Studying forward looking bubbles in Bitcoin/USD exchange rates

Stefano Bistarelli
stefano.bistarelli@unipg.it
University of Perugia

Gianna Figá Talamanca
gianna.figatalamanca@unipg.it
University of Perugia

Francesco Lucarini
francesco.lucarini@studenti.unipg.it
University of Perugia

Ivan Mercanti
ivan.mercanti@imtlucca.it
IMT School for Advanced Studies, Lucca

Abstract

Although Bitcoin is a relatively new subject in Economics, contributions in this topic are growing very fast. Several papers evidenced a bubble behaviour in exchange rates between Bitcoin and traditional currencies. In this paper we explore and give validation to such conjecture, proving also that the bubble effect is due to confidence in Bitcoin future values. This means that Bitcoin price/exchange rate is influenced both by future and past events, but that the bubble behaviour is strictly connected to trust on the future of the Bitcoin system.

CCS Concepts • **Applied computing** → **Digital cash**; • **Information systems** → *Record storage systems*; • **Networks** → Network performance evaluation.

Keywords Bitcoin, Causal-Noncausal Autoregressive models

ACM Reference Format:
Stefano Bistarelli, Gianna Figá Talamanca, Francesco Lucarini, and Ivan Mercanti. 2019. Studying forward looking bubbles in Bitcoin/USD exchange rates. In *23rd International Database Engineering & Applications Symposium (IDEAS'19), June 10–12, 2019, Athens, Greece.* ACM, New York, NY, USA, 9 pages. https://doi.org/10.1145/3331076.3331106

1 Introduction

Bitcoin is a relatively new subject in Economics and Finance, however, such digital currency is fostering a lot of studies, and contributions in this topic are growing very fast. Some of the studies go in the direction of understanding the reasons of special activities in the market. In particular, several papers evidenced a bubble behaviour in exchange rates between Bitcoin (BTC henceforth) and traditional currencies (Euro or Dollars usually) [9, 17]. The aim of this paper is to explore the conjecture that the bubble effect is due to confidence in Bitcoin future values so that its price/exchange rate is influenced both by past events and by views about future ones.

Traditional econometrics models within the class of AutoRegressive Integrated Moving Average (ARIMA) are *backward looking* since the only time-dependence admitted regards the past [6] and are usually referred to as causal models. Recently, models known as Mixed causal-noncausal AutoRegressive (MAR) have been introduced in order to extend time dependence to the future [7, 10, 18] thus reflecting a *backward-forward looking* behaviour.

The paper by Gouriéroux & Hencic [15] represents a valid anchor to refer to, at least in this area of study, as it undertakes a non-causal analysis of the BTC/USD rates in order to predict its future evolution. The present study shares with [15] both the same decomposition of the BTC/USD price in a bubble and in a fundamental part, and the observed time series; though, here the main objective is to investigate whether confidence in future values of the BTC/USD rate (i.e. the *forward looking* part) is the one responsible of the bubble effect, while in [15] the focus was on forecasting future rates. If this is the case, a significant change in the estimated parameters should be detected when the MAR model is estimated separately in the observed time series and in the bubble component. In particular the *forward looking* parameters should be stronger in the bubble part than in the observed price.

The rest of the paper is structured as follows: the firt part is devoted to the economic explanation of our conjecture about the relation of the speculative bubble in BTC/USD exchange rates with the monetary policy of the Bitcoin system; then, in Section 3 the theory behind the *Mixed Causal–Noncausal autoregressive models* is briefly described. Section 4 describes the dataset and Section 5 summarizes the results of the estimation of the MAR model on the observed data. Section 6 gives conclusions and final remarks.

IDEAS'19, June 10–12, 2019, Athens, Greece

ACM ISBN 978-1-4503-6249-8/19/06…$15.00
https://doi.org/10.1145/3331076.3331106

2 The speculative bubble in BTC/USD rates

By simply watching the trajectory of the BTC/USD exchange rate time series it's easy to notice how often its pattern surges and bursts rapidly mimicking the one of speculative bubbles. The definition of speculative bubble considered in this paper is the one proposed by Blanchard [5] in the framework of rational expectations models where it is assumed that the economic variable of interest, say x_t, has two components: the first one depicts the fundamental path of x_t, while the second represents the bubble effect. In this context a bubble results from the departure of x_t from it's *fundamental path*. In Fig. 1 one of the major bubbles occurred in 2013 for the BTC/USD rate is recorded.

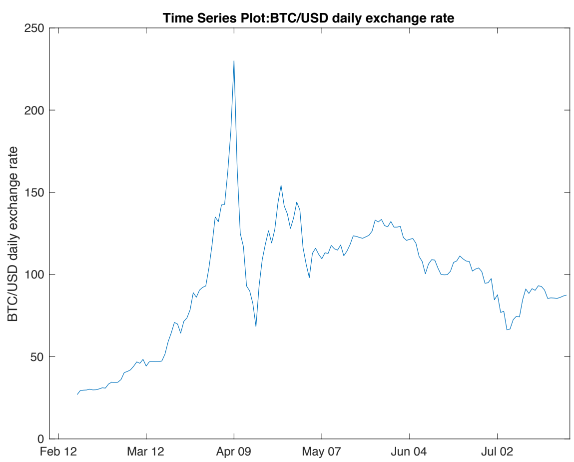

Figure 1. Bitcoin/USD observed time series

Bitcoins are produced through a "mining" process which involves computers (nodes from now on) solving complex mathematical problems (cryptography) to keep the system secure; when the node find a solution to the problem it is rewarded with an amount of Bitcoins which is referred to as "Block reward". The protocol running Bitcoin is programmed to halve every 4 years the "Block reward" by suitably increasing the difficulty of the mathematical problems to be solved. Hence, the volume of new coins will decay to zero with time and the long-term monetary supply will be fixed.

