1) i) $\iiint_B xyz^2 \, dV \quad B = \{(x,y,z) \mid 0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}$

$$\int_0^1 \int_{-1}^2 \int_0^3 xyz^2 \, dz\,dy\,dx$$

$$\int_0^1 \int_{-1}^2 xy \frac{z^3}{3}\Big]_0^3 \, dy\,dx$$

$$\int_0^1 \int_{-1}^2 \frac{27}{3} xy \, dy\,dx$$

$$\frac{27}{3} \int_0^1 x \frac{y^2}{2}\Big]_{-1}^2$$

$$\frac{27}{\cancel{6}2} \times \frac{x^2}{2}\Big]_0^1 = \frac{27}{4}$$

ii) $\iiint_E (xz - y^3) \, dV \quad E = \{(x,y,z) \mid -1 \le x \le 1, 0 \le y \le 2, 0 \le z \le 1\}$

$$\int_{-1}^1 \int_0^2 \int_0^1 (xz - y^3) \, dz\,dy\,dx$$

$$\int_{-1}^1 \int_0^2 \left( x\frac{z^2}{2}\Big]_0^1 - y^3 z\Big]_0^1 \right) dy\,dx$$

$$= \int_{-1}^{1}\int_{0}^{2} \left(\frac{x}{2} - y^3\right) dy\, dx$$

$$\int_{-1}^{1}\left(\frac{x}{2} \times y\Big]_0^2 - \frac{y^4}{4}\Big]_0^2\right) dx$$

$$= \int_{-1}^{1} (x - 4)\, dx$$

$$\frac{x^2}{2}\Big]_{-1}^{1} - 4[x]_{-1}^{1} \quad = \frac{1}{2} - \frac{(-1)}{2} - 4(2)$$

$$= \cancel{\frac{1}{2}} + \cancel{-\frac{1}{2}} - 4[1+1]$$

$$= -8$$

iii) $\iiint_E \frac{z}{x^2+z^2}\, dv \quad E = \{(x,y,z) \mid 1 \le y \le 4 \quad y \le z \le 4 \quad 0 \le x \le 2\}$

$$\int_{1}^{4}\int_{y}^{4}\int_{0}^{2} \frac{z}{x^2+z^2}\, dx\, dz\, dy$$

$$\int_{1}^{4}\int_{y}^{4} \cancel{\frac{z}{z}} \tan^{-1}\frac{x}{z}\Big]_0^2\, dz\, dy$$

$$\int_{1}^{4}\int_{y}^{4} \frac{\pi}{4}\, dz\, dy \Rightarrow \int_{1}^{4} \frac{\pi}{4}(4-y)\, dy = \pi \times [y]_1^4 - \frac{y^2}{8}\Big]_1^4$$

$$= \pi\left(3 - 2 + \frac{1}{8}\right) = \frac{9\pi}{8}$$

2.)

$$\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a} (x^2+y^2+z^2)\, dz\, dy\, dx.$$

$$\int_{-c}^{c}\int_{-b}^{b} x^2 z\Big]_{-a}^{a} + y^2 z\Big]_{-a}^{a} + \frac{z^3}{3}\Big]_{-a}^{a} dy\, dx$$

$$\int_{-c}^{c}\int_{-b}^{b} x^2(2a) + y^2(2a) + \left(\frac{a^3}{3}+\frac{a^3}{3}\right) dy\, dx$$

$$2a\int_{-c}^{c}\int_{-b}^{b} \left(x^2 + y^2 + \frac{a^2}{3}\right) dy\, dx$$

$$= 2a\int_{-c}^{c} \left(x^2 y\Big]_{-b}^{b} + \frac{y^3}{3}\Big]_{-b}^{b} + \frac{a^2}{3} y\Big]_{-b}^{b}\right) dx$$

$$= 2a\int_{-c}^{c} \left(x^2(2b) + \frac{2b^3}{3} + \frac{2a^2 b}{3}\right) dx$$

