Numerical Linear Algebra Libraries

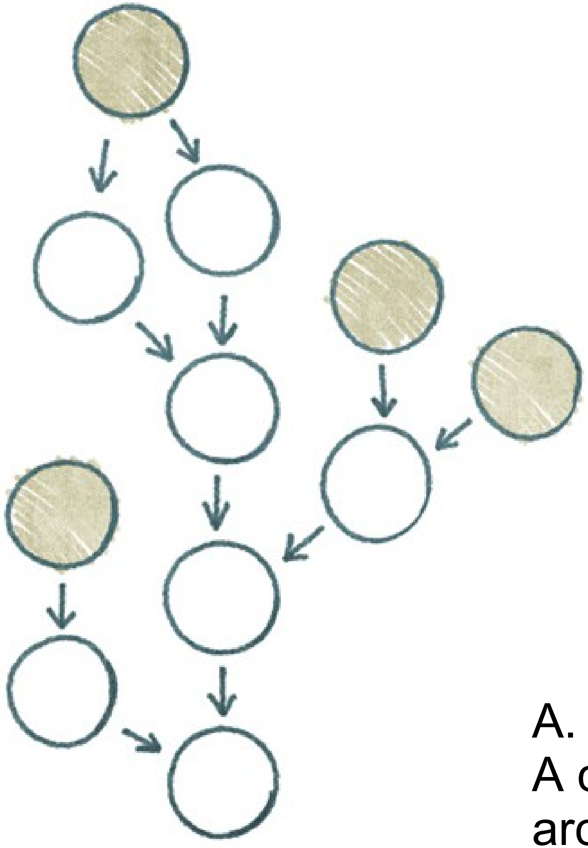
for

Multicore and Many-core Systems

with

PLASMA

PLASMA Concept and Design



- dense linear algebra for multicore
  - dataflow scheduling
  - tile matrix layout
  - tile algorithms

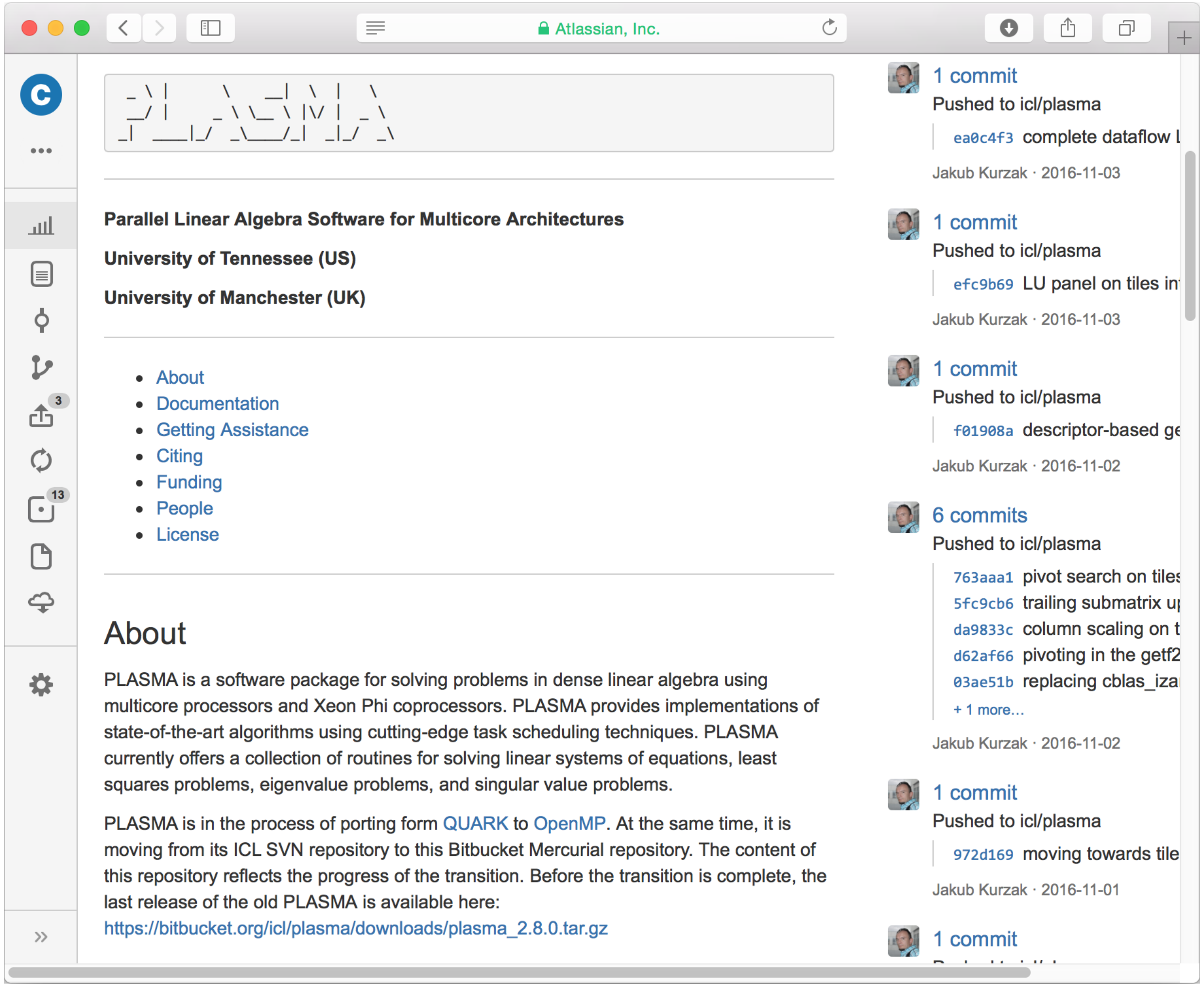
A. Buttari, J. Langou, J. Kurzak, J. Dongarra,
A class of parallel tiled linear algebra algorithms for multicore architectures,
Parallel Computing, 35(1):38-53, 2009. DOI: 10.1016/j.parco.2008.10.002

http://icl.cs.utk.edu/plasma/

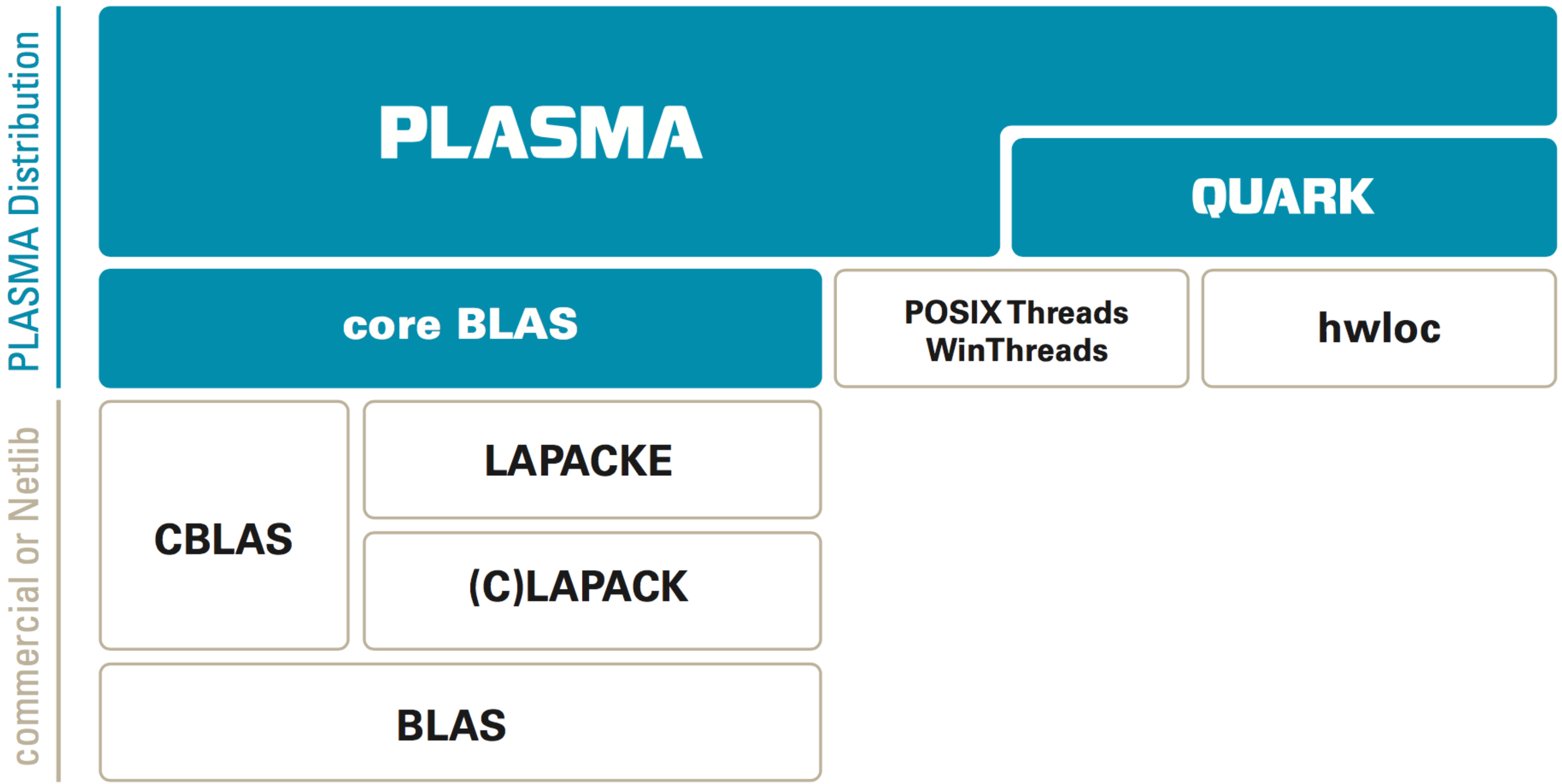
https://bitbucket.org/icl/plasma

→ Documentation (Doxygen)