This paper aims at investigating whether the peculiar "deflationary" mechanism running the system's monetary issuance is the main responsible of the formation, and the subsequent crash, of speculative bubbles (against the USD and other currencies). Indeed economic agents, before undertaking any action within the system, already include the system's monetary issuance in their preferences/expectations, *i.e.* they already know that monetary issuance will never diverge from their expectations inasmuch the system has a unalterable monetary policy programmed to ever decrease the monetary issue over time. Therefore as the system grows (think of it as the Gross Domestic Product of a national economy) the demand of Bitcoins will increase, boosting upwards its price against other traditional currencies, given the *ex-ante* fixed monetary supply.

The reason why this issuance mechanism is hereby defined "deflationary" is that as long as the general belief that the system as a whole will keep growing stands, the price against other currencies will inevitably increase, increasing the inter-temporal opportunity cost of spending any given amount of BTC. As a matter of fact since the agents know that the price will increase then they are encouraged to withhold any transaction in BTC and increase their *savings* in BTC. A very interesting effect of such mechanism is that any steep fall in the price may boost the awareness of BTC as a system, potentially increasing it's diffusion among the general public, thus incrementing the aforementioned self–sustaining dynamic [17].

It must be noticed that if the system had a flexible monetary policy, where changes are not known *ex-ante*, then the economic agents within and without the system wouldn't be able to include it in their preferences, thus neutralizing the aforementioned self–sustaining mechanism, even if the bitcoin system is flourishing. After the explanation given above it must be clear now the reason why it is expected and tested below in this study that the speculative bubble in BTC/USD rates is a *forward–looking* phenomena.

3 Mixed causal-noncausal autoregressive models

For a long time, as mentioned by Gouriéroux & Hencic [15], speculative bubbles were considered as nonstationary phenomena and treated similarly to the explosive, stochastic trends due to unit roots. Gouriéroux & Zakoian [11] propose a different approach and assume that the bubbles are rather short-lived explosive patterns caused by extreme valued shocks in a noncausal, *stationary* process. In particular they assume a noncausal AR(1) (Auto Regressive) model, strictly *forward looking*, with Cauchy distributed errors.

A useful feature of such models is that shocks are nonfundamental, combining this trait with the extended time dependance (to the future) allows these models to perfectly fit the peculiar pattern of the aforementioned (definition of) speculative bubble.

3.1 Introduction to noncausality

Let y_t be the observed time series onto which estimate the traditional autoregressive model:

$$\begin{aligned} a(L)\, y_t &= \varepsilon_t \\ (1 - a_1 L - \cdots - a_p L^{-p})\, y_t &= \varepsilon_t \end{aligned} \tag{1}$$

with L being the backshift operator, *i.e.*, $L y_t = y_{t-1}$ gives lags and $L^{-1} y_t = y_{t+1}$ produces leads and a the autoregressive parameters. It is known [18] that if s out of p of the polynomial's $(a(L))$ *zeros* are inside the unit circle, then the model is *non stationary* causing the impossibility to estimate the traditional autoregressive model. ε_t represent the usual error term of the model.

In Lanne & Saikkonen [18] it is shown that when $p = r + s$, with r being the zeros outside the unit circle, one can factor the polynomial $a(z)$ as[1]:

$$a(z) = \varphi^*(z)\,\phi(z) \tag{2}$$

where $\phi(z)$ is the usual causal polynomial of the autoregressive parameters and $\varphi^*(z)$ has its zeros inside the unit circle.

The polynomial $\varphi^*(z)$ can be expressed as:

$$\begin{aligned}\varphi^*(z) &= 1 - \varphi_1^* z - \cdots - \varphi_s^* z^s \\ &= -\varphi_s^* z^s \left(1 + \frac{\varphi_{s-1}^*}{\varphi_s^*} z^{-1} + \cdots + \frac{\varphi_1^*}{\varphi_s^*} z^{1-s} - \frac{1}{\varphi_s^*} z^{-s}\right) \\ &= -\varphi_s^* z^s \varphi(z^{-1})\end{aligned} \tag{3}$$

where $\varphi(z^{-1}) = 1 - \varphi_1 z - \cdots - \varphi_s z^s$ in view of the fact that $\varphi_{s-j}^*/\varphi_s^* = -\varphi_j$ for $j = 1, \ldots, s$ and $1/\varphi_s^* = \varphi_s$.

Because the zeros of $\varphi^*(z)$ lie inside the unit circle those of $\varphi(z)$ lie outside of the unit circle. Thus, (1) can be written as:

$$\phi(L)\,[-\varphi_s^* L^s \varphi(L^{-1})] = \varepsilon_t$$

given the decomposition shown in (3). Also, the latter expression can be rearranged as:

$$\phi(L)\,\varphi(L^{-1})\,y_t = \epsilon_t \tag{4}$$

where $\epsilon_t = -(1/\varphi_s^*)\,L^{-s}\varepsilon_t = -(1/\varphi_s^*)\,\varepsilon_{t+s}$. It is important to notice that $E_t[\epsilon_t] \neq \epsilon_t$ since this variable is not determined by any informations available at time point t (see above).

