Chapter 4: Electromagnetics

Magnetic Fields: Analysis Model: Particle in a Field (Magnetic), Motion of a Charged Particle in a Uniform Magnetic Field, Applications Involving Charged Particles Moving in a Magnetic Field, Magnetic Force Acting on a Current-Carrying Conductor, Torque on a Current Loop in a Uniform Magnetic Field, The Hall Effect . Sources of the Magnetic Field: The Biot–Savart Law, The Magnetic Force Between Two Parallel Conductors, Ampere's Law, The Magnetic Field of a Solenoid, Gauss's Law in Magnetism, Magnetism in Matter. Faraday's Law: Faraday's Law of Induction, Motional emf, Lenz's Law, Induced emf and Electric Fields Generators and Motors, Eddy Currents. **10 hrs**

Magnetic Fields

Up until now, we have been discussing electrostatics, which deals with physics of the electric field created by static charges. We will now look into a different phenomenon, that of production and properties of magnetic field, whose source is steady current, i.e., of charges in motion. An essential difference between the electrostatics and magnetostatics is that electric charges can be isolated, i.e., there exist positive and negative charges which can exist by themselves. Unlike this situation, magnetic charges (which are known as magnetic monopoles) cannot exist in isolation, every north magnetic pole is always associated with a south pole, so that the net magnetic charge is always zero. We emphasize that there is no physical reason as to why this must be so. Nevertheless, in spite of best attempt to isolate them, magnetic monopoles have not been found. This, as we will see, brings some asymmetry to the physical laws with respect to electricity and magnetism.

Hans Christian Oersted
Danish Physicist and Chemist (1777–1851)
Oersted is best known for observing that a compass needle deflects when placed near a wire carrying a current. This important discovery was the first evidence of the connection between electric and magnetic phenomena. Oersted was also the first to prepare pure aluminum.

The relationship between magnetism and electricity was discovered in 1819 when, during a lecture demonstration, *Hans Christian Oersted* found that an electric current in a wire deflected a nearby compass needle. In the 1820^{s}, further connections between electricity and magnetism were demonstrated independently by *Faraday* and *Joseph Henry* (1797–1878). They showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field. Years later, theoretical work by Maxwell showed that the reverse is also true: a changing electric field creates a magnetic field. This chapter examines the forces that act on moving charges and on current-carrying wires in the presence of a magnetic field.

Analysis Model: Particle in a Field (Magnetic)

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any electric charge. In addition to containing an electric field, the region of space surrounding any *moving* electric charge also contains a **magnetic field.**

A magnetic field also surrounds a magnetic substance making up a permanent magnet. Historically, the symbol $\vec{B}$ has been used to represent a magnetic field. The direction of the magnetic field $\vec{B}$ at any location is the direction in which a compass needle points at that location. As with the electric field, we can represent the magnetic field by means of drawings with *magnetic field lines.*

Figure 1 shows how the magnetic field lines of a bar magnet can be traced with the aid of a compass. Notice that the magnetic field lines outside the magnet point away from the North Pole and toward the South Pole. One can display magnetic field patterns of a bar magnet using small iron filings as shown in Figure 2.

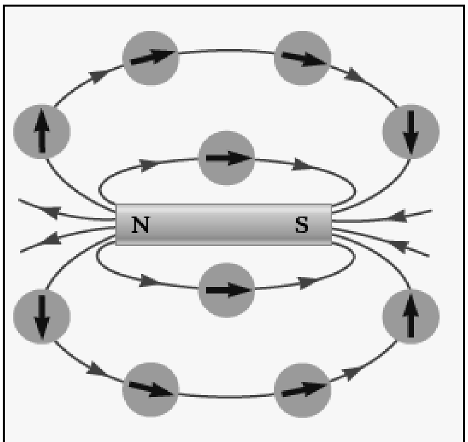

Figure 1 Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet.

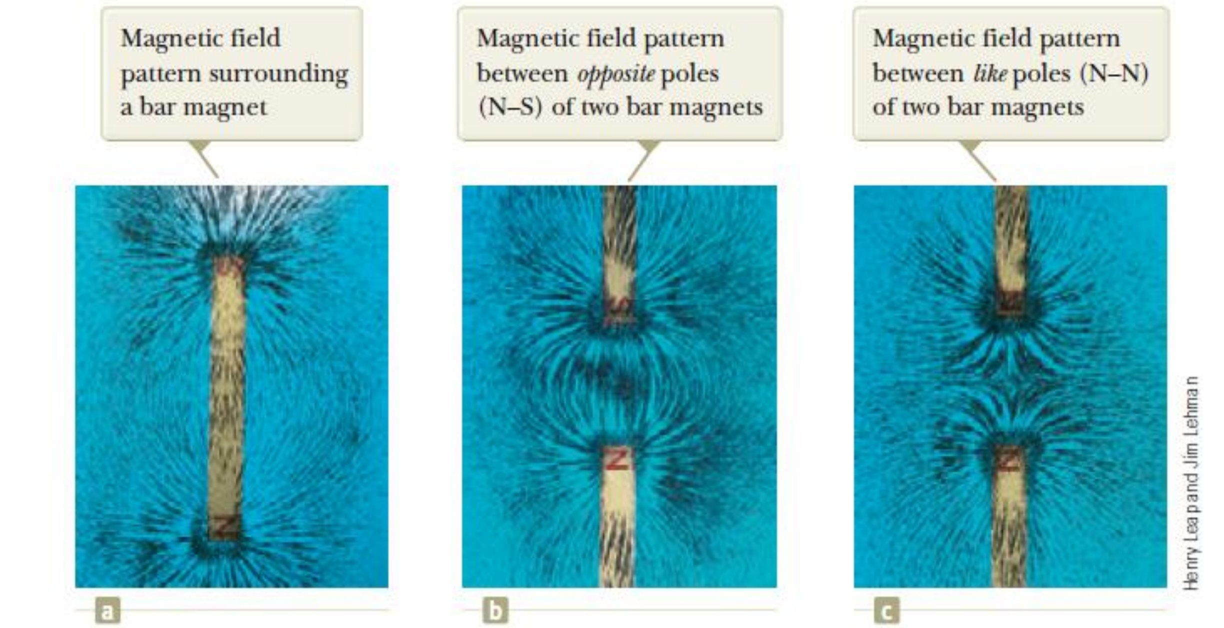

Figure 2. Magnetic field patterns can be displayed with iron filings sprinkled on paper near magnets.

We can quantify the magnetic field $\vec{B}$ by using our model of a particle in a field. The existence of a magnetic field at some point in space can be determined by measuring the **magnetic force** $\vec{F}_B$ exerted on an appropriate test particle placed at that point. If we perform such an experiment by placing a particle with charge q in the magnetic field, we find the following results:

- The magnetic force is proportional to the charge q of the particle.
- The magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction.
- The magnetic force is proportional to the magnitude of the magnetic field vector $\vec{B}$.
- The magnetic force is proportional to the speed v of the particle.
- If the velocity vector makes an angle θ with the magnetic field, the magnitude of the magnetic force is proportional to sin θ.
- When a charged particle moves *parallel* to the magnetic field vector, the magnetic force on the charge is zero.
- When a charged particle moves in a direction *not* parallel to the magnetic field vector, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$ that is, the magnetic force is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$**.**

These results show that the magnetic force on a particle is more complicated than the electric force. The magnetic force is distinctive because it depends on the velocity of the particle and because its direction is perpendicular to both $\vec{v}$ and $\vec{B}$.

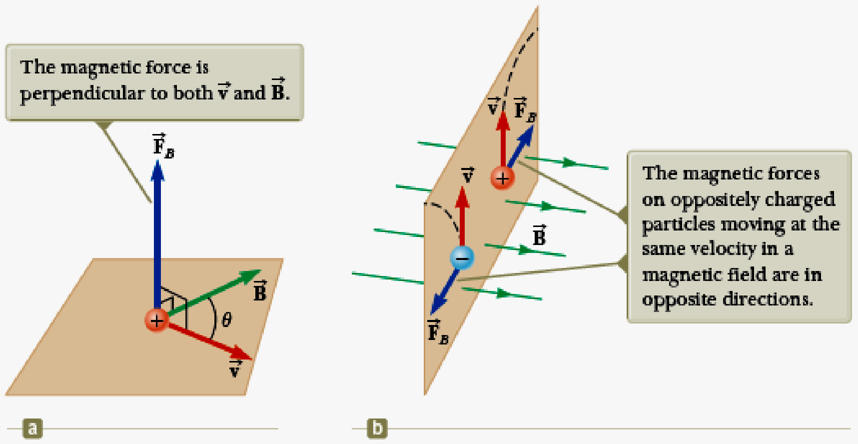

Figure 3 (a) The direction of the magnetic force $\vec{F}_B$ acting on a charged particle moving with a velocity in the presence of a magnetic field $\vec{B}$.

(b) Magnetic forces on positive and negative charges. The dashed lines show the paths of the particles.

Figure 3 shows the details of the direction of the magnetic force on a charged particle. Despite this complicated behaviour, these observations can be summarized in a compact way by writing the magnetic force in the form

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

where $\vec{F}_B$, $\vec{v}$ and $\vec{B}$ are mutually perpendicular to each other. This equation is the mathematical representation of the magnetic version of the **particle in a field** analysis model.

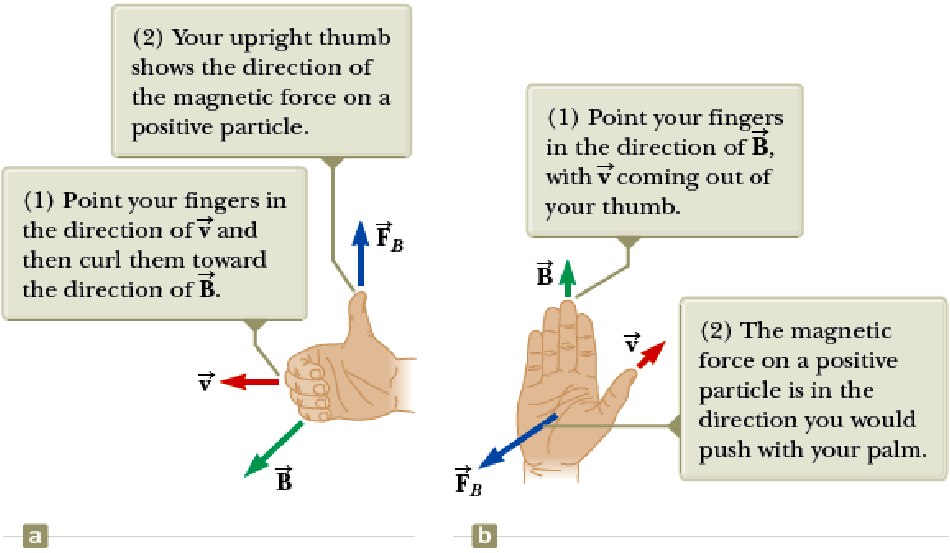

Figure 4. Two right-hand rules for determining the direction of the magnetic force $\vec{F}_B = q\vec{v} \times \vec{B}$ acting on a particle with charge q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$. (a) In this rule, the magnetic force is in thedirection in which your thumb points. (b) In this rule, the magnetic force is in the direction of your palm, as if you are pushing the particle with your hand.

Figure 4 reviews two right-hand rules for determining the direction of the cross product $\vec{v} \times \vec{B}$ and determining the direction of $\vec{F}_B$. Point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. Your extended thumb, which is at a right angle to your fingers, points in the direction of $\vec{v} \times \vec{B}$. Because $\vec{F}_B = q\, \vec{v} \times \vec{B}$, $\vec{F}_B$ is in the direction of your thumb if q is positive and is opposite the direction of your thumb if q is negative.

An alternative rule is shown in Figure 4b. Here the thumb points in the direction of $\vec{v}$ and the extended fingers in the direction of $\vec{B}$. Now, the force $\vec{F}_B$ on a positive charge extends outward from the palm. The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand: outward from your palm. The force on a negative charge is in the opposite direction. You can use either of these two right-hand rules.

The magnitude of the magnetic force on a charged particle is

$$\boxed{F_B = |q| vB \sin\theta}$$

where θ is the smaller angle between $\vec{v}$ and $\vec{B}$. From this expression, we see that F_B is zero when $\vec{v}$ is parallel or antiparallel to $\vec{B}$ ($\theta = 0$ or 180^0) and maximum when $\vec{v}$ is perpendicular to $\vec{B}$ ($\theta = 90^0$).

The SI unit of magnetic field is the newton per coulomb-meter per second, which is called the **tesla** (T):

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{C \cdot m/s}}$$

Because a coulomb per second is defined to be an ampere,

$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A \cdot m}}$$

A non-SI magnetic-field unit in common use, called the *gauss* (G), is related to the tesla through the conversion 1 T = 10^4 G

Table 1. Some typical values of magnetic fields.

SI. No.	Source of Field	Field Magnitude (T)
1.	Strong superconducting laboratory magnet	30
2.	Strong conventional laboratory magnet	2
3.	Medical MRI unit	1.5
4.	Bar magnet	10^{-2}
5.	Surface of the Sun	10^{-2}
6.	Surface of the Earth	0.5×10^{-4}
7.	Inside human brain (due to nerve impulses)	10^{-13}

Important differences between the electric and magnetic forces of the particle in a field model:

Si. N	ELECTRIC FORCE	MAGNETIC FORCE
1.	The electric force vector is along the direction of the electric field	The magnetic force vector is perpendicular to the magnetic field
2.	The electric force acts on a charged particle regardless of whether the particle is moving	The magnetic force acts on a charged particle only when the particle is in motion.
3.	The electric force does work in displacing a charged particle	The magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application.

From the last statement, we conclude that the kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. The field can alter the direction of the velocity vector, but it cannot change the speed or kinetic energy of the particle.

Motion of a Charged Particle in a Uniform Magnetic Field:

To indicate the direction of $\vec{B}$ in illustrations, we sometimes present perspective views such as those in **Figure 5**.

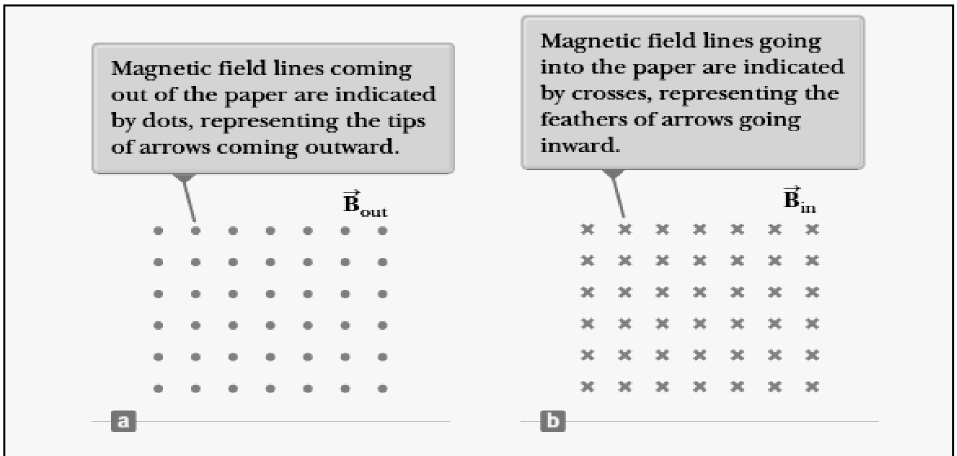

Figure 5 Representations of magnetic field lines perpendicular to the page.