$$= 4ab\int_{-c}^{c} \left(x^2 + \frac{b^2}{3} + \frac{a^2}{3}\right) dx$$

$$= 4ab \times \left(\frac{x^3}{3}\Big]_{-c}^{c} + \frac{b^2}{3} x\Big]_{-c}^{c} + \frac{a^2}{3} x\Big]_{-c}^{c}\right)$$

$$= 4ab\left(\frac{2c^3}{3} + \frac{2b^2 c}{3} + \frac{2a^2 c}{3}\right)$$

$$= 8abc\left(\frac{c^2 + b^2 + a^2}{3}\right)$$

ii) $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{a} (r^2 \sin(\theta))\, dr\, d\theta\, d\phi$

$= \int_0^{2\pi}\int_0^{\pi/4} \sin\theta \left.\frac{r^3}{3}\right]_0^{a} d\theta\, d\phi$

$= \int_0^{2\pi}\int_0^{\pi/4} \left(\frac{a^3}{3}\sin\theta\right) d\theta\, d\phi$

$\frac{a^3}{3} \int_0^{2\pi} \left. -\cos\theta \right]_0^{\pi/4} d\phi$

$= \frac{a^3}{3} \int_0^{2\pi} \left(-\frac{1}{\sqrt{2}} + 1\right) d\phi$

$= \frac{a^3}{3} \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) \int_0^{2\pi} 1\, d\phi$

$= \frac{a^3}{3} \times \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) \times (2\pi)$

$= \frac{2\pi a^3 (\sqrt{2}-1)}{3\sqrt{2}}$ //

iii) $$\int_0^1 \int_x^{2x} \int_0^y 2xyz \, dz \, dy \, dx$$

$$\int_0^1 \int_x^{2x} 2xy \frac{z^2}{2} \Big]_0^y \, dy \, dx$$

$$\int_0^1 \int_x^{2x} \frac{\cancel{2}x \, y^3}{\cancel{2}} \, dy \, dx$$

$$\int_0^1 x \, \frac{y^4}{4} \Big]_x^{2x} \, dx$$

$$\int_0^1 x \left( \frac{16x^4}{4} - \frac{x^4}{4} \right)$$

$$\int_0^1 \frac{15x^5}{4} \, dx$$

$$= \frac{15}{4} \times \frac{x^6}{6} \Big]_0^1$$

$$= \frac{15}{24} = \frac{5}{8}$$

iv) $$\int_0^2 \int_0^{z^2} \int_0^{y-z} (2x-y) \, dx \, dy \, dz$$

$$= \int_0^2 \int_0^{z^2} x^2 \Big]_0^{y-z} - y \, x \Big]_0^{y-z}$$

$$= \int_0^2 \int_0^{z^2} \left( (y-z)^2 - y(y-z) \right) dy\, dz$$

$$= \int_0^2 \int_0^{z^2} \cancel{y^2} + z^2 - 2yz - \cancel{y^2} + yz$$

$$= \int_0^2 \int_0^{z^2} (z^2 - yz)\, dy\, dz$$

$$= \int_0^2 z^2 y \Big]_0^{z^2} - z \frac{y^2}{2} \Big]_0^{z^2}$$

$$= \int_0^2 z^4 - \frac{z^5}{2}$$

$$= \frac{z^5}{5}\Big]_0^2 - \frac{z^6}{12}\Big]_0^2$$

$$= \frac{32}{5} - \frac{64}{12}$$

$$= \frac{16}{15}$$

v) $$\int_0^{\pi/2} \int_0^{y} \int_0^{x} \cos(x+y+z)\, dz\, dx\, dy.$$

$$\int_0^{\pi/2}\int_0^{y} \left[\sin(x+y+z)\right]_0^{x} dx\,dy$$

$$\int_0^{\pi/2}\int_0^{y} \left[\sin(2x+y) - \sin(x+y)\right] dx\,dy$$

$$\int_0^{\pi/2} \left[\left[\frac{-\cos(2x+y)}{2}\right]_0^{y} + \left[\cos(x+y)\right]_0^{y}\right] dy$$

