Electrostatics

Syllabus: Review on vectors: Coordinate Systems, Vector and Scalar Quantities, Properties of Vectors, Components of a Vector and Unit Vectors, vector operations: gradient, divergence and curl. vector integrals, Stokes and Green's theorem Electric Fields: Properties of Electric Charges, Charging Objects by Induction, Coulomb's Law, Analysis Model: Particle in a Field (Electric), Electric Field of a Continuous Charge Distribution, Electric Field Lines Motion of a Charged Particle in a Uniform Electric Field Gauss's Law: Electric Flux, Gauss's Law, Application of Gauss's Law to Various Charge Distributions, Conductors in Electrostatic Equilibrium Electric Potential: Electric Potential and Potential Difference, Potential Difference in a Uniform Electric Field, Electric Potential and Potential Energy Due to Point Charges, Obtaining the Value of the Electric Field from the Electric Potential, Electric Potential Due to Continuous Charge Distributions Electric Potential Due to a Charged Conductor, Applications of Electrostatics Capacitance and Dielectrics: Definition of Capacitance, Calculating Capacitance, Combinations of Capacitors, Energy Stored in a Charged Capacitor, Capacitors with Dielectrics, Electric Dipole in an Electric Field, An Atomic Description of Dielectrics

Vectors

A physical quantity that is fully described by a single number (with units) is called a **scalar quantity.** Mass, temperature, volume pressure, density, energy, charge and voltage are all scalars.

Our universe has three dimensions, so some quantities also need a direction for a full description. If you ask someone for directions to the post office, the reply "Go three blocks" will not be very helpful. A full description might be, "Go three blocks south." A quantity having both a magnitude and a direction is called a vector quantity. Vector is a quantity having both magnitude and direction.

Vector Operations:

Geometric representation of a vector:

Figure 1 shows that the *geometric representation* of a vector is an arrow, with the tail of the arrow (not its tip!) placed at the point where the measurement is made. The vector then seems to radiate outward from the point to which it is attached. An arrow makes a natural representation of a vector because it inherently has both a length and a direction. Here we label vectors by drawing a small arrow over the letter that represents the vector: $\vec{r}$ for position, $\vec{v}$ for velocity, $\vec{a}$ for acceleration, and so on. Magnitude of a vector cannot be a negative number; it must be positive or zero, with appropriate units.

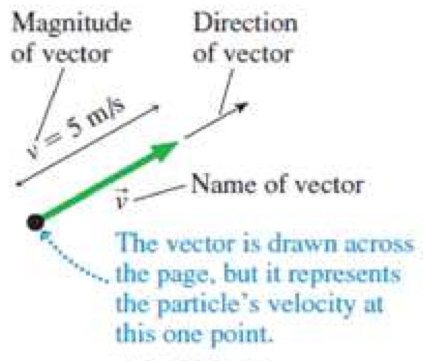


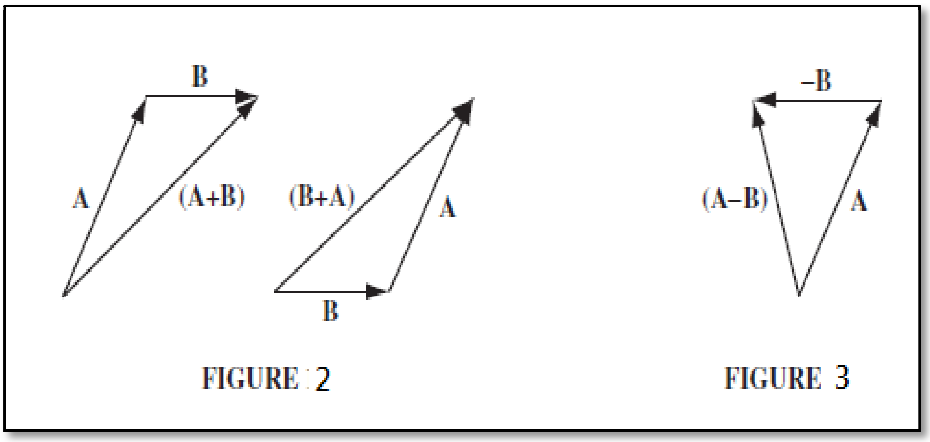
FIGURE 1

We define four vector operations: addition and three kinds of multiplication.

1. Addition of two vectors.
2. Multiplication by a scalar
3. Dot product of two vectors
4. Cross product of two vectors

1. Addition of two vectors.

a) Tip-to-tail method:



Place the tail of **B** at the head of **A**; the sum, **A** + **B**, is the vector from the tail of **A** to the head of **B** (Fig. 2). (This rule generalizes the obvious procedure for combining two displacements.) Addition is *commutative:*

A+B = B+C

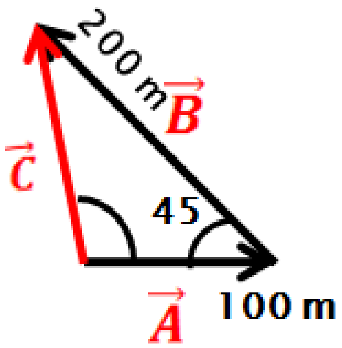
3 miles east followed by 4 miles north gets you to the same place as 4 miles north followed by 3 miles east. Addition is also *associative:*

(A + B) + C = A + (B + C)

To subtract a vector, add its opposite (Fig. 3)

A – B = A + (-B)

Ex: A bird flies 100 m east and then 200 m NW at 45 degree, what is the net displacement and direction?



Net displacement, $\vec{C} = \vec{A} + \vec{B}$

Using the law of cosines,

$C^2 = A^2 + B^2 - 2AB\cos\theta$

$= 100^2 + 200^2 - 2*100*200\cos 45^0$

$C = 147$ m

To find ϕ $B^2 = A^2 + C^2 - 2AC\cos\phi$

$\phi = \cos^{-1}\left(\frac{A^2+C^2-B^2}{2AC}\right)$

$\phi = 106^0$ with respect to x-axis

b) Parallelogram law of vector addition:

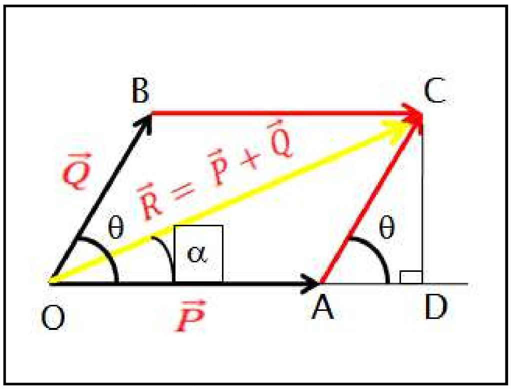
The magnitude and direction of two vectors can be determined using parallelogram law of vector addition

From right angle triangle ODC

$$R = CO = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

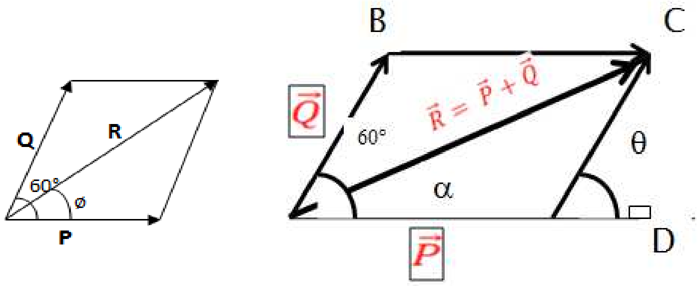
Direction of the resultant, OC

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{(OA + AD)} = \frac{Q\sin\theta}{(P + Q\cos\theta)}$$



Ex: Two forces of magnitude 6N and 10N are inclined at an angle of 60° with each other. Calculate the magnitude of resultant and the angle made by resultant with 6N force.

So, P = 6N, Q = 10N and θ = 60°



We have,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$
$$or, R = \sqrt{6^2 + 10^2 + 2.6.10\cos 60^\circ}$$
$$\therefore R = \sqrt{196} = 14N$$

which is the required magnitude

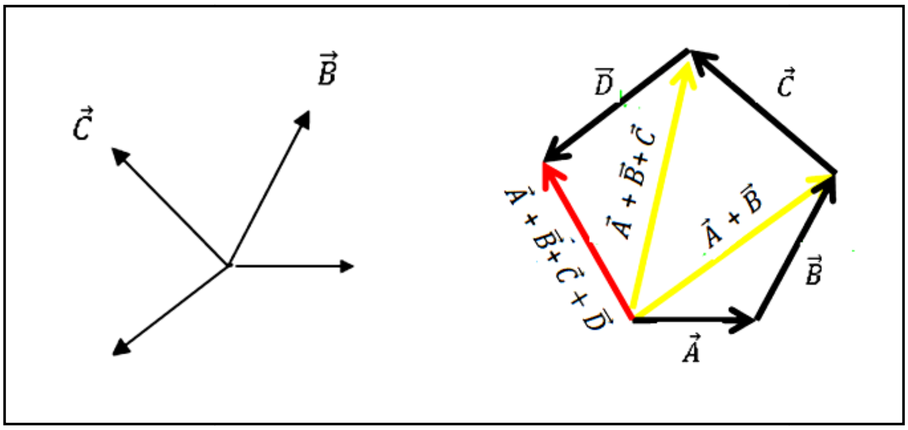
Let ø be the angle between **P** and **R**. Then,

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta}$$
$$or, \tan\phi = \frac{10\sin 60^\circ}{6 + 10\cos 60^\circ}$$
$$or, \tan\phi = \frac{5\sqrt{3}}{11}$$
$$\therefore \phi = \tan^{-1}(\frac{5\sqrt{3}}{11})$$

which is the required angle.

c) **Polygon law of vector addition:**

If two or more vectors are represented by adjacent sides of a polygon taken in same order in both direction and magnitude, then the resultant is given by closing the side of polygon in opposite direction.



2. Multiplication by a scalar

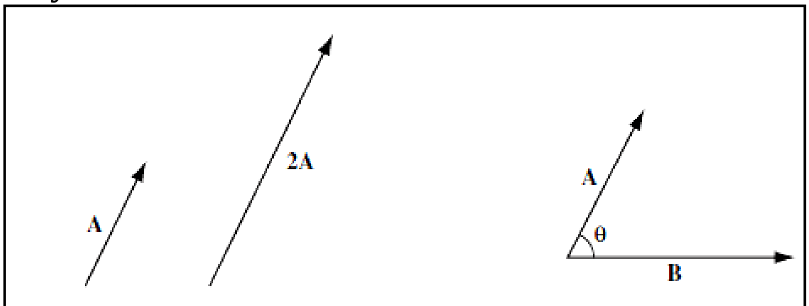


FIGURE 4 FIGURE 5

Multiplication of a vector by a positive scalar a multiplies the *magnitude* but leaves the direction unchanged (Fig. 4). (If a is negative, the direction is reversed.) Scalar multiplication is *distributive*

$$a\,(\mathbf{A}+\mathbf{B}) = a\mathbf{A} + a\mathbf{B}$$

3. Dot product of two vectors

The dot product of two vectors is defined by

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$$

where θ is the angle they form when placed tail-to-tail (Fig. 5). Note that $\mathbf{A}\cdot\mathbf{B}$ is itself a *scalar* (hence the alternative name **scalar product**). The dot product is *commutative*,

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$$

and *distributive*

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

Geometrically, $\mathbf{A}\cdot\mathbf{B}$ is the product of A times the projection of $\mathbf{B}$ along $\mathbf{A}$ (or the product of B times the projection of $\mathbf{A}$ along $\mathbf{B}$). If the two vectors are parallel, then $\mathbf{A}\cdot\mathbf{B} = AB$. In particular, for any vector $\mathbf{A}$,

$$\mathbf{A}\cdot\mathbf{A} = A^2$$

If $\mathbf{A}$ and $\mathbf{B}$ are perpendicular, then $\mathbf{A}\cdot\mathbf{B} = 0$.