3.2 Mixed Causal-Noncausal Autoregressive Model

The univariate mixed causal-noncausal autoregressive model, denoted MAR(r, s), shown with equation 4 is usually written as:

$$(1 - \phi_1 L - \cdots - \phi_r L^r)(1 - \varphi_1 L^{-1} - \cdots - \varphi_s L^{-s})\,y_t = \epsilon_t \tag{5}$$

When $\varphi_1 = \cdots = \varphi_s = 0$, the process y_t represent a purely causal autoregressive process denoted AR$(r, 0)$:

$$(1 - \phi_1 L - \cdots - \phi_r L^r)\,y_t = \epsilon_t \tag{6}$$

where y_t is regressed on past values, giving the process y_t a *backward looking* autoregressive dynamic.

The process y_t is *purely noncausal* when $\phi_1 = \cdots = \phi_r = 0$, hence defined as:

$$(1 - \varphi_1 L^{-1} - \cdots - \varphi_s L^{-s})\,y_t = \epsilon_t. \tag{7}$$

usually referred to as *forward looking* AR$(0, s)$ process, being the exact counterpart of the model specification given in (6), since it's regressed on future values rather than past ones.

Models containing both lags and leads of the dependent variable are called *mixed causal–noncausal models.*

[1] In order to maintain the same notation as in Lanne & Saikkonen [18] the polynomial $a(L)$ will be referred to as $a(z)$ for the following proof.

Assuming that the roots of the causal and noncausal polynomial are outside the unit circle, that is:

$$\phi(z) = 0 \quad \text{per} \quad |z| > 1 \quad \text{e} \quad \varphi(z) = 0 \quad \text{per} \quad |z| > 1 \tag{8}$$

than these conditions imply that the series y_t admits a two-sided Moving Average (MA) representation:

$$y_t = \sum_{j=-\infty}^{\infty} \psi_j\,\epsilon_{t-j} \tag{9}$$

such that $\varphi_j = 0$ for all $j < 0$ implies a purely causal process t and a purely noncausal model when $\varphi_j = 0$ for all $j > 0$ [19]. More in detail, the ψ_jâĂŹs are the coefficients of an infinite order polynomial in positive and negative powers of the Lag operator and such that $\Xi(z) = \sum_{j=-\infty}^{\infty} \psi_j z^j = [\Psi(z^{-1})]^{-1}\,[\Phi(z)]^{-1}$.

Error terms ϵ_t are assumed *iid* non-Gaussian with $E(|\varepsilon_t|^{\delta}) < \infty \;\forall\, \delta \in (0, 1)$ [11]. Following Gouriéroux & Jasiak [10] the unobserved causal and noncausal components of the process y_t are defined as follows:

$$u_t \equiv \phi(L)\,y_t \;\leftrightarrow\; \varphi(L^{-1})u_t = \epsilon_t, \tag{10}$$

$$v_t \equiv \varphi(L^{-1})\,y_t \leftrightarrow \phi(L)\,v_t = \epsilon_t \tag{11}$$

The specification of these values will prove useful for the following part regarding the estimation of mixed causal-noncausal processes.

The non-Gaussianity assumption for the error term ensures the identifiability of the causal and the noncausal part. Most papers by Lanne & Saikkonen et al. use Student's t_v distributions, with $v \geq 2$ while Gouriéroux et al. rely on the Cauchy or a mixture of Cauchy and Normal distributions. As shown by Hecq et al. [14] it emerges that the Cauchy has too strong fat tails features and many series would have a degree of freedom between 1.5 and 2.5[2].

Figure 2. TradeBitcoin data price download panel

[2] Notice that when $v < 2$ then the Student's t expected value is undefined.

4 The Data

The sample consists in 151 observation of the BTC/USD price spanning from February 20 to July 20 2013. The dynamic of the data is shown in Fig.1, where it is possible to notice the speculative bubble behaviour of the BTC/USD path, boosting and bursting rapidly around the month of April. In fact, in the April 2013 there was a famous bubble, commonly called simply the April bubble, that was a rally, all-time high and subsequent crash of the bitcoin exchange rate. The bubble resulted in a momentary all-time high of $266 USD per bitcoin on Mt. Gox[3] on 10th April 2013. Then Mt. Gox suspended trading on 11th April 2013 until 12th April 2013 2 am UTC for a "market cooldown". The value of a single bitcoin fell to a low of $55.59 after the resumption of trading before stabilizing above $100[4] (a price decline of 61%).

The data is obtained from our application, TradeBitcoin [1], part of the suite BlockChainVis[5] [2–4] used for Bitcoin analysis and visualization, is based on finding the price options on the Bitcoin exchange and writing possible arbitrage operations on a database to see if it is possible to correctly perform an arbitrage on the Bitcoin market. It also collects all this data price from 17 different exchanges and it allows to download that data with a detection time of 1 day or 1 hour or 15 minutes (Fig. 2).

4.1 Price decomposition

As a first issue it is important to disentangle the fundamental component from the bubble component of the BTC/USD prices. The fundamental value of the Bitcoin is still under debate. While in [8] it is argued that this fundamental value is zero, in [9] it is linked to the reputation of the Bitcoin system measured by internet queries, moreover it is suggested (still in [9]) that the production cost of Bitcoin, due to the mining process, should be considered as the lower limit of the fundamental value of Bitcoin. Since this study assumes that Bitcoin has a fundamental value indeed, the price will be firstly decomposed following the approach in [15], where the fundamental path of the BTC/USD rate is assumed to be a nonlinear deterministic trend modelled as a 3rd degree polynomial in time and the bubble part is obtained by subtraction from the observed prices, as it is done still in [15]. The other decomposition that will be undertaken builds upon the suggestion made in [9], by setting apart the production cost of Bitcoin and the bubble component using the cost of production model shown in [13].