→ Get Assistance (Google Group)



PLASMA Old Design

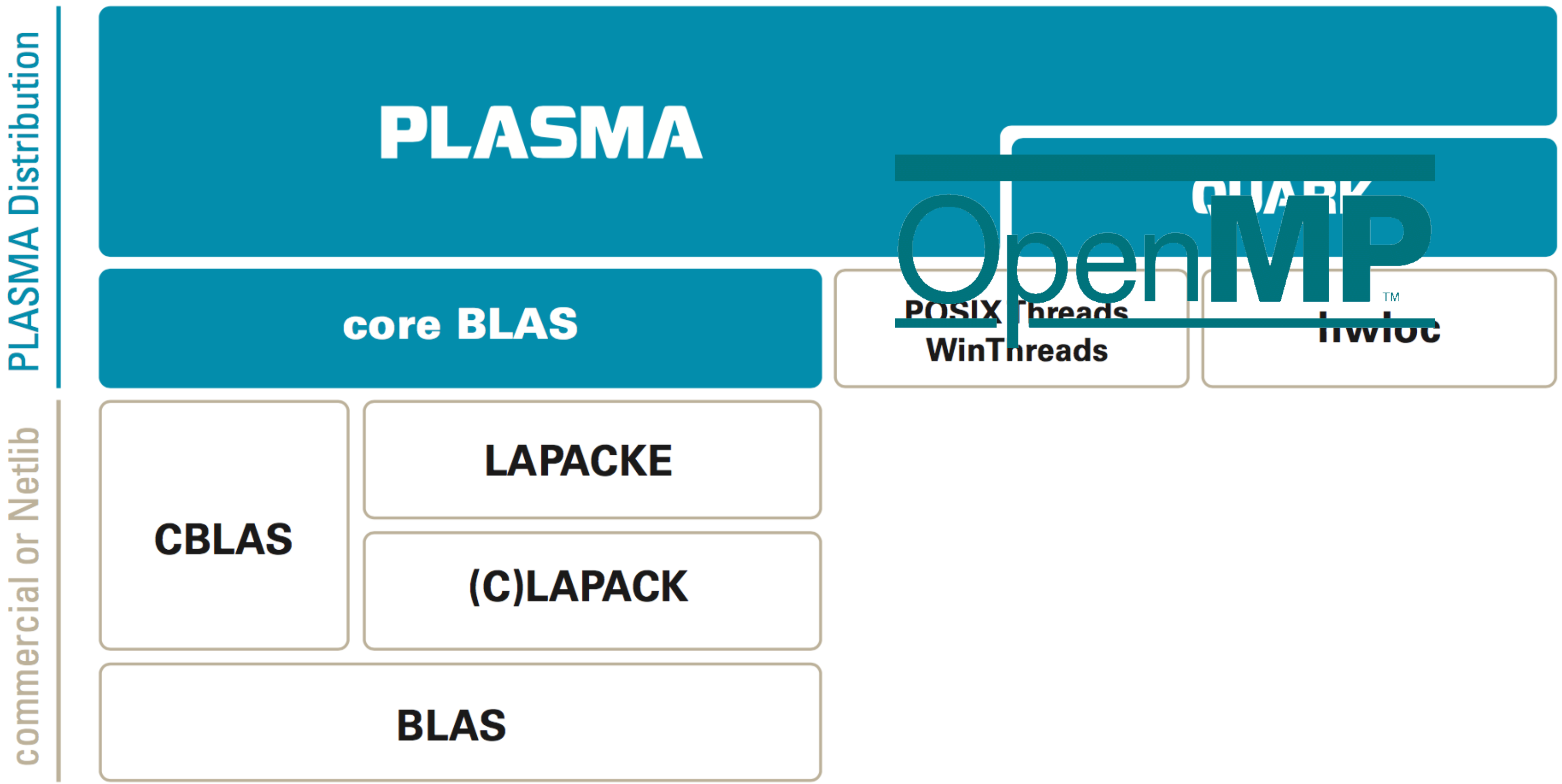


A. YarKhan, J. Kurzak, P. Luszczek, J. Dongarra, Porting the PLASMA Numerical Library to the OpenMP Standard, International Journal of Parallel Programming, 1-22, 2016. DOI: 10.1007/s10766-016-0441-6

Dynamic Scheduling, OpenMP, GNU GCC

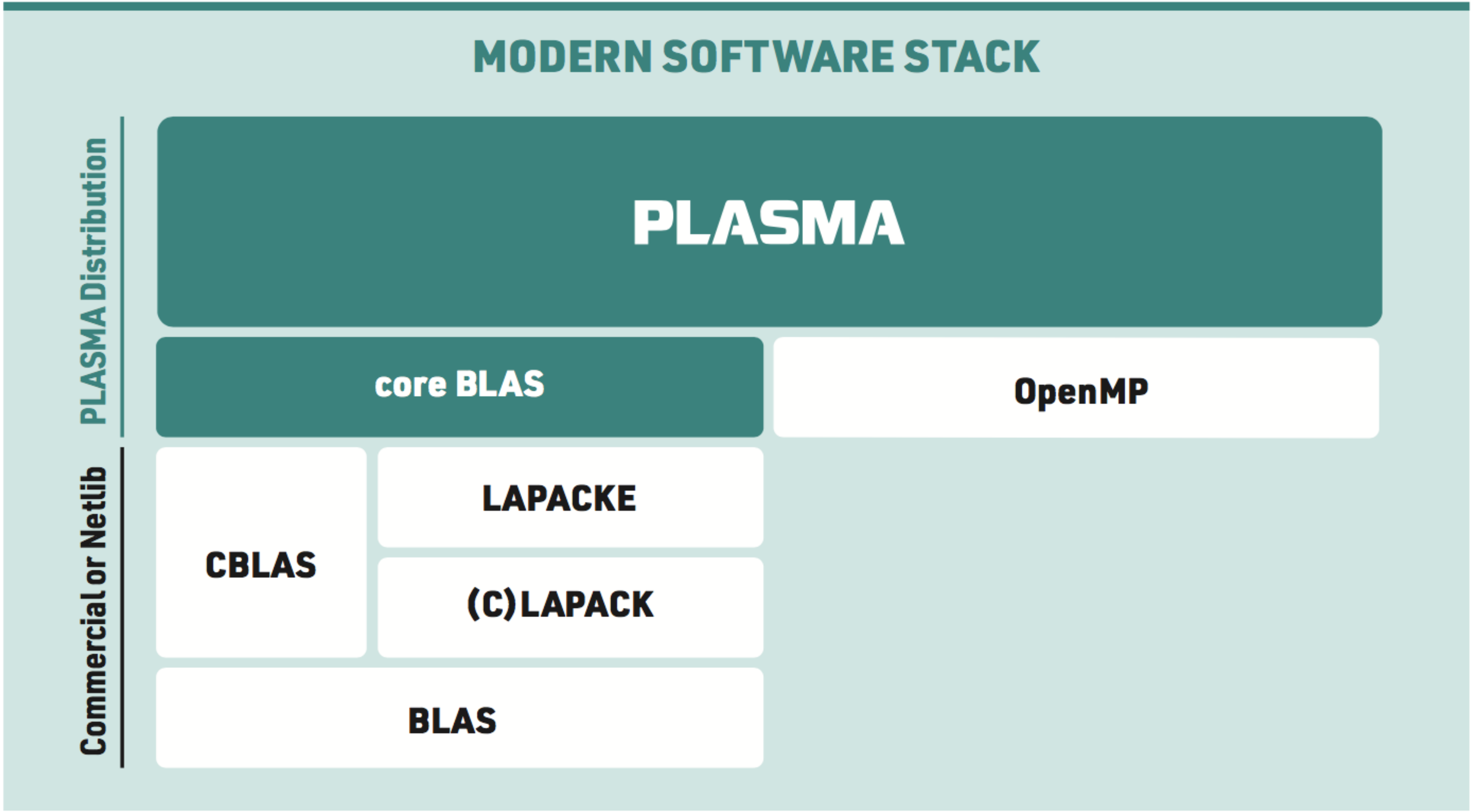
May 2008	OpenMP 3.0		
April 2009		GCC 4.4	`#pragma omp task`
July 2013	OpenMP 4.0		
April 2014		GCC 4.9	`#pragma omp task depend`
Nov. 2015	OpenMP 4.5		
April 2016		GCC 6.1	`#pragma omp task priority`

