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ENGINEERING MECHANICS

UNIT-II Chapter – 5

STATIC FRICTION

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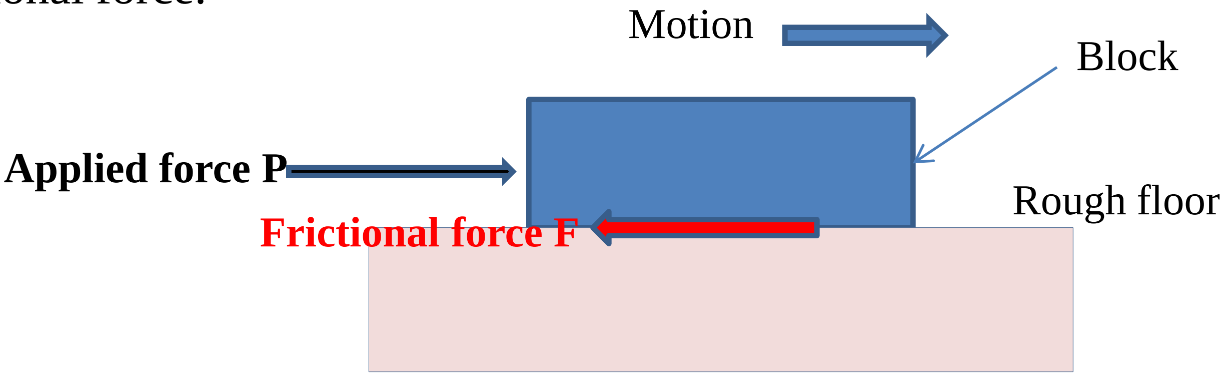
What is Friction?

When a body slide or tend to slide on a surface it is resting , a resisting force opposing the motion is produced at the contact surface. This resisting force is called friction or frictional force.



In the absence of friction, it would be impossible for us to walk, ride a vehicle or pick some object from floor.

In some cases frictional force causes energy loss, which leads to wear & tear of components which are in contact. In such cases frictional force must be reduced.

❖To reduce friction, lubricants are used in machines.

❖Fine powder on the carrom board reduces friction.

Thus,

“Friction can be our friend or it can be an evil depending upon the circumstance”

Types of friction

In general, friction can be classified as :

I. Dry Friction: It is the force that opposes motion of one solid surface sliding across another solid surface which are in contact.

II. Fluid friction: It is the force that resists motion either within the fluid itself or of another medium moving through the fluid.

Further **Dry Friction** is classified into following two types

1. Static friction
2. Dynamic friction

    i. Sliding friction

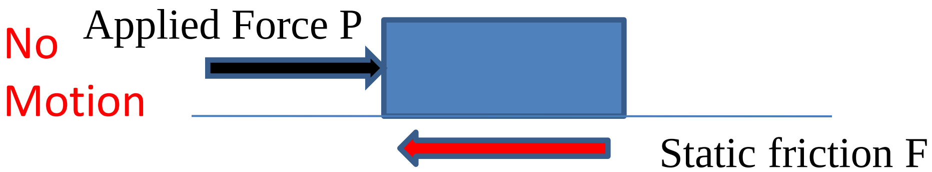
    ii. Rolling friction

1. **Static friction**

Friction experienced by the body when it is at rest is called static friction.

Here applied force is less than the limiting friction, hence the body remains at rest.

Ex: Pushing the wall.



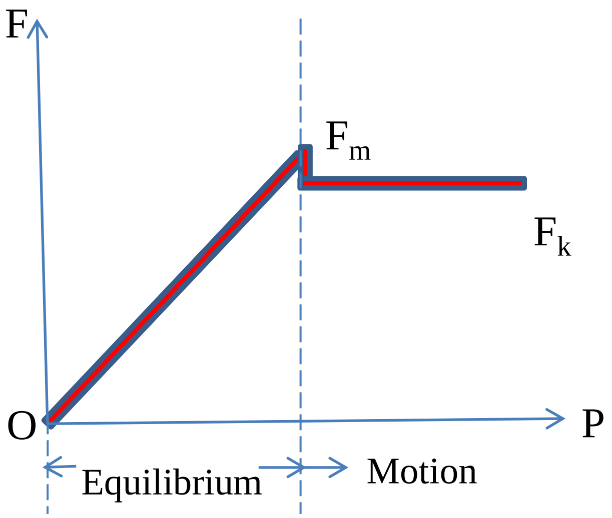
Limiting friction

The maximum friction force that can be developed at the contact surface, when body is just on the point of moving is called as limiting friction.

Small horizontal force P applied to block will produce static frictional force F and block remain stationary, in equilibrium.

As P increases, the static-frictional force F increases as well until it reaches a maximum value F_m which is limiting frictional force.

Further increase in P causes the block to begin to move as F drops to a smaller dynamic frictional force F_k. and it remains constant further.



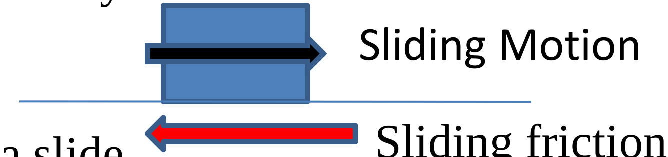
2. Dynamic friction (F_k)

Friction experienced by the body when it is in motion is called dynamic friction.

Here applied force exceeds the limiting friction, hence the body starts moving over another body.

i. Sliding friction

Friction experienced by the body when it sides over the other body is called as sliding friction.



Ex: A person sliding down a slide.

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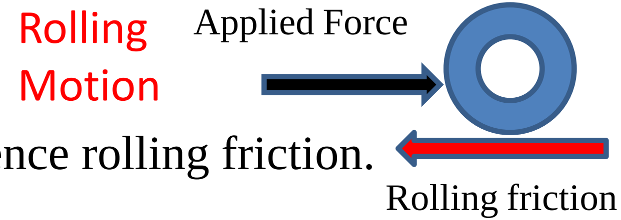
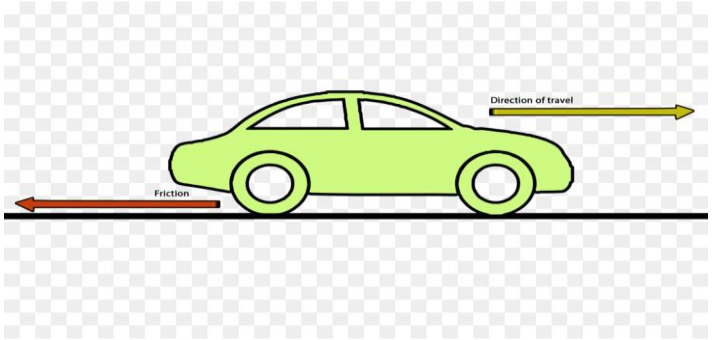
ii. Rolling friction

Friction experienced by the body when it rolls over the other body is called as rolling friction.



Ex: When car moves on the floor, wheels experience rolling friction.

Roller skates used for skating.

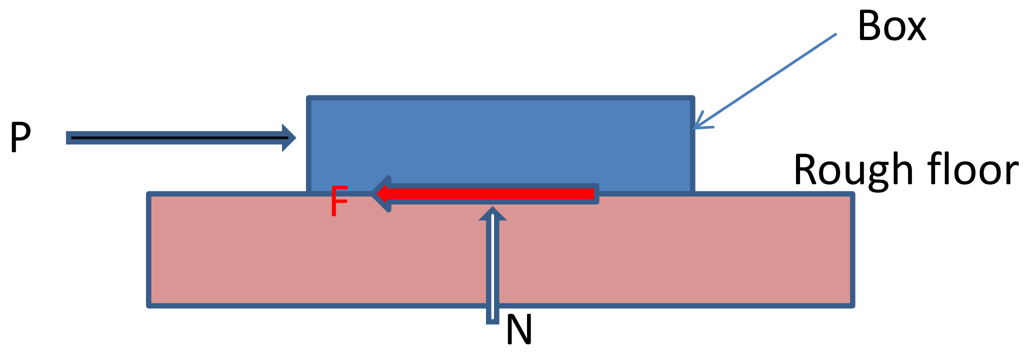


Coefficient of friction (μ):-

It is the ratio of limiting frictional force and the normal reaction between the objects. Mathematically it is expressed as;

Coefficient of friction = F/N

μ = F/N



P = Applied force

F = Frictional force

N = Normal reaction

Laws of friction:

Coulomb conducted some experiments & the results of these experiments are known as laws of dry friction or Coulomb’s laws of friction.

1. The frictional force always acts in the direction opposite to that in which the body tends to move.
2. Till the limiting value is reached, the magnitude of frictional force is exactly equal to the applied force which tends to move the body.

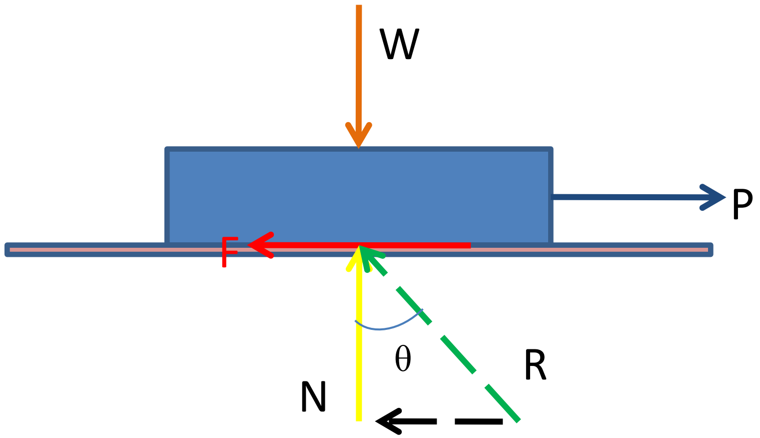
3. The magnitude of liming friction bears a constant ratio to the normal reaction between the two surfaces of contact and this ratio is called coefficient of friction.

4. The force of friction depends on roughness or smoothness of the surfaces.

5. The force of friction is independent of the area of contact between the two surfaces.

6. After the body starts moving , the dynamic friction comes into play, the magnitude of which is less than that of liming friction and it bears a constant ratio with normal reaction. This ratio is called **coefficient of dynamic friction**.

Angle of Friction:



Consider a block subjected to pull ‘P’ as shown in the above figure. Let ‘F’ be the frictional force developed and ‘N’ be the normal reaction. At contact surface, the reactions are ‘F’ & ‘N’. These two reactions can be combined to get the resultant reaction ‘R’ which acts at angle ‘θ’ to the normal reaction.

Thus, $\tan\theta = F/N$

As the frictional force increases, the angle θ increases & it can reach a maximum value 'α' when limiting value of friction is reached.

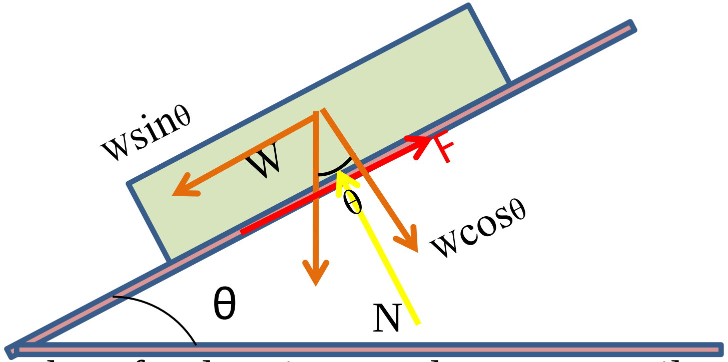
Thus, $\tan\alpha = F_m/N = \mu$

Therefore, $\tan\alpha = \mu$ -----(a)

This value of 'α' is called as **angle of friction.**

The angle of friction is defined as that angle made by the resultant reaction 'R' with the normal component of reaction 'N' when friction reaches its maximum value..

Angle of Repose:



We know that when grains such as food grains, sand, cement, soil etc are heaped, there exists a limit for the inclination of the heap. Beyond which the grains starts rolling down. The limiting angle up to which the grains repose is called as angle of repose.

Consider a block of weight ‘W’ which is resting on an inclined plane which makes an ‘θ’ with the horizontal.

When ‘θ’ is small, block rests on an inclined plane. As ‘θ’ is increased gradually, at one particular stage, the block starts sliding. The angle made by the plane with the horizontal is called angle of repose for the contact surfaces. Thus “the maximum inclination of the plane on which a body free from external forces, can repose is called **Angle of Repose”**.

Now ***to prove that angle of repose is equal to limiting angle of friction*** consider equilibrium of block which is show in the above figure.

Applying equilibrium equations:

$\sum Fx = 0$ i.e Forces parallel to plane = 0

$F - W\sin\theta = 0$

$F = W\sin\theta$ -------- (i)

$\sum Fy = 0$ i.e Forces normal to plane = 0

$N - W\cos\theta = 0$

$N = W\cos\theta$ ------- (ii)

Let ϕ be the value of 'θ' such that the maximum frictional force F_m is called into play. and the motion is impending hence

$$\tan\phi = F_m/N$$

$$\tan\phi = W\sin\phi / W\cos\phi = \mu$$

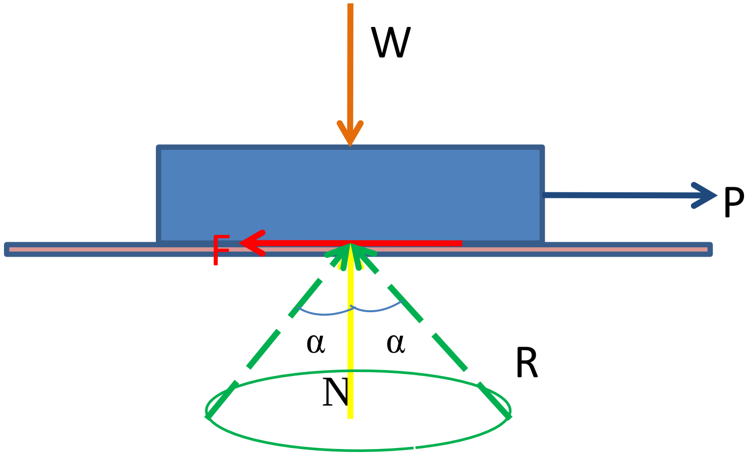
$$\tan\phi = \mu \text{ ------(b)}$$

From (a) and (b) we have $\tan\alpha = \tan\phi$

=> $\alpha = \Phi$

Thus value of limiting angle of friction is same as the angle of repose

Cone of friction:



When the body is having impending motion in the direction of applied force ‘P’, the frictional force will be limiting friction & the resultant reaction ‘R’ will make limiting frictional angle α with the normal reaction.

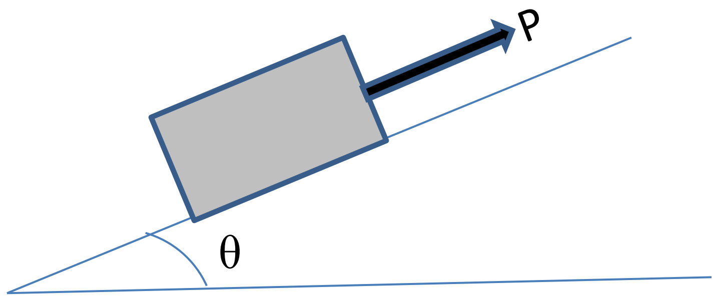
If the body is having impending motion in some other direction, the resultant reaction ‘R’ will make limiting frictional angle α with the normal reaction.

Thus, if the direction of force ‘P’ is gradually changed through 360^0, the resultant ‘R’ generates a right circular cone with semi central angle equal to ‘α’.