4.1.1 Nonlinear deterministic trend

As mentioned above the BTC/USD rate is defined as follows:

$$rate_t = trend_t + y_t, \tag{12}$$

with $rate_t$ being the observed prices, $trend_t$ the fundamental component and y_t the bubble component and the estimated trend is given by:

$$trend_t = 0.000073\, t^3 - 0.0316\, t^2 + 3.6590\, t - 3.2951.$$

The corresponding time-series are plotted in Fig. 3.

4.1.2 Production cost as the lower limit of the fundamental value

The Bitcoin production cost model shown in [13] assumes the perspective of a generic *miner* that is deciding whether to mine or not for Bitcoin. The *miner* will decide to join the mining process in case of positive profit expectations and to abandon it on the contrary case. The variables considered to be influencing the mining process and hence the production cost in [13] are: the *block reward* β, the *hashing power* (computational power) of the mining hardware equipment ρ, the *difficulty* set by the network δ, the *cost* per kilowatt-hour[6] $\$\,kW/h$ and the *average energy efficiency* $W\,GH/s$ of the mining hardware deployed.

As shown in [13] the expected number of cryptocurrency coins to be mined per day on average given the difficulty and block reward (number of coins issued per successful mining attempt) per unit of hashing power is given by:

$$BTC/day = \frac{\beta\,\rho\,sec_{hr}}{\delta\,2^{32}}\,hr_{day}$$

sec_{hr}= 3600 being the seconds in 1 hour and hr_{day}= 24 being the hours in a day.

The cost of mining can be expressed as:

$$E_{day} = (\rho/1000)\,(\$\,kW/h \;\; W\,GH/s \;\; hr_{day})$$

with $\$\ kW/h$ being the electricity cost and $W\ GH/s$ the average energy efficiency. Bitcoin production cost estimates over the considered time span (Feb.-Jul. 2013) are shown in Fig. 4, where it is assumed an average energy efficiency of $W\ GH/s = 500$ as suggested by Garcia et al. [9], a computational power[7] of $GH/s = 1000$, an average global electricity cost of $\$\,kW/h = 0.115$. In 2013 the block reward set by the network was $\beta = 25\ BTC$, the values of the ever changing difficulty over the considered time-period can be found in the public database https://blockchain.info.

Assuming the lower limit of the fundamental value, given by the aforementioned definition of production cost, as the actual fundamental value, therefore the BTC/USD rate is

[3] https://en.bitcoin.it/wiki/Mt._Gox.
[4] https://bitcoincharts.com/charts/mtgoxUSD#rg5zczsg2013-04-10zeg2013-04-12ztgSzm1g10zm2g25zl.
[5] http://normandy.dmi.unipg.it/blockchainvis/.

[6] In this study we consider the same average cost for electricity that is considered in [13], although it must be noticed that it changes depending on the geographical location of the miner and therefore on the national electricity supplier.
[7] It must be noticed that in this example varying the computational power does not change the cost, *i.e.*, the cost of 1 BTC in USD is only affected by the difficulty, the electricity cost and the average energy efficiency of the mining hardware, increasing the scale of production in this case doesn't lead to economies of scale.

defined similarly as in 12:

$$rate_t = cost_t + yc_t, \tag{13}$$

with $rate_t$ being the observed prices, yc_t the bubble component and where the fundamental component in this case is given by the aforementioned production cost $cost_t$. Assuming that the production cost is correctly estimated, it must be noticed that the bubble component could be considered as the *added market value*.

