Gate 2023 EC 58

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EE23BTECH11026

Problem Statement

Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\, dt$ is _____.

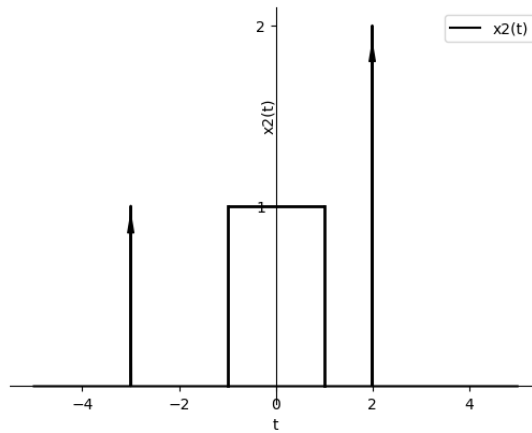Fig. 1. Figure

Solution

Input Parameters

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Function	**Expression**	**Description**
$x_1(t)$	$u(t + 1.5) - u(t - 1.5)$	Step function with delay and width parameters.
$X_1(\omega)$		Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2)$	Impulse function followed by a rectangle and two impulses.
$X_2(\omega)$		Fourier Transform of $x_2(t)$.

$$x_1(t) = u(t + 1.5) - u(t - 1.5) \tag{1}$$

$$x_1(t) = \text{rect}\left(\frac{t}{3}\right) \tag{2}$$

$$\text{rect}\left(\frac{t}{a}\right) \xrightarrow{f} f(\omega) = a \times \sin(\omega\frac{a}{2}) \tag{3}$$

$$X_1(\omega) = 3\text{sinc}(1.5\omega) \tag{4}$$

$$x_2(t) = \delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2) \tag{5}$$

$$X_2(\omega) = e^{3j\omega} + 2\text{sinc}(\omega) + 2e^{-2j\omega} \tag{6}$$

$$y(t) = x_1(t) * x_2(t) \tag{7}$$

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega) \tag{8}$$

We know: $\tag{9}$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt \tag{10}$$

$$\int_{-\infty}^{\infty} y(t)dt = Y(0) \tag{11}$$

$$Y(0) = X_1(0) \cdot X_2(0) \tag{12}$$

$$= 3(1 + 2 + 2) \tag{13}$$

$$= 15 \tag{14}$$