Studying forward looking bubbles in Bitcoin/USD exchange rates

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Abstract

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special activities in the market. In particular, several papers

Although Bitcoin is a relatively new subject in Economics, contributions in this topic are growing very fast. Several papers evidenced a bubble behaviour in exchange rates be tween Bitcoin and traditional currencies. In this paper we explore and give validation to such conjecture, proving also that the bubble effect is due to confidence in Bitcoin future values. This means that Bitcoin price/exchange rate is influ enced both by future and past events, but that the bubble behaviour is strictly connected to trust on the future of the Bitcoin system.

*CCS Concepts* • Applied computing → Digital cash; • Information systems → *Record storage systems*; • Net works → Network performance evaluation.

*Keywords* Bitcoin, Causal-Noncausal Autoregressive mod els

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1 Introduction

Bitcoin is a relatively new subject in Economics and Finance, however, such digital currency is fostering a lot of studies, and contributions in this topic are growing very fast. Some of the studies go in the direction of understanding the reasons of

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evidenced a bubble behaviour in exchange rates between Bitcoin (BTC henceforth) and traditional currencies (Euro or Dollars usually) [9, 17]. The aim of this paper is to explore the conjecture that the bubble effect is due to confidence in Bitcoin future values so that its price/exchange rate is influenced both by past events and by views about future ones.

Traditional econometrics models within the class of Au toRegressive Integrated Moving Average (ARIMA) are *back ward looking* since the only time-dependence admitted re gards the past [6] and are usually referred to as causal models. Recently, models known as Mixed causal-noncausal AutoRe gressive (MAR) have been introduced in order to extend time dependence to the future [7, 10, 18] thus reflecting a *backward-forward looking* behaviour.

The paper by Gouriéroux & Hencic [15] represents a valid anchor to refer to, at least in this area of study, as it under takes a non-causal analysis of the BTC/USD rates in order to predict its future evolution. The present study shares with [15] both the same decomposition of the BTC/USD price in a bubble and in a fundamental part, and the observed time series; though, here the main objective is to investi gate whether confidence in future values of the BTC/USD rate (i.e. the *forward looking* part) is the one responsible of the bubble effect, while in [15] the focus was on forecast ing future rates. If this is the case, a significant change in the estimated parameters should be detected when the MAR model is estimated separately in the observed time series and in the bubble component. In particular the *forward looking* parameters should be stronger in the bubble part than in the observed price.

The rest of the paper is structured as follows: the firt part is devoted to the economic explanation of our conjecture about the relation of the speculative bubble in BTC/USD exchange rates with the monetary policy of the Bitcoin system; then, in Section 3 the theory behind the *Mixed Causal–Noncausal autoregressive models* is briefly described. Section 4 describes the dataset and Section 5 summarizes the results of the esti mation of the MAR model on the observed data. Section 6 gives conclusions and final remarks.

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2 The speculative bubble in BTC/USD rates By simply watching the trajectory of the BTC/USD exchange rate time series it’s easy to notice how often its pattern surges and bursts rapidly mimicking the one of speculative bubbles. The definition of speculative bubble considered in this paper is the one proposed by Blanchard [5] in the framework of rational expectations models where it is assumed that the economic variable of interest, say *xt*, has two components: the first one depicts the fundamental path of *xt*, while the second represents the bubble effect. In this context a bubble results from the departure of *xt* from it’s *fundamental path*. In Fig. 1 one of the major bubbles occurred in 2013 for the BTC/USD rate is recorded.

**Time Series Plot:BTC/USD daily exchange rate**

250

200

e

t

a

r

e

(think of it as the Gross Domestic Product of a national econ omy) the demand of Bitcoins will increase, boosting upwards its price against other traditional currencies, given the *ex ante* fixed monetary supply.

The reason why this issuance mechanism is hereby de fined “deflationary” is that as long as the general belief that the system as a whole will keep growing stands, the price against other currencies will inevitably increase, increasing the inter-temporal opportunity cost of spending any given amount of BTC. As a matter of fact since the agents know that the price will increase then they are encouraged to with hold any transaction in BTC and increase their *savings* in BTC. A very interesting effect of such mechanism is that any steep fall in the price may boost the awareness of BTC as a system, potentially increasing it’s diffusion among the general public, thus incrementing the aforementioned self– sustaining dynamic [17].