The corresponding time series are plotted in Fig. 4

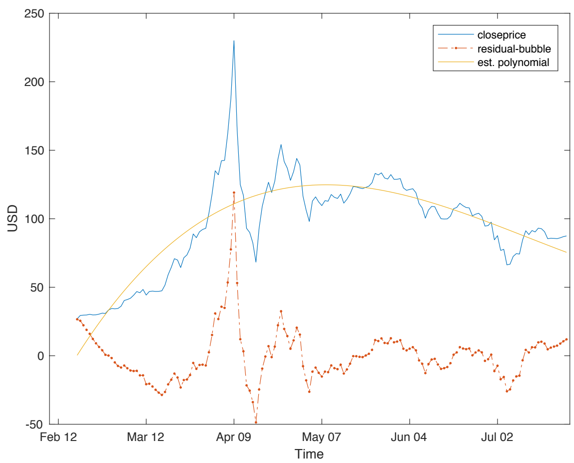

Figure 3. Bitcoin/USD price decomposition

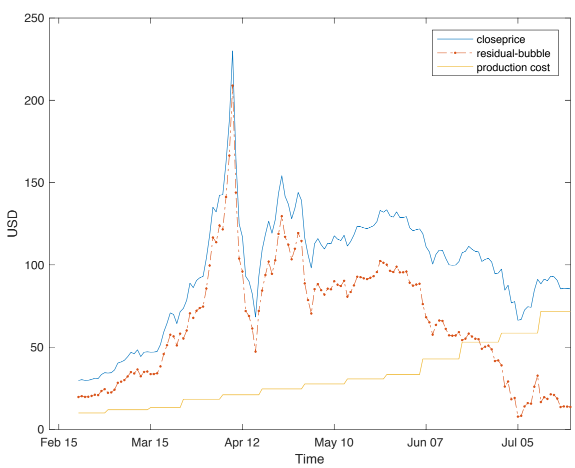

Figure 4. Bitcoin/USD price vs. production cost

5 Estimated models

The following part of the study undertakes a mixed causal/noncausal analysis by estimating MAR models on the BTC/USD price and on the bubble component according to the two different definitions of the fundamental part. As already discussed in the introduction it is expected that the *forward looking* dependence is stronger in the isolated bubble component than in the observed price. The model specifications in what follows are chosen by applying *information criteria* which are useful tools to select the number of lags (and leads) to be included in the model. The information criteria hereby considered are the *Akaike information criterion* (AIC), the *Bayesian information criterion* (BIC) and the *Hannan-Quinn information criterion* (HQ) (for a general review see the book by Hamilton [12]). Once the number of lags/leads have been detected, models are estimated by maximizing the approximated log-likelihood function based on the Student's t density function for the error term; a detailed description of the procedure may be found in Hecq et al. [14]. The related Matlab routines used in this work are kindly provided by the authors of the above quoted paper.

Table 1. AR(1) model's estimated parameters

AR(1) Model		***t* distribution**	
φ_1	Std. Dev.	λ	ν
0.8066	0.0234	4.3928	2.5013

Table 2. BDS test results, purely noncausal model AR(1).

m	w	$p-value$	m	w	$p-value$
2	5,978547545	1,13E-09	9	13,3666165	0
3	6,525463574	3,39E-11	10	14,65204326	0
4	7,420797806	5,82E-14	11	15,91260972	0
5	8,615265114	0	12	17,42915916	0
6	9,743131337	0	13	19,3469674	0
7	10,83386529	0	14	21,49415105	0
8	12,07560832	0	15	24,05813227	0

5.1 Noncausal analysis of the bubble component

Firstly is considered a *strictly noncausal* AR(1) (forward looking):

$$y_t = \varphi_1 y_{t+1} + \epsilon_t.$$

where ϵ_t are *iid* Student's t distributed errors, with location 0 and scale parameter λ, $\epsilon_t \sim (0, \lambda)$. Estimated parameters are reported in Table 1. The residuals of the models are shown in Fig 5. In order to test the model's goodness of fit, the results of the BDS test (Brock, William, Davis Dechert & Scheinkman, 1987) [16], used to test whether the residual are truly a sequence of *iid* Student's t random variables, are reported in Table 2. The test fails to accept the null hypothesis of *iid* distributed residuals, this implies that the present model must be discarded.

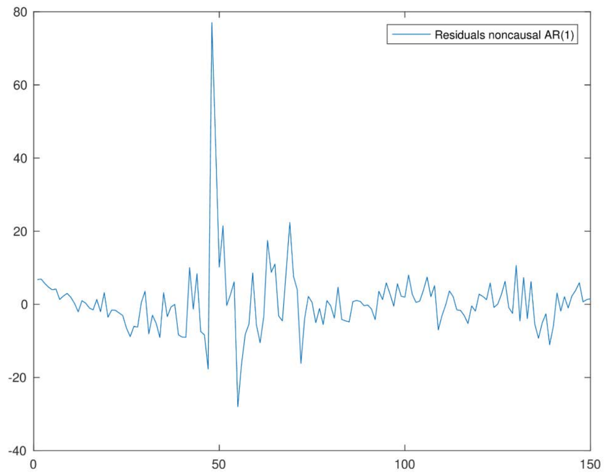

Figure 5. Noncausal AR(1) model residuals

Table 3. Information Criteria

p	BIC	AIC	HQ	p	BIC	AIC	HQ
0	6,0119	5,9565	5,9649	5	4,8862	4,5537	4,6042
1	4,7057	4,5949	4,6117	6	4,914	4,526	4,585
2	4,6823	4,5161	4,5413	7	4,9494	4,506	4,5734
3	4,7513	4,5296	4,5633	8	4,987	4,4882	4,5639
4	4,818	4,5409	4,583				

Table 4. MAR(1,1) estimated parameters.

Parameter	Estimate	Confidence bounds	
ϕ_1	0.5255	0.4585	0.5925
φ_1	0.6503	0.5897	0.7110

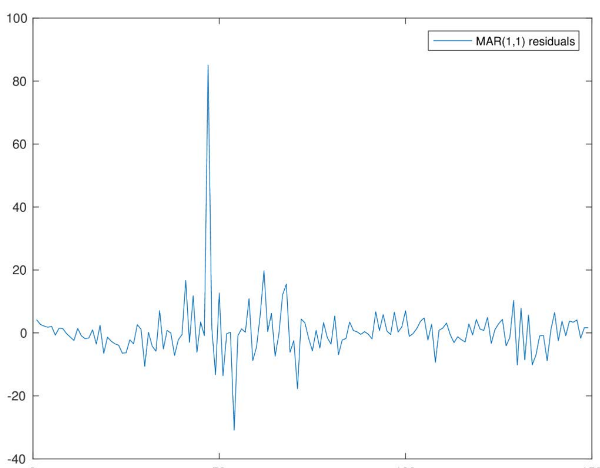

Figure 6. *Mixed causal-noncausal* MAR(1,1) residuals

Table 5. BDS test results for the MAR(1,1) model residuals on y_t.