If the resultant reaction ‘R’ lies on the surface of this inverted cone whose semi central angle is limiting frictional angle ‘α’ the motion of the body is impending. If the resultant is within this cone the body is stationary. This inverted cone with semi central angle equal to limiting frictional angle ‘α’ is called **Cone of Friction.**

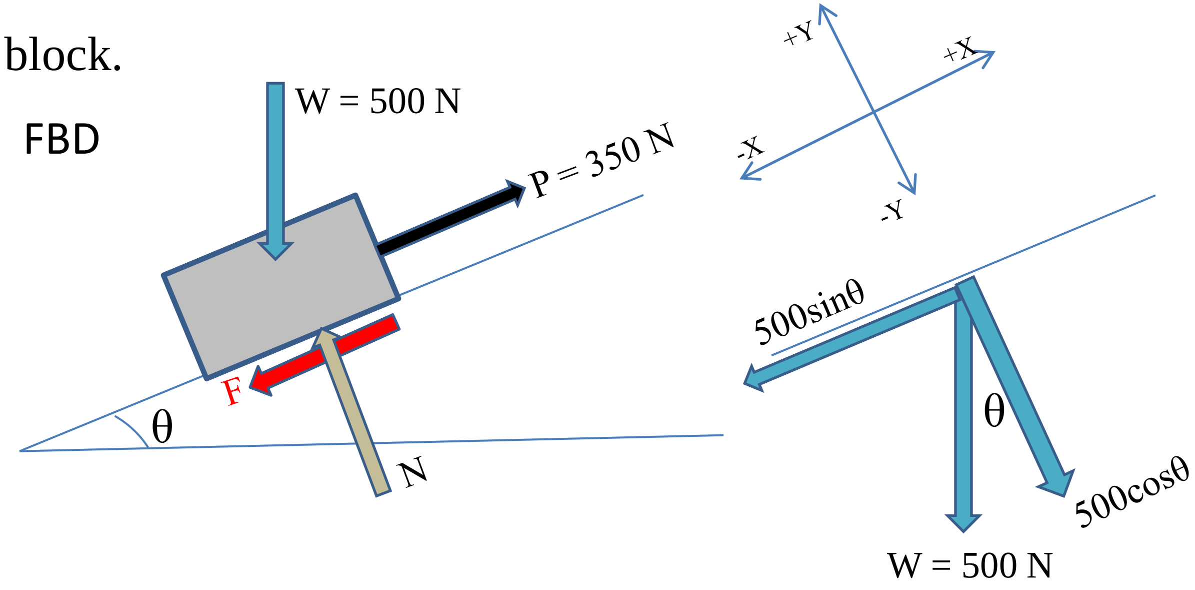
Problem 1.0

A body of weight W = 500 N is pulled up an inclined plane by a force of P = 350 N. The inclination of the plane is $\theta = 30^0$ to the horizontal & the force is applied parallel to the plane. Determine the coefficient of friction.



Solution:

Let us first draw the Free body diagram (FBD) and understand different forces and reactions acting on the block.



Since the applied force is pulling the bock up the plane it is at impending motion. Resolving forces in X and Y directions.

$\sum Fx = 0$

$P – F – 500\sin\theta = 0$

$350 – F – 500\sin 30 = 0$

$F = 100$ N

$\sum Fy = 0$

$-500\cos 30 + N = 0$

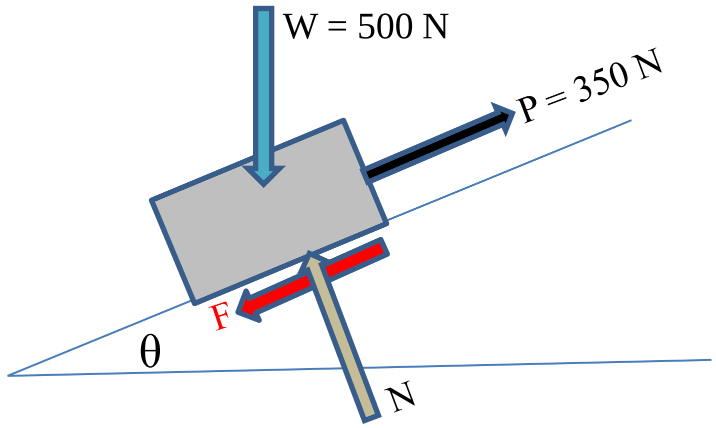
$N = 433.012$ N

At limiting equilibrium condition

Coefficient of friction, **μ = F/N**

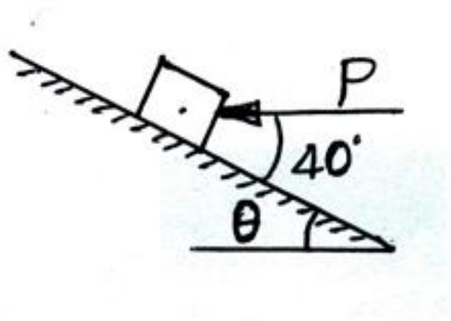
$\mu = 100/433.012$

$\mu = 0.2309$



Problem 2.0

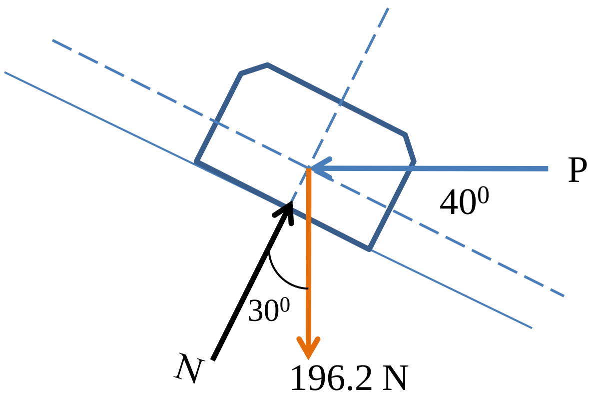
The coefficient of friction between 20 kg block and incline are $\mu_S = 0.4$ and $\mu_K = 0.3$. Determine whether the block is in equilibrium if P= 400 N & $\theta = 30°$ as shown and find the magnitude & direction of friction force.



Solution:

Let us first draw the Free body diagram (FBD) and understand different forces and reactions acting on the block.

FBD



Net force acting along the plane,

196.2sin30 – 400cos40

= -208.31 N (acting up the plane)

Hence the body is having tendency to move up the plane.

For equilibrium frictional force required is F_{req} = 208.31 N acting Down the plane.

Net force acting perpendicular to the plane,

-196.2cos30 – 400sin40 + N = 0

N = 427 N

Frictional force at limiting equilibrium condition is

$F_m = \mu_s N$

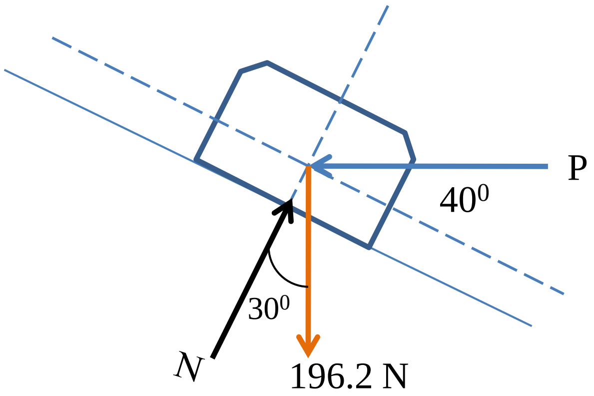
$F_m = 0.4*427$

$= 170.80$ N

Since $F_m < F_{req}$ the block will not be in equilibrium and is moving up the plane.

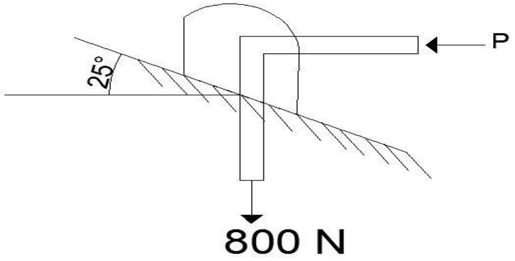
Hence, the frictional force during the motion, $F_k = \mu_k N = 0.3 * 427 = 128.10$ N down the plane.

FBD



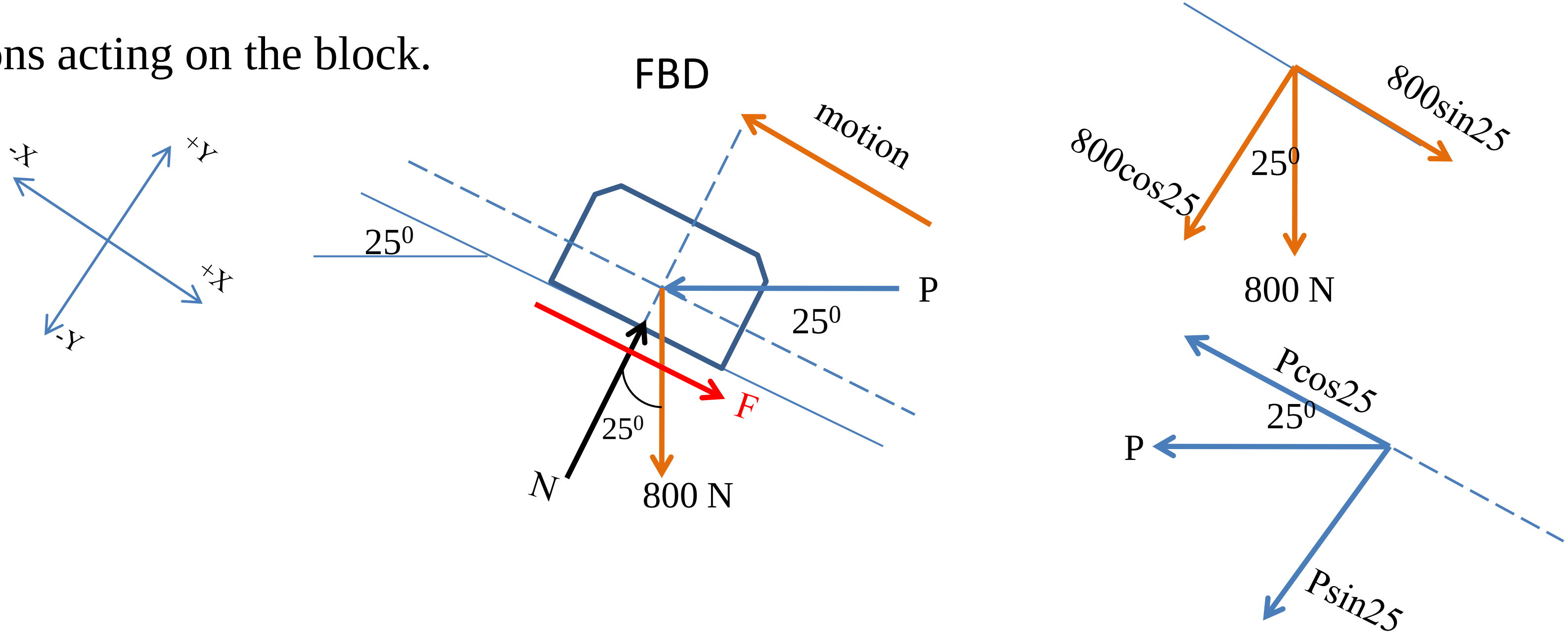
Problem 3.0

A block is acted upon by two forces as shown. Knowing that the coefficients of friction between the block and the incline are $\mu_S = 0.35$ and $\mu_K = 0.25$, determine the force P required (a) to start the block moving up the incline, (b) to keep it moving up, (c) to prevent it from sliding down.



Solution: (a) to start the block moving up the incline

Let us first draw the Free body diagram (FBD) and understand different forces and reactions acting on the block.



The body is in static condition hence $F_m = \mu_s N$

$\sum Fx = 0$

$F + 800\sin25 – P\cos25 = 0$

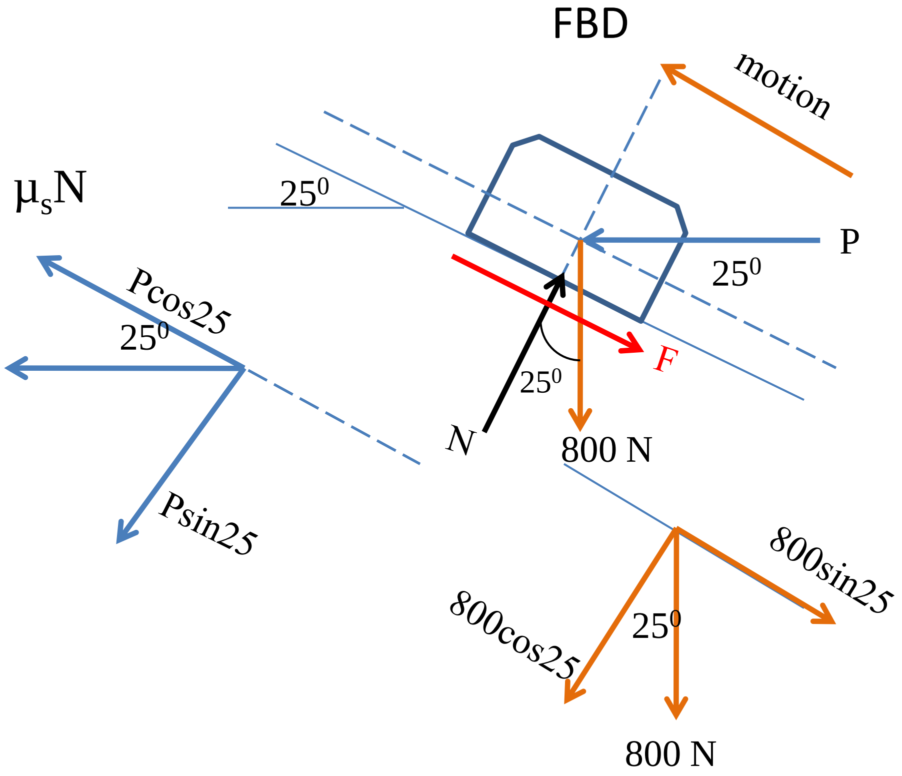
$F = P\cos25 – 800\sin25$

WKT $F = \mu_s N = 0.35N$

$0.35N = P\cos25 – 800\sin25$ --------(i)

$\sum Fy = 0$

$N – 800\cos25 – P\sin25 = 0$



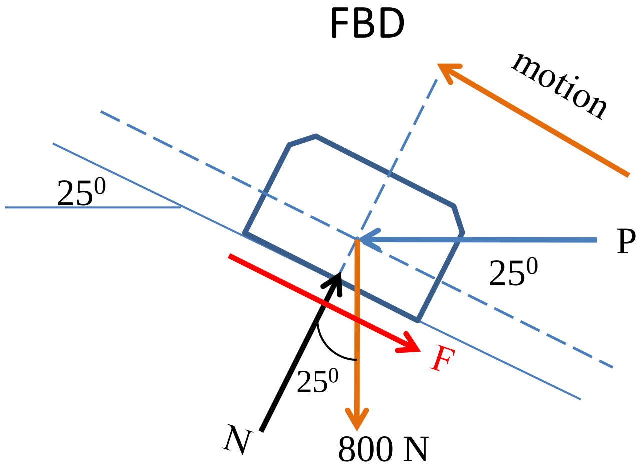
$N = 800\cos25 + P\sin25$ -----(ii)

Substituting (ii) in (i)

$0.35[800\cos25 + P\sin25] = P\cos25 – 800\sin25$

$0.758P = 591.86$

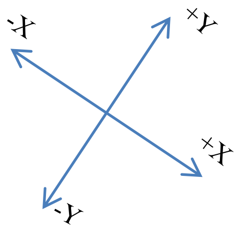
$P = 780.82$ N



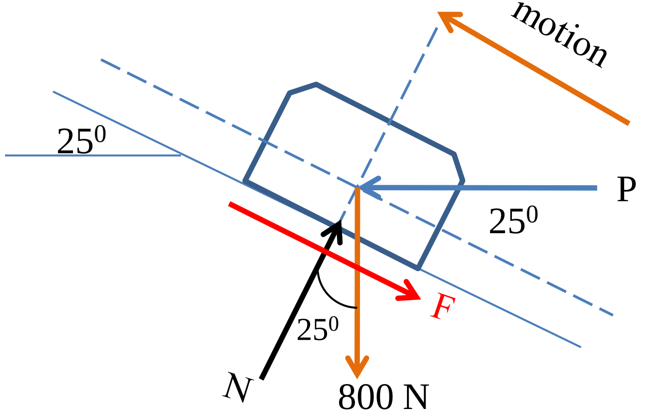
(b) To keep it moving up:

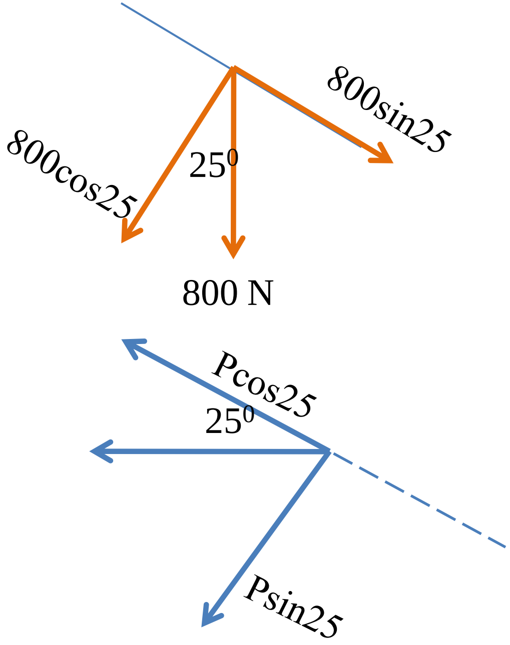
Since body is moving up, coefficient of dynamic friction should be considered.