PLASMA New Design



A. YarKhan, J. Kurzak, P. Luszczek, J. Dongarra, Porting the PLASMA Numerical Library to the OpenMP Standard, International Journal of Parallel Programming, 1-22, 2016. DOI: 10.1007/s10766-016-0441-6

Current Software Stack of PLASMA



Structure of Typical PLASMA Function

User Application

PLASMA

plasma_dgemm

```
#pragma omp parallel
#pragma omp master
{
  plasma_omp_dgemm()
}
```

plasma_omp_dgemm

plasma_pdgemm

CORE Layer

core_omp_dgemm

```
#pragma omp task depend()
{
  core_dgemm()
}
```

core_dgemm

BLAS, LAPACK

dgemm

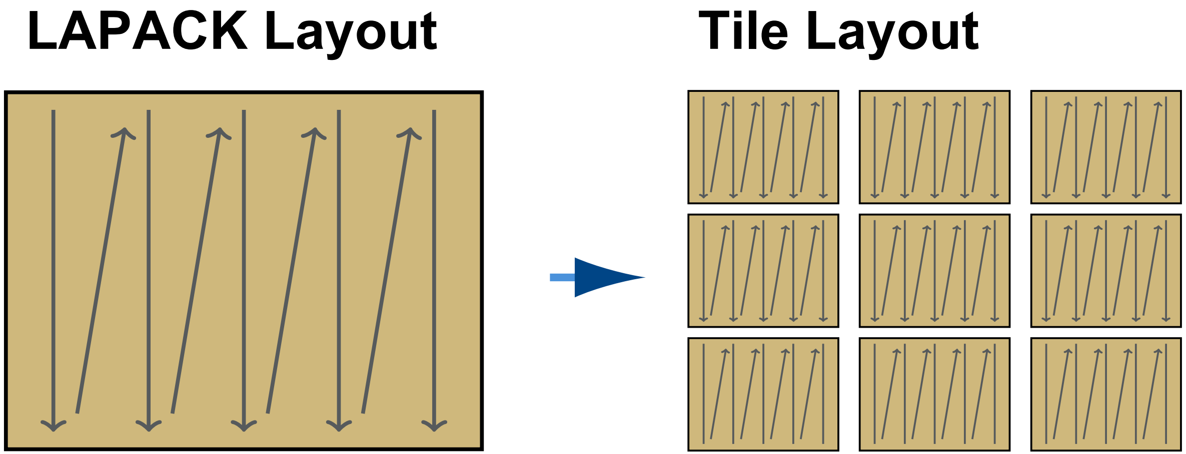
Calling PLASMA – Basic Layout

```
// A is the matrix (pointer)
info = LAPACKE_dpotrf   (layout, uplo, n, A, lda);
// A is the matrix (pointer)
info = plasma_dpotrf    (layout, uplo, n, A, lda);
 // A is a descriptor (matrix object)
       plasma_omp_dpotrf(        uplo, A, sequence, request);
```

Calling PLASMA – Full Example

```
plasma_init();
plasma_desc_general_create(..., &A);
plasma_sequence_create(&sequence);
plasma_request_t request = PlasmaRequestInitializer;

#pragma omp parallel
#pragma omp master
{
    plasma_omp_zge2desc(pA, lda, A, sequence, &request);
    plasma_omp_dpotrf(uplo, A, sequence, &request);
    plasma_omp_zdesc2ge(A, pA, lda, sequence, &request);
}

plasma_desc_destroy(&A);
plasma_sequence_destroy(sequence);
plasma_finalize();
```

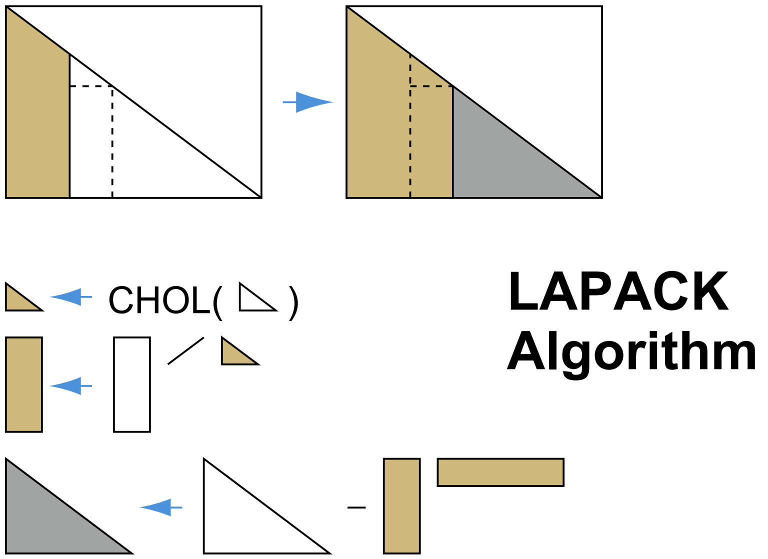
PLASMA and Tile Matrix Layout

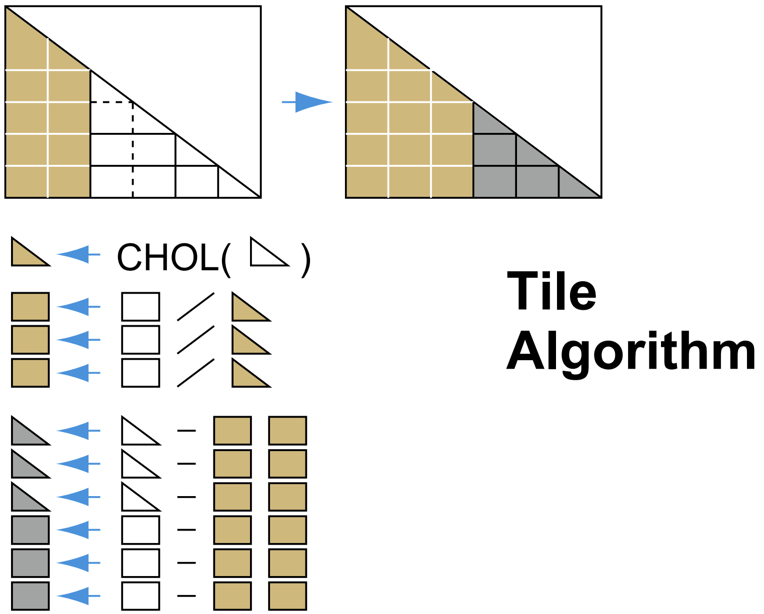


- defines the unit of parallelism
- enables dataflow scheduling
- helps memory efficiency
- simplifies communication

F. Gustavson, L. Karlsson, Bo Kågström, Parallel and Cache-Efficient In-Place Matrix Storage Format Conversion, ACM Transactions on Mathematical Software (TOMS), 38(3), Article No. 17, 2012. DOI: 10.1145/2168773.2168775

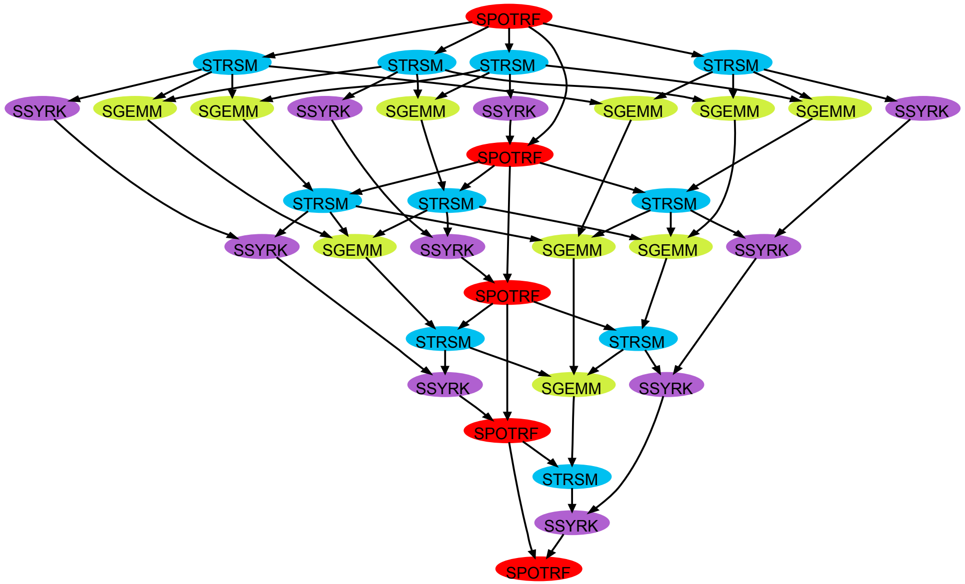
PLASMA Tile Algorithms





H. Bouwmeester,
Tiled Algorithms for Matrix Computations on Multicore Architectures,
PhD dissertation, University of Colorado Denver, 2012.
arxiv.org/abs/1303.3182

