Linear Algebra

Hiba Shafqat FA19-BCS-027

Assignment # 2

Q1. What is the determinant of a Matrix?

The determinant is a special number that can be calculated from a matrix. Matrix has to be a square matrix for a determinant. The determinant helps us to find the inverse of a matrix, tells us things about the matrix that are useful in systems of linear equations, calculus and more.

For $n \geq 2$, the determinant of an $n \times n$ matrix $A = [a_{ij}]$, is the sum of n terms of the form $\pm a_{1j} \det A_{1j}$, with plus and minus signs alternating, where the entries $a_{11}, a_{12} \cdots a_{1n}$ are from the first row of A. In symbols

$$\det A = a_{11} \det A_{11} - a_{12} \det A_{12} + \cdots + (-1)^{1+n} a_{1n} \det A_{1n}$$
$$= \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j}$$

Q2. What are the properties of a determinant? Explain each property with the help of an example?

Properties of Determinants

- Row Operations

Let A be a square matrix

1. If a multiple of one row of A is added to another row to produce a matrix B, then

$$\det B = \det A$$

e.g.

let $A = \begin{bmatrix} 9 & 4 \\ 6 & 5 \end{bmatrix}$

~~Add~~ 2, Multiply 2 with row 2 and add it to row 1.

The matrix will be

$B = \begin{bmatrix} 21 & 14 \\ 6 & 5 \end{bmatrix}$

$$\det A = \det B$$
$$45 - 24 = 105 - 84$$
$$21 = 21$$

Hence proved

2. Two rows of a matrix A are interchanged with matrix B, then

$$\det B = -\det A$$

e.g.

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 6 & 8 \\ 2 & 4 \end{bmatrix}$$

$$-(\det A) = \det B$$
$$16 - 24 = 24 - 16$$
$$-8 = 8$$

Hence proved

3. If one row of A is multiplied by k to produce B, then

$$\det B = k \cdot \det A.$$

e.g. let $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

multiplying row 1 with 2,

$$B = \begin{bmatrix} 2 & 6 \\ 2 & 4 \end{bmatrix}$$

$$\det A = k \cdot \det B.$$
$$4 - 6 = 8 - 12$$
$$-2 = -4$$

Hence proved

Column Operations

If A is a $n \times n$ matrix, then

$$\det A^T = \det A$$

e.g.

let $A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$A^t = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$\det A^t = \det A$$
$$6 - 4 = 6 - 4$$
$$2 = 2$$

Hence Proved

Multiplicative Property:

If A and B are $n \times n$ matrix, then

$$\det AB = (\det A)(\det B)$$

e.g. $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [\cancel{13}$$

$$= \begin{bmatrix} (4 \times 1) + (3 \times 3) & (4 \times 2) + (3 \times 4) \\ (1 \times 1) + (2 \times 3) & (1 \times 2) + (2 \times 4) \end{bmatrix}$$

$$= \begin{bmatrix} 4 + 9 & 8 + 12 \\ 1 + 6 & 2 + 8 \end{bmatrix} = \begin{bmatrix} 13 & 20 \\ 7 & 10 \end{bmatrix}$$

$$\det AB = (\det A)(\det B)$$

$$130 - 140 = (8-3)(4-6)$$

$$-10 = (5)(-2)$$

$$= -10$$

Hence proved