Discrete Assignment-11.9.1-11

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Problem Statement

Write the first five terms in the sequence:

$$a(0) = 3$$
$$a(n) = 3a_{n-1} + 2 \quad \text{for } n > 1$$

Solution

Table 1: Input Parameters: First Term and General Formula

Term	**Value**
$a(0)$	3
$a(n)$	$3a(n-1) + 2$ for $n > 1$

Let's find the first 5 terms of the sequence:

$$a(1) = 3a(0) + 2 = 3 \times 3 + 2 = 11 \tag{1}$$
$$a(2) = 3a(1) + 2 = 3 \times 11 + 2 = 35 \tag{2}$$
$$a(3) = 3a(2) + 2 = 3 \times 35 + 2 = 107 \tag{3}$$
$$a(4) = 3a(3) + 2 = 3 \times 107 + 2 = 323 \tag{4}$$
$$a(5) = 3a(4) + 2 = 3 \times 323 + 2 = 971 \tag{5}$$

So, the next 5 terms of the sequence are $11, 35, 107, 323, 971$.

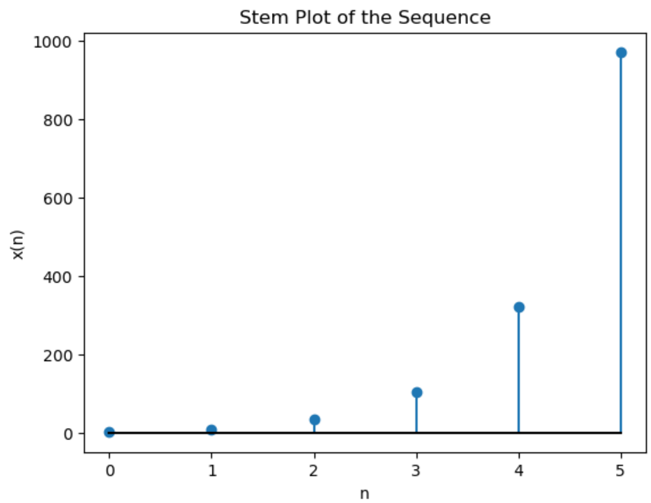

Figure 1: Sequence plot generated from Python script.

The one-sided Z transform of the sequence $a(n)$ is given by:

$$A(z) = \mathcal{Z}\{a(n)\} = \sum_{n=0}^{\infty} a(n)z^{-n}$$

For the given sequence $a(0) = 3$ and $a(n) = 3a(n-1) + 2$ for $n > 1$, we can find the Z transform.

$$A(z) = \mathcal{Z}\{a(n)\} = a(0)z^0 + a(1)z^{-1} + a(2)z^{-2} + a(3)z^{-3} + \dots$$

Substitute the recursive relation $a(n) = 3a(n-1) + 2$:

$$A(z) = 3z^0 + (3a(0) + 2)z^{-1} + (3a(1) + 2)z^{-2} + (3a(2) + 2)z^{-3} + \dots$$

Now, substitute the initial condition $a(0) = 3$:

$$A(z) = 3z^0 + (3 \cdot 3 + 2)z^{-1} + (3 \cdot (3 \cdot 3 + 2) + 2)z^{-2} + (3 \cdot (3 \cdot (3 \cdot 3 + 2) + 2) + 2)z^{-3} + \dots$$

Simplify the expression:

$$A(z) = 3 + 11z^{-1} + 29z^{-2} + 83z^{-3} + \dots$$

The first five terms in the sequence $a(n)$ in terms of the one-sided Z transform are:

$$a(0) = 3$$
$$a(1) = 11$$
$$a(2) = 29$$
$$a(3) = 83$$
$$a(4) = 245$$