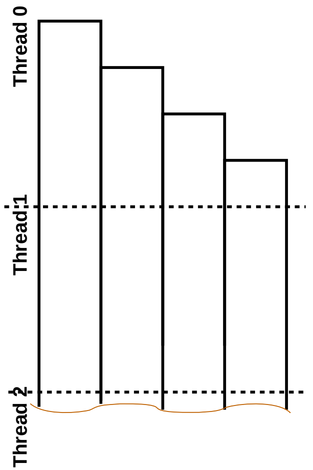
PLASMA and Dataflow Scheduling

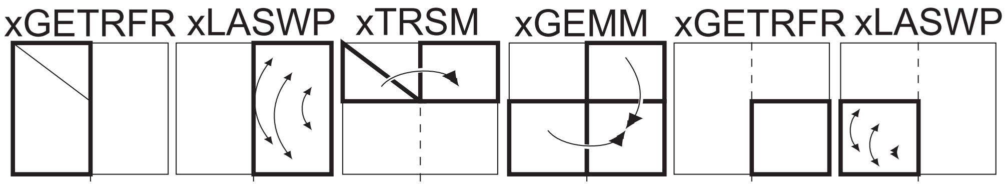


- exploit parallelism
- balance load
- maximize data locality

J. Kurzak, H. Ltaief, J. Dongarra, R.M. Badia, Scheduling dense linear algebra operations on multicore processors, Concurrency and Computation: Practice and Experience, 22(1):15-44, 2010. DOI: 10.1002/cpe.1467

PLASMA Algorithms: LU Factorization





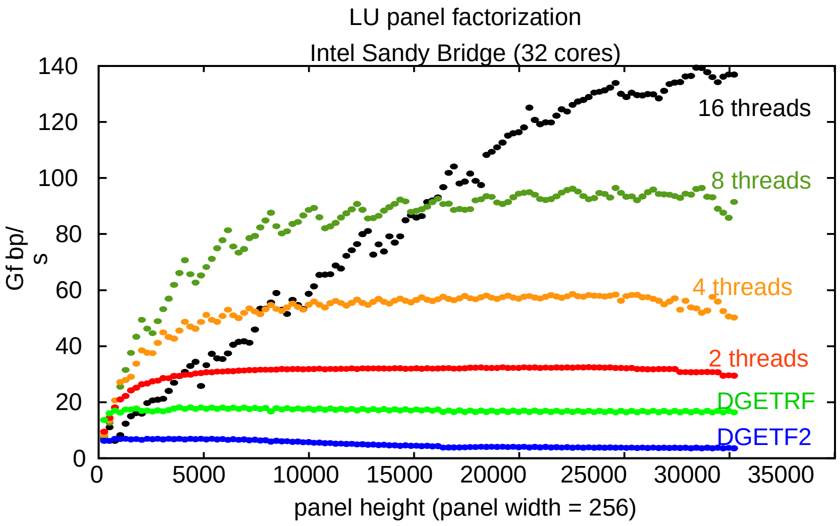
<u>Fast & scalable panel factorization</u>

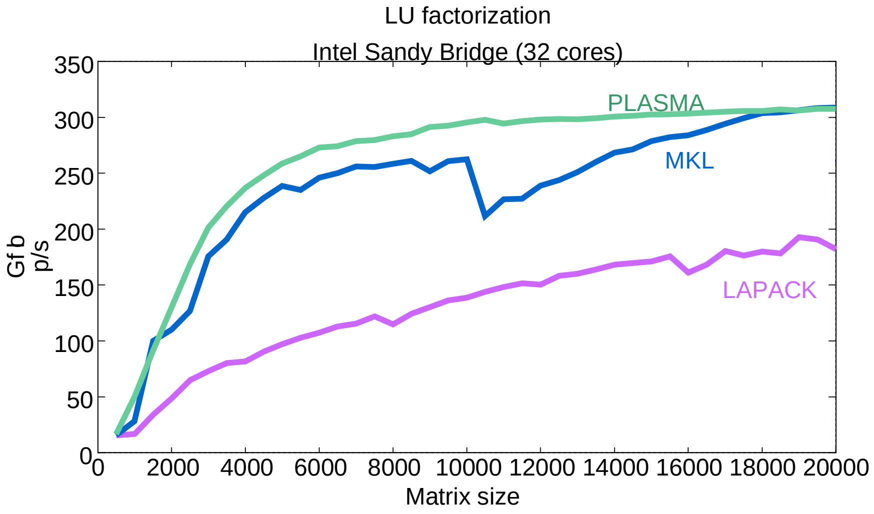
- multithreaded
- cache resident
- recursive

A.M. Castaldo, R.C. Whaley, S. Samuel, Scaling LAPACK panel operations using parallel cache assignment, ACM Transactions on Mathematical Software (TOMS), 39(4), Article No. 22, 2013. DOI: 10.1145/2491491.2491492

J. Dongarra, M. Faverge, H. Ltaief, P. Luszczek, Achieving numerical accuracy and high performance using recursive tile LU factorization with partial pivoting, Concurrency and Computation: Practice and Experience, 26(7):1408-1431, 2014. DOI: 10.1002/cpe.3110

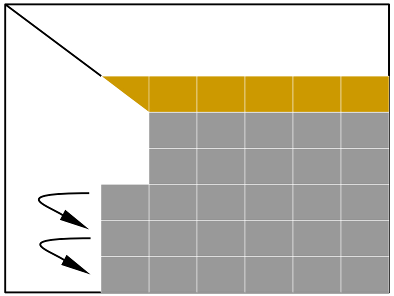
PLASMA Algorithms: LU Performance



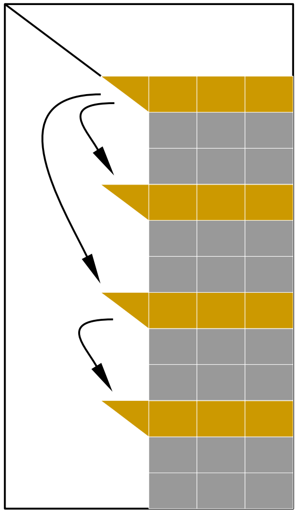


S. Donfack, J. Dongarra, M. Faverge, M. Gates, J. Kurzak, P. Luszczek, I. Yamazaki, A survey of recent developments in parallel implementations of Gaussian elimination, Concurrency and Computation: Practice and Experience, 27(5):1292–1309, 2015. DOI: 10.1002/cpe.3110

PLASMA – Algorithms – QR



Tile QR
(incremental)



TSQR / CAQR
(tree reduction)

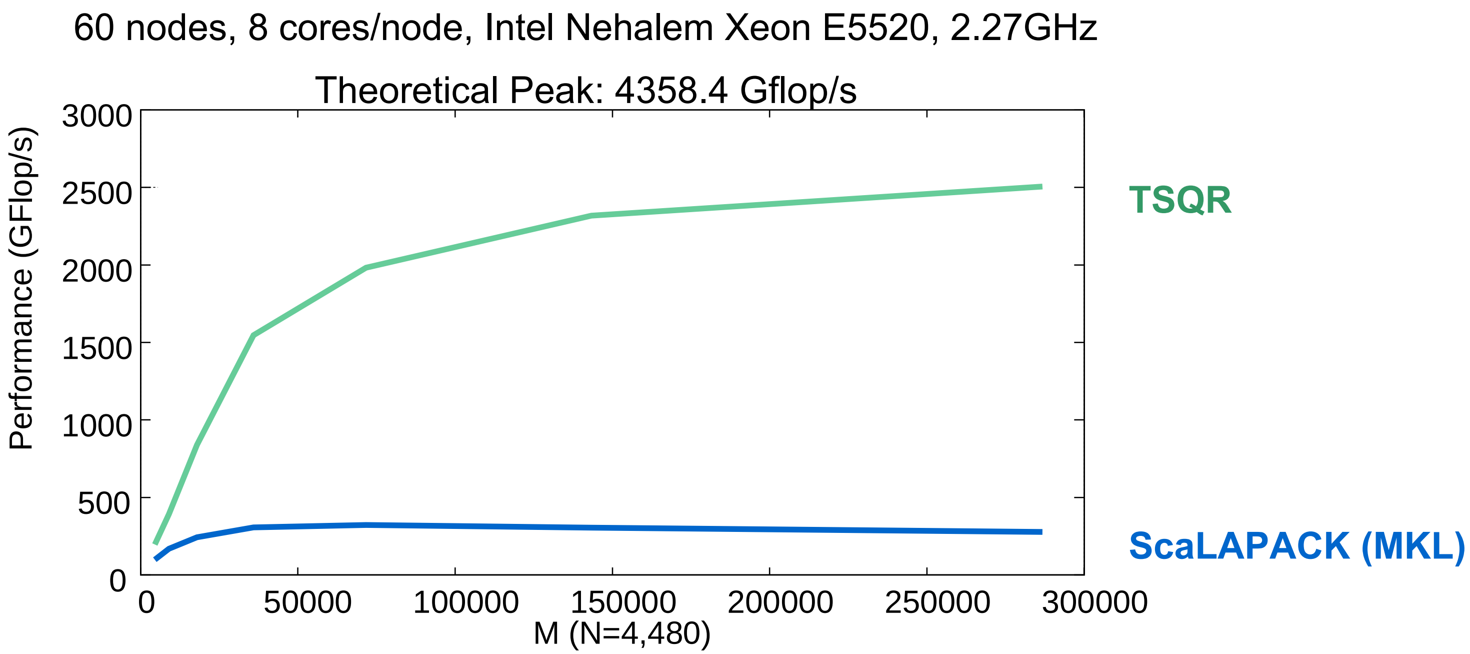
Tile QR

- great for square matrices
- great for multicore processors

TSQR / CAQR

- great for tall and skinny matrices
- great for distributed memory

PLASMA – Algorithms – QR – Performance



PLASMA QR Algorithms' References

J. Demmel, L. Grigori, M. Hoemmen, J. Langou, Communication-optimal Parallel and Sequential QR and LU Factorizations, SIAM Journal on Scientific Computing, 34(1):A206-A239, 2012. DOI: 10.1137/080731992