It must be noticed that if the system had a flexible mone tary policy, where changes are not known *ex-ante*, then the economic agents within and without the system wouldn’t be able to include it in their preferences, thus neutralizing

150

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Feb 12 Mar 12 Apr 09 May 07 Jun 04 Jul 02 Figure 1. Bitcoin/USD observed time series

the aforementioned self–sustaining mechanism, even if the bitcoin system is flourishing. After the explanation given above it must be clear now the reason why it is expected and tested below in this study that the speculative bubble in BTC/USD rates is a *forward–looking* phenomena.

3 Mixed causal-noncausal autoregressive models

For a long time, as mentioned by Gouriéroux & Hencic [15], speculative bubbles were considered as nonstationary phe nomena and treated similarly to the explosive, stochastic

Bitcoins are produced through a “mining” process which involves computers (nodes from now on) solving complex mathematical problems (cryptography) to keep the system secure; when the node find a solution to the problem it is rewarded with an amount of Bitcoins which is referred to as “Block reward”. The protocol running Bitcoin is programmed to halve every 4 years the “Block reward” by suitably increas ing the difficulty of the mathematical problems to be solved. Hence, the volume of new coins will decay to zero with time and the long-term monetary supply will be fixed.

This paper aims at investigating whether the peculiar “deflationary” mechanism running the system’s monetary issuance is the main responsible of the formation, and the subsequent crash, of speculative bubbles (against the USD and other currencies). Indeed economic agents, before under taking any action within the system, already include the sys tem’s monetary issuance in their preferences/expectations, *i.e.* they already know that monetary issuance will never diverge from their expectations inasmuch the system has a unalterable monetary policy programmed to ever decrease the monetary issue over time. Therefore as the system grows

trends due to unit roots. Gouriéroux & Zakoian [11] pro pose a different approach and assume that the bubbles are rather short-lived explosive patterns caused by extreme val ued shocks in a noncausal, *stationary* process. In particular they assume a noncausal AR(1) (Auto Regressive) model, strictly *forward looking*, with Cauchy distributed errors.

A useful feature of such models is that shocks are non fundamental, combining this trait with the extended time dependance (to the future) allows these models to perfectly fit the peculiar pattern of the aforementioned (definition of) speculative bubble.

3.1 Introduction to noncausality

Let *yt* be the observed time series onto which estimate the traditional autoregressive model:

*a* (*L*)*yt* = *εt*

(1 − *a*1 *L* − · · · − *ap L*−*p*)*yt* = *εt*(1) with *L* being the backshift operator, *i.e.*, *Lyt* = *yt*−1 gives lags and *L*−1 *yt* = *yt*+1 produces leads and *a* the autoregres sive parameters. It is known [18] that if *s* out of *p* of the polynomial’s (*a* (*L*)) *zeros* are inside the unit circle, then the

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model is *non stationary* causing the impossibility to estimate the traditional autoregressive model. *εt* represent the usual error term of the model.

In Lanne & Saikkonen [18] it is shown that when *p* = *r* +*s*, with *r* being the zeros outside the unit circle, one can factor the polynomial *a* (*z*) as1:

*a*(*z*) = *φ*∗(*z*)*ϕ*(*z*) (2)

Assuming that the roots of the causal and noncausal poly nomial are outside the unit circle, that is:

*ϕ*(*z*) = 0 per |*z*| > 1 e *φ*(*z*) = 0 per |*z*| > 1 (8)

than these conditions imply that the series *yt* admits a two sided Moving Average (MA) representation:

Õ∞

*yt* =

where *ϕ*(*z*) is the usual causal polynomial of the autoregres

*j*=−∞

*ψj ϵt*−*j*(9)

sive parameters and *φ*∗(*z*) has its zeros inside the unit circle. The polynomial *φ*∗(*z*) can be expressed as:

such that *φj* = 0 for all *j* < 0 implies a purely causal process *t* and a purely noncausal model when *φj* = 0 for all *j* > 0

*φ*∗(*z*) = 1 − *φ*∗1*z* − · · · − *φ*∗*szs*

1 +*φ*∗*s*−1

*~~φ~~*∗*sz*1−*s* −1*~~φ~~*∗*sz*−*s*

[19]. More in detail, the *ψj* âĂŹs are the coefficients of an infinite order polynomial in positive and negative powers

of the Lag operator and such that Ξ(*z*) =Í∞*j*=−∞ *ψj zj* =

= −*φ*∗*szs*

*~~φ~~*∗*sz*−1 + · · · +*φ*∗1

= −*φ*∗*szs φ*(*z*−1)(3) where *φ*(*z*−1) = 1 − *φ*1 *z* − · · · − *φs zs*in view of the fact that *φ*∗*s*−*j*/*φ*∗*s* = −*φj* for *j* = 1, . . . ,*s* and 1/*φ*∗*s* = *φs*.