$$\int_0^{\pi/2} -\frac{\cos(3y)}{2} + \frac{\cos y}{2} + \cos(2y) - \cos y$$

$$\int_0^{\pi/2} -\frac{\cos 3y}{2} + \cos 2y - \frac{\cos y}{2}$$

$$= -\left.\frac{\sin 3y}{6}\right]_0^{\pi/2} + \left.\frac{\sin(2y)}{2}\right]_0^{\pi/2} - \left.\frac{\sin y}{2}\right]_0^{\pi/2}$$

$$= \frac{+1}{6} + 0 - \frac{1}{2}$$

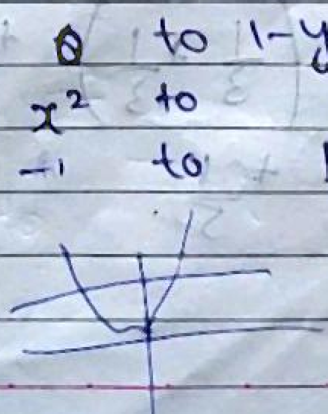
$$= -\frac{1}{3} \;//$$

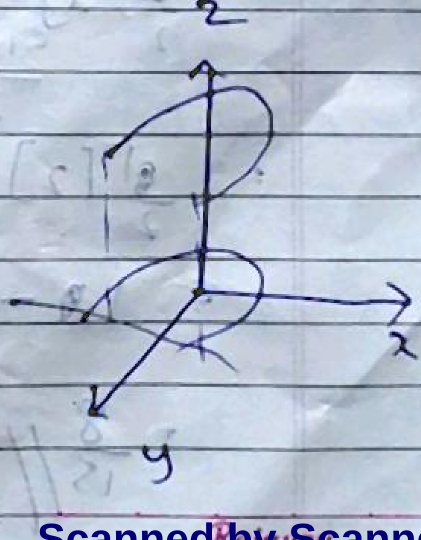
3) i) $y = x^2$, $z=0$, $y+z=1$.

$z$ varies from $0$ to $1-y$

$y$ " " $x^2$ to $1$

$x$ " " $-1$ to $1$





$$V = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\, dy\, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} z\Big]_{0}^{1-y} dy\, dx$$

$$= \int_{-1}^{1}\int_{x^2}^{1} (1-y)\, dy\, dx$$

$$= \int_{-1}^{1}\left( y\Big]_{x^2}^{1} - \frac{y^2}{2}\Big]_{x^2}^{1} \right) dx$$

$$= \int_{-1}^{1} 1 - x^2 - \left( \frac{1}{2} - \frac{x^4}{2} \right)$$

$$= \int_{-1}^{1} \frac{1}{2} - x^2 + \frac{x^4}{2}$$

$$= \frac{1}{2}x\Big]_{-1}^{1} - \frac{x^3}{3}\Big]_{-1}^{1} + \frac{x^5}{10}\Big]_{-1}^{1}$$

$$= \frac{1}{2}[2] - \left(\frac{1}{3} + \frac{1}{3}\right) + \left[\frac{1}{10} + \frac{1}{10}\right]$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