m	w	$p-value$	m	w	$p-value$
2	1,703389269	0,0442	9	6,924875969	2,18E-12
3	2,249277819	0,0122	10	7,294838916	1,50E-13
4	3,342684301	4,15E-04	11	7,639695322	1,09E-14
5	4,384330303	5,82E-06	12	8,143710723	2,22E-16
6	5,191782337	1,04E-07	13	8,892820344	0
7	5,887119414	1,96E-09	14	9,500629834	0
8	6,412692984	7,15E-11	15	10,18321732	0

5.1.1 Mixed causal-noncausal AR model

The following specification of the model is derived by the suggestions of the *information criteria*, these are very useful tools to determine the time dependencies to be included in the model, *i.e.* they are used to determine the order of the autoregressive polynomial (see equation 1) p. The information criteria hereby considered are the *Akaike information criteria* AIC, the *Bayesian information criteria* and the *Hannan-Quinn* information criterion HQ [14], Hecq et al. [14] show that simulation results would favour the use of BIC. As reported in Table 3 the information criteria suggest setting $p = 2$.

When $p = 2$ the estimated *Mixed causal-noncausal* model is a MAR(1,1):

$$(1 - \phi_1 L)(1 - \varphi_1 L^{-1})\, y_t = \varepsilon_t.$$

Table 4 shows the estimated parameters of the model. Fig. 6 shows the sequence of the MAR(1,1) model's estimated residuals $\hat{\varepsilon}_t$.

As shown in Table 5, the BDS test for independence fails to accept the null hypothesis of *iid* distributed residuals for most of the tested *embedded dimensions*, thus suggesting to discard the model just now estimated.

Table 6. Information Criteria, MAR model on $rate_t$

p	BIC	AIC	HQ	p	BIC	AIC	HQ
0	7,0358	6,9803	6,9888	5	4,9587	4,6262	4,6767
1	4,7483	4,6374	4,6543	6	5,0137	4,6257	4,6847
2	4,7546	4,5883	4,6136	7	5,0781	4,6348	4,7021
3	4,8215	4,5998	4,6335	8	5,0705	4,5716	4,6474
4	4,8906	4,6135	4,6556				

5.2 Noncausal analysis of the observed price BTC/USD

Since the interpretation of the aforementioned estimated parameters can be rather misleading and therefore hard to be extended to the market reality, given the arbitrary choice for

Table 7. MAR(0,1) estimated autoregressive parameters on $rate_t$

Parameter	Estimate	Confidence bounds	
ϕ_1	0.9809	0.9740	0.9878

Table 8. Estimated parameters of the Student's t error distribution, MAR(0,1) model on $rate_t$

λ	ν
3.3073	1.4863

Table 9. BDS test results for the MAR(0,1) model residuals

m	w	H_0	m	w	H_0
2	6,45069	1	9	16,22165	1
3	8,26635	1	10	18,05194	1
4	9,30074	1	11	20,14559	1
5	10,38450	1	12	22,50443	1
6	11,66036	1	13	25,59976	1
7	13,03694	1	14	29,31705	1
8	14,59404	1	15	33,57504	1

Table 10. MAR(1,1) estimated autoregressive parameters on $rate_t$

Parameter	Estimate	Confidence bounds	
ϕ_1	0.9747	0.9650	0.9844
φ_1	0.2781	0.2077	0.3485

the fundamental component, it is of great interest to estimate the MAR model directly on the BTC/USD time series ($rate_t$).

The aforementioned information criteria, in application to the BTC/USD time series, suggest to set the order of the autoregressive polynomial to $p = 1$ (see Eq. 1) or $p = 2$ depending on the selected criterion (see Table 6).

5.2.1 Estimated MAR model, case $p = 1$

When $p = 1$ the estimated model that best fits the observed time series $rate_t$ is a *purely causal* AR(1,0). Table 7 and Table 8 display the estimated parameters of the autoregressive polynomial and of the error distribution, respectively.

Since the distribution's degrees of freedom $\nu = 1.4863 < 2$, then the estimated sequence of error terms $\hat{\epsilon}_t$ cannot be likened to the case $\epsilon_t \sim iid\ t_\nu(0, \lambda)$, given the fact that when $\nu < 2$ the expected value of the distribution is not defined. Anyhow the BDS test (Table 12) for independence does not

Table 11. Estimated parameters of the Student's t error distribution, MAR(0,1) model on $rate_t$

λ	ν
3.3901	1.6043

Table 12. BDS test results for the MAR(1,0) model residuals on $rate_t$.

m	w	$p-value$	m	w	$p-value$
2	6,450686379	5,57E-11	9	16,22165279	0
3	8,266350334	1,11E-16	10	18,05194201	0
4	9,300742867	0	11	20,14558908	0
5	10,38450476	0	12	22,50443038	0
6	11,66035838	0	13	25,5997597	0
7	13,03694212	0	14	29,31704918	0
8	14,59403959	0	15	33,57504129	0

Table 13. BDS test results for the MAR(1,1) model residuals on $rate_t$.

m	w	$p-value$	m	w	$p-value$
2	5,121150647	1,52E-07	9	15,28926444	0
3	7,537452115	2,40E-14	10	16,97636905	0
4	8,912767967	0	11	18,98947641	0
5	10,07953357	0	12	21,39229563	0
6	11,15351875	0	13	24,08437913	0
7	12,36324983	0	14	27,23839452	0
8	13,78408522	0	15	30,78448895	0

accept the null hypothesis of *iid* distributed residuals, thus suggesting once again to discard the estimated model.