Because the zeros of *φ*∗(*z*) lie inside the unit circle those of *φ*(*z*) lie outside of the unit circle. Thus, (1) can be written as:

*ϕ*(*L*) [−*φ*∗*s Lsφ*(*L*−1)] = *εt*

given the decomposition shown in (3). Also, the latter ex pression can be rearranged as:

*ϕ*(*L*)*φ*(*L*−1)*yt* = *ϵt*(4)

where *ϵt* = −(1/*φ*∗*s*) *L*−*sεt* = −(1/*φ*∗*s*)*εt*+*s*. It is important to notice that *Et* [*ϵt* ] , *ϵt* since this variable is not determined by any informations available at time point *t* (see above).

3.2 Mixed Causal-Noncausal Autoregressive Model The univariate mixed causal-noncausal autoregressive model, denoted MAR(*r*,*s*), shown with equation 4 is usually written as:

(1 −*ϕ*1 *L* − · · · −*ϕr Lr*)(1 −*φ*1 *L*−1 − · · · −*φs L*−*s*)*yt* = *ϵt*(5)

When *φ*1 = · · · = *φs* = 0, the process *yt* represent a purely causal autoregressive process denoted AR(*r*, 0):

(1 − *ϕ*1 *L* − · · · − *ϕr Lr*)*yt* = *ϵt*(6)

where *yt*is regressed on past values, giving the process *yt* a *backward looking* autoregressive dynamic.

The process*yt*is *purely noncausal* when *ϕ*1 = · · · = *ϕr* = 0, hence defined as:

(1 − *φ*1 *L*−1 − · · · − *φs L*−*s*)*yt* = *ϵt*. (7) usually referred to as *forward looking* AR(0,*s*) process, being the exact counterpart of the model specification given in (6), since it’s regressed on future values rather than past ones.

Models containing both lags and leads of the dependent variable are called *mixed causal–noncausal models*.

1In order to maintain the same notation as in Lanne & Saikkonen [18] the polynomial *a*(*L*) will be referred to as *a*(*z*) for the following proof.

[Ψ(*z*−1)]−1[Φ(*z*)]−1.

Error terms *ϵt* are assumed *iid* non-Gaussian with *E*(|*εt*|*δ*) < ∞ ∀ *δ* ∈ (0, 1) [11]. Following Gouriéroux & Jasiak [10] the unobserved causal and noncausal components of the process *yt* are defined as follows:

*ut* ≡ *ϕ*(*L*)*yt* ↔ *φ*(*L*−1)*ut* = *ϵt*, (10)

*vt* ≡ *φ*(*L*−1)*yt* ↔ *ϕ*(*L*)*vt* = *ϵt*(11)

The specification of these values will prove useful for the following part regarding the estimation of mixed causal noncausal processes.

The non-Gaussianity assumption for the error term en sures the identifiability of the causal and the noncausal part. Most papers by Lanne & Saikkonen et al. use Student’s *tν* distributions, with *ν* ≥ 2 while Gouriéroux et al. rely on the Cauchy or a mixture of Cauchy and Normal distributions. As shown by Hecq et al. [14] it emerges that the Cauchy has too strong fat tails features and many series would have a degree of freedom between 1.5 and 2.52.

Figure 2. TradeBitcoin data price download panel 2Notice that when *ν* < 2 then the Student’s *t* expected value is undefined.

