ENGINEERING MECHANICS

UNIT-II Chapter – 4

COPLANAR NON-CONCURRENT FORCE SYSTEM(EQUILIBRIUM)

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Chapter Content:

- Conditions of Equilibrium, Types of supports and reactions for 2D structures, statically determinate beam, loads acting on the beam.
- Determination of support reactions and Numerical problems on equilibrium of coplanar non-concurrent force system for unknown forces.

Conditions of equilibrium:

When several coplanar non-current forces acts on a body, the body may attain different states. They are as follows:

1. The body may move in any direction.
2. Body may move in any direction + rotate also.
3. Body may rotate about itself without motion.
4. Body may be at rest.

According to the 1st state, to achieve equilibrium the resultant R should be equal to zero. So the resultant will be zero when,

$$\sum F_x = 0, \sum F_y = 0$$

According to the 2nd state, to achieve equilibrium the resultant R should be equal to zero and moment should also be equal to zero.

$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

According to the 3rd state, to achieve equilibrium moment should be equal to zero.

$$\sum M = 0$$

According to the 4th state, the body is already in equilibrium because, the resultant R is equal to zero and moment also is equal to zero. Therefore the following conditions are satisfied.

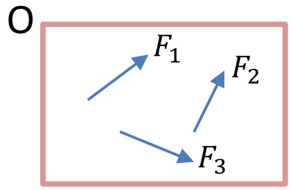
$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

To summarise all the 4 states we can say that, to attain the equilibrium of the body:

$$\boxed{\sum \boldsymbol{F_x} = \mathbf{0}, \sum \boldsymbol{F_y} = \mathbf{0} \textbf{ and } \sum \boldsymbol{M} = \mathbf{0}}$$

Law of moments:

If several coplanar forces acts on a system which is in equilibrium, then according to the law of moments " the sum of clockwise moments is equal to the sum of anticlockwise moments" or " the algebraic sum of moments of all the forces about any point on the plane of the forces should be zero."



Sum of clockwise moments = Sum of anticlockwise moments

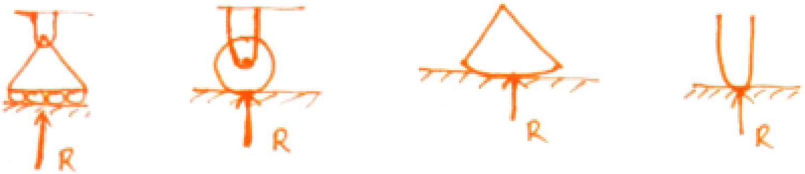
or

$$\sum M_o = F_1 X d_1 + F_2 X d_2 + F_3 X d_3 = 0$$

Types of supports and their reactions:

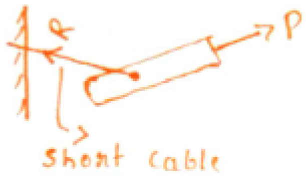
1. Reactions equivalent to a force with known line of action.

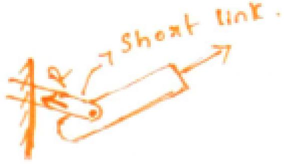
- Roller.
- Rocker.
- Smooth surface.



In all the above supports the movement of the body is restrained in only vertical direction therefore there will be only one reaction.

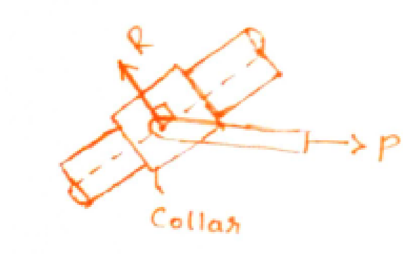
- Short cable.
- Short link.

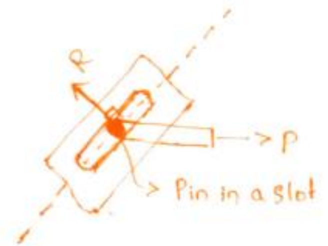




In all the above supports the movement of the body is restrained in only one direction therefore there will be only one reaction along the cable or link.

- Collar on frictionless a rod.
- Frictionless pin in a slot.

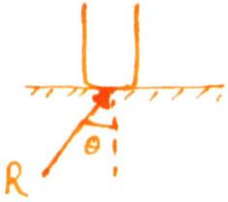


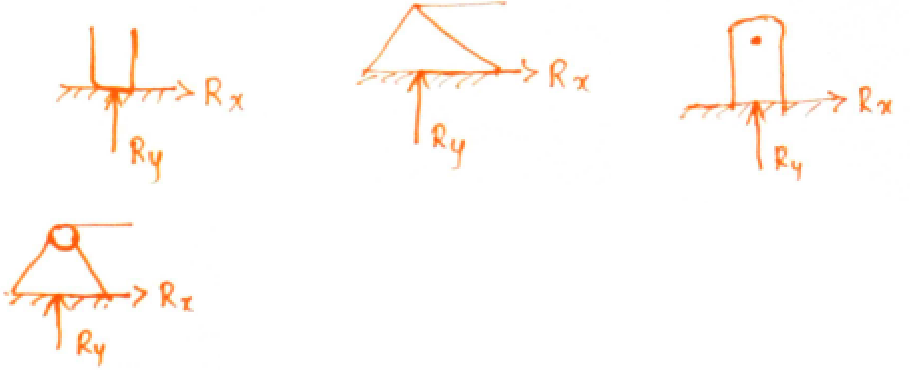


In all the above supports the movement of the body is restrained in only one direction therefore there will be only one reaction normal to the axis of the rod..

2. Reactions equivalent to a force of unknown direction.

- Rough surface.
- Hinges.
- Frictionless pins in a fitted hole.