$F_k = \mu_k N$



FBD





$\sum Fx = 0$

$F + 800\sin25 - P\cos25 = 0$

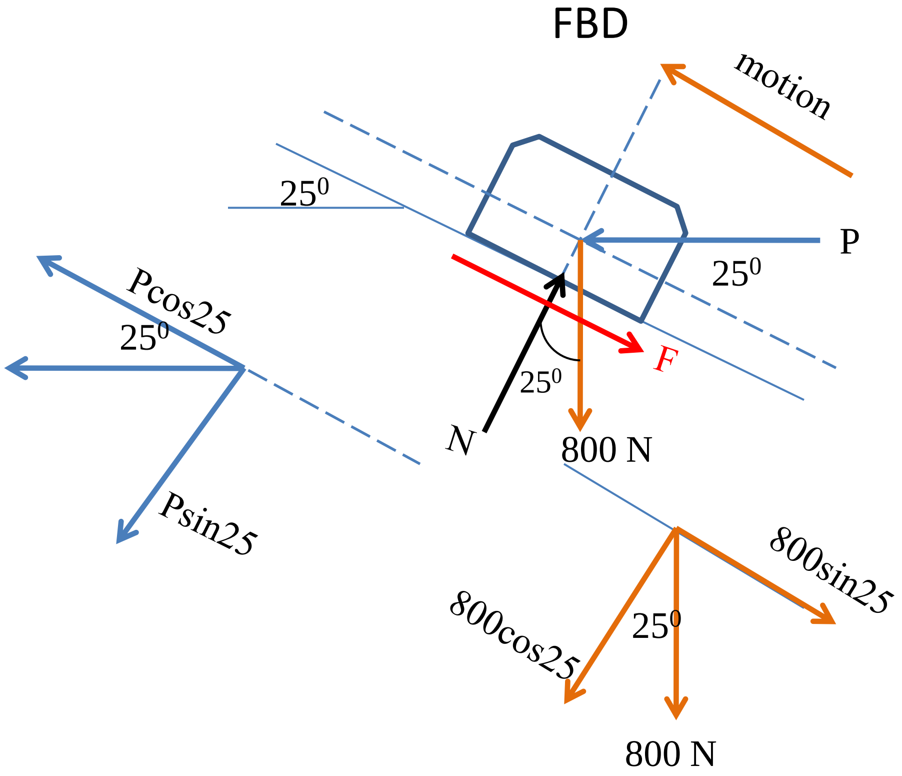
$F = P\cos25 - 800\sin25$

WKT $F = \mu_k N = 0.25N$

$0.25N = P\cos25 - 800\sin25$ --------(i)

$\sum Fy = 0$

$N - 800\cos25 - P\sin25 = 0$



$$N = 800\cos 25 + P\sin 25 \quad \text{-----(ii)}$$

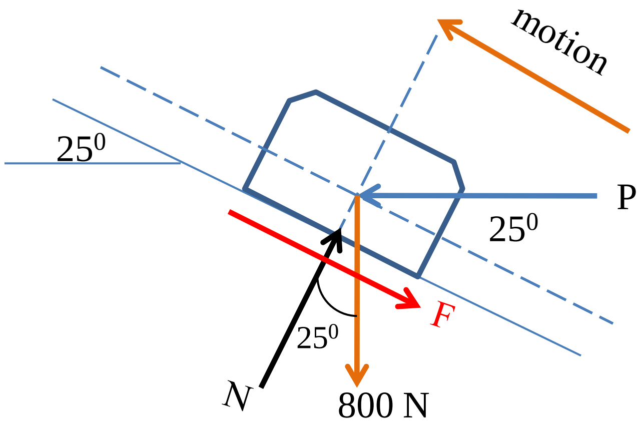
Substituting (ii) in (i)

$$0.25[800\cos 25 + P\sin 25] = P\cos 25 - 800\sin 25$$

$$0.8P = 519.351$$

$$P = 649.188\ N$$

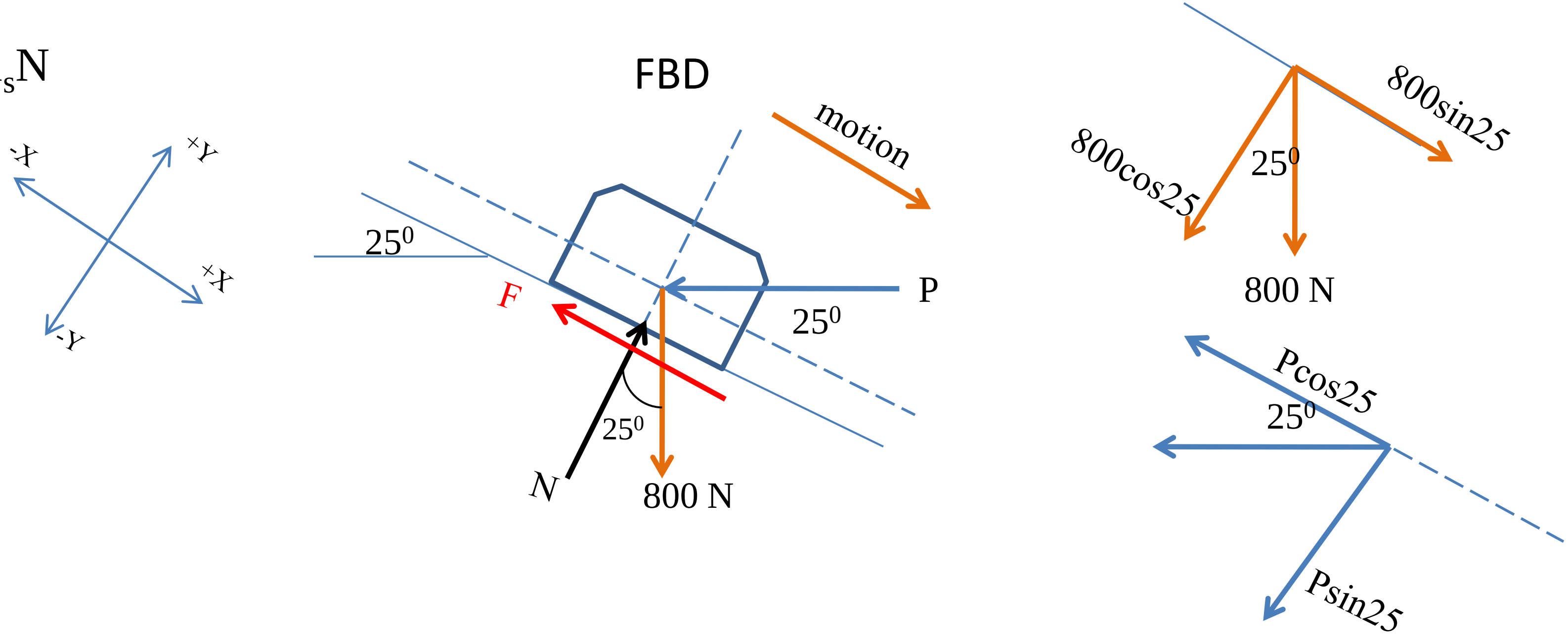
FBD



(c) To prevent it from sliding down:

To prevent it from sliding down means the block should be under static condition.

$F_m = \mu_s N$



$\sum Fx = 0$

$-F + 800\sin25 – P\cos25 = 0$

$F = 800\sin25 - P\cos25$

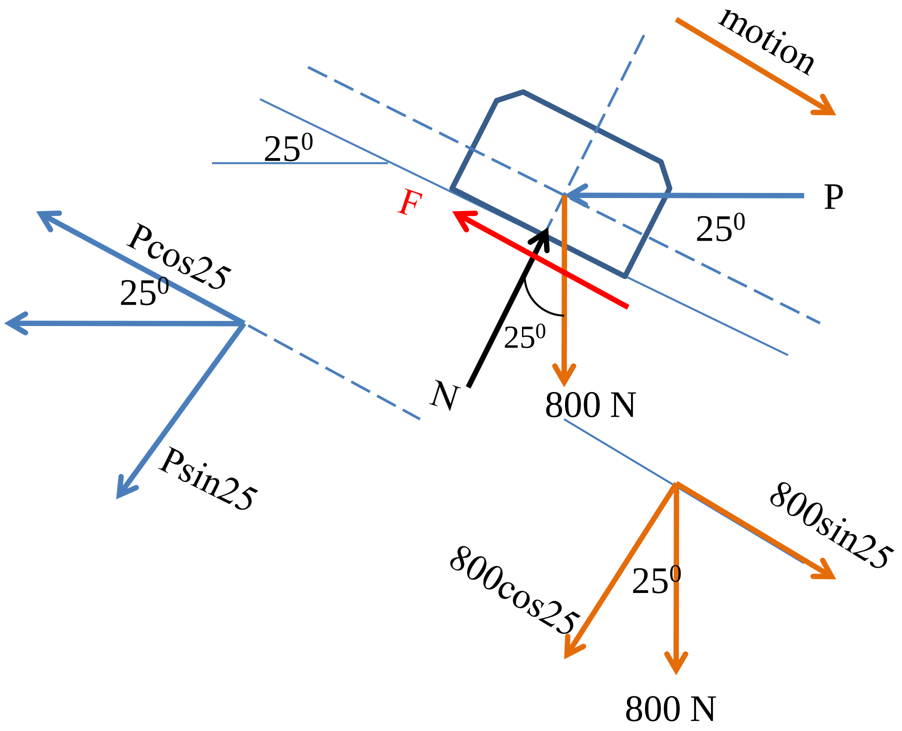
WKT $F = \mu_s N = 0.35N$

$0.35N = 800\sin25 - P\cos25$ ---------(i)

$\sum Fy = 0$

$N – 800\cos25 – P\sin25 = 0$

FBD



N = 800cos25 + Psin25   -----(ii)

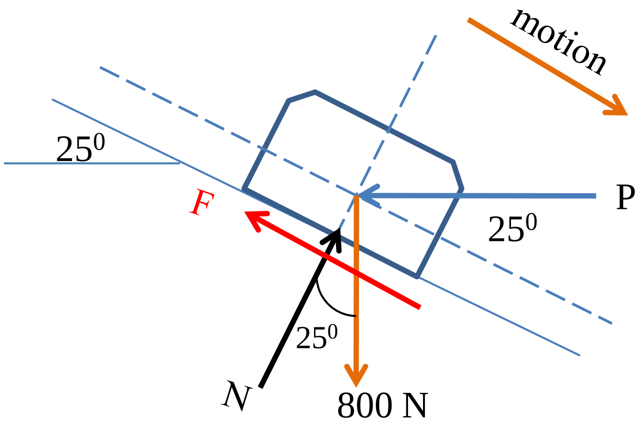
Substituting (ii) in (i)

0.25[800cos25 + Psin25] = 800sin25 - Pcos25

1.054P = 84.328

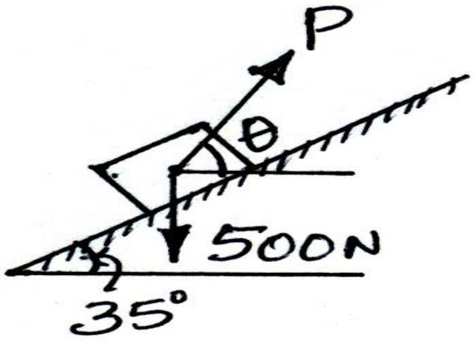
P = 80 N

FBD



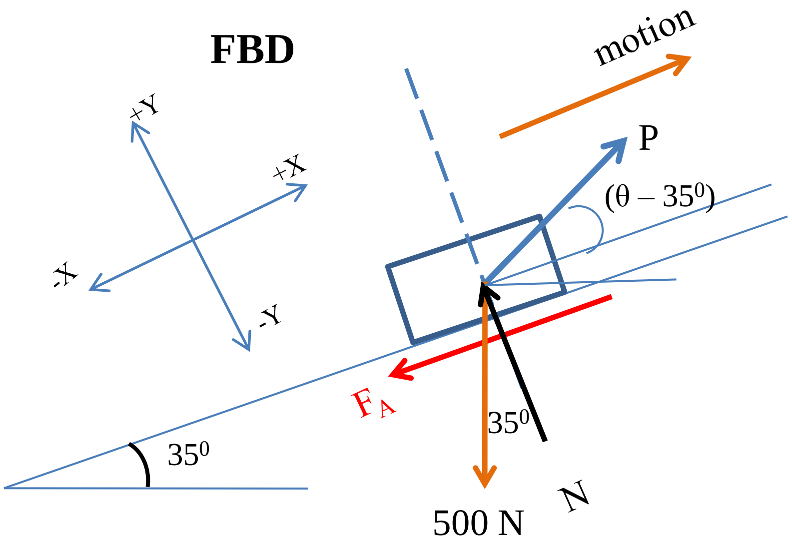
Problem 4.0

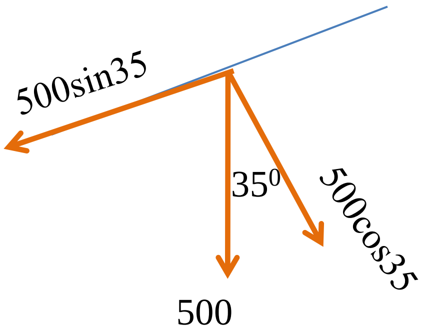
The coefficients of friction between the block and the rail are $\mu_S = 0.3$ & $\mu_K = 0.25$. Find the magnitude and direction of the smallest force P required to (i) start the block up the rail and (ii) to keep it from moving down.



Solution: Case(i) to start the block moving up the incline

Considering the limiting equilibrium, the FBD of block is as shown. Since the motion impends up the plane the frictional force acts down the plane.





