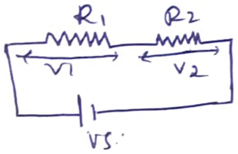
CHAPTER 2 :- D. C. Circuits.

1) Voltage Divider Rule:



$V = IR$

$V_s = V_1 + V_2$

$IR_s = IR_1 + IR_2$

$\boxed{R_s = R_1 + R_2}$

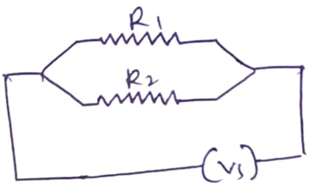
$V_1 = IR_1$

$= \frac{V_s \times R_1}{R_1 + R_2}$

$V_2 = IR_2$

$V_2 = \frac{V_s \, R_2}{(R_1 + R_2)}$

2) Current divider Rule



$V_p = V_1 = V_2$

$I = I_1 + I_2$

$\frac{V}{R_p} = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

$\frac{\cancel{V}}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

$I_1 = \frac{V}{R_1}$

$= \frac{I \, (\cancel{R_1} R_2)}{\cancel{R_1} \, (R_1 + R_2)} = \frac{IR_2}{(R_1 + R_2)}$

$I_2 = \frac{V}{R_2} = \frac{I \, (R_1 \cancel{R_2})}{(R_1 + R_2) \cancel{R_2}} = \frac{IR_1}{(R_1 + R_2)}$

Kirchoffs laws

1) Kirchoff's Junction Law

at particular junction, sum of incoming currents is equal to out going current.

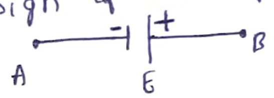
$I_3 \quad I_2 \quad I_1 \rightarrow I_1 + I_2 + I_3 = 0$

2) Kirchoff's voltage Law.

at any instant in a closed loop the algebraic sum of EMF's acting around the loop is equal to the algebraic sum of potential drop/difference round the loop.

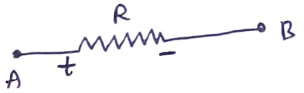
→ **Sign Conventions**

a) Sign of battery EMF



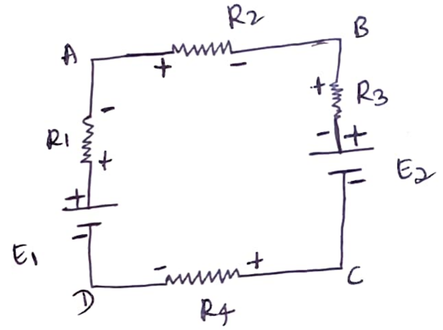
$A \longrightarrow B$ E $A \longleftarrow B$ $-E$

b) Sign of voltage drop across resistor



$A \longrightarrow B$ $-IR$. $A \longleftarrow B$ IR.

Example:



$-IR_2 - IR_3 - E_2 - IR_4 + E_1 - IR_1 = 0$

$-I(R_1+R_2+R_3+R_4) = E_2 - E_1$

× by (−)

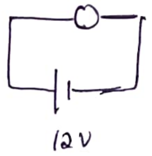
$I(R_1+R_2+R_3+R_4) = E_1 - E_2$

Numericals:

A Simple Circuit is formed using 12V lead acid battery and automobile headlight If the battery delivers a total energy
460.8 J watt hour over an 8 hr. discharge period.

a) How much power is delivered to the headlight.

b) what is the current flowing through the bulb (assume battery voltage remains const during disch



a) power $= \dfrac{E}{t} = \dfrac{460.8 \text{ watt hour}}{8 \text{ hr}}$

$P = 57.6\,W$.

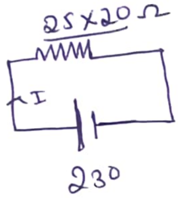
b) $P = V \times I$

$57.6 = 12 \times I$

$I = \dfrac{57.6}{12}$ = 4.8

$I = 4.8\,A$.

2) The lamps in a set of Christmas are connected in series and each lamp has resistance of 25 Ω. If there are 20 lamps and each lamp. Calculate the (a) total resistance of set of lamps and hence calculate the current and power taken from 230V supply



a) $T_R = 500\,\Omega$.

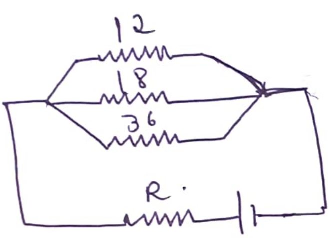
b) current $I = \frac{230}{500} = 0.46$ A

c) power $= V \times I = 2.30 \times 0.46$

$= 23 \times 4.6$

$= 105.8$ W.

