Gate 2023 EC 58

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Problem Statement

Let $x_1(t) = u(t+1.5) - u(t-1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is ________.

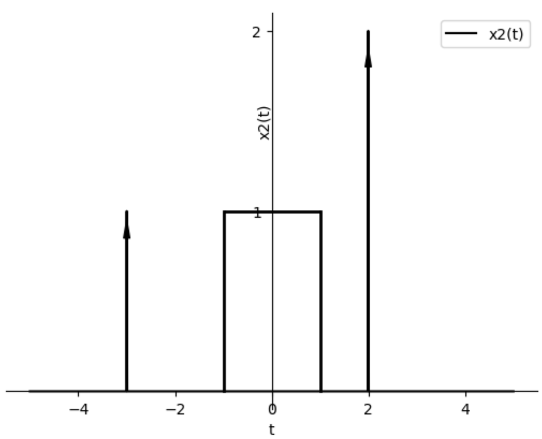

Fig. 1. Figure

Solution

Input Parameters

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Function	**Expression**	**Description**
$x_1(t)$	$u(t+1.5) - u(t-1.5)$	Step function with delay and width parameters.
$x_1(\omega)$	$3\mathrm{sinc}(1.5\omega)$	Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t+3) + \mathrm{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$	Impulse function followed by a rectangle and two impulses.
$x_2(\omega)$	$e^{3j\omega} + 2\mathrm{sinc}(\omega) + 2e^{-2j\omega}$	Fourier Transform of $x_2(t)$.

$$x_1(t) = u(t+1.5) - u(t-1.5)$$
$$x_1(t) = \mathrm{rect}\left(\frac{t}{3}\right)$$
$$x_1(t) = \mathrm{recr}\left(\frac{t}{3}\right)$$
$$\mathrm{rect}(t) = \mathrm{rect}\left(\frac{t}{a}\right) \longleftrightarrow a \cdot \mathrm{sinc}(a\omega)$$
$$X1(\omega) = 3\mathrm{sinc}(1.5\omega)$$
$$x_2(t) = \delta(t+3) + \mathrm{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$$
$$X_2(\omega) = e^{3j\omega} + 2\mathrm{sinc}(\omega) + 2e^{-2j\omega}$$
$$y(t) = x_1(t) * x_2(t)$$
$$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$$

We know:

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt$$
$$\int_{-\infty}^{\infty} y(t) = Y(0)$$
$$\begin{aligned} Y(0) &= X_1(0) \cdot X_2(0) \\ &= 3(1+2+2) \\ &= 15 \end{aligned}$$