Resolving the forces perpendicular to the plane,

$\sum Fy = 0$

$N - 500\cos35 + P\sin(\theta - 35^0) = 0$

$N = 409.57 - P\sin(\theta - 35^0)$

But $F = \mu_s N = 0.3[409.57 - P\sin(\theta - 35^0)]$

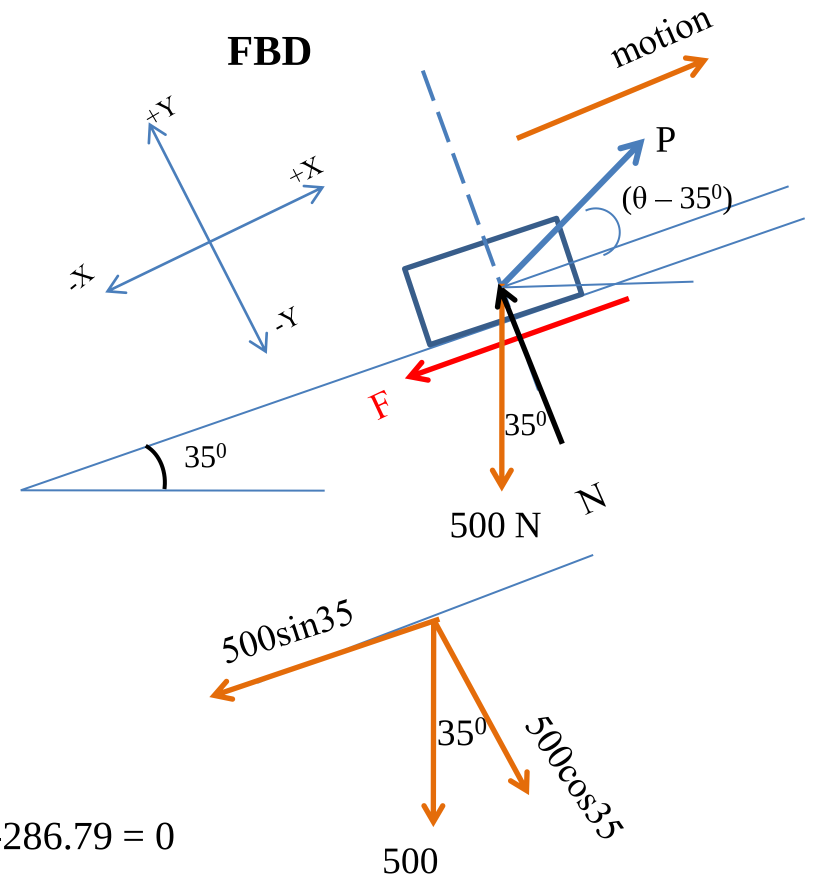
$F = 122.87 - 0.3P\sin(\theta - 35^0)$ --------(i)

Resolving the forces along the plane,

$\sum Fx = 0$

$P\cos(\theta - 35^0) - F - 500\sin35 = 0$

Therefore, $P\cos(\theta - 35^0) - 122.87 + 0.3P\sin(\theta - 35^0) - 286.79 = 0$



$$P[\cos(\theta - 35^0) + 0.3\sin(\theta - 35^0)] = 409.66$$

$$P = 409.66/[\cos(\theta - 35^0) + 0.3\sin(\theta - 35^0)] \quad \text{----(A)}$$

For P to be minimum, $\cos(\theta - 35^0) + 0.3\sin(\theta - 35^0)$

Should be maximum i.e

$$d/d\theta\ [\cos(\theta - 35^0) + 0.3\sin(\theta - 35^0)] = 0$$

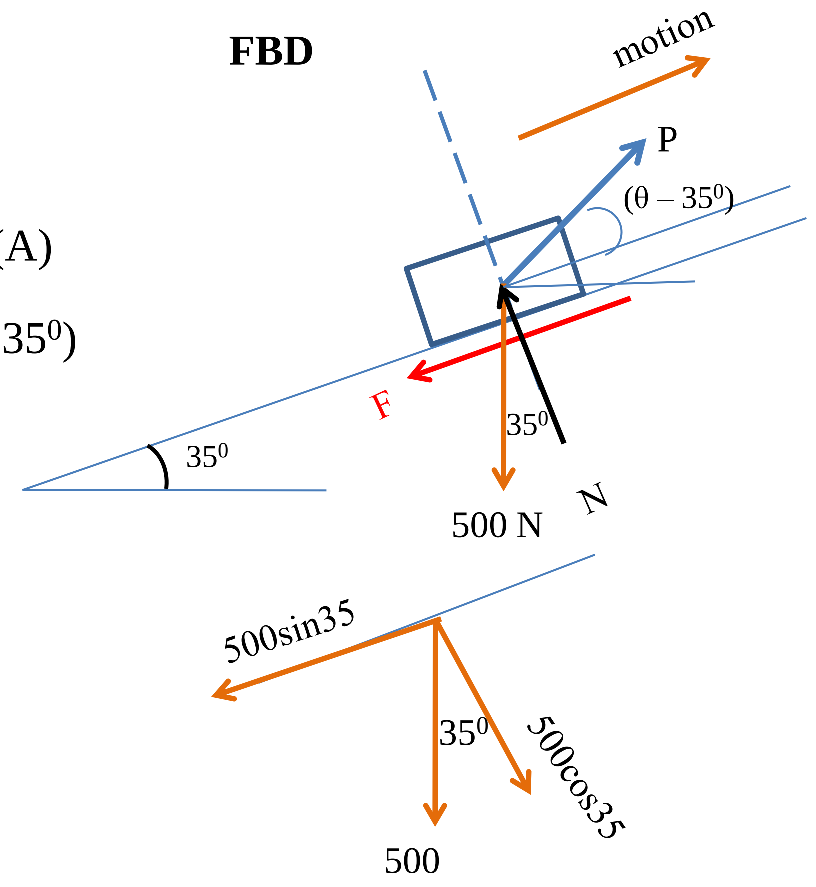
Therefore $-\sin(\theta - 35^0) + 0.3\cos(\theta - 35^0) = 0$

$$\sin(\theta - 35^0)/\cos(\theta - 35^0) = 0.3$$

$$\tan(\theta - 35^0) = 0.3$$

$$\theta = 51.7^0$$

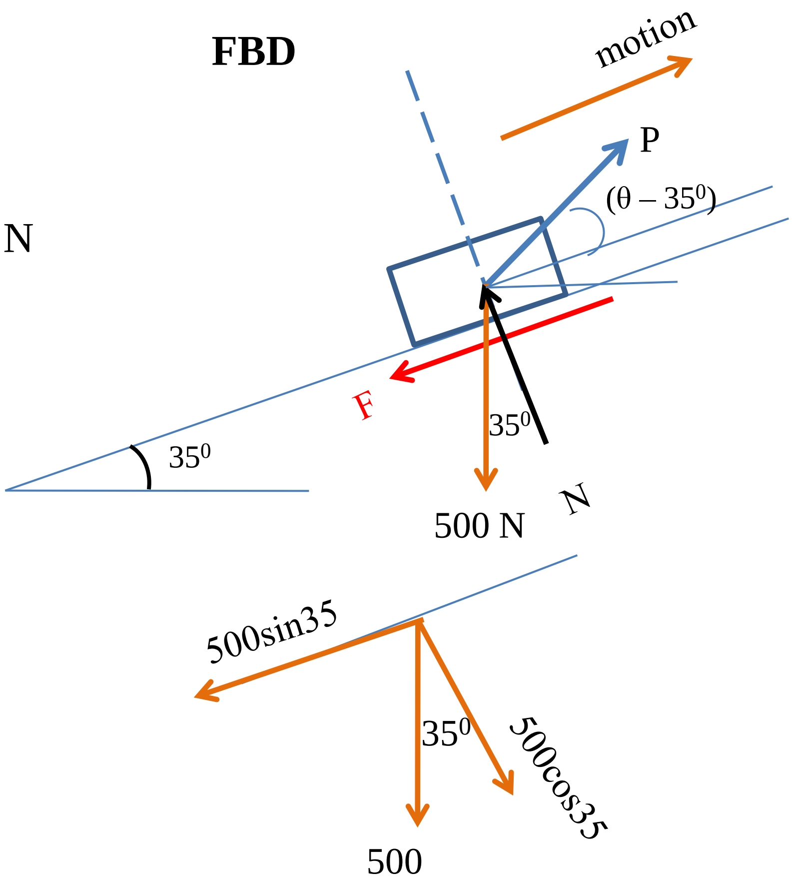
FBD



From (A)

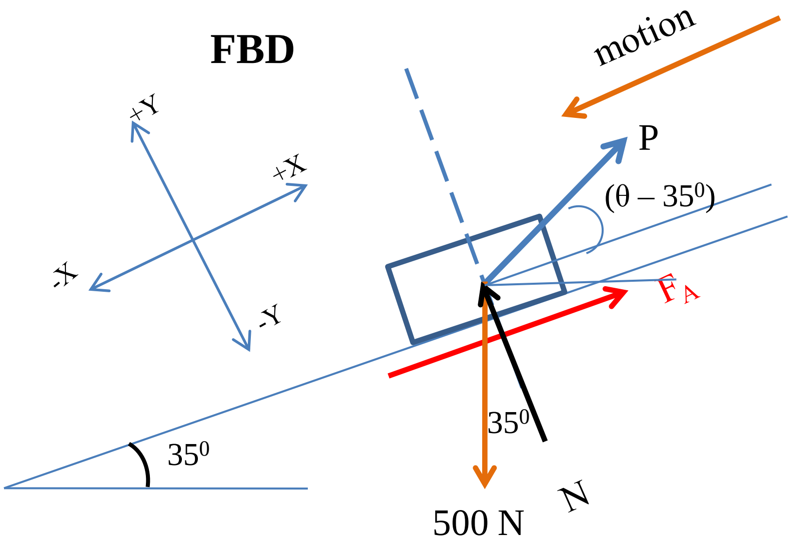
P_{min} = 409.66/[cos(16.7^0) + 0.3sin(16.7^0)] = 392.38 N

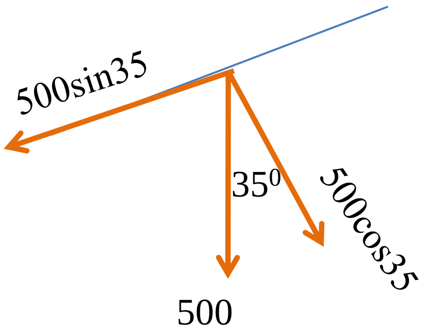
P_{min} = 392.38 N



Case (ii) to keep it from moving down

Since the block tends to move down the plane, the frictional force will be acting up the plane. i.e $\mu_S = 0.3$





From case(i) $F = 122.87 - 0.3P\sin(\theta - 35^0)$

Resolving the forces along the plane,

$P\cos(\theta - 35^0) + F - 500\sin35 = 0$

$P\cos(\theta - 35^0) + 122.87 - 0.3P\sin(\theta - 35^0) - 286.79 = 0$

$P\cos(\theta - 35^0) - 0.3P\sin(\theta - 35^0) = 163.92$

$P[\cos(\theta - 35^0) - 0.3\sin(\theta - 35^0)] = 163.92$-----(B)

For P to be minimum,

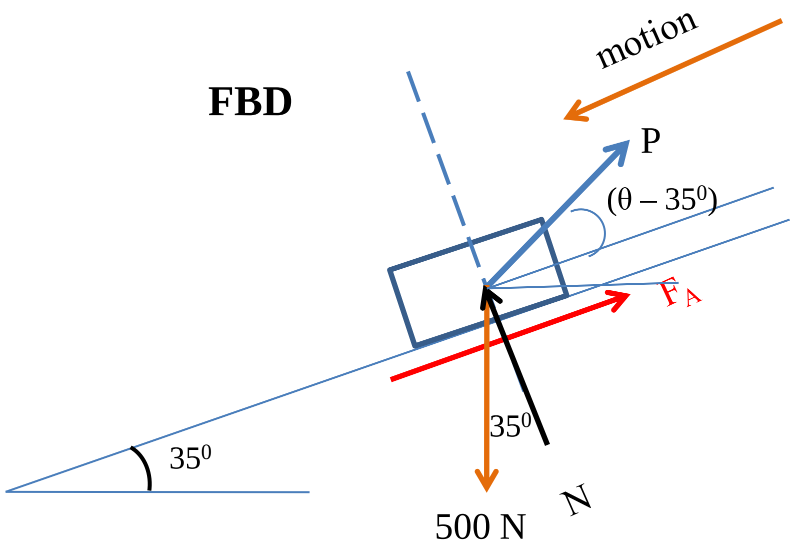
$d/d\theta\ [\cos(\theta - 35^0) - 0.3\sin(\theta - 35^0)] = 0$

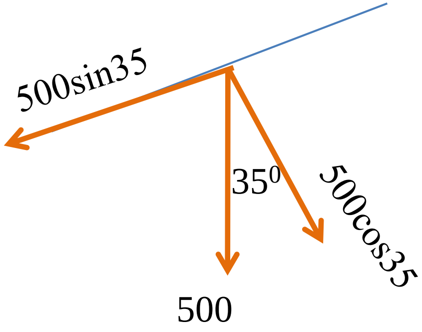
i.e $-\sin(\theta - 35^0) - 0.3\cos(\theta - 35^0) = 0$

$\tan(\theta - 35^0) = -0.3$

$\theta = 18.3^0$

FBD



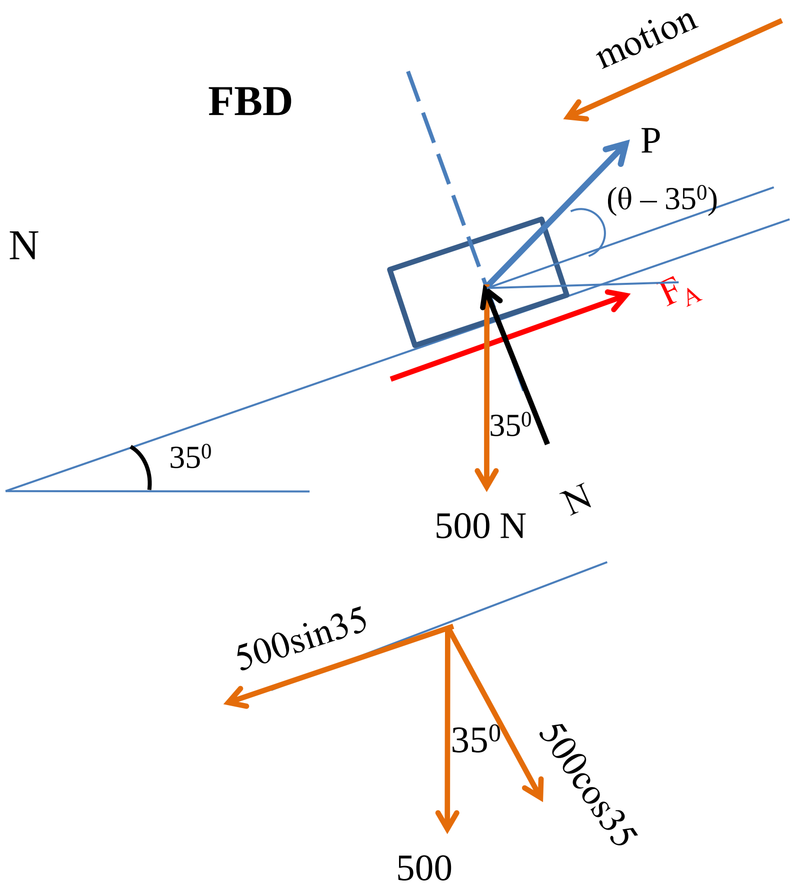


From (B)

P_{min} = 163.92/[cos(-16.7) – 0.3sin(-16.7)] = 157.0 N

P_{min} = 157.0 N

FBD



Problem 5.0

A body resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just removed it. It was found that a push of 220 N inclined at 30° to the plane just removed the body. Determine the weight of the body and the coefficient of friction.

Solution: Let 'W' be the weight of body in N. When a pull of 180 N is applied, its FBD is as shown.