3) A circuit comprising of 3 resistances, 12 Ω, 18 Ω, 36 Ω joined in parallel is connected in series with 4th resistance. The whole circuit is supplied at 60V and it is found that the power dissipated in the 12 Ω resistance is 36W. determine value of 4th resistance and total power dissipated in the group



$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{18} + \frac{1}{36}$.

$\frac{1}{R_{eq}} = \frac{3+2+1}{36} = \frac{6}{36} = \frac{1}{6}$

$R_{eq} = 6\,\Omega$

$I = V/(R+6)$

$V = I(R+6)$

$60 = I(R+6)$

$I_1 12 = 18 I_2 = 36 I_3$

$2I_1 = 3I_2 = 6I_3$.

$I_2 = I_1 \frac{2}{3}$, $I_3 = \frac{I_1}{3}$.

P in 12 Ω

$P = V_1 \times I_1$

$36 = 6 I \times I_1$

$i_1 = \frac{6}{I}$.

$60 = IR + I(6)$

$60 = I(6+R)$

$I = \frac{60}{(6+R)}$

$P_{12} = \frac{V^2}{R}$

$36 = \frac{V^2}{12}$

$36 \times 12 = V^2$

$V = \sqrt{36 \times 12}$

$= 6\sqrt{12}$

$V = 12\sqrt{3}\,\Omega$

$i_1 = \frac{12\sqrt{3}}{12} = 1.73$.

$i_2 = \frac{12\sqrt{3}}{18} = 1.154$

$i_3 = \frac{12\sqrt{3}}{36} = \frac{1}{\sqrt{3}} = 0.57$

$I = 3.45$.

$P_T = V \times I$

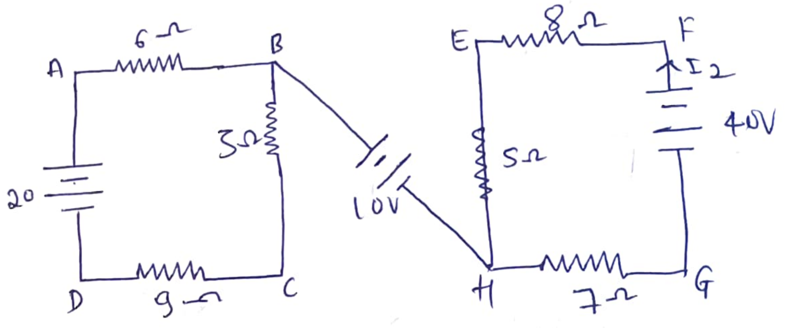
$= 60 \times 3.45$.

$= 207$ W.

power dessipated in parallel group $= 12\sqrt{3} * 3.45$

$= 71.706$ W.

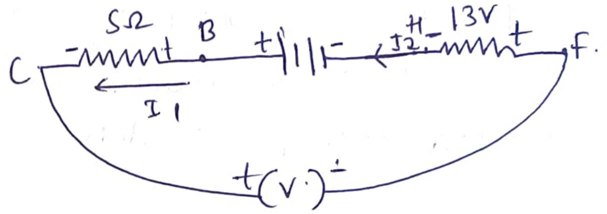
4)



Circuit.

For the given, voltage across points C & F and A & G

1) C and F



$$V_{CF} + I_1 * 5 + 13 I_2 - 10 = 0$$

$$V_{CF} + I_1 * 5 + 13 I_2 = 10.$$

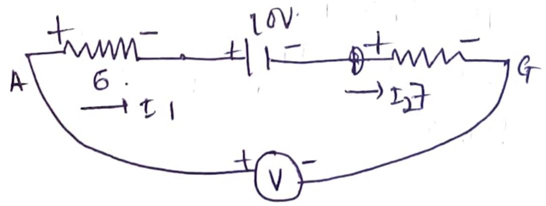
$$V_{CF} + 5*1 + 13 \times 2 = 10$$

$$V_{CF} = 10 - 26 - 5.$$

$$V_{CF} = 10 - 31$$

$$\boxed{V_{CF} = -21 \text{ V}}$$

2) A and G



$$V_{AG} - 6 I_1 - 10 - I_2 * 7 = 0$$

$$V_{AG} = 6 I_1 + 10 + 7 I_2.$$

$$= 6*1 + 10 + 7*2$$

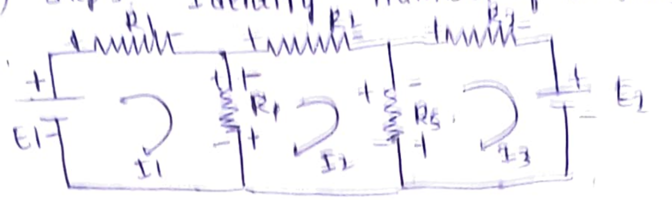
$$= 6 + 10 + 14$$

$$= 16 + 14$$

$$\boxed{V_{AG} = 30 \text{ V}}$$

Maxwells circulating current Method / Maxwells loop current Method.