If $\vec{B}$ lies in the plane of the page or is present in a perspective drawing, we use vectors or field lines with arrowheads. In nonperspective illustrations, we depict a magnetic field perpendicular to and directed out of the page with a series of dots, which represent the tips of arrows coming toward you (see Fig. 5a). In this case, the field is labeled $\vec{B}_{out}$.

If $\vec{B}$ is directed perpendicularly into the page, we use crosses, which represent the feathered tails of arrows fired away from you, as in Figure 5b. In this case, the field is labeled $\vec{B}$ in, where the subscript "in" indicates "into the page." The same notation with crosses and dots is also used for other quantities that might be perpendicular to the page such as forces and current directions.

We found that the magnetic force acting on a charged particle moving in a magnetic field is perpendicular to the particle's velocity and consequently the work done by the magnetic force on the particle is zero. Now consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field. Let's assume the direction of the magnetic field is into the page as in **Figure 6**. The particle in a field model tells us that the magnetic force on the particle is perpendicular to both the magnetic field lines and the velocity of the particle. The fact that there is a force on the particle tells us to apply the particle under a net force model to the particle. As the particle changes the direction of its velocity in response to the magnetic force, the magnetic force remains perpendicular to the velocity. As we know, if the force is always perpendicular to the velocity, the path of the particle is a circle! **Figure 6** shows the particle moving in a circle in a plane perpendicular to the magnetic field. Although magnetism and magnetic forces may be new and unfamiliar to you now, we see a magnetic effect that results in something with which we are familiar: the particle in uniform circular motion model!

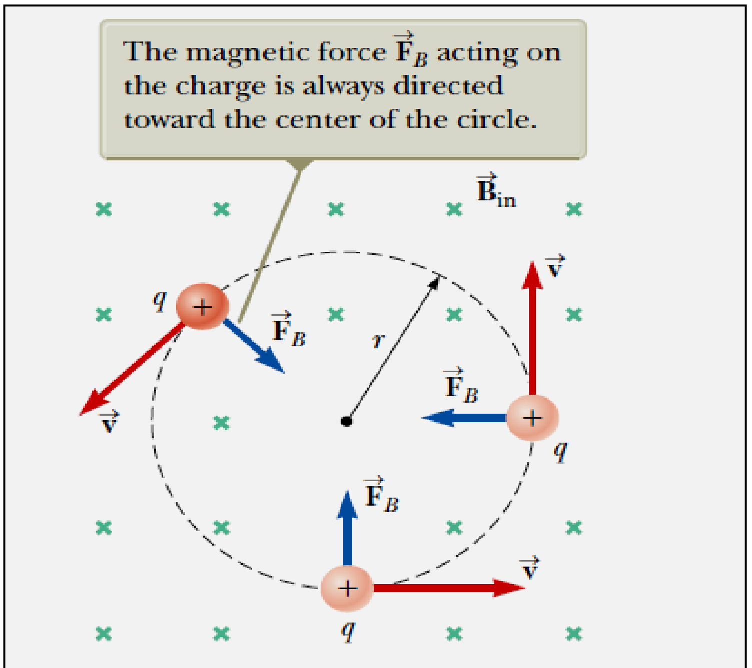

Figure 6. When the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to $\vec{B}$.

The particle moves in a circle because the magnetic force $\vec{F}_B$ is perpendicular to $\vec{v}$ and $\vec{B}$ and has a constant magnitude qvB. As **Figure 6** illustrates, the rotation is counterclockwise for a positive charge in a magnetic field directed into the page. If q were negative, the rotation would be clockwise. We use the particle under a net force model to write Newton's second law for the particle:

$$\boxed{\sum F = F_B = ma}$$

Because the particle moves in a circle, we also model it as a particle in uniform circular motion and we replace the acceleration with centripetal acceleration:

$$\boxed{F_B = qvB = \frac{mv^2}{r}}$$

This expression leads to the following equation for the radius of the circular path:

$$\boxed{r = \frac{mv}{qB}}$$

That is, the radius of the path is proportional to the linear momentum mv of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field. The angular speed of the particle is

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}}$$

The period of the motion (the time interval the particle requires to complete one revolution) is equal to the circumference of the circle divided by the speed of the particle:

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}}$$

These results show that the angular speed of the particle and the period of the circular motion do not depend on the speed of the particle or on the radius of the orbit. The angular speed v is often referred to as the **cyclotron frequency** because charged particles circulate at this angular frequency in the type of accelerator called a *cyclotron.*

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to $\vec{B}$, its path is a helix. For example, if the field is directed in the x direction as shown in **Figure 7**, there is no component of force in the x direction. As a result, $a_x = 0$, and the x component of velocity remains constant. The charged particle is a particle in equilibrium in this direction. The magnetic force **q** $\vec{v}$ **x** $\vec{B}$ causes the components v_y and v_z to change in time, however, and the resulting motion is a helix whose axis is parallel to the magnetic field. The projection of the path onto the yz plane (viewed along the x axis) is a circle. (The projections of the path onto the *xy* and *xz* planes are sinusoids!) and *v* is replaced by $v_\perp = \sqrt{v_y^2 + v_z^2}$.

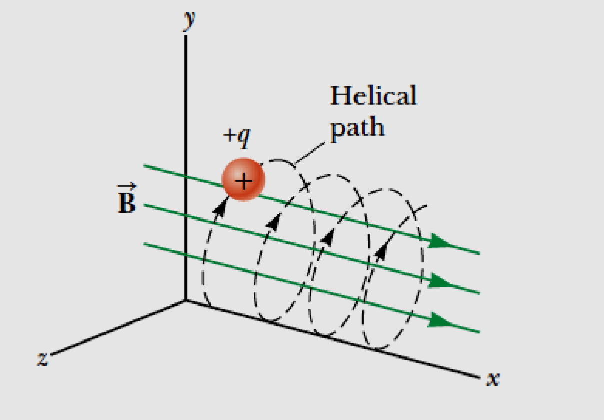

Figure 7. A charged particle having a velocity vector that has a component parallel to a uniform magnetic field moves in a helical path.

Applications Involving Charged Particles Moving in a Magnetic Field:

A charge moving with a velocity $\vec{v}$ in the presence of both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ is described by two particles in a field models. It experiences both an electric force q$\vec{E}$ and a magnetic force **q** $\vec{v}$ **x** $\vec{B}$. The total force (called the Lorentz force) acting on the charge is

$$\boxed{\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}}$$

Velocity Selector

In many experiments involving moving charged particles, it is important that all particles move with essentially the same velocity, which can be achieved by applying a combination of an electric field and a magnetic field oriented as shown in **Figure 8**. A uniform electric field is directed to the right (in the plane of the page in Fig. 8), and a uniform magnetic field is applied in the direction perpendicular to the electric field (into the page in Fig. 8).

If q is positive and the velocity $\vec{v}$ is upward, the magnetic force **q** $\vec{v}$ **x** $\vec{B}$ is to the left and the electric force **q**$\vec{E}$ is to the right. When the magnitudes of the two fields are chosen so that $qE = qvB$, the forces cancel.

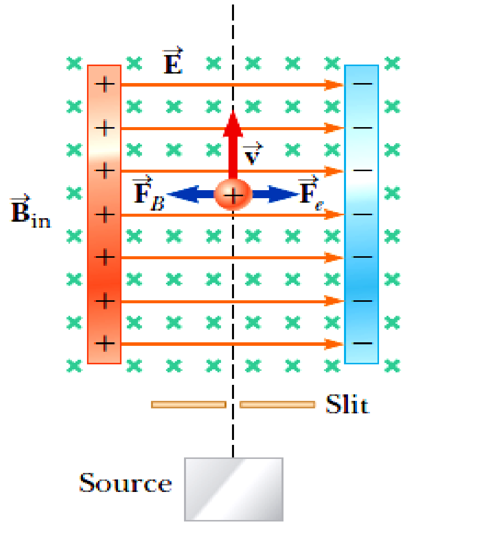

Figure 8 A velocity selector. When a positively charged particle is moving with velocity $\vec{v}$ in the presence of a magnetic field directed into the page and an electric field directed to the right, it experiences an electric force $q\vec{E}$ to the right and a magnetic force $q\vec{v} \times \vec{B}$ to the left.

The charged particle is modeled as a particle in equilibrium and moves in a straight vertical line through the region of the fields. From the expression $qE = qvB$, we find that

$$v = \frac{E}{B}$$

Only those particles having this speed pass undeflected through the mutually perpendicular electric and magnetic fields. The magnetic force exerted on particles moving at speeds greater than that is stronger than the electric force, and the particles are deflected to the left. Those moving at slower speeds are deflected to the right.

The Mass Spectrometer

A **mass spectrometer** separates ions according to their mass-to-charge ratio. In one version of this device, known as the *Bainbridge mass spectrometer,* a beam of ions first passes through a velocity selector and then enters a second uniform magnetic field $\vec{B}_0$ that has the same direction as the magnetic field in the selector (**Fig. 9**).

Upon entering the second magnetic field, the ions are described by the particle in uniform circular motion model. They move in a semicircle of radius ***r*** before striking a detector array at ***P***. If the ions are positively charged, the beam deflects to the left as **Figure 9** shows. If the ions are negatively charged, the beam deflects to the right. From Equation $r = \frac{mv}{qB}$, we can express the ratio m/q as

$$\boxed{\frac{m}{q} = \frac{rB_0}{v}}$$

substitute $v = \frac{E}{B}$, we get

$$\boxed{\frac{m}{q} = \frac{rB_0 B}{E}}$$

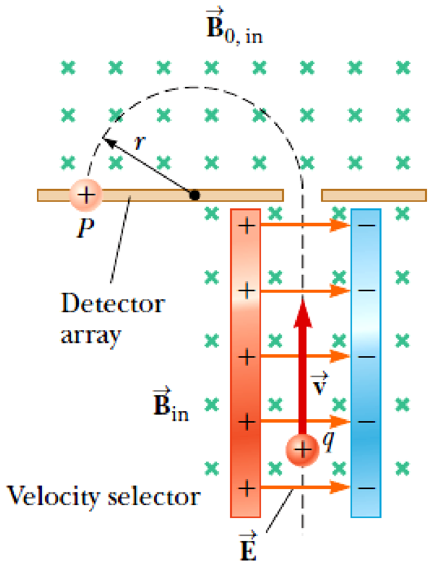

Figure 9. A mass spectrometer. Positively charged particles are sent first through a velocity selector and then into a region where the magnetic field $\vec{B}_0$ causes the particles to move in a semicircular path and strike a detector array at *P*.

Therefore, we can determine m/q by measuring the radius of curvature and knowing the field magnitudes B, B_0, and E. In practice, one usually measures the masses of various isotopes of a given ion, with the ions all carrying the same charge q. In this way, the mass ratios can be determined even if q is unknown.

A variation of this technique was used by J. J. Thomson (1856–1940) in 1897 to measure the ratio e/m_e for electrons**. Figure 10a** shows the basic apparatus he used. Electrons are accelerated from the cathode and pass through two slits. They then drift into a region of perpendicular electric and magnetic fields. The magnitudes of the two fields are first adjusted to produce an undeflected beam. When the magnetic field is turned off, the electric field produces a measurable beam deflection that is recorded on the fluorescent screen. From the size of the deflection and the measured values of E and B, the charge-to-mass ratio can be determined. The results of this crucial experiment represent the discovery of the electron as a fundamental particle of nature.

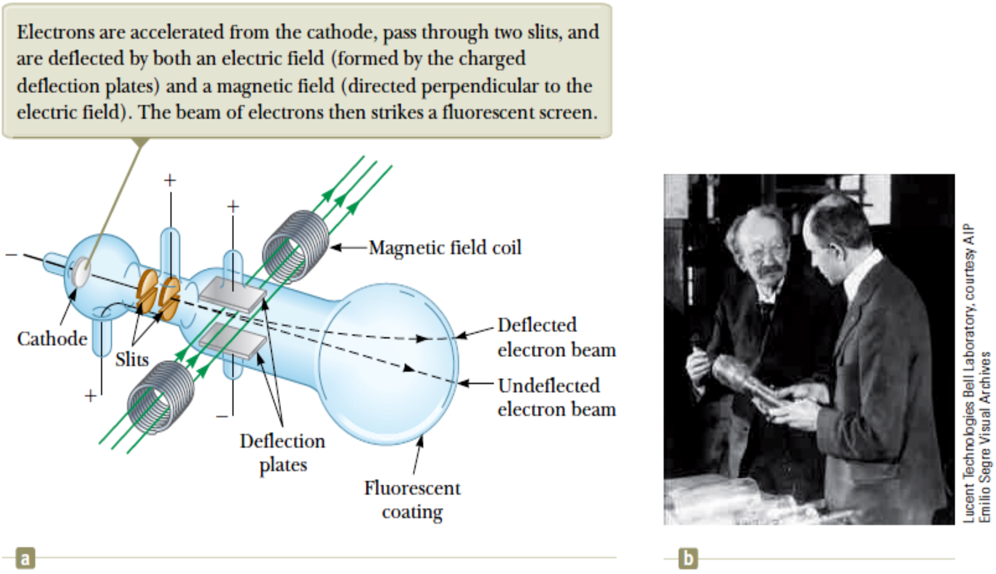

Figure 10. (a) Thomson's apparatus for measuring e/m_e. (b) J. J. Thomson *(left)* in the Cavendish Laboratory, University of Cambridge. The man on the right, Frank Baldwin Jewett, is a distant relative of John W. Jewett, Jr., coauthor of this text.

The Cyclotron

A **cyclotron** is a device that can accelerate charged particles to very high speeds. The energetic particles produced are used to bombard atomic nuclei and thereby produce nuclear reactions of interest to researchers. A number of hospitals use cyclotron facilities to produce radioactive substances for diagnosis and treatment. Both electric and magnetic forces play key roles in the operation of a cyclotron, a schematic drawing of which is shown in **Figure 11a**. The charges move inside two semicircular containers D_1 and D_2, referred to as *dees* because of their shape like the letter D. A high-frequency alternating potential difference is applied to the dees, and a uniform magnetic field is directed perpendicular to them. A positive ion released at P near the center of the magnet in one dee moves in a semicircular path (indicated by the dashed black line in the drawing) and arrives back at the gap in a time interval $T/2$, where T is the time interval needed to make one complete trip around the two dees, given by equation $\boldsymbol{T = \frac{2\pi m}{qB}}$.