$\sum Fx = 0$

$180\cos 30^0 - F = 0$

$F = 155.88$ N

$\sum Fy = 0$

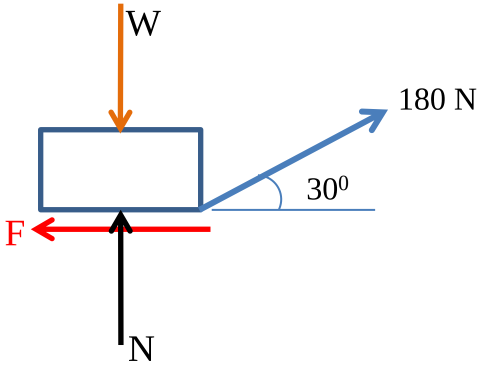
$N + 180\sin 30^0 - W = 0$

$N = W - 90$

But $F = \mu N$

$155.88 = \mu(W - 90)$ -----(i)

FBD



When a push of 220 N is applied

$\sum Fx = 0$

$F - 220\cos30^0 = 0, \quad F = 190.52$ N

$\sum Fy = 0$

$N - 220\sin30^0 – W = 0$

$N = W + 110 \qquad$ But $F = \mu N$

$190.52 = \mu(W + 110)$ -----(ii)

Dividing (i) by (ii)

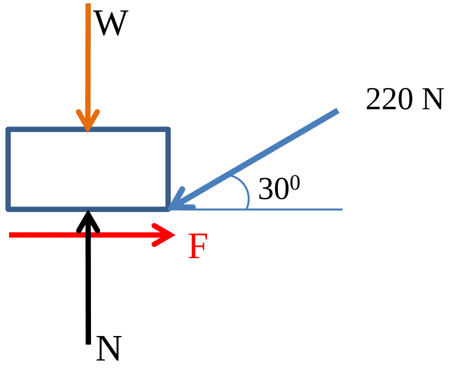
$(155.88 / 190.52) = \mu(W – 90) / \mu(W + 110)$

$0.818W + 90 = W – 90$

$0.181W = 180$ or $W = 989$ N

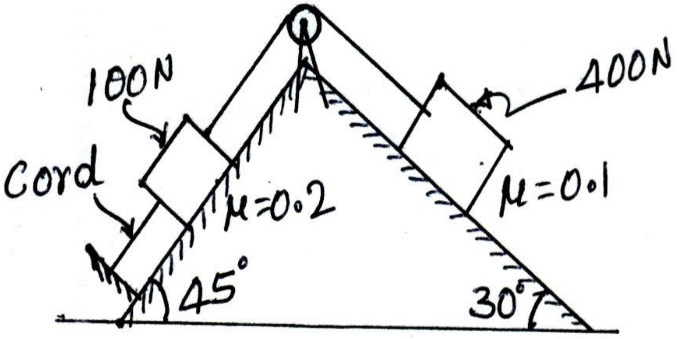
Now from (ii) $\mu = 0.173$

FBD



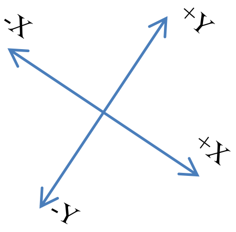
Problem 6.0

Find the maximum tension in the cord shown in figure, if the bodies have developed full friction.

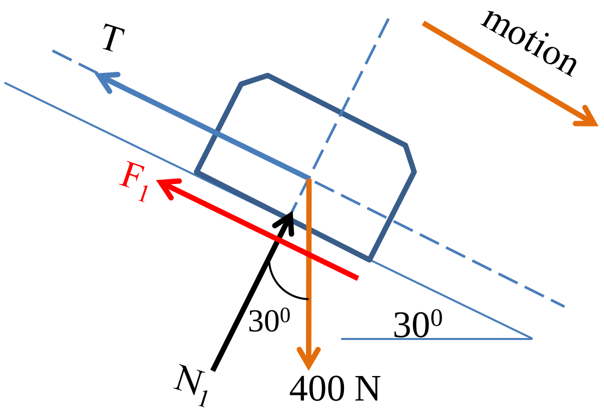


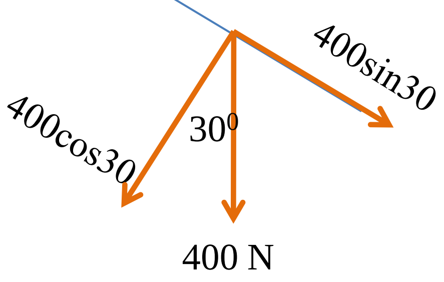
Solution: for the tension to occur in the cord, 100 N block should move up the plane and 400 N block should move down the plane.

Considering the equilibrium of the block resting on 30^0 incline plane its FBD is as shown.



FBD





Resolving all the forces perpendicular to the plane

$N_1 – 400\cos30 = 0$

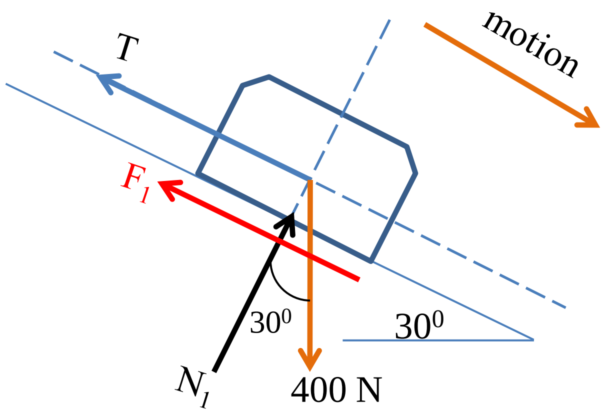
i.e $N_1 = 346.41$ N

Resolving all the forces along the plane

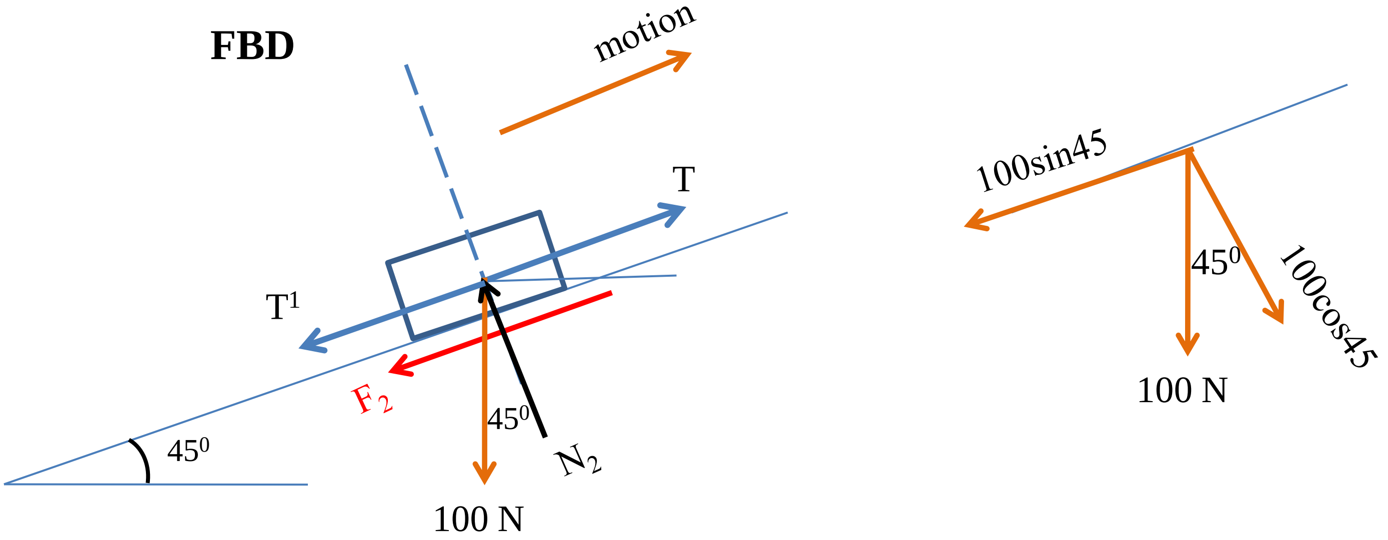
$400\sin30 – F_1 – T = 0$

$T = 165.36$ N

FBD



Now considering equilibrium of 100 N block, the tension in the cord T^l and frictional forces are shown in the figure.



Resolving all the forces perpendicular to the plane

$N_2 - 100\cos 45 = 0$

i.e $N_2 = 70.70$ N

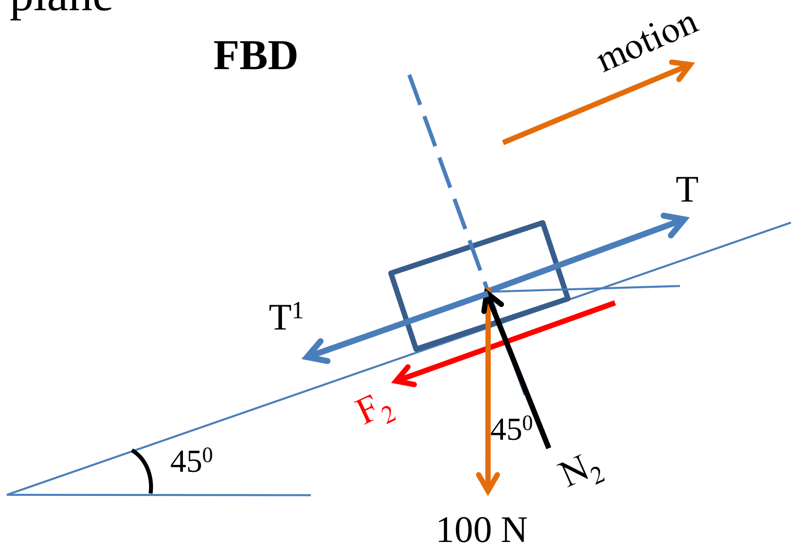
Resolving all the forces along the plane

$T - T^l - F_2 - 100\sin 45 = 0$

But $F_2 = \mu N_2 = 14.14$ N

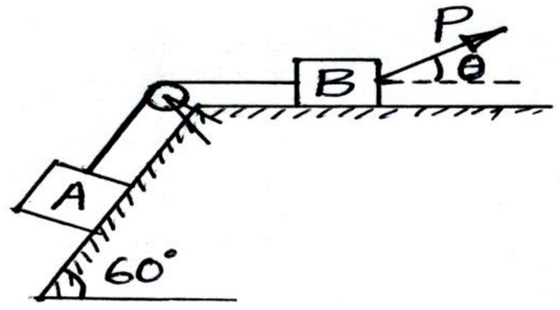
$T^1 = 80.52$ N

FBD



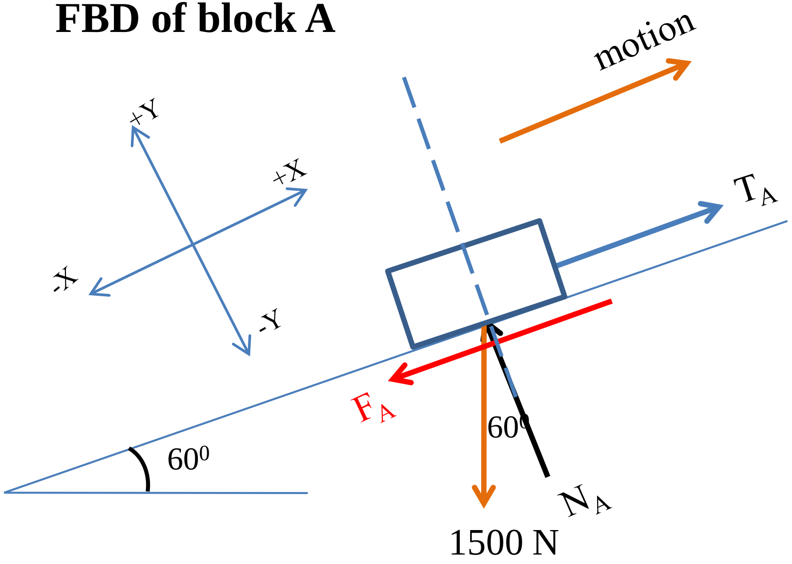
Problem 7.0

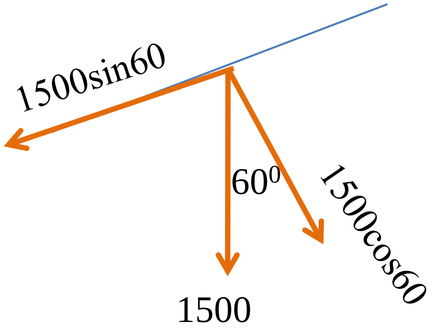
Two blocks A and B weighing 1500 N & 1000 N, and connected by a string passing over a smooth pulley, rest on a 60° inclined plane and a horizontal plane, the block B being subjected to an external force P as shown in figure. If the coefficient of friction be 0.25 under the blocks, find the least value of P to drag the block A up the plane.



Solution: Consider the equilibrium of block A. since it has tendency to move up the plane, frictional force acts down the plane. Let T_A be the tension in the string.

FBD of block A





Consider equilibrium of block A

Resolving the forces perpendicular to the plane

$\sum Fy = 0$

$N_A - 1500\cos 60 = 0$

$N_A = 1500\cos 60 = 750$ N

WKT $F_A = \mu_A N_A = 0.25 * 750$

$F_A = 187.5$ N

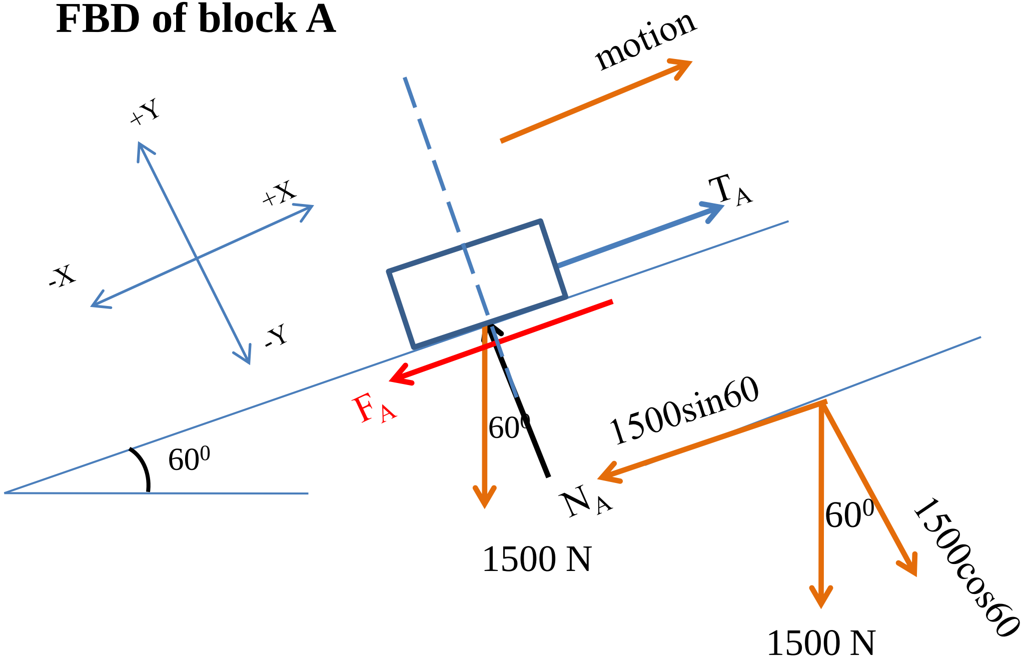
Resolving the forces parallel to the plane

