

# Discreet 12.9.5.24

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## PROBLEM STATEMENT

If  $S_1$ ,  $S_2$ ,  $S_3$  are the sum of the first  $n$  natural numbers, their squares, and their cubes, respectively, show that

$$9(S_2)^2 = (S_3)(1+8(S_1))$$

## SOLUTION

Sequence	Expression	Description
$s_1$	$\frac{n(n+1)}{2}$	sum of n natural numbers
$s_2$	$\frac{n(n+1)(2n+1)}{6}$	sum of squares
$s_3$	$\left(\frac{n(n+1)}{2}\right)^2$	sum of cubes
$x_1$	$x_1(n) = nu(n)$	
$x_2$	$x_2(n) = n^2u(n)$	
$x_3$	$x_3(n) = n^3u(n)$	

TABLE I  
INPUT EQUATIONS

$$1) \quad nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1$$

$$\begin{aligned} x_1(n) &\xleftrightarrow{Z} X_1(z) \\ X_1(z) &= \frac{z^{-1}}{(1-z^{-1})^2}, |z| > 1 \\ y_1(x) &= x_1(n) * u(n) \\ Y_1(z) &= X_1(z) \cdot u(z) \\ Y_1(z) &= \frac{z^{-1}}{(1-z^{-1})^3} \\ y_1(n) &= (n) \frac{(n+1)}{2} u(n) \end{aligned}$$

$$2) \quad n^2u(n) \xleftrightarrow{Z} \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, |z| > 1$$

$$\begin{aligned} x_2(n) &\xleftrightarrow{Z} X_2(z) \\ X_2(z) &= \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^3}, |z| > 1 \\ y_2(x) &= x_2(n) * u(n) \\ Y_2(z) &= X_2(z) \cdot u(z) \\ Y_2(z) &= \frac{z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} \\ y_2(n) &= \frac{(n)(n+1)(2n+1)}{6} u(n) \end{aligned}$$

$$3) \quad n^3u(n) \xleftrightarrow{Z} \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}, |z| > 1$$

$$\begin{aligned} x_3(n) &\xleftrightarrow{Z} X_3(z) \\ X_3(z) &= \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}, |z| > 1 \\ y_3(x) &= x_3(n) * u(n) \\ Y_3(z) &= X_3(z) \cdot u(z) \\ Y_3(z) &= \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} \\ y_3(n) &= \left(\frac{(n)(n+1)}{2}\right)^2 u(n) \end{aligned}$$

$$y_2^2 = (y_3)(1 + 8(y_1)) \quad (1)$$

$$\text{LHS} = 9(y_2)^2 = 9 \left( \frac{(n+1)(n)(2n+1)}{6} \right)^2 u(n) \quad (2)$$

$$= n^6 u(n)^2 + 3n^5 u(n)^2 + \frac{13}{4} n^4 u(n)^2 + \frac{3}{2} n^3 u(n)^2 + \frac{1}{4} n^2 u(n)^2 \quad (3)$$

$$= n^6 + 3n^5 + \frac{13}{4} n^4 + \frac{3}{2} n^3 + \frac{1}{4} n^2 \quad n \geq 0 \quad (4)$$

$$\text{RHS} = (y_3)(1 + 8(y_1)) = \left( \frac{(n+1)(n+2)}{2} \right)^2 u(n) \left( 1 + 8 \left( \frac{(n+1)(n+2)}{2} \right) u(n) \right) \quad (5)$$

$$= n^6 u(n)^3 + 3n^5 u(n)^3 + 3n^4 u(n)^3 + \frac{1}{4} n^4 u(n)^2 + n^3 u(n)^3 + \frac{1}{2} n^3 u(n)^2 + \frac{1}{4} n^2 u(n)^2 \quad (6)$$

$$= n^6 + 3n^5 + \frac{13}{4} n^4 + \frac{3}{2} n^3 + \frac{1}{4} n^2 \quad n \geq 0 \quad (7)$$

LHS=RHS

$$9(S_2)^2 = (S_3)(1 + 8(S_1))$$

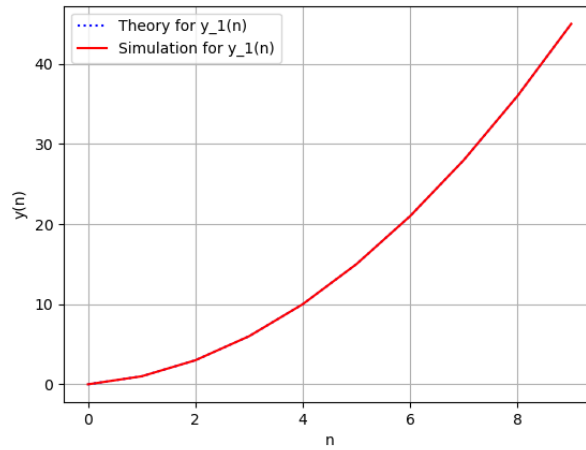


Fig. 1. Simulation vs Theory for  $y_1(n) = (n(n+1)u(n))/2$

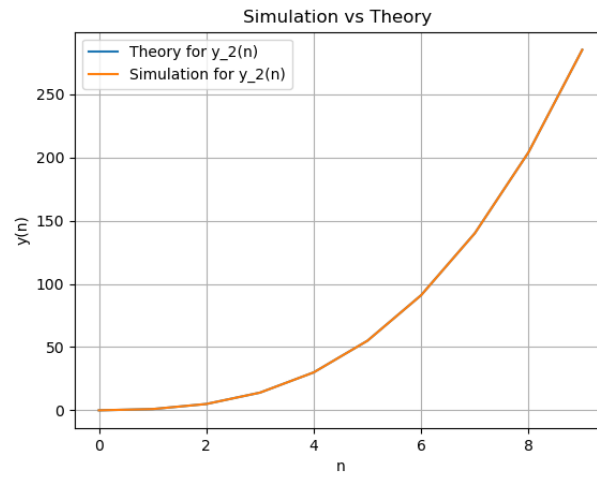


Fig. 2. Simulation vs Theory for  $y_2(n)$

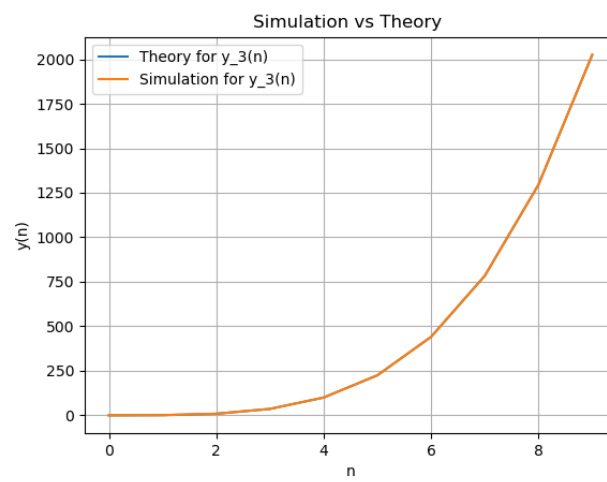


Fig. 3. Simulation vs Theory for  $y_2(n)$