J. Dongarra, M. Faverge, T. Herault, M. Jacquelin, J. Langou, Y. Robert, Hierarchical QR factorization algorithms for multi-core clusters, 39(4-5):212–232, 2013. DOI: 10.1016/j.parco.2013.01.003

H. Bouwmeester, M. Jacquelin, J. Langou, Y. Robert, Tiled QR factorization algorithms, Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis (SC), 2011. arxiv.org/abs/1104.4475

B. Hadri, H. Ltaief, E. Agullo, J. Dongarra, Tile QR factorization with parallel panel processing for multicore architectures. Proceedings of the International Parallel & Distributed Processing Symposium (IPDPS), 2010. DOI: 10.1109/IPDPS.2010.5470443

J. Demmel, L. Grigori, M. Hoemmen, J. Langou, Communication-avoiding parallel and sequential QR factorizations, Technical Report No. UCB/EECS-2008-74, University of California Berkeley, 2008. digitalassets.lib.berkeley.edu/techreports/ucb/text/EECS-2008-74.pdf

PLASMA Algorithms' References

A.M. Castaldo, R.C. Whaley, S. Samuel, *Scaling LAPACK panel operations using parallel cache assignment*, ACM Transactions on Mathematical Software (TOMS), 39(4), Article No. 22, 2013. DOI: 10.1145/2491491.2491492

J. Dongarra, M. Faverge, H. Ltaief, P. Luszczek, *Achieving numerical accuracy and high performance using recursive tile LU factorization with partial pivoting*, Concurrency and Computation: Practice and Experience, 26(7):1408-1431, 2014. DOI: 10.1002/cpe.3110

S. Donfack, J. Dongarra, M. Faverge, M. Gates, J. Kurzak, P. Luszczek, I. Yamazaki, A survey of recent developments in parallel implementations of Gaussian elimination, Concurrency and Computation: Practice and Experience, 27(5):1292–1309, 2015. DOI: 10.1002/cpe.3110

J. Demmel, L. Grigori, M. Hoemmen, J. Langou, Communication-optimal Parallel and Sequential QR and LU Factorizations, SIAM Journal on Scientific Computing,
34(1):A206-A239, 2012. DOI: 10.1137/080731992

J. Dongarra, M. Faverge, T. Herault, M. Jacquelin, J. Langou, Y. Robert, Hierarchical QR factorization algorithms for multi-core clusters, 39(4-5):212–232, 2013. DOI: 10.1016/j.parco.2013.01.003

H. Bouwmeester, M. Jacquelin, J. Langou, Y. Robert, Tiled QR factorization algorithms, Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis (SC), 2011. arxiv.org/abs/1104.4475

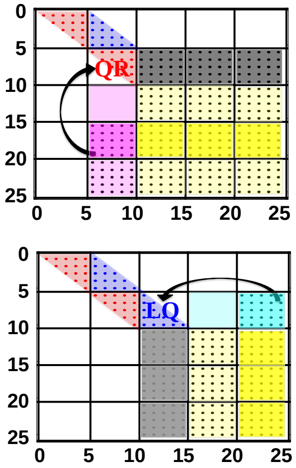
B. Hadri, H. Ltaief, E. Agullo, J. Dongarra, Tile QR factorization with parallel panel processing for multicore architectures. Proceedings of the International Parallel & Distributed Processing Symposium (IPDPS), 2010. DOI: 10.1109/IPDPS.2010.5470443

J. Demmel, L. Grigori, M. Hoemmen, J. Langou, Communication-avoiding parallel and sequential QR factorizations, Technical Report No. UCB/EECS-2008-74, University of California Berkeley, 2008. digitalassets.lib.berkeley.edu/techreports/ucb/text/EECS-2008-74.pdf

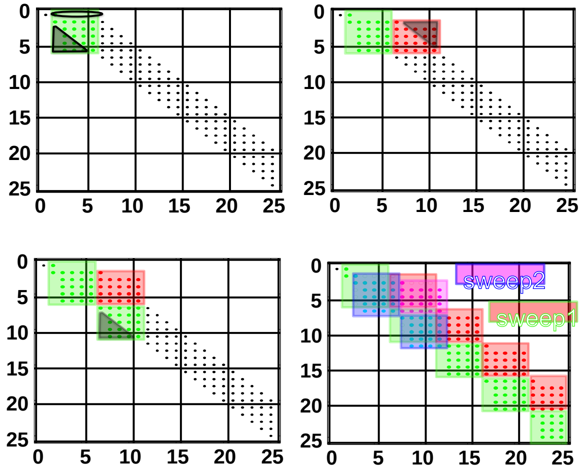
A. Haidar, J. Kurzak, P. Luszczek, An improved parallel singular value algorithm and its implementation for multicore hardware, Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis (SC), Article No. 90, 2013. DOI: 10.1145/2503210.2503292

G. Pichon, A. Haidar, M. Faverge, J. Kurzak, Divide and Conquer Symmetric Tridiagonal Eigensolver for Multicore Architectures, Proceedings of the International Parallel and Distributed Processing Symposium (IPDPS), pages 51-60, 2015. DOI: 10.1109/IPDPS.2015.51

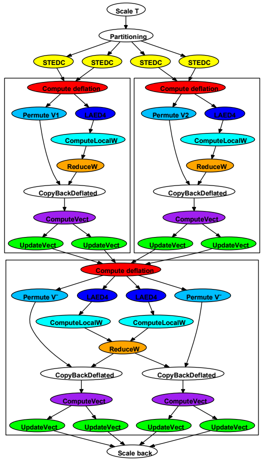
PLASMA – Algorithms – SVD/EVP (symmetric)



reduction to band
parallel & cache efficient
tile algorithm

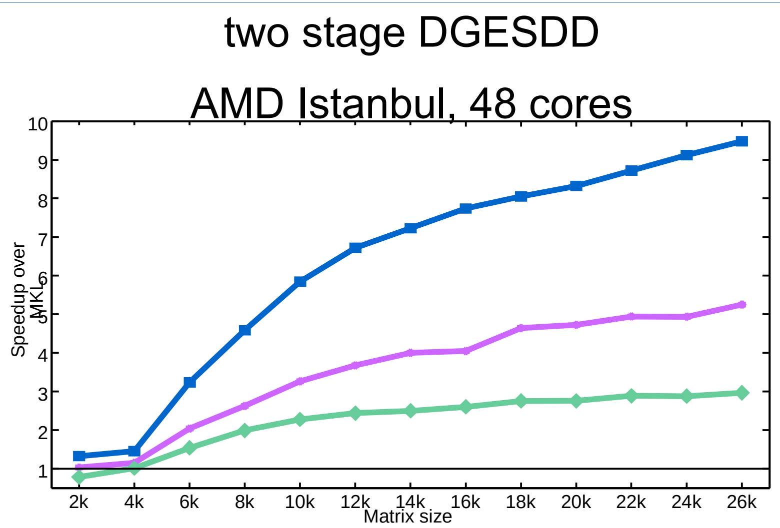


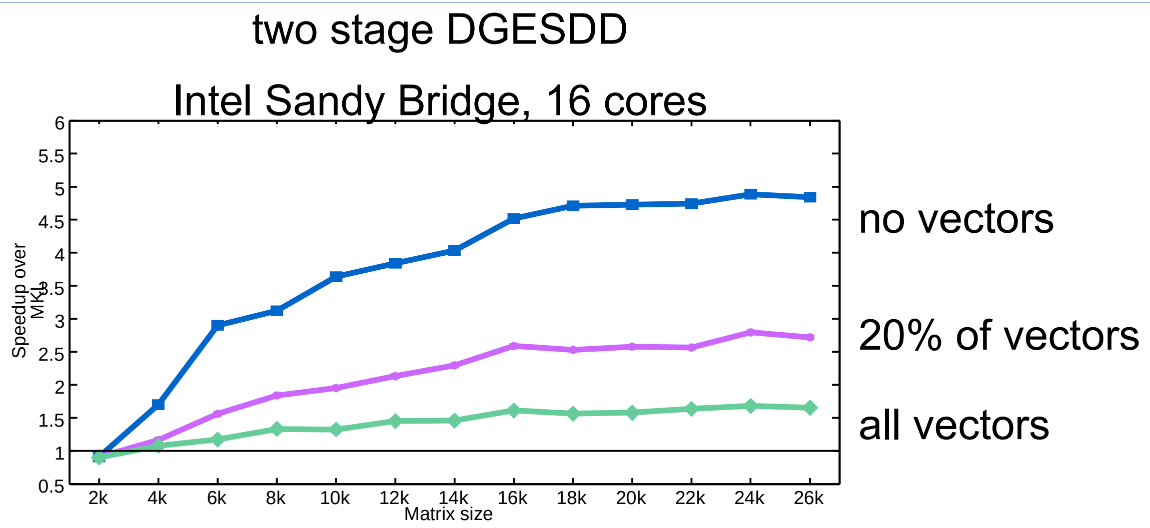
band reduction
parallel & cache efficient
a flavor of communication avoiding