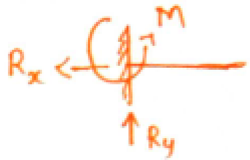


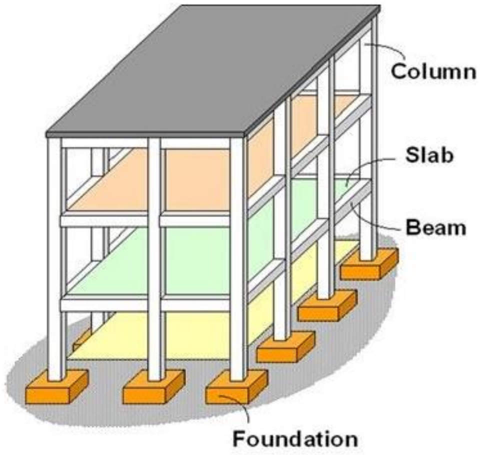


In all the above supports the movement of the body is restrained in two directions therefore there will be two reactions.

3. Reactions equivalent to a force of unknown direction and a couple.

- Fixed supports.





In the above supports the movement of the body is restrained in three directions therefore there will be three reactions.

Sign conventions:

All the upward reactions are positive and downward reactions are negative.

All the right-side going reactions are positive and left side going reactions are negative.

All the clockwise moments are positive and anticlockwise moments are negative.

Statically determinate beam:

It is the beam which can be solved for the unknown reactions completely using only three equilibrium equations.

$$\sum F_x = 0, \sum F_y = 0 \text{ and } \sum M = 0$$

Ex: simply supported beam.

Note: Statically indeterminate beams require additional equations along with three equilibrium equations to get all the unknown reactions.

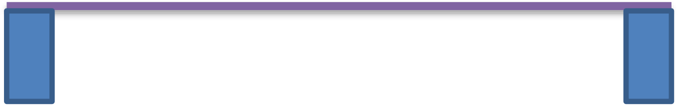
Types of beams:

Based on the support conditions we have following types of beams.

1. **Simply supported beam:**

The beam which has simple supports at either end is known as simply supported beam.

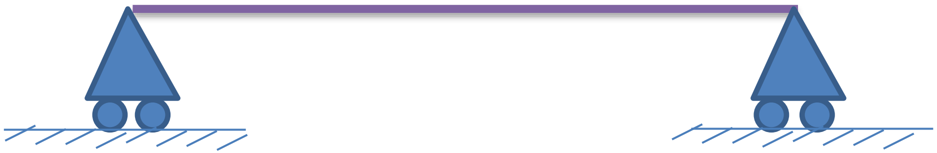
Ex:



2. Roller supported beam:

The beam which has roller supports at either end is known as roller supported beam.

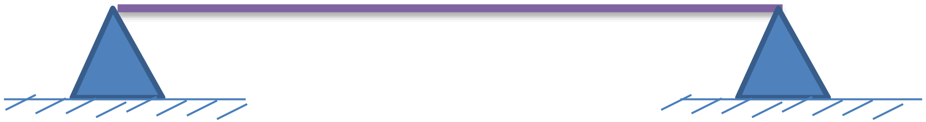
Ex:



3. Hinged/pinned supported beam:

The beam which has hinged or pinned supports at either end is known as hinged/pinned supported beam.

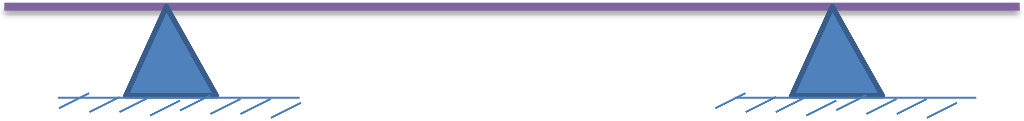
Ex:



4. Overhanging beam

The beam in which certain length is extended beyond the supports is called as overhanging beam.

Ex:

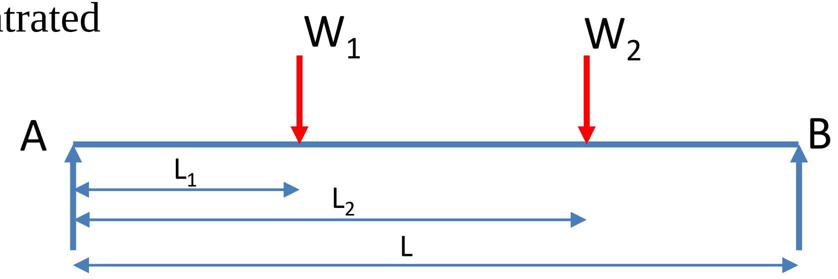


Types of loads:

1. Point load/concentrated load:

If the area occupied by the load is very less compared to the total area on which the load is applied is called as point load/concentrated

Ex:



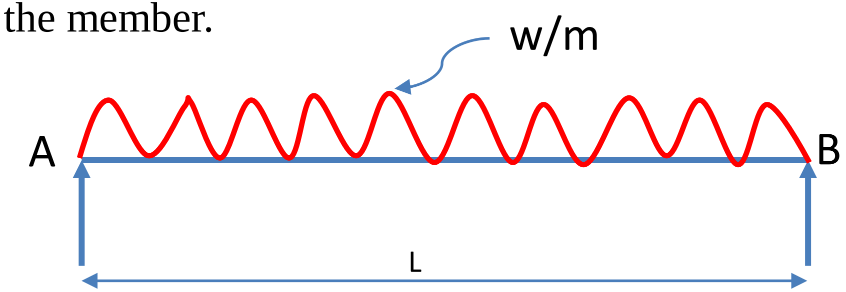
Here W_1 and W_2 are two point loads acting on the beam of length ‘L’. The unit of point load may be N or kN.

Sometimes point load can be inclined also. Finding moment of point loads about any point.

Considering A as reference point, $M_A = W_1XL_1 + W_2XL_2$

2. Uniformly distributed load (UDL):

The load which is spread over a member in such a way that, the intensity of this load remains constant over each unit length of the member.



