Discreet 12.9.5.24

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PROBLEM STATEMENT

If S_1, S_2, S_3 are the sum of the first n natural numbers, their squares, and their cubes, respectively, show that

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$

SOLUTION

Sequence	Expression	Description
s_1	$\frac{n(n+1)}{2}$	sum of n natural numbers
s_2	$\frac{n(n+1)(2n+1)}{6}$	sum of squares
s_3	$\left(\frac{n(n+1)}{2}\right)^2$	sum of cubes
x_1	$x_1(n) = nu(n)$	
x_2	$x_2(n) = n^2 u(n)$	
x_3	$x_3(n) = n^3 u(n)$	

TABLE I

INPUT EQUATIONS

$$nu(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{z^{-1}}{(1 - z^{-1})^2}, \qquad |z| > 1 \qquad (1)$$

$$x_1(n) \overset{\mathcal{Z}}{\leftrightarrow} X_1(z) \qquad (2)$$

$$X_1(z) = \frac{z^{-1}}{(1 - z^{-1})^2}, \qquad |z| > 1 \qquad (3)$$

$$y_1(x) = x_1(n) * u(n) \qquad (4)$$

$$Y_1(z) = X_1(z) \cdot u(z) \qquad (5)$$

$$Y_1(z) = \frac{z^{-1}}{(1 - z^{-1})^3} \qquad (6)$$

$$y_1(n) = \frac{(n + 1)}{2} (n) \, u(n) \qquad (7)$$

$$n^2 u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{z^{-1} \left(z^{-1} + 1 \right)}{(1 - z^{-1})^3}, \qquad |z| > 1 \qquad (8)$$

$$x_2(n) \overset{\mathcal{Z}}{\leftrightarrow} X_2(z) \qquad (9)$$

$$X_2(z) = \frac{z^{-1}(z^{-1} + 1)}{(1 - z^{-1})^3}, \qquad |z| > 1 \qquad (10)$$

$$y_2(x) = x_2(n) * u(n) \qquad (11)$$

$$Y_2(z) = X_2(z) \cdot u(z) \qquad (12)$$

$$Y_2(z) = \frac{z^{-1}(z^{-1} + 1)}{(1 - z^{-1})^4} \qquad (13)$$

$$y_2(n) = \frac{(n)(n + 2)(2n + 1)}{6} u(n) \qquad (14)$$

$$n^3 u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{z^{-1} \left(1 + 4z^{-1} + z^{-2} \right)}{(1 - z^{-1})^4}, \qquad |z| > 1 \qquad (15)$$

$$x_3(n) \overset{\mathcal{Z}}{\leftrightarrow} X_3(z) \qquad (16)$$

$$X_3(z) = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}, \qquad |z| > 1 \qquad (17)$$

$$y_3(x) = x_3(n) * u(n) \qquad (18)$$

$$Y_3(z) = X_3(z) \cdot u(z) \qquad (19)$$

$$Y_3(z) = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^5} \qquad (20)$$

$$y_3(n) = \left(\frac{(n)(n + 1)}{2} \right)^2 u(n) \qquad (21)$$

$$y_1^2 = (y_3)(1 + 8(y_1)) \tag{22}$$

$$\text{LHS} = 9(y_2)^2 = 9\left(\frac{(n + 1)(n)(2n + 1)}{6}\right)^2 u(n) \tag{23}$$

$$= n^6 u(n)^2 + 3n^5 u(n)^2 + \frac{13}{4}n^4 u(n)^2 + \frac{3}{2}n^3 u(n)^2 + \frac{1}{4}n^2 u(n)^2 \tag{24}$$

$$= n^6 + 3n^5 + \frac{13}{4}n^4 + \frac{3}{2}n^3 + \frac{1}{4}n^2 \qquad\qquad n >= 0 \tag{25}$$

$$\text{RHS} = (y_3)(1 + 8(y_1)) = \left(\frac{(n + 1)(n + 2)}{2}\right)^2 u(n)(1 + 8\left(\frac{(n + 1)(n + 2)}{2}\right)u(n)) \tag{26}$$

$$= n^6 u(n)^3 + 3n^5 u(n)^3 + 3n^4 u(n)^3 + \frac{1}{4}n^4 u(n)^2 + n^3 u(n)^3 + \frac{1}{2}n^3 u(n)^2 + \frac{1}{4}n^2 u(n)^2 \tag{27}$$

$$= n^6 + 3n^5 + \frac{13}{4}n^4 + \frac{3}{2}n^3 + \frac{1}{4}n^2 \qquad\qquad n >= 0 \tag{28}$$

LHS=RHS

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$