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4 The Data

The sample consists in 151 observation of the BTC/USD price spanning from February 20 to July 20 2013. The dynamic of the data is shown in Fig.1, where it is possible to notice the speculative bubble behaviour of the BTC/USD path, boosting and bursting rapidly around the month of April. In fact, in the April 2013 there was a famous bubble, commonly called simply the April bubble, that was a rally, all-time high and subsequent crash of the bitcoin exchange rate. The bubble resulted in a momentary all-time high of $266 USD per bitcoin on Mt. Gox3 on 10th April 2013. Then Mt. Gox suspended trading on 11th April 2013 until 12th April 2013 2 am UTC for a "market cooldown". The value of a single bitcoin fell to a low of $55.59 after the resumption of trading before stabilizing above $1004(a price decline of 61%).

The data is obtained from our application, TradeBitcoin [1], part of the suite BlockChainVis5[2–4] used for Bitcoin anal ysis and visualization, is based on finding the price options on the Bitcoin exchange and writing possible arbitrage op erations on a database to see if it is possible to correctly perform an arbitrage on the Bitcoin market. It also collects all this data price from 17 different exchanges and it allows to download that data with a detection time of 1 day or 1 hour or 15 minutes (Fig. 2).

4.1 Price decomposition

As a first issue it is important to disentangle the fundamen tal component from the bubble component of the BTC/USD prices. The fundamental value of the Bitcoin is still under debate. While in [8] it is argued that this fundamental value is zero, in [9] it is linked to the reputation of the Bitcoin sys tem measured by internet queries, moreover it is suggested (still in [9]) that the production cost of Bitcoin, due to the mining process, should be considered as the lower limit of the fundamental value of Bitcoin. Since this study assumes that Bitcoin has a fundamental value indeed, the price will be firstly decomposed following the approach in [15], where the fundamental path of the BTC/USD rate is assumed to be a nonlinear deterministic trend modelled as a 3rd degree polynomial in time and the bubble part is obtained by sub traction from the observed prices, as it is done still in [15]. The other decomposition that will be undertaken builds upon the suggestion made in [9], by setting apart the production cost of Bitcoin and the bubble component using the cost of production model shown in [13].

4.1.1 Nonlinear deterministic trend

As mentioned above the BTC/USD rate is defined as follows: *ratet* = *trendt* + *yt*, (12)

3https://en.bitcoin.it/wiki/Mt.\_Gox.

4https://bitcoincharts.com/charts/mtgoxUSD#rg5zczsg2013-04- 10zeg2013-04-12ztgSzm1g10zm2g25zl.

5http://normandy.dmi.unipg.it/blockchainvis/.

with *ratet* being the observed prices, *trendt* the fundamental component and *yt* the bubble component and the estimated trend is given by:

*trendt* = 0.000073 *t*3 − 0.0316 *t*2 + 3.6590 *t* − 3.2951. The corresponding time-series are plotted in Fig. 3.

4.1.2 Production cost as the lower limit of the fundamental value

The Bitcoin production cost model shown in [13] assumes the perspective of a generic *miner* that is deciding whether to mine or not for Bitcoin. The *miner* will decide to join the mining process in case of positive profit expectations and to abandon it on the contrary case. The variables considered to be influencing the mining process and hence the produc tion cost in [13] are: the *block reward β*, the *hashing power* (computational power) of the mining hardware equipment *ρ*, the *difficulty* set by the network *δ*, the *cost* per kilowatt hour6$ *kW* /*h* and the *average energy efficiency W GH*/*s* of the mining hardware deployed.

As shown in [13] the expected number of cryptocurrency coins to be mined per day on average given the difficulty and block reward (number of coins issued per successful mining attempt) per unit of hashing power is given by:

*BTC*/*day* =*β ρ sechr*

*δ* 232 *hrday*

*sechr* = 3600 being the seconds in 1 hour and *hrday* = 24 being the hours in a day.

The cost of mining can be expressed as:

*Eday* = (*ρ*/1000) ($ *kW* /*h W GH*/*s hrday* )

with $ *kW* /*h* being the electricity cost and *W GH*/*s* the av erage energy efficiency. Bitcoin production cost estimates over the considered time span (Feb.-Jul. 2013) are shown in Fig. 4, where it is assumed an average energy efficiency of *W GH*/*s* = 500 as suggested by Garcia et al. [9], a computa tional power7 of *GH*/*s* = 1000, an average global electricity cost of $ *kW* /*h* = 0.115. In 2013 the block reward set by the network was *β* = 25 *BTC*, the values of the ever changing difficulty over the considered time-period can be found in the public database https://blockchain.info.