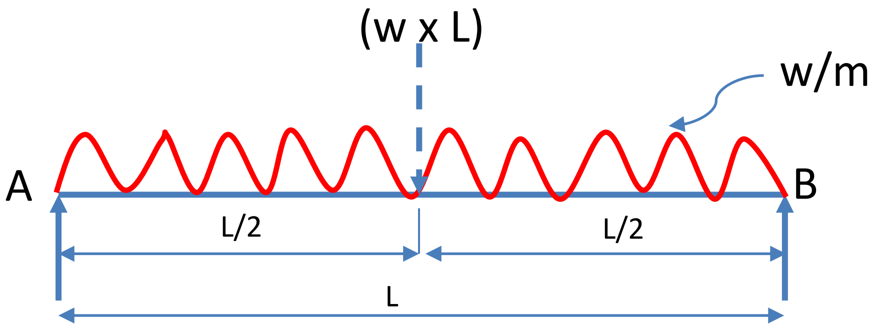
Consider the above figure in which a simply supported beam is loaded with UDL over its entire length ‘L’.

For UDL two things are very important:

1. Intensity of load = ‘w’ per unit length either in N/m of kN/m (depending on the unit of force)
2. Span of the load = l in ‘m’

For the above figure span of the UDL is equal to span of the beam

$$\text{i.e. } l = L$$



To find out moment of UDL about any point its very important to find its equivalent concentrated load (ECL).

Equivalent concentrated load of UDL can be found by multiplying intensity of load with **span of the load**.

Equivalent concentrated load (ECL) = w X L (here l = L)

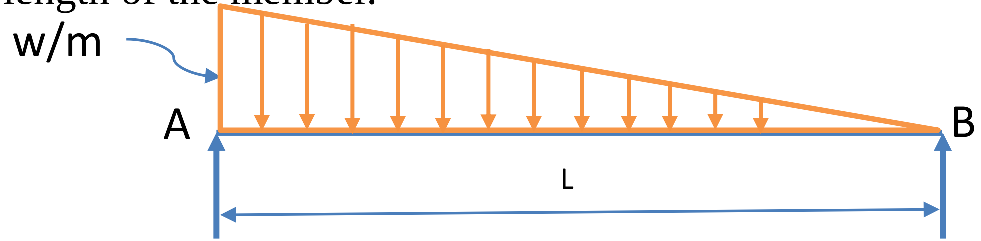
This ECL acts at the midpoint of span of the UDL (L/2).

To calculate moment of UDL about any point:

$M_A = (wXL)XL/2$ $\qquad\qquad$ $M_B = -(wXL)XL/2$

3. Uniformly varying load (UVL): Triangular variation UVL

The load which is spread over a member in such a way that, the intensity of this load varies uniformly over each unit length of the member.



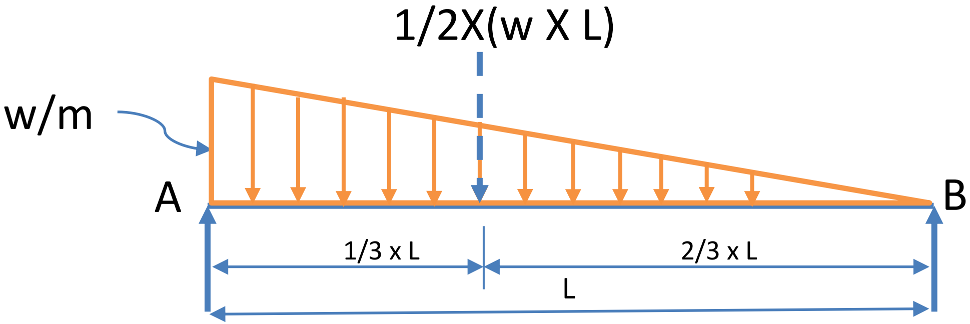
Consider the above figure in which a simply supported beam is loaded with UVL over its entire length ‘L’.

For UVL two things are very important:

1. Intensity of load = ‘w’ per unit length either in N/m of kN/m (depending on the unit of force)
2. Span of the load = l

For the above figure span of the UVL is equal to span of the beam

i.e. l = L



To find out moment of UVL about any point its very important to find its equivalent concentrated load(ECL).

ECL of UVL can be found by applying area of triangle, (1/2 X intensity of UVL X **span of the UVL**).

Equivalent concentrated load(ECL)= $\frac{1}{2} X w X L$ (here l = L)

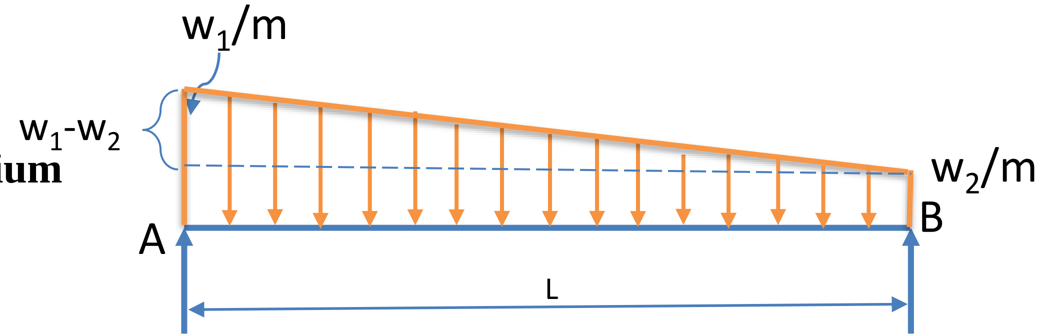
This equivalent point load acts at the centroid of the area of the triangle.

Here the area of UVL resembles the shape of the triangle. Hence its centroidal distances are $\frac{1}{3} xL \; and \; \frac{2}{3} xL$ from base and apex respectively.

To calculate moment of UVL about any point:

$M_A = \frac{1}{2}(wXL)X\frac{1}{3}XL$ $M_B = -\frac{1}{2}(wXL)X\frac{2}{3}XL$

UVL can also be in the shape of trapezium



Consider the above figure in which a simply supported beam is loaded with UVL over its entire length 'L'.