divide and conquer
task-based
dataflow

PLASMA – Algorithms –SVD/EVP – Performance





A. Haidar, J. Kurzak, P. Luszczek, An improved parallel singular value algorithm and its implementation for multicore hardware, Proceedings of the International Conference on High Performance Computing, Networking, Storage and Analysis (SC), Article No. 90, 2013. DOI: 10.1145/2503210.2503292

G. Pichon, A. Haidar, M. Faverge, J. Kurzak, Divide and Conquer Symmetric Tridiagonal Eigensolver for Multicore Architectures, Proceedings of the International Parallel and Distributed Processing Symposium (IPDPS), pages 51-60, 2015. DOI: 10.1109/IPDPS.2015.51

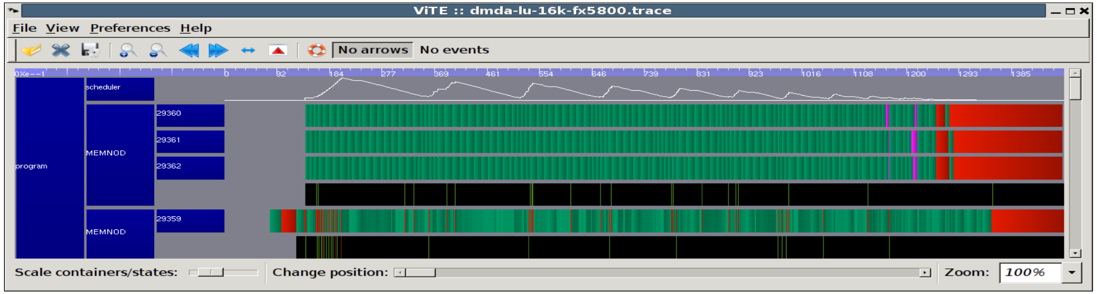
Dynamic Scheduling Projects

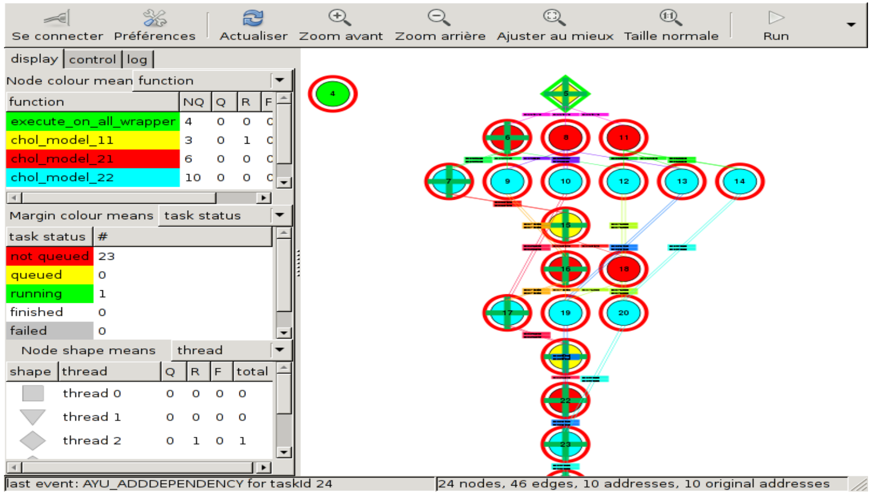
• Jade	Stanford University
• Legion	Stanford University
• SMPSs / OMPSs	Barcelona Supercomputer Center
• StarPU	INRIA Bordeaux
• PaRSEC	University of Tennessee
• QUARK	University of Tennessee
• SuperGlue & DuctTEiP	Uppsala University

http://suif.stanford.edu/jade.html
https://pm.bsc.es/ompss
http://starpu.gforge.inria.fr
http://icl.utk.edu/parsec/
http://icl.cs.utk.edu/quark/
https://github.com/tillenius/superglue

Dynamic Scheduling – Tools

- DAG visualization
- tracing
- MPI support
- accelerator support
- simulation





Dynamic Scheduling – References

M. C. Rinard, M. S. Lam, The design, implementation, and evaluation of Jade, ACM Transactions on Programming Languages and Systems,
20(1):1-64, 1998. DOI: 10.1145/291889.291893

A. Duran, E. Ayguadé, R. M. Badia, J. Labarta, L. Martinell, X. Martorell, J. Planas, OmpSs: A proposal for programming heterogeneous multi-core architectures, Parallel Processing Letters, 21(02):173-193, 2011. DOI: 10.1142/S0129626411000151

C. Augonnet, S. Thibault, R. Namyst, P.-A. Wacrenier, StarPU: A unified platform for task scheduling on heterogeneous multicore architectures, Concurrency and Computation: Practice and Experience, Special Issue: Euro-Par 2009, 23(2):187–198, 2011, DOI: 10.1002/cpe.1631

A. YarKhan, Dynamic Task Execution on Shared and Distributed Memory Architectures, PhD dissertation, University of Tennessee, 2012. http://trace.tennessee.edu/utk_graddiss/1575

M. Tillenius, SuperGlue: A shared memory framework using data versioning for dependency-aware task-based parallelization, SIAM Journal on Scientific Computing, 37(6):C617–C642, 2015. DOI:10.1137/140989716

A. Zafari, M. Tillenius, E. Larsson, Programming models based on data versioning for dependency-aware task-based parallelisation, Computational Science and Engineering (CSE), 2012 IEEE 15th International Conference on. IEEE, 2012. DOI: 10.1109/ICCSE.2012.45

PLASMA Cholesky with QUARK

```
for (k = 0; k < nt; k++) {
    QUARK_CORE_dpotrf(
        plasma->quark, &task_flags,
        PlasmaLower,
        nb, nb,
        A(k, k), nb,
        sequence, request, nb*k);

    for (m = k+1; m < nt; m++) {
        QUARK_CORE_dtrsm(
            plasma->quark, &task_flags,
            PlasmaRight, PlasmaLower,
            PlasmaTrans, PlasmaNonUnit,
            nb, nb, nb,
            1.0, A(k, k), nb,
                 A(m, k), nb);
    }
    for (m = k+1; m < nt; m++) {
        QUARK_CORE_dsyrk(
            plasma->quark, &task_flags,
            PlasmaLower, PlasmaNoTrans,
            nb, nb, nb,
            -1.0, A(m, k), nb,
             1.0, A(m, m), nb);

        for (n = k+1; n < m; n++) {
            QUARK_CORE_dgemm(
                plasma->quark, &task_flags,
                PlasmaNoTrans, PlasmaTrans,
                nb, nb, nb, nb,
                -1.0, A(m, k), nb,
                      A(n, k), nb,
                 1.0, A(m, n), nb);
        }
    }
}
```

PLASMA Cholesky with QUARK and Dependent Routines

```
for (k = 0; k < nt; k++) {
  QUARK_CORE_dpotrf(
    plasma->quark, &task_flags,
    PlasmaLower,
    nb, nb,
    A(k, k), nb,
    sequence, request, nb*k);
  for (m = k+1; m < nt; m++) {
    QUARK_CORE_dtrsm(
      plasma->quark, &task_flags,
      PlasmaRight, PlasmaLower,
      PlasmaTrans, PlasmaNonUnit,
      nb, nb, nb,
      1.0, A(k, k), nb,
      A(m, k), nb);
  }
for (m = k+1; m < nt; m++) {
  QUARK_CORE_dsyrk(
    plasma->quark, &task_flags,
    PlasmaLower, PlasmaNoTrans,
    nb, nb, nb,
    -1.0, A(m, k), nb,
    1.0, A(m, m), nb);
    for (n = k+1; n < m; n++) {
      QUARK_CORE_dgemm(
        plasma->quark, &task_flags,
        PlasmaNoTrans, PlasmaTrans,
        nb, nb, nb, nb,
        -1.0, A(m, k), nb, A(n, k), nb,
         1.0, A(m, n), nb);
    }
  }
}
```

```
void QUARK_CORE_dpotrf(
  Quark *quark, Quark_Task_Flags *task_flags,
  PLASMA_enum uplo,
  int n, int nb,
  double *A, int lda,
  PLASMA_sequence *sequence, PLASMA_request *request,
  int iinfo)
{
  QUARK_Insert_Task(
    quark, CORE_dpotrf_quark, task_flags,
    sizeof(PLASMA_enum),  &uplo,  VALUE,
    sizeof(int),  &n,  VALUE,
    sizeof(double)*nb*nb,  A,  INOUT,
    sizeof(int),  &lda,  VALUE,
    sizeof(PLASMA_sequence*),  &sequence, VALUE,
    sizeof(PLASMA_request*),  &request,  VALUE,
    sizeof(int),  &iinfo,  VALUE,
    0);
}
```

