

Gate 2023 EC 58

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PROBLEM STATEMENT

Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t) dt$ is _____.

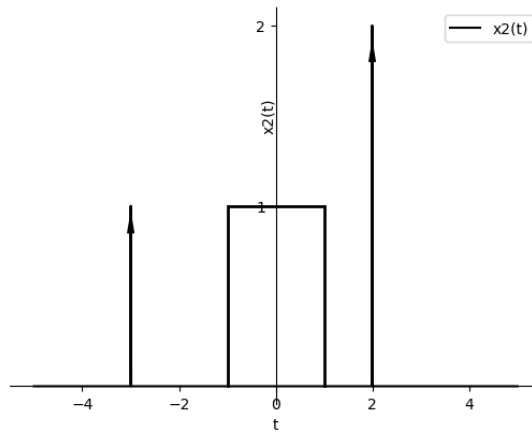


Fig. 1. Figure

SOLUTION

INPUT PARAMETERS

Input Parameters		
Function	Expression	Description
$x_1(t)$	$u(t + 1.5) - u(t - 1.5)$	Step function with delay and width parameters.
$X_1(f)$		Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t + 3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t - 2)$	Impulse function followed by a rectangle and two impulses.
$X_2(f)$		Fourier Transform of $x_2(t)$.

$$x_1(t) = u(t + 1.5) - u(t - 1.5) \quad (1)$$

$$x_1(t) = \text{rect}\left(\frac{t}{3}\right) \quad (2)$$

$$\text{rect}\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{F}} a \times \sin(2\pi f \frac{a}{2}) \quad (3)$$

$$\text{where } \text{rect}\left(\frac{t}{a}\right) = \begin{cases} 1 & \text{if } |t| < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

The Fourier Transform of $\text{rect}\left(\frac{t}{a}\right)$:

$$F(f) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{a}\right) e^{-j2\pi ft} dt \quad (5)$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-j2\pi ft} dt \quad (6)$$

$$= \frac{1}{-j2\pi f} e^{-j2\pi ft} \Big|_{-\frac{a}{2}}^{\frac{a}{2}} \quad (7)$$

$$= \frac{1}{-j2\pi f} (e^{-j2\pi f \frac{a}{2}} - e^{j2\pi f \frac{a}{2}}) \quad (8)$$

$$= \frac{2}{2\pi f} \sin\left(\frac{2\pi fa}{2}\right) \quad (9)$$

$$= \boxed{F(f) = a \text{ sinc}\left(\frac{2\pi fa}{2}\right)} \quad (10)$$

$$X_1(f) = 3\text{sinc}(1.5 \cdot 2\pi f) \quad (11)$$

$$x_2(t) = \delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2) \quad (12)$$

$$X_2(f) = e^{3j2\pi f} + 2\text{sinc}(2\pi f) + 2e^{-2j2\pi f} \quad (13)$$

$$y(t) = x_1(t) * x_2(t) \quad (14)$$

$$y(t) = \int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) d\tau \quad (15)$$

$$= \int_{-\infty}^{\infty} \text{rect}\left(\frac{\tau}{3}\right) \cdot \left(\delta(t+3-\tau) + \text{rect}\left(\frac{t-\tau}{2}\right) + 2\delta(t-\tau-2)\right) d\tau \quad (16)$$

$$= \delta(t+3) + 2\text{rect}\left(\frac{t}{2}\right) + 4\delta(t-2) \quad (17)$$

$$Y(f) = X_1(f) \cdot X_2(f) \quad (18)$$

$$= 3\text{sinc}(1.52\pi f) \cdot (e^{3j2\pi f} + 2\text{sinc}(2\pi f) + 2e^{-2j2\pi f}) \quad (19)$$

$$\text{as its not periodic } f=0 \quad (20)$$

$$Y(0) = 3\text{sinc}(0) \cdot (e^0 + 2\text{sinc}(0) + 2e^0) \quad (21)$$

$$= 3 \cdot (1 + 2 + 2) \quad (22)$$

$$= 15 \quad (23)$$

Therefore, the value of $\int_{-\infty}^{\infty} y(t) dt$ is 15