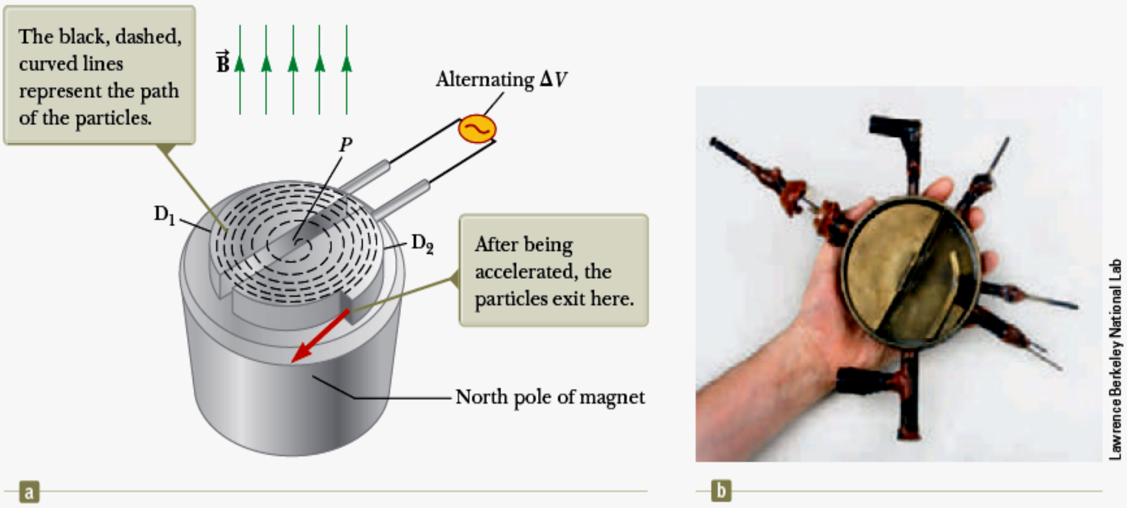

Figure 11. (a) A cyclotron consists of an ion source at *P*, two dees D_1 and D_2 across which an alternating potential difference is applied, and a uniform magnetic field. (The south pole of the magnet is not shown.) (b) The first cyclotron, invented by E. O. Lawrence and M. S. Livingston in 1934.

The frequency of the applied potential difference is adjusted so that the polarity of the dees is reversed in the same time interval during which the ion travels around one dee. If the applied potential difference is adjusted such that D_1 is at a lower electric potential than D_2 by an amount ΔV, the ion accelerates across the gap to D_1 and its kinetic energy increases by an amount $q\ \Delta V$. It then moves around D_1 in a semicircular path of greater radius (because its speed has increased). After a time interval $T/2$, it again arrives at the gap between the dees. By this time, the polarity across the dees has again been reversed and the ion is given another "kick" across the gap. The motion continues so that for each half-circle trip around one dee, the ion gains additional kinetic energy equal to $q\ \Delta V$. When the radius of its path is nearly that of the dees, the energetic ion leaves the system through the exit slit. The cyclotron's operation depends on T being independent of the speed of the ion and of the radius of the circular path.

We can obtain an expression for the **kinetic energy** of the ion when it exits the cyclotron in terms of the radius R of the dees.

We know that $v = \frac{qBR}{m}$. Hence, the kinetic energy is

$$\boxed{K = \tfrac{1}{2}mv^2 = \frac{q^2B^2R^2}{2m}}$$

When the energy of the ions in a cyclotron exceeds about 20 MeV, relativistic effects come into play. Observations show that T increases and the moving ions do not remain in phase with the applied potential difference. Some accelerators overcome this problem by modifying the period of the applied potential difference so that it remains in phase with the moving ions.

Magnetic Force Acting on a Current- Carrying Conductor:

If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field. The current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.

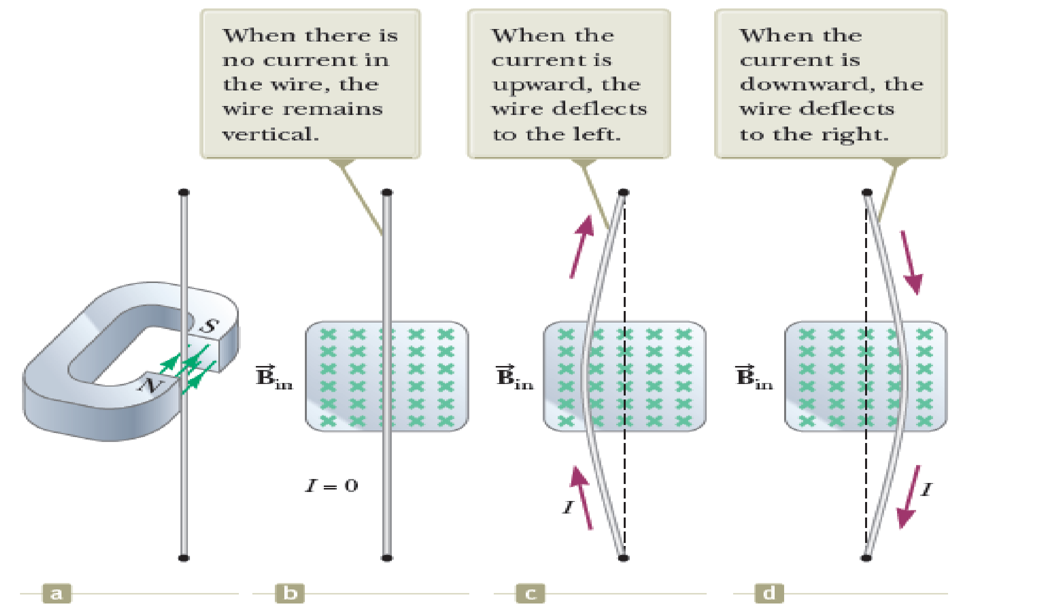

Figure 12. (a) A wire suspended vertically between the poles of a magnet. (b)–(d) The setup shown in (a) as seen looking at the south pole of the magnet so that the magnetic field (green crosses) is directed into the page.

One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet as shown in **Figure 12a**. For ease in visualization, part of the horseshoe magnet in part (a) is removed to show the end face of the south pole in parts (b) through (d) of **Figure 12**. The magnetic field is directed into the page and covers the region within the shaded squares. When the current in the wire is zero, the wire remains vertical as in **Figure 12b**. When the wire carries a current directed upward as in **Figure 12c**, however, the wire deflects to the left. If the current is reversed as in **Figure 12d**, the wire deflects to the right.

Let's quantify this discussion by considering a straight segment of wire of length L and cross-sectional area A carrying a current I in a uniform magnetic field $\vec{B}$ as in **Figure 13**. According to the magnetic version of the particle

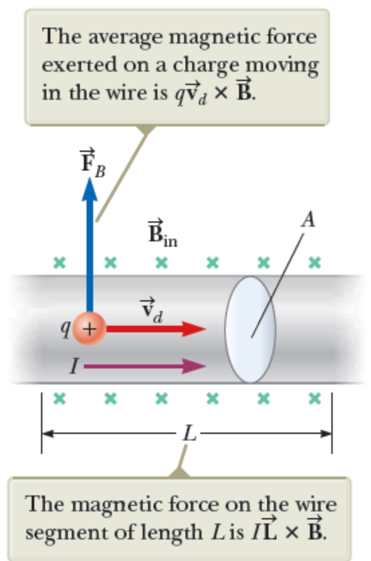

Figure 13. A segment of a current-carrying wire in a magnetic field $\vec{B}$.

in a field model, the magnetic force exerted on a charge q moving with a drift velocity $\vec{v}_d$ is $q\,\vec{v}_d \times \vec{B}$. To find the total force acting on the wire, we multiply the force $q\,\vec{v}_d \times \vec{B}$ exerted on one charge by the number of charges in the segment. Because the volume of the segment is AL, the number of charges in the segment is nAL, where n is the number of mobile charge carriers per unit volume. Hence, the total magnetic force on the segment of wire of length L is

$$\boxed{\vec{\mathbf{F}}_B = (q\vec{\mathbf{v}}_d \times \vec{\mathbf{B}})nAL}$$

We can write this expression in a more convenient form by noting that, the current in the wire is $\boldsymbol{I = nqvdA}$. Therefore,

$$\boxed{\vec{\mathbf{F}}_B = I\,\vec{\mathbf{L}} \times \vec{\mathbf{B}}}$$

where $\vec{L}$ is a vector that points in the direction of the current $\boldsymbol{I}$ and has a magnitude equal to the length $\boldsymbol{L}$ of the segment. This expression applies only to a straight segment of wire in a uniform magnetic field.

Now consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field as shown in **Figure 14**. It follows from above equation, the magnetic force exerted on a small segment of vector length $d\vec{s}$ in the presence of a field $\vec{B}$ is

$$\boxed{d\vec{\mathbf{F}}_B = I\,d\vec{\mathbf{s}} \times \vec{\mathbf{B}}}$$

where $d\vec{\mathbf{F}}_{\mathbf{B}}$ is directed out of the page for the directions of $\vec{B}$ and $d\vec{s}$ in **Figure 14**. We can define the magnetic field $\vec{B}$ in terms of a measurable force exerted on a current element, where the force is a maximum when $\vec{B}$ is perpendicular to the element and zero when $\vec{B}$ is parallel to the element.

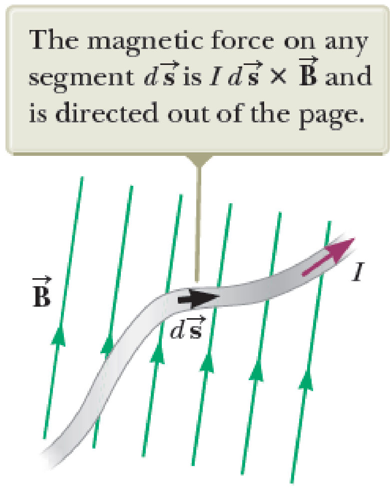

Figure 14. A wire segment of arbitrary shape carrying a current ***I*** in a magnetic field $\vec{B}$ experiences a magnetic force.

To calculate the total force $\vec{\mathbf{F}}_{\mathbf{B}}$ acting on the wire shown in **Figure 14**, we integrate above equation over the length of the wire:

$$\boxed{\vec{\mathbf{F}}_B = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}}$$

where a and b represent the endpoints of the wire. When this integration is carried out, the magnitude of the magnetic field and the direction the field makes with the vector $d\vec{s}$ may differ at different points.

Torque on a Current Loop in a Uniform Magnetic Field

Consider a rectangular loop carrying a current *I* in the presence of a uniform magnetic field directed parallel to the plane of the loop as shown in **Figure 15a**. No magnetic forces act on sides **1** and **3** because these wires are parallel to the field; hence, $\vec{L} \times \vec{B} = 0$ for these sides. Magnetic forces do, however, act on sides **2** and **4** because these sides are oriented perpendicular to the field. The magnitude of these forces is, can be written as,

$$\boxed{F_2 = F_4 = IaB}$$

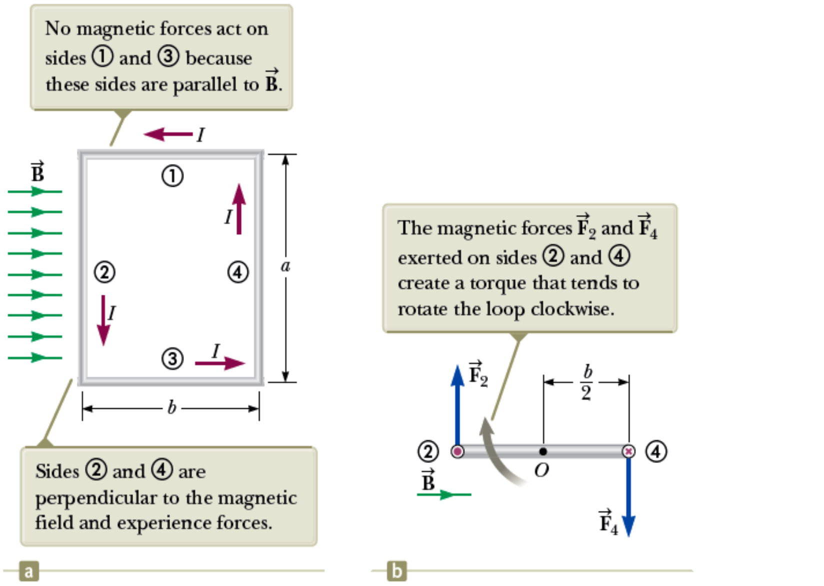

Figure 15. (a) Overhead view of a rectangular current loop in a uniform magnetic field. (b) Edge view of the loop sighting down sides ② and ④. The purple dot in the left circle represents current in wire ② coming toward you; the purple cross in the right circle represents current in wire ④ moving away from you.

The direction of $\vec{F}_2$, the magnetic force exerted on wire 2, is out of the page in the view shown in **Figure 15a** and that of $\vec{F}_2$, the magnetic force exerted on wire 4, is into the page in the same view. If we view the loop from side c and sight along sides x and v, we see the view shown in **Figure 15b**, and the two magnetic forces $\vec{F}_2$ and $\vec{F}_4$ are directed as shown. Notice that the two forces point in opposite directions but are *not* directed along the same line of action. If the loop is pivoted so that it can rotate about point *O*, these two forces produce about *O* a torque that rotates the loop clockwise. The magnitude of this torque τ_{max} is

$$\tau_{\max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2} = IabB$$

where the moment arm about *O* is *b*/2 for each force. Because the area enclosed by the loop is *A* = *ab*, we can express the maximum torque as

$$\boxed{\tau_{\max} = IAB}$$

This maximum-torque result is valid only when the magnetic field is parallel to the plane of the loop. The sense of the rotation is clockwise when viewed from side **3** as indicated in **Figure15b.** If the current direction were reversed, the force directions would also reverse and the rotational tendency would be counterclockwise.