4. Cross product of two vectors

The cross product of two vectors is define by

$$\mathbf{A} \times \mathbf{B} = \mathbf{AB} \sin \theta \, \hat{\boldsymbol{n}}$$

where $\hat{\boldsymbol{n}}$ is a **unit vector** (vector of magnitude 1) pointing perpendicular to the plane of **A** and **B**. (hat (ˆ) is used to denote unit vectors.) Of course, there are *two* directions perpendicular to any plane: "in" and "out." The ambiguity is resolved by the **right-hand rule**: let your fingers point in the direction of the first vector and curl around (via the smaller angle) toward the second; then your thumb indicates the direction of $\hat{\boldsymbol{n}}$. (In Fig. 6, **A** × **B** points *into* the page; **B** × **A** points *out* of the page.) Note that **A** × **B** is itself a *vector* (hence the alternative name **vector product**). The cross product is *distributive,*

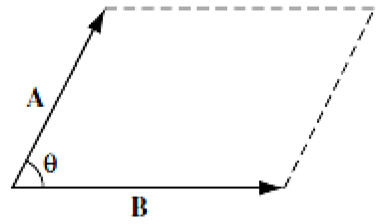


FIGURE 6

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C})$$

but *not commutative.* In fact,

$$(\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})$$

Geometrically, |**A** × **B**| is the area of the parallelogram generated by **A** and **B** (Fig. 6). If two vectors are parallel, their cross product is zero. In particular,

$$\mathbf{A} \times \mathbf{A} = \mathbf{0}$$

for any vector **A**. (Here **0** is the **zero vector**, with magnitude 0.)

Vector Algebra: Component Form

In practice, A vector can be identified by specifying its three Cartesian components: Let $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$ be unit vectors parallel to the *x*, *y*, and *z* axes, respectively (Fig. 7(a)). An arbitrary vector **A** can be expanded in terms of these **basis vectors** (Fig. 7(b)):

$$\vec{\boldsymbol{A}} = \boldsymbol{A_X}\hat{\boldsymbol{X}} + \boldsymbol{A_Y}\hat{\boldsymbol{Y}} + \boldsymbol{A_Z}\hat{\boldsymbol{Z}}$$

The numbers A_X, A_Y, and A_Z, are the "components" of **A**; geometrically, they are the projections of **A** along the three coordinate axes (A_X = **A** · $\hat{\boldsymbol{x}}$, A_Y = **A** · $\hat{\boldsymbol{Y}}$, A_Z = **A** · $\hat{\boldsymbol{Z}}$).

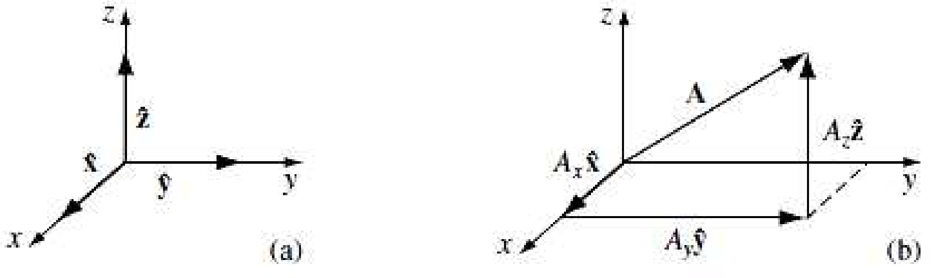


FIGURE7

1. **Addition of two vectors:** $\vec{A} + \vec{B} = (A_X\hat{X} + A_Y\hat{Y} + A_Z\hat{Z}) + (B_X\hat{X} + B_Y\hat{Y} + B_Z\widehat{Z})$
$= (A_X + B_X)\,\hat{X} + (A_Y + B_Y)\hat{Y} + (A_Z + B_Z)\hat{Z}$

2. **Multiplication of two vectors by a scalar (dot product):**

To multiply a vector by a scalar, multiply each Component. ($\hat{\boldsymbol{X}}$, $\hat{\boldsymbol{Y}}$ and $\hat{\boldsymbol{Z}}$ are mutually perpendicular unit vectors.)

$\hat{\boldsymbol{X}} \cdot \hat{\boldsymbol{X}} = \hat{\boldsymbol{Y}} \cdot \hat{\boldsymbol{Y}} = \hat{\boldsymbol{Z}} \cdot \hat{\boldsymbol{Z}} = \mathbf{1}$; $\qquad \hat{\boldsymbol{X}} \cdot \hat{\boldsymbol{Y}} = \hat{\boldsymbol{X}} \cdot \hat{\boldsymbol{Z}} = \hat{\boldsymbol{Y}} \cdot \hat{\boldsymbol{Z}} = \mathbf{0}$

($\hat{\boldsymbol{X}}\cdot\hat{\boldsymbol{X}}$ - Parallel components, cos 0=1; $\hat{\boldsymbol{X}}\cdot\hat{\boldsymbol{Y}}$ -perpendicular components, cos 90=0)
To calculate the dot product, multiply like components, and add
Ex: $\vec{A}\cdot\vec{A} = A_x^2 + A_y^2 + A_z^2$
$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
(The dot product of $\vec{A}$ with any unit vector is the component of $\vec{A}$ along that direction.
$\vec{A}\cdot\hat{x} = A_x$; $\vec{A}\cdot\hat{y} = A_y$; $\vec{A}\cdot\hat{z} = A_z$)

3. Multiplication of two vectors by a vector (cross product) :

$\hat{\mathbf{x}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}\times\hat{\mathbf{z}} = 0$; (sin0=0; ref $\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}} = |\vec{\boldsymbol{A}}|\,|\vec{\boldsymbol{B}}|\sin\theta\,\hat{\boldsymbol{n}} = AB\sin\theta\,\hat{\boldsymbol{n}}$)

$\hat{\mathbf{x}}\times\hat{\mathbf{y}} = -\hat{\mathbf{y}}\times\hat{\mathbf{x}} = \hat{\mathbf{z}}$

$\hat{\mathbf{y}}\times\hat{\mathbf{z}} = -\hat{\mathbf{z}}\times\hat{\mathbf{y}} = \hat{\mathbf{x}}$

$\hat{\mathbf{z}}\times\hat{\mathbf{x}} = -\hat{\mathbf{x}}\times\hat{\mathbf{z}} = \hat{\mathbf{y}}$

Therefore,

$$\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}} = (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}})\times(B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}})$$

$$= (A_yB_z - A_zB_y)\,\hat{\mathbf{x}} + (A_zB_x - A_xB_z)\,\hat{\mathbf{y}} + (A_xB_y - A_yB_x)\,\hat{\mathbf{z}}$$

This expression can be written as a determinant form: $\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

(To calculate the cross product, form the determinant whose first row is $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ whose second row is $\vec{\boldsymbol{A}}$ (in component form), and whose third row is $\vec{\boldsymbol{B}}$.

Position, Displacement, and Separation Vectors:

The location of a point in three dimensions can be described by listing its Cartesian coordinates *(x, y, z)*. The vector to that point from the origin (*O*) is called the **position vector** (Fig. 8):

r ≅ x $\hat{x}$ + y $\hat{y}$ + z $\hat{z}$ and magnitude $r = \sqrt{x^2 + y^2 + z^2}$its distance from the origin

and $\hat{\boldsymbol{r}} = \frac{\vec{\boldsymbol{r}}}{|\vec{r}|} = \frac{\mathbf{x}\,\hat{\boldsymbol{x}} + \mathbf{y}\,\hat{\boldsymbol{y}} + \mathbf{z}\,\hat{\boldsymbol{z}}}{\sqrt{x^2+y^2+z^2}}$

is a unit vector pointing radially outward. The infinitesimal displacement vector, from (x, y, z) to (x+dx, y+dy, z+dz), is

$$\mathbf{dl} = \mathbf{dx}\,\hat{\boldsymbol{x}} + \mathbf{dy}\,\hat{\boldsymbol{y}} + \mathbf{dz}\,\hat{\boldsymbol{z}}$$

(We could call this *d*r, since that's what it *is*, but it is useful to have a special notation for infinitesimal displacements.)

Source point $\mathbf{r}'$, where a electric charge is located, and a field point $\boldsymbol{r}$ at which one has to calculate the electric filed or magnetic field. The separation vector from the source point $\boldsymbol{r}'$, and field point $\boldsymbol{r}$ is given by (Fig. 9).

$$\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$$

Its magnitude, $\boldsymbol{R} = |\boldsymbol{r} - \boldsymbol{r}'|$

And a unit vector in the direction from r′ to r is

$$\widehat{\boldsymbol{R}} = \frac{\vec{\boldsymbol{R}}}{|\vec{\boldsymbol{R}}|} = \frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

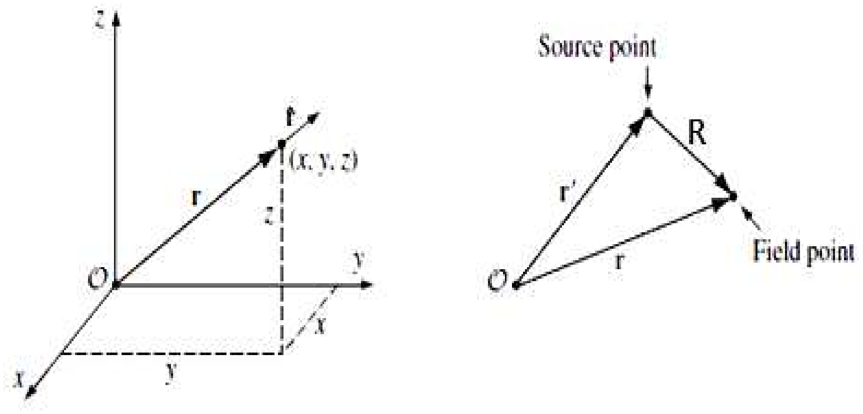


FIGURE 8 FIGURE 9

Differential vector calculus:

Suppose we have a function of one variable: *f (x)*, then $\frac{df}{dx}$ provides us with information on how quickly a function of one variable, *f*(*x*), changes. For instance, when the argument changes by an infinitesimal amount, from *x* to *x*+*dx*, *f* changes by df, given by

$$df = \frac{df}{dx}\,dx$$

In three dimensions, the function *f* will in general be a function of *x*, *y*, and *z*: *f*(*x*, *y*, *z*). The change in *f* is equal to

$$df = \frac{\partial f}{\partial x}dx + \left(\frac{\partial f}{\partial y}\right)dy + \left(\frac{\partial f}{\partial z}\right)dz$$

$$= \left(\left(\frac{\partial f}{\partial x}\right)\hat{\mathbf{x}} + \left(\frac{\partial f}{\partial y}\right)\hat{\mathbf{y}} + \left(\frac{\partial f}{\partial z}\right)\hat{\mathbf{z}}\right).\,(\mathrm{d}x\hat{\mathbf{x}} + \mathrm{d}y\hat{\mathbf{y}} + \mathrm{d}z\hat{\mathbf{z}})$$

$$= (\vec{\nabla} f).\,(d\vec{\mathbf{l}})$$

The vector derivative operator $\vec{\nabla}$ ("del") is given by

$$\vec{\nabla} = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

$\vec{\nabla}$ is a vector, it produces a vector when it operates on scalar function *f*(*x*, *y*, *z*).
There are three kinds of vector derivatives, corresponding to the three kinds of multiplication possible with vectors.

1. **Gradient**, on a scalar function *f* the analogue of multiplication by a scalar. $\vec{\nabla} f$ (Result is vector)
2. **Divergence**, on a vector function, like the scalar (dot) product. $\vec{\nabla}.\boldsymbol{v}$ (Result is scalar)
3. **Curl**, on a vector function **v** which corresponds to the vector (cross) product. $\vec{\nabla}\times\boldsymbol{v}$ (Result is vector)

1. Gradient :

The result of applying the vector derivative operator on a scalar function *f* is called the gradient of *f*:

$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x}\hat{\mathbf{x}}\right) + \left(\frac{\partial f}{\partial y}\right)\hat{\mathbf{y}} + \left(\frac{\partial f}{\partial z}\right)\hat{\mathbf{z}}$$

$\vec{\nabla} f$ is a vector quantity, with three components.

Geometrical interpretation (physical) of gradient:

Like vector, gradient has magnitude and direction. Rewriting the equation in terms of dot product $df = (\vec{\nabla} f).(d\vec{\mathbf{l}})$.

$$df = \vec{\nabla} f . d\vec{\mathbf{l}} = |\vec{\nabla} f||d\vec{l}| \cos\theta$$

Where θ is the angle between $\vec{\nabla} f$ and $d\vec{l}$.

If we fix the magnitude $|d\vec{l}|$ and search around in various direction (that is, vary θ), the maximum change in *f* evidentially occurs when θ=0 (cosθ=1).That is, for a fixed distance $|d\vec{l}|$, $\vec{\nabla} f$ is greatest if it moves in the same direction as $\vec{\nabla} f$.

The gradient $\vec{\nabla} f$ points in the direction of maximum increase of the function f .The magnitude $|\vec{\nabla} f|$ gives the slope (rate of increase) along this maximal direction.

Ex.1.