1) Step 1: Identify number of meshes



2) Step 2: Assign the current in clockwise direction for each mesh.

Step 3: Apply KVL for each mesh

Mesh 1:

$$-I_1R_1 - I_1R_4 + I_2R_4 + E_1 = 0$$

$$E_1 = I_1(R_1 + R_4) - I_2R_4$$

Mesh 2:

$$-I_2R_2 - I_2R_5 + I_3R_5 - I_2R_4 + I_1R_4 = 0$$

$$0 = I_1R_4 + I_3R_5 - I(R_2 + R_5 + R_4)$$

Mesh 3:

$$-I_3R_3 - E_2 - I_3R_5 + I_2R_5 = 0$$

$$-E_2 = I_3R_5 + I_3R_3 - I_2R_5$$

$$-E_2 = I_3(R_5 + R_3) - I_2R_5$$

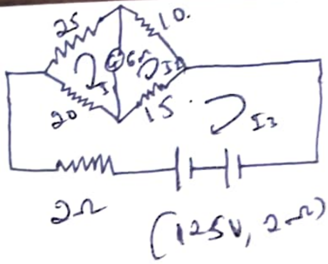
Step 4:

Matrix form

$$V = RI$$

$$\begin{bmatrix} E_1 \\ 0 \\ E_2 \end{bmatrix} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} R_1+R_4 & -R_4 & -0 \\ -R_4 & R_2+R_4+R_5 & -R_5 \\ -0 & & R_3+R_5 \end{bmatrix} \end{array} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

1)



find the current through the galvanometer 'G' in the wheatstone bridge in figure. Assume $G_R = 5\Omega$

$V = RI$

$$\begin{bmatrix} 0 \\ 0 \\ 25 \end{bmatrix} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 25+5+20, & -5 & -20 \\ 2 & -5 & 10+20 & -15 \\ 3 & -20 & -15 & 20+15+4 \end{array} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 25 \end{bmatrix} = \begin{bmatrix} 50, & -5, & -20 \\ -5 & 30 & -15 \\ -20 & -15 & 39 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

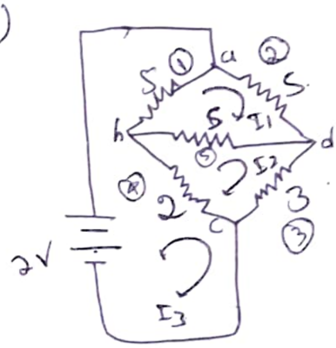
$I_1 = 0.548 \quad I_2 = 0.68, \quad I_3 = 1.18.$

$I_2 > I_1$

$I_G = 0.68 - 0.54 = 0.14A.$

$\boxed{I_G = 0.14A}$

2)



Use mesh analysis to determine the current through each resistor and determine the voltage across A and C and b&d.

$V = RI$

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 15 & -5 & -5 \\ -5 & 10 & -2 \\ -5 & -2 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$I_1 = 0.26 A$

$I_2 = 0.24 A$

$I_3 = 0.54 A.$

current through 1st resistor

$I_1 - I_3 = 0.26 - 0.54 = -0.28$.

Current through 4th Resistor

$I_3 - I_2 = 0.54 - 0.24$

$= 0.30\ A$.

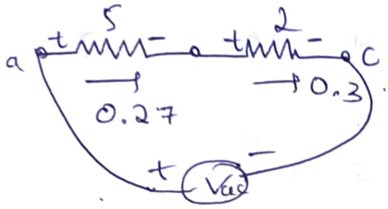
Current through 5th resistor

$I_1 - I_2 = 0.02A$.

current through 2nd Resistor & 3rd Resistor

$I_1 = 0.26\ A$. $I_3 = 0.24\ A$

V. across A & C & b & d.



$V_{ac} - 5 \times 0.27 - 0.3 \times 2 = 0$

$V_{ac} = 5 \times 0.27 + 0.3 \times 2$

$= 1.35 + 0.6$

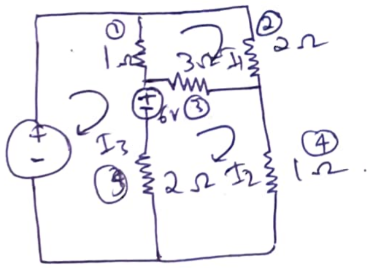
$\boxed{V_{ac} = 1.95\ V}$

$V_{Bd} = (I_1 - I_2)\ 5$

$= 0.02 \times 5$

$= 0.10\ V$.