Now suppose the uniform magnetic field makes an angle θ, 90^0 with a line perpendicular to the plane of the loop as in **Figure 16**. For convenience, let's assume $\vec{B}$ is perpendicular to sides **2** and **4**. In this case, the magnetic forces $\vec{F}_1$ and $\vec{F}_3$ exerted on sides **1** and **3** cancel each other and produce no torque because they act along the same line. The magnetic forces $\vec{F}_2$ and $\vec{F}_4$ acting on sides **2** and **4**, however, produce a torque about *any point*. Referring to the edge view shown in **Figure 16**, we see that the moment arm of $\vec{F}_2$ about the point *O* is equal to $(b/2)\sin\theta$. Likewise, the moment arm of $\vec{F}_4$ about *O* is also equal to $(b/2)$ sin u. Because $F_2 = F_4 = IaB$, the magnitude of the net torque about *O* is

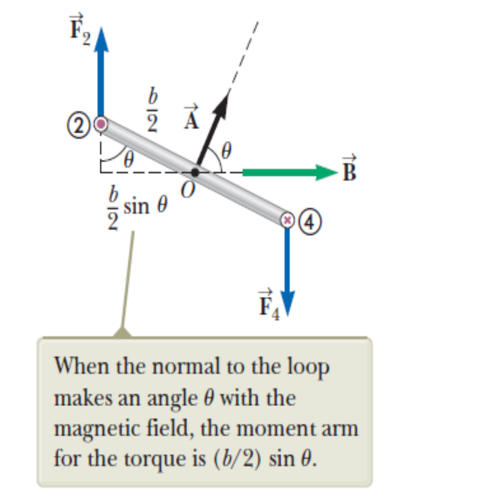

Figure 16. An edge view of the loop in Figure 15 with the normal to the loop at an angle θ with respect to the magnetic field.

$$\begin{aligned}\tau &= F_2\frac{b}{2}\sin\theta + F_4\frac{b}{2}\sin\theta \\ &= IaB\left(\frac{b}{2}\sin\theta\right) + IaB\left(\frac{b}{2}\sin\theta\right) = IabB\sin\theta \\ &= IAB\sin\theta\end{aligned}$$

where $A = ab$ is the area of the loop. This result shows that the torque has its maximum value *IAB* when the field is perpendicular to the normal to the plane of the loop ($\theta = 90^0$) as discussed with regard to **Figure 16** and is zero when the field is parallel to the normal to the plane of the loop ($\theta = 0$).

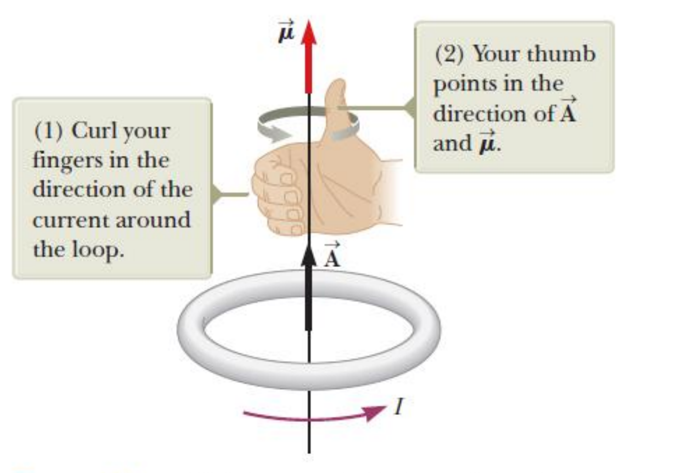

Figure 17. Right-hand rule for determining the direction of the vector $\vec{A}$ for a current loop. The direction of the magnetic moment $\vec{\mu}$ is the same as the direction of $\vec{A}$.

A convenient vector expression for the torque exerted on a loop placed in a uniform magnetic field $\vec{B}$ is

$$\vec{\tau} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

where $\vec{A}$, the vector shown in **Figure 16**, is perpendicular to the plane of the loop and has a magnitude equal to the area of the loop. To determine the direction of $\vec{A}$, use the right-hand rule described in **Figure 17.** When you curl the fingers of your right hand in the direction of the current in the loop, your thumb points in the direction of $\vec{A}$. **Figure 16** shows that the loop tends to rotate in the direction of decreasing values of θ (that is, such that the area vector $\vec{A}$ rotates toward the direction of the magnetic field).

The product $I\vec{A}$ is defined to be the **magnetic dipole moment,** $\vec{\mu}$ (often simply called the "magnetic moment") of the loop:

$$\vec{\mu} \equiv I\vec{\mathbf{A}}$$

The SI unit of magnetic dipole moment is the ampere-meter^2 (A^2m^2). If a coil of wire contains N loops of the same area, the magnetic moment of the coil is

$$\vec{\mu}_{\text{coil}} = NI\vec{\mathbf{A}}$$

we can express the torque exerted on a current-carrying loop in a magnetic field $\vec{B}$ as

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}}$$

Sources of the Magnetic Field

The Biot–Savart Law:

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Jean-Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{s}$ of a wire carrying a steady current I (**Fig. 18**):

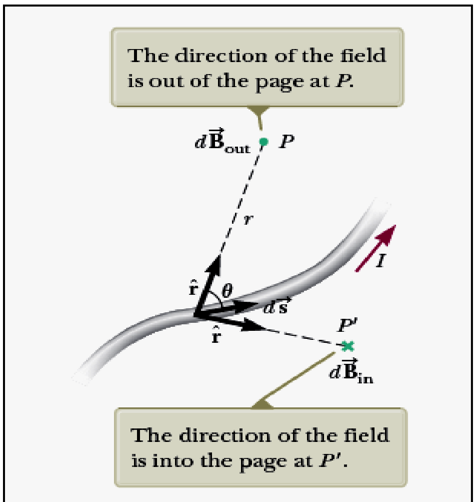

Figure 18. The magnetic field $d\vec{B}$ at a point due to the current I through a length element $d\vec{s}$ is given by the Biot–Savart law.

- The vector $d\vec{\mathbf{B}}$ is perpendicular both to $d\vec{\mathbf{s}}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\vec{\mathbf{s}}$ toward P.
- The magnitude of $d\vec{\mathbf{B}}$ is inversely proportional to r^2, where r is the distance from $d\vec{\mathbf{s}}$ to P.
- The magnitude of $d\vec{\mathbf{B}}$ is proportional to the current I and to the magnitude ds of the length element $d\vec{\mathbf{s}}$.
- The magnitude of $d\vec{\mathbf{B}}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$.

These observations are summarized in the mathematical expression known today as the **Biot–Savart law:**

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \quad \text{——— (1)}$$

where μ_0 is a constant called the **permeability of free space:**

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}$$

Notice that the field $d\vec{B}$ in above **equation 1** is the field created at a point by the current in only a small length element $d\vec{s}$ of the conductor. To find the *total* magnetic field $\vec{B}$ created at some point by a current of finite size, we must sum up contributions from all current elements $I\,d\vec{s}$ that make up the current. That is, we must evaluate $\vec{B}$ by integrating the above equation:

$$\boxed{\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}} \quad \text{——— (2)}$$

where the integral is taken over the entire current distribution. This expression must be handled with special care because the integrand is a cross product and therefore a vector quantity.

Although the Biot–Savart law was discussed for a current-carrying wire, it is also valid for a current consisting of charges flowing through space such as the particle beam in an accelerator. In that case**,** $d\vec{s}$ represents the length of a small segment of space in which the charges flow.

The magnitude of the magnetic field varies as the inverse square of the distance from the source, as does the electric field due to a point charge. The directions of the two fields are quite different, however. The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{s}$ and the unit vector $\hat{r}$ as described by the cross product in Eq. 1. Hence, if the conductor lies in the plane of the page as shown in **Figure 18**, $d\vec{B}$ points out of the page at P and into the page at P′.

Another difference between electric and magnetic fields is related to the source of the field. An electric field is established by an isolated electric charge. The Biot–Savart law gives the magnetic field of an isolated current element at some point, but such an isolated current element cannot exist the way an isolated electric charge can. A current element *must* be part of an extended current distribution because a complete circuit is needed for charges to flow.

Therefore, the Biot–Savart law (Eq. 1) is only the first step in a calculation of a magnetic field; it must be followed by integration over the current distribution as in Eq. 2.

The Magnetic Force between Two Parallel Conductors:

As we know that the magnetic force that acts on a current-carrying conductor placed in an external magnetic field. Because a current in a conductor sets up its own magnetic field, it is easy to understand that two current-carrying conductors exert magnetic forces on each other. One wire establishes the magnetic field and the other wire is modeled as a collection of particles in a magnetic field. Such forces between wires can be used as the basis for defining the ampere and the coulomb.

Consider two long, straight, parallel wires separated by a distance a and carrying currents I_1 and I_2 in the same direction as in **Figure 19** Let's determine the force exerted on one wire due to the magnetic field set up by the other wire. Wire 2, which carries a current I_2 and is identified arbitrarily as the source wire, creates a magnetic field $\vec{B_2}$ at the location of wire 1, the test wire. The magnitude of this magnetic field is the same at all points on wire 1. The direction of $\vec{B_2}$ is perpendicular to wire 1 as shown in **Figure 19**. According to Equation $\vec{F_B} = I\vec{L}\,x\,\vec{B}$, the magnetic force on a length , of wire 1 is $\vec{F_1} = I_1\vec{\ell}\,x\,\vec{B_2}$. Because $\vec{\ell}$ is perpendicular to $\vec{B_2}$ in this situation, the magnitude of $\vec{F_1}$ is $F_1 = I_1\,\ell\,B_2$. Because the magnitude of $\vec{B_2}$ is given by equation $\mathbf{B} = \frac{\mu_0 I}{2\pi a}$,

$$F_1 = I_1 \ell B_2 = I_1 \ell \left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}\ell \qquad (3)$$

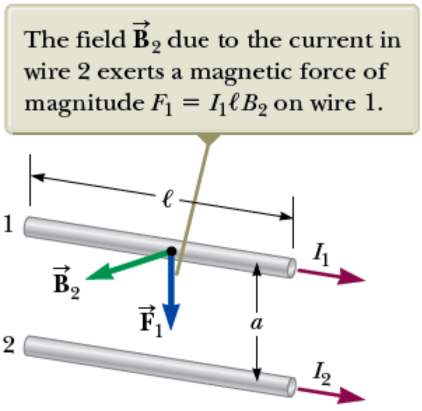

Figure 19. Two parallel wires that each carry a steady current exert a magnetic force on each other. The force is attractive if the currents are parallel (as shown) and repulsive if the currents are antiparallel.

The direction of $\vec{F_1}$ is toward wire 2 because $\vec{\ell}\,x\,\vec{B_2}$ is in that direction. When the field set up at wire 2 by wire 1 is calculated, the force $\vec{F_2}$ acting on wire 2 is found to be equal in magnitude and opposite in direction to $\vec{F_1}$, which is what we expect because Newton's third law must be obeyed. When the currents are in opposite directions (that is, when one of the currents is reversed in Fig. 19), the forces are reversed and the wires repel each other. Hence, parallel conductors carrying currents in the *same* direction *attract* each other, and parallel conductors carrying currents in *opposite* directions *repel* each other. Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B. We can rewrite this magnitude in terms of the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \qquad (4)$$

The force between two parallel wires is used to define the **ampere** as follows:

Definition of the ampere: When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2 x 10^{-7} N/m, the current in each wire is defined to be 1 A.

The value 2 x 10^{-7} N/m is obtained when $I_1 = I_2 = 1$ A and $a = 1$ m. Because this definition is based on a force, a mechanical measurement can be used to standardize the ampere. For instance, the National Institute of Standards and Technology uses an instrument called a *current balance* for primary current measurements. The results are then used to standardize other, more conventional instruments such as ammeters.

The SI unit of charge, the **coulomb,** is defined in terms of the ampere: When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

In deriving Equations 3 and 4, we assumed both wires are long compared with their separation distance. In fact, only one wire needs to be long. The equations accurately describe the forces exerted on each other by a long wire and a straight, parallel wire of limited length.

Ampère's Law:

Figure 20 is a perspective view of the magnetic field surrounding a long, straight, current-carrying wire. Because of the wire's symmetry, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire. The magnitude of $\vec{B}$ is constant on any circle of radius a and is given by Equation $B = \frac{\mu_0 I}{2\pi a}$. A convenient rule for determining the direction of $\vec{B}$ is to grasp the wire with the right hand, positioning the thumb along the direction of the current. The four fingers wrap in the direction of the magnetic field.

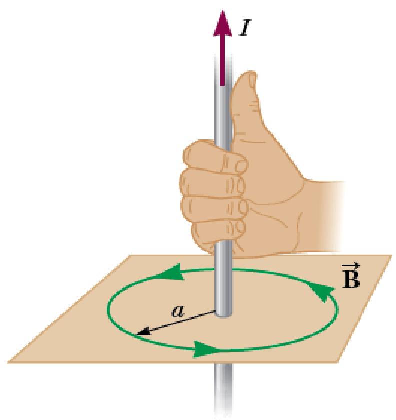

Figure 20. The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current. Notice that the magnetic field lines form circles around the wire.

Figure 20 also shows that the magnetic field line has no beginning and no end. Rather, it forms a closed loop. That is a major difference between magnetic field lines and electric field lines, which begin on positive charges and end on negative charges.

Oersted's 1819 discovery about deflected compass needles demonstrates that a current-carrying conductor produces a magnetic field. **Figure 21a** shows how this effect when several compass needles are placed in a horizontal plane nears a long, vertical wire. When no current is present in the wire, all the needles point in the same direction (that of the horizontal component of the Earth's magnetic field) as expected. When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle as in Figure **21b**. These observations demonstrate that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule described in **Figure 20**. When the current is reversed, the needles in **Figure 21b** also reverse.

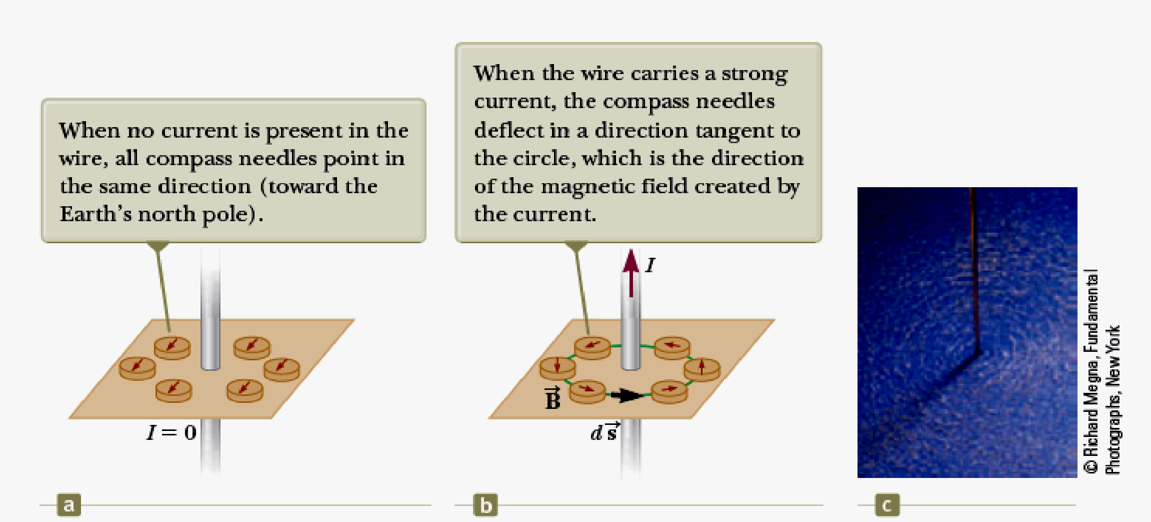

Figure 21. (a) and (b) Compasses show the effects of the current in a nearby wire. (c) Circular magnetic field lines surrounding a current-carrying conductor, displayed with iron filings.

