Discreet 12.9.5.24

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Problem Statement

If S_1, S_2, S_3 are the sum of the first n natural numbers, their squares, and their cubes, respectively, show that

$$9(S_2)^2=(S_3)(1+8(S_1))$$

Solution

Sequence	Expression	Description
s_1	$\frac{n(n+1)}{2}$	sum of n natural numbers
s_2	$\frac{n(n+1)(2n+1)}{6}$	sum of squares
s_3	$\left(\frac{n(n+1)}{2}\right)^2$	sum of cubes
x_1	$x_1(n) = nu(n)$	
x_2	$x_2(n) = n^2u(n)$	
x_3	$x_3(n) = n^3u(n)$	

TABLE I
Input Equations

1) $nu(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}}{(1-z^{-1})^2},\ |z| > 1$
$x_1(n) \xleftrightarrow{\mathcal{Z}} X_1(z)$
$X_1(z) = \frac{z^{-1}}{(1-z^{-1})^2},\ |z| > 1$
$y_1(x) = x_1(n) * u(n)$
$Y_1(z) = X_1(z) \cdot u(z)$
$Y_1(z) = \frac{z^{-1}}{(1-z^{-1})^3}$
$y_1(n) = (n)\,\frac{(n+1)}{2}u(n)$

2) $n^2u(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3},\ |z| > 1$
$x_2(n) \xleftrightarrow{\mathcal{Z}} X_2(z)$
$X_2(z) = \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3},\ |z| > 1$
$y_2(x) = x_2(n) * u(n)$
$Y_2(z) = X_2(z) \cdot u(z)$
$Y_2(z) = \frac{z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4}$
$y_2(n) = \frac{(n)(n+1)(2n+1)}{6}u(n)$

3) $n^3u(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4},\ |z| > 1$
$x_3(n) \xleftrightarrow{\mathcal{Z}} X_3(z)$
$X_3(z) = \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4},\ |z| > 1$
$y_3(x) = x_3(n) * u(n)$
$Y_3(z) = X_3(z) \cdot u(z)$
$Y_3(z) = \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^5}$
$y_3(n) = \left(\frac{(n)(n+1)}{2}\right)^2 u(n)$

$$y_1^2 = (y_3)(1 + 8(y_1)) \tag{1}$$

$$\text{LHS} = 9(y_2)^2 = 9\left(\frac{(n+1)(n)(2n+1)}{6}\right)^2 u(n) \tag{2}$$

$$= n^6u(n)^2 + 3n^5u(n)^2 + \frac{13}{4}n^4u(n)^2 + \frac{3}{2}n^3u(n)^2 + \frac{1}{4}n^2u(n)^2 \tag{3}$$

$$= n^6 + 3n^5 + \frac{13}{4}n^4 + \frac{3}{2}n^3 + \frac{1}{4}n^2 \qquad n >= 0 \tag{4}$$

$$\text{RHS} = (y_3)(1 + 8(y_1)) = \left(\frac{(n+1)(n+2)}{2}\right)^2 u(n)(1 + 8\left(\frac{(n+1)(n+2)}{2}\right)u(n)) \tag{5}$$

$$= n^6u(n)^3 + 3n^5u(n)^3 + 3n^4u(n)^3 + \frac{1}{4}n^4u(n)^2 + n^3u(n)^3 + \frac{1}{2}n^3u(n)^2 + \frac{1}{4}n^2u(n)^2 \tag{6}$$

$$= n^6 + 3n^5 + \frac{13}{4}n^4 + \frac{3}{2}n^3 + \frac{1}{4}n^2 \qquad n >= 0 \tag{7}$$

LHS=RHS

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$

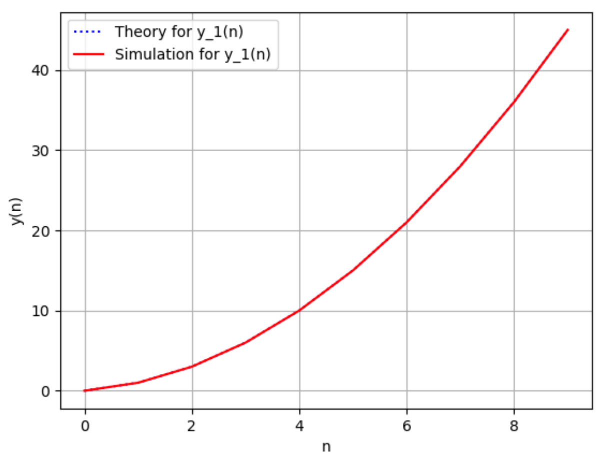

Fig. 1. Simulation vs Theory for y1(n) = (n(n+1)u(n))/2

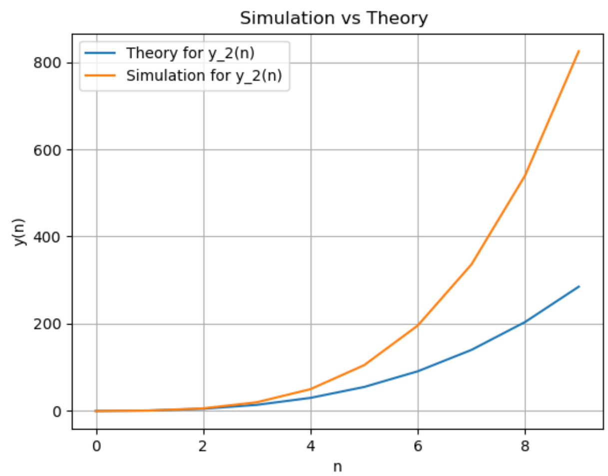

Fig. 2. Simulation vs Theory for y2(n)

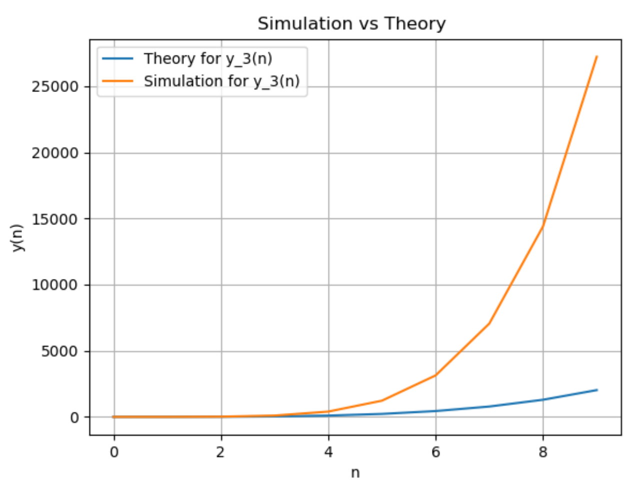

Fig. 3. Simulation vs Theory for y2(n)