We can solve the above problem conveniently by splitting the UVL into triangular load of intensity $(w_1 - w_2)/m$ and UDL of intensity w_2/m.

To calculate moment of UVL about any point:

$$M_A = \frac{1}{2}(w_1 - w_2)XLX\frac{1}{3}XL + (w_2XL)X\frac{L}{2}$$

$$M_B = -\frac{1}{2}(w_1 - w_2)XLX\frac{2}{3}XL - (w_2XL)X\frac{L}{2}$$

Some examples on various loadings:

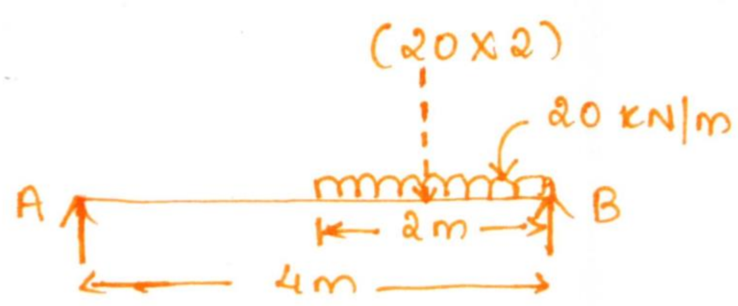


Fig. ‘a’

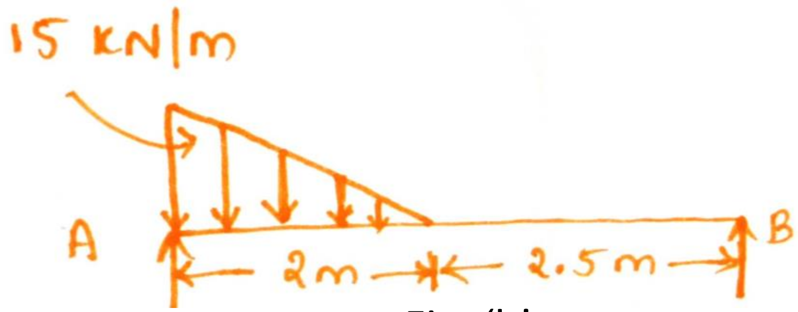


Fig. ‘b’

To find $\boldsymbol{M_A}$ *and* $\boldsymbol{M_B}$:

For the beam given in Fig. ‘a’ $M_A = (20X2)X\left(2+\frac{2}{2}\right)$ = 120 kNm

$M_B = -(20X2)X\left(\frac{2}{2}\right)$ = - 40 kNm

For the beam given in Fig. ‘b’ $M_A = \left(\frac{1}{2}X15X2\right)X\left(\frac{1}{3}X2\right)$ = 10 kNm

$M_B = -\left(\frac{1}{2}X15X2\right)X\left(\frac{2}{3}X2+2.5\right)$ = -57.5 kNm

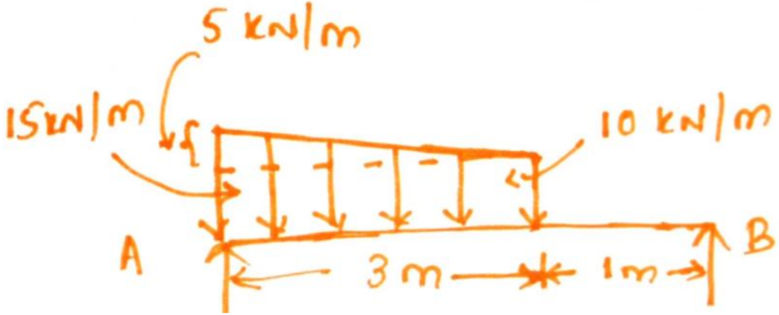


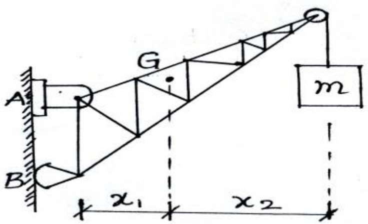
Fig. ‘c’

To find M_A and M_B:

For the beam given in Fig. ‘c’ $M_A = (10X3)X\left(\frac{3}{2}\right) + \left(\frac{1}{2}X5X3\right)X(\frac{1}{3}X3)$= 52.5 kNm

$M_B = -(10X3)X\left(\frac{3}{2}+1\right)$ -$\left(\frac{1}{2}X5X3\right)X(\frac{2}{3}X3+1)$= - 97.5 kNm

Problem 1:A fixed crane has a mass of 1000 kg and is used to lift a crate of mass m = 2400 kg. It is held in place by a pin at A and rocker at B. The CG of the crane is located at G. determine the components of the reactions at A & B. Take AB = 1.5 m, x_1 = 2 m, x_2 = 4 m.



Solution: weight of crane: 1000 X 9.81

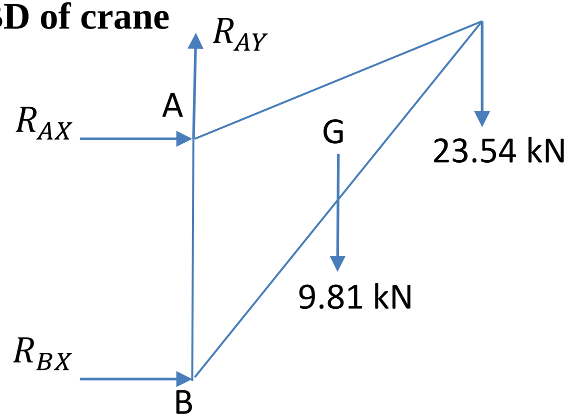
= 9.81 kN

Weight of crate = 2400 X 9.81

= 23.54 kN

Let the reactions at ‘A’ & ‘B’ are as shown in FBD.