Find the gradient of $v(x,y) = (x^2 - y)\hat{x} + (x + y^2)\hat{y}$

Figure 1.8 indicates gradient of $v(x,y) = (x^2 - y)\hat{x} + (x + y^2)\hat{y}$:

$$\nabla(x,y) = 2x\hat{x} + 2y\hat{y}$$

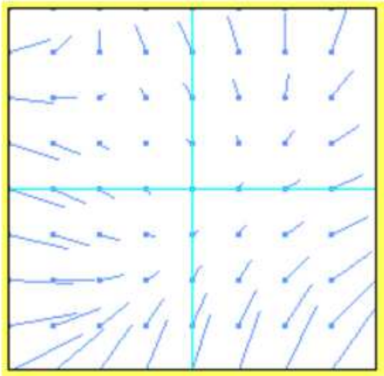


FIGURE 10

(Courtesy: Image by Eric Carlen, School of Mathematics, Georgia Institute of Technology)

Ex.2.

Imagine we are standing on a hillside,the direction of the steepest ascent,is the direction of the gradient.The measurement of slope in that direction is the magnitude of the gradient.

Ex.3.

The concept of stationary point can be understood using the gradient principle.

$v(x, y, z)$ is a scalar function. The $\vec{\nabla}v(x, y, z)$=0 at (x, y, z) indicates $dv = 0$ for small displacements about the point (x, y, z).

In particular to locate the extreme of a function of three variables,set its gradient equal to zero.

Ex.4.

Gradient of the 2-d function (Fig. 11), $f\ x,=x\ e^{-x^2-y^2}$

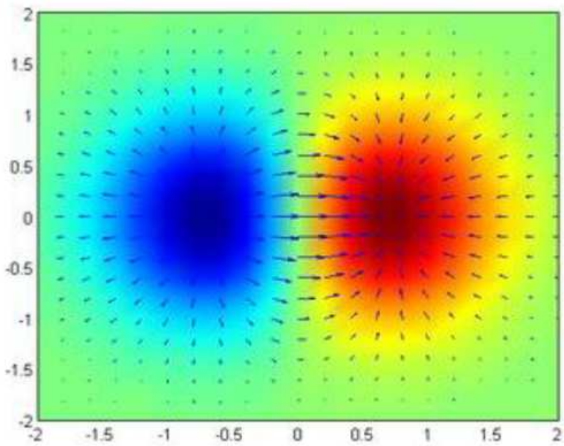


FIGURE 11

(Courtesy: Image by Eric Carlen, School of Mathematics, Georgia Institute of Technology)

2. Divergence:

The scalar product of the vector derivative operator and a vector function is called the **divergence** of the vector.

$$\vec{\nabla}.\vec{\mathbf{v}} = (\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial x} + \hat{\mathbf{z}}\frac{\partial}{\partial x}).(v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

The divergence of a vector function is a **scalar**.

What is the divergence? If two objects following the direction specified by the vector function increase their separation, the divergence of the vector function is positive. If their separation decreases, the divergence of the vector function is negative.

Geometrical interpretation (physical) of divergence: The name divergence is a measure of how much the vector spreads out (diverge) from the point in question (Fig. 12 and 13).

Ex.1:

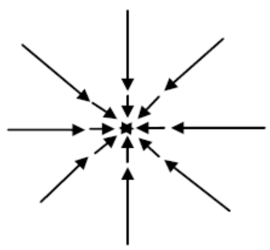


FIGURE 13

The vector function in Fig: 13 has a negative divergence (arrows pointed in)

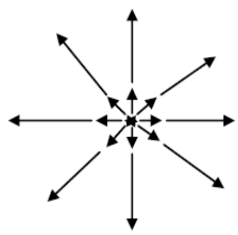


FIGURE 12

The vector function in Fig: 12 has a positive divergence (arrows pointed out)

If the material spreads out at a point is the positive divergence, while material collects at a point is negative divergence.

3. The Curl:

The curl of a vector function **v** is $\vec{\nabla} \times \vec{\boldsymbol{v}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$

$$= \hat{\mathbf{x}}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

The curl of a vector function $\vec{v}$ is a vector. (**Cross product of vector is a vector function**)

Geometrical Interpretation: The name **curl** is also well chosen, for $\nabla \times \mathbf{v}$ is a measure of how much the vector **v** swirls around the point in question. In the Fig.14 (a) functions have a substantial curl, pointing in the *z* direction, as the natural right-hand rule would suggest.

Imagine (again) you are standing at the edge of a pond. Float a small paddlewheel (a cork with toothpicks pointing out radially would do); if it starts to rotate, then you placed it at a point of nonzero *curl.* A whirlpool would be a region of large curl.

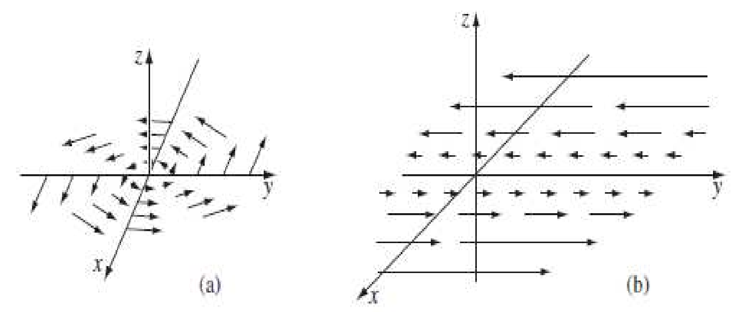


FIGURE 14

INTEGRAL CALCULUS

Line (or path) integral:

A line integral is an expression of the form

$$\int_{a}^{b} \vec{v} \cdot \overrightarrow{dl}$$

Where $\boldsymbol{V}$ is a vector function, d$\boldsymbol{l}$ is the infinitesimal displacement vector, and the integral is to be carried out along a prescribed path *P* from point a to b (Fig. 15)

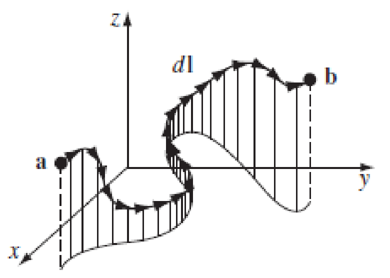


FIGURE 15

If the path in question forms a closed loop (that is, if b=a) i.e. $\oint \vec{v} \cdot \overrightarrow{dl}$.

At each point on the path, we take the dot product of **v** (evaluated at that point) with the displacement *d***l** to the next point on the path. To a physicist, the most familiar example of a line integral is the work done by a force **F**: $W = \int F \cdot dl$

Ordinarily, the value of a line integral depends critically on the path taken from **a** to **b**, but there is an important special class of vector functions for which the line integral is *independent* of path and is determined end entirely by the points.

2. Surface Integrals.

A surface integral is an expression of the form

$$\int_{S} \vec{v} \cdot \overrightarrow{da}$$

where $\vec{v}$ is again some vector function, and the integral is over a specified surface *S*. Here $\overrightarrow{da}$ is an infinitesimal patch of area, with direction perpendicular to the surface (Fig. 16).

Ex: If vector $\boldsymbol{V}$ describes the flow of a fluid (mass per unit area per unit time), then $\boldsymbol{V}\,.d\boldsymbol{a}$ represents the total mass per unit time passing through the surface (flux).

The surface integral depends on the particular surface chosen, but there is a particular special class of vector function for which it is independent of the surface, and it is determined entirely by the boundary line.

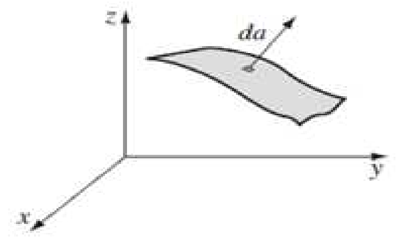


FIGURE 16

Volume Integrals

A volume integral is an expression of the form

$$\int_v T d\tau$$

Where T a scalar function and dτ is an infinitesimal volume element. In Cartesian coordinates, $d\tau = dx\, dy\, dz$.

Ex: If T is the density of a substance (which might vary from point to point), then the volume integral would give the total mass. Occasionally we shall encounter volume integrals of *vector* functions

$$\int \mathbf{v}\, d\tau = \int (v_x\,\hat{\mathbf{x}} + v_y\,\hat{\mathbf{y}} + v_z\,\hat{\mathbf{z}})d\tau = \hat{\mathbf{x}}\int v_x d\tau + \hat{\mathbf{y}}\int v_y d\tau + \hat{\mathbf{z}}\int v_z d\tau;$$

because the unit vectors ($\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$) are constants, they come outside the integral.

The Fundamental Theorem of Calculus

F (x) is a function of one variable. The **fundamental theorem of calculus** says:

$$\int_a^b \frac{df}{dx} dx = f(b) - f(a) \text{ or } \int_a^b F(x)dx = f(b) - f(a)$$

In vector calculus, there are three different kinds of derivatives – gradient, divergence and curl – so there are three different analogues of the fundamental theorem of calculus:

1. The Fundamental Theorem for Gradients

Suppose we have a scalar function of three variables $T\,(x, y, z)$. Starting at point **a**, we move a small distance $d\mathbf{l}_1$ (Fig. 17). The function T will change by an amount

$$dT = (\nabla T).dl_1$$

Now little further, let us take small displacement $d\mathbf{l}_2$; the incremental change in T will be $(\nabla T) \cdot d\mathbf{l}_2$. In this manner, proceeding by infinitesimal steps, we make the journey to point **b**. At each step we compute the gradient of T (at that point) and dot it into the displacement $d\mathbf{l}$. . . this gives us the change in T . Evidently the *total* change in T in going from **a** to **b** (along the path selected) is

$$\int_a^b (\overrightarrow{\nabla T}).\overrightarrow{dl} = T(b) - T(a)$$

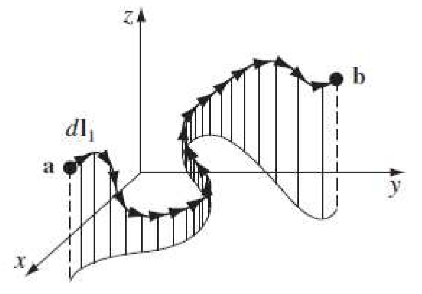


FIGURE 17

This is the **fundamental theorem for gradients**; like the “ordinary” fundamental theorem, it says that the integral (here a *line* integral) of a derivative (here the *gradient*) is given by the value of the function at the boundaries (**a** and **b**).

2. The Fundamental Theorem for Divergences

Statement: The integral of a derivative (divergence) over a region (volume) is equal to the value of the function at the boundary (surface that bounds the volume)

The fundamental theorem for divergences states that:

$$\int_{v} (\nabla \cdot \mathrm{v})d\tau = \oint_{s} \mathrm{v} \cdot \mathrm{da}$$

The left hand side is the sum of all the **sources** of $\boldsymbol{v}$ within the volume. Again, in the example of fluid flow, this would be the sum of the output of the “faucets,” which in turn are places where the divergence of $\boldsymbol{v}$ is high.

The right-hand side is the **flux** of the vector function $\boldsymbol{v}$ through the surface $\boldsymbol{S}$. For example, if $\boldsymbol{v}$ were the velocity of a fluid, this would tell us the rate of flow of the fluid through the surface.

Ex: $\int$(faucets within the volume) = $\oint$(flow out through the surface)

{Difference of numbers of faucets and drains within volume}= {Total flow through bounding surface}

3. The Fundamental Theorem for Curls (Stokes’Theorem)

Statement: The integral of a derivative (here, the curl) over a region (here, a patch of surface) is equal to the value of the function at the boundary. (Here, the perimeter of the patch).Fig:18

$$\int_{s} (\nabla\times\mathrm{v})\cdot\mathrm{da} = \oint_{p} \mathrm{v} \cdot \mathrm{dl}$$

The left-hand side is the **circulation** of the vector function $\boldsymbol{v}$ on curve *C*. Again the name comes from the fluid-flow analogy.

The right-hand side is the sum of the **sources** of circulation with the area bounded by *C*. If high divergence corresponds to a productive water faucet within *V*, high curl corresponds to a good stirring-rod within *S*

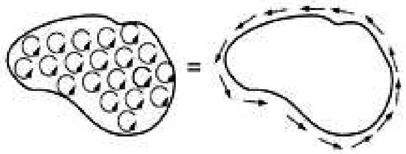


FIGURE 18
Illustration of the theorem

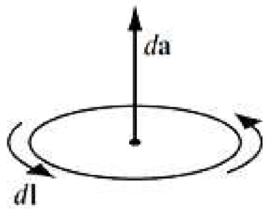


FIGURE 19
If your figure point in the direction of the line integral, then the thumb fixes the direction of $d\mathbf{a}$

Ex: Twist a paper clip into a loop, and dip it in soapy water. The soap film constitutes a surface, with the wire loop as its boundary. If you blow on it, the soap film will expand, making a larger surface, with the same boundary.

