

Discreet 12.9.5.24

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PROBLEM STATEMENT

If S_1 , S_2 , S_3 are the sum of the first n natural numbers, their squares, and their cubes, respectively, show that

$$9(S_2)^2 = (S_3)(1+8(S_1))$$

SOLUTION

Sequence	Expression	Description
s_1	$\frac{n(n+1)}{2}$	sum of n natural numbers
s_2	$\frac{n(n+1)(2n+1)}{6}$	sum of squares
s_3	$\left(\frac{n(n+1)}{2}\right)^2$	sum of cubes
x_1	$x_1(n) = nu(n)$	
x_2	$x_2(n) = n^2u(n)$	
x_3	$x_3(n) = n^3u(n)$	

TABLE I
INPUT EQUATIONS

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1 \quad (1)$$

$$x_1(n) \xleftrightarrow{Z} X_1(z) \quad (2)$$

$$X_1(z) = \frac{z^{-1}}{(1-z^{-1})^2}, \quad |z| > 1 \quad (3)$$

$$s_1(x) = \sum_{n=-\infty}^{\infty} x_1(n)n = x_1(n) * u(n) \quad (4)$$

$$S_1(z) = X_1(z) \cdot u(z) \quad (5)$$

$$S_1(z) = \frac{z^{-1}}{(1-z^{-1})^3} \quad (6)$$

$$s_1(n) = \delta(n-2) \quad (7)$$

$$n^2 u(n) \xleftrightarrow{Z} \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, |z| > 1 \quad (8)$$

$$x_2(n) \xleftrightarrow{Z} X_2(z) \quad (9)$$

$$X_2(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, \quad |z| > 1 \quad (10)$$

$$s_2(x) = \sum_{n=-\infty}^{\infty} x_2(n)n = x_2(n) * u(n) \quad (11)$$

$$S_2(z) = X_2(z) \cdot u(z) \quad (12)$$

$$S_2(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} \quad (13)$$

$$s_2(n) = -u(n-1) + 3u(n-2) - 6u(n-3) + 4u(n-4) \quad (14)$$

$$n^3 u(n) \xleftrightarrow{Z} \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}, |z| > 1 x_3(n) \quad \xleftrightarrow{Z} X_3(z) \quad (15)$$

$$X_3(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}, \quad |z| > 1 \quad (16)$$

$$s_3(x) = \sum_{n=-\infty}^{\infty} x_3(n)n = x_3(n) * u(n) \quad (17)$$

$$S_3(z) = X_3(z) \cdot u(z) \quad (18)$$

$$S_3(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} \quad (19)$$

$$s_3(n) = -\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4) \quad (20)$$

$$9(s_2)^2 = (s_3)(1 + 8(s_1)) \quad (21)$$

$$9(-u(n-1) + 3u(n-2) - 6u(n-3) + 4u(n-4))^2 = (-\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4))(1 + 8(\delta(n-2))) \quad (22)$$

$$9(-u(n-1) + 3u(n-2) - 6u(n-3) + 4u(n-4))^2 = 9u(n-1)^2 - 54u(n-1)u(n-2) + 108u(n-1)u(n-3) - 72u(n-1)u(n-4) + 9u(n-2)^2 - 54u(n-2)u(n-3) + 36u(n-2)u(n-4) - 36u(n-3)^2 + 72u(n-3)u(n-4) - 16u(n-4)^2 \quad (23)$$

$$(-\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4))(1 + 8(\delta(n-2))) = -\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4) - 2u(n-1)\delta(n-2) - 3u(n-2)\delta(n-2) - 2u(n-3)\delta(n-2) + 8\delta(n-4)\delta(n-2) \quad (24)$$

the coefficients of the terms on both sides are the same, which means that the identity holds.

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$