Now let's evaluate the product $\vec{B}$ x $d\,\vec{s}$ for a small length element $d\,\vec{s}$ on the circular path defined by the compass needles and sum the products for all elements over the closed circular path. Along this path, the vectors $d\,\vec{s}$ and $\vec{B}$ are parallel at each point (see Fig. 21b), so $\vec{B}\,.\,d\,\vec{s} = B\,ds$. Furthermore, the magnitude of $\vec{B}$ is constant on this circle and is given by Equation $B = \frac{\mu_0 I}{2\pi a}$. Therefore, the sum of the products $B\,ds$ over the closed path, which is equivalent to the line integral of $\vec{B}\,.\,d\,\vec{s}$, is

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

where $\oint ds = 2\pi r$ is the circumference of the circular path of radius r. Although this result was calculated for the special case of a circular path surrounding a wire of infinite length, it holds for a closed path of *any* shape (an *amperian loop*) surrounding a current that exists in an unbroken circuit. The general case, known as **Ampère's law,** can be stated as follows:

The line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I \qquad (5)$$

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

The Magnetic Field of a Solenoid:

A **solenoid** is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire-which we shall call the *interior* of the solenoid-when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop; the net magnetic field is the vector sum of the fields resulting from all the turns.

Figure 22 shows the magnetic field lines surrounding a loosely wound solenoid. The field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is strong and almost uniform.

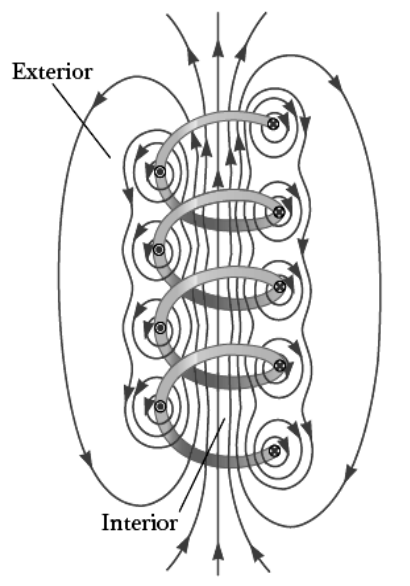

Figure 22. The magnetic field lines for a loosely wound solenoid.

If the turns are closely spaced and the solenoid is of finite length, the external magnetic field lines are as shown in **Figure 23a**. This field line distribution is similar to that surrounding a bar magnet (**Fig. 23b**). Hence, one end of the solenoid behaves like the north pole of a magnet and the opposite end behaves like the South Pole. As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes weaker. An *ideal solenoid* is approached when the turns are closely spaced and the length is much greater than the radius of the turns. **Figure 24** shows a longitudinal cross section of part of such a solenoid carrying a current ***I***. In this case, the external field is close to zero and the interior field is uniform over a great volume.

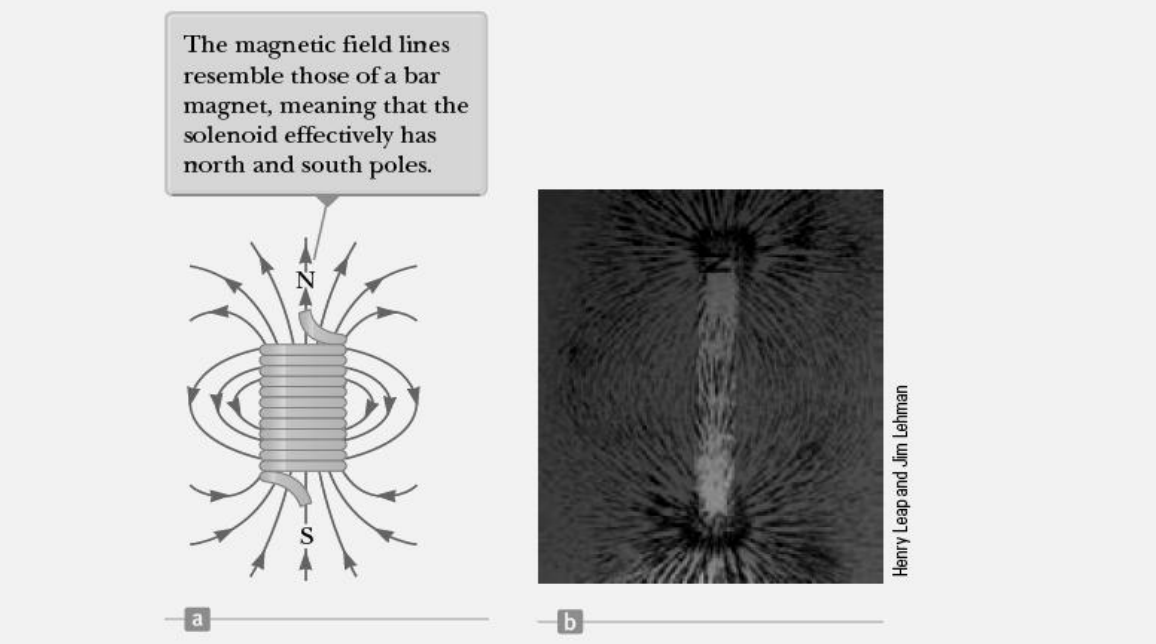

Figure 23. (a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is strong and nearly uniform. (b) The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper.

Consider the amperian loop (loop 1) perpendicular to the page in **Figure 24**, surrounding the ideal solenoid. This loop encloses a small current as the charges in the wire move coil by coil along the length of the solenoid. Therefore, there is a nonzero magnetic field outside the solenoid. It is a weak field, with circular field lines, like those due to a line of current as in **Figure 20.** For an ideal solenoid, this weak field is the only field external to the solenoid.

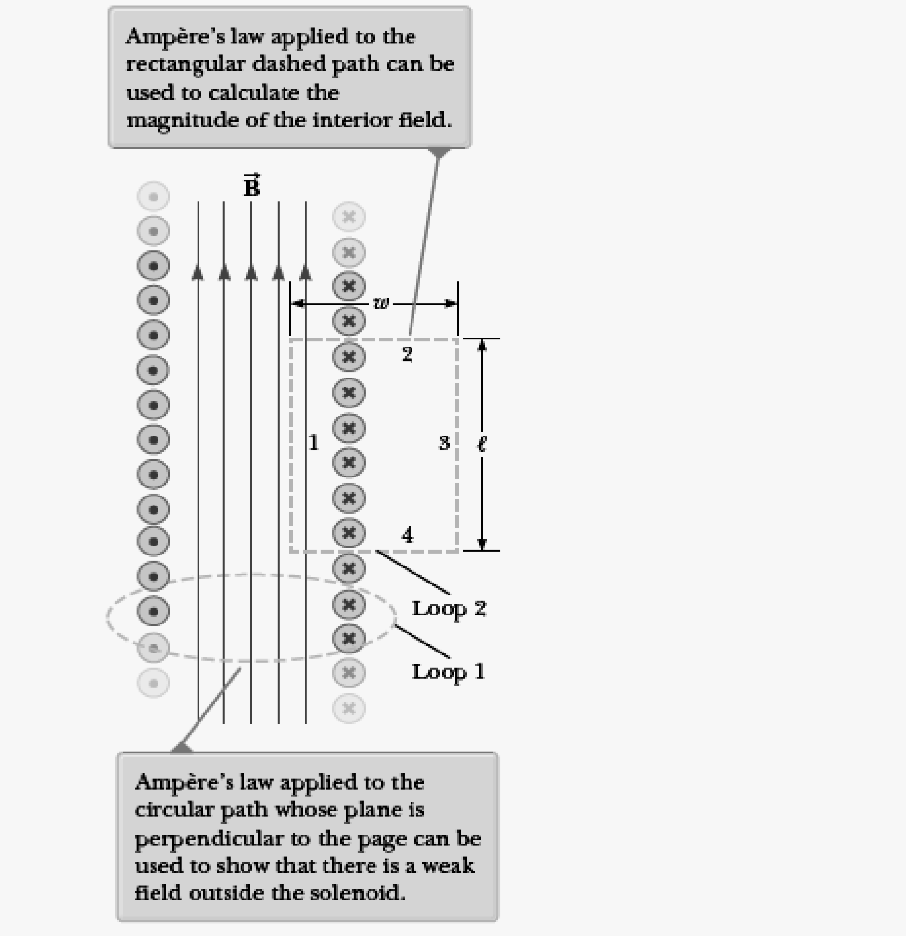

Figure 24. Cross-sectional view of an ideal solenoid, where the interior magnetic field is uniform and the exterior field is close to zero.

We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in an ideal solenoid. Because the solenoid is ideal, $\vec{B}$ in the interior space is uniform and parallel to the axis and the magnetic field lines in the exterior space form circles around the solenoid. The planes of these circles are perpendicular to the page. Consider the rectangular path (loop 2) of length, and width *w* shown in **Figure 24**. Let's apply Ampère's law to this path by evaluating the integral of $\vec{B}.\, d\vec{s}$ over each side of the rectangle. The contribution along side 3 is zero because the external magnetic field lines are perpendicular to the path in this region. The contributions from sides 2 and 4 are both zero, again because $\vec{B}$ is perpendicular to $d\vec{s}$ along these paths, both inside and outside the solenoid. Side 1 gives a contribution to the integral because along this path $\vec{B}$ is uniform and parallel to $d\vec{s}$. The integral over the closed rectangular path is therefore

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path 1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path 1}} ds = B\ell$$

The right side of Ampère's law involves the total current ***I*** through the area bounded by the path of integration. In this case, the total current through the rectangular path equals the current through each turn multiplied by the number of turns. If ***N*** is the number of turns in the length, the total current through the rectangle is ***NI***. Therefore, Ampère's law applied to this path gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 NI$$

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I \quad \text{——— (6)}$$

where $n = N/\ell$ is the number of turns per unit length.

Equation 6 is valid only for points near the center (that is, far from the ends) of a very long solenoid. As you might expect, the field near each end is smaller than the value given by **Equation 6**. As the length of a solenoid increases, the magnitude of the field at the end approaches half the magnitude at the center.

Gauss's Law in Magnetism:

The flux associated with a magnetic field is defined in a manner similar to that used to define electric flux. Consider an element of area ***dA*** on an arbitrarily shaped surface as shown in **Figure 25**. If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is $\vec{B} \cdot d\vec{A}$,, where $d\vec{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area ***dA***. Therefore, the total magnetic flux $\mathbf{F_B}$ through the surface is

$$\Phi_B \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \quad \text{——— (7)}$$

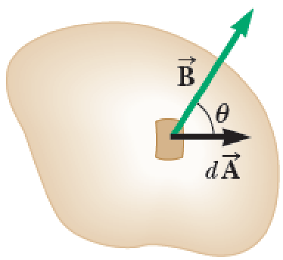

Figure 25. The magnetic flux through an area element dA is $\vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = B\,dA\cos\theta$, where $d\vec{\mathbf{A}}$ is a vector perpendicular to the surface.

Consider the special case of a plane of area A in a uniform field $\vec{B}$ that makes an angle θ with ***d*****A.** The magnetic flux through the plane in this case is

$$\Phi_B = BA\cos\theta \quad \text{——— (8)}$$

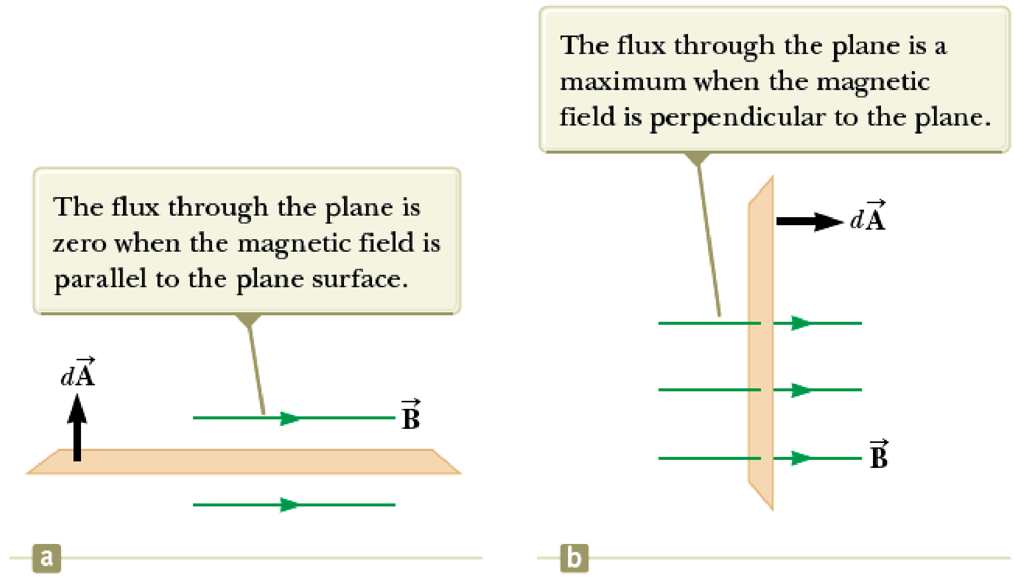

Figure 26. Magnetic flux through a plane lying in a magnetic field.

If the magnetic field is parallel to the plane as in **Figure 26a**, then $\theta = 90^0$ and the flux through the plane is zero. If the field is perpendicular to the plane as in **Figure 26b**, then $\theta = 0$ and the flux through the plane is $\boldsymbol{B\,A}$ (the maximum value).
The unit of magnetic flux is **T. m²**, which is defined as a *weber* (Wb); 1 Wb = 1 T. m^2.

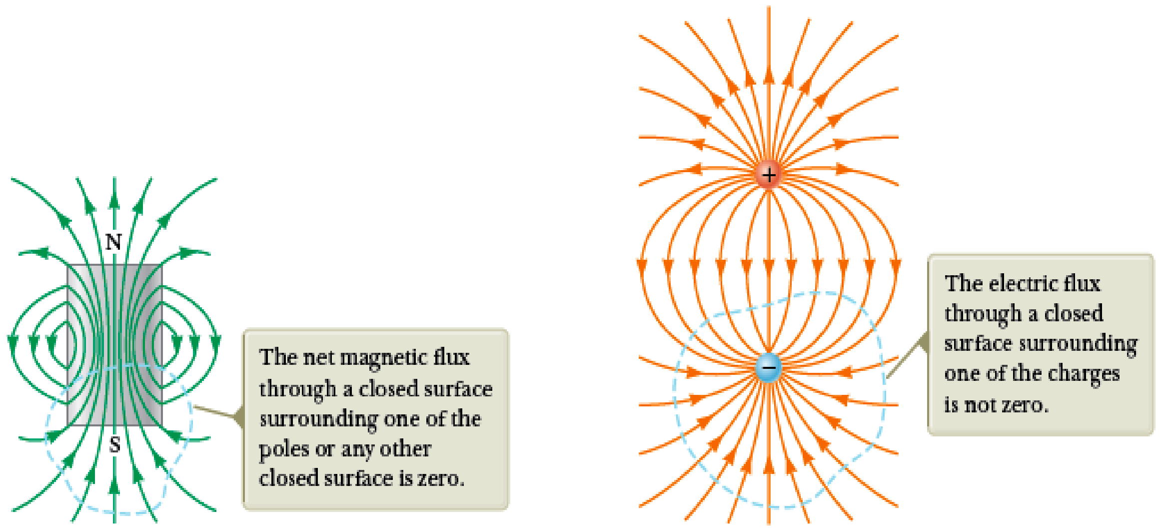

Figure 27. The magnetic field lines of a bar magnet form closed loops. (The dashed line represents the intersection of a closed surface with the page.)