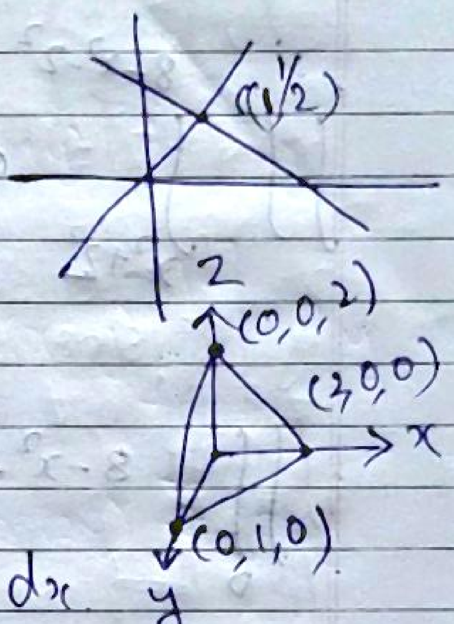
$$= \frac{8}{15}$$

iii) $x+2y+z=2$, $x=2y$, $x=0$ and $z=0$

z varies from 0 to $2-x-2y$

y " " $\frac{x}{2}$ to $1-\frac{x}{2}$

x varies from 0 to 1




$$\int_0^1 \int_{x/2}^{1-x/2} \int_0^{2-x-2y} dz\, dy\, dx$$

$$V = \int_0^1 \int_{x/2}^{1-x/2} (2-x-2y)\, dy\, dx$$

$$= \int_0^1 \left\{ 2y \Big]_{x/2}^{1-x/2} - x\, y \Big]_{x/2}^{1-x/2} - y^2 \Big]_{x/2}^{1-x/2} \right\}$$

$$= \int_0^1 2(1-x) - x(1-x) - \left(1 + \frac{x^4}{4} - x - \frac{x^4}{4}\right)$$

$$= \int_0^1 x^2 - 2x + 1$$

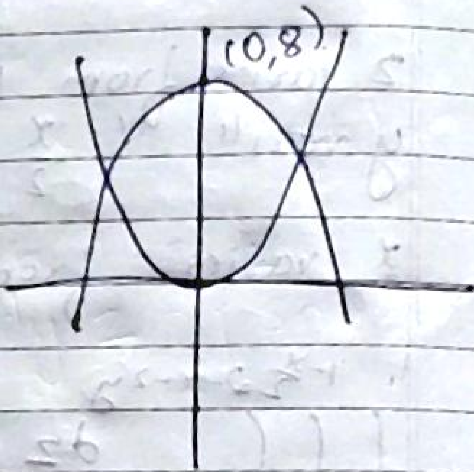
$$= \frac{x^3}{3}\Big]_0^1 - 2\frac{x^2}{2}\Big]_0^1 + x\Big]_0^1$$

$$= \frac{1}{3} - 1 + 1$$

$$= 1/3$$

ii) $y = x^2+z^2$ , $y = 8-x^2-z^2$

$$\iint \int_{x^2+z^2}^{8-x^2-z^2} dy\; dA$$




$$\iint_D 8-x^2-z^2-x^2-z^2 \; dA.$$

Using polar con'

$x^2+z^2 = 8-x^2-z^2$

$x^2+z^2=4$

$\therefore \boxed{r=2}$ put $dx\,dy = r\,dr\,d\theta$.

$$\int_0^{2\pi}\int_0^{2} \overset{8-2(x^2+z^2)}{(8-2r^2)}\, r\,dr\,d\theta$$

$$\cancel{8[2-0] - \frac{2}{3}r^3\Big]_0^2}$$

$$\iint_0^{2\pi}\left[\frac{8r^2}{2} - \frac{2r^4}{4}\right]_0^2 d\theta.$$

$$\int_0^{2\pi} (16-8)\,\theta\Big] = 8\theta\Big]_0^{2\pi} = 16\pi$$

4) i) $\iiint_B (x+y+z)\,dx.dy\,dz$

$\cancel{z = 4 - x^2 - y^2}$

$\cancel{\rho\cos\phi}$

$\cancel{\text{put } x = \rho\sin\theta\cos\theta\sin\phi}$

$\cancel{y = \rho\sin\theta\sin\phi}$

$\cancel{z = \rho\cos\phi}$

$\cancel{dx\,dy\,dz = \rho^2 \sin}$

$$\iint_B \int_0^{4-x^2-y^2} (x+y+z)\,dz\,dA.$$

$\boxed{r=2}$

$$\iint_D x(4-x^2-y^2) + y(4-x^2-y^2) + \frac{z^2}{2}\Big]_0^{4-x^2-y^2}$$

$$\iint 4x - x^3 - xy^2 + 4y - x^2y - y^3 + \frac{(4-x^2-y^2)^2}{2}$$

put $x = r\cos\theta$. $y = r\sin\theta$ $dx\,dy = r\,dr\,d\theta$.