Assuming the lower limit of the fundamental value, given by the aforementioned definition of production cost, as the actual fundamental value, therefore the BTC/USD rate is

6In this study we consider the same average cost for electricity that is considered in [13], although it must be noticed that it changes depending on the geographical location of the miner and therefore on the national electricity supplier.

7It must be noticed that in this example varying the computational power does not change the cost, *i.e.*, the cost of 1 BTC in USD is only affected by the difficulty, the electricity cost and the average energy efficiency of the mining hardware, increasing the scale of production in this case doesn’t lead to economies of scale.

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defined similarly as in 12:

*ratet* = *costt* + *yct*, (13)

with *ratet* being the observed prices, *yct* the bubble compo nent and where the fundamental component in this case is given by the aforementioned production cost *costt*. Assum ing that the production cost is correctly estimated, it must be noticed that the bubble component could be considered as the *added market value*.

The corresponding time series are plotted in Fig. 4

250

closeprice

residual-bubble

est. polynomial

200

150

price and on the bubble component according to the two different definitions of the fundamental part. As already dis cussed in the introduction it is expected that the *forward looking* dependence is stronger in the isolated bubble com ponent than in the observed price. The model specifications in what follows are chosen by applying *information crite ria* which are useful tools to select the number of lags (and leads) to be included in the model. The information criteria hereby considered are the *Akaike information criterion* (AIC), the *Bayesian information criterion* (BIC) and the *Hannan Quinn information criterion* (HQ) (for a general review see the book by Hamilton [12]). Once the number of lags/leads have been detected, models are estimated by maximizing the approximated log-likelihood function based on the Student’s *t* density function for the error term; a detailed description of the procedure may be found in Hecq et al. [14]. The related Matlab routines used in this work are kindly provided by the authors of the above quoted paper.

D S

U

100

Table 1. AR(1) model’s estimated parameters

50

0

-50

Feb 12 Mar 12 Apr 09 May 07 Jun 04 Jul 02 Time

AR(1) Model *φ*1 Std. Dev. 0.8066 0.0234

*t* distribution *λ ν*

4.3928 2.5013

D S

U

Figure 3. Bitcoin/USD price decomposition

250

closeprice

residual-bubble

production cost

200

150

100

50

0

Table 2. BDS test results, purely noncausal model AR(1).

*m w p* −*value m w p* −*value* 2 5,978547545 1,13E-09 9 13,3666165 0 3 6,525463574 3,39E-11 10 14,65204326 0 4 7,420797806 5,82E-14 11 15,91260972 0 5 8,615265114 0 12 17,42915916 0 6 9,743131337 0 13 19,3469674 0 7 10,83386529 0 14 21,49415105 0 8 12,07560832 0 15 24,05813227 0

5.1 Noncausal analysis of the bubble component Firstly is considered a *strictly noncausal* AR(1) (forward look ing):

*yt* = *φ*1 *yt*+1 + *ϵt*.

where *ϵt* are *iid* Student’s *t* distributed errors, with location

Feb 15 Mar 15 Apr 12 May 10 Jun 07 Jul 05

Time

Figure 4. Bitcoin/USD price vs. production cost

5 Estimated models

The following part of the study undertakes a mixed causal/non causal analysis by estimating MAR models on the BTC/USD

0 and scale parameter *λ*, *ϵt* ∼ (0, *λ*). Estimated parame ters are reported in Table 1. The residuals of the models are shown in Fig 5. In order to test the model’s goodness of fit, the results of the BDS test (Brock, William, Davis Dechert & Scheinkman, 1987) [16], used to test whether the residual are truly a sequence of *iid* Student’s *t* random variables, are reported in Table 2. The test fails to accept the null hypothe sis of *iid* distributed residuals, this implies that the present model must be discarded.

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Table 5. BDS test results for the MAR(1,1) model residuals

on *yt*.