FBD of crane



Using $\sum M = 0$ & taking moment about ‘A’,

$$\sum M_A = 23.54X6 + 9.81X2 - R_{BX}X1.5 = 0$$

$$R_{BX} = \frac{160.884}{1.5} = 107.26\,kN$$

Using $\sum F_x = 0$

Using $\sum F_x = 0$,

$R_{AX} + R_{BX} = 0$ or $R_{BX} = -R_{AX} = -107.26\ kN$

(-ve sign indicates that R_{AX} acts towards left)

Applying $\sum F_y = 0$,

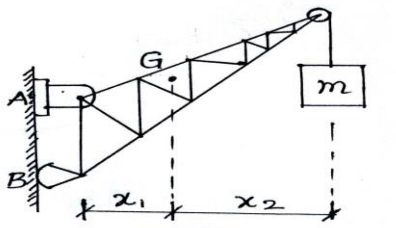
$R_{AY} - 9.81 - 23.54 = 0$ or $R_{AY} = 33.354\ kN$

Resultant reaction at A, $R_A = \sqrt{(107.26^2 + 33.354^2)}$

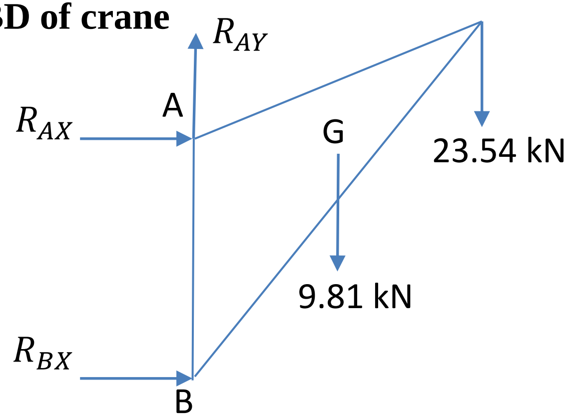
$R_A = 112.33\ kN$

Direction; $\tan\theta = \frac{33.354}{107.26} = 0.3109$

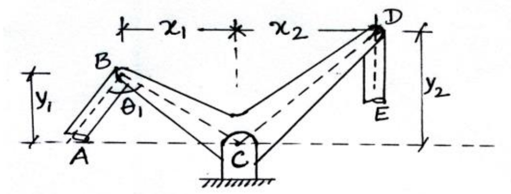
$\theta = 17.27^0$



FBD of crane



Problem 2:Two links AB & DE are connected by a bell crank as shown in figure. Knowing that the tension in the link AB is 180 N, determine (i) tension in the link DE and (ii) the reaction at C. $\theta 1 = 90°$, and $x_1 = 40$ cm, $x_2 = 60$ cm, $y_1 = 30$ cm, & $y_2 = 45$ cm



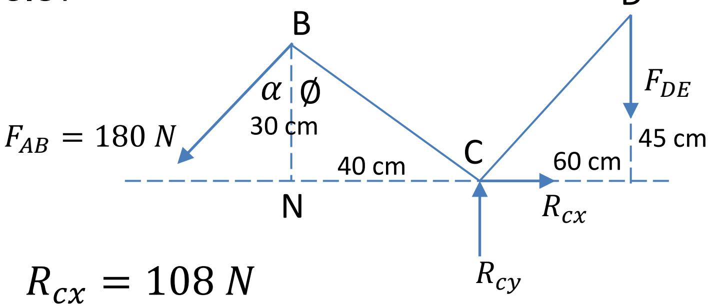
Solution: The forces and the reaction at different points are as shown in FBD.

Consider $\sum M_c = 0$

From triangle BCN, using Pythagoras theorem,

BC = 50 cm. $\cos\emptyset = \frac{30}{50}$ or $\emptyset = 53.13^0$ & $\alpha = 36.87^0$

FBD of crank



$$\sum M_c = -180 \text{ X } 50 + F_{DE} X60 = 0$$

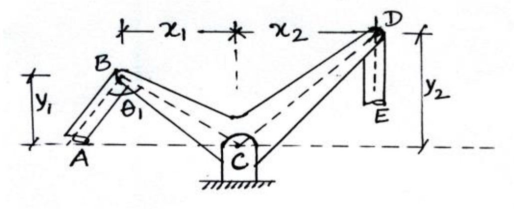
$$F_{DE} = \frac{180\ X50}{60} = 150\ N$$

Applying $\sum F_x = 0,\ R_{cx} - 180sin36.87^0 = 0$ $\quad R_{cx} = 108\ N$

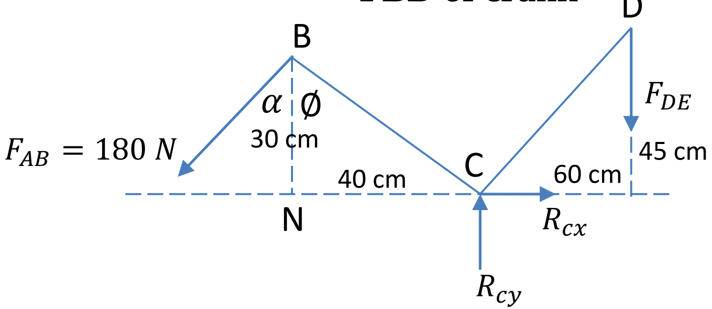
Applying $\sum F_y = 0,\ R_{cy} - 180cos36.87^0 - F_{DE} = 0$ $\quad R_{cy} = 294\ N$

$$R_A = \sqrt{(108^2 + 294^2)} = 313.21 \text{ N}$$

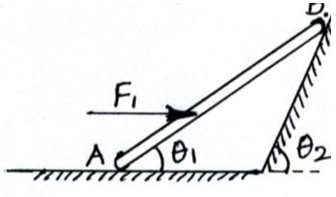
$$\text{Direction } \tan\theta = \frac{294}{108} \quad \theta = 69.82^0$$



FBD of crank



Problem 3:Find the distance ‘x’ measured along AB at which a horizontal force F1 = 60 N should be applied to hold the uniform bar AB in the position shown in figure. Bar AB is 3.0 m long and weighs 140 N. the incline and the floor are smooth. $\theta 1$ = 36.87°, θ2 = 56.3°.



