

Gate 2023 EC 58

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PROBLEM STATEMENT

Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t) dt$ is _____.

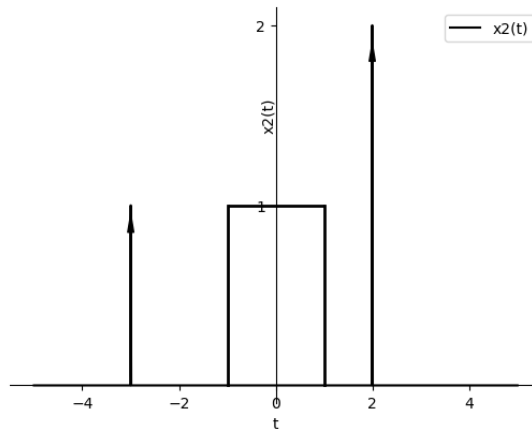


Fig. 1. Figure

SOLUTION

INPUT PARAMETERS

Input Parameters		
Function	Expression	Description
$x_1(t)$	$u(t + 1.5) - u(t - 1.5)$	Step function with delay and width parameters.
$X_1(\omega)$		Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2)$	Impulse function followed by a rectangle and two impulses.
$X_2(\omega)$		Fourier Transform of $x_2(t)$.

$$x_1(t) = u(t + 1.5) - u(t - 1.5) \quad (1)$$

$$x_1(t) = \text{rect}\left(\frac{t}{3}\right) \quad (2)$$

$$\text{rect}\left(\frac{t}{a}\right) \xrightarrow{f} f(\omega) = a \times \text{sinc}\left(\omega \frac{a}{2}\right) \quad (3)$$

$$\text{where } \text{rect}\left(\frac{t}{a}\right) = \begin{cases} 1 & \text{if } |t| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

The Fourier Transform of $\text{rect}\left(\frac{t}{a}\right)$:

$$F(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{a}\right) e^{-j\omega t} dt \quad (5)$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-j\omega t} dt \quad (6)$$

$$= \frac{1}{-j\omega} e^{-j\omega t} \Big|_{-\frac{a}{2}}^{\frac{a}{2}} \quad (7)$$

$$= \frac{1}{-j\omega} (e^{-j\omega \frac{a}{2}} - e^{j\omega \frac{a}{2}}) \quad (8)$$

$$= \frac{2}{\omega} \sin\left(\frac{\omega a}{2}\right) \quad (9)$$

$$= \boxed{F(\omega) = a \operatorname{sinc}\left(\frac{\omega a}{2}\right)} \quad (10)$$

$$X_1(\omega) = 3\operatorname{sinc}(1.5\omega) \quad (11)$$

$$x_2(t) = \delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2) \quad (12)$$

$$X_2(\omega) = e^{3j\omega} + 2\operatorname{sinc}(\omega) + 2e^{-2j\omega} \quad (13)$$

$$y(t) = x_1(t) * x_2(t) \quad (14)$$

$$Y(\omega) = X_1(\omega) \cdot X_2(\omega) \quad (15)$$

$$\text{We know:} \quad (16)$$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt \quad (17)$$

$$\int_{-\infty}^{\infty} y(t) dt = Y(0) \quad (18)$$

$$Y(0) = X_1(0) \cdot X_2(0) \quad (19)$$

$$= 3(1 + 2 + 2) \quad (20)$$

$$= 15 \quad (21)$$