For Stokes' theorem says that $\int(\nabla \times \mathbf{v}) \cdot d\mathbf{a}$ is equal to the line integral of $\mathbf{v}$ around the boundary, and the latter makes no reference to the specific surface you choose.

Electrostatics

Properties of Electric Charges:

- Charges of the same sign repel one another and charges with opposite signs attract one another.
- Electric charge is always conserved in an isolated system. The algebraic sum of charges (net charge) of an isolated system never changes.
- Charge is quantized. The smallest unit of charge observed in nature is that on the proton or electron. All charged bodies have charges that are integer multiples of this quantum of charge.
- Positive charge has diverging field around it and negative charge has converging field around it.
- Charge is a scalar quantity. Charge is additive in nature.

Charging Objects by Induction:

We can classify materials in terms of the ability of electrons to move through the material when they are charged. Materials such as glass, rubber, and dry wood fall into the category of electrical insulators. When such materials are charged by rubbing, only the area rubbed becomes charged and the charged particles are unable to move to other regions of the material. In contrast, materials such as copper, aluminium, and silver are good electrical conductors. When such materials are charged in some small region, the charge readily distributes (moves) itself over the entire surface of the material.

Charging an object by induction requires no contact with the object inducing the charge. That is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.

THE ELECTRIC FIELDS

Coulomb's Law:

Coulomb law is used to measure the magnitudes of electric forces between charged objects or particles. From Coulomb's experiments, we can generalize the properties of the electric force (sometimes called the *electrostatic force*) between two stationary charged particles. We use the term point charge to refer to a charged particle of zero size. The electrical behaviour of electrons and protons is very well described by modelling them as point charges. From experimental observations, we find that the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges is given by **Coulomb's law**

$$\boldsymbol{F_e = k_e \frac{|q_1||q_2|}{r^2}} \text{ -----------------(1)}$$

where k_e is a constant called the **Coulomb constant.** The Coulomb constant k_e in SI units has the value ke =8.987 6 x 10^9 N m^2/C^2**.** This constant is also written in the form $k_e = 1/4\pi\varepsilon_0$, where the constant ε_0 is known as the permittivity of free space and has the value ε_0 = 8.8542 x 10^{-12} $C^2/N.\ m^2$.

Remember that a positive value of F_e indicates a repulsive force directed along the line joining the charges, and a negative value indicates an attractive force. Coulomb's Law applies to particles—electrons and protons—and also to any small charged bodies, provided that the sizes of these bodies are much smaller than the distances between them; such bodies are called point charges. Equation 1 obviously resembles Newton's Law for the gravitational force; the constant k is analogous to the gravitational constant G, and the electric charges are analogous to the gravitating masses.

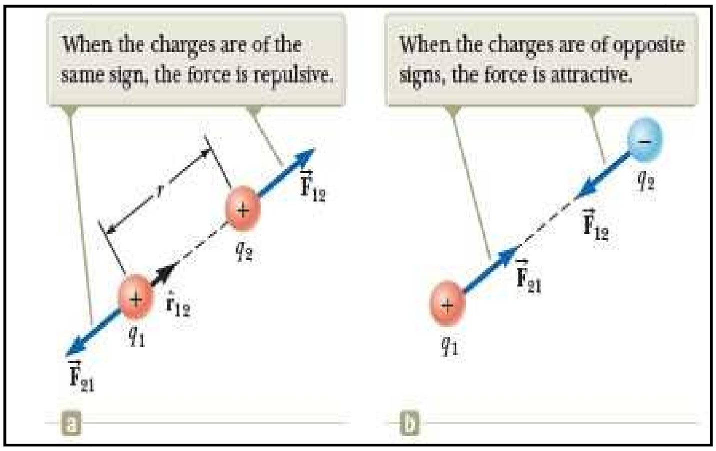


Figure 20 Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\overrightarrow{F_{12}}$ exerted by q2 on q1 is equal in magnitude and opposite in direction to the force F_{21} exerted by q1 on q2.

Coulomb's law in vector form:

Coulomb's law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2 (Fig. 20) written $\vec{F}_{12}$ **is**

$$\vec{F}_{12} = k_e \frac{|q_1||q_2|}{r^2} \hat{r}_{12} \text{---------------(2)}$$

where $\hat{r}_{12}$ is a unit vector directed from q_1 toward q_2 as shown in Figure. Because the electric force obeys Newton's third law, the electric force exerted by q_2 on q_1 is equal in magnitude to the force exerted by q_1 on q_2 and in the opposite direction; that is

$$\vec{F}_{12} = -\vec{F}_{21}$$

Equation 2 shows that if q_1 and q_2 have the same sign as in above Figure, the product q_1q_2 is positive and the electric force on one particle is directed away from the other particle. If q_1 and q_2 are of opposite sign as shown in Figure … the product q_1q_2 is negative and the electric force on one particle is directed toward the other particle.

The superposition of electric forces

Statement: The interaction between any two charges is completely unaffected by the presence of others.

As shown in Figure 21, if we have several point charges (Source charges) q_1, q_2, q_3,........ and test charge Q. To determine the force on Q, we can first compute the force $\mathbf{F}_1$, due to q_1 alone (ignoring all the others); then we compute the force $\mathbf{F}_2$, due to q_2 alone; and so on. Finally, we take the vector sum of all these individual forces:

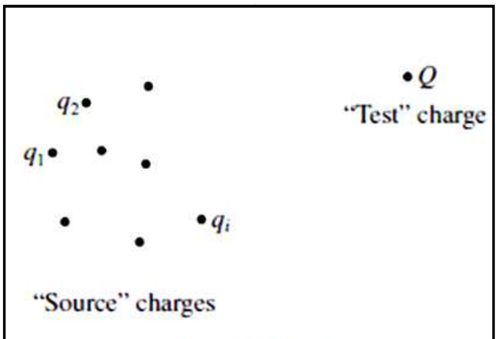


FIGURE 21

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots \text{---------------(3)}$$

This simple combination law is an important empirical fact about electric forces. Since the contact forces of everyday experience, such as the normal force and the friction force, arise from electric forces between the atoms, they will likewise obey the Superposition Principle, and they can be combined with simple vector addition.

Factors which depend force between q and Q,

1. Separation distance r between the charges.
2. Both their velocities and on the acceleration of q

The Electric Field

If we have *several* point charges $q_1, q_2, \ldots, q_n$, at distances $r_1, r_2, \ldots, r_n$ from Q, the total force on Q is evidently

$$F = F_1 + F_2 + \ldots\ldots\ldots\ldots$$
$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \frac{q_2 Q}{r_2^2}\hat{r}_2 + \ldots\ldots\ldots\right)$$
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \ldots\ldots\ldots\ldots\right)$$

or $F = QE$

where $$\mathbf{E(r)} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i \quad \text{----------------(4)}$$

E is called the electric field of the source charges. Notice that it is a function of position (**r**), because the separation vectors $\boldsymbol{r}_i$ depend on the location of the field point P (Fig. 22). But it makes no reference to the test charge Q. The electric field is a vector quantity that varies from point to point and is determined by the configuration of source charges; physically, **E**(**r**) is the force per unit charge that would be exerted on a test charge, if you were to place one at P.

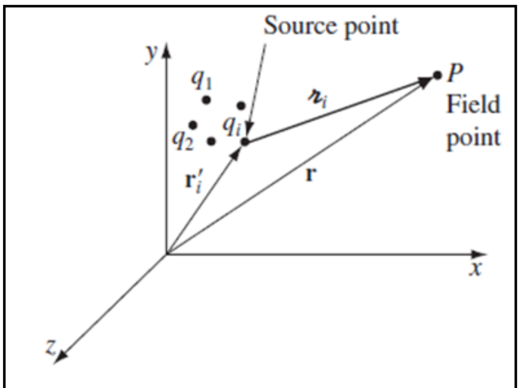


FIGURE22

Example: Find the electric field a distance z above the midpoint between two equal charges (q), a distance d apart (Fig. 23a)

Let $\mathbf{E}_1$ be the field of the left charge alone, and $\mathbf{E}_2$ that of the right charge alone (Fig. 23b). Adding them (vectorially), the horizontal components cancel and the vertical components conspire:.

$$E_z = 2\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta$$

Here $r = \sqrt{z^2 + (\frac{d}{2})^2}$ and $\cos\theta = \frac{z}{r}$, so

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2qz}{\left(\sqrt{z^2+(\frac{d}{2})^2}\right)^{3/2}}\hat{z}$$

Check: When z>>d you are so far away that it just looks like a single charge 2q, so the field should reduce to $E = \frac{1}{4\pi\varepsilon_0}\frac{2qz}{(z)^2}\hat{z}$.

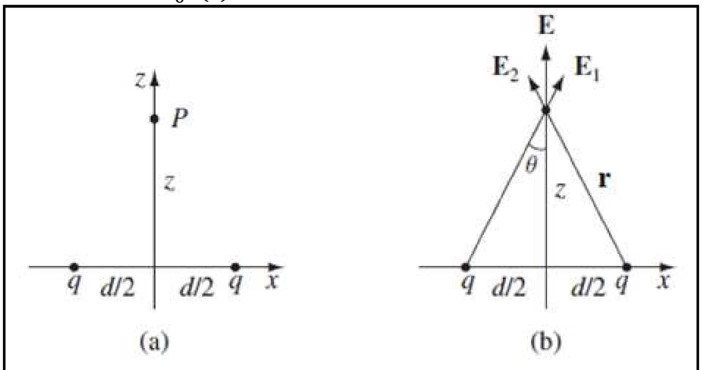


FIGURE 23

Particle in a Electric Field: Model

As pointed out earlier, field forces can act through space, producing an effect even when no physical contact occurs between interacting objects. Such an interaction can be modeled as a two-step process: a source particle establishes a field, and then a charged particle interacts with the field and experiences a force. Imagine an object with charge that we call a *source charge*. The source charge establishes an **electric field** $\vec{E}$ throughout space. Now imagine, if an arbitrary charge q is placed in an electric field $\vec{E}$, it experiences an electric force given by (Fig. 24)

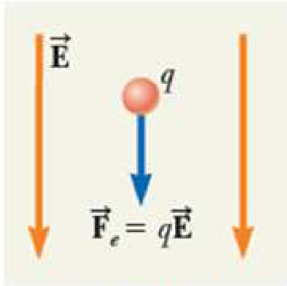
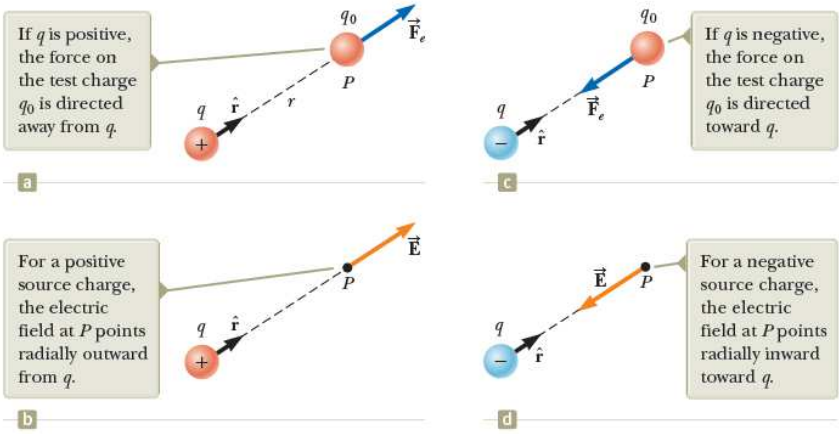


FIGURE 24

$$\vec{F}_e = q\,\vec{E} \quad \text{----------(5)}$$

This equation is the mathematical representation of the electric version of the **particle in a field** analysis model. If q is positive, the force is in the same direction as the field. If q is negative, the force and the field are in opposite directions.

To determine the direction of an electric field, consider a point charge q as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge q_0 is placed at point P, a distance r from the source charge, as in **Figure 25a**. We imagine using the test charge to determine the direction of the electric force and therefore that of the electric field. According to Coulomb's law, the force exerted by q on the test charge is



FIGUREY 25 (a), (c) When a test charge q_0 is placed near a source charge q, the test charge experiences a force. (b), (d) At a point P near a source charge q, there exists an electric field

$$\vec{F}_e = k_e \frac{q q_0}{r^2}\hat{r} \qquad \text{-------------(6)}$$

where $\hat{\boldsymbol{r}}$ is a unit vector directed from q toward q_0. This force in Figure 23a is directed away from the source charge q. Because the electric field at P, the position of the test charge, is defined by $\vec{E} = \vec{F}_e / q_0$, the electric field at P created by q is

$$\vec{E} = k_e \frac{q}{r^2}\hat{r} \qquad \text{-------------(7)}$$

If the source charge q is positive, Figure 25b shows the situation with the test charge removed: the source charge sets up an electric field at P, directed away from q. If q is negative as in Figure 25c, the force on the test charge is toward the source charge, so the electric field at P is directed toward the source charge as in Figure 25d.

