Discrete Assignment-11.9.1-11

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Problem Statement

Write the first five terms in the sequence:

$$a_0 = 3$$
$$a_n = 3a_{n-1} + 2 \quad \text{for } n > 0$$

Solution

Table 1: Input Parameters: First Term and General Formula

Term	**Value**
$x(0)$	3
$x(n)$	$3x(n-1)+2$

Let's find the first 5 terms of the sequence:

$$x(1) = 3x(0) + 2 = 3 \times 3 + 2 = 11$$
$$x(2) = 3x(1) + 2 = 3 \times 11 + 2 = 35$$
$$x(3) = 3x(2) + 2 = 3 \times 35 + 2 = 107$$
$$x(4) = 3x(3) + 2 = 3 \times 107 + 2 = 323$$
$$x(5) = 3x(4) + 2 = 3 \times 323 + 2 = 971$$

So, the first 5 terms of the sequence are $3, 11, 35, 107, 323$.

Consider the difference equation $x(n) = 3x(n-1) + 2u(n)$. The Z-transform of this difference equation is

$$\begin{aligned} X(z) &= \frac{2}{(1-z^{-1})(1-3z^{-1})} \\ &= \frac{A_1}{1-z^{-1}} + \frac{A_2}{1-3z^{-1}} \end{aligned}$$

To find the values of A_1 and A_2, multiply through by the common denominator:

$$1 = A_1(1-3z^{-1}) + A_2(1-z^{-1}) \tag{1}$$

Equating coefficients, solve for A_1 and A_2:

$$\begin{aligned} A_1 &= -1 \\ A_2 &= 3 \end{aligned}$$

Substitute these values back into the modified partial fraction decomposition:

$$X(z) = -\frac{1}{1-z^{-1}} + \frac{3}{1-3z^{-1}}$$

Now, find the inverse Z-transform of each term using the property $Z^{-1}\left[\frac{1}{1-cz^{-1}}\right] = c^n u_n$. The result is:

$$x_n = -u_n + 3(3^n u_n) \tag{2}$$

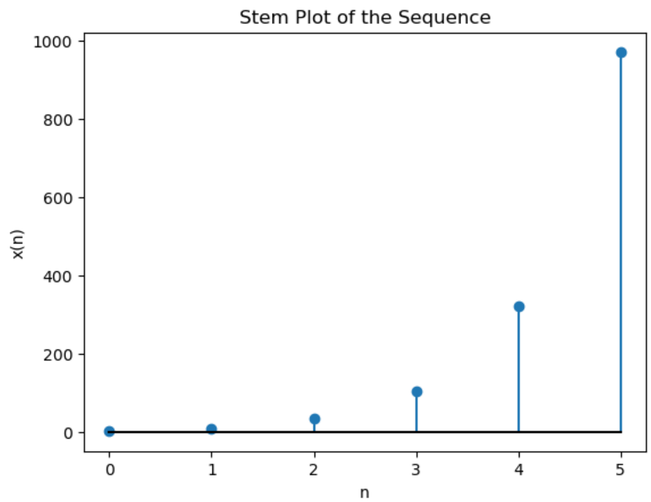

Figure 1: Sequence plot generated from the Python script.

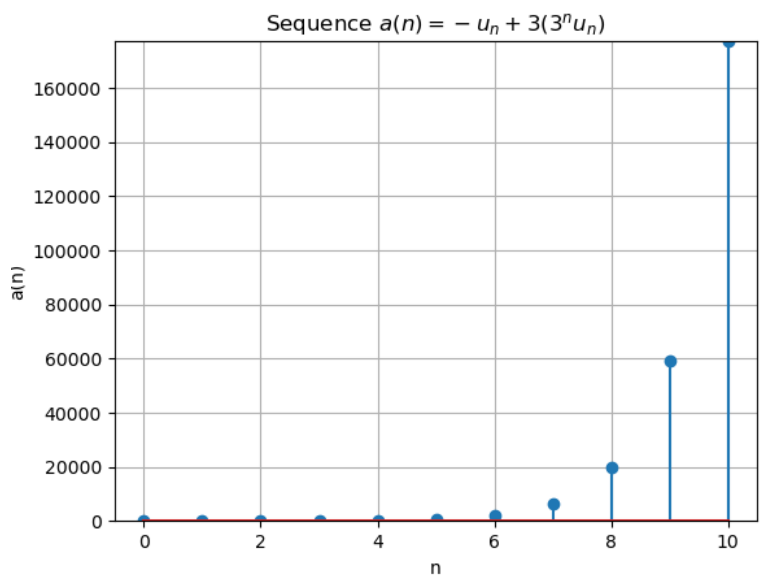

Figure 2: Plot of the sequence $a_n = -u_n + 3(3^n u_n)$