Gate 2023 EC 58

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EE23BTECH11026

PROBLEM STATEMENT

Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is _____.
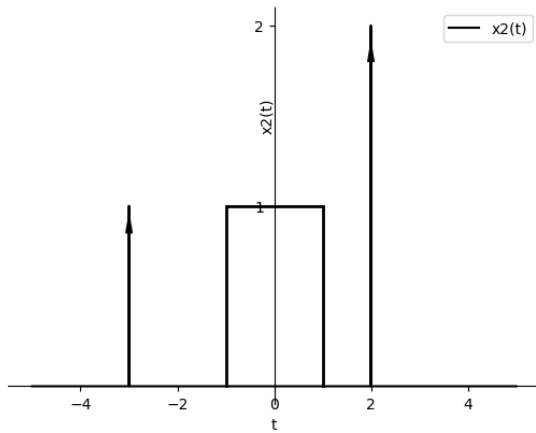

Fig. 1. Figure

$x_1(t) = u(t + 1.5) - u(t - 1.5)$

$x_1(t) = \text{rect}\left(\frac{t}{3}\right)$

$x_1(t) = \text{recr}\left(\frac{t}{3}\right)$ Fourier transform $= 3\text{sinc}(1.5\omega)$

$x_2(t) = \delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2)$

Taking Fourier transform:

$X_2(\omega) = e^{3j\omega} + 2\text{sinc}(\omega) + 2e^{-2j\omega}$

$y(t) = x_1(t) * x_2(t)$

$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$

We know:

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}\,dt$$

$$\int_{-\infty}^{\infty} y(t) = Y(0)$$

$Y(0) = X_1(0) \cdot X_2(0)$

$\qquad = 3(1 + 2 + 2)$

$\qquad = 15$

SOLUTION

INPUT PARAMETERS

Input Parameters		
Function	**Expression**	**Description**
$x_1(t)$	$u(t + 1.5) - u(t - 1.5)$	Step function with delay and width parameters.
$x_2(t)$	$\delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2)$	Impulse function followed by a rectangle and two impulses.