Monte Carlo Analysis – Asian Option price

Monte Carlo analysis is a cornerstone for implementing financial models in the industry. These simulations have many advantages, including the ease of implementation and applicability to multi-dimensional problems commonly encountered in finance. Option pricing can be represented as expectations and Monte Carlo analysis is very attractive and commonly used. An example is an Asian Option Call, Which is a financial contract dependant on the average security price over discrete dates in the future. The assent price at some time, t, in the future follows the classic Black-Scholes model as follows.

$$S_t = S_0 \, e^{(r - 0.5\,\sigma)\,t + \sigma\, Wt}$$

Where r is the risk-free rate of return, σ is volatility of the asset price and dWt is the increment of standard Brownian motion. The price of this option is a function of the strike price, K, and option maturity, T shown as follows

$$P\,(T, K) = e^{-rT}\mathbb{E}\,\{\max(S_a - K, 0) \mid S_0 = s_0\}$$

Where the average asset price is

$$S_a = \sum_{i=1}^{n} S_{ti}$$

The combination of these equations does not have a closed form solution. To solve this pricing problem, A Monte Carlo simulation is used.

However, for risk management, hedging, and stress testing of a portfolio, the price sensitivity as a function of changes to model inputs, the “greeks” as they are commonly known, becomes quite valuable, One Greek of interest is Vega; the option price sensitivity to changes in the securities volatility, which is as follows

$$\text{vega} = \frac{dP(T,\ K)}{d\sigma}$$

The price and vega calculation using Monte Carlo techniques is very time consuming for several reasons. For simulation accuracy, many Brownian motion trajectories are used for price determination. For each option simulation, there are several contract dates during the option maturity; monthly dates for an annual contract. In addition, an accurate picture of price volatility is achieved by rerunning the simulation with many different values for the volatility, σ. The option trader faced with minimizing risk to their client-base and portfolio may look to have this price and volatility analysis before making trades or in post-closing analysis, For very large, multi-commodity portfolios, analysts frequently wait hours for model simulations affecting their ability to respond in real-time to a dynamic market or to complete risk analysis before the next day trading

Reference: http://www.interactivesupercomputing.com/success/pdf/caseStudy_financialmodeling.pdf