A thin plate covers the triangular region bounded by x-axis and the lines x=1 and y = 2x in the first quadrant. The plate's density at the point (x, y) is $\rho(x,y) = 6(x+y+1)$. Find the plates mass about the coordinate's axis. 
16	Triple Integrals 1 Evaluate $\int_0^1 \int_0^{x^2} \int_0^{x+y} (x-2y+z)\,dzdydx$ 2.Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ 3. Evaluate $\int\int\int y\sin(x) + z\cos(x)$ over the region $0 \le x \le \pi$, $0 \le y \le 1$, and $-1 \le z \le 1$. 4. Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)\,dxdydz$
17	Suppose the temperature at a point is given by T=xyz. Find the average temperature in the cube with opposite corners at (0,0,0) and (2,2,2)
18	Find the volume of the cylinder $x^2 + y^2 = a^2$; z=0; z=h. 
19	Find the volume of the solid common to the two cylinder $x^2 + y^2 = a^2,\ x^2 + z^2 = a\text{^}2.$ 

20	Find the volume of the portion of the sphere $x^2+y^2+z^2=a^2$ lying inside the cylinder $x^2+y^2=ax$ 
21	Find the volume of the tetrahedron bounded by the plane passing through the points A(1,0,0), B(0,2,0), C(0,0,3) and the three coordinate planes . 
22	Find the volume of the solid formed by two paraboloids: $z_1=x^2+y^2$ and $z_2=1-x^2-y^2$ 
23	Calculate the volume of the solid bounded by the paraboloid $z=2-x^2-y^2$ and the conic surface $z=\sqrt{x^2+y^2}$ 
24	A cube has sides of length 4. Let one corner be at the origin and the adjacent corners be on the positive x, y, and z axes. 
25	Let W be the pyramid bounded by the planes $z=0, z=4-2x, z=2-y, z=2x$ and $z=2+y$. Mass of the pyramid is the triple integral of the density which is given by $\iiint f(x,y,z)dV$
26	Use cylindrical coordinates to find the volume of a curved wedge cut out from a cylinder $(x-2)^2+y^2=4$ by the planes z=0 and z=-y.

27	Find the volume of the region bounded by the cone $z = \sqrt{3(x^2 + y^2}$ and the hemisphere $z = \sqrt{(4 - x^2 - y^2)}$ 
28	Plot of gradient field $f = \frac{1}{2}(x^2 + y^2)$
29	Plot gradient vector field together with contour map of $f(x, y) = x^2 + y^2$ and how they are related to each other?
30	Finding directional derivative in given direction Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$ (i) Find the rate of change of the potential at P(3,4,5) in the direction of the vector $a = i + j - k$ (ii) In which direction does V change most rapidly at P? (iii) What is the maximum rate of change at P?
31	Evaluate line integral $\int_c y^2 dx + x dy$, where (a) $C = C_1$is the line segment from (-5,-3) to (0,2) and (b) $C = C_2$ is the arc of the parabola $x = 4 - y^2$ from (-5,-3) to (0,2)
32	Application of line integrals i) Find the workdone by the force field $F = x^2 i - xyj$ in moving a partical along the quater circle $r(t) = cos t i + sin t j, 0 \leq t \leq \frac{\pi}{2}$ ii) A 160-lb man carries a 25-lb can of paint up a helical staircase that encircles a silo with a radius of 20 ft. If the silo is 90 ft high and the man makes exactly three complete revolutions, how much work is done by the man against the gravity in climbing the top? path of integration 
33	**<u>Greens Theorem</u>** Use Greens theorem to evaluate the line integral along the given positively oriented curve $\int_C (y + e^{\sqrt{x}})dx + (2x + \cos y^2)dy$; C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.
34	**<u>Application of Greens Theorm</u>** (to find area) 2.Using Green's theorem, find the area of the region in the first quadrant bounded by the curves $y = x, y = \frac{1}{x}, y = \frac{x}{4}$
35	**Divergence theorem** i) Use divergence theorem to calculate the flux of $F = 3xy^2 i + xe^z j + z^3 k$ across surface S of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes x=-1 and x=2 with upward orientation. ii) A fluid has density 1500 and velocity field $v = -yi + xj + 2zk$. Find the rate of flow outward through the sphere $x^2 + y^2 + z^2 = 25$. iii) The temperature at the point (x, y, z) in a substance with conductivity K=6.5 is $u = 2y^2 + 2z^2$. Find the rate of heat flow inward across the cylindrical surface $y^2 + z^2 = 6, 0 \leq x \leq 4$.

DEPARTMENT OF MATHEMATICS

36	**Stokes Theorem** i) Use Stokes theorem to evaluate surface integral $F = yzi + xzj + xyk$; S is surface of paraboloid $z = 9 - x^2 - y^2$ that lies above the plane z=5 ii) Use Stokes theorem to evaluate line integral $F = e^{-x}i + e^{x}j + e^{z}k$. , C is boundary of the part of the plane 2x+y+2z=2 in the first octant.
37	A string with a mass of 2kg has natural length 0.5m. A force of 25.6 N is required to maintain it stretched to a length of 0.7m and then released with initial velocity 0, find the position of the mass at any time t. Soln-From hooke's law, the force required to stretch the spring is k(0.2)=25.6, so k=128. 
	Suppose that the spring of the above example is immersed in a fluid with damping constant c=40. Find the position of the mass at any time t if it starts from the equilibrium position and is given a push to start it with an initial velocity of 0.6m/s.
38	A string with a mass of 2kg has natural length 0.5m. A force of 25.6 N is required to maintain it stretched to a length of 0.7m .Suppose that the spring is immersed in a fluid with damping constant c=40. Find the position of the mass at any time t if it starts from the equilibrium position and is given a push to start it with an initial velocity of 0.6m/s.
39	A string has mass of 1 kg and its spring constant is k=100.The spring is released at a point 0.1m above its equilibrium position. Graph the position function for thr following values of the damping constant c:10 ,15, 20 ,25 , 30.What type of damping occurs in each case
40	A series circuit contains a register with R =24Ω an inductor with L=2 H, a capacitor with C=0.005F, and a 12-V battery .The initial charge is Q=0.001 C and the initial current is 0. a) Find the charge and current at time t b) Graph the charge and current functions.
41	The figure shows a pendulum with length L and the angle θ from the vertical to the pendulum. It can be shown that θ ,as a function of time , satisfies the nonlinear differential equation $\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$ Where g is the acceleration due to gravity. For small values of θ we can use the linear approximation $\sin\theta \approx \theta$ and then the differential equation becomes linear.  a) Find the equation of motion of a pendulum with length 1 m if θ is initially 0.2 rad and the initial angular velocity is $\frac{d\theta}{dt} = 1$ rad/s. b) What is the maximum angle from the vertical? c) What is the period of pendulum (that is, the time to complete one back-and-swing)? d) When will the pendulum first be vertical? e) What is the angular velocity when the pendulum is vertical?