Embedded Convex Optimization for Control of Synchronous machines

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PhD Defense, November 26, 2025

Problem Statement

The embedded approach

Implement advanced control laws synthesis on low-cost hardware to drive real-world systems.

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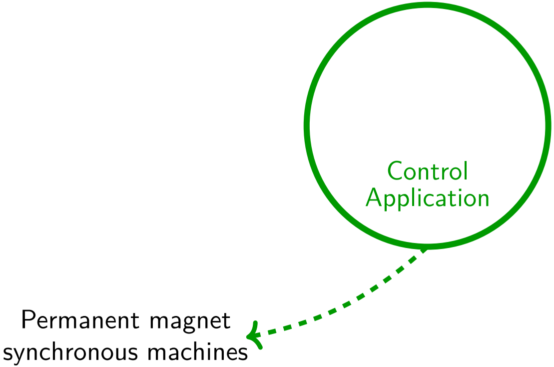


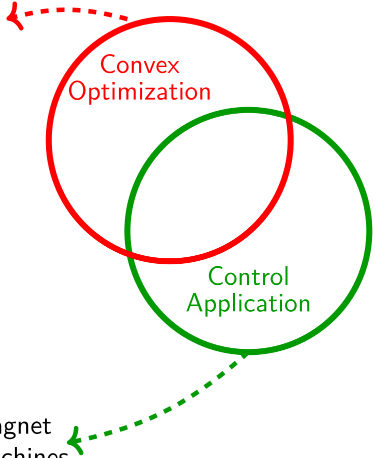
Problem Statement

The embedded approach

Implement advanced control laws synthesis on low-cost hardware to drive real-world systems.

Trajectory generation

Feedback control



Permanent magnet synchronous machines

Problem Statement
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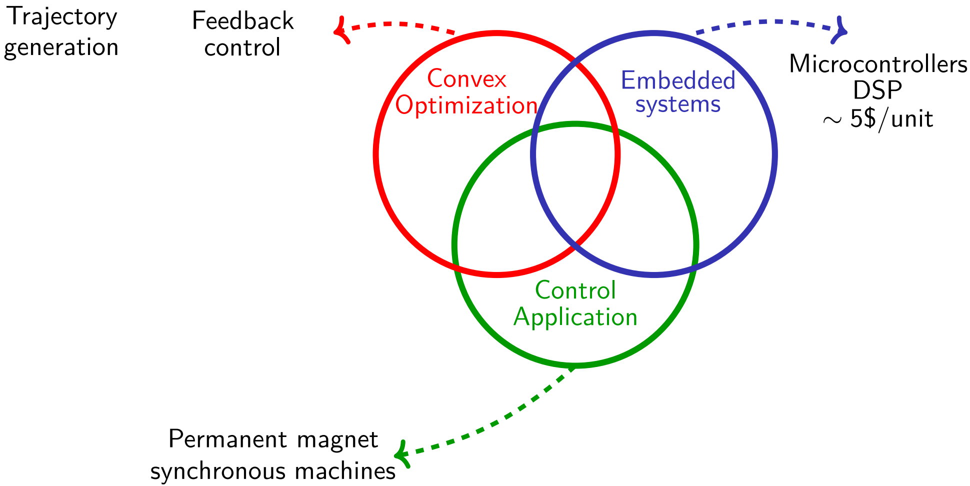
Implement advanced control laws synthesis on low-cost hardware to drive real-world systems.



Problem Statement

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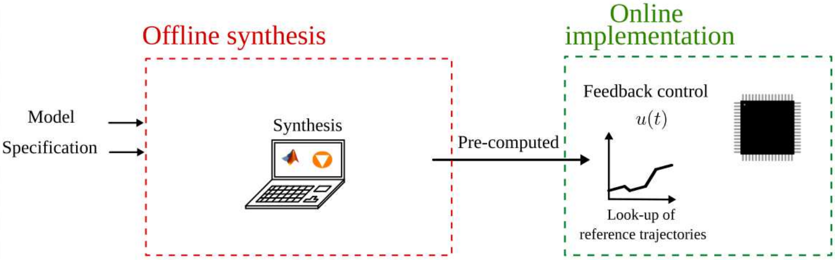

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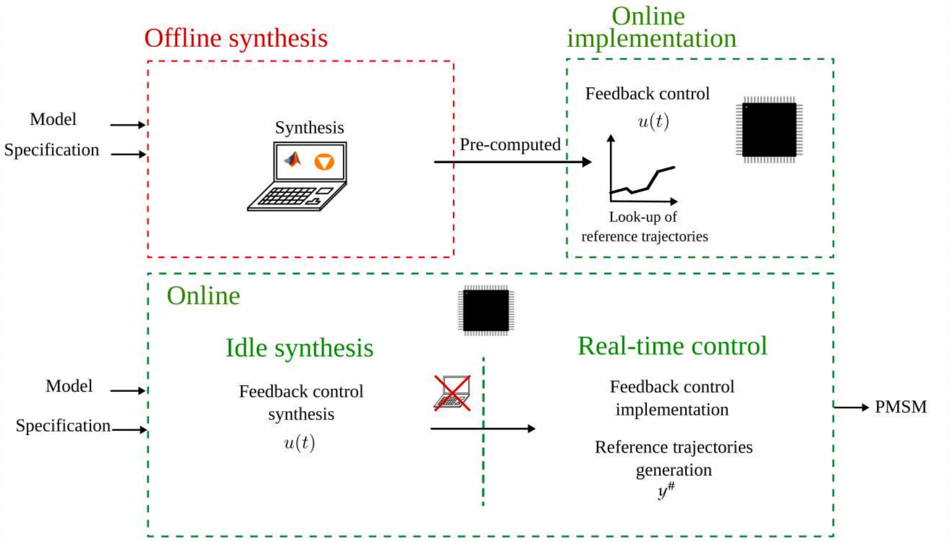

Problem Statement

Permanent Magnet Synchronous Motors (PMSM)

Widely used in industry

Automotive

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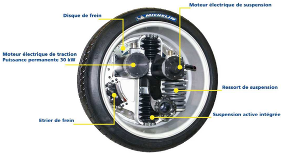

Robotics

Problem Statement

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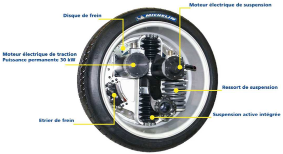

Robotics

Power generation

Problem Statement

Permanent Magnet Synchronous Motors (PMSM)

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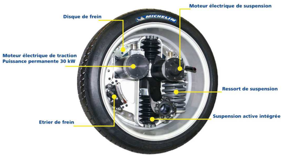


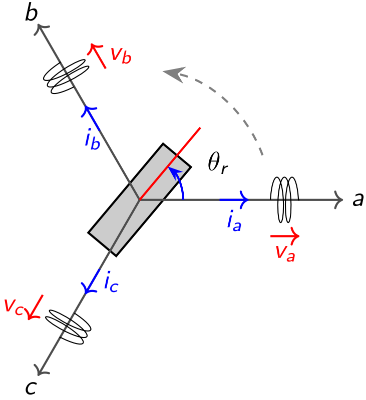
Robotics

Power generation

Goal: control torque and speed, while minimizing losses, taking into consideration the physical limitations.

Problem statement

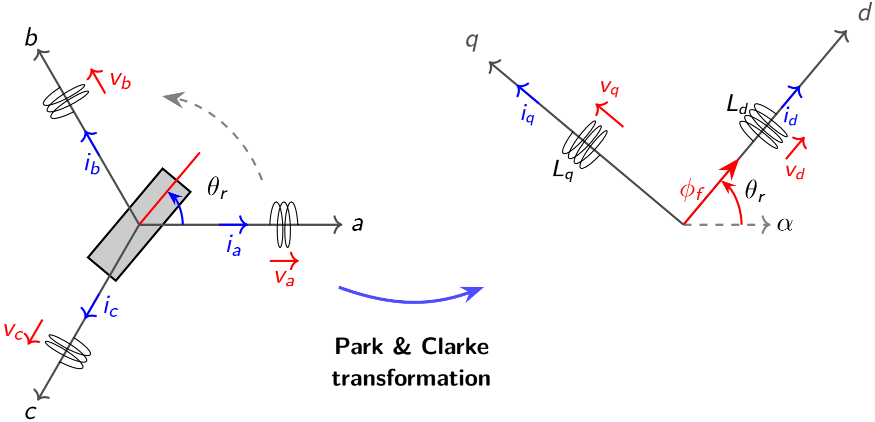
Simple representation of 3-phase PMSM



Static 3-coordinate system

Problem statement

Simple representation of 3-phase PMSM



Static 3-coordinate system **Rotating 2-coordinate system**

Model of the PMSM

The nonlinear model of the system in the dq frame can be represented as

$$
\begin{aligned}
L_d \frac{di_d}{dt} &= v_d - Ri_d + pL_q \omega i_q, \\
L_q \frac{di_q}{dt} &= v_q - Ri_q - pL_d \omega i_d - p\phi_f \omega, \\
J \frac{d\omega}{dt} &= \tau_{em} - f\omega - \tau_l.
\end{aligned}
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The system is subject to physical constraints:

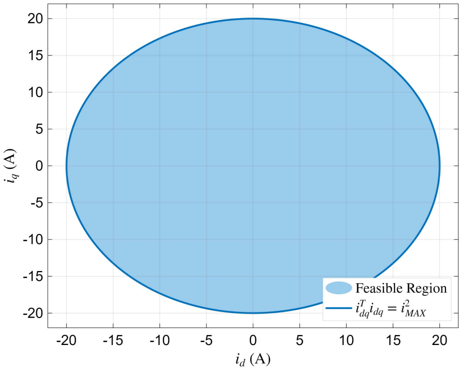
$$\begin{gathered} i_{dq}^\top i_{dq} - i_{max}^2 \leqslant 0, \\ v_{dq}^\top v_{dq} - v_{max}^2 \leqslant 0, \\ \tau_{em} = \frac{3}{2} p \big(\phi_f + (L_d - L_q) i_d\big) i_q. \end{gathered}$$

Problem statement

System constraint

$$
\begin{aligned}
\boxed{i_{dq}^{\top} i_{dq} - i_{max}^2 \leqslant 0} \\
v_{dq}^{\top} v_{dq} - v_{max}^2 \leqslant 0, \\
\tau_{em} - \frac{3}{2} p \left(\phi_f + (L_d - L_q) i_d\right) i_q = 0.
\end{aligned}
$$

Current constraint in the i_{dq} frame



Problem statement
System constraint

$$i_{dq}^{\top} i_{dq} - i_{max}^2 \leqslant 0$$
$$\boxed{v_{dq}^{\top} v_{dq} - v_{max}^2 \leqslant 0,}$$
$$\tau_{em} - \frac{3}{2} p \big(\phi_f + (L_d - L_q) i_d \big) i_q = 0.$$

Voltage constraint in the i_{dq} frame

$$\cancel{L_d \frac{di_d}{dt}} = v_d - R i_d + p L_q \omega i_q = 0,$$
$$\cancel{L_q \frac{di_q}{dt}} = v_q - R i_q - p L_d \omega i_d - p \phi_f \omega = 0,$$

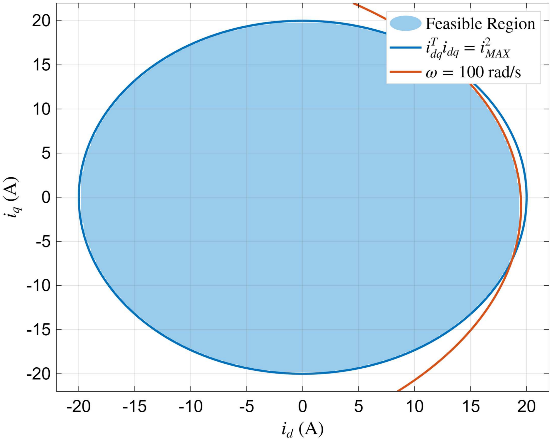
Problem statement

System constraint

$$i_{dq}^{\top} i_{dq} - i_{max}^2 \leqslant 0$$

$$\boxed{\begin{array}{c} A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + \\ D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0 \end{array}}$$

$$\tau_{em} - \frac{3}{2} p \left(\phi_f + (L_d - L_q) i_d \right) i_q = 0.$$



Problem statement

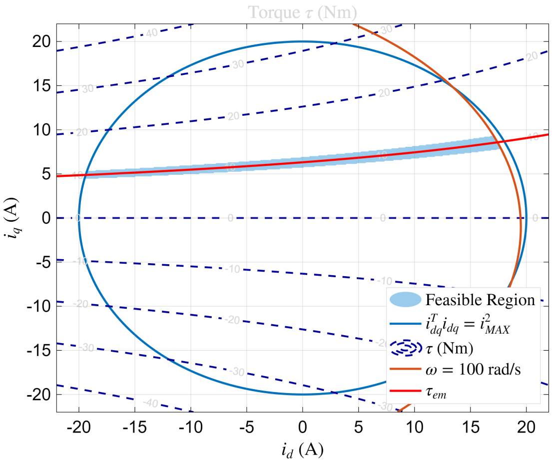
System constraint

$$i_{dq}^{\top} i_{dq} - i_{max}^2 \leqslant 0$$

$$A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0$$

$$\boxed{\tau_{em} - \frac{3}{2} p \left(\phi_f + (L_d - L_q) i_d\right) i_q = 0.}$$

Torque constraint in the i_{dq} frame

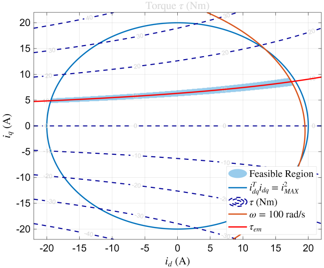


Problem statement

Goals:

Part 1: Optimal currents $i_d^\#$, $i_q^\#$ to produce desired torque τ_{em} and speed ω

Part1: Optimal references



Output: $i_d^\#(\omega, \tau_{em})$, $i_q^\#(\omega, \tau_{em})$

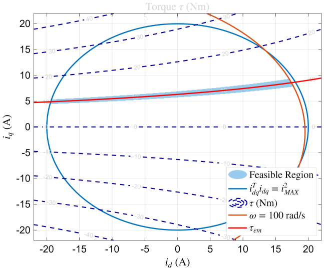
Problem statement

Goals:

Part 1: Optimal currents $i_d^{\#}$, $i_q^{\#}$ to produce desired torque τ_{em} and speed ω

Part 2: Embedded closed-loop control synthesis

Part1: Optimal references



Output: $i_d^{\#}(\omega, \tau_{em})$, $i_q^{\#}(\omega, \tau_{em})$

Part 2: Embedded closed-loop control synthesis

Find $u(t) = Kx(t)$ such that:

- Tracks $i_d^{\#}$, $i_q^{\#}$ trajectories
- Stable, easy-to-tune
- minimise the energy consumption

Problem statement

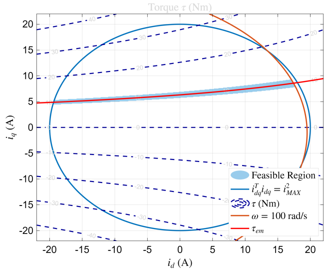
Goals:

Part 1: Optimal currents $i_d^{\#}$, $i_q^{\#}$ to produce desired torque τ_{em} and speed ω

Part 2: Embedded closed-loop control synthesis

Part 3: New control apporachs

Part1: Optimal references



Output: $i_d^{\#}(\omega, \tau_{em})$, $i_q^{\#}(\omega, \tau_{em})$

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Find $u(t) = Kx(t)$ such that:

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Part 3: Other closed-loop control strategies

Find $u(t) = K(x(t))$ such that:

- Robust to measurement noise
- Robust to parametric uncertainty

Problem statement

Goals:

Part 1: Optimal currents $i_d^{\#}$, $i_q^{\#}$ to produce desired torque τ_{em} and speed ω

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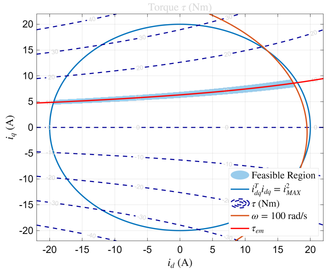
Part1: Optimal references



Output: $i_d^{\#}(\omega, \tau_{em})$, $i_q^{\#}(\omega, \tau_{em})$

Part 2: Embedded closed-loop control synthesis

Find $u(t) = Kx(t)$ such that:

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- Robust to parametric uncertainty

Embedded

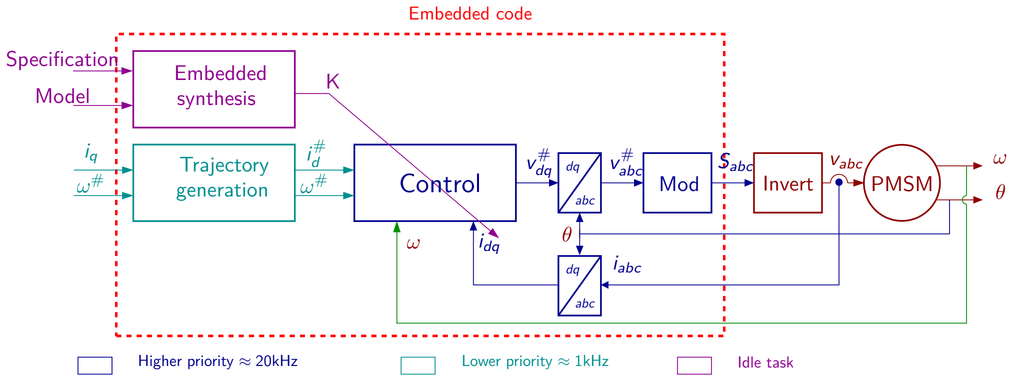
Outline

Objectives:

Part 1: Optimal currents $i_d^\#$, $i_q^\#$ to produce desired torque τ_{em} and speed ω ?

Part 2: Embedded closed-loop control synthesis

Part 3: New control apporachs



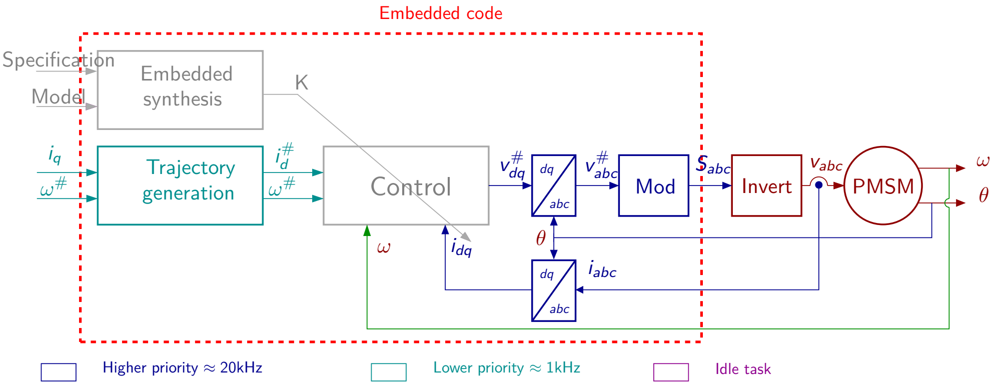
Outline

Objectives:

Part 1: Optimal currents $i_d^{\#}$, $i_q^{\#}$ to produce desired torque τ_{em} and speed ω ?