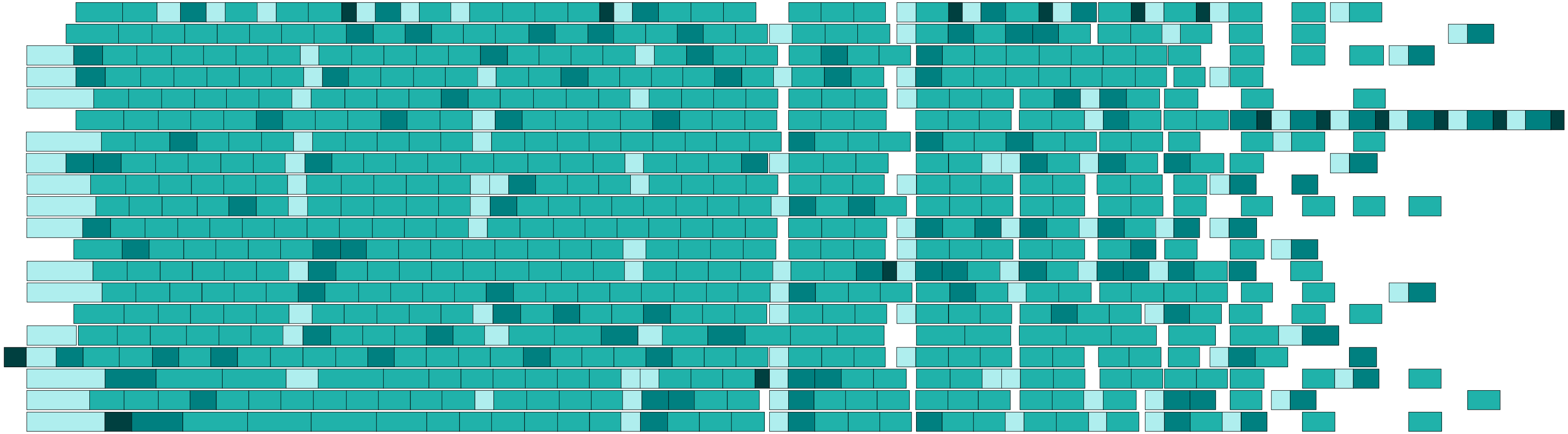
```
void CORE_dpotrf_quark(Quark *quark) {
  PLASMA_enum uplo;
  int n;
  double *A;
  int lda;
  PLASMA_sequence *sequence;
  PLASMA_request *request;
  int iinfo, info;
  quark_unpack_args_7(quark, uplo, n, A, lda, sequence, request, iinfo)
  info = LAPACKE_dpotrf_work(
  LAPACK_COL_MAJOR,
  lapack_const(uplo),
  n, A, lda);
  if (sequence->status == PLASMA_SUCCESS && info != 0)
  plasma_sequence_flush(quark, sequence, request, iinfo+info);
}
```

PLASMA Cholesky with OpenMP

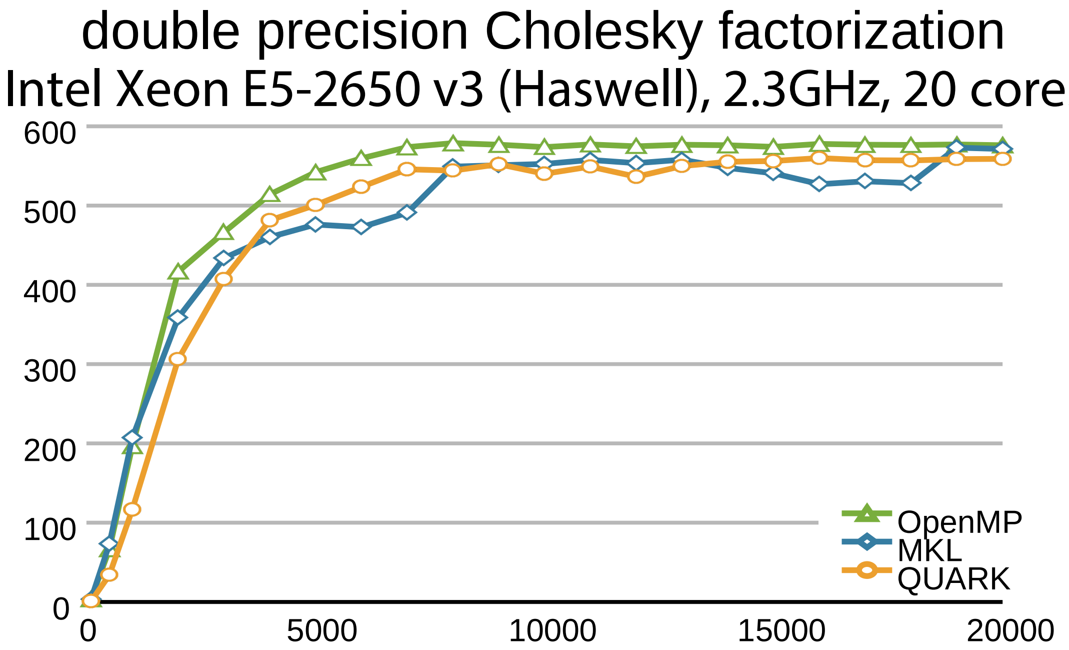
```
#pragma omp parallel
#pragma omp master
{
  for (k = 0; k < nt; k++) {
    #pragma omp task depend(inout:A(k,k)[0:nb*nb])
    info = LAPACKE_dpotrf_work(
      LAPACK_COL_MAJOR,
      lapack_const(PlasmaLower),
      nb, A(k,k), nb);
    for (m = k+1; m < nt; m++) {
  #pragma omp task depend(in:A(k,k)[0:nb*nb]) depend(inout:A(m,k)[0:nb*nb])
      cblas_dtrsm(CblasColMajor,
        CblasRight, CblasLower, CblasTrans, CblasNonUnit,
        nb, nb,
        1.0, A(k,k), nb,
        A(m,k), nb);
    }
    for (m = k+1; m < nt; m++) {
      #pragma omp task depend(in:A(m,k)[0:nb*nb]) depend(inout:A(m,m)[0:nb*nb])
      cblas_dsyrk( CblasColMajor,
        CblasLower, CblasNoTrans,
        nb, nb,
        -1.0, A(m,k), nb,
        1.0, A(m,m), nb);
      for (n = k+1; n < m; n++) {
        #pragma omp task depend(in:A(m,k)[0:nb*nb]) \
        depend(in:A(n,k)[0:nb*nb]) depend(inout:A(m,n)[0:nb*nb])
        cblas_dgemm( CblasColMajor,
          CblasNoTrans, CblasTrans,
          nb, nb, nb,
          -1.0, A(m,k), nb,
          A(n,k), nb, 1.0, A(m,n), nb);
      }
    }
  }
}
```

PLASMA OpenMP Trace

PLASMA Cholesky factorization using OpenMP
Intel Xeon E5-2650 v3 (Haswell) 2.3GHz 20 cores
tiles of size 224 x 224, matrix of size 20 x 20 tiles (4480 x 4480)

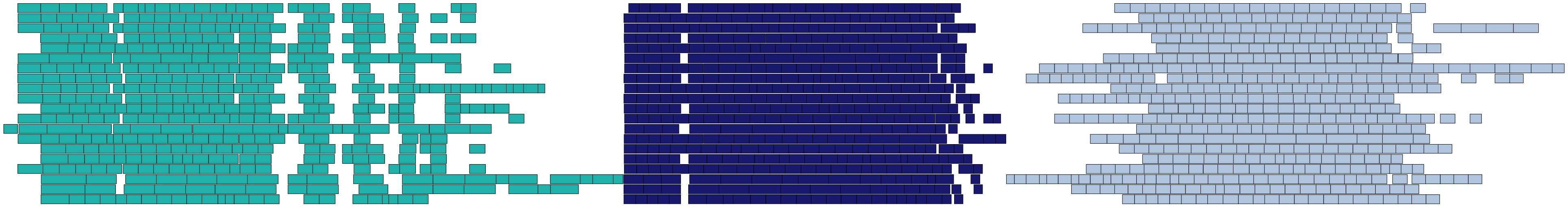


PLASMA OpenMP Cholesky Performance



PLASMA OpenMP Cholesky Inversion Trace

PLASMA Cholesky inversion using OpenMP
Intel Xeon E5-2650 v3 (Haswell) 2.3GHz 20 cores
tiles of size 224 x 224, matrix of size 13 x 13 tiles (2912 x 2912)