$$\int_0^{\pi/2}\int_0^{2} 4r\cos\theta - r^3\cos^3\theta - r^3\cos\theta\sin^2\theta + 4r\sin\theta - r^3\cos^2\theta\sin\theta - r^3\sin^3\theta + \frac{(4-r)^2}{2}$$

$$\int_0^{\pi/2} \frac{64}{15}\cos\theta + \frac{64}{15}\sin\theta + \frac{16}{2}\,d\theta$$

$$= \frac{64}{15}\sin\theta - \frac{64}{15}\cos\theta + \frac{16}{3}\theta\Big]_0^{\pi/2}$$

$$= \frac{64}{15} + \frac{16\pi}{6} + \frac{64}{15} = \frac{128}{15} + 8\pi/3.$$

3 ii) $\iiint_B x e^{x^2+y^2+z^2} dV$

$x^2+y^2+z^2 \leq 1$ $\qquad \rho = 1$

$x = \rho \cos\theta \sin\phi$ $\qquad dx\,dy\,dz = \rho^2 \sin\phi\, \partial\rho\, d\phi\, d\theta$

$y = \rho \sin\theta \sin\phi$

$z = \rho \cos\phi$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1} \rho\cos\theta \sin\phi\, e^{\rho^2} \rho^2 \sin\phi\, \partial\rho\, d\theta\, d\phi$$

$\rho^3 = t$

$3\rho^2 \frac{d\rho}{dt} = 1$

$dt = \partial\rho$

$$\int_0^{\pi/2}\int_0^{\pi/2} t\cos\theta \sin^2\phi\, e^{1/t}\, \partial t\, d\theta\, d\phi$$

3) ii) $\iiint x^2+y^2$ , $x^2+y^2+z^2=4$ , $x^2+y^2+z^2=9$

$$\int_0^{\pi}\int_0^{2\pi}\int_2^{3} \rho^2\sin^2\phi \cdot \rho^2\sin\phi \, d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi}\int_0^{2\pi} \left[\frac{\rho^5}{5}\right]_2^3 \sin^3\phi \, d\rho\, d\theta\, d\phi$$

$$\frac{211}{5}\int_0^{\pi} \sin^3\phi \left[\theta\right]_0^{2\pi} d\phi$$

$$\frac{422\pi}{5}\int_0^{\pi} \sin^3\phi \, d\phi$$

$$\cancel{\int_0^{\pi}\left(\frac{1+\cos 2\theta}{2}\right)}$$

$$\frac{422\pi}{5}\int_0^{\pi} \sin\phi\,(1-\cos^2\phi)$$

$t=\cos\phi$ $\quad dt=-\sin\phi\, d\phi$

$t=1,-1$

$$\frac{422\pi}{5}\int_1^{-1} -(1-t^2)\, dt$$

$$\frac{422\pi}{5}\int_1^{-1} t^2-1$$

$$\frac{422\pi}{5}\left[\frac{t^3}{3} - t\right]_1^{-1} \Rightarrow \frac{422\pi}{5}\times\left(\frac{4}{3}\right)$$

$$=\frac{1688\pi}{15}$$

6) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ $\rho = kxyz$.

z varies from 0 to $c\left(1 - \frac{x}{a} - \frac{y}{b}\right)$

y varies from 0 to $b\left(1 - \frac{x}{a}\right)$

x varies from 0 to a.