*m w p* −*value m w p* −*value*

2 1,703389269 0,0442 9 6,924875969 2,18E-12

3 2,249277819 0,0122 10 7,294838916 1,50E-13

4 3,342684301 4,15E-04 11 7,639695322 1,09E-14

5 4,384330303 5,82E-06 12 8,143710723 2,22E-16

6 5,191782337 1,04E-07 13 8,892820344 0

7 5,887119414 1,96E-09 14 9,500629834 0

8 6,412692984 7,15E-11 15 10,18321732 0

5.1.1 Mixed causal-noncausal AR model

The following specification of the model is derived by the

Figure 5. Noncausal AR(1) model residuals Table 3. Information Criteria

*p* BIC AIC HQ *p* BIC AIC HQ 0 6,0119 5,9565 5,9649 5 4,8862 4,5537 4,6042 1 4,7057 4,5949 4,6117 6 4,914 4,526 4,585 2 4,6823 4,5161 4,5413 7 4,9494 4,506 4,5734 3 4,7513 4,5296 4,5633 8 4,987 4,4882 4,5639 4 4,818 4,5409 4,583

Table 4. MAR(1,1) estimated parameters.

Parameter Estimate Confidence bounds *ϕ*1 0.5255 0.4585 0.5925 *φ*1 0.6503 0.5897 0.7110

Figure 6. *Mixed causal-noncausal* MAR(1,1) residuals

suggestions of the *information criteria*, these are very useful tools to determine the time dependencies to be included in the model, *i.e.* they are used to determine the order of the au toregressive polynomial (see equation 1) *p*. The information criteria hereby considered are the *Akaike information criteria* AIC, the *Bayesian information criteria* and the *Hannan-Quinn* information criterion HQ [14], Hecq et al. [14] show that simulation results would favour the use of BIC. As reported in Table 3 the information criteria suggest setting *p* = 2.

When *p* = 2 the estimated *Mixed causal-noncausal* model is a MAR(1,1):

(1 − *ϕ*1 *L*) (1 − *φ*1 *L*−1)*yt* = *εt*.

Table 4 shows the estimated parameters of the model. Fig. 6 shows the sequence of the MAR(1,1) model’s estimated resid uals *ε*ˆ*t*.

As shown in Table 5, the BDS test for independence fails to accept the null hypothesis of *iid* distributed residuals for most of the tested *embedded dimensions*, thus suggesting to discard the model just now estimated.

Table 6. Information Criteria, MAR model on *ratet*

*p* BIC AIC HQ *p* BIC AIC HQ 0 7,0358 6,9803 6,9888 5 4,9587 4,6262 4,6767 1 4,7483 4,6374 4,6543 6 5,0137 4,6257 4,6847 2 4,7546 4,5883 4,6136 7 5,0781 4,6348 4,7021 3 4,8215 4,5998 4,6335 8 5,0705 4,5716 4,6474 4 4,8906 4,6135 4,6556

5.2 Noncausal analysis of the observed price BTC/USD

Since the interpretation of the aforementioned estimated parameters can be rather misleading and therefore hard to be extended to the market reality, given the arbitrary choice for

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Table 7. MAR(0,1) estimated autoregressive parameters on *ratet*

Parameter Estimate Confidence bounds

*ϕ*1 0.9809 0.9740 0.9878

Table 8. Estimated parameters of the Student’s *t* error dis tribution, MAR(0,1) model on *ratet*

*λ ν*

3.3073 1.4863

Table 9. BDS test results for the MAR(0,1) model residuals

*m w H*0 *m w H*0

2 6,45069 1 9 16,22165 1

3 8,26635 1 10 18,05194 1

4 9,30074 1 11 20,14559 1

5 10,38450 1 12 22,50443 1

6 11,66036 1 13 25,59976 1

7 13,03694 1 14 29,31705 1

8 14,59404 1 15 33,57504 1

Table 10. MAR(1,1) estimated autoregressive parameters on *ratet*

Parameter Estimate Confidence bounds

*ϕ*1 0.9747 0.9650 0.9844

*φ*1 0.2781 0.2077 0.3485

the fundamental component, it is of great interest to estimate the MAR model directly on the BTC/USD time series (*ratet*). The aforementioned information criteria, in application to the BTC/USD time series, suggest to set the order of the autoregressive polynomial to *p* = 1 (see Eq. 1) or *p* = 2 depending on the selected criterion (see Table 6).

5.2.1 Estimated MAR model, case *p* = 1

When *p* = 1 the estimated model that best fits the observed time series *ratet*is a *purely causal* AR(1,0). Table 7 and Ta ble 8 display the estimated parameters of the autoregressive polynomial and of the error distribution, respectively.