Solution: Let the reactions and forces acting on the bar are as shown in FBD.

Applying $\sum F_x = 0$, 60-$R_B sin56.3^0 = 0$

$R_B = 72.12\ N$

Applying $\sum F_y = 0,\ \ R_A - 140 + R_B cos56.3^0 = 100\ N$

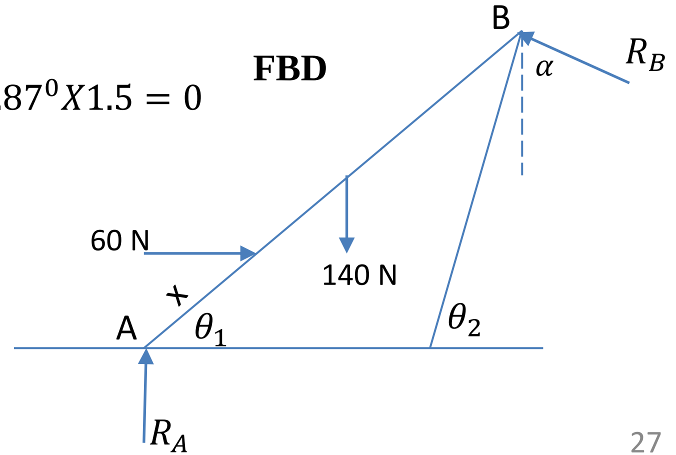
Applying $\sum M = 0$ & $taking\ moment\ about\ B$

$$\sum M_B = 100X3cos36.87^0 - 60X(3-x)sin36.87^0 - 140cos36.87^0 X1.5 = 0$$

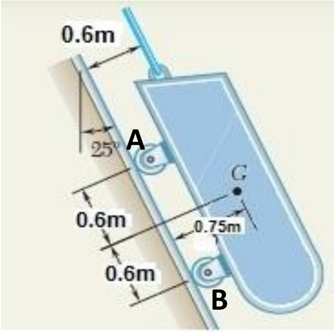
= 240-108+36x-168=0

x = 1 m.

FBD



Problem 4:A loading car is at rest on a track forming an angle of 25° with the vertical. The gross weight of the car and its load is 25 kN, and it is applied at a point 0.75 m from the track, halfway between the two axles. The car is held by a cable attached 0.6 m from the track. Determine the tension in the cable and the reaction at each pair of wheels.



Solution: : Let the tension in the rope, reactions and forces acting on the loading car are as shown in FBD

Applying $\sum F_x = 0$, -T+25cos25= 0

T = 22.657 kN.

Applying $\sum F_y = 0$, $R_A + R_B - 25sin25 = 0$

$R_A + R_B = 10.565\ kN$ -------(i)

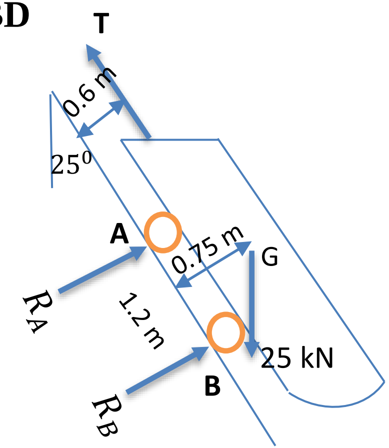
Applying $\sum M = 0\ \&\ taking\ moment\ about\ A$

$$\sum M_A = -22.657X0.6 - R_BX1.2 + 25cos25X0.75$$
$$+25sin25X0.6 = 0$$

$R_B = 8.115\ kN$, from (i)

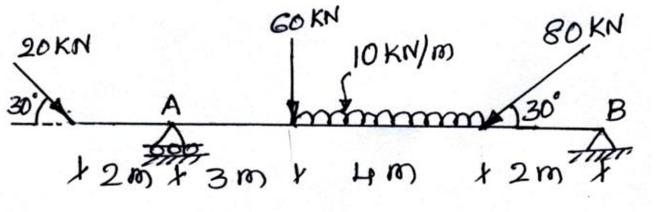
$R_A = 2.45\ kN$

FBD



Problem 5: Find the support reactions at A & B for the beam carrying loads as shown in the figure.

Solution: : Let the reaction at A & B are as shown in FBD.



Applying $\sum M_A = 0$,

60X3+10X4X5+80sin30X7-$R_{By}X$9-20sin30X2=0

R_{By}=71.11 kN

Applying $\sum F_y = 0$,

$R_{Ay} + R_{By} = 20sin30 + 60 + 10X4 + 80sin30$

R_{Ay}=78.9 kN

Applying $\sum F_x = 0$, 20cos30-80cos30-$R_{Bx} = 0$

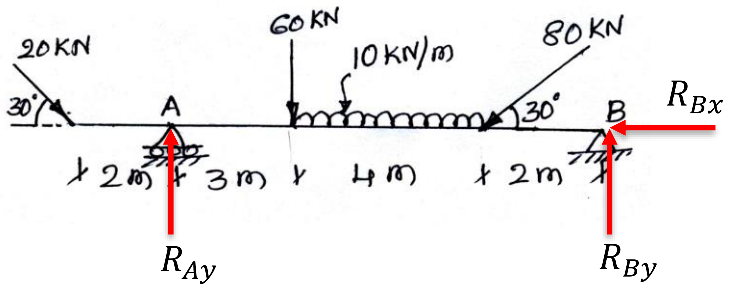
R_{Bx}=-51.96 kN (i.e. sense of R_{Bx} is to be changed)

Resultant reaction at B, $R_B = \sqrt{(71.11^2 + 51.96^2)}$ = 88.07 kN

Direction, tan$\theta = \frac{71.11}{51.96} = 1.368$

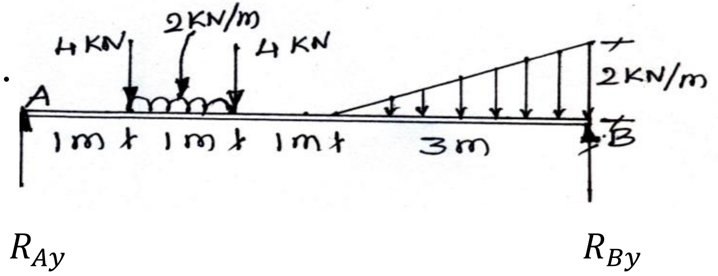
$\theta = 53.83^0$

FBD



Problem 6: A simply supported beam AB of 6 m span is subjected to loading as shown. Find the support reactions at A and B.