$\sum Fx = 0$

$T_A - F_A - 1500\sin 60 = 0$

$T_A = 187.5 + 1500\sin 60$

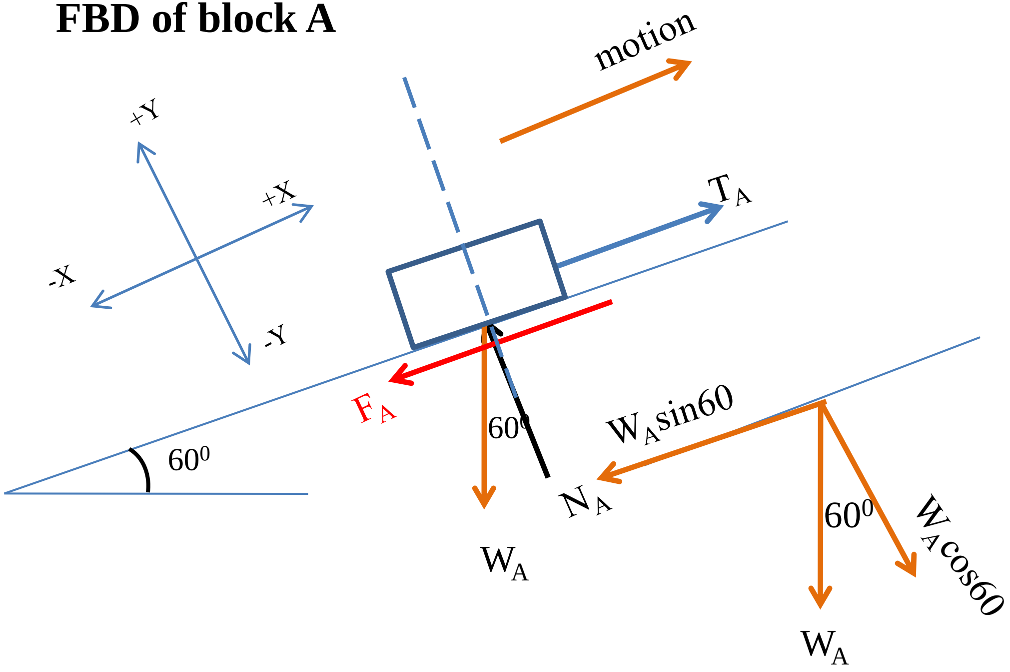
FBD of block A



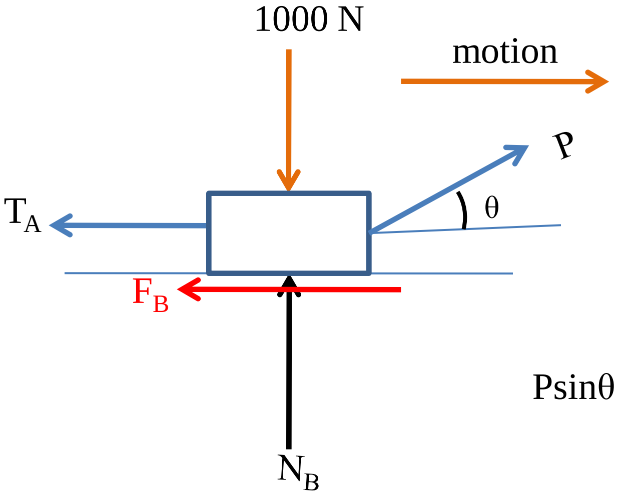
$T_A = 187.5 + 1299$

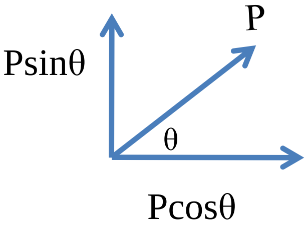
$T_A = 1486.5\ N$

FBD of block A



FBD of block B





Consider equilibrium of block B

Resolving the forces parallel to the plane

$\sum Fx = 0$

$-T_A - F_B + P\cos\theta = 0$

WKT $\quad F_B = \mu_B N_B$

$-1486.5 - 0.25N_B + P\cos\theta = 0$ -----(i)

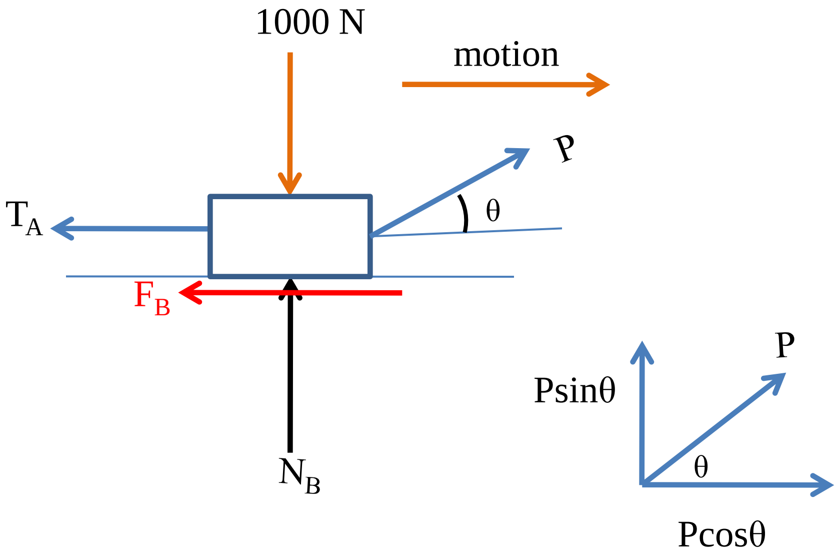
Resolving the forces perpendicular to the plane

$\sum Fy = 0$

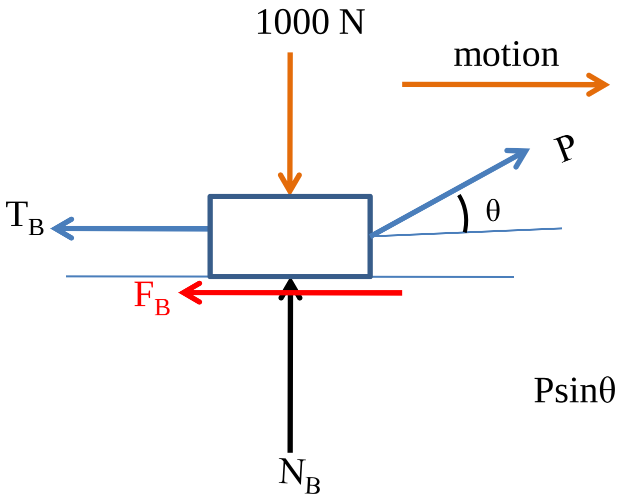
$N_B - 1000 + P\sin\theta = 0$

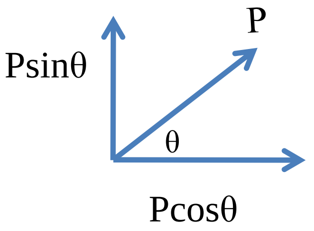
$N_B = 1000 - P\sin\theta$ -----(ii)

FBD of block B



FBD of block B





Substituting (ii) in (i)

$-1486.5 – 0.25(1000 – P\sin\theta) + P\cos\theta = 0$

$P\cos\theta + 0.25P\sin\theta = 1736.54$

Therefore $P = 1736.5/(\cos\theta + 0.25\sin\theta)$

For P to be minimum , $(\cos\theta + 0.25\sin\theta)$ should be maximum

i.e, $d/d\theta\ (\cos\theta + 0.25\sin\theta) = 0$

i.e $– \sin\theta + 0.25\cos\theta = 0$

Or $\sin\theta = 0.25\cos\theta$

Or $\tan\theta = 0.25$

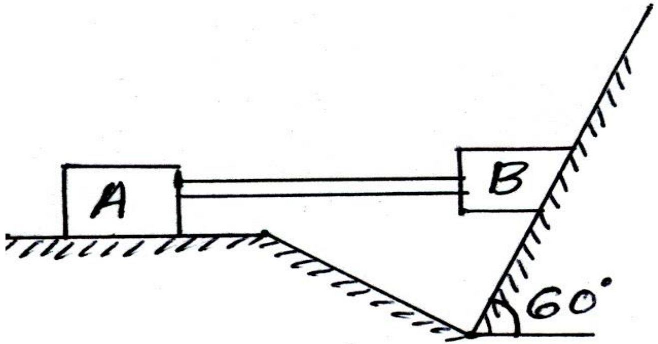
$\theta = 14.03^0$

$P_{min} = 1736.5/(\cos\theta + 0.25\sin\theta)$

$P_{min} = 1684.65$ N

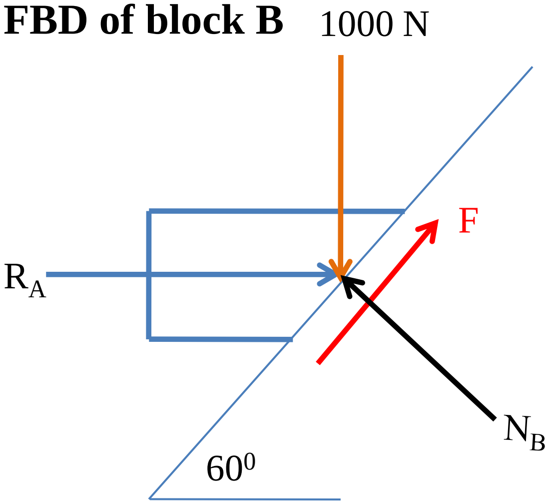
Problem 8.0

Two blocks A & B, connected by a horizontal rod by frictionless hinges, are supported on two rough planes as shown in figure. The coefficients of friction are 0.3 between block A and the horizontal surface & 0.4 between block B and the inclined surface. Block B weighs 1000 N. What is the smallest weight of block A that will hold the system in equilibrium ?



Solution: Since the block B has a tendency to move down the plane, frictional force acts up the plane. There will be reaction from block A through the rod. FBD of block B is as shown

FBD of block B



Resolving all the forces perpendicular to the plane

$N_B - 1000\cos60 - R_A\sin60 = 0$

i.e $N_B = 500 + 0.866R_A$

But $F = \mu N_B = 0.4(500 + 0.866R_A)$

$F = 200 + 0.3464R_A$-----(i)

Resolving all the forces along the plane

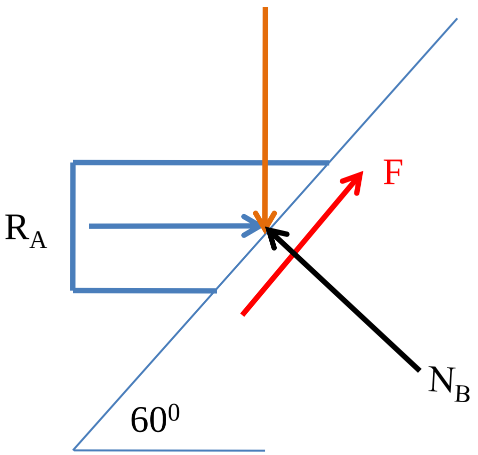
$F - 1000\sin60 + R_A\cos60 = 0$

From (i) putting the value of F

$200 + 0.3464R_A - 866 + 0.5R_A = 0$

$R_A = 786.86$ N

FBD of block B



Considering equilibrium of block A its FBD is as shown in figure.

Let W be the weight of block A

Resolving all the forces perpendicular to the plane

$\sum F_y = 0$

$N_A = W$

Resolving all the forces along the plane

$\sum F_x = 0$

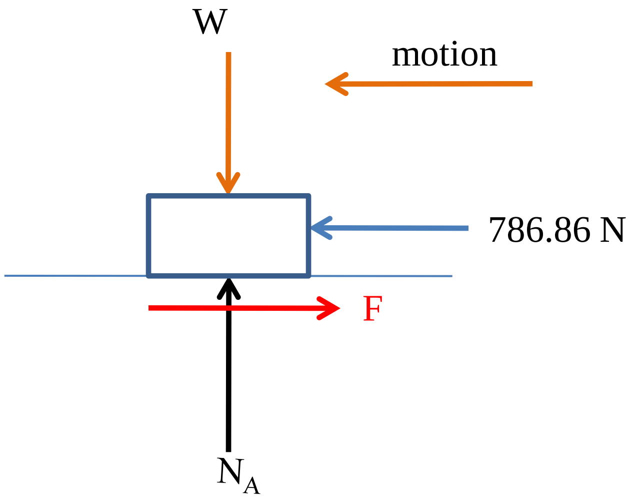
$F = 786.86$ N

But $F = \mu N_A$

$786.86 = 0.3W$

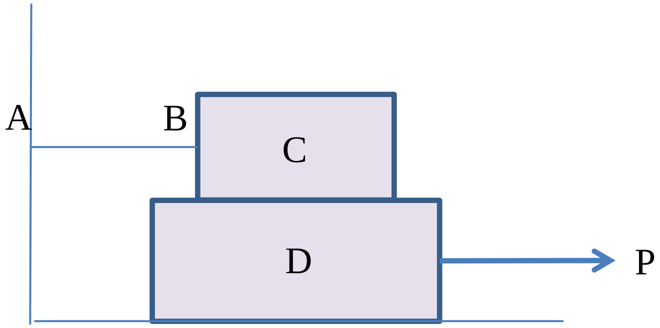
$W = 2622.87$ N

FBD of block A



Problem 9.0

The coefficients of friction is 0.30 between all the surfaces of contact. Block C & D weigh 1000 N & 1500 N respectively. Determine the smallest force required to start block D moving if (i) Block C is restrained by cable AB as shown in figure.(ii) cable AB is removed.



Solution: Case(i) Consider the equilibrium of block C. when block D moves to the right, because of the cable AB. Block C moves to the left.

Hence the friction force acts to the right. If T is the tension

In AB, its FBD is as shown .

Applying $\sum Fy = 0$, $N_D = 1000$ N.

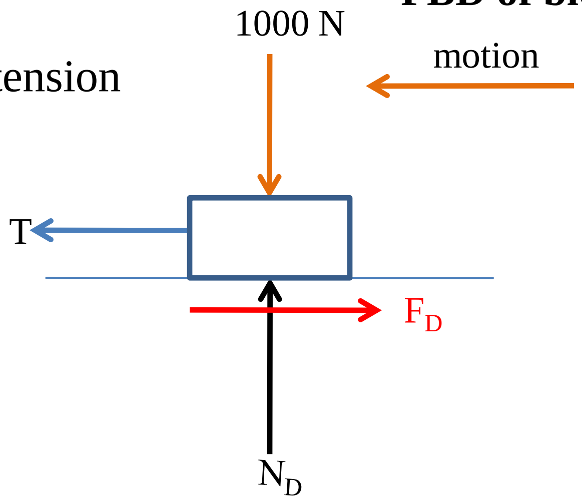
$\sum Fx = 0$, $F_D = T$

$\mu N_D = T$

$0.3 * 1000 = T$

$T = 300$ N.

FBD of block C



Consider the equilibrium of block D.

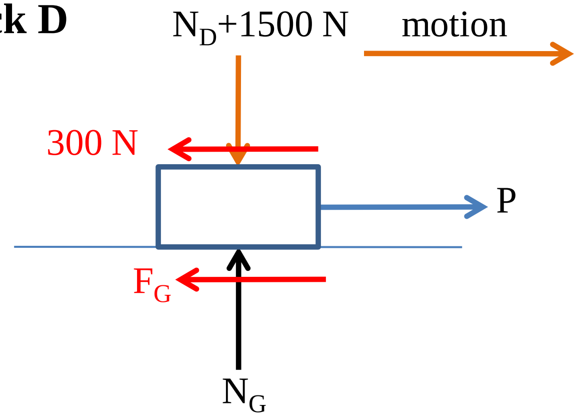
Applying $\sum Fy = 0$, $N_G = 2500$ N.

But $F_G = \mu N_G = 0.3 * 2500 = 750$ N

$\sum Fx = 0$, $P - 300 - F_G = 0$

$P = 300 + 750 = 1050$ N

FBD of block D



Case(ii) when cable AB is absent

Applying $\sum Fy = 0$, $N_G = 2500$ N.

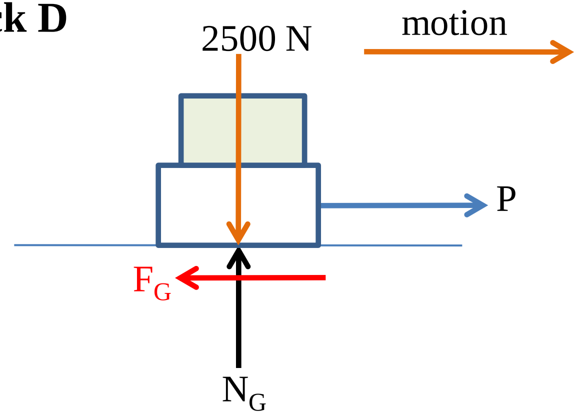
But $F_G = \mu N_G = 0.3 * 2500 = 750$ N

$\sum Fx = 0$,

$P - F_G = 0$

$P = 750$ N

FBD of block D



Application to ladder problems.