5.2.2 MAR model, p=2

When $p = 2$ the estimated model is a *Mixed causal-noncausal* MAR; estimates of the model are displayed in Table 10 and 11. Once again the estimated t distribution's degrees of freedom is $\nu = 1.6043 < 2$, therefore the estimated sequence of error terms cannot be likened to the case $\epsilon_t \sim iid\ t_\nu(0, \lambda)$, suggesting to discard the model once again. In any case, the BDS test(Table 12) for independence does not accept the null hypothesis of *iid* distributed residuals, thus suggesting once again to discard the estimated model.

5.3 Residual analysis

To sum up, MAR models are estimated directly on the BTC/USD time series ($rate_t$) and then on the bubble part (y_t, yc_t); the aforementioned information criteria suggest to set the order of the autoregressive polynomial to $p = 1$ or $p = 2$ depending on the selected criterion, both for the BTC/USD rate and for the bubble terms. In the former case, $p = 1$, a *strictly causal backward looking* AR(1) is the preliminary reference specification for both the full rate $rate_t$ and the bubble component yc_t whereas a *strictly non-causal forward looking* AR(1) is the preliminary reference for the bubble component y_t. For the latter case, $p = 2$, a MAR(1,1) model is found to be fitting all the time series $rate_t$, y_t and yc_t.

The estimation results are reported in Table 14.

Table 14. MAR(r,s) estimated parameters on $rate_t$, y_t & yc_t

T. Series	MAR(r,s)	Par.	Est.	Conf. bounds		Par.	Est.	Conf. bounds	
$rate_t$	MAR(1,0)	ϕ_1	0.9809	0.9740	0.9878	φ_1	-	-	-
	MAR(1,1)	ϕ_1	0.9747	0.9650	0.9844	φ_1	0.2781	0.2077	0.3485
y_t	MAR(0,1)	-	-	-	-	φ_1	0.8066	0.8028	0.8103
	MAR(1,1)	ϕ_1	0.5255	0.4585	0.5925	φ_1	0.6503	0.5897	0.7110
yc_t	MAR(1,0)	ϕ_1	0.9803	0.9702	0.9904	φ_1	-	-	-
	MAR(1,1)	ϕ_1	0.3424	0.2604	0.4245	φ_1	0.9396	0.9216	0.9576

It is evident from the results in Table 14 that there is a very strong *backward looking* dependence in one lagged value, for the BTC/USD rate and for the bubble component yc_t; conversely, for the isolated bubble term y_t, there is a very strong *forward looking* dependence in one led value.

The estimation of a *Mixed causal/non-causal* MAR(1,1) gives further insights on the *backward* and *forward* dependence; outcomes are summed up in Table 14 respectively for the full rate $rate_t$, the bubble term y_t and the bubble term yc_t.

Particularly interesting is the difference in the parameter ϕ_1 and φ_1 when estimating the MAR(1,1) model separately on the bubble component y_t and on the original time series $rate_t$. As shown in Table 14 the non-causal parameters (*forward looking*) φ are stronger in the bubble component y_t than in the observed price $rate_t$, whereas the causal parameter ϕ is much stronger in the observed price $rate_t$ than in the bubble component y_t. This is consistent with the conjecture made in the introduction, that the speculative bubble is rather a forward looking phenomena than a past one, since the *forward looking* estimated parameters on the bubble part are stronger than the ones on the observed BTC/USD price $rate_t$. This evidence is strengthen by the MAR(1,1) estimated parameters on the bubble part yc_t, indeed it can be noticed that the value of the forward looking and backward looking components almost trade places when estimating the model on the full price time series $rate_t$ and the bubble component yc_t respectively. As mentioned in Section 3.2, if the model is correctly specified then the model residuals ϵ_t should be a sequence of *Independent Identically Distributed* Student's t observations. In this study the *IID* hypothesis is tested through the BDS test for independence. This test is based on the correlation dimension, with m *embedded dimension*, since it can be shown [16] that the test statistic w is asymptotically

Table 15. BDS test results for the MAR(1,0) model residuals on yc_t.

m	w	$p-value$	m	w	$p-value$
2	5,528256527	1,62E-08	9	12,29309902	0
3	7,009012288	1,20E-12	10	13,36389728	0
4	7,773551059	3,77E-15	11	15,15913039	0
5	8,506760111	0	12	17,1200305	0
6	9,287013257	0	13	19,26464437	0
7	10,17111148	0	14	21,78528929	0
8	11,24555355	0	15	24,56128819	0

Table 16. BDS test results for the MAR(1,1) model residuals on yc_t.

m	w	$p-value$	m	w	$p-value$
2	4,236007075	1,14E-05	9	10,32346963	0
3	5,315716933	5,31E-08	10	11,13612446	0
4	6,272423899	1,78E-10	11	12,51271091	0
5	7,320422719	1,24E-13	12	14,01078341	0
6	8,132958153	2,22E-16	13	15,68841368	0
7	8,838578863	0	14	17,59084596	0
8	9,612705164	0	15	19,49717776	0

normally distributed $\sim \mathcal{N}(0, 1)$, it is quite feasible to obtain $p - values$. The Tables 15 and 16 reporting the outcome of performing such test on the residuals ϵ_t of the yc_t estimated models. As shown in Table 5 it can be noticed that the only model for which the null hypothesis of *IID* residuals cannot be rejected is the MAR(1,1) model on y_t, and only for $m = 1$ or $m = 2$, depending on the selected confidence bound width.