To calculate the electric field at a point P due to a small number of point charges, we first calculate the electric field vectors at P individually using Eqn.7 and then add them vectorially. In other words, at any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges. This superposition principle applied to fields follows directly from the vector addition of electric forces. Therefore, the electric field at point P due to a group of source charges can be expressed as the vector sum

$$\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i \qquad \text{-------------(8)}$$

where r_i is the distance from the i^{th} source charge q_i to the point P and $\hat{r}_i$ is a unit vector directed from q_i toward P.

Electric Field of a Continuous Charge Distribution

Equation 8 is useful for calculating the electric field due to a small number of charges. In many cases, we have a continuous distribution of charge rather than a collection of discrete charges. The charge in these situations can be described as continuously distributed along some line, over some surface, or throughout some volume.

To set up the process for evaluating the electric field created by a continuous charge distribution, let's use the following procedure. First, divide the charge distribution into small elements, each of which contains a small charge Δq as shown in Figure 24. Next, use Equation 8 to calculate the electric field due to one of these elements at a point P. Finally, evaluate the total electric field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).

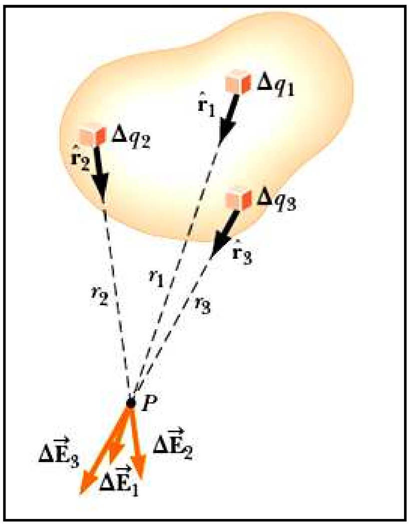


FIGURE 24 The electric field at P due to a continuous charge distribution is the vector sum of the fields $\Delta \vec{E}_i$ due to all the elements $\Delta \vec{q}_i$ of the charge distribution. Three sample elements are shown.

The electric field at P due to one charge element carrying charge Δq is

$$\boldsymbol{\Delta\vec{E} = k_e \frac{\Delta q}{r^2} \hat{r}} \qquad \text{-----------------(9)}$$

where r is the distance from the charge element to point P and $\hat{\boldsymbol{r}}$ is a unit vector directed from the element toward P. The total electric field at P due to all elements in the charge distribution is approximately

$$\boldsymbol{\vec{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i} \qquad \text{-----------------(10)}$$

where the index i refers to the i^{th} element in the distribution. Because the number of elements is very large and the charge distribution is modelled as continuous, the total field at P in the limit $\Delta q_i \to 0$ is

$$\boldsymbol{\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i = k_e \int \frac{dq}{r^2} \hat{r}} \text{ -------------(11)}$$

where the integration is over the entire charge distribution. The integration in Eqn. 11 is a vector operation and must be treated appropriately. Let's illustrate this type of calculation with several examples in which the charge is distributed on a line, on a surface, or throughout a volume. When performing such calculations, it is convenient to use the concept of a *charge density* along with the following notations:

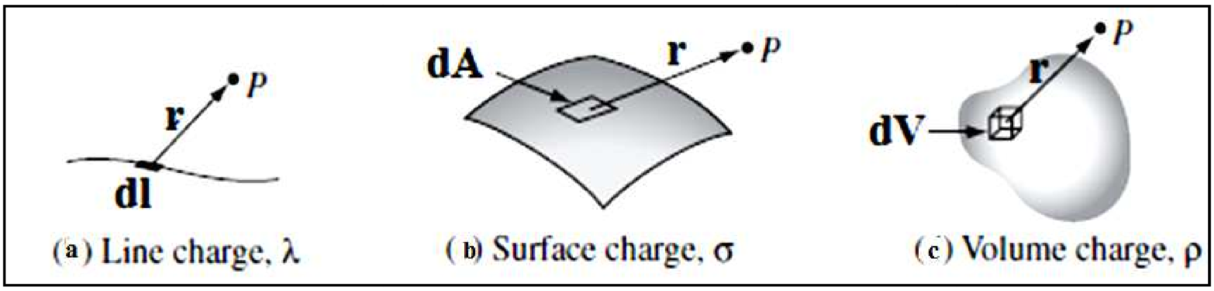


Figure 25. Various charge distribution

Linear charge density (λ): If a charge *Q* is uniformly distributed along a line of length, the linear charge density λ is defined by(Ref: Fig 25a)

$$\lambda = dq / d\ell$$

Then Eqn. 11. becomes

$$\vec{\mathbf{E}} = \boldsymbol{k_e} \int \frac{\lambda \, d\ell}{r_i^2} \, \hat{\mathbf{r}}_i \quad \text{………………….(12)}$$

This is the field due to continuous line distribution of charges

Surface charge density(σ):

If a charge q is uniformly distributed on a surface area A the surface charge density σ is defined by (Ref: Fig 25b) **σ = dq / dA**

where σ has units of coulombs per square meter (C/m^2).

Then Eqn. 11 becomes

$$\vec{\mathbf{E}} = \boldsymbol{k_e} \int \frac{\sigma \, dA}{r_i^2} \, \hat{\mathbf{r}}_i \text{………………….(13)}$$

This is the field due to continuous surface distribution of charges

Volume charge density (ρ):

If a charge q is uniformly distributed throughout a volume V, the volume charge density ρ is defined by(Ref: Fig 25c) **ρ = dq / dV**

where ρ has units of coulombs per cubic meter (C/m^3).

Then Eqn. 11 becomes

$$\vec{\mathbf{E}} = \boldsymbol{k_e} \int \frac{\rho \, dV}{r_i^2} \, \hat{\mathbf{r}}_i \quad \text{………………….(14)}$$

This is the field due to continuous distribution of charges throughout a volume V

Examples of particle in a field:

• An electron moves between the deflection plates of a cathode ray oscilloscope and is deflected
  from its original path
• Charged ions experience an electric force from the electric field in a velocity selector before entering a mass spectrometer
• An electron moves around the nucleus in the electric field established by the proton in a hydrogen atom as modelled by the Bohr theory
• A hole in a semiconducting material moves in response to the electric field established by applying a voltage to the material

EXAMPLES OF CONTINUOUS CHARGE DISTRIBUTION

1: The Electric Field Due to a Charged Rod

A rod of length ℓ , has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.

Solution :

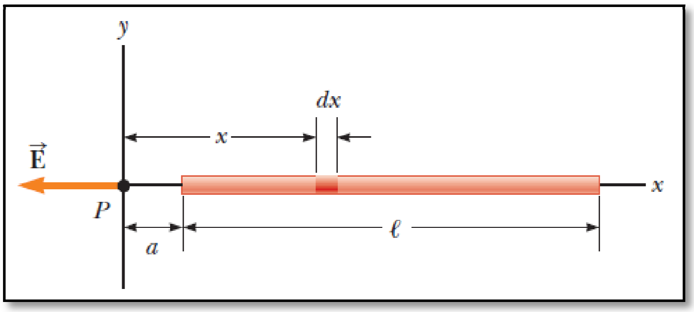


Figure 26: The electric field at *P* due to a uniformly charged rod lying along the *x* axis.

Conceptualize: The field $d\vec{E}$ at P due to each segment of charge on the rod is in the negative x direction because every segment carries a positive charge. Fig. 26 shows the appropriate geometry. In our result, we expect the electric field to become smaller as the distance a becomes larger because point P is farther from the charge distribution.

Categorize: Because the rod is continuous, we are evaluating the field due to a continuous line charge distribution rather than a group of individual charges. Because every segment of the rod produces an electric field in the negative x direction, the sum of their contributions can be handled without the need to add vectors.

Analyze: Let's assume the rod is lying along the x axis, *dx* is the length of one small segment, and dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$

The magnitude of the electric field at P due to one segment of the rod having a charge *dq*

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda dx}{x^2}$$

The total field at P, using Eqn. 12 is

$$\mathrm{E} = \int_a^{a+l} k_e \frac{\lambda dx}{x^2}$$

Noting that k_e and λ = Q /ℓ, are constants and can be removed from the integral, valuate the integral

$$\mathrm{E} = k_e \lambda \int_a^{a+l} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x}\right]_a^{a+l}$$

$$\mathrm{E} = k_e \lambda \left(\frac{1}{a} - \frac{1}{a+l}\right)$$

$$\mathrm{E} = \frac{k_e \lambda l}{a(a+l)}$$

$$\mathbf{E} = \frac{\boldsymbol{k_e Q}}{\boldsymbol{a(a+l)}}$$

Therefore $\mathbf{E} = \frac{\boldsymbol{k_e Q}}{\boldsymbol{a(\ell+a)}}$ is the field at a point due to charged rod**.**

Finalize: We see that our prediction is correct; if 'a' becomes larger, the denominator of the fraction grows larger, and E becomes smaller. On the other hand, if a → 0, which

corresponds to sliding the bar to the left until its left end is at the origin, then E $\rightarrow\infty$, That represents the condition in which the observation point P is at zero distance from the charge at the end of the rod, so the field becomes infinite.
If, point P is very far away from the rod (a>> ℓ), then ℓ in the denominator can be neglected and $E \sim k_e Q/a^2$
That is exactly the form you would expect for a point charge. Therefore, at large values of a/ℓ, the charge distribution appears to be a point charge of magnitude Q; the point P is so far away from the rod we cannot distinguish that it has a size.

2. The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring

Solution :

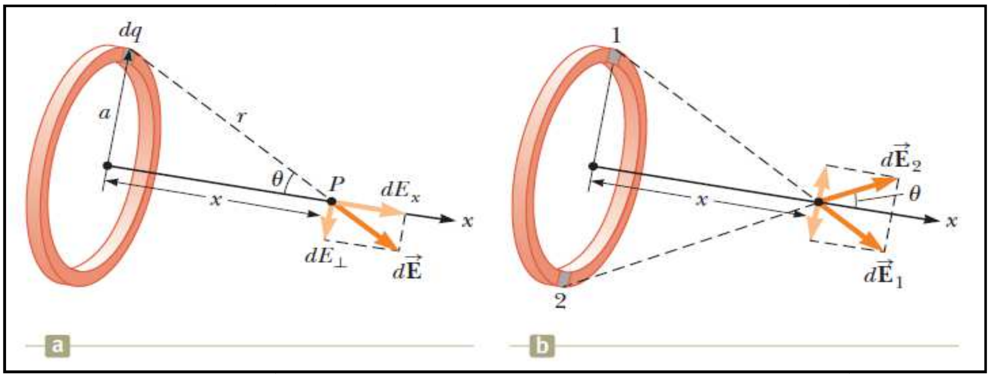


Figure 27. A uniformly charged ring of radius a. (a) The field at P on the x axis due to an element of charge dq. (b) The total electric field at P is along the x axis. The perpendicular component of the field at P due to segment 1 is cancelled by the perpendicular component due to segment 2

Conceptualize: Figure 27a. shows the electric field contribution dE at P due to a single segment of charge at the top of the ring. This field vector can be resolved into components dE_x parallel to the axis of the ring and $dE_\perp$ perpendicular to the axis. Figure 27b, shows the electric field contributions from two segments on opposite sides of the ring. Because of the symmetry of the situation, the perpendicular components of the field cancel. That is true for all pairs of segments around the ring, so we can ignore the perpendicular component of the field and focus solely on the parallel components, which simply add.

Categorize: Because the ring is continuous, we are evaluating the field due to a continuous charge distribution rather than a group of individual charges.

Analyze: Evaluate the parallel component of an electric field contribution from a segment of charge dq on the ring

$$dE_x = k_e \frac{dq}{r^2}\cos\theta = k_e \frac{dq}{a^2 + x^2}\cos\theta \qquad \text{......................(1)}$$

From the geometry in Figure a, evaluate cos θ

$$\cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}} \qquad \text{........................(2)}$$

Substitute Equation (2) into Equation (1)

$$dE_x = k_e \frac{dq}{a^2 + x^2}\left[\frac{x}{(a^2 + x^2)^{1/2}}\right] = \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq$$

All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Integrate over the circumference of the ring to obtain the total field at P

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}}\, dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

$$\boxed{E = \frac{k_e x}{(a^2 + x^2)^{3/2}}\, Q} \qquad \text{…………………(3)}$$

Finalize: This result shows that the field is zero at x = 0. Is that consistent with the symmetry in the problem? Furthermore,notice that Eqn.3 reduces to

$E = k_e Q / x^2$ if x>>a, so the ring acts like a point charge for locations far away from the ring. From a faraway point, we cannot distinguish the ring shape of the charge.