A ladder resting on ground and leaning against wall is a typical case of friction problem in non concurrent coplanar system. Here three equations of equilibrium are available,

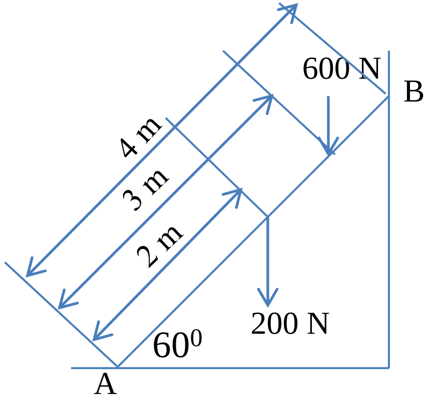
$$\sum \mathbf{Fx = 0}; \quad \sum \mathbf{Fy = 0}; \quad \sum \mathbf{M = 0.}$$

From law of static friction we have **μ = F/N**

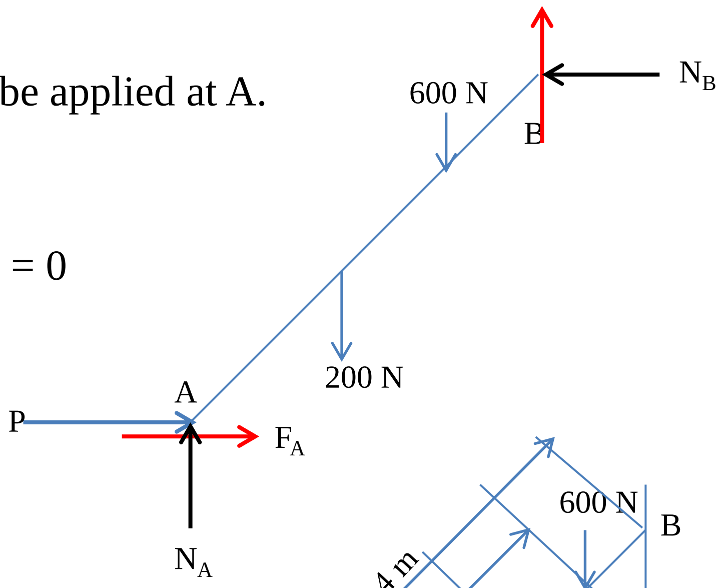
Thus using these three equations, problems on ladder is solved.

Problem 10

A ladder of length 4.0 m and weighing 200N is placed against a vertical wall as shown in figure. The coefficient of friction between the wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. the ladder in addition to its own weight has to support a man weighing 600 N at a distance of 3.0 m from A. calculate the minimum horizontal force to be applied at A to prevent slipping.



FBD



FBD of ladder is as shown in figure. Let P be the force to be applied at A.

$\sum M_A = 0,$

$200 * 2\cos 60 + 600 * 3\cos 60 – N_B * 4\sin 60 – F_B * 4\cos 60 = 0$

But $F_B = \mu N_B = 0.2\ N_B$

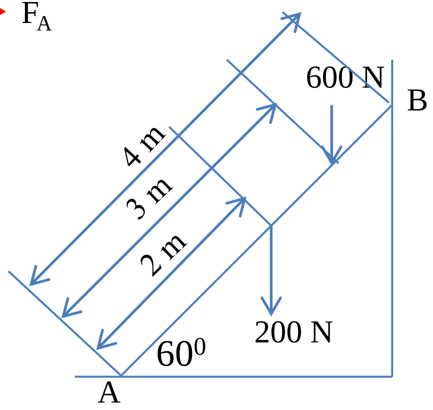
Therefore $200 + 900 – 3.464N_B - 0.4N_B = 0$

$3.864N_B = 1100$ or $N_B = 284.68$ N

$F_B = 0.2 * 284.68 = 56.93$ N

$\sum Fy = 0,\ N_A + F_B – 200 – 600 = 0$

$N_A = 743.07$ N

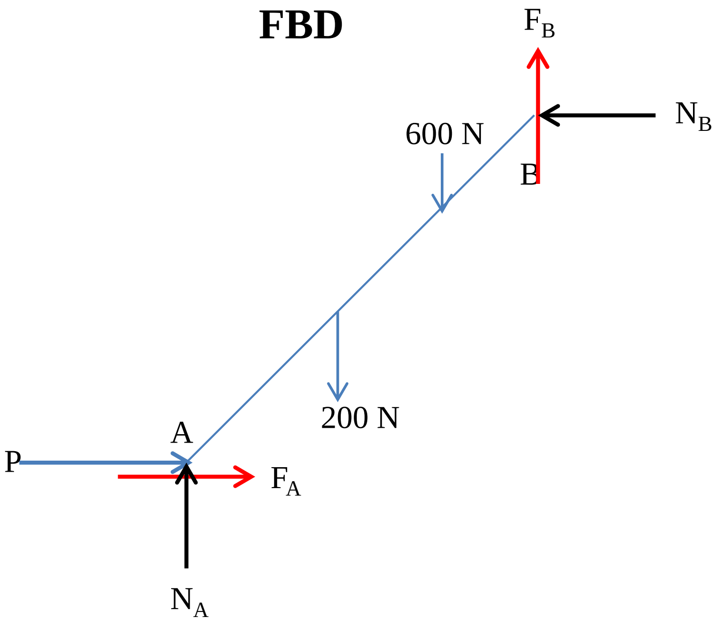


$\sum Fx = 0$, $P + F_A - N_B - = 0$

$P = 284.68 – 0.3 * 743.07 = 61.76$ N

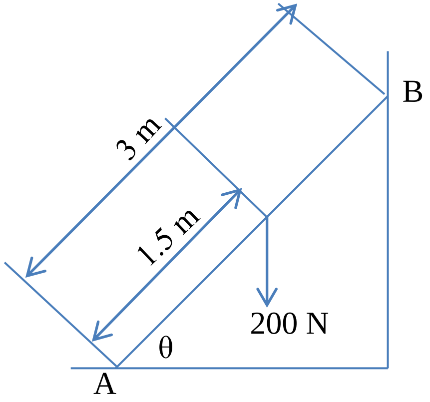
$P = 61.76$ N

FBD



Problem 11

A uniform ladder AB of length 3.0 m and weight 200 N rests with the end B against rough vertical wall and the end A on the level ground. If the wall and the ground are equally rough and the coefficient of friction is 0.5, find the limiting position of equilibrium.



Let the ladder makes an angle θ with horizontal in limiting equilibrium condition.

FBD of the ladder is as shown

Applying $\sum Fx = 0$, $F_A - N_B = 0$ or $F_A = N_B$

But $F_A = \mu N_A$

therefore $\mu N_A = N_B$

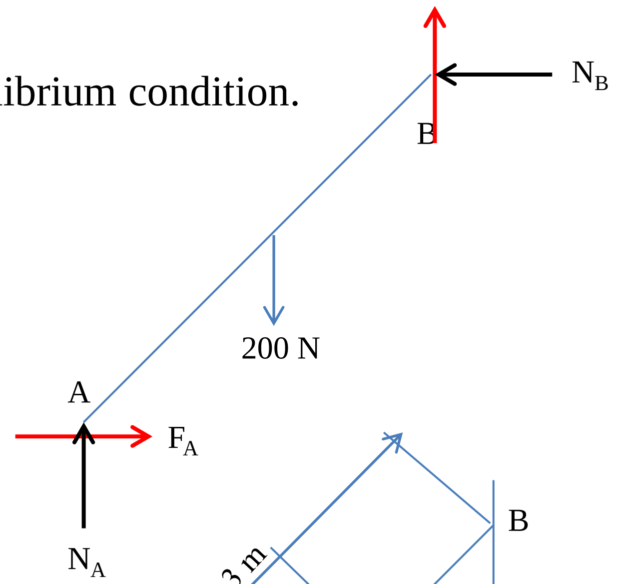
$N_A = N_B / 0.5 = 2N_B$ -----(i)

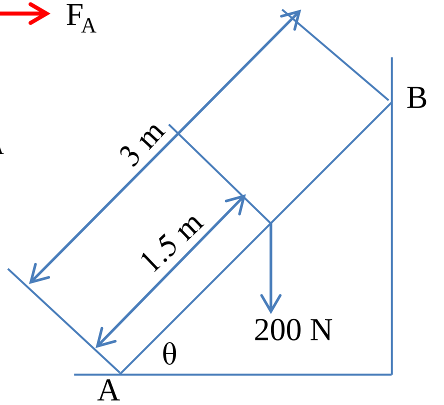
$\sum Fy = 0$,

$N_A - 200 + F_B = 0$ or $2N_B + 0.5N_B = 200$

$N_B = 80$ N

FBD





Applying $\sum M_A = 0$,

$200 * 1.5\cos\theta - N_B * 3\sin\theta - 0.5N_B * 3\cos\theta = 0$

$300\cos\theta - 240\sin\theta - 120\cos\theta = 0$

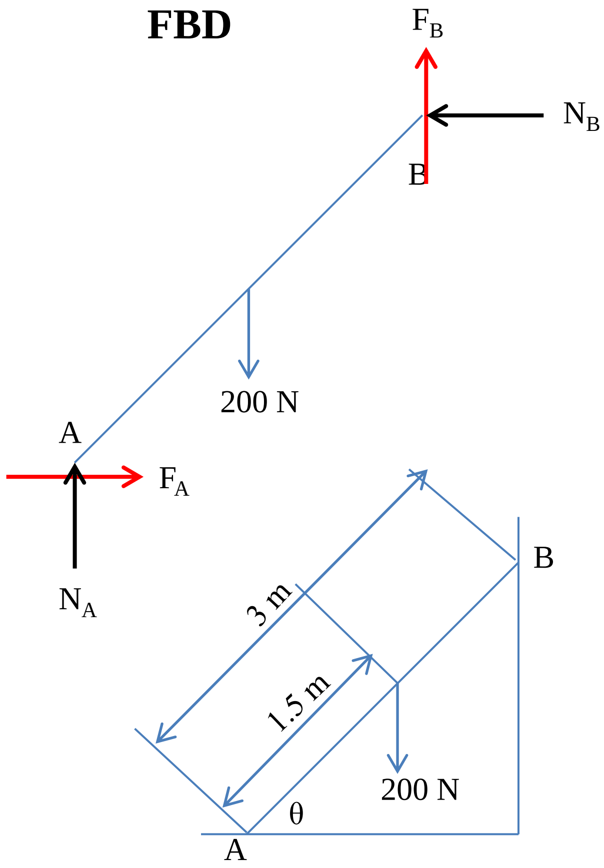
$180\cos\theta = 240\sin\theta$

$\tan\theta = 180/240 = 0.75$

Therefore $\theta = 36.86^0$

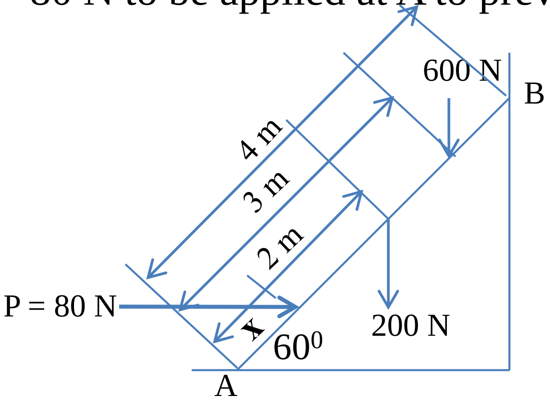
The ladder will be in limiting equilibrium when it makes 36.86^0 with horizontal.

FBD



Problem

A ladder of length 4.0 m and weighing 200N is placed against a vertical wall as shown in figure. The coefficient of friction between the wall and the ladder is 0.2 and that between the floor and the ladder is 0.3. the ladder in addition to its own weight has to support a man weighing 600 N at a distance of 3.0 m from A. calculate the distance x measured along the ladder from A at which horizontal force P = 80 N to be applied at A to prevent slipping.



$\sum Fx = 0, \quad 80 + F_A - N_B = 0$

$F_A - N_B = -80$

$0.3\,N_A - N_B = -80$ ------(i)

$\sum Fy = 0,\ N_A + F_B - 200 - 600 = 0$

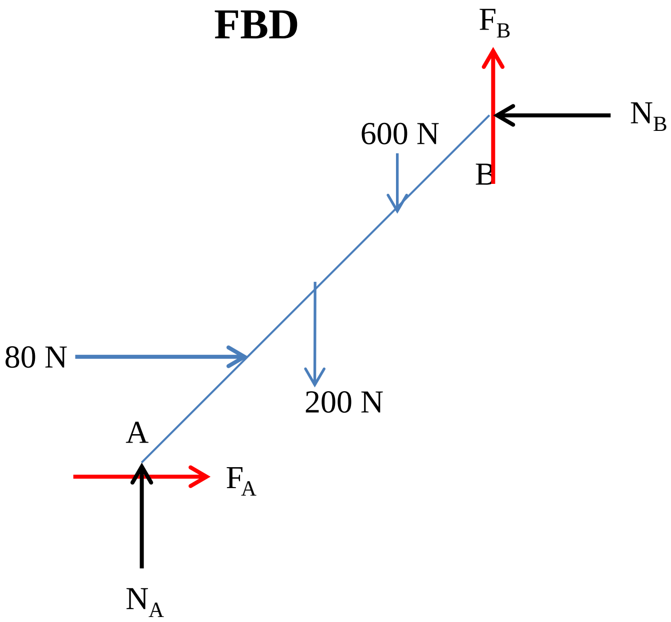
$N_A + F_B = 800$

$N_A + 0.2\,N_B = 800$ ------(ii)

Solving (i) & (ii) we get

$N_A = 739.62$ N & $N_B = 301.88$ N

FBD



$F_A = 0.3 * 739.62 = 221.88$ N

$F_B = 0.2 * 301.88 = 60.37$ N

$\sum M_A = 0,$

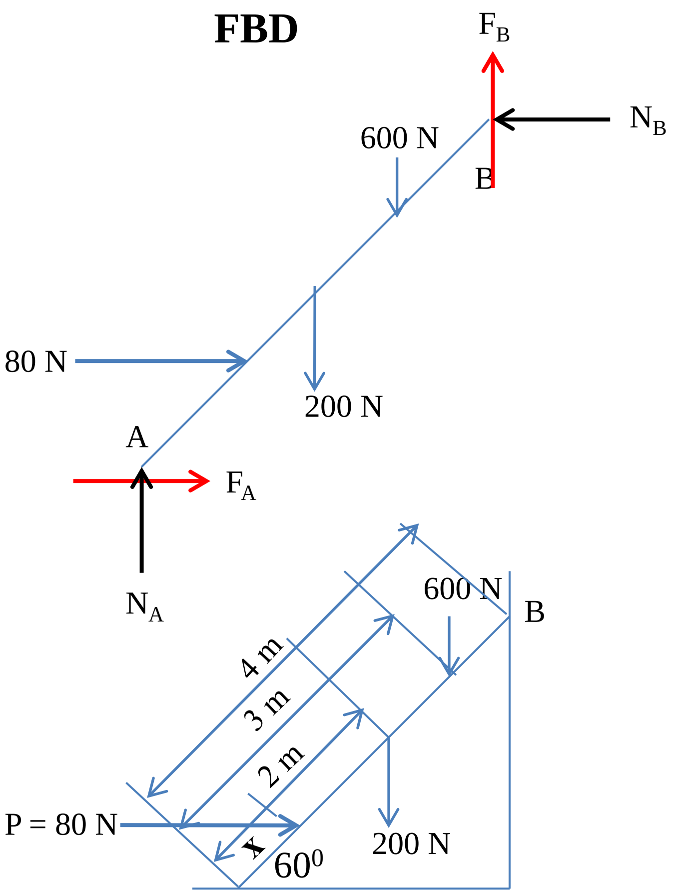
$200 * 2\cos 60 + 600 * 3\cos 60 - N_B * 4\sin 60 - F_B * 4\cos 60$

$+ 80 * x\sin 60 = 0$

Therefore $200 + 900 - 1045.74 - 120.74 + x 80\sin 60 = 0$

$x = 66.48/(80\sin 60)$

x = 0.96 m



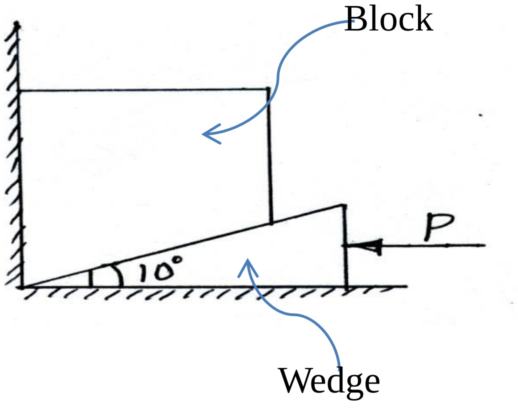
Applications to wedge problems

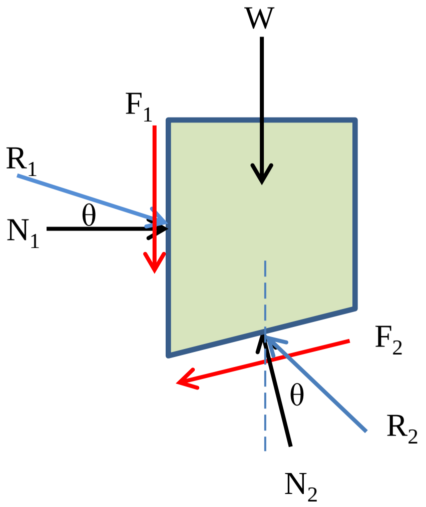
Wedges are small piece of hard materials with two of their opposite surfaces are not parallel to each other. They are used to slightly lift heavy blocks, machinery, precast beams etc. for making final alignment or to make place for inserting lifting devices.

