E5: solving LP problems with Simplex

Exercise 5:

maximize $z = x_1 + 2x_2$

subject to
$$-x_1 + x_2 \leq 2$$
$$x_1 + 2x_2 \leq 8$$
$$x_1 \leq 6$$
$$x_1, x_2 \geq 0$$

→ the Simplex Tableau:

	z	x_1	x_2	s	t	v	b
z	1	-1	-2	0	0	0	0
s	0	-1	1	1	0	0	2
t	0	1	2	0	1	0	8
v	0	1	0	0	0	1	6

after putting it in this form:

$$-x_1 - 2x_2 + z = 0$$
$$-x_1 + x_2 + s = 2$$
$$x_1 + 2x_2 + t = 8$$
$$x_1 + v = 6$$

- We choose the most negative coefficient in the objective row, here it is $x_2 = -2$.
- We divide b by the positive coefficients in the entering variable's (x_2) column to find the pivot row.
- Pivot: turn the pivot column into a unit column
- Repeat the pivot until there are no more negative coefficients in the first row for the most optimal solution

	z	x_1	x_2	s	t	u	b	
z	1	-3	0	2	0	0	4	$\leftarrow R_1+2R_2$
x_2	0	-1	1	1	0	0	2	
t	0	3	0	-2	0	0	4	$\leftarrow R_3-2R_2$
u	0	1	0	0	0	1	6	

	z	x_1	x_2	s	t	u	b	
z	1	0	0	0	1	0	8	R_1+3R_3
x_2	0	0	1	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{10}{3}$	R_2+R_3
x_1	0	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{4}{3}$	
u	0	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	1	$\frac{14}{3}$	R_4-R_3

- The <u>optimal solution</u> found is:

$$x_2 = \frac{10}{3} \; ; \; x_1 = \frac{4}{3} \; ; \; u = \frac{14}{3}$$

Exercice 6

maximize $z = 3x_1 + x_2$

subject to
$$x_1 + x_2 \geq 4$$
$$-x_1 + x_2 \leq 4$$
$$-x_1 + 2x_2 \geq -4$$
$$x_1, x_2 \geq 0$$

the standard form:

$$-3x_1 - x_2 + z = 0 \;/\; \max\ 3x_1 + x_1$$

$$x_1 + x_2 - s = 4$$
$$-x_1 + x_2 + t = 4$$
$$x_1 - 2x_2 + u = 4$$

Auxiliary problem: $\max z = -R_1 \iff z + R_1 = 0$

$$x_1 + x_2 - s + R_1 = 4$$
$$-x_1 + x_2 + t = 4$$
$$x_1 - 2x_2 + u = 4$$

	z	x_1	x_2	s	t	u	R_1	b
z	1	0	0	0	0	0	1	0
R_1	0	$\boxed{1}$	1	-1	0	0	1	4
t	0	-1	1	0	1	0	0	4
u	0	1	-2	0	0	1	0	4

	z	x_1	x_2	s	t	u	R_1	b
z	1	0	0	0	0	0	1	0
x_1	0	1	1	-1	0	0	1	4
t	0	0	2	-1	1	0	1	8
u	0	0	-3	1	0	1	1	0

	z	x_1	x_2	s	t	u	b
z	1	-3	-1	0	0	0	0
x_1	0	$\boxed{1}$	1	-1	0	0	4
t	0	0	2	-1	1	0	8
u	0	0	-3	1	0	1	0

	z	x_1	x_2	s	t	u	b
z	1	0	2	-3	0	0	12
x_1	0	1	1	-1	0	0	4
t	0	0	2	-1	1	0	8
u	0	0	-3	$\boxed{1}$	0	1	0

	z	x_1	x_2	s	t	u	b
z	1	0	-7	0	0	3	12
x_1	0	1	-2	0	0	1	4
t	0	0	-1	0	1	1	8
u	0	0	-3	1	0	1	0

full negative column → no feasible solution

Exercice 7

maximize $z = 3x_1 + x_2$

subject to
$$-x_1 + x_2 \geq 4$$
$$-x_1 + 2x_2 \leq -4$$
$$x_1, x_2 \geq 0$$

Standard form: $-x_1 + x_2 - s = 4$

$x_1 - 2x_2 - t = 4$

$z - 3x_1 - x_2 = 0$

Auxiliary problem: $z + R_1 + R_2 = 0$

$-x_1 + x_2 - s + R_1 = 4$

$x_1 - 2x_2 - t + R_2 = 4$

	z	x_1	x_2	s	t	R_1	R_2	b
z	1	0	0	0	0	1	1	0
R_1	0	-1	1	-1	0	1	0	4
R_2	0	1	-2	0	-1	0	1	4

	z	x_1	x_2	s	t	R_1	R_2	b
z	1	0	1	1	1	0	0	-8
R_1	0	-1	1	-1	0	1	0	4
R_2	0	1	-2	0	-1	0	1	4

First phase completed but there are artificial variables in the base with values > 0

$\Rightarrow$ the problem has no feasible solution