3)



we mesh analysis to find current in each Resistors

$V = RI$

$$\begin{bmatrix} 0 \\ 6V \\ 1 \end{bmatrix} = \begin{bmatrix} 3+2+1 & -3 & -1 \\ -3 & 3+2+1 & -2 \\ -1 & -2 & 1+2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 & -1 \\ -3 & 6 & -2 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$I_1 = 2$ $I_2 = 3$ $I_3 = 3$.

I in 1st R = 3 A − 2 = 1 A

2nd R = 2 A

3rd R = $I_1 - I_2 = 2 - 3 = -1A$.

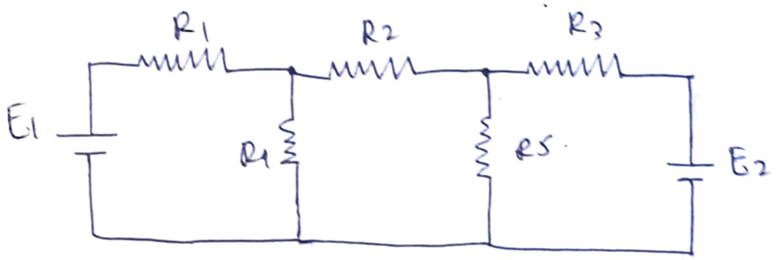
4th R = 3A

5th R = $I_3 - I_2$

$= 3 - 2$

$= 1A$.

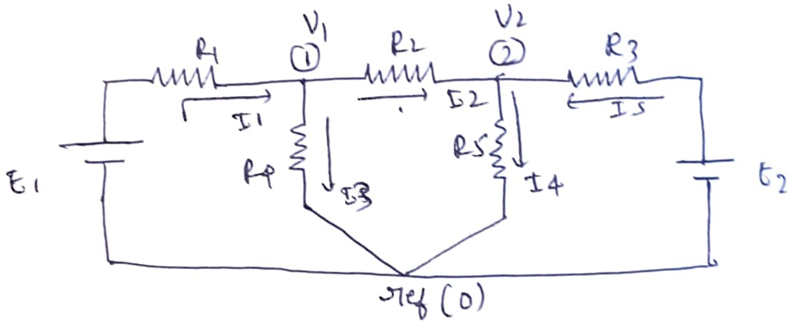
Nodal Analysis :



Step 1: Identify the number of Junctions.

Step 2: Assign Voltage to the Junctions and branch currents.

Step 3: Apply KCl at each Junction



Junction 1,

$$I_1 = I_2 + I_3$$

$$\frac{E - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_4}$$

$$\frac{E}{R_1} = \frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + \frac{V_1}{R_4}$$

$$= V_1\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right] + \left(\frac{-V_2}{R_2}\right] \quad (1)$$

Junction 2.

$$I_4 = I_2 + I_5$$

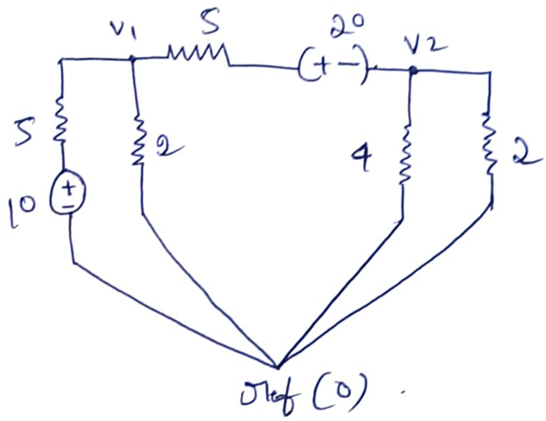
$$I_5 = I_4 - I_2$$

$$\frac{E_2 - V_2}{R_3} = \frac{V_2}{R_5} - \left(\frac{V_1 - V_2}{R_2}\right)$$

$$\frac{E_2}{R_3} = \frac{V_2}{R_3} + \frac{V_2}{R_5} - \frac{V_1}{R_2} + \frac{V_2}{R_2} \rightarrow \frac{E_2}{R_3} = V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) +$$

$$\begin{bmatrix} \frac{E1}{R_1} \\ \frac{E2}{R_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4} & \left[-\frac{1}{R_2}\right] \\ -\left[\frac{1}{R_2}\right] & \left[\frac{1}{R_5}+\frac{1}{R_3}+\frac{1}{R_2}\right] \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

①



$$\begin{bmatrix} \frac{10}{5}+\frac{20}{5} \\ -20/5 \end{bmatrix} = \begin{bmatrix} \frac{1}{5}+\frac{1}{5}+\frac{1}{2} & -1/5 \\ -1/5 & \frac{1}{5}+\frac{1}{2}+1/4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ -4 \end{bmatrix} = \begin{bmatrix} 2/5+1/2 & -1/5 \\ -1/5 & \frac{1}{5}+1/2+1/4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$