Figure 28. The electric field lines surrounding an electric dipole begin on the positive charge and terminate on the negative charge.

In electrostatics chapter, we found that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gauss's law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This behavior exists because electric field lines originate and terminate on electric charges. The situation is quite different for magnetic fields, which are continuous and form closed loops. In other words, as illustrated by the magnetic field lines of a current in **Figure 20** and of a bar magnet in **Figure 27**, magnetic field lines do not begin or end at any point. For any closed surface such as the one outlined by the dashed line in **Figure 27**, the number of lines entering the surface equals the number leaving the surface; therefore, the net magnetic flux is zero. In contrast, for a closed surface surrounding one charge of an electric dipole (**Fig. 28**), the net electric flux is not zero.

Gauss's law in magnetism states that

the net magnetic flux through any closed surface is always zero:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad (9)$$

This statement represents that isolated magnetic poles (monopoles) have never been detected and perhaps do not exist. Nonetheless, scientists continue the search because certain theories that are otherwise successful in explaining fundamental physical behavior suggest the possible existence of magnetic monopoles.

Magnetism in Matter

The Magnetic Moments of Atoms:

Let's begin our discussion with a classical model of the atom in which electrons move in circular orbits around the much more massive nucleus. In this model, an orbiting electron constitutes a tiny current loop (because it is a moving charge), and the magnetic moment of the electron is associated with this orbital motion. Although this model has many deficiencies, some of its predictions are in good agreement with the correct theory, which is expressed in terms of quantum physics.

In our classical model, we assume an electron is a particle in uniform circular motion: it moves with constant speed v in a circular orbit of radius r about the nucleus as in **Figure 29.** The current I associated with this orbiting electron is its charge e divided by its period T. Therefore the particle in uniform circular motion model, $\boldsymbol{T} = \mathbf{2\pi r/v}$, gives

$$I = \frac{e}{T} = \frac{ev}{2\pi r} \qquad (10)$$

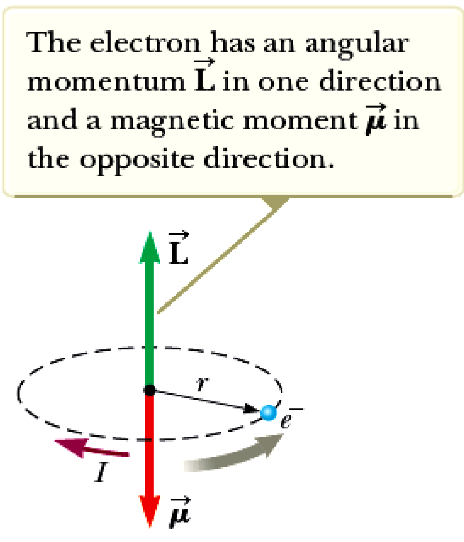

Figure 29. An electron moving in the direction of the gray arrow in a circular orbit of radius r. Because the electron carries a negative charge, the direction of the current due to its motion about the nucleus is opposite the direction of that motion.

The magnitude of the magnetic moment associated with this current loop is given by $\boldsymbol{\mu} = \boldsymbol{IA}$, where $\boldsymbol{A} = \pi\, \boldsymbol{r}^2$ is the area enclosed by the orbit.

Therefore,

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \tfrac{1}{2}evr \quad \text{———} (11)$$

Because the magnitude of the orbital angular momentum of the electron is given by $L = m_e vr$, the magnetic moment can be written as

$$\mu = \left(\frac{e}{2m_e}\right)L \quad \text{———} (12)$$

This result demonstrates that the magnetic moment of the electron is proportional to its orbital angular momentum. Because the electron is negatively charged, the vectors $\vec{\mu}$ and $\vec{L}$ point in *opposite* directions. Both vectors are perpendicular to the plane of the orbit as indicated in **Figure 29.**

A fundamental outcome of quantum physics is that orbital angular momentum is quantized and is equal to multiples of **$\hbar$ = $h/2\pi$ = 1.05x 10^{-34} J.s**, where h is Planck's constant. The smallest nonzero value of the electron's magnetic moment resulting from its orbital motion is

$$\mu = \sqrt{2}\,\frac{e}{2m_e}\,\hbar \quad \text{———} (13)$$

Because all substances contain electrons, you may wonder why most substances are not magnetic. The main reason is that, in most substances, the magnetic moment of one electron in an atom is canceled by that of another electron orbiting in the opposite direction. The net result is that, for most materials, the magnetic effect produced by the orbital motion of the electrons is either zero or very small. In addition to its orbital magnetic moment, an electron (as well as protons, neutrons, and other particles) has an intrinsic property called **spin** that also contributes to its magnetic moment. Classically, the electron might be viewed as spinning about its axis as shown in **Figure 30**, but you should be very careful with the classical interpretation. The magnitude of the angular momentum $\vec{S}$ associated with spin is on the same order of magnitude as the magnitude of the angular momentum $\vec{L}$ due to the orbital motion. The magnitude of the spin angular momentum of an electron predicted by quantum theory is

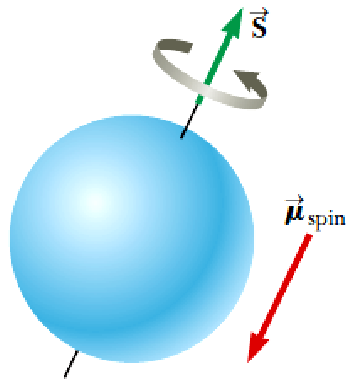

Figure 30 Classical model of a spinning electron. We can adopt this model to remind ourselves that electrons have an intrinsic angular momentum. The model should not be pushed too far, however; it gives an incorrect magnitude for the magnetic moment, incorrect quantum numbers, and too many degrees of freedom.

$$S = \frac{\sqrt{3}}{2}\hbar \quad \text{———— (14)}$$

The magnetic moment characteristically associated with the spin of an electron has the value

$$\mu_{\text{spin}} = \frac{e\hbar}{2m_e} \quad \text{———— (15)}$$

This combination of constants is called the **Bohr magneton** μ_B:

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ \text{J/T} \quad \text{———— (16)}$$

Therefore, atomic magnetic moments can be expressed as multiples of the Bohr magneton. (Note that $1\ \text{J/T} = 1\ \text{A} \cdot \text{m}^2$.)

In atoms containing many electrons, the electrons usually pair up with their spins opposite each other; therefore, the spin magnetic moments cancel. Atoms containing an odd number of electrons, however, must have at least one unpaired electron and therefore some spin magnetic moment. The total magnetic moment of an atom is the vector sum of the orbital and spin magnetic moments, and a few examples are given in **Table 2**. Notice that helium and neon have zero moments because their individual spin and orbital moments cancel. The nucleus of an atom also has a magnetic moment associated with its constituent protons and neutrons. The magnetic moment of a proton or neutron, however, is much smaller than that of an electron and can usually be neglected. We can understand this smaller value by inspecting **Equation 16** and replacing the mass of the electron with the mass of a proton or a neutron. Because the masses of the proton and neutron are much greater than that of the electron, their magnetic moments are on the order of 10^3 times smaller than that of the electron.

Table 2. Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

Faraday's Law

Experiments conducted by Michael Faraday in England in 1831 and independently Joseph Henry in the United States that same year showed that an emf can be induced in a circuit by a changing magnetic field. The results of these experiments led to a very basic and important law of electromagnetism known as ***Faraday's law of induction***. An emf (and therefore a current as well) can be induced in various processes that involve a change in a magnetic flux.

Faraday's Law of Induction:

To see how an emf can be induced by a changing magnetic field, consider the experimental results obtained when a loop of wire is connected to a sensitive ammeter as illustrated in **Figure 31**. When a magnet is moved toward the loop, the reading on the ammeter changes from zero to a nonzero value, arbitrarily shown as negative in **Figure 31.a**. When the magnet is brought to rest and held stationary relative to the loop (**Fig. 31.b**), a reading of zero is observed. When the magnet is moved away from the loop, the reading on the ammeter changes to a positive value as shown in **Fig. 31.c**. Finally, when the magnet is held stationary and the loop is moved either toward or away from it, the reading changes from zero. From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field. Therefore, it seems that These results are quite remarkable because a current is set up even though no batteries are present in the circuit! We call such a current an *induced current* and say that it is produced by an *induced emf*.

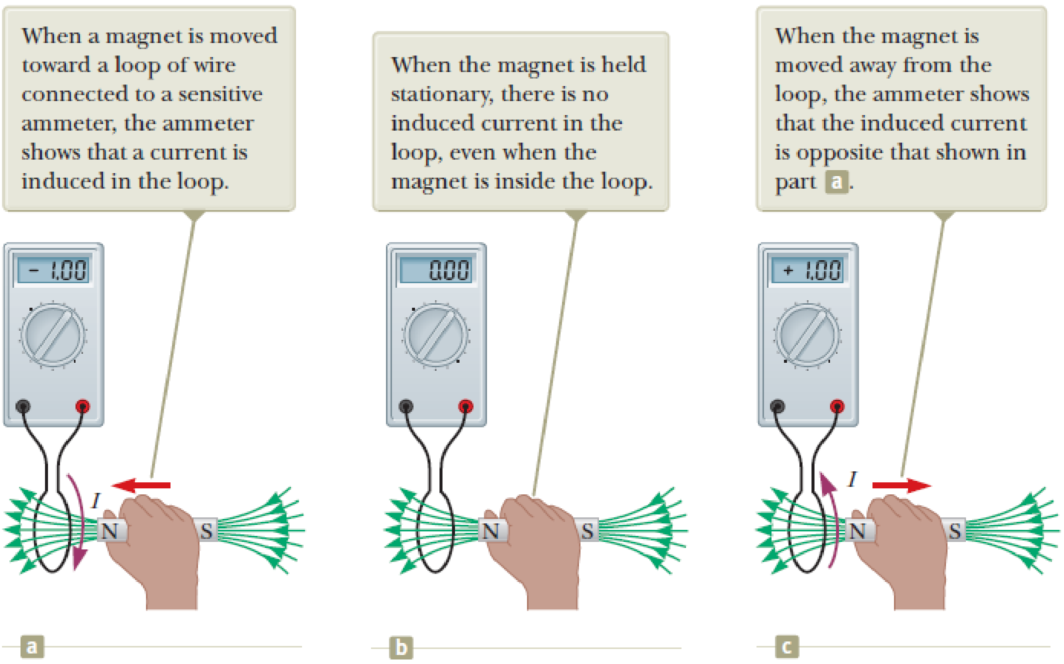

Figure 31. A simple experiment showing that a current is induced in a loop when a magnet is moved toward or away from the loop.

Now let's describe an experiment conducted by Faraday and illustrated in **Figure 32**. A primary coil is wrapped around an iron ring and connected to a switch and a battery. A current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a sensitive ammeter. No battery is present in the secondary circuit, and the secondary coil is not electrically connected to the primary coil. Any current detected in the secondary circuit must be induced by some external agent.

Initially, you might guess that no current is ever detected in the secondary circuit. Something quite amazing happens when the switch in the primary circuit is either opened or thrown closed, however. At the instant the switch is closed, the ammeter reading changes from zero momentarily and then returns to zero. At the instant the switch is opened, the ammeter changes to a reading with the opposite sign and again returns to zero. Finally, the ammeter reads zero when there is either a steady current or no current in the primary circuit. To understand what happens in this experiment, note that when the switch is closed, the current in the primary circuit produces a magnetic field that penetrates the secondary circuit. Furthermore, when the switch is thrown closed, the magnetic field produced by the current in the primary circuit changes from zero to some value over some finite time, and this changing field induces a current in the secondary circuit. Notice that no current is induced in the secondary coil even when a steady current exists in the primary coil. It is a *change* in the current in the primary coil that induces a current in the secondary coil, not just the *existence* of a current.

As a result of these observations, Faraday concluded that an electric current can be induced in a loop by a changing magnetic field. The induced current exists only while the magnetic field through the loop is changing. Once the magnetic field reaches a steady value, the current in the loop disappears. In effect, the loop behaves as though a source of emf were connected to it for a short time. It is customary to say that an induced emf is produced in the loop by the changing magnetic field.

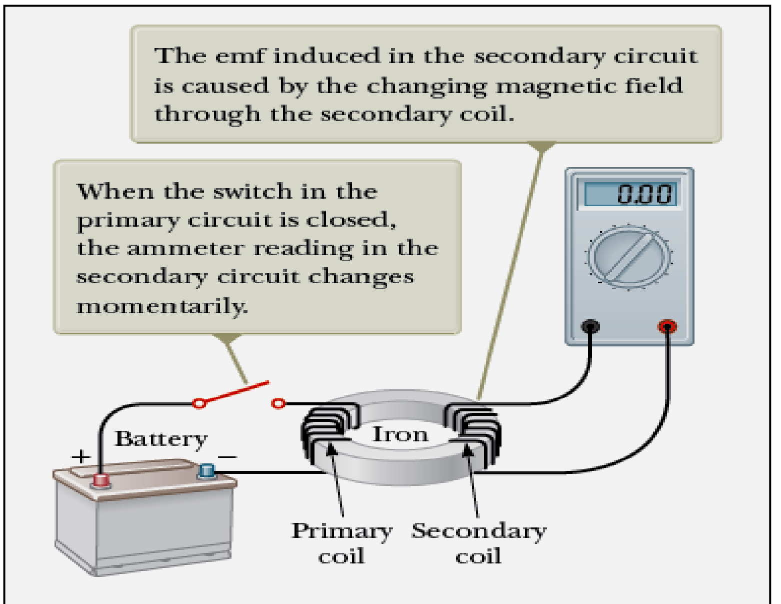

Figure 32. Faraday's experiment.

The experiments shown in **Figures 31 & 32** have one thing in common: in each case, an emf is induced in a loop when the magnetic flux through the loop changes with time. In general, this emf is directly proportional to the time rate of change of the magnetic flux through the loop.

This statement can be written mathematically as **Faraday's law of induction:**

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}} \qquad \text{(17)}$$

where $\phi_B = \int \vec{\mathbf{B}}.\vec{\mathbf{dA}}$ is the magnetic flux through the loop. If a coil consists of ***N*** loops with the same area and ϕ_B is the magnetic flux through one loop, an emf is induced in every loop. The loops are in series, so their emfs add; therefore, the total induced emf in the coil is given by

$$\boxed{\mathcal{E} = -N\frac{d\Phi_B}{dt} \qquad \text{(18)}}$$

The negative sign in **Equations 17** and **18** is of important physical significance.