Suppose a negative charge is placed at the center of the ring in Figure 27 and displaced slightly by a distance x <<a, along the x axis, then

$$\boxed{E_x = \frac{k_e Q}{a^3}\, x}$$

Therefore, the force on a charge 2*q* placed near the center of the ring is

$$\boxed{F_x = -\frac{k_e q Q}{a^3}\, x}$$

Because this force has the form of Hooke's law, the motion of the negative charge is described with the particle in simple harmonic motion model!

Electric Field Lines

A convenient way of visualizing the electric field is by a pictorial representation of drawing lines, called electric field lines and first introduced by Faraday, that are related to the electric field in a region of space in the following manner:

- The electric field vector $\vec{E}$ is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead that is the same as that of the electric field vector. The direction of the line is that of the force on a positive charge placed in the field according to the particle in a field model.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Therefore, the field lines are close together where the electric field is strong and far apart where the field is weak.

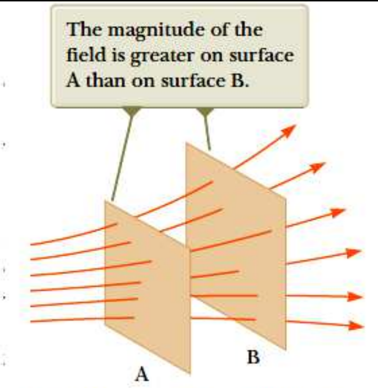


FIGURE 28. Electric field lines penetrating two surfaces.

These properties are illustrated in Figure 28. The density of field lines through surface A is greater than the density of lines through surface B. Therefore, the magnitude of the electric field is larger on surface A than on surface B. Furthermore, because the lines at different locations point in different directions, the field is nonuniform.

Properties of Electric field lines:

Electric Field Lines are not Paths of Particles! Electric field lines represent the field strength at various locations. The lines must begin on a positive charge and terminate on a negative Charge (Fig. 29). The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge. No two field lines can cross. Figure 30 shows the two-dimensional drawing. Only the field lines that lie in the plane containing the point charge. The lines are actually directed radially outward from the charge in all directions. The lines are along the radial direction and extend all the way to infinity. The lines become closer together as they approach the charge, indicating that the strength of the field increases as we move toward the source charge.

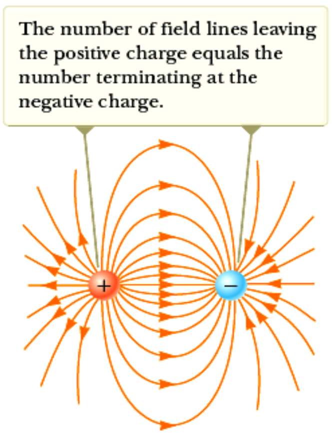


Figure 29 The electric field lines for two point charges of equal magnitude and opposite sign (an electric dipole).

Electric field lines are not real, Electric field lines are not material objects. They are used only as a pictorial representation to provide a qualitative description of the electric field. The field, in fact, is continuous, existing at every point.

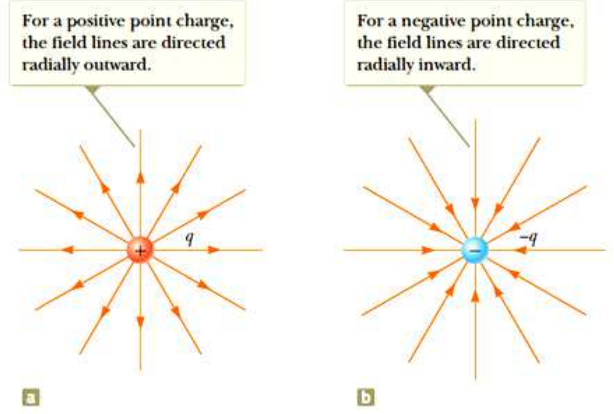


FIGURE 30 The electric field lines for a point charge. Notice that the figures show only those field lines that lie in the plane of the page.

Motion of a Charged Particle in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field $\vec{E}$, the electric force exerted on the charge is $q\vec{E}$ in the particle in a field model. If that is the only force exerted on the particle, it must be the net force, and it causes the particle to accelerate according to the particle under a net force model. Therefore,

$$\mathbf{F= q\vec{E}=m\vec{a}}$$

and the acceleration of the particle is $\mathbf{\vec{a} = q\,\vec{E} / m}$

If $\vec{E}$ is uniform (that is, constant in magnitude and direction), and the particle is free to move, the electric force on the particle is constant and we can apply the particle under constant acceleration model to the motion of the particle. Therefore, the particle in this situation is described by three analysis models: particle in a field, particle under a net force, and particle under constant acceleration! If the particle has a positive charge, its acceleration is in the direction of the electric

field. If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

Electric Flux

Consider an electric field that is uniform in both magnitude and direction as shown in Figure.31. The field lines penetrate a rectangular surface of area whose plane is oriented perpendicular to the field The number of lines crossing per unit area (in other words, the line density) is proportional to the magnitude of the electric field. Therefore, the total number of lines penetrating the surface is proportional to the product EA. This product of the magnitude of the electric field and surface area perpendicular to the field is called the electric flux

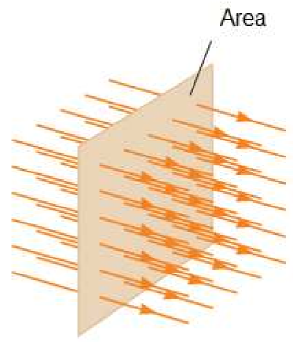
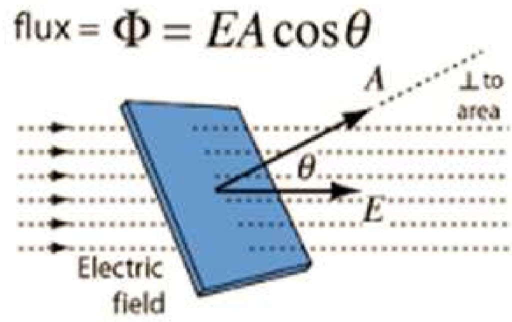


FIGURE 31. Field lines representing a uniform electric field penetrating a plane of area perpendicular to the field.

Figure 32. Field lines representing a uniform electric field penetrating a plane of area perpendicular to the field ($\theta = 0^0$)



FIRURE 32

Electric flux Φ is proportional to the number of electric field lines penetrating some surface.

$$\mathbf{\Phi = EA}$$

Electric flux is a scalar quantity, it has units of newton meters squared per coulomb (N. m^2/C)

If the surface under consideration is not perpendicular to the field, the flux through it must be less than that given by

In vector form $\mathbf{\Phi} = \vec{E}.\overrightarrow{dA}$ = **EA cos θ**

Where the normal to the surface of area A is at an angle θ to the uniform electric field. Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θ with the normal to a surface of area A, the electric flux through the inclined surface is

$$\mathbf{\Phi = EA\cos\theta}$$

From this result, we see that the flux through a surface of fixed area A has a maximum value EA when the surface is perpendicular to the field (when the normal to the surface is parallel to the field, that is, when $\theta = 0^0$)

As shown in the Fig. 33, the flux is zero when the surface is parallel to the field (when the normal to the surface is perpendicular to the field, that is, when $\theta = 90^0$).

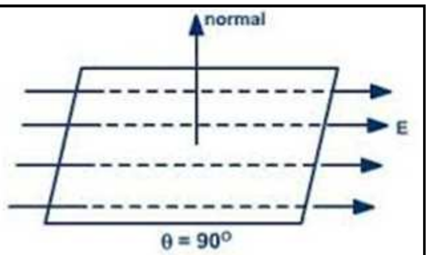


FIGURE 33

In this discussion, the angle θ is used to describe the orientation of the Surface of area A. We can also interpret the angle as that between the electric field vector and the normal to the surface. In this case, the product E cos θ in Equation Φ = EA cos θ is the component of the electric field perpendicular to the surface.

So far we assumed a uniform electric field in the preceding discussion. In more general situations, the electric field may vary over a large surface. Therefore, the definition of flux given by the relation $\Phi = EA\cos\theta$ has meaning only for a small element of area over which the field is approximately constant. A shown in Fig. 34, consider a general surface divided into a large number of small elements, each of area $\Delta\vec{A}_i$.

It is convenient to define a vector $\Delta\vec{A}i$ whose magnitude represents the area of the ith element of the large surface and whose direction is defined to be perpendicular to the surface element as shown in Figure .The electric field $\vec{E}_i$ at the location of this element makes an angle θi with the vector $\Delta\vec{A}i$.

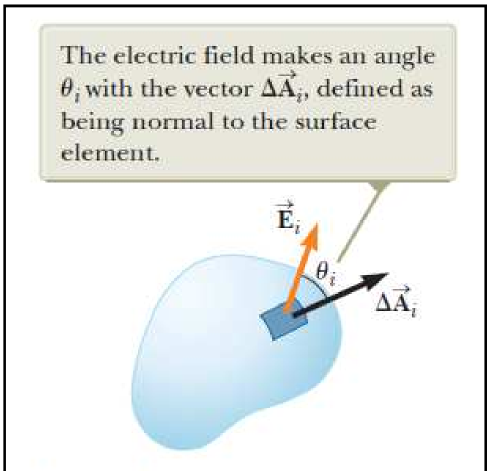


FIGURE 34. A small element of surface area ΔA_i in an electric field.

The electric flux Φ_i through this element is

$$\boldsymbol{\Phi_i = \vec{E}_i\ \Delta A_i \cos\theta = \vec{E}_i \,.\, \Delta\vec{A}i}$$

where $\Delta\vec{A}i$ is a small element of surface area in an electric field Here we have used the definition of the scalar product of two vectors ($\vec{A}\,.\,\vec{B} = \overrightarrow{AB}\cos\theta$). Summing the contributions of all elements gives an approximation to the total flux through the surface.

If the area of each element approaches zero, the number of elements approaches infinity and the sum is replaced by an integral. Therefore, the general definition of electric flux is

$$\boldsymbol{\Phi = \int_{surface} \vec{E}\,.\,d\vec{A}}$$

This is a surface integral, which means it must be evaluated over the surface in question. In general, the value of **Φ** depends both on the field pattern and on the surface

We often evaluate the flux through a closed surface, which is defined as a surface that divides space into an inside and an outside region so that one cannot move from one region to the other without crossing the surface. The surface of a sphere, for example, is a closed surface.

Consider the closed surface in Figure 35. The vectors $\boldsymbol{\Delta\vec{A}i}$ point in different directions for the various surface elements, but for each element they are normal to the surface and point outward.

At the element labelled 1, the field lines are crossing the surface from the inside to the outside and $\theta < 90^0$; hence, the flux $\boldsymbol{\Phi}_1 = \vec{E}.\Delta\boldsymbol{\vec{A}_1}$ through this element is positive.

For element 2, the field lines graze the surface (perpendicular to $\Delta\boldsymbol{\vec{A}_2}$) therefore, $\theta = 90^0$ and the flux is zero.

For elements such as 3, where the field lines are crossing the surface from outside to inside, $180^0 > \theta > 90^0$ and the flux is negative because cos θ is negative.

The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number of lines leaving the surface minus the number of lines entering the surface. If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative. Using the symbol $\oint$ to represent an integral over a closed surface, we can write the net flux **Φ** through a closed surface as

$$\boldsymbol{\Phi = \oint \overrightarrow{E}\,.\,d\vec{A}}$$

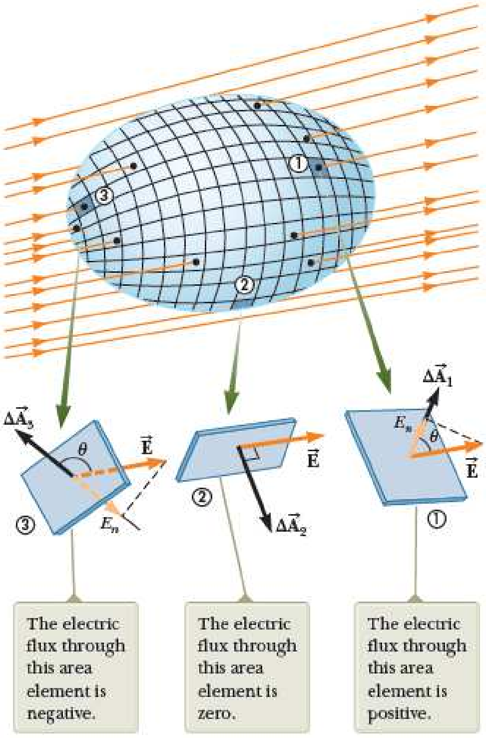


Figure 35. A closed surface in an electric field. The area vectors are, by convention, normal to the surface and point outward.

