Gate 2023 EC 58

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Problem Statement

Let $x_1(t) = u(t+1.5) - u(t-1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)\,dt$ is __________.

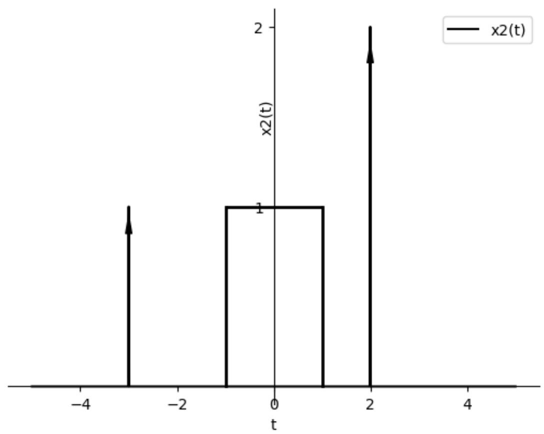

Fig. 1. Figure

Solution

Input Parameters

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Function	**Expression**	**Description**
$x_1(t)$	$u(t+1.5) - u(t-1.5)$	Step function with delay and width parameters.
$X_1(\omega)$		Fourier Transform of $x_1(t)$.
$x_2(t)$	$\delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$	Impulse function followed by a rectangle and two impulses.
$X_2(\omega)$		Fourier Transform of $x_2(t)$.

$x_1(t) = u(t+1.5) - u(t-1.5)$
$x_1(t) = \text{rect}\left(\frac{t}{3}\right)$
$\text{rect}\left(\frac{t}{a}\right) \xrightarrow{f} f(\omega) = a \times \sin(\omega\frac{a}{2})$
$X_1(\omega) = 3\text{sinc}(1.5\omega)$
$x_2(t) = \delta(t+3) + \text{rect}\left(\frac{t}{2}\right) + 2\delta(t-2)$
$X_2(\omega) = e^{3j\omega} + 2\text{sinc}(\omega) + 2e^{-2j\omega}$
$y(t) = x_1(t) * x_2(t)$
$Y(\omega) = X_1(\omega) \cdot X_2(\omega)$
We know:
$Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt$
$\int_{-\infty}^{\infty} y(t)dt = Y(0)$
$$\begin{aligned} Y(0) &= X_1(0) \cdot X_2(0) \\ &= 3(1+2+2) \\ &= 15 \end{aligned}$$