Suppose a loop enclosing an area ***A*** lies in a uniform magnetic field $\vec{\mathbf{B}}$ as in **Figure 33.** The magnetic flux through the loop is equal to ***BA* cosθ**, where u is the angle between the magnetic field and the normal to the loop; hence, the induced emf can be expressed as

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta) \qquad \text{(19)}$$

From this expression, we see that an emf can be induced in the circuit in several ways:

- The magnitude of $\vec{\mathbf{B}}$ can change with time.
- The area enclosed by the loop can change with time.
- The angle **θ** between $\vec{\mathbf{B}}$ and the normal to the loop can change with time.
- Any combination of the above can occur.

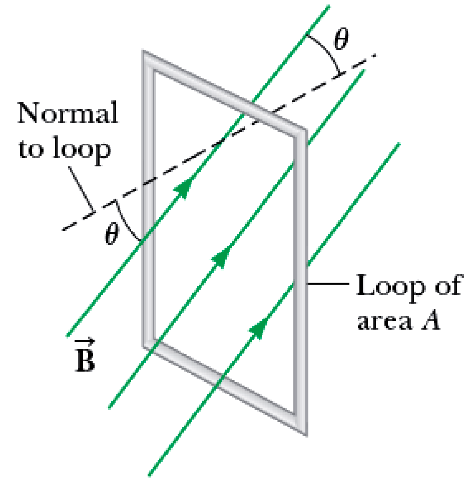

Figure 33. A conducting loop that encloses an area A in the presence of a uniform magnetic field $\vec{\mathbf{B}}$. The angle between $\vec{\mathbf{B}}$ and the normal to the loop is θ.

Motional emf:

So far, we considered cases in which an emf is induced in a stationary circuit placed in a magnetic field when the field changes with time. In this section, we describe **motional emf,** the emf induced in a conductor moving through a constant magnetic field.

The straight conductor of length, shown in **Figure 34** is moving through a uniform magnetic field directed into the page. For simplicity, let's assume the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. From the magnetic version of the particle in a field model, the electrons in the conductor experience a force $\vec{F_B} = q\vec{v}\,x\,\vec{B}$ that is directed along the length ,, perpendicular to both $\vec{v}$ and $\vec{B}$. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field $\vec{E}$ is produced inside the conductor. Therefore, the electrons are also described by the electric version of the particle in a field model. The charges accumulate at both ends until the downward magnetic force qvB on charges remaining in the conductor is balanced by the upward electric force qE. The electrons are then described by the particle in equilibrium model. The condition for equilibrium requires that the forces on the electrons balance:

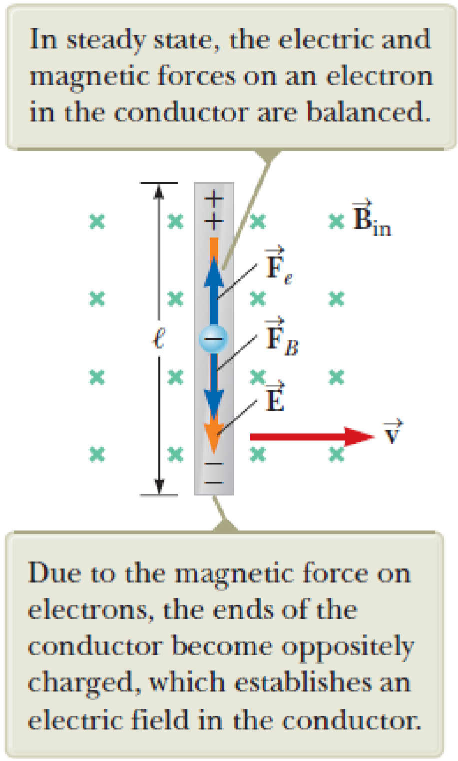

Figure 34. A straight electrical conductor of length ℓ moving with a velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ directed perpendicular to $\vec{v}$.

$$qE = qvB \quad \text{or} \quad E = vB$$

The magnitude of the electric field produced in the conductor is related to the potential difference across the ends of the conductor according to the relationship
$\Delta V = E\ell$. Therefore, for the equilibrium condition,

$$\Delta V = E\ell = B\ell v$$

where the upper end of the conductor in **Figure 34** is at a higher electric potential than the lower end. Therefore, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field. If the direction of the motion is reversed, the polarity of the potential difference is also reversed.

A more interesting situation occurs when the moving conductor is part of a closed conducting path. This situation is particularly useful for illustrating how a changing magnetic flux causes an induced current in a closed circuit. Consider a circuit consisting of a conducting bar of length ℓ sliding along two fixed, parallel conducting rails as shown in

Figure 35.a. For simplicity, let's assume the bar has zero resistance and the stationary part of the circuit has a resistance $\boldsymbol{R}$. A uniform and constant magnetic field $\vec{\boldsymbol{B}}$ is applied perpendicular to the plane of the circuit. As the bar is pulled to the right with a velocity $\vec{\boldsymbol{v}}$ under the influence of an applied force $\vec{\boldsymbol{F}}_{\boldsymbol{app}}$, free charges in the bar are moving particles in a magnetic field that experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. In this case, the rate of change of magnetic flux through the circuit and the corresponding induced motional emf across the moving bar are proportional to the change in area of the circuit.

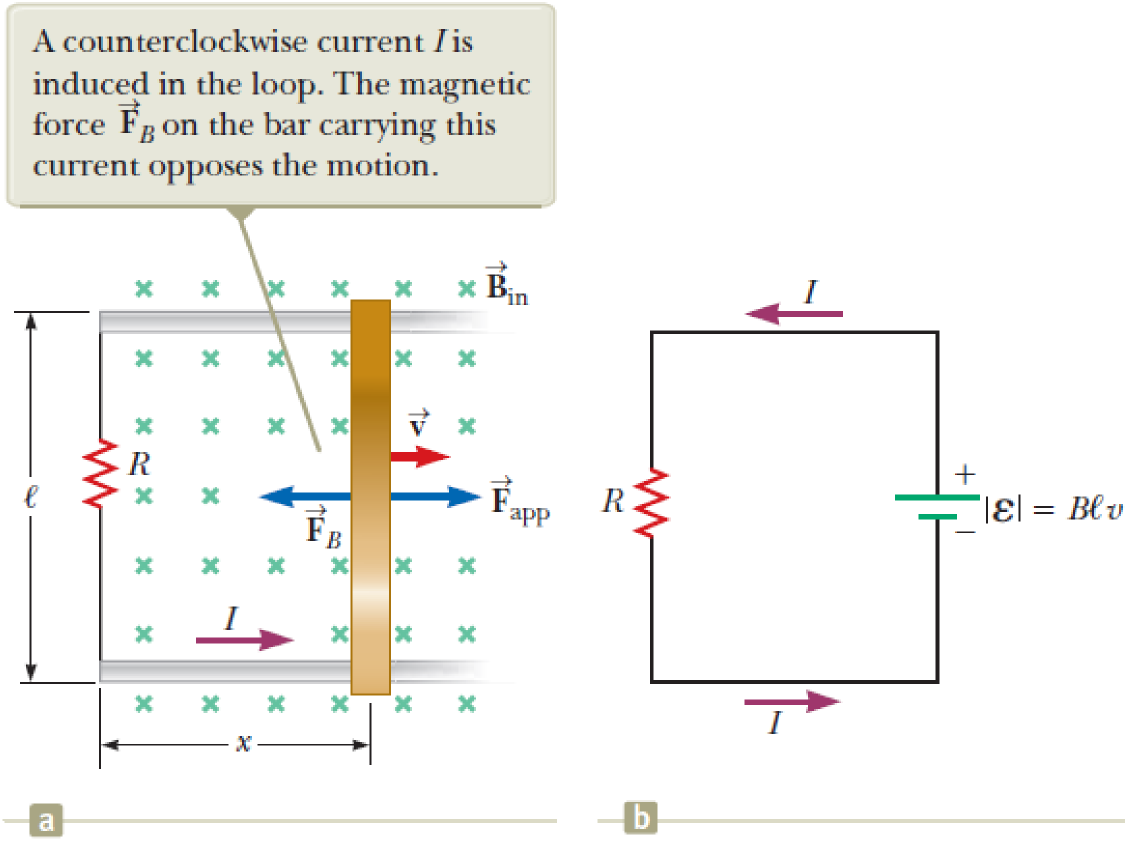

Figure 35. (a) A conducting bar sliding with a velocity $\vec{v}$ along two conducting rails under the action of an applied force $\vec{F}_{app}$.

(b) The equivalent circuit diagram for the setup shown in (a).

Because the area enclosed by the circuit at any instant is ℓx, where x is the position of the bar, the magnetic flux through that area is

$$\Phi_B = B\ell x$$

Using Faraday's law and noting that x changes with time at a rate $dx/dt = v$, we find that the induced motional emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt}$$

$$\mathcal{E} = -B\ell v \qquad \text{———(20)}$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R} \qquad \text{———(21)}$$

Lenz's Law

Faraday's law (**Eq. 17**) indicates that the induced emf and the change in flux have opposite algebraic signs. This feature has a very real physical interpretation that has come to be known as **Lenz's law.**

Lenz's law Statement*: The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.*

That is, the induced current tends to keep the original magnetic flux through the loop from changing. We shall show that this law is a consequence of the law of conservation of energy.

To understand Lenz's law, let's return to the example of a bar moving to the right on two parallel rails in the presence of a uniform magnetic field (the *external* magnetic field, shown by the green crosses in **Fig. 36a**). As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases. Lenz's law states that the induced current must be directed so that the magnetic field it produces opposes the change in the external magnetic flux. Because the magnetic flux due to an external field directed into the page is increasing, the induced current—if it is to oppose this change-must produce a field directed out of the page. Hence, the induced current must be directed counterclockwise when the bar moves to the right. (Use the right-hand rule to verify this direction.). If the bar is moving to the left as in **Figure 36b**, the external magnetic flux through the area enclosed by the loop decreases with time. Because the field is directed into the page, the direction of the induced current must be clockwise if it is to produce a field that also is directed into the page. In either case, the induced current attempts to maintain the original flux through the area enclosed by the current loop.

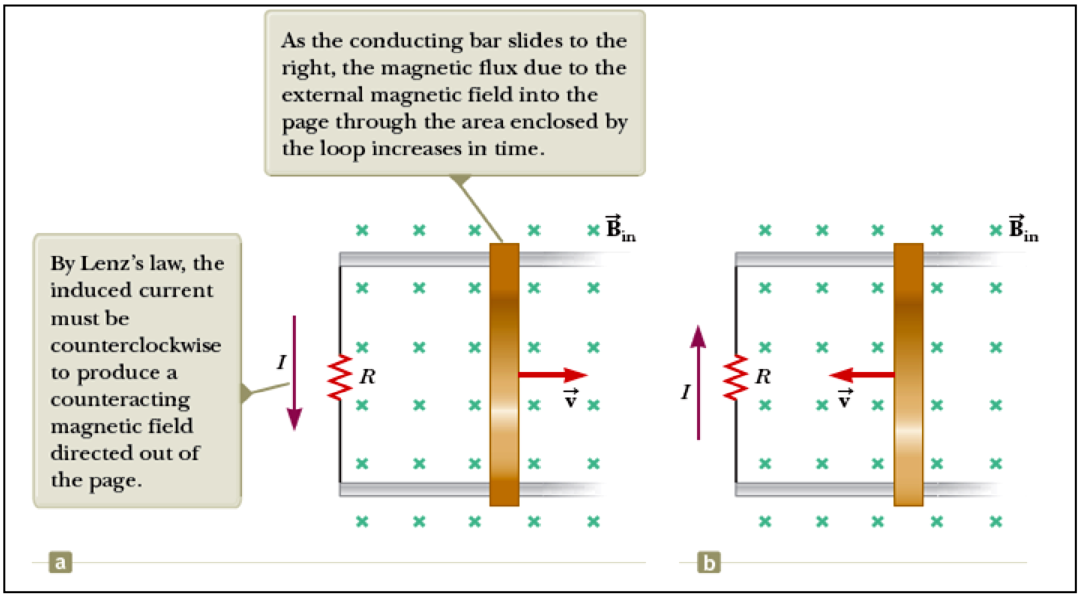

Figure 36 (a) Lenz's law can be used to determine the direction of the induced current. (b) When the bar moves to the left, the induced current must be clockwise. Why?

Let's examine this situation using energy considerations. Suppose the bar is given a slight push to the right. In the preceding analysis, we found that this motion sets up a counterclockwise current in the loop. What happens if we assume the current is clockwise such that the direction of the magnetic force exerted on the bar is to the right? This force would accelerate the rod and increase its velocity, which in turn would cause the area enclosed by the loop to increase more rapidly. The result would be an increase in the induced current, which would cause an increase in the force, which would produce an increase in the current, and so on. In effect, the system would acquire energy with no input of energy. This behavior is clearly inconsistent with all experience and violates the law of conservation of energy. Therefore, the current must be counterclockwise.

Induced emf and Electric Fields:

We have seen that a changing magnetic flux induces an emf and a current in a conducting loop. In our study of electricity, we related a current to an electric field that applies electric forces on charged particles. In the same way, we can relate an induced current in a conducting loop to an electric field by claiming that an electric field is created in the conductor as a result of the changing magnetic flux. We also noted in our study of electricity that the existence of an electric field is independent of the presence of any test charges. This independence suggests that even in the absence of a conducting loop, a changing magnetic field generates an electric field in empty space.

This induced electric field is *nonconservative*, unlike the electrostatic field produced by stationary

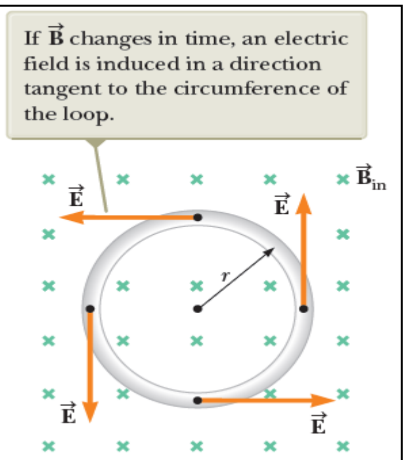

Figure 37 A conducting loop of radius r in a uniform magnetic field perpendicular to the plane of the loop.

charges. To illustrate this point, consider a conducting loop of radius ***r*** situated in a uniform magnetic field that is perpendicular to the plane of the loop as in **Figure 37.** If the magnetic field changes with time, an emf $\varepsilon = - d\phi_B / dt$ is, according to Faraday's law (Eq. 17), induced in the loop. The induction of a current in the loop implies the presence of an induced electric field $\vec{\boldsymbol{E}}$, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force. The work done by the electric field in moving a charge ***q*** once around the loop is equal to $q\varepsilon$. Because the electric force acting on the charge is $\boldsymbol{q\vec{E}}$, the work done by the electric field in moving the charge once around the loop is $\boldsymbol{qE\ (2\pi r)}$, where $\boldsymbol{2\pi r}$ is the circumference of the loop. These two expressions for the work done must be equal; therefore,

$$q\varepsilon = qE\ (2\pi r)$$

$$E = \frac{\varepsilon}{2\pi r}$$

Using this result along with Equation 17 and that $\Phi_B = BA = B\pi r^2$ for a circular loop, the induced electric field can be expressed as

$$E = -\frac{1}{2\pi r}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt} \qquad (22)$$

If the time variation of the magnetic field is specified, the induced electric field can be calculated from Equation 22.