Part 2: Embedded closed-loop control synthesis

Part 3: New control approaches



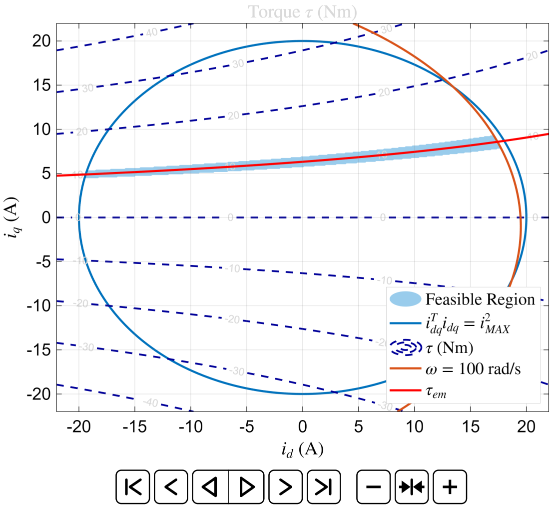
Optimal torque control

System constraint

$$i_{dq}^{\top} i_{dq} - i_{max}^2 \leqslant 0$$

$$A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0$$

$$\tau_{em} - \frac{3}{2} p \left(\phi_f + (L_d - L_q) i_d \right) i_q = 0.$$



Optimal torque control

System constraint

Objective: Minimize Joule losses

$$i_{dq}^{\top} i_{dq} - i_{max}^2 \leqslant 0$$

$$A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0$$

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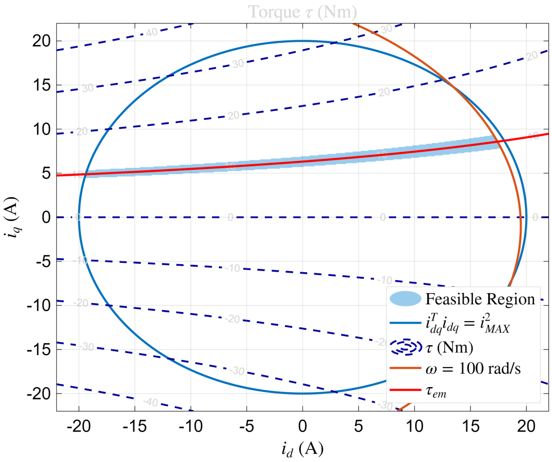

Optimal torque control

System constraint

$$\min_{i_d, i_q} \quad i_{dq}^\top i_{dq}, \quad \textbf{such that}$$

$$i_{dq}^\top i_{dq} - i_{max}^2 \leqslant 0$$

$$A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0$$

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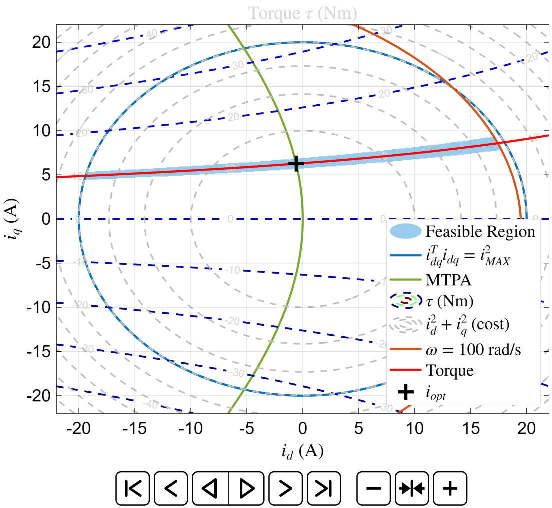

Optimal torque control

System constraint

$$\min_{i_d, i_q} \quad i_{dq}^{\top} i_{dq}, \quad \textbf{such that}$$

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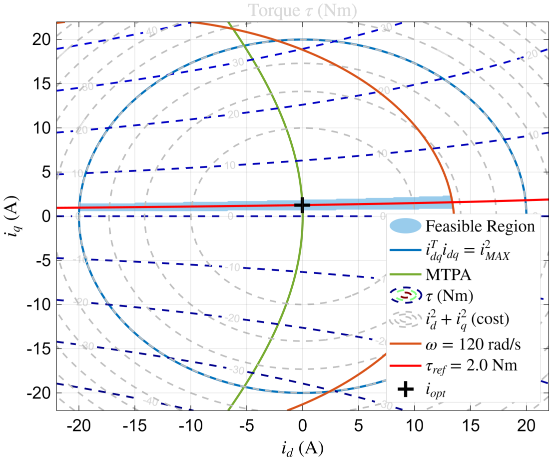

Optimal torque control

Literature

The classical optimal torque control problem:

$$\min_{i_d, i_q} \quad i_{dq}^\top i_{dq}, \quad \textbf{such that}$$

$$i_{dq}^\top i_{dq} - i_{max}^2 \leqslant 0$$

$$A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0$$

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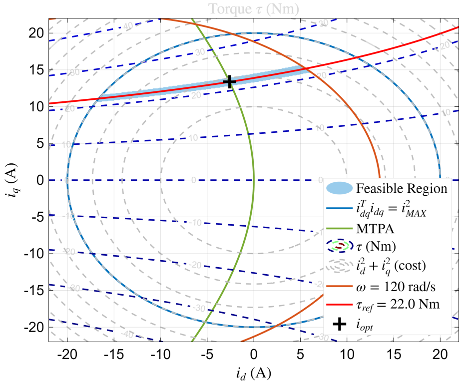

Challenge: the torque constraint is nonconvex

Optimal torque control

Classical approach: Decision tree [1]

Literature approach: Decision tree

Operating zones:

- MTPC: No constraint active
- Maximum Current: Current constraint active
- Field Weakening: Voltage constraint active
- MTPV: Both constraints active

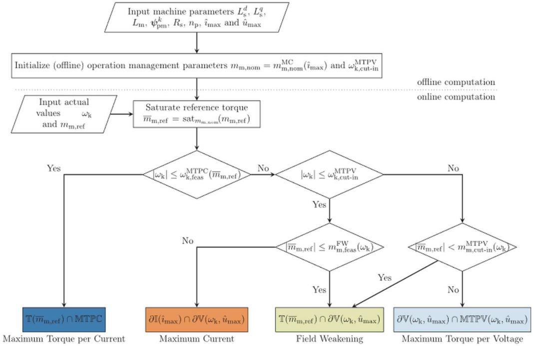

Figure: Optimal torque control decision tree

[1] H. Eldeeb et al., "**A unified theory for optimal feedforward torque control of anisotropic synchronous machines**," *International Journal of Control*, pp. 2273–2302, Oct. 2018.

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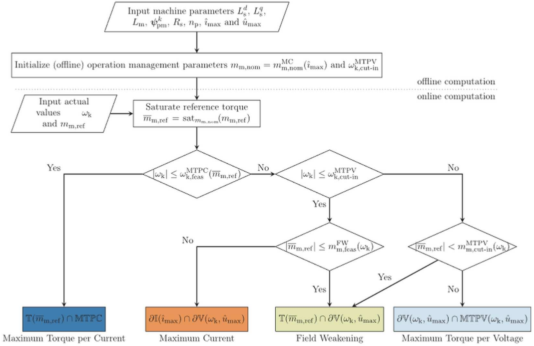

Figure: Optimal torque control decision tree

The proposed approach: look for a convex formulation.

[1] H. Eldeeb et al., "**A unified theory for optimal feedforward torque control of anisotropic synchronous machines**," *International Journal of Control*, pp. 2273–2302, Oct. 2018.

Optimal torque problem

Advantages of the proposed convex formulation [2]

The convex formulation

- **Solvers with polynomial complexity**
- **Scales efficiently** to complex scenarios
- **Easy to add constraints**
- **Guaranteed convergence** to global optimum
- **Easy to integrate**
- **Unified solution**

Decision tree approach:

- Combinatorial complexity when adding constraints
- Difficult to scale up the problem
- Case-by-case analysis for each operating region

[2] S. Boyd and L. Vandenberghe, "**Convex Optimization**," Cambridge University Press, 2004.

The classical problem:

$$\min_{i_d, i_q} \quad i_{dq}^\top i_{dq}, \quad \textbf{such that}$$

$$i_{dq}^\top i_{dq} - i_{max}^2 \leqslant 0$$

$$A(\omega) i_d^2 + B(\omega) i_d i_q + C(\omega) i_q^2 + D(\omega) i_d + E(\omega) i_q + F(\omega) \leqslant 0$$

$$\tau_{em} - \frac{3}{2} p \big(\phi_f + (L_d - L_q) i_d\big) i_q = 0.$$

How to make the problem convex ?

The classical problem:

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How to make the problem convex ?