Solution: : Let the reaction at A & B are as shown in FBD.



Applying $\sum M_A = 0$,

$$-R_{BY}X6 + \left(\frac{1}{2}X2X3\right)X\left(3+\frac{2}{3}X3\right) + 4X2 + 4X1 +$$

$$(2X1)X(1+0.5) = 0$$

$$R_{By} = 5\ kN$$

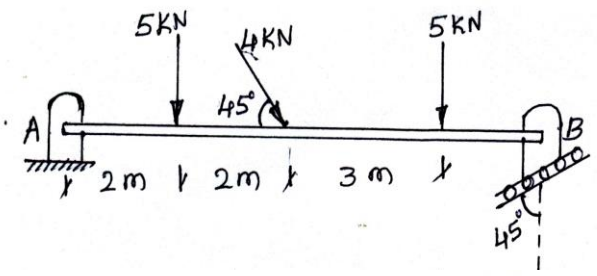
Applying $\sum F_y = 0$,

$$R_{Ay} + R_{By} - 4 - (2X1) - 4 - \left(\frac{1}{2}X2X3\right) = 0$$

R_{Ay} = 8 kN

Problem 7: A beam AB 8.5 m long is hinged at A and supported on rollers over a smooth surface inclined at 45^0 to the vertical at B. the beam is loaded as shown. Determine the reactions at A and B.

Solution: : Let the reaction at A & B are as shown in FBD.



Applying $\sum M_A = 0$,

$-R_B cos45X8.5 + 5X7 + 4sin45X4 + 5X2 = 0$

$R_B = 9.369\ kN$

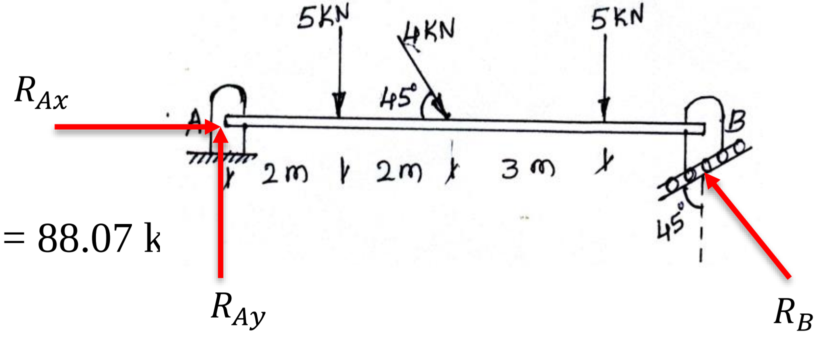
Applying $\sum F_x = 0$, $R_{Ax} + 4cos45 - 9.369sin45 = 0$

R_{Ax}=3.796 kN

Applying $\sum F_y = 0$,

$R_{Ay} + R_B cos45 - 5 - 4sin45 - 5 = 0$

R_{Ay} = 6.203 kN

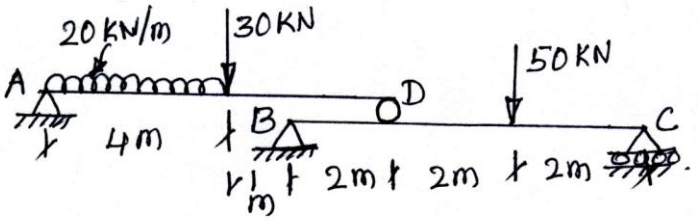


Resultant reaction at A, $R_A = \sqrt{(3.796^2 + 6.203^2)}$ = 88.07 k

Direction, $\tan\theta = \frac{6.203}{3.796} = 1.634$

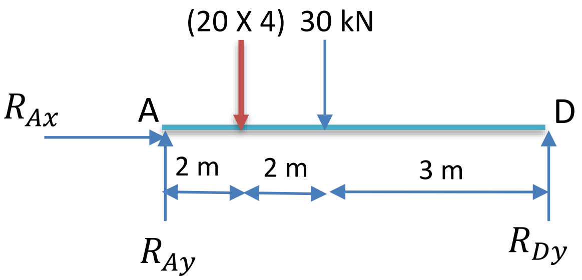
$\theta = 53.53^0$

Problem 8: Determine the support reactions at A, B & C for the arrangement of forces as shown in figure .



