# Math Assignment

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## Problem Statement

If $S_1$, $S_2$, $S_3$ are the sum of the first $n$ natural numbers, their squares, and their cubes, respectively, show that

$$9(S_{2)^2=(S3)(1+8(S1))}$$

## Solution

$$S_1 = \frac{n(n+1)}{2}$$
$$S_2 = \frac{n(n+1)(2n+1)}{6}$$
$$S_3 = \left(\frac{n(n+1)}{2}\right)^2$$

## Input Equations

Now, let's substitute these expressions into the given equation $9(S_2)^2 = (S_3)(1+8(S_1))$ and simplify:

Table 1: Input Equations

| **Equation** | **Expression** |
|---|---|
| $S_1$ | $\frac{n(n+1)}{2}$ |
| $S_2$ | $\frac{n(n+1)(2n+1)}{6}$ |
| $S_3$ | $\left(\frac{n(n+1)}{2}\right)^2$ |

$$
\begin{aligned}
9\left(\frac{n(n+1)(2n+1)}{6}\right)^2 &= \left(\frac{n(n+1)}{2}\right)^2\left(1+8\cdot\frac{n(n+1)}{2}\right)\\
\frac{9}{36}\left(n(n+1)(2n+1)\right)^2 &= \frac{1}{4}\left(n(n+1)\right)^2\left(1+4n(n+1)\right)\\
\frac{1}{4}\left(n(n+1)(2n+1)\right)^2 &= \frac{1}{4}\left(n(n+1)\right)^2\left(4n(n+1)+1\right)\\
\left(n(n+1)(2n+1)\right)^2 &= \left(n(n+1)\right)^2\left(4n(n+1)+1\right)
\end{aligned}
$$

The last equation holds true, which verifies that $9(S_2)^2 = (S_3)(1+8(S_1))$ for the given expressions of $S_1$, $S_2$, and $S_3$.