Φ is scalar, can be +ve , -ve or zero

1. For $\theta<90^0$, flux $\mathbf{\Phi}$ is +ve because cos θ=+ve i.e field lines are crossing the surface from the inside to the outside,

2. For $\theta=90^0$, flux $\mathbf{\Phi}$ is zero, because $\cos 90^0=0$, i.e the field lines graze the Surface

3. For $180^0>\theta>90^0$, flux $\mathbf{\Phi}$ is -ve because cos θ=-ve i.e the field lines are crossing the surface from outside to inside.

SOLVE

Flux Through a Cube

Consider a uniform electric field $\vec{E}$ oriented in the x direction in empty space. A cube of edge length ℓ is placed in the field, oriented as shown in Figure 24.5. Find the net electric flux through the surface of the cube.

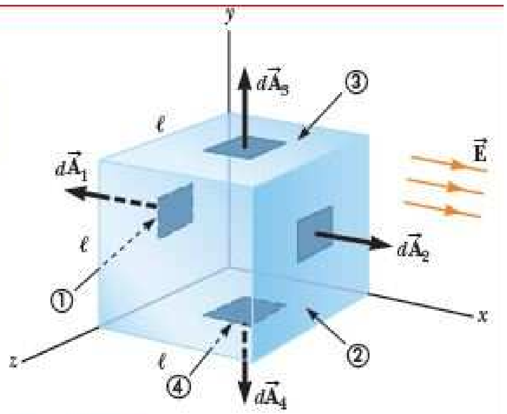


Figure 24.5 (Example 24.1) A closed surface in the shape of a cube in a uniform electric field oriented parallel to the x axis. Side ④ is the bottom of the cube, and side ① is opposite side ②.

SOLUTION

Conceptualize Examine Figure 24.5 carefully. Notice that the electric field lines pass through two faces perpendicularly and are parallel to four other faces of the cube.

Categorize We evaluate the flux from its definition, so we categorize this example as a substitution problem.

The flux through four of the faces (③, ④, and the unnumbered faces) is zero because $\vec{E}$ is parallel to the four faces and therefore perpendicular to $d\vec{A}$ on these faces.

Write the integrals for the net flux through faces ① and ②:

$$\Phi_E = \int_1 \vec{E}\cdot d\vec{A} + \int_2 \vec{E}\cdot d\vec{A}$$

For face ①, $\vec{E}$ is constant and directed inward but $d\vec{A}_1$ is directed outward ($\theta = 180°$). Find the flux through this face:

$$\int_1 \vec{E}\cdot d\vec{A} = \int_1 E(\cos 180°)\,dA = -E\int_1 dA = -EA = -E\ell^2$$

For face ②, $\vec{E}$ is constant and outward and in the same direction as $d\vec{A}_2$ ($\theta = 0°$). Find the flux through this face:

$$\int_2 \vec{E}\cdot d\vec{A} = \int_2 E(\cos 0°)\,dA = E\int_2 dA = +EA = E\ell^2$$

Find the net flux by adding the flux over all six faces:

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

Gauss's Law :

Gauss's law, is of fundamental importance in the study of electric fields, it describe a general relationship between the net electric flux through a closed surface (often called a Gaussian surface) and the charge enclosed by the surface

Statement: The net flux through any closed surface surrounding a point charge q is given by q/ε_0 and is independent of the shape of that surface.

$$\mathbf{\Phi} = q/\varepsilon_0$$

Derivation / Explanation:

Consider a positive point charge q located as shown in Figure 36. Consider a point p at a distance r from charge q. To find the field at point p, draw a Gaussian surface passing through point p. we know that the magnitude of the electric field everywhere on the surface of the sphere is same given by

$$E = k_e q / r^2$$

The field lines are directed radially outward and hence are perpendicular to the surface at every point on the surface.

That is, at each surface point, $\vec{E}$ is parallel to the vector $\Delta\vec{\boldsymbol{A}}\boldsymbol{i}$ (surface element) representing a local element of area ΔA_i surrounding the surface point p. Therefore, Flux at p is the dot product of $\vec{E}$ and $\Delta\vec{\boldsymbol{A}}\boldsymbol{i}$

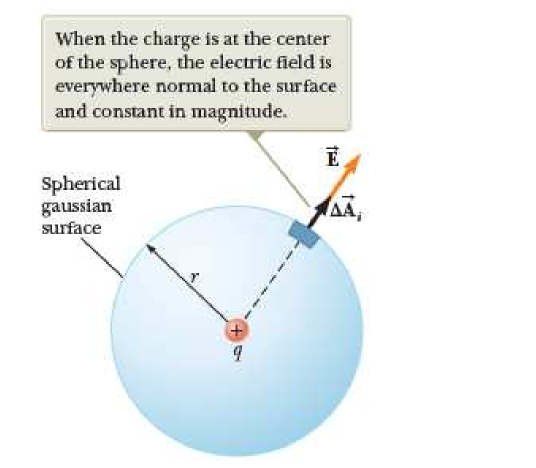


Figure 36 A spherical gaussian surface of radius r surrounding a positive point charge q.

$$\mathbf{\Phi} = \vec{E} \,.\, \Delta\vec{A}i = \vec{E}\, \Delta A_i \cos\theta$$

Since $\theta = 0$, $\cos 0 = 1$

$$\mathbf{\Phi} = \vec{E} \,.\, \Delta\vec{A}i = \vec{E}\, \Delta A_i$$

The net flux through the whole surface Gaussian surface is

$$\mathbf{\Phi} = \oint \vec{\boldsymbol{E}}\ \Delta A_i$$

The value of $\vec{E}$ is constant at all points on the surface given by $E = k_e q / r^2$.

Therefore $\mathbf{\Phi} = \boldsymbol{E} \oint \Delta A_i$

The Gaussian surface is spherical with surface area given by $4\pi r^2$

Substitute $E = k_e q / r^2$.and $\Delta A_i = 4\pi r^2$, in flux equation

$$\Phi = \frac{k\,q}{r^2}(4\pi r^2)$$

$$\Phi = 4\pi k q$$

$$\boxed{\Phi = q/\varepsilon_0} \qquad \text{where } 1/\varepsilon_0 = 4\pi k \text{ or } k = 1/4\pi\varepsilon_0$$

This shows that the net flux through the spherical surface is proportional to the charge inside the surface. The flux is independent of the radius r because the area of the spherical surface is proportional to r^2, whereas the electric field is proportional to $1/r^2$. Therefore, in the product of area and electric field, the dependence on r cancels.

Now consider several closed surfaces surrounding a charge q as shown in Figure.37 Surface S_1 is spherical, but surfaces S_2 and S_3 are not. Flux is proportional to the number of electric field lines passing through a surface. The Fig.37 shows that the number of lines through S_1 is equal to the number of lines through the nonspherical surfaces S_2 and S_3.

Therefore**, the net flux through any closed surface surrounding a point charge q is given by q/ε_0 and is independent of the shape of that surface and radius.**

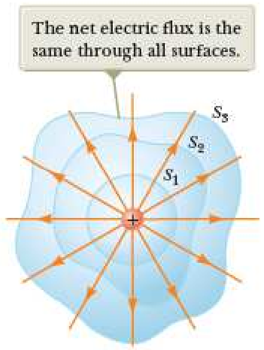


Figure 37. Closed surfaces of various shapes surrounding a positive charge.

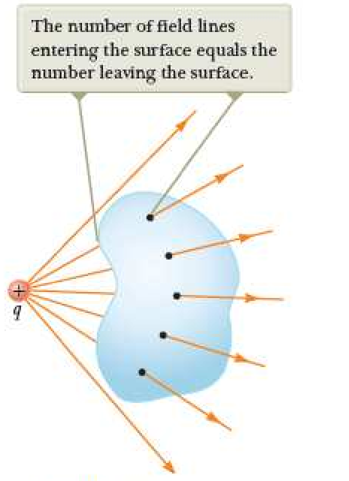


Figure 38. A point charge located *outside* a closed surface.

Consider a point charge located outside a closed surface of arbitrary shape as shown in Figure 38. As can be seen from this construction, any electric field line entering the surface leaves the surface at another point. The number of electric field lines entering the surface equals the number leaving the surface. Therefore, **the net electric flux through a closed surface that surrounds no charge is zero**. So we see that the net flux through the cube is zero because there is no charge inside the cube.

Multiple Charges

Electric field due to many charges is the vector sum of the electric fields produced by the individual charges. Therefore, the flux through any closed surface can be expressed as

$$\boxed{\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint (\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots) \cdot d\vec{\mathbf{A}}}$$

where $\vec{E}$ is the total electric field at any point on the surface produced by the vector addition of the electric fields at that point due to the individual charges.

For the system of charges shown in Figure 39., the net flux through S is q_1/ε_0. The flux through S due to charges q_2, q_3, and q_4 outside it is zero because each electric field line from these charges that enters S at one point leaves it at another.

The net flux through S′ it is $(q_2 + q_3)/\ \varepsilon_0$.

The net flux through surface *S″* is zero because there is no charge inside this surface.

Charge q_4 does not contribute to the net flux through any of the surfaces. The net electric flux through any closed surface depends only on the charge inside that surface.

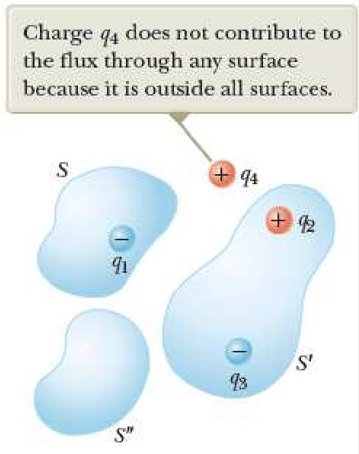


FIGURE 39. The net electric flux through any closed surface depends only on the charge *inside* that surface. The net flux through surface S is q_1/ϵ_0, the net flux through surface S' is $(q_2 + q_3)/\epsilon_0$, and the net flux through surface S'' is zero.

The mathematical form of Gauss's law is a generalization of what we have just described and states that the net flux through any closed surface is

$$\boxed{\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\epsilon_0}}$$

where $\vec{E}$ represents the electric field at any point on the surface and q_{in} represents the net charge inside the surface.

Note : Zero Flux is Not Zero Field : In two situations, there is zero flux through a closed surface: either (1) there are no charged particles enclosed by the surface or (2) there are charged particles enclosed, but the net charge inside the surface is zero. For either situation, it is incorrect to conclude that the electric field on the surface is zero. Gauss's law states that he electric flux is proportional to the enclosed charge, not the electric field.

Application of Gauss's Law to Various Charge Distributions

Gauss's law is useful for determining electric fields when the charge distribution is highly symmetric. The following examples demonstrate ways of choosing the Gaussian surface over which the surface integral given by Equation $\mathbf{\Phi = \oint \vec{E} .\ d\vec{A} = q / \varepsilon_0}$ can be simplified and the electric field determined. In choosing the surface, always take advantage of the symmetry of the charge distribution so that E can be removed from the integral. The goal in this type of calculation is to determine a surface for which each portion of the surface satisfies one or more of the following conditions:

1. Gauss's law applies only to a closed surface, called a Gaussian surface.
2. A Gaussian surface is not a physical surface. It need not coincide with the boundary of any physical object (although it could if we wished). It is an imaginary, mathematical surface in the space surrounding one or more charges.
3. We can't find the electric field from Gauss's law alone. We need to apply Gauss's law in situations where, from symmetry and superposition, we already can guess the shape of the field.