Since the distribution’s degrees of freedom *ν* = 1.4863 < 2, then the estimated sequence of error terms *ϵ*ˆ*t* cannot be likened to the case *ϵt* ∼ *iid tν* (0, *λ*), given the fact that when *ν* < 2 the expected value of the distribution is not defined. Anyhow the BDS test (Table 12) for independence does not

Table 11. Estimated parameters of the Student’s *t* error dis tribution, MAR(0,1) model on *ratet*

*λ ν*

3.3901 1.6043

Table 12. BDS test results for the MAR(1,0) model residuals on *ratet*.

*m w p* −*value m w p* −*value* 2 6,450686379 5,57E-11 9 16,22165279 0 3 8,266350334 1,11E-16 10 18,05194201 0 4 9,300742867 0 11 20,14558908 0 5 10,38450476 0 12 22,50443038 0 6 11,66035838 0 13 25,5997597 0 7 13,03694212 0 14 29,31704918 0 8 14,59403959 0 15 33,57504129 0

Table 13. BDS test results for the MAR(1,1) model residuals on *ratet*.

*m w p* −*value m w p* −*value* 2 5,121150647 1,52E-07 9 15,28926444 0 3 7,537452115 2,40E-14 10 16,97636905 0 4 8,912767967 0 11 18,98947641 0 5 10,07953357 0 12 21,39229563 0 6 11,15351875 0 13 24,08437913 0 7 12,36324983 0 14 27,23839452 0 8 13,78408522 0 15 30,78448895 0

accept the null hypothesis of *iid* distributed residuals, thus suggesting once again to discard the estimated model.

5.2.2 MAR model, p=2

When *p* = 2 the estimated model is a *Mixed causal-noncausal* MAR; estimates of the model are displayed in Table 10 and 11. Once again the estimated *t* distribution’s degrees of freedom is *ν* = 1.6043 < 2, therefore the estimated sequence of error terms cannot be likened to the case *ϵt* ∼ *iid tν* (0, *λ*), suggesting to discard the model once again. In any case, the BDS test(Table 12) for independence does not accept the null hypothesis of *iid* distributed residuals, thus suggesting once again to discard the estimated model.

5.3 Residual analysis

To sum up, MAR models are estimated directly on the BTC/USD time series (*ratet*) and then on the bubble part (*yt*, *yct*); the aforementioned information criteria suggest to set the order of the autoregressive polynomial to *p* = 1 or *p* = 2 depending on the selected criterion, both for the BTC/USD rate and for

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the bubble terms. In the former case, *p* = 1, a *strictly causal backward looking* AR(1) is the preliminary reference specifi cation for both the full rate *ratet* and the bubble component *yct* whereas a *strictly non-causal forward looking* AR(1) is the preliminary reference for the bubble component *yt*. For the latter case, *p* = 2, a MAR(1,1) model is found to be fitting all the time series *ratet*, *yt* and *yct*.

The estimation results are reported in Table 14. Table 14. MAR(r,s) estimated parameters on *ratet*, *yt* & *yct*

T. Series MAR(r,s) Par. Est. Conf. bounds Par. Est. Conf. bounds *r atet*MAR(1,0) *ϕ*1 0.9809 0.9740 0.9878 *φ*1 - - - MAR(1,1) *ϕ*1 0.9747 0.9650 0.9844 *φ*1 0.2781 0.2077 0.3485 *yt*MAR(0,1) - - - - *φ*1 0.8066 0.8028 0.8103 MAR(1,1) *ϕ*1 0.5255 0.4585 0.5925 *φ*1 0.6503 0.5897 0.7110 *yct*MAR(1,0) *ϕ*1 0.9803 0.9702 0.9904 *φ*1 - - - MAR(1,1) *ϕ*1 0.3424 0.2604 0.4245 *φ*1 0.9396 0.9216 0.9576

It is evident from the results in Table 14 that there is a very strong *backward looking* dependence in one lagged value, for the BTC/USD rate and for the bubble component *yct*; conversely, for the isolated bubble term *yt*, there is a very strong *forward looking* dependence in one led value.

The estimation of a *Mixed causal/non-causal* MAR(1,1) gives further insights on the *backward* and *forward* depen dence; outcomes are summed up in Table 14 respectively for the full rate *ratet*, the bubble term *yt* and the bubble term *yct*.

