Discrete Assignment-11.9.1-11

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Problem Statement

Write the first five terms in the sequence:

$$a_0 = 3$$
$$a_n = 3a_{n-1} + 2 \quad \text{for } n > 0$$

Solution

Table 1: Input Parameters: First Term and General Formula

Term	**Value**
$x(0)$	3
$x(n)$	$3x(n-1) + 2$ for $n > 0$

Let's find the first 5 terms of the sequence:

$$x(1) = 3x(0) + 2 = 3 \times 3 + 2 = 11 \tag{1}$$
$$x(2) = 3x(1) + 2 = 3 \times 11 + 2 = 35 \tag{2}$$
$$x(3) = 3x(2) + 2 = 3 \times 35 + 2 = 107 \tag{3}$$
$$x(4) = 3x(3) + 2 = 3 \times 107 + 2 = 323 \tag{4}$$
$$x(5) = 3x(4) + 2 = 3 \times 323 + 2 = 971 \tag{5}$$

So, the next 5 terms of the sequence are $11, 35, 107, 323, 971$.

Solution using Z Transform

Let's find the Z transform of the sequence $a(n)$:

$$\begin{aligned} X(z) &= \mathcal{Z}\{x(n)\} = \sum_{n=0}^{\infty} x(n) z^{-n} \\ &= x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \dots \\ &= 3 + (3x(0) + 2)z^{-1} + (3x(1) + 2)z^{-2} + (3x(2) + 2)z^{-3} + \dots \\ &= 3 + (3 \cdot 3 + 2)z^{-1} + (3 \cdot (3 \cdot 3 + 2) + 2)z^{-2} + (3 \cdot (3 \cdot (3 \cdot 3 + 2) + 2) + 2)z^{-3} + \dots \\ &= 3 + 11z^{-1} + 35z^{-2} + 107z^{-3} + \dots \quad \text{(ROC: } \mathrm{Re}\{z\} > 0) \end{aligned}$$

So, the Z transform of the sequence $x(n)$ is given by $X(z) = 3 + 11z^{-1} + 35z^{-2} + 107z^{-3} + \dots$ with the Region of Convergence (ROC) specified as $\mathrm{Re}\{z\} > 0$.

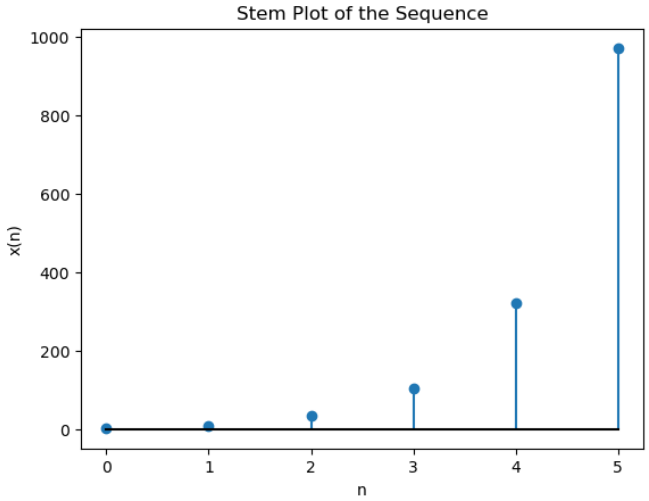

Figure 1: Sequence plot generated from Python script.