Solution: The above problem can be solved conveniently by considering span AD and BC separately

FBD of beam AD is as shown



Applying $\sum F_x = 0, R_{Ax} = 0$

Applying $\sum F_y = 0,$

$R_{Ay} + R_{Dy} - (20X4) - 30 = 0$

$R_{Ay} + R_{Dy} = 120$ -------(i)

Applying $\sum M_A = 0$,

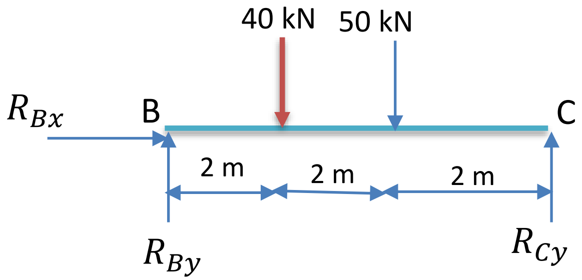
$-R_{Dy}X7 + 80X2 + 30X4 = 0$

$R_{Dy} = 40\ kN$

From (i) $R_{Ay} = 80\ kN$

The reaction R_{Dy} will act as a point load on span BC

FBD of beam BC is as shown



Applying $\sum F_x = 0, R_{Bx} = 0$

Applying $\sum F_y = 0,$

$R_{By} + R_{Cy} - 40 - 50 = 0$

$R_{By} + R_{Cy} = 90$ -------(ii)

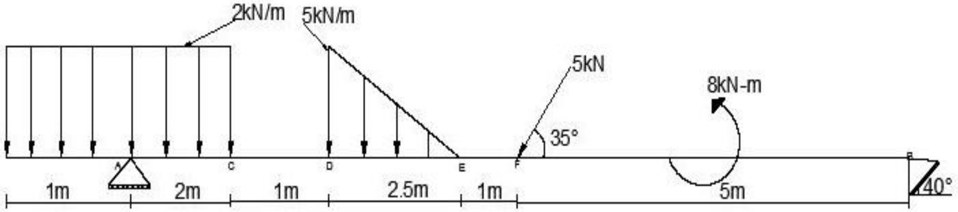
Applying $\sum M_B = 0\,,$

$-R_{Cy}X6 + 40X2 + 50X4 = 0$

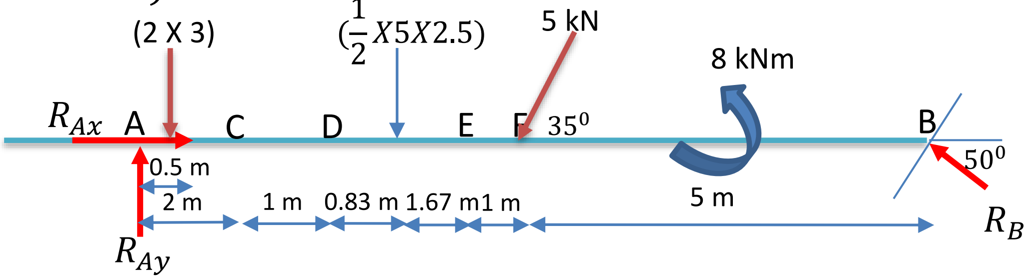
$R_{Cy} = 46.66\ kN$

From (ii) $R_{By} = 43.33\ kN$

Problem 9: Find the reaction at the supports A and B of the beam carrying the loads as shown in figure.



Solution: The FBD diagram of the beam is as shown.



Applying $\sum M_A = 0$,

$-R_B sin50X11.5 - 8 + 5sin35X6.5 + (0.5X5X2.5)X3.83$

+6X0.5= 0

$R_B = 4.265\ kN$

Applying $\sum F_x = 0$,

$R_{Ax} - 5cos35 - 4.265cos50 = 0$

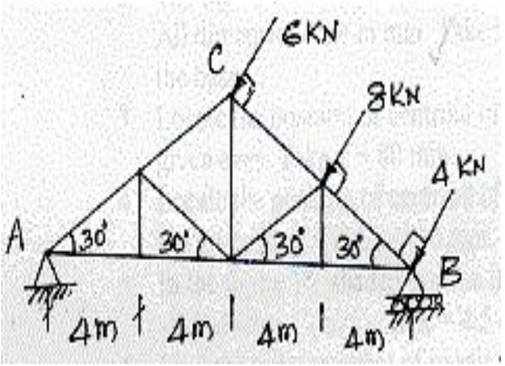
R_{Ax}=6.83 kN

Applying $\sum F_y = 0$,

$R_{Ay} + 4.265sin50 - 6 - (0.5X5X2.5) - 5 = 0$

$R_{Ay} = 13.982\ kN$

Problem 10: Determine the reactions at the supports of the truss shown in fig below.



Solution: The FBD diagram of the truss is as shown

Applying $\sum M_A = 0$,

$-R_B X16 + 4sin60X16 + 8sin60X12 + 6sin60X8 - 8cos60X2.309$

-6cos60X4.61= 0

$R_B = 9.816\ kN$

Applying $\sum F_x = 0,$

$R_{Ax} - 4cos60 - 8cos60 - 6cos60 = 0$

$R_{Ax} = 9\ kN$

Applying $\sum F_y = 0,$

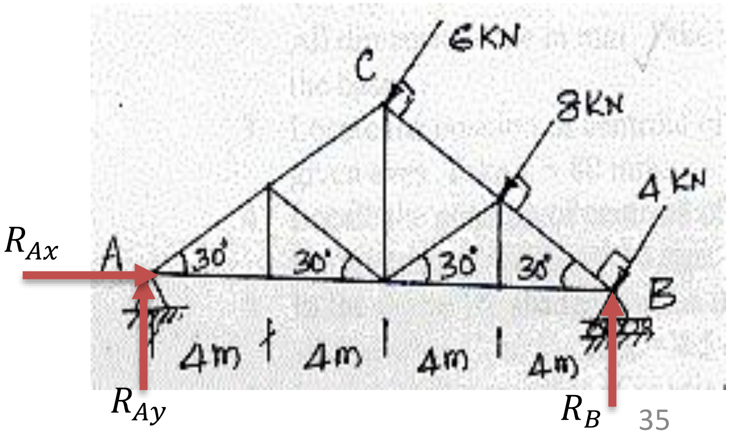
$R_{Ay} - 4sin60 - 8sin60 - 6sin60 + 9.816 = 0$

$R_{Ay} = 5.77\ kN$

Resultant reaction at A, $R_A = \sqrt{(9^2 + 5.77^2)}$ = 10.69 kN

Direction, $\tan\theta = \frac{5.77}{9} = 0.6411$

$\theta = 32.66^0$



Thank You