Gate 2023 EC 58

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EE23BTECH11026

Problem Statement

Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t) \, dt$ is _____.
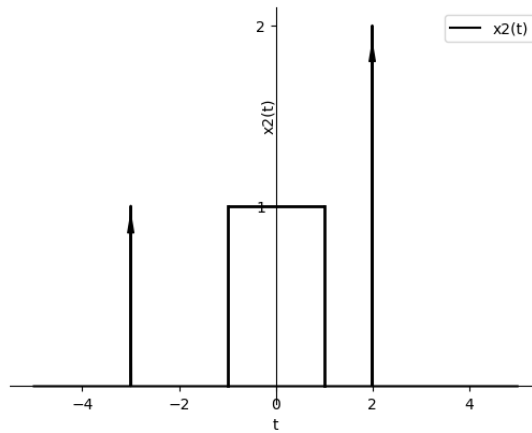

Fig. 1. Figure

Solution

Input Parameters

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Function	**Expression**	**Description**
$x_1(t)$	$u(t + 1.5) - u(t - 1.5)$	Step function with delay and width parameters.
$x_1(\omega)$	$3\,\text{sinc}(1.5\omega)$	Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2)$	Impulse function followed by a rectangle and two impulses.
$x_2(\omega)$	$e^{3j\omega} + 2\,\text{sinc}(\omega) + 2e^{-2j\omega}$	Fourier Transform of $x_2(t)$.

$x_1(t) = u(t + 1.5) - u(t - 1.5)$

$x_1(t) = \text{rect}\left(\dfrac{t}{3}\right)$

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$\text{rect}\left(\dfrac{t}{a}\right) \xrightarrow{F} f(\omega) = \dfrac{\sin(a\omega)}{2}$

$X_1(\omega) = 3\text{sinc}(1.5\omega)$

$x_2(t) = \delta(t + 3) + \text{rect}\left(\dfrac{t}{2}\right) + 2\delta(t - 2)$

$X_2(\omega) = e^{3j\omega} + 2\text{sinc}(\omega) + 2e^{-2j\omega}$

$y(t) = x_1(t) * x_2(t)$

$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$

We know:

$Y(\omega) = \displaystyle\int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt$

$\displaystyle\int_{-\infty}^{\infty} y(t)dt = Y(0)$

$\begin{aligned}
Y(0) &= X_1(0) \cdot X_2(0) \\
&= 3(1 + 2 + 2) \\
&= 15
\end{aligned}$