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Problem Statement

If S_1, S_2, S_3 are the sum of the first n natural numbers, their squares, and their cubes, respectively, show that

$$9(S\,2)^2=(S\,3)(1+8(S\,1))$$

Solution

Sequence	Expression	Description
s_1	$\frac{n(n+1)}{2}$	sum of n natural numbers
s_2	$\frac{n(n+1)(2n+1)}{6}$	sum of squares
s_3	$\left(\frac{n(n+1)}{2}\right)^2$	sum of cubes
x_1	$x_1(n) = nu(n)$	
x_2	$x_2(n) = n^2u(n)$	
x_3	$x_3(n) = n^3u(n)$	

TABLE I
Input Equations

$$nu\left(n\right) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{\left(1-z^{-1}\right)^{2}}, |z| > 1 \tag{1}$$

$$x_1\left(n\right) \overset{\mathcal{Z}}{\longleftrightarrow} X_1(z) \tag{2}$$

$$X_1(z) = \frac{z^{-1}}{(1-z^{-1})^2}, \qquad |z| > 1 \tag{3}$$

$$y_1(x) = x_1(n) * u(n) \tag{4}$$

$$Y_1(z) = X_1(z) \cdot u(z) \tag{5}$$

$$Y_1(z) = \frac{z^{-1}}{(1-z^{-1})^3} \tag{6}$$

$$y_1(n) = \frac{(n+1)}{2}\left(2+n\right)u(n) \tag{7}$$

$$n^2u\left(n\right) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^{3}}, |z| > 1 \tag{8}$$

$$x_2\left(n\right) \overset{\mathcal{Z}}{\longleftrightarrow} X_2(z) \tag{9}$$

$$X_2(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, \qquad |z| > 1 \tag{10}$$

$$y_2(x) = x_2(n) * u(n) \tag{11}$$

$$Y_2(z) = X_2(z) \cdot u(z) \tag{12}$$

$$Y_2(z) = \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} \tag{13}$$

$$y_2(n) = \frac{(n+1)\left(n+2\right)\left(2n+3\right)}{6}u(n) \tag{14}$$

$$n^3u\left(n\right) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^{4}}, |z| > 1 \tag{15}$$

$$x_3\left(n\right) \overset{\mathcal{Z}}{\longleftrightarrow} X_3(z) \tag{16}$$

$$X_3(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}, \qquad |z| > 1 \tag{17}$$

$$y_3(x) = x_3(n) * u(n) \tag{18}$$

$$Y_3(z) = X_3(z) \cdot u(z) \tag{19}$$

$$Y_3(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} \tag{20}$$

$$y_3(n) = (\frac{(n+1)\left(n+2\right)}{2})^2u(n) \tag{21}$$

$$y_1^2 = (y_3)(1 + 8(y_1)) \qquad n >= 0 \tag{22}$$

$$\text{LHS} = 9(y_2)^2 = 9\left(\frac{(n+1)(n+2)(2n+3)}{6}u(n)\right)^2 \tag{23}$$

$$= n^6 + 9n^5 + \frac{133}{4}n^4 + \frac{129}{2}n^3 + \frac{277}{4}n^2 + 39n + 9 \tag{24}$$

$$\text{RHS} = (y_3)(1 + 8(y_1)) = \left(\frac{(n+1)(n+2)}{2}\right)^2 u(n)(1 + 8\left(\frac{(n+1)(n+2)}{2}\right)u(n)) \tag{25}$$

$$= n^6 + 9n^5 + \frac{133}{4}n^4 + \frac{129}{2}n^3 + \frac{277}{4}n^2 + 39n + 9 \tag{26}$$

LHS=RHS

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$