A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius a has a uniform volume charge density r and carries a total positive charge Q.
(A)Calculate the magnitude of the electric field at a point outside the sphere.
(B) Find the magnitude of the electric field at a point inside the sphere

Solution : A)

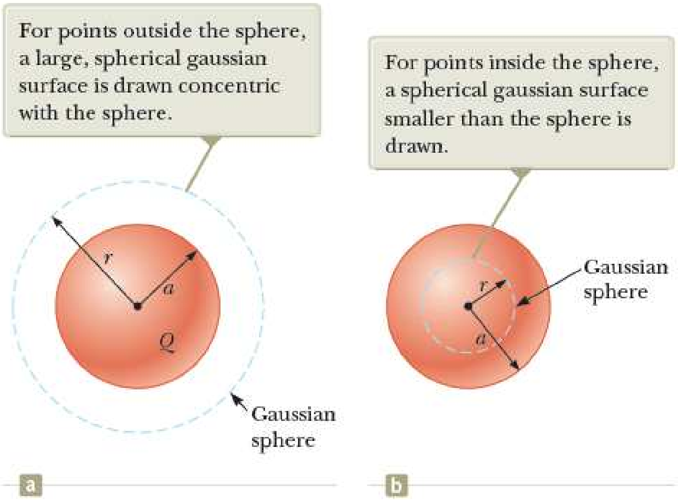


FIGURE 40. A uniformly charged insulating sphere of radius *a* and total charge *Q*. In diagrams such as this one, the dotted line represents the intersection of the gaussian surface with the plane of the page.

Conceptualize: In Chapter Electric fields, We found the field for various distributions of charge by integrating over the distribution. Here we find the electric field using Gauss's law.
Categorize: Because the charge is distributed uniformly throughout the sphere (Figure 40), the charge distribution has spherical symmetry and we can apply Gauss's law to find the electric field.
Analyze: To reflect the spherical symmetry, let's choose a spherical gaussian surface of radius r, concentric with the sphere, as shown in Figure a.

We use the equation $\boldsymbol{\Phi} = \oint \vec{\boldsymbol{E}} \,.\, d\vec{A} = \oint \vec{E}\, \Delta A_i \cos\theta$

As $\theta = 0$, $\cos 0 = 1$, $\Phi = \oint \vec{E} \,.\, \Delta\vec{A}i = \oint \vec{E}\, \Delta A_i$

From gauss law $\Phi = q/\varepsilon_0$

Equate the two above equations

$$\oint \vec{E}\, \Delta A_i = q/\varepsilon_0$$

By symmetry, E has the same value everywhere on the surface, so we can remove E from the integral

$$\vec{E} \oint \Delta Ai = q/\varepsilon 0$$

$$\vec{E}(4\pi r^2) = q/\varepsilon 0$$

$$E = \frac{1}{4\pi\varepsilon 0}\frac{q}{r^2}$$

$$\mathbf{E} = \frac{kq}{r^2} \qquad \textbf{for (r > a) i.e outside sphere}$$

Finalize: This field is identical to that for a point charge. Therefore, the electric field due to a uniformly charged sphere in the region external to the sphere is equivalent to that of a point charge located at the center of the sphere.

As r increases, E decreases. As r tends to zero, E tends to infinity, which is not physically possible, therefore we should have another equation for field inside the sphere

b) Analyze: In this case, let's choose a spherical Gaussian surface having radius r, a, concentric with the insulating sphere as shown in figure b. Let $V^{/l}$ be the volume of this smaller sphere. To apply Gauss's law in this situation, recognize that the charge q_{in} within the gaussian suface of volume $V^{/l}$ is less than Q. Calculate q_{in} by using

$$q_{in} = \rho V^{/} = \rho \frac{4}{3}\pi r^3$$

At the point on the surface, $\Phi = \oint \overrightarrow{E} . d\vec{A} = \Phi = E \oint d\vec{A}$

$$\Phi = E.\ 4\pi r^2$$

Apply Gauss's law , $\Phi = \frac{1}{\varepsilon 0} q_{in}$

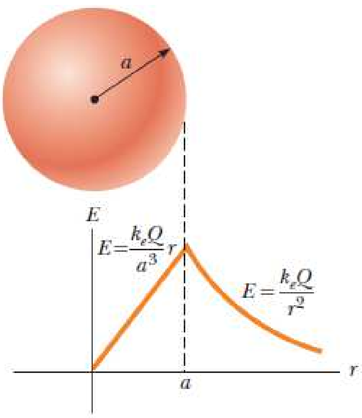
equating the two equations

$$E = \frac{\rho}{\varepsilon 0} r$$

Now $\rho = \frac{Q}{\frac{4}{3}\pi a^3}$

substitute $\rho = \frac{Q}{\frac{4}{3}\pi a^3}$ and $\varepsilon_0 = \frac{1}{4\pi k}$ in above eqn. we get

$$\boxed{\vec{E} = k \frac{Q}{a^3} r} \qquad (\text{ for } r < a)$$



E varies linearly with r, as r tends to zero, E tends to zero.

If r=a, i.e if point is on the surface, both the equations are applicable.

Using eqn $E = \frac{kq}{r^2}$, $E = \frac{kq}{a^2}$ for r = a

Using eqn $\vec{E} = k\frac{Q}{a^3} r$ $E = \frac{kq}{a^2}$ for r = a

A Cylindrically Symmetric Charge Distribution

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ

Solution:

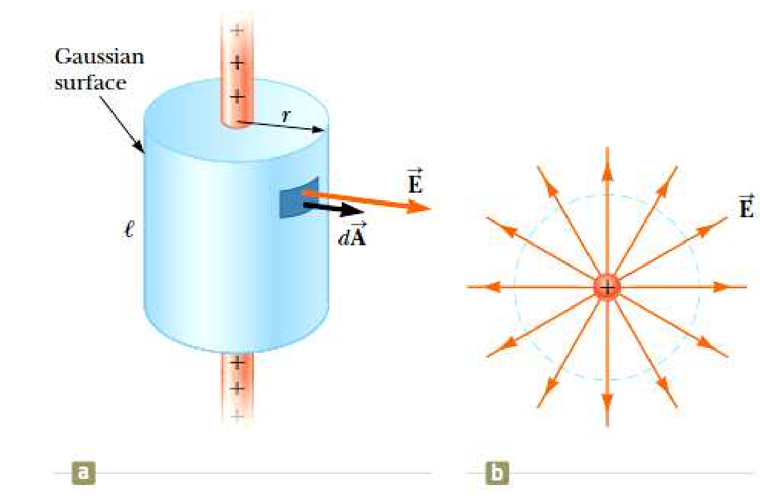


FIGURE 41. (a) An infinite line of charge surrounded by a cylindrical gaussian surface concentric with the line. (b) An end view shows that the electric field at the cylindrical surface is constant in magnitude and perpendicular to the surface.

Conceptualize: The line of charge is infinitely long. Therefore, the field is the same at all points equidistant from the line, regardless of the vertical position of the point in Figure 41, We expect the field to become weaker as we move farther away from the line of charge.

Categorize: Because the charge is distributed uniformly along the line, the charge distribution has cylindrical symmetry and we can apply Gauss's law to find the electric field.

Analyze: To reflect the symmetry of the charge distribution, let's choose a cylindrical Gaussian surface of radius r and length ℓ, that is coaxial with the line charge. For the curved part of this surface, $\vec{E}$ is constant in magnitude and perpendicular to the surface at each point. Furthermore, the flux through the ends of the Gaussian cylinder is zero because $\vec{E}$ is parallel to these surfaces.

Therefore $\vec{\mathbf{E}}.\overrightarrow{\mathbf{dA}}$ **is zero for the flat ends of the cylinder.**

We must take the surface integral in Gauss's law over the entire Gaussian surface.

On the curved Gaussian surface,

$$\boldsymbol{\Phi} = \oint \overrightarrow{\boldsymbol{E}} .\ d\vec{A} = \boldsymbol{\Phi} = \boldsymbol{E} \oint \ d\vec{A} = EA \quad \text{since E is constant on the g.surface.}$$

A is the total area of g.surface A= 2πr ℓ

$$\boldsymbol{\Phi} = EA = E\ 2\pi r\ \ell \qquad \ldots\ldots(i)$$

Applying gauss law, $\boldsymbol{\Phi} = \frac{\mathbf{1}}{\boldsymbol{\varepsilon 0}} Q$, where Q is total charge of a line charge, Q= λℓ

$$\boldsymbol{\Phi} = \frac{\mathbf{1}}{\boldsymbol{\varepsilon 0}} \lambda\ell \qquad \ldots\ldots..(ii)$$

Equate i and ii

$$\boxed{\overrightarrow{\boldsymbol{E}} = \mathbf{2k}\,\frac{\boldsymbol{\lambda}}{\boldsymbol{r}}}$$

Finalize: This result shows that the electric field due to a cylindrically symmetric charge distribution varies as $1/r$

A Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ.

Solution:

Conceptualize: The plane of charge is infinitely large. Therefore, the electric field should be the same at all points equidistant from the plane.

Categorize: Because the charge is distributed uniformly on the plane, the charge distribution is symmetric; hence, we can use Gauss's law to find the electric field.

Analyze: By symmetry, $\overrightarrow{\boldsymbol{E}}$ must be perpendicular to the plane at all points. The direction of $\overrightarrow{\boldsymbol{E}}$ is away from positive charges, indicating that the direction of $\overrightarrow{\boldsymbol{E}}$ on one side of the plane must be opposite its direction on the other side as shown in Figure 42. A Gaussian surface that reflects the symmetry is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area A and are equidistant from the plane. Because $\overrightarrow{\boldsymbol{E}}$ is parallel to the curved surface of the cylinder and therefore perpendicular to $\overrightarrow{d\ A}$ at all points on this surface, and there is no contribution to the surface integral from

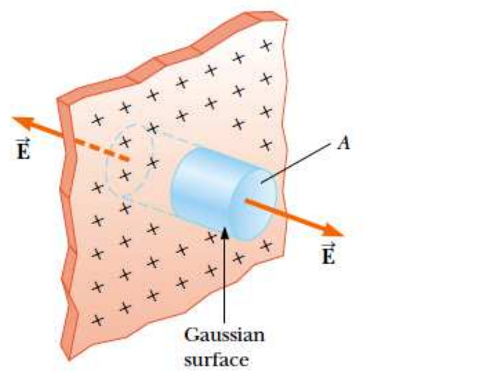


FIGURE 42 A cylindrical gaussian surface penetrating an infinite plane of charge. The flux is *EA* through each end of the gaussian surface and zero through its curved surface.

this surface. For the flat ends of the Cylinder, the flux through each end of the cylinder is EA; hence, the total flux through the entire Gaussian surface is

Φ = flux at the curved surface + flux at end A + flux at opposite end of A

$$= \vec{E}.\overrightarrow{dA}\cos 90^o + \vec{E}.\overrightarrow{dA}\cos 0^o + \overrightarrow{E}.\overrightarrow{dA}\cos 0^o$$

$$= 0 + \vec{E}.\overrightarrow{dA} + \overrightarrow{E}.\overrightarrow{dA}$$

$$= EA + EA$$

$$\Phi = 2EA \qquad \text{…………………….(i)}$$

Applying gauss law, $\Phi = \frac{1}{\varepsilon 0} Q$,

Here Q = σ A hence $\Phi = \frac{1}{\varepsilon 0} \sigma A$ ………………….(ii)

Equate i and ii

$$E = \frac{1}{2\varepsilon_0}\sigma$$

Finalize: Because the distance from each flat end of the cylinder to the plane does not appear in above Equation, we conclude that $E = \frac{1}{2\varepsilon_0}\sigma$ is applicable at any distance from the plane. That is, the field is uniform everywhere. Figure 42 shows this uniform field due to an infinite plane of charge

Conductors in Electrostatic Equilibrium

A good electrical conductor contains charges (electrons) that are not bound to any atom and therefore are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in **electrostatic equilibrium.** A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor, whether the conductor is solid or hollow.
2. If the conductor is isolated and carries a charge, the charge resides on its surface.
3. The electric field at a point just outside a charged conductor is perpendicular to the surface of the conductor and has a magnitude

   $$E=\sigma/\varepsilon_0,$$

   where σ is the surface charge density at that point.
4. On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature of the surface is smallest.

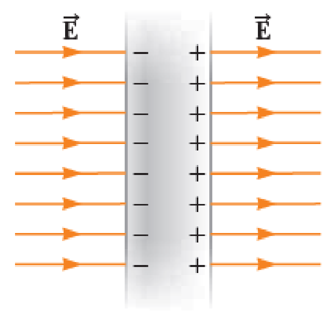


FIGURE 43. A conducting slab in an external electric field $\vec{E}$. The charges induced on the two surfaces of the slab produce an electric field that opposes the external field, giving a resultant field of zero inside the slab.

We can understand the first property by considering a conducting slab placed in an external field $\vec{E}$ (Fig. 43). The electric field inside the conductor *must* be zero, assuming electrostatic equilibrium exists. If the field were not zero, free electrons in the conductor would experience an electric force ($\vec{F} = q\vec{E}$) and would accelerate due to this force. This motion of electrons, however, would mean that the conductor is not in electrostatic equilibrium. Therefore, the existence of electrostatic equilibrium is consistent only with a zero field in the conductor.