Discrete Assignment-11.9.1-11

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Problem Statement

Write the first five terms in the sequence:

$$a_0 = 3$$
$$a_n = 3a_{n-1} + 2 \quad \text{for } n > 0$$

Solution

Table 1: Input Parameters: First Term and General Formula

Term	Value
$a(0)$	3
$a(n)$	$3a(n-1) + 2$ for $n > 0$

Let's find the first 5 terms of the sequence:

$$a(1) = 3a(0) + 2 = 3 \times 3 + 2 = 11 \tag{1}$$
$$a(2) = 3a(1) + 2 = 3 \times 11 + 2 = 35 \tag{2}$$
$$a(3) = 3a(2) + 2 = 3 \times 35 + 2 = 107 \tag{3}$$
$$a(4) = 3a(3) + 2 = 3 \times 107 + 2 = 323 \tag{4}$$
$$a(5) = 3a(4) + 2 = 3 \times 323 + 2 = 971 \tag{5}$$

So, the first 5 terms of the sequence are $3, 11, 35, 107, 323$.

Solution using Z Transform

$$A(z) = \frac{2}{(1 - z^{-1})(1 - 3z^{-1})}$$
$$= \frac{A_1}{1 - z^{-1}} + \frac{A_2}{1 - 3z^{-1}}$$

Now, find the values of A_1 and A_2. Multiply through by the common denominator:

$$1 = A_1(1 - 3z^{-1}) + A_2(1 - z^{-1}) \tag{6}$$

Equating coefficients, you can solve for A_1 and A_2:

$$A_1 = -1 \tag{7}$$
$$A_2 = 3 \tag{8}$$

Now, substitute these back into the modified partial fraction decomposition:

$$A(z) = -\frac{1}{1 - z^{-1}} + \frac{3}{1 - 3z^{-1}}$$
$$= -\frac{1}{1 - z^{-1}} + \frac{3}{1 - 3z^{-1}}$$

Now, you can find the inverse Z-transform of each term using the property $Z^{-1}\left[\frac{1}{1-cz^{-1}}\right] = c^n u_n$. The result should be:

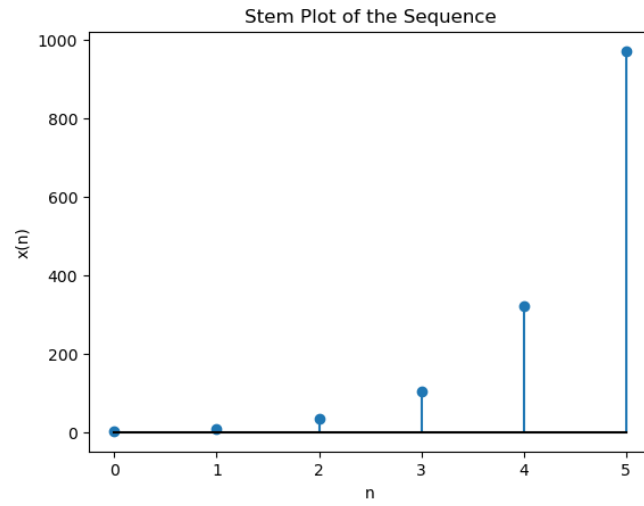$$a_n = -u_n + 3(3^n u_n) \tag{9}$$
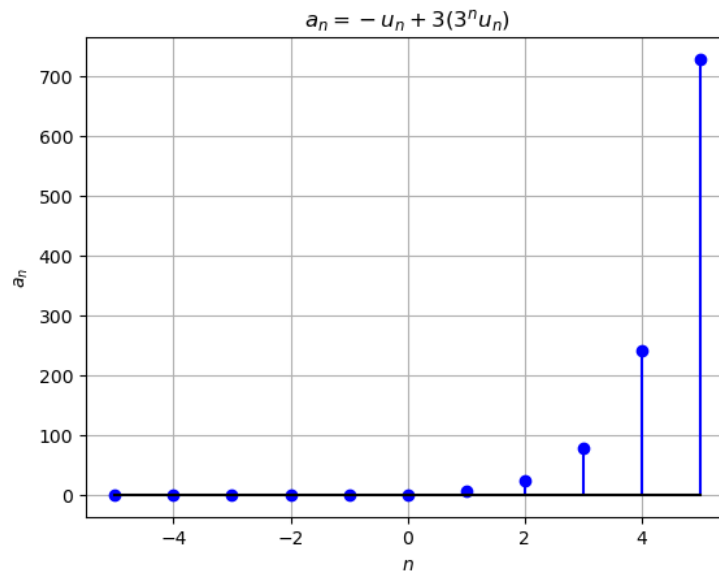
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Figure 1: Sequence plot generated from the Python script.

Figure 2: Plot of the sequence $a_n = -u_n + 3(3^n u_n)$

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