# Discreet 12.9.5.24

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## Problem Statement

If $S_1$, $S_2$, $S_3$ are the sum of the first $n$ natural numbers, their squares, and their cubes, respectively, show that

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$

## Solution

| Sequence | Expression | Description |
|---|---|---|
| $s_1$ | $\frac{n(n+1)}{2}$ | sum of n natural numbers |
| $s_2$ | $\frac{n(n+1)(2n+1)}{6}$ | sum of squares |
| $s_3$ | $\left(\frac{n(n+1)}{2}\right)^2$ | sum of cubes |
| $x_1$ | $x_1(n) = nu(n)$ | |
| $x_2$ | $x_2(n) = n^2u(n)$ | |
| $x_3$ | $x_3(n) = n^3u(n)$ | |

TABLE I
INPUT EQUATIONS

$$nu\,(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{\left(1-z^{-1}\right)^{2}}, |z|>1 \tag{1}$$

$$x_1\,(n) \overset{\mathcal{Z}}{\longleftrightarrow} X_1(z) \tag{2}$$

$$X_1(z) = \frac{z^{-1}}{(1-z^{-1})^2}, \qquad |z|>1 \tag{3}$$

$$y_1(x) = \sum_{n=-\infty}^{\infty} x_1(n)n = x_1(n) * u(n) \tag{4}$$

$$Y_1(z) = X_1(z)\cdot u(z) \tag{5}$$

$$Y_1(z) = \frac{z^{-1}}{(1-z^{-1})^3} \tag{6}$$

$$y_1(n) = \frac{(n+1)}{2}\,(2+n)\,u(n) \tag{7}$$

$$n^2u\,(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^{3}}, |z|>1 \tag{8}$$

$$x_2\,(n) \overset{\mathcal{Z}}{\longleftrightarrow} X_2(z) \tag{9}$$

$$X_2(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, \qquad |z|>1 \tag{10}$$

$$y_2(x) = \sum_{n=-\infty}^{\infty} x_2(n)n = x_2(n) * u(n) \tag{11}$$

$$Y_2(z) = X_2(z)\cdot u(z) \tag{12}$$

$$Y_2(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} \tag{13}$$

$$y_2(n) = -u(n-1) + 3u(n-2) - 6u(n-3) + 4u(n-4) \tag{14}$$

$$n^3u\,(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^{4}}, |z|>1 \tag{15}$$

$$x_3\,(n) \overset{\mathcal{Z}}{\longleftrightarrow} X_3(z) \tag{16}$$

$$X_3(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}, \qquad |z|>1 \tag{17}$$

$$y_3(x) = \sum_{n=-\infty}^{\infty} x_3(n)n = x_3(n) * u(n) \tag{18}$$

$$Y_3(z) = X_3(z)\cdot u(z) \tag{19}$$

$$Y_3(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} \tag{20}$$

$$y_3(n) = -\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4) \tag{21}$$

$$9(y_2)^2 = (y_3)(1 + 8(y_1)) \tag{22}$$

$$9(-u(n-1) + 3u(n-2) - 6u(n-3) + 4u(n-4))^2 = (-\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4))(1 + 8(\delta(n-2))) \tag{23}$$

$$9(-u(n-1) + 3u(n-2) - 6u(n-3) + 4u(n-4))^2 = 9u(n-1)^2 - 54u(n-1)u(n-2) + 108u(n-1)u(n-3) - 72u(n- \tag{24}$$

$$(-\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4))(1 + 8(\delta(n-2))) = -\frac{1}{4}u(n-1) - \frac{3}{8}u(n-2) - \frac{1}{4}u(n-3) + \delta(n-4) - \tag{25}$$

the coefficients of the terms on both sides are the same, which means that the identity holds.

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$