The emf for any closed path can be expressed as the line integral of $\vec{\boldsymbol{E}}.\overrightarrow{\boldsymbol{ds}}$ over that path $\varepsilon = \oint \vec{E}.\overrightarrow{ds}$. In more general cases, *E* may not be constant and the path may not be a circle. Hence, Faraday's law of induction, $\varepsilon = - d\Phi_B / dt$, can be written in the general form

$$\oint \vec{E}.\overrightarrow{ds} = -\frac{d\Phi_B}{dt} \qquad (23)$$

The induced electric field $\vec{\boldsymbol{E}}$ in Equation 23 is a nonconservative field that is generated by a changing magnetic field. The field $\vec{\boldsymbol{E}}$ that satisfies Equation 23 cannot possibly be an electrostatic field because were the field electrostatic and hence conservative, the line integral of $\vec{\boldsymbol{E}}.\overrightarrow{\boldsymbol{ds}}$ over a closed loop would be zero, which would be in contradiction to Equation 23**.**

Generators and Motors:

Electric generators are devices that take in energy by work and transfer it out by electrical transmission. To understand how they operate, let us consider the **Alternatingcurrent (AC) generator:**

In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field (**Fig. 38a**). In commercial power plants, the energy required to rotate the loop can be derived from a variety of sources. For example, in a hydroelectric plant, falling water directed against the blades of a turbine produces the rotary motion; in a coalfired plant, the energy released by burning coal is used to convert water to steam, and this steam is directed against the turbine blades.

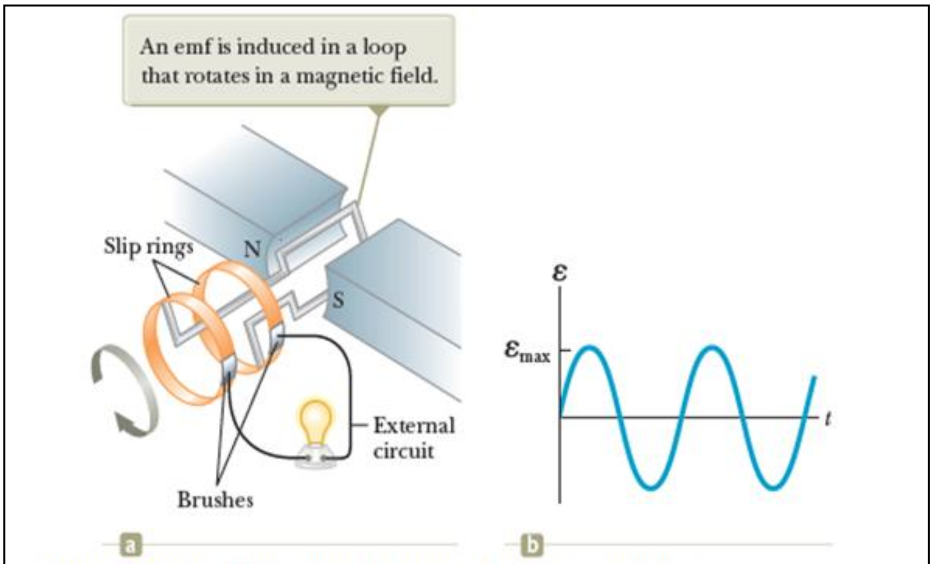

Figure 38. (a) Schematic diagram of an AC generator.
(b) The alternating emf induced in the loop plotted as a fuction of time.

As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time, and this change induces an emf and a current in the loop according to Faraday's law. The ends of the loop are connected to slip rings that rotate with the loop. Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary metallic brushes in contact with the slip rings. Instead of a single turn, suppose a coil with *N* turns (a more practical situation), with the same area *A*, rotates in a magnetic field with a constant angular speed v. If u is the angle between the magnetic field and the normal to the plane of the coil as in **Figure 39**, the magnetic flux through the coil at any time *t* is

$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

where we have used the relationship $\theta = \omega t$ between angular position and angular speed. (We have set the clock so that $t = 0$ when $\theta = 0$.) Hence, the induced emf in the coil is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t) = NBA\omega\sin\omega t \qquad (24)$$

This result shows that the emf varies sinusoidally with time as plotted in **Figure 38b**. Equation 24 shows that the maximum emf has the value

$$\mathcal{E}_{max} = NBA\omega$$

which occurs when $\omega t = 90^0$ or 270^0. In other words, $\varepsilon = \varepsilon_{max}$ when the magnetic field is in the plane of the coil and the time rate of change of flux is a maximum. Furthermore, the emf is zero when $\omega t = 0$ or 180^0, that is, when $\vec{B}$ is perpendicular to the plane of the coil and the time rate of change of flux is zero. The frequency for commercial generators in the United States and Canada is 60 Hz, whereas in some European countries it is 50 Hz. (Recall that $\omega = 2\pi f$, where f is the frequency in hertz.)

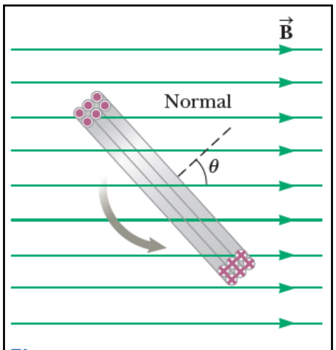

Figure 39. A cutaway view of a loop enclosing an area *A* and containing *N* turns, rotating with constant angular speed ω in a magnetic field. The emf induced in the loop varies sinusoidally in time.

Direct-current (DC) generator:

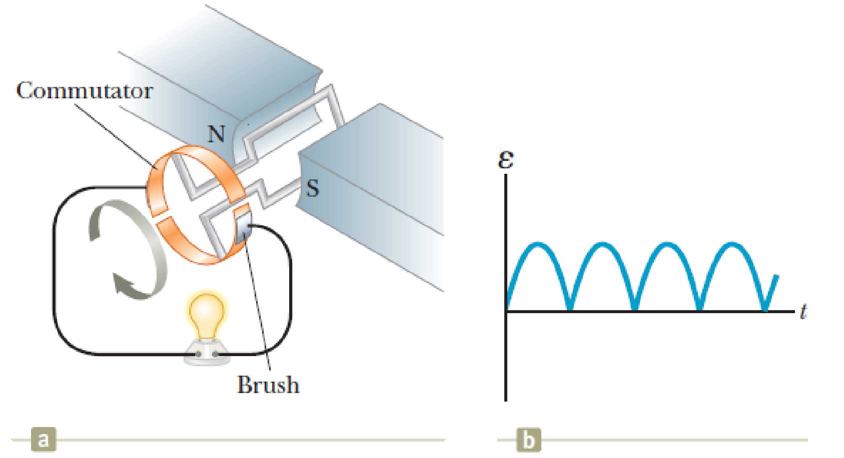

Figure 40. (a) Schematic diagram of a DC generator.
(b) The magnitude of the emf varies in time, but the polarity never changes.

The **direct-current (DC) generator** is illustrated in **Figure 40a**. Such generators are used, for instance, in older cars to charge the storage batteries. The components are essentially the same as those of the AC generator except that the contacts to the rotating coil are made using a split ring called a *commutator*.

In this configuration, the output voltage always has the same polarity and pulsates with time as shown in **Figure 40b**. We can understand why by noting that the contacts to the split ring reverse their roles every half cycle. At the same time, the polarity of the induced emf reverses; hence, the polarity of the split ring (which is the same as the polarity of the output voltage) remains the same.

A pulsating DC current is not suitable for most applications. To obtain a steadier DC current, commercial DC generators use many coils and commutators distributed so that the sinusoidal pulses from the various coils are out of phase. When these pulses are superimposed, the DC output is almost free of fluctuations.

A **motor** is a device into which energy is transferred by electrical transmission while energy is transferred out by work. A motor is essentially a generator operating in reverse. Instead of generating a current by rotating a coil, a current is supplied to the coil by a battery, and the torque acting on the current-carrying coil causes it to rotate.

Useful mechanical work can be done by attaching the rotating coil of a motor to some external device. As the coil rotates in a magnetic field, however, the changing magnetic flux induces an emf in the coil; consistent with Lenz's law, this induced emf always acts to reduce the current in the coil. The back emf increases in magnitude as the rotational speed of the coil increases. (The phrase *back emf* is used to indicate an emf that tends to reduce the supplied current.) Because the voltage available to supply current equals the difference between the supply voltage and the back emf, the current in the rotating coil is limited by the back emf.
When a motor is turned on, there is initially no back emf, and the current is very large because it is limited only by the resistance of the coil. As the coil begins to rotate, the induced back emf opposes the applied voltage and the current in the coil decreases. If the mechanical load increases, the motor slows down, which causes the back emf to decrease. This reduction in the back emf increases the current in the coil and therefore also increases the power needed

from the external voltage source. For this reason, the power requirements for running a motor are greater for heavy loads than for light ones. If the motor is allowed to run under no mechanical load, the back emf reduces the current to a value just large enough to overcome energy losses due to internal energy and friction. If a very heavy load jams the motor so that it cannot rotate, the lack of a back emf can lead to dangerously high current in the motor's wire. A modern application of motors in automobiles is seen in the development of *hybrid drive systems*. In these automobiles, a gasoline engine and an electric motor are combined to increase the fuel economy of the vehicle and reduce its emissions. **Figure 41** shows the engine compartment of a Toyota Prius, one of the hybrids available in the United States. In this automobile, power to the wheels can come from either the gasoline engine or the electric motor. In normal driving, the electric motor accelerates the vehicle from rest until it is moving at a speed of about 15 mi/h (24 km/h). During this acceleration period, the engine is not running, so gasoline is not used and there is no emission. At higher speeds, the motor and engine work together so that the engine always operates at or near its most efficient speed. The result is a significantly higher gasoline mileage than that obtained by a traditional gasoline-powered automobile. When a hybrid vehicle brakes, the motor acts as a generator and returns some of the vehicle's kinetic energy back to the battery as stored energy. In a normal vehicle, this kinetic energy is not recovered because it is transformed to internal energy in the brakes and roadway.

Figure 41. The engine compartment of a Toyota Prius, a hybrid vehicle.

Eddy Currents:

As we have seen, an emf and a current are induced in a circuit by a changing magnetic flux. In the same manner, circulating currents called **eddy currents** are induced in bulk pieces of metal moving through a magnetic field. This phenomenon can be demonstrated by allowing a flat copper or aluminum plate attached at the end of a rigid bar to swing back and forth through a magnetic field (**Fig. 42**). As the plate enters the field, the changing magnetic flux induces an emf in the plate, which in turn causes the free electrons in the plate to move, producing the swirling eddy currents. According to Lenz's law, the direction of the eddy currents is such that they create magnetic fields that oppose the change that causes the currents. For this reason, the eddy currents must produce effective magnetic poles on the plate, which are repelled by the poles of the magnet; this situation gives rise to a repulsive force that opposes the motion of the

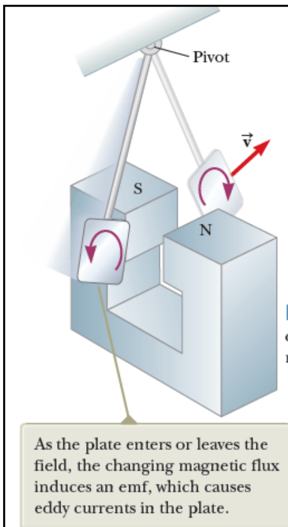

Figure 42. Formation of eddy currents in a conducting plate moving through a magnetic field.

plate. (If the opposite were true, the plate would accelerate and its energy would increase after each swing, in violation of the law of conservation of energy.)

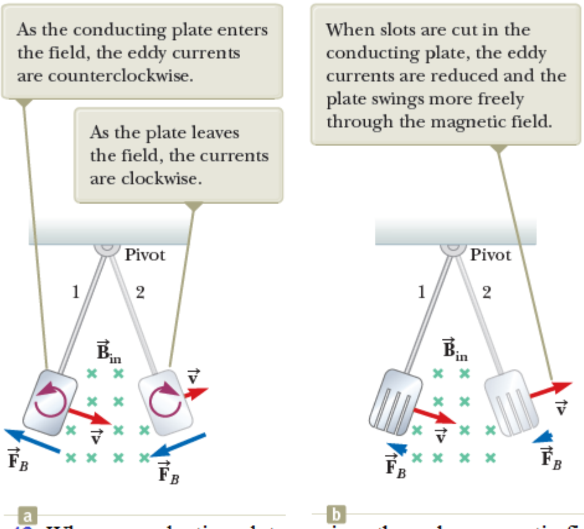

Figure 43. When a conducting plates swings through a magnetic field, edddy currents are induced and the magnetic force F ob the plate opposes its velocity, causing it to eventually come to rest.

As indicated in **Figure 43a**, with $\vec{B}$ directed into the page, the induced eddy current is counterclockwise as the swinging plate enters the field at position 1 because the flux due to the external magnetic field into the page through the plate is increasing. Hence, by Lenz's law, the induced current must provide its own magnetic field out of the page. The opposite is true as the plate leaves the field at position 2, where the current is clockwise. Because the induced eddy current always produces a magnetic retarding force $\vec{F}_B$when the plate enters or leaves the field, the swinging plate eventually comes to rest.

If slots are cut in the plate as shown in **Figure 43b**, the eddy currents and the corresponding retarding force are greatly reduced. We can understand this reduction in force by realizing that the cuts in the plate prevent the formation of any large current loops.

The braking systems on many subway and rapid-transit cars make use of electromagnetic induction and eddy currents. An electromagnet attached to the train is positioned near the steel rails. (An electromagnet is essentially a solenoid with an iron core.) The braking action occurs when a large current is passed through the electromagnet. The relative motion of the magnet and rails induces eddy currents in the rails, and the direction of these currents produces a drag force on the moving train. Because the eddy currents decrease steadily in magnitude as the train slows down, the braking effect is quite smooth. As a safety measure, some power tools use eddy currents to stop rapidly spinning blades once the device is turned off.

Eddy currents are often undesirable because they represent a transformation of mechanical energy to internal energy. To reduce this energy loss, conducting parts are often laminated; that is, they are built up in thin layers separated by a nonconducting material such as lacquer or a metal oxide. This layered structure prevents large current loops and effectively confines the currents to small loops in individual layers. Such a laminated structure is used in transformer cores and motors to minimize eddy currents and thereby increase the efficiency of these devices.