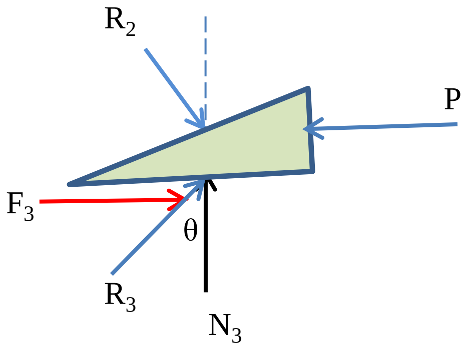
Here weight of the wedge is small compared to the weight lifted. Hence in all the problems weight of the wedge is neglected, unless it is specified.

In the analysis instead of considering normal reaction & frictional force independently, it is advantageous to take the **resultant** of these two forces.

If ‘**F**’ is the limiting friction, ‘**N**’ b the normal reaction, then ‘**R**’ will be the resultant making limiting Angle of friction $'\theta'$with the normal.

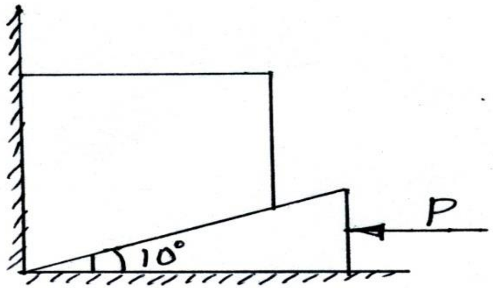






Problem 12

A block weighing 1500 N is to be raised by means of a 10° wedge. Assuming the coefficient of friction between all the contacting surfaces to be 0.30, determine what minimum horizontal force P should be applied to raise the block?



Solution: Coefficient of friction $\mu = 0.3$

At limiting equilibrium $\tan\theta = \mu$

$\tan\theta = 0.3$

$\theta = 16.69^0$

Consider the equilibrium of block when the wedge

is pushed inside, the block slides upwards & towards right also

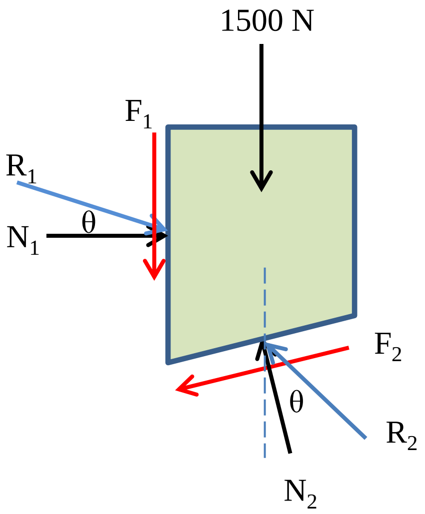
FBD of block is as shown.

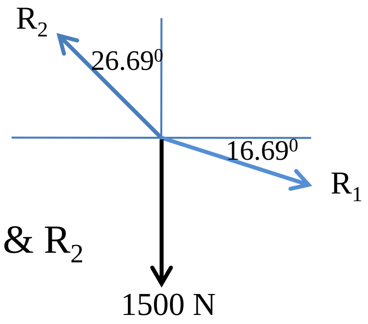
R_1 makes an angle θ with horizontal & R_2 makes an angle $(\theta + 10) = 26.69^0$

with vertical.

Since the three forces are concurrent we can apply Lami's theorem to get R_1 & R_2

FBD





$$R_1/\sin(180\text{-}26.69) = R_2/\sin(90\text{-}16.69) = 1500/\sin(90+26.69+16.69)$$

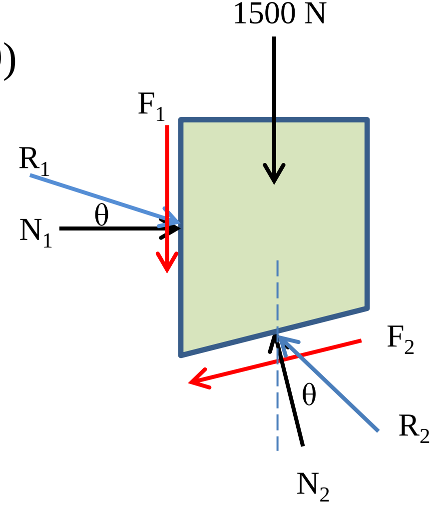
$$R_1/\sin(153.31) = 1500/\sin(133.38)$$

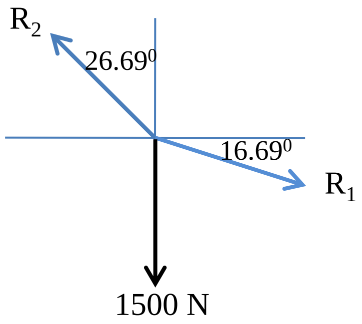
$$R_1 = 926.98\ \text{N}$$

$$R_2/\sin(73.39) = 1500/\sin(133.38)$$

$$R_2 = 1976.97\ \text{N}$$

FBD





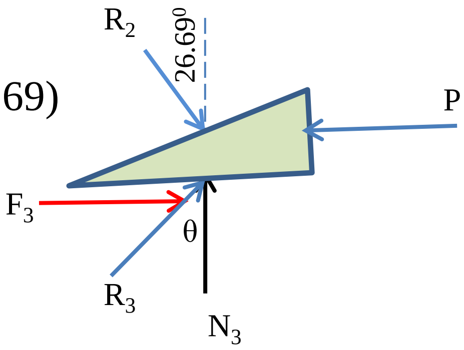
FBD

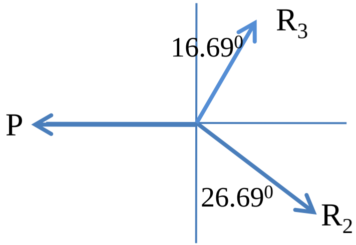
Now consider the equilibrium of wedge

$R_2/\sin(90+16.69) = R_3/\sin(90+26.69) = P/\sin(180-26.69-16.69)$

$1976.97/\sin(106.69) = P/\sin(136.62)$

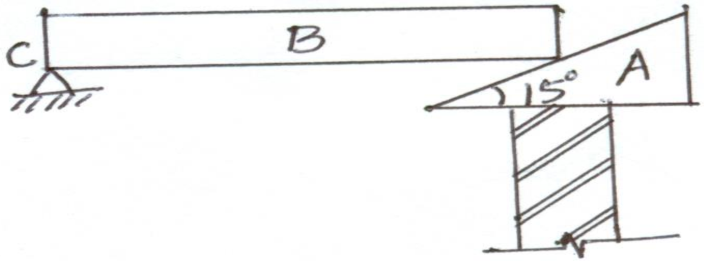
$P = 1417.56$ N





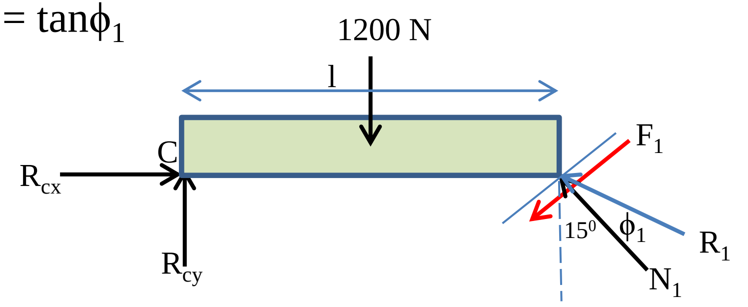
Problem 13

One end of a beam B, of uniform cross section weighing 1200 N is hinged & the other end is lifted by a 15° wedge. The lower face of the wedge A is horizontal on a horizontal support. The coefficient of friction is 0.36 at the upper surface of the wedge & is 0.25 at the lower surface. Find the horizontal force required to push the wedge under the beam.



FBD

Solution: On the upper surface of the wedge, $\mu_1 = 0.36 = \tan\phi_1$



$\phi_1 = 19.79^0$

On lower surface of the wedge, $\mu_2 = 0.25 = \tan\phi_2$

$\phi_2 = 14.03^0$

Consider the equilibrium of beam. The reaction of wedge R_1 on the beam makes an angle of $19.79^0 + 15^0 = 34.79^0$ with vertical as shown in FBD.

$\sum M_c = 0,\ 1200 * (l/2) - R_1\cos 34.79^0 * l = 0$

$R_1 = 600/\cos 34.79^0 = 730.30$ N

Now Consider the equilibrium of wedge. FBD is as shown,

Applying Lamis's theorem

$R_2/\sin(90+34.79) = 730.6/\sin(90+14.03) = P/\sin(180-34.79-14.03)$

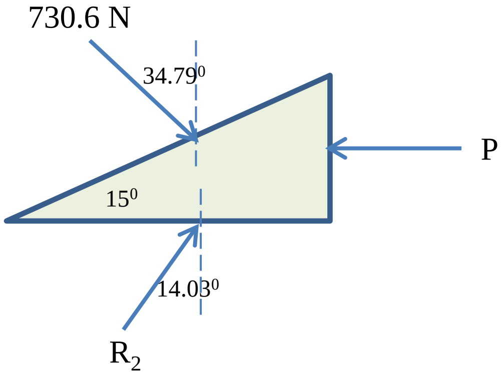
$R_2/\sin(90+34.79) = 730.6/\sin(90+14.03)$

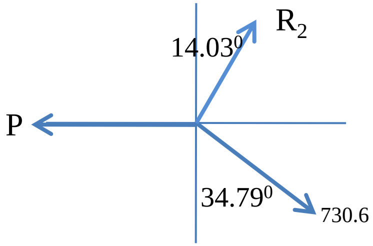
$R_2 = 618.44$ N

$730.6/\sin(90+14.03) = P/\sin(180-34.79-14.03)$

P = 566.78 N

FBD



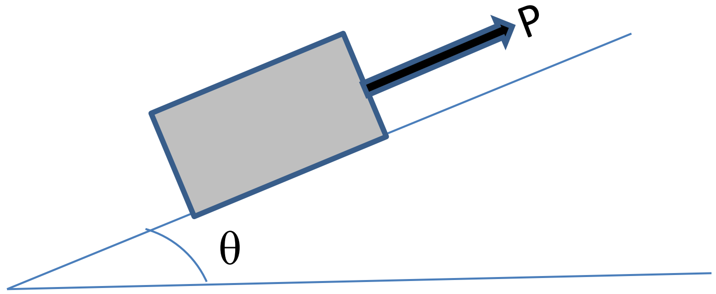


Problem

Determine the horizontal force P to start 4000 N wedge, to the right. The angle of friction is 20^0 at all the contact surfaces. Refer fig. also find the reaction from inclined surface.

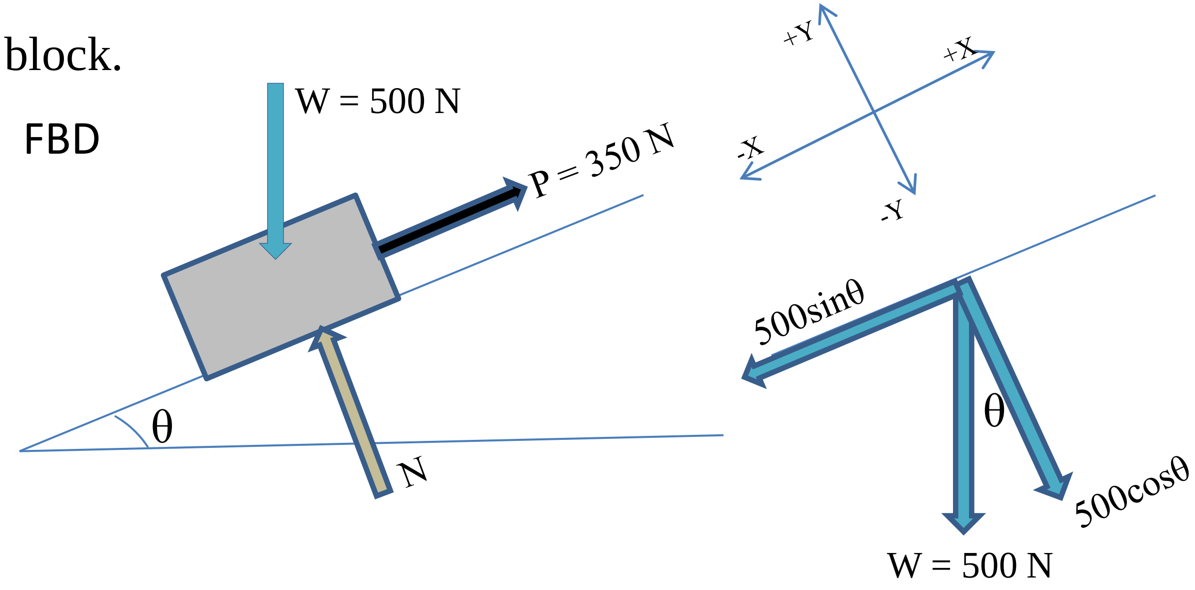
Problem 2.0

A force P = 350 N acts as shown on the block of weight W = 500 N which is resting on an inclined plane. The coefficients of friction between block and inclined plane are μ_s= 0.4 & μ_k= 0.3 Determine whether the block is in equilibrium and find the value of the frictional force. Take $\theta = 30^0$



Solution:

Let us first draw the Free body diagram (FBD) and understand different forces and reactions acting on the block.



Net force acting along the plane,

$350 - 500\sin\theta$

$= 100$ N (acting up the plane)

Hence the body is having tendency to move up the plane.

For equilibrium frictional force required is $F_{req} = 100$ N acting Down the plane.

Net force acting perpendicular to the plane,

$-500\cos30 + N = 0$

$N = 433.01$ N

Frictional force at limiting equilibrium condition is

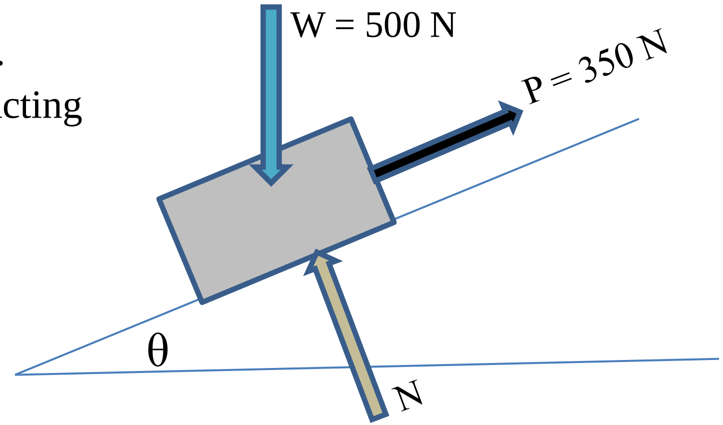
$F_m = \mu_s N$

$F_m = 0.4*433.01$

$= 173.20$ N

Since $F_m > F_{req}$ the block will be in equilibrium.

value of the frictional force; $F_s = 173.20$ N and it acts in the downward direction.



Thank You