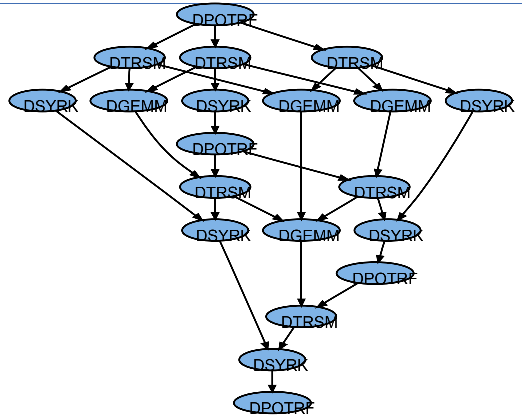


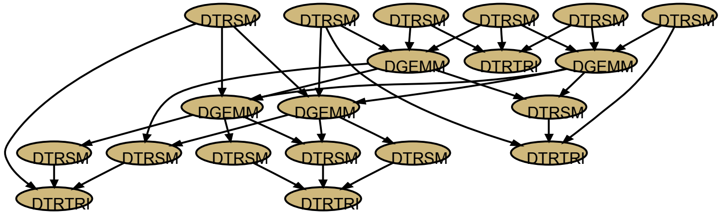
```
plasma_dpotrf(uplo, n, pA, lda);
plasma_dlauum(uplo, n, pA, lda);
plasma_dtrtri(uplo, diag, n, pA, lda);
```

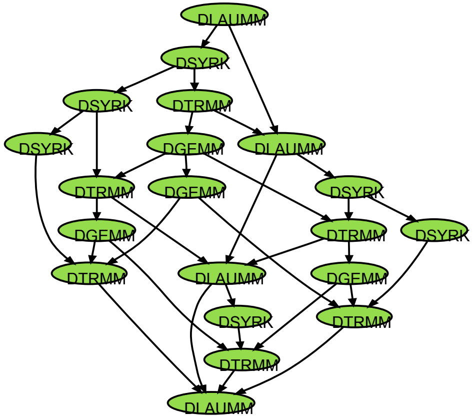
PLASMA OpenMP Cholesky Inversion Code

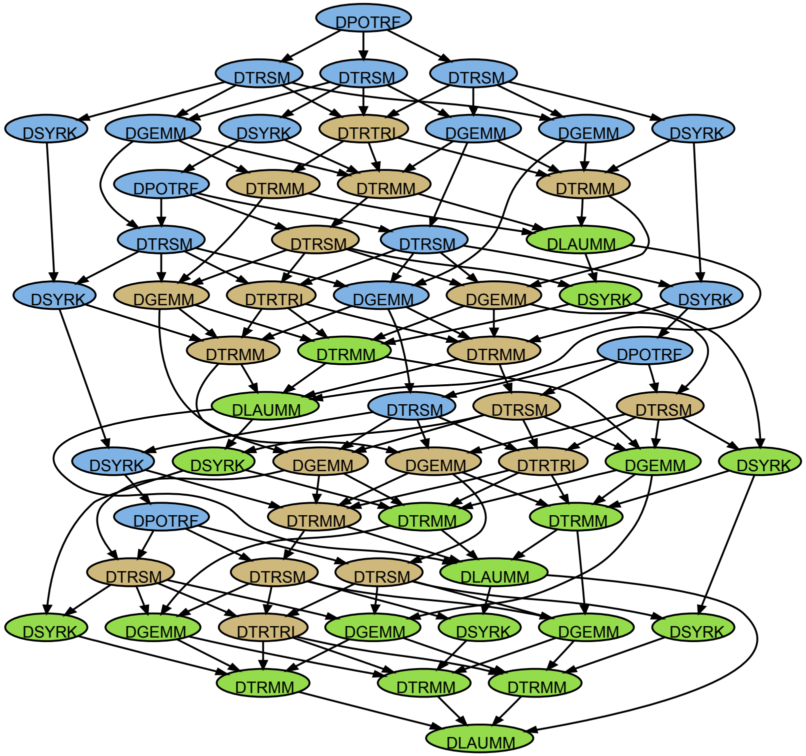
```
#pragma omp parallel
#pragma omp master
{
    plasma_omp_zge2desc(pA, lda, A, sequence, &request);

    plasma_omp_dpotrf(uplo, A, sequence, &request);
    plasma_omp_zlauum(uplo, A, sequence, &request);
    plasma_omp_ztrtri(uplo, diag, A, sequence, &request);

    plasma_omp_zdesc2ge(A, pA, lda, sequence, &request);
}
```

PLASMA OpenMP Cholesky Inversion DAG



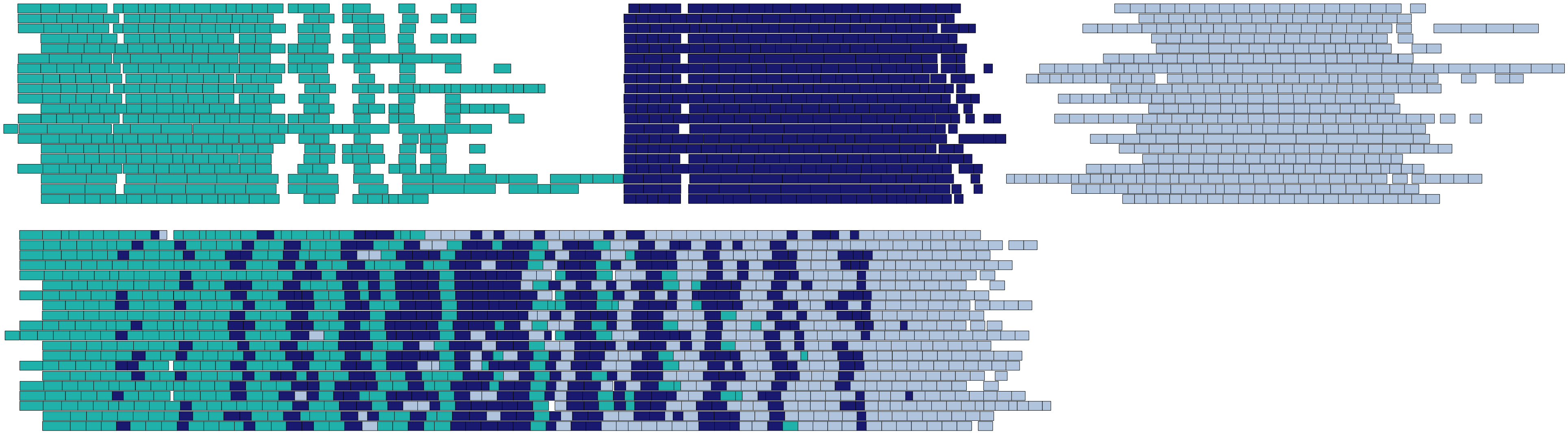




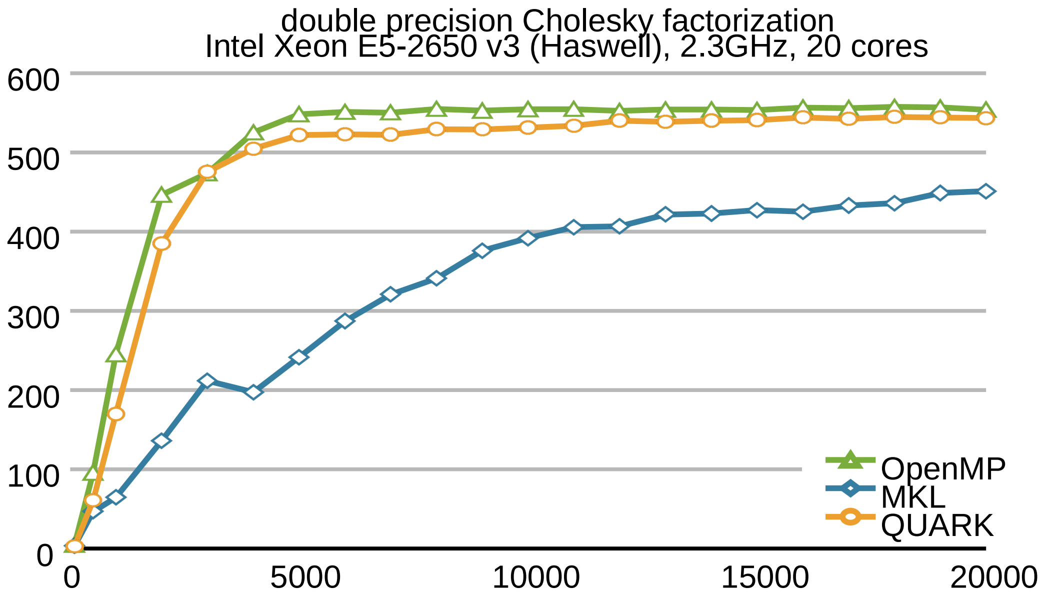


PLASMA OpenMP Cholesky Inversion Traces