6 Conclusions

This study undertook a *Mixed causal-noncausal* analysis of the BTC/USD exchange rates time series, over the period February-July 2013, to test whether the bubble effect disentangled on observed data may be explained by a *forward looking* behaviour of the economic agents. In the introduction it was noticed that given the system's monetary issuance, the exchange rate of one Bitcoin with respect to a traditional currency should be influenced by agents's future expectations and that classical ARIMA models, *backward looking* by definition, are not suitable to describe the dynamics of the Bitcoin price given the fact that the only time dependence admitted by these model regards the past. *Mixed backward forward looking* MAR models are hence considered both for the BTC/USD exchange rate and for the isolated bubbles.

The conjecture underlying this study is that the forward looking parameters should be stronger in the bubble part than in the observed price. Indeed this turns out to be the case, when estimating the model on the observed data, however the residuals analysis, conducted by performing the BDS test for independence, suggests not to consider this models valid but for one case (partially). Since the results of this test are asymptotical (for $n \to \infty$) and given the low entity of the residuals a more extensive residual analysis could be performed in order to assess the capability of the chosen model to describe the dynamics of BTC/USD rate and/or the isolated bubble term (y_t, yc_t). Several techniques are available such as the classical *Ljung-Box-Q* test on residuals autocorrelation (see [12]). Although the focus of this study is not to come across the true Data Generating Process for the Bitcoin, a deeper investigation of this issue is beyond the scope of the present study and will be tackled in future research.

In the future we plan to evaluate the possibility of proposing cross-evaluation techniques, and propose complementary validation with regression metrics such as RMSE, MAE, RMSD and others.

Acknowledgments

This work is supported by project "Agrichain" (funded by Fondazione Cassa di Risparmio di Perugia 2018-2020) and project "ASIA" (funded by INDAM-GNCS).

References

[1] Stefano Bistarelli, Alessandra Cretarola, Gianna Figà-Talamanca, Ivan Mercanti, and Marco Patacca. Is arbitrage possible in the bitcoin market? (work-in-progress paper). In *GECON*, volume 11113 of *Lecture Notes in Computer Science*, pages 243–251. Springer, 2018.

[2] Stefano Bistarelli, Ivan Mercanti, and Francesco Santini. A suite of tools for the forensic analysis of bitcoin transactions: Preliminary report. In *Euro-Par Workshops*, volume 11339 of *Lecture Notes in Computer Science*, pages 329–341. Springer, 2018.

[3] Stefano Bistarelli, Matteo Parroccini, and Francesco Santini. Visualizing bitcoin flows of ransomware: Wannacry one week later. In *ITASEC*, volume 2058 of *CEUR Workshop Proceedings*. CEUR-WS.org, 2018.

[4] Stefano Bistarelli and Francesco Santini. Go with the -bitcoin- flow, with visual analytics. In *ARES*, pages 38:1–38:6. ACM, 2017.

[5] Olivier Blanchard. Speculative bubbles, crashes and rational expectations. *Economics Letters*, 3(4):387–389, 1979.

[6] George Edward Pelham Box and Gwilym Jenkins. *Time Series Analysis, Forecasting and Control*. Holden-Day, Inc., San Francisco, CA, USA, 1990.

[7] F.Jay Breid, Richard A Davis, Keh-Shin Lh, and Murray Rosenblatt. Maximum likelihood estimation for noncausal autoregressive processes. *Journal of Multivariate Analysis*, 36(2):175 – 198, 1991.

[8] Gerald Dwyer. The economics of bitcoin and similar private digital currencies. *Journal of Financial Stability*, 17(C):81–91, 2015.

[9] David García, Claudio Juan Tessone, Pavlin Mavrodiev, and Nicolas Perony. The digital traces of bubbles: feedback cycles between socio-economic signals in the bitcoin economy. *CoRR*, abs/1408.1494, 2014.

[10] Christian Gourieroux and Joann Jasiak. Filtering, prediction and simulation methods for noncausal processes. *Journal of Time Series Analysis*, 37(3):405–430, 2016.

[11] Christian GouriÃĺroux and Jean-Michel Zakoian. Explosive Bubble Modelling by Noncausal Process. Working Papers 2013-04, Center for Research in Economics and Statistics, February 2013.

[12] James D Hamilton. Time series econometrics. *Princeton U. Press, Princeton*, 1994.

[13] Adam Hayes. Cryptocurrency value formation: An empirical study leading to a cost of production model for valuing bitcoin. *Telematics and Informatics*, 34:1308–1321, 11 2017.

[14] Alain Hecq, Lenard Lieb, and Sean Telg. Identification of mixed causal-noncausal models in finite samples. *Annals of Economics and Statistics*, 123/124:307–331, 2016.

[15] Andrew Hencic and Christian Gouriéroux. Noncausal autoregressive model in application to bitcoin/usd exchange rates. In *Econometrics of Risk*, volume 583 of *Studies in Computational Intelligence*, pages 17–40. Springer, 2015.

[16] Ludwig Kanzler. Very fast and correctly sized estimation of the bds statistic. *Available at SSRN 151669*, 1999.

[17] Ladislav Kristoufek. What are the main drivers of the bitcoin price? evidence from wavelet coherence analysis. *PLOS ONE*, 10(4):1–15, 04 2015.

[18] Markku Lanne and Pentti Saikkonen. Modeling expectations with noncausal autoregressions. *Available at SSRN 1210122*, 2008.

[19] Markku Lanne and Pentti Saikkonen. Noncausal autoregressions for economic time series. *Journal of Time Series Econometrics*, 3(3), 2011.