The proposed reformulation:

A change of variable

$$i_q = \frac{2}{3p} \frac{\tau_{em}}{\phi_f + (L_d - L_q) i_d}.$$

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$$\min_{i_d, i_q} \quad i_{dq}^\top i_{dq}, \quad \textbf{such that}$$

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$$i_q = \frac{2}{3p} \frac{\tau_{em}}{\phi_f + (L_d - L_q) i_d}.$$

The new problem is

$$\min_{i_d} \quad f\big(i_d, \tau_{em}\big) \quad \textbf{such that}$$

$$c_i\big(i_d, \tau_{em}\big) \leqslant 0$$

$$c_v\big(i_d, \omega, \tau_{em}\big) \leqslant 0$$

Optimal torque control

The proposed approach

The classical problem:

$$\min_{i_d, i_q} \quad i_{dq}^\top i_{dq}, \quad \textbf{such that}$$

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$$\tau_{em} - \frac{3}{2} p \big(\phi_f + (L_d - L_q) i_d\big) i_q = 0.$$

How to make the problem convex ?

Is the problem convex? over the set
$\mathcal{S} = \{(i_d, \omega, \tau_{em}) \in \mathbb{R}^3 \mid -i_{max} \leqslant i_d \leqslant 0,\ \omega \geqslant 0,\ \tau_{em} \geqslant 0\}$

The proposed reformulation:

A change of variable

$$i_q = \frac{2}{3p} \frac{\tau_{em}}{\phi_f + (L_d - L_q) i_d}.$$

The new problem is

$$\min_{i_d} \quad f(i_d, \tau_{em}) \quad \textbf{such that}$$

$$c_i(i_d, \tau_{em}) \leqslant 0$$

$$c_v(i_d, \omega, \tau_{em}) \leqslant 0$$

Optimal torque control problem

Convexity of the proposed formulation

Question: Is the reformulated problem convex over $\mathcal{S}$?

$$\min_{i_d} \quad f(i_d, \tau_{em}) \quad \textbf{s.t.} \quad c_i(i_d, \tau_{em}) \leqslant 0, \quad c_v(i_d, \omega, \tau_{em}) \leqslant 0$$

Analysis over $\mathcal{S}$:

- ✓ $f(i_d, \tau_{em})$ is **convex**
- ✓ $c_i(i_d, \tau_{em}) \leqslant 0$ is **convex**
- **?** $c_v(i_d, \omega, \tau_{em}) \leqslant 0$ is **convex** ?

Key question: Is $\dfrac{\partial^2 c_v}{\partial i_d^2}(i_d, \omega, \tau_{em}) > 0$ over $\mathcal{S}$?

Optimal torque control problem

Convexity analysis: Voltage constraint

Key question: Is $\dfrac{\partial^2 c_v}{\partial i_d^2}(i_d, \omega, \tau_{em}) > 0$ over $\mathcal{S}$?

Approach: Multiply by a positive term to simplify

$$h(i_d, \omega, \tau_{em}) = \underbrace{(K_1 + K_2 i_d)^4}_{>0} \cdot \frac{\partial^2 c_v}{\partial i_d^2}(i_d, \omega, \tau_{em})$$

Optimal torque control problem

Convexity analysis: Voltage constraint

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This gives a polynomial:

$$h(i_d, \omega, \tau_{em}) = a_4 i_d^4 + a_3 i_d^3 + a_2 i_d^2 + a_1 i_d + a_0$$

where $a_i = a_i(\omega, \tau_{em})$

Optimal torque control problem

Convexity analysis: Voltage constraint

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This gives a polynomial:

$$h(i_d, \omega, \tau_{em}) = a_4 i_d^4 + a_3 i_d^3 + a_2 i_d^2 + a_1 i_d + a_0$$

where $a_i = a_i(\omega, \tau_{em})$

Goal: show $h(i_d, \omega, \tau_{em}) > 0$ over $\mathcal{S}$

Optimal torque control problem

Sum-of-Squares (SOS) decomposition

Recall the goal:

Goal: show

$$h(i_d, \omega, \tau_{em}) > 0 \quad \forall\, i_d, \omega, \tau_{em} \in \mathcal{S}$$

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Recall the goal:

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Proposed approach: Show that h admits a **SoS decomposition** over $\mathcal{S}$:

$$h(i_d, \omega, \tau_{em}) = s_0 + s_1 \underbrace{(i_d + i_{max})}_{\geq 0} + s_2 \underbrace{(-i_d)}_{\geq 0} + s_3 \underbrace{\omega}_{\geq 0} + s_4 \underbrace{\tau_{em}}_{\geq 0} \qquad \textbf{(Positivstellensatz)}$$

where each s_j is a **SoS polynomial**

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What is Sum-of-Squares (SOS)?

- Each s_j is a **sum of squares**: $s_j = \sum_k p_{j,k}^2 \geqslant 0$. *Ex:* $s_0 = (i_d + \omega)^2 + (2\tau_{em} - 1)^2$

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- SOS found via **semidefinite programming** (YALMIP + MOSEK)

Optimal torque control problem
Convexity via hybrid analytical-SoS approach

Proposition

For a given PMSM, the reformulated problem is **convex** *if one of the following holds:*

1. ***Direct SoS approach:*** *h admits a SoS decomposition*

$$h(i_d, \omega, \tau_{em}) = s_0 + s_1(i_d + i_{max}) + s_2(-i_d) + s_3\omega + s_4\tau_{em}$$

where each s_j is a ***SoS polynomial****.*

2. ***Hybrid approach:*** *$h = h_1 + h_2$ with*

$$\begin{cases} h_1(i_d, \omega, \tau_{em}) \geqslant 0 \text{ (proven analytically)}, & \forall (i_d, \omega, \tau_{em}) \in \mathcal{S} \\ h_2(i_d, \omega, \tau_{em}) = s_0 + s_1(i_d + i_{max}) + s_2(-i_d) + s_3\omega + s_4\tau_{em} \end{cases}$$

where each s_j is a SoS polynomial, ensuring $h \geqslant 0$ over $\mathcal{S}$.

where $h(i_d, \omega, \tau_{em}) = (K_1 + K_2 i_d)^4 \cdot c_v''(i_d, \omega, \tau_{em})$ and
$\mathcal{S} = \{(i_d, \omega, \tau_{em}) \in \mathbb{R}^3 \mid -i_{max} \leqslant i_d \leqslant 0,\ \omega \geqslant 0,\ \tau_{em} \geqslant 0\}$.

Proposition

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Optimal torque control problem

A practical example PMSM

Consider the PMSM test bench available in the Ampère Laboratory:

Parameter	PMSM
Resistance R (Ω)	0.9
d-axis inductance L_d (mH)	0.012
q-axis inductance L_q (mH)	0.020
Flux constant ϕ_f (mWb)	0.264
Pole pairs p	4
DC voltage v_{DC} (V)	600
Maximum power P (kW)	15

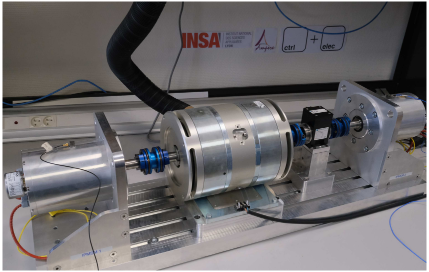


Figure: PMSM test bench.

$$h(i_d, \omega, \tau_{em}) = \underbrace{a_4(\omega, \tau_{em}) i_d^4 + a_3(\omega, \tau_{em}) i_d^3 + a_2(\omega, \tau_{em}) i_d^2}_{h_1(i_d, \omega, \tau_{em})} + \underbrace{a_1(\omega, \tau_{em}) i_d + a_0(\omega, \tau_{em})}_{h_2(i_d, \omega, \tau_{em})}$$

- $h_1(i_d, \omega, \tau_{em}) > 0 \quad \forall\, i_d,\ \omega,\ \tau_{em} \in \mathcal{S}$ is shown analytically
- $h_2(i_d, \omega, \tau_{em}) > 0 \quad \forall\, i_d,\ \omega,\ \tau_{em} \in \mathcal{S}$ is shown through SoS decomposition

Optimal torque control problem

A practical example PMSM

The optimal torque control problem

$$
\begin{aligned}
\min_{i_d} \quad & f(i_d, \tau_{em}) \quad \textbf{such that} \\
& c_i(i_d, \tau_{em}) \leqslant 0 \\
& c_v(i_d, \omega, \tau_{em}) \leqslant 0
\end{aligned}
$$

over the domain of definition $\mathcal{S}$ is **convex**

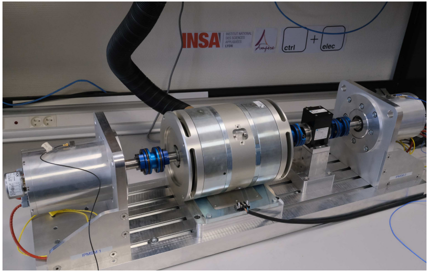


Figure: PMSM test bench.