Particularly interesting is the difference in the parameter *ϕ*1 and *φ*1 when estimating the MAR(1,1) model separately on the bubble component*yt* and on the original time series*ratet*. As shown in Table 14 the non-causal parameters (*forward looking*) *φ* are stronger in the bubble component *yt* than in the observed price *ratet*, whereas the causal parameter *ϕ* is much stronger in the observed price *ratet* than in the bubble component *yt*. This is consistent with the conjecture made in the introduction, that the speculative bubble is rather a forward looking phenomena than a past one, since the *forward looking* estimated parameters on the bubble part are stronger than the ones on the observed BTC/USD price *ratet*. This evidence is strengthen by the MAR(1,1) estimated parameters on the bubble part *yct*, indeed it can be noticed that the value of the forward looking and backward looking components almost trade places when estimating the model on the full price time series *ratet* and the bubble component *yct* respectively. As mentioned in Section 3.2, if the model

is correctly specified then the model residuals *ϵt* should be a sequence of *Independent Identically Distributed* Student’s *t* observations. In this study the *IID* hypothesis is tested through the BDS test for independence. This test is based on the correlation dimension, with*m embedded dimension*, since it can be shown [16] that the test statistic*w* is asymptotically

Table 15. BDS test results for the MAR(1,0) model residuals on *yct*.

*m w p* −*value m w p* −*value* 2 5,528256527 1,62E-08 9 12,29309902 0 3 7,009012288 1,20E-12 10 13,36389728 0 4 7,773551059 3,77E-15 11 15,15913039 0 5 8,506760111 0 12 17,1200305 0 6 9,287013257 0 13 19,26464437 0 7 10,17111148 0 14 21,78528929 0 8 11,24555355 0 15 24,56128819 0

Table 16. BDS test results for the MAR(1,1) model residuals on *yct*.

*m w p* −*value m w p* −*value* 2 4,236007075 1,14E-05 9 10,32346963 0 3 5,315716933 5,31E-08 10 11,13612446 0 4 6,272423899 1,78E-10 11 12,51271091 0 5 7,320422719 1,24E-13 12 14,01078341 0 6 8,132958153 2,22E-16 13 15,68841368 0 7 8,838578863 0 14 17,59084596 0 8 9,612705164 0 15 19,49717776 0

normally distributed ∼ N (0, 1), it is quite feasible to obtain *p* −*values*. The Tables 15 and 16 reporting the outcome of performing such test on the residuals *ϵt* of the *yct* estimated models. As shown in Table 5 it can be noticed that the only model for which the null hypothesis of *IID* residuals cannot be rejected is the MAR(1,1) model on *yt*, and only for *m* = 1 or*m* = 2, depending on the selected confidence bound width.

6 Conclusions

This study undertook a *Mixed causal-noncausal* analysis of the BTC/USD exchange rates time series, over the period February-July 2013, to test whether the bubble effect disen tangled on observed data may be explained by a *forward looking* behaviour of the economic agents. In the introduc tion it was noticed that given the system’s monetary issuance, the exchange rate of one Bitcoin with respect to a traditional currency should be influenced by agents’s future expecta tions and that classical ARIMA models, *backward looking* by definition, are not suitable to describe the dynamics of the Bitcoin price given the fact that the only time dependence admitted by these model regards the past. *Mixed backward forward looking* MAR models are hence considered both for the BTC/USD exchange rate and for the isolated bubbles.

The conjecture underlying this study is that the forward looking parameters should be stronger in the bubble part than in the observed price. Indeed this turns out to be the

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case, when estimating the model on the observed data, how ever the residuals analysis, conducted by performing the BDS test for independence, suggests not to consider this models valid but for one case (partially). Since the results of this test are asymptotical (for *n* → ∞) and given the low entity of the residuals a more extensive residual analysis could be performed in order to assess the capability of the chosen model to describe the dynamics of BTC/USD rate and/or the isolated bubble term (*yt*, *yct*). Several techniques are available such as the classical *Ljung-Box-Q* test on resid uals autocorrelation (see [12]). Although the focus of this study is not to come across the true Data Generating Process for the Bitcoin, a deeper investigation of this issue is beyond the scope of the present study and will be tackled in future research.

In the future we plan to evaluate the possibility of propos ing cross-evaluation techniques, and propose complemen tary validation with regression metrics such as RMSE, MAE, RMSD and others.

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