$$K \int_0^a \int_0^{b(1-\frac{x}{a})} \int_0^{c(1-\frac{x}{a}-\frac{y}{b})} xyz \, dz\,dy\,dx$$

$$\frac{xy}{2} \left. z^2 \right]_0^{c\left(1-\frac{x}{a}-\frac{y}{b}\right)}$$

$$= \frac{xy}{2}\left(c^2\left(1 - \frac{x}{a} - \frac{y}{b}\right)^2\right) dy\,dx$$

$$= \frac{Kc^2}{2} \iint xy\left(1 - \frac{x}{a} - \frac{y}{b}\right)^2 dy\,dx$$

$$= \frac{Kc^2}{2} \iint xy\left(\left(1-\frac{x}{a}\right)^2 + \frac{y^2}{b^2} - 2\left(1-\frac{x}{a}\right)\left(\frac{y}{b}\right)\right)$$

$$= \frac{Kc^2}{2} \iint x\left(\left(1-\frac{x^2}{a}\right)y + \frac{y^3}{b^2} - 2\left(1-\frac{x}{a}\right)\left(\frac{y^2}{b}\right)\right]$$

$$= \frac{Kc^2}{2} \int \left[x\left(\left(1-\frac{x^2}{a}\right)\frac{y^2}{2} + \frac{y^4}{4b^2} - 2\left(1-\frac{x}{a}\right)\frac{y^3}{3b}\right]_0^{b\left(1-\frac{x}{a}\right)}$$

$$= \frac{kc^2}{2} \int x\left(1-\frac{x}{a}\right)^4 \left[\frac{b^2}{2} + \frac{b^2}{4} - \frac{2}{3} b^2\right) dx$$

$$= \frac{kc^2 b^2}{12 \times 2} \int_0^a x\left(1-\frac{x}{a}\right)^4 dx$$

put $1-\frac{x}{a} = t$ $\quad -\frac{dx}{a} = dt$ $\quad x=0 \;\; t=1$, $\; x=a \;\; t=0$

$$\frac{kc^2b^2}{24} \int_1^0 t^4 (a(1-t)) \cdot -a\,dt.$$

$$= -\frac{kc^2b^2a^2}{24} \int_1^0 t^4 - t^5$$

$$= -\frac{kc^2b^2a^2}{24} \left[ \left[\frac{t^5}{5}\right]_1^0 - \left[\frac{t^6}{6}\right]_1^0 \right]$$

$$= -\frac{k a^2 b^2 c^2}{24} \left[ -\frac{1}{5} + \frac{1}{6} \right]$$

$$= -\frac{k a^2 b^2 c^2}{24} \times \left(-\frac{1}{30}\right)$$

$$= \frac{k a^2 b^2 c^2}{720}$$

7) $\int_0^1\int_0^1\int_0^1 xyz\,dx\,dy\,dz$

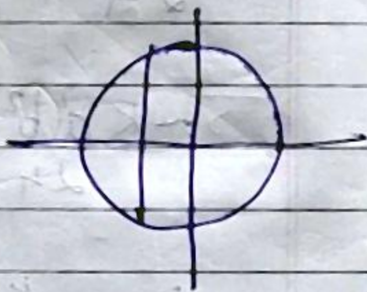
$m = \int_0^1\int_0^1 yz \left.\frac{x^2}{2}\right]_0^1 dy\,dz$

$m = \frac{1}{2}\int_0^1 z \left.\frac{y^2}{2}\right]_0^1 dz$

$= \frac{1}{4} \left.\frac{z^2}{2}\right]_0^1$

$= \frac{1}{8}$

Q $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{+\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2)\,dz\,dy\,dx$



$= \int_0^{2\pi}\int_0^2\int_r^2 r^3\,dz\,dr\,d\theta$

$= \int_0^{2\pi}\int_0^2 r^3 \left. z\right]_r^2 dr\,d\theta$

$= \int_0^{2\pi}\int_0^2 r^3(2-r)\,dr\,d\theta \Rightarrow \int_0^{2\pi} \left.\frac{2r^4}{4}\right]_0^2 - \left.\frac{r^5}{5}\right]_0^2$

$$= \left[\frac{8}{2}\theta\right]_0^{5\pi} = \frac{8}{2}\times 5\pi = \frac{16\pi}{2}$$