Optimal torque control problem

A practical example PMSM

The optimal torque control problem

$$\min_{i_d} \quad f(i_d, \tau_{em}) \quad \textbf{such that}$$
$$c_i(i_d, \tau_{em}) \leqslant 0$$
$$c_v(i_d, \omega, \tau_{em}) \leqslant 0$$

over the domain of definition $\mathcal{S}$ is **convex**

We implement:

Interior-point embedded solver

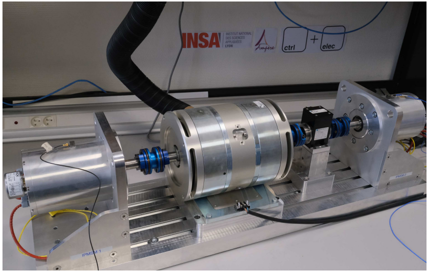
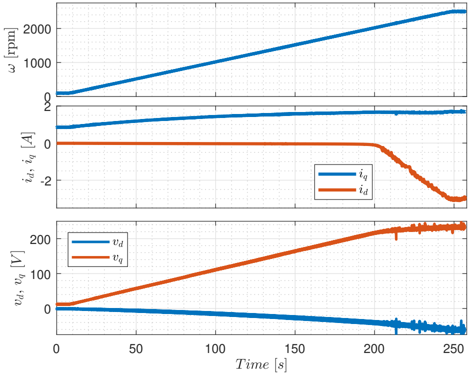
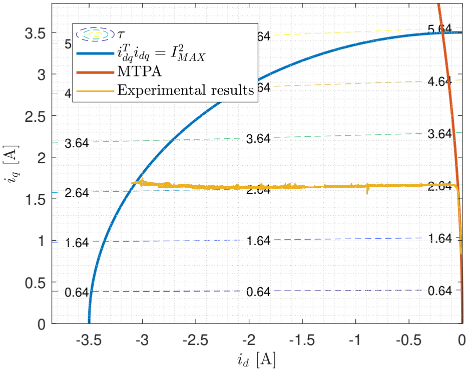


Figure: PMSM test bench.

Optimal torque control problem

Experiment results





Result Summary

Challenge: A nonconvex Optimal torque control problem

Theoretical contributions:

- Change of variable
- New formulation of OTC problem
- Proof of Convexity via Sum-of-Squares programming

Implementation:

- Interior-point embedded solver

Advantages:

+ Polynomial complexity solvers
+ Scales well
+ Easy integration into larger convex problems
+ Unified solution for all operating regions
+ Experimental validation on PMSM test bench

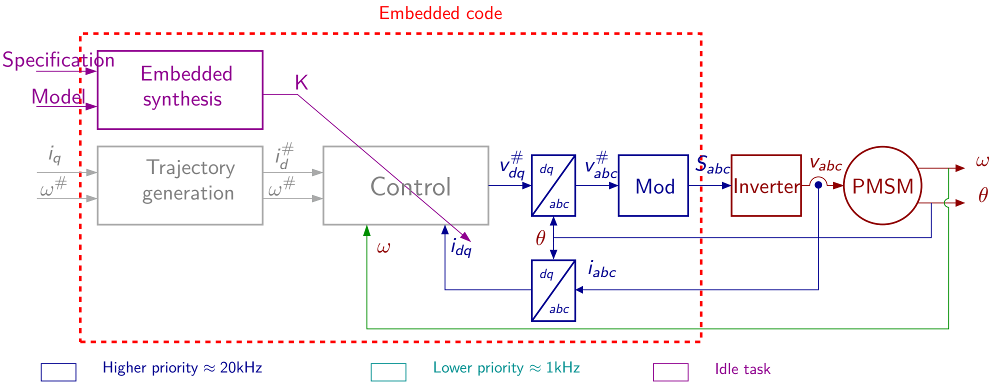
Outline

Objectives:

Part 1: Optimal currents $i_d^{\#}$, $i_q^{\#}$ to produce desired torque τ_{em} and speed ω ?

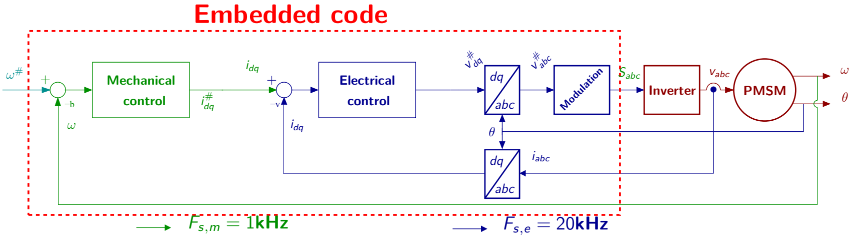
Part 2: Embedded closed-loop control synthesis

Part 3: New control approaches



Embedded control synthesis

Literature: nested PI control [3]



Classical approach:

- Linearize the PMSM model
- Second-order closed-loop dynamics:
  - Electrical dynamics (i_d, i_q)
  - Mechanical dynamics (ω)
- Nested PI controllers

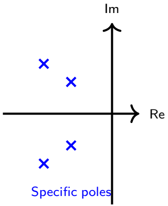
[3] M. Bodson et al., "**High-performance non-linear feedback control of a permanent magnet stepper motor**," *IEEE Transactions on Control Systems Technology*, pp. 5–14, 1993.

Embedded control synthesis

Control synthesis challenges [4]

PI controllers:

- ✓ Easy to tune: relate t_r, overshoot to eigenvalues
- ✓ Specific pole locations
- × Too restrictive: no performance optimization



[4] B. Stellato et al., "**OSQP: an operator splitting solver for quadratic programs**," *Mathematical Programming Computation*, vol. 12, no. 4, pp. 637–672, **2020**.

Embedded control synthesis

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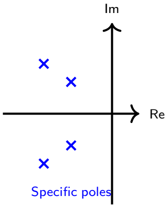


LQR/MPC:

- ✓ Soft constraints
  $$\min \int_0^{\infty} (x^\top Q x + u^\top R u) dt$$
- × Difficult to tune Q, R
- × Difficult to explicitly guarantee robustness

Note: Both methods already embedded in microcontrollers

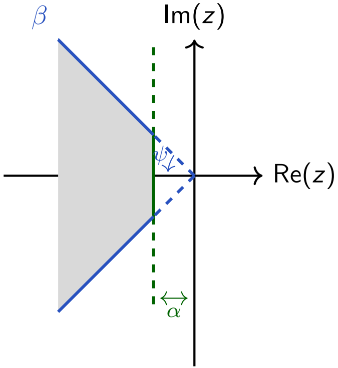
[4] B. Stellato et al., "**OSQP: an operator splitting solver for quadratic programs**," *Mathematical Programming Computation*, vol. 12, no. 4, pp. 637–672, **2020**.

Embedded control synthesis

Pole-constrained $\mathcal{H}_2$ control via LMI

For the same linearized model:

Goal: **Both hard and soft constraints** + robustness



Hard constraints: (tuning similar to PI controller)

- $\mathbb{D}_\alpha$: $\alpha \to$ decay rate
- $\mathbb{D}_\beta$: $\beta \to$ damping ratio

Soft constraint: (similar to LQR/MPC cost)

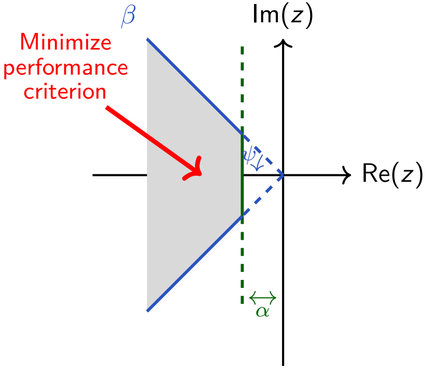
$$\min \int_0^\infty (x^\top Q x + u^\top R u) dt$$

Tuning recommendation: $Q = \mathbf{0}$

Tool: Linear Matrix Inequalities

Embedded control synthesis

LMI capabilities

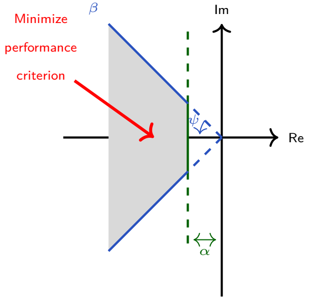


Why LMIs?:

- Convex optimization problems
- Hard and soft constraint formulation
- Robustness to parametric uncertainty
- Anti-windup control design
- Formal guarantees via Lyapunov certificates

Embedded control synthesis

Pole-constrained $\mathcal{H}_2$ optimization problem



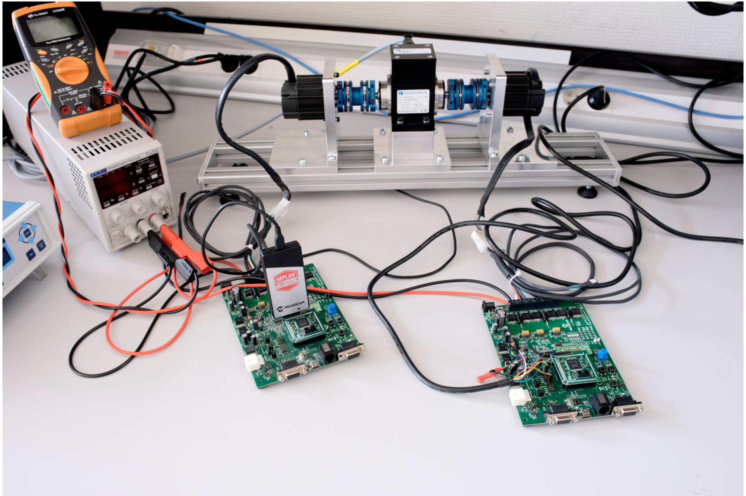
Complete LMI formulation:

$$\min_{X,Y,W,\gamma} \ \gamma \quad \text{subject to}$$

$$\left.\begin{aligned}
&\begin{bmatrix} \langle AX + BY \rangle_S & (*)^\top \\ (C_z X + D_z Y) & -I \end{bmatrix} \prec 0 \\
&\begin{bmatrix} W & B_w^\top \\ B_w & X \end{bmatrix} \succ 0 \\
&\operatorname{trace}(W) < \gamma
\end{aligned}\right\} \ \mathcal{H}_2 \text{ synthesis constraints}$$

$$\left.\begin{aligned}
&\langle AX + BY \rangle_S + 2\alpha X \prec 0 \\
&\begin{bmatrix} \beta \langle AX + BY \rangle_S & (*)^\top \\ (AX + BY)^\top - (AX + BY) & \beta \langle AX + BY \rangle_S \end{bmatrix} \prec 0
\end{aligned}\right\} \ \text{Pole placement constraints}$$

Embedded control synthesis

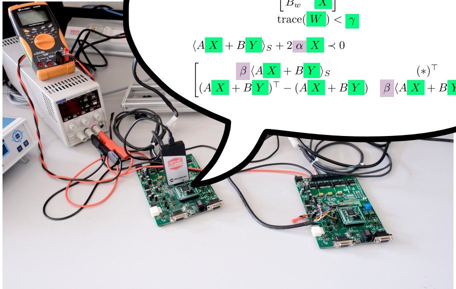


Embedded control synthesis

How to solve an LMI problem?

Solution methodology:

1. **Parametrization**
2. **Feasibility**
3. **Optimization**



$$\min_{X,Y,W,\gamma} \gamma \quad \text{subject to}$$

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Embedded control synthesis

How to solve an LMI problem?

Solution methodology:

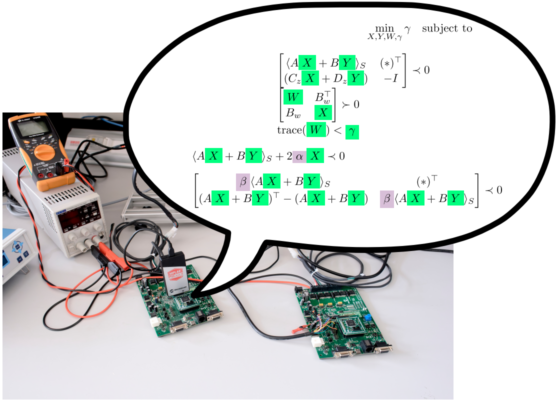
1. **Parametrization** : express decision variables as linear combinations of basis matrices

$$X = X(\xi) = \sum_{i=1}^{n(n+1)/2} \xi_i X_i$$

The LMI constraints become:

$$F(\xi) = F_0 + \sum_{i=1}^{\mu+1} \xi_i F_i \succ 0$$

2. **Feasibility**
3. **Optimization**



Embedded control synthesis

How to solve an LMI problem?

Solution methodology:

1. **Parametrization** The LMI constraints become:

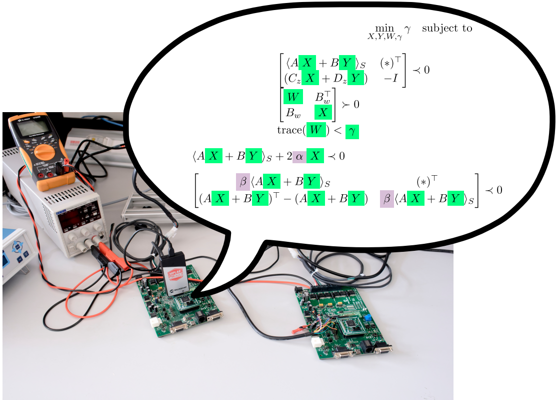
$$F(\xi) = F_0 + \sum_{i=1}^{\mu+1} \xi_i F_i \succ 0$$

2. **Feasibility**: find an initial feasible point

$$\min_{\xi=[\xi_e,\gamma],\lambda} \lambda \text{ s.t.}$$

$$F(\xi) + \lambda I \succ 0$$

3. **Optimization**



Embedded control synthesis

How to solve an LMI problem?

Solution methodology:

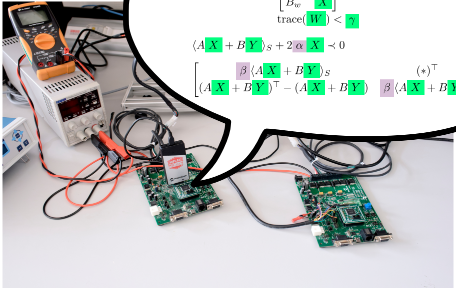
1. **Parametrization** The LMI constraints become:

$$F(\xi) = F_0 + \sum_{i=1}^{\mu+1} \xi_i F_i \succ 0$$

2. **Feasibility**
3. **Optimization** Minimize the objective function

$$\min_{\xi=[\xi_e,\gamma],\lambda} \gamma \text{ s.t.}$$

$$F(\xi) + \lambda I \succ 0$$



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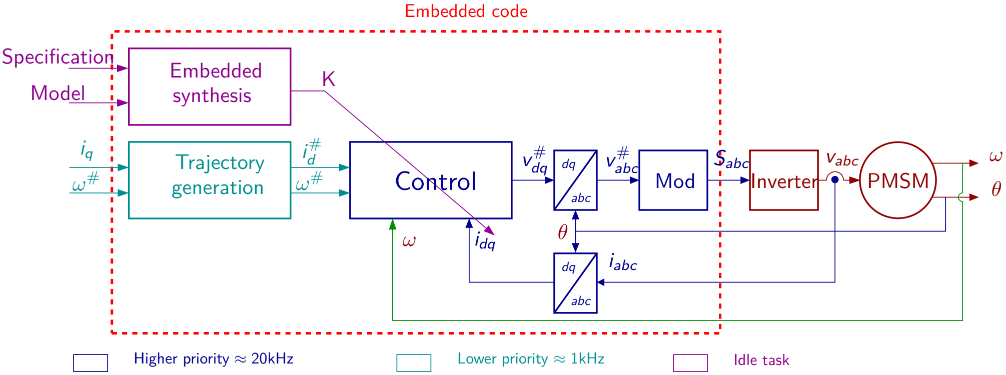
Practical implementation: Real-time scheduler

Hardware platforms:

- dsPIC33AK512 DSC ∼\$3

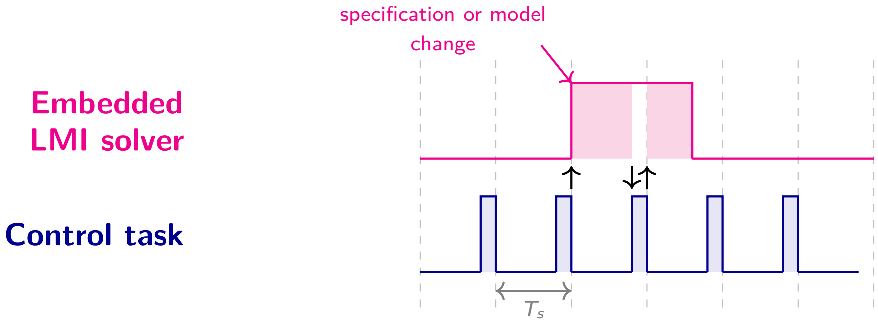
Two concurrent tasks:



Embedded control synthesis

Rate-monotonic scheduler

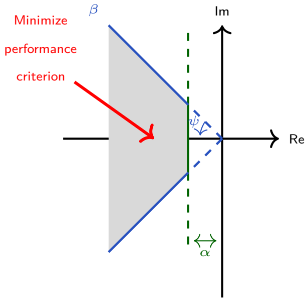
Multi-rate scheduling: High-priority tasks preempt low-priority tasks

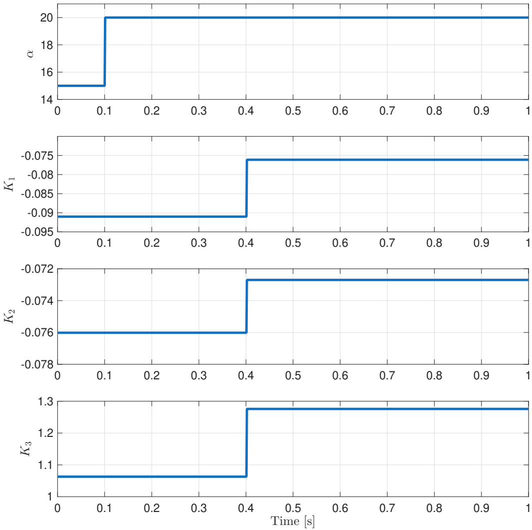


- High-priority task (20 kHz): Control loop runs deterministically
- Low-priority task (idle): LMI solver interrupted when needed

Embedded control synthesis

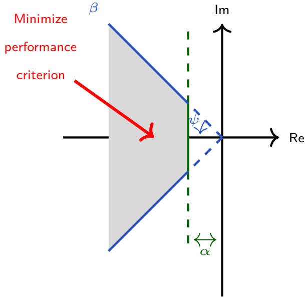
Experimental validation

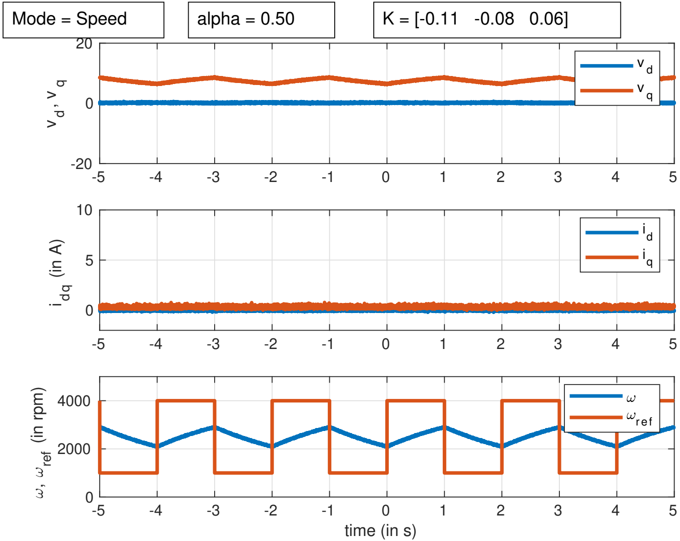




Embedded control synthesis

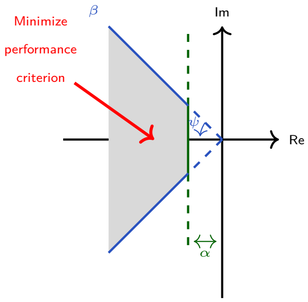
Experimental validation (No laod)

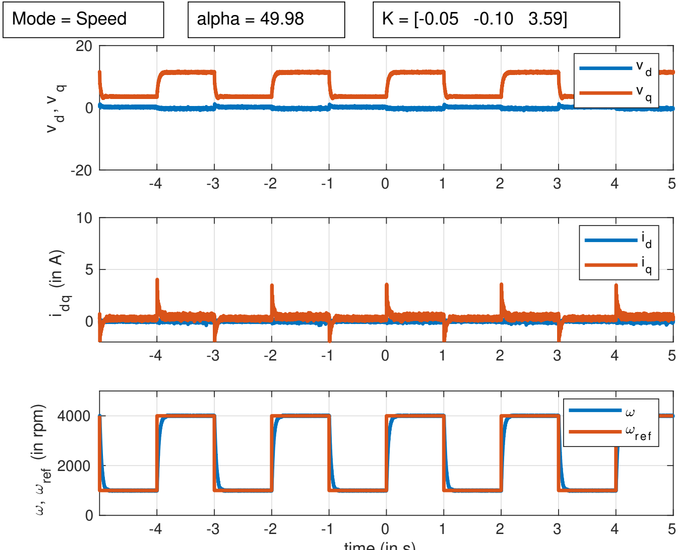




Embedded control synthesis

Experimental validation (no load)





Experimental validation confirms:

- ✓ **Solver operates correctly as idle task**
- ✓ **No need for external intervention**
- ✓ **Computational time for this application is 0.3 s**
  - Dependent on the hardware, the size of the problem, or on the low priority task load.

Remark:

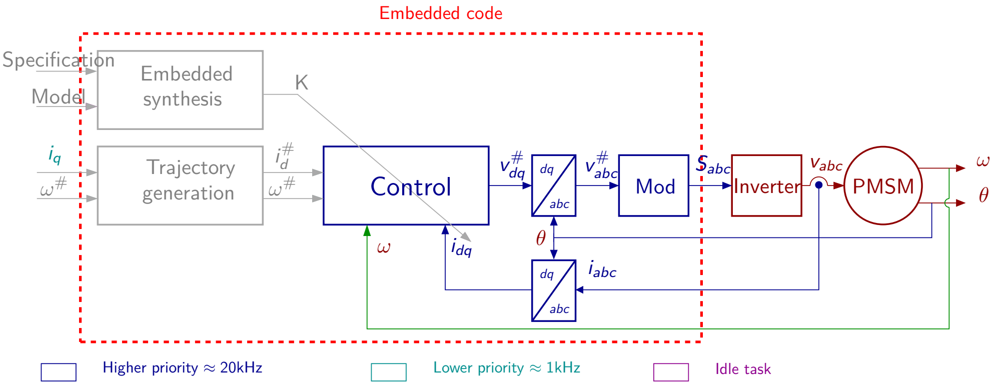
- the LMI solver is generic

Embedded LMI-based control synthesis is feasible on resource-constrained hardware

Outline

Objectives:

1. What are the optimal currents $i_d^\#$, $i_q^\#$ to produce desired torque τ and speed ω ?
2. Embedded closed-loop control synthesis
3. **New control approaches**



Nonlinear PMSM dynamics:

$$L_d \frac{di_d}{dt} = v_d - Ri_d + pL_q \omega i_q$$
$$L_q \frac{di_q}{dt} = v_q - Ri_q - pL_d \omega i_d - p\phi_f \omega$$

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Feedback linearization:

$$\begin{cases} v_d = u_d - pL_q\omega i_q \\ v_q = u_q + pL_d\omega i_d \end{cases}$$

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Linearized system:

$$\dot{x} = Ax + B_u u$$

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Augmented state:

$$\varepsilon_{i_d} = \int_0^t (i_d^{\#} - i_d)d\tau$$
$$\varepsilon_{i_q} = \int_0^t (i_q^{\#} - i_q)d\tau$$

Linear parameter varying control

Classical approach

Nonlinear PMSM dynamics:

$$L_d \frac{di_d}{dt} = v_d - Ri_d + pL_q\omega i_q$$
$$L_q \frac{di_q}{dt} = v_q - Ri_q - pL_d\omega i_d - p\phi_f\omega$$

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$$\dot{x} = Ax + B_u u$$

Augmented state:

$$\varepsilon_{i_d} = \int_0^t (i_d^\# - i_d)d\tau$$
$$\varepsilon_{i_q} = \int_0^t (i_q^\# - i_q)d\tau$$

PI control law:

$$u = Kx$$

Nonlinear PMSM dynamics:

$$L_d \frac{di_d}{dt} = v_d - Ri_d + pL_q \omega i_q$$
$$L_q \frac{di_q}{dt} = v_q - Ri_q - pL_d \omega i_d - p\phi_f \omega$$

Classical approach:

$$\xcancel{\begin{cases} v_d = u_d - pL_q \omega i_q \\ v_q = u_q + pL_d \omega i_d \end{cases}}$$

Nonlinear PMSM dynamics:

$$
\begin{aligned}
L_d \frac{di_d}{dt} &= v_d - Ri_d + pL_q\omega i_q \\
L_q \frac{di_q}{dt} &= v_q - Ri_q - pL_d\omega i_d - p\phi_f\omega
\end{aligned}
$$

Classical approach:

$$
\cancel{\begin{cases} v_d = u_d - pL_q\omega i_q \\ v_q = u_q + pL_d\omega i_d \end{cases}}
$$

LPV model:

$$
\begin{aligned}
\dot{x} &= A(\omega)x + B_u u \\
\omega &\in [\omega_{\text{min}}, \omega_{\text{max}}]
\end{aligned}
$$

Nonlinear PMSM dynamics:

$$L_d \frac{di_d}{dt} = v_d - Ri_d + pL_q\omega i_q$$
$$L_q \frac{di_q}{dt} = v_q - Ri_q - pL_d\omega i_d - p\phi_f\omega$$

Classical approach:

$$\cancel{\begin{cases} v_d = u_d - pL_q\omega i_q \\ v_q = u_q + pL_d\omega i_d \end{cases}}$$

LPV model:

$$\dot{x} = A(\omega)x + B_u u$$
$$\omega \in [\omega_{\min}, \omega_{\max}]$$

Augmented state:

$$x = \begin{bmatrix} i_d & i_q & \varepsilon_{i_d} & \varepsilon_{i_q} \end{bmatrix}^\top$$

Nonlinear PMSM dynamics:

$$L_d \frac{di_d}{dt} = v_d - Ri_d + pL_q\omega i_q$$
$$L_q \frac{di_q}{dt} = v_q - Ri_q - pL_d\omega i_d - p\phi_f\omega$$

Classical approach:

$$\cancel{\begin{cases} v_d = u_d - pL_q\omega i_q \\ v_q = u_q + pL_d\omega i_d \end{cases}}$$

LPV model:

$$\dot{x} = A(\omega)x + B_u u$$
$$\omega \in [\omega_{\min}, \omega_{\max}]$$

Augmented state:

$$x = \begin{bmatrix} i_d & i_q & \varepsilon_{i_d} & \varepsilon_{i_q} \end{bmatrix}^\top$$

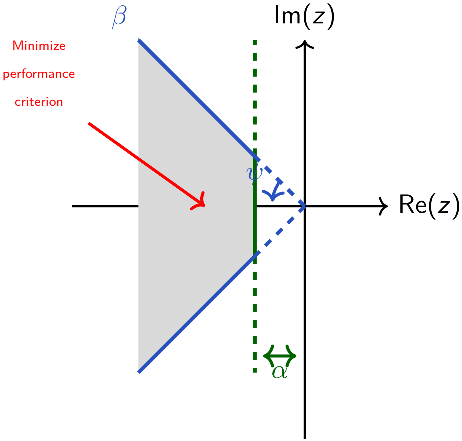
Parameter dependant control:

$$v_{dq} = K(\omega)x$$

Linear parameter varying control

LPV specification: constrained-$\mathcal{H}_2$

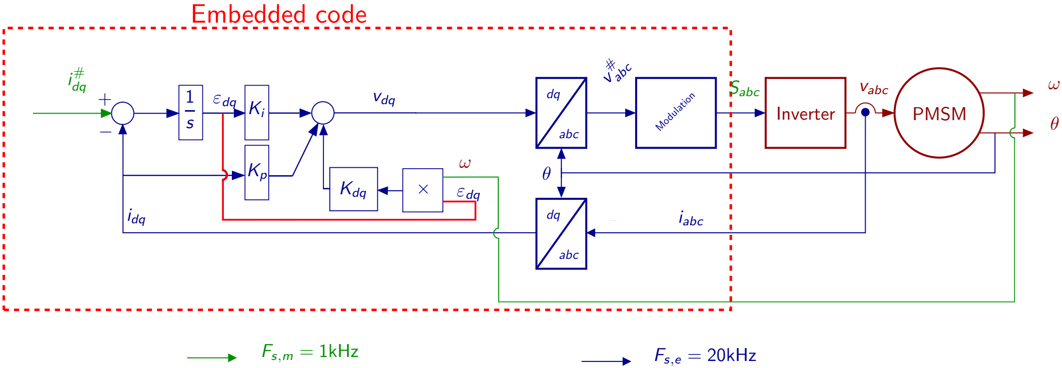
Control synthesis problem: Design a parameter-dependent controller $K(\omega)$ that ensures closed-loop stability and performance for all speed trajectories $\omega(t)$.



Linear parameter varying control

Proposed implementation: Constrained $\mathcal{H}_2$ LPV control

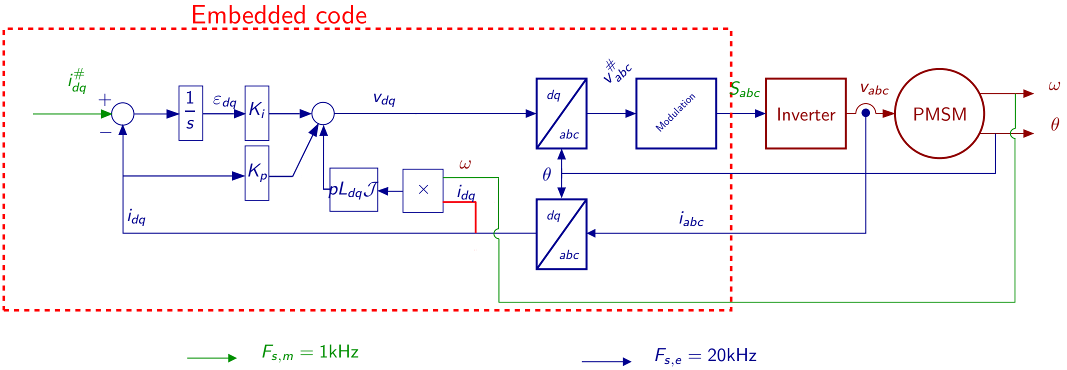
Proposed approach: LPV controller with "frozen" pole placement and $\mathcal{H}_2$ performance optimization.



Linear parameter varying control

Classical implementation: PI + Feedback linearization

Classical approach: PI controller with feedback linearization to cancel nonlinearities.



Linear parameter varying control

Linearization-free approach

Proposed approach: Linearization-free control for PMSM through LMI-based synthesis.

PI + Feedback linearization

$$\begin{cases} v_d = u_d - pL_q\omega i_q \\ v_q = u_q + pL_d\omega i_d \end{cases}$$

- – Direct current injection
- – High sensitivity to noise
- – No formal guarantees

Constrained-$\mathcal{H}_2$ LPV

$$\begin{cases} v_d = u_d - K_d\omega\varepsilon_{i_q} \\ v_q = u_q + K_q\omega\varepsilon_{i_d} \end{cases}$$

- \+ Reduced noise sensitivity
- \+ Stability via Lyapunov function
- \+ Performance guarantees

Key difference: ωi_{dq} (noisy) $\rightarrow$ $\omega\varepsilon_{i_{dq}}$ (filtered)

Linear parameter varying control

Experimental validation

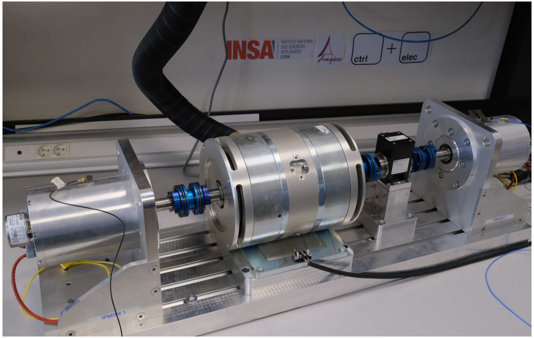
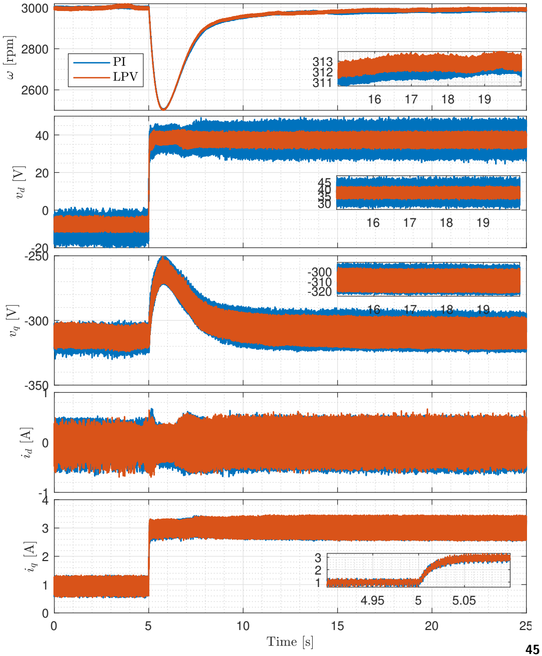


Figure: PMSM test bench.



Contributions:

- **Linearization-free control:** No exact cancellation needed
- **Performance guarantees:** Lyapunov certificate for time-varying ω
- **Noise reduction:** filtering effect due to the $\mathcal{H}_2$ optimization
- **Experimental validation:** Confirmed on PMSM test bench

Other work:

- Robustness to parametric uncertainty $\rightarrow$ norm-bounded parametric uncertainty

Conclusions and Perspectives

Conclusions Summary

Optimal Torque Control

- Change of variable
- New formulation of OTC problem
- Convexity proof via Sum-of-Squares programming
- Interior-point solver for optimal torque trajectories

Embedded Synthesis

- Pole-constrained $\mathcal{H}_2$ synthesis
- Embedded LMI solver (idle task)
- 0.3s computation time

LPV Control

- Linearization-free control
- Lyapunov certificate for time-varying ω (LMI-based synthesis)
- Noise reduction at no additional embedded computational cost

Experimental Validation

What is next ?

Short-term perspectives

Optimal Torque Control

- Extend the convex formulation to other types of machines
- Generic embedded solver for all synchronous machines

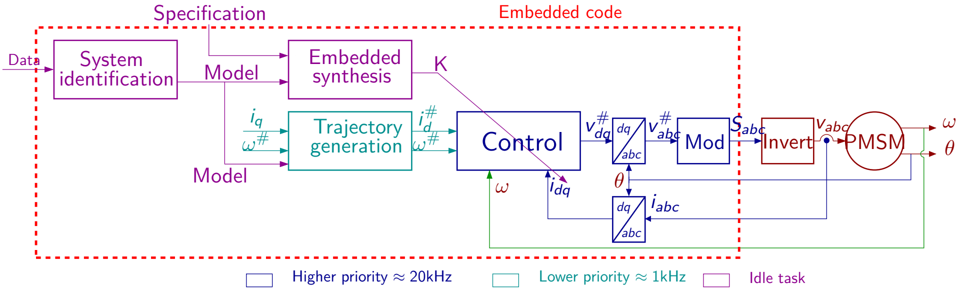
Embedded Synthesis

- Exploit the sparsity of the matrices
- Better tuning of the interior-point method (ADMM...)

LPV Control

- Explore other nonlinear types of control (Sum-of-Squares)
- Include input and state saturations in the LMI synthesis

Long-term perspectives



Vision: Fully embedded, plug-and-play control system integrating parameter estimation, control synthesis, and real-time optimization

Publications

[C.1] **A Karush-Kuhn-Tucker approach to field-weakening for Surface-Mounted Permanent Magnets Synchronous Motors**
Hiba Houmsi, Federico Bribiesca-Argomedo, Paolo Massioni, Romain Delpoux.
IEEE International Conference on Control, Automation and Diagnosis (ICCAD), Rome, Italy, 2023

[C.2] **Embedded Controller Optimization for Efficient Electric Motor Drive**
Hiba Houmsi, Paolo Massioni, Federico Bribiesca-Argomedo, Romain Delpoux.
IEEE Vehicle Power and Propulsion Conference (VPPC), Milan, Italy, 2023

[C.3] **Robust Pole-constrained $\mathcal{H}_2$ Controller for Permanent Magnet Synchronous Motors**
Hiba Houmsi, Paolo Massioni, Federico Bribiesca-Argomedo, Romain Delpoux.
IEEE 23rd European control conference, Thessaloniki, Greece, 2025

[C.4] **Real-Time Interior-Point Solver for Optimal Torque Control of Interior-permanent Magnet Synchronous machines**
Hiba Houmsi, Federico Bribiesca-Argomedo, Paolo Massioni, Romain Delpoux.
IEEE 60th Industrial Application Sociaty Annual Meeting, New Taipei City, Taiwan,2025

Submitted articles

[J.1] **On-chip Embedded Convex Optimization LMI Solver for Rapid Control Tuning on Industrial Targets**
Hiba Houmsi, Paolo Massioni, Federico Bribiesca-Argomedo, Romain Delpoux.
Ongoing work

[J.2] **Linear parameter-varying (LPV) control of synchronous machines: an energy-efficient intuitive gain tuning approach**
Hiba Houmsi, Paolo Massioni, Federico Bribiesca-Argomedo, Romain Delpoux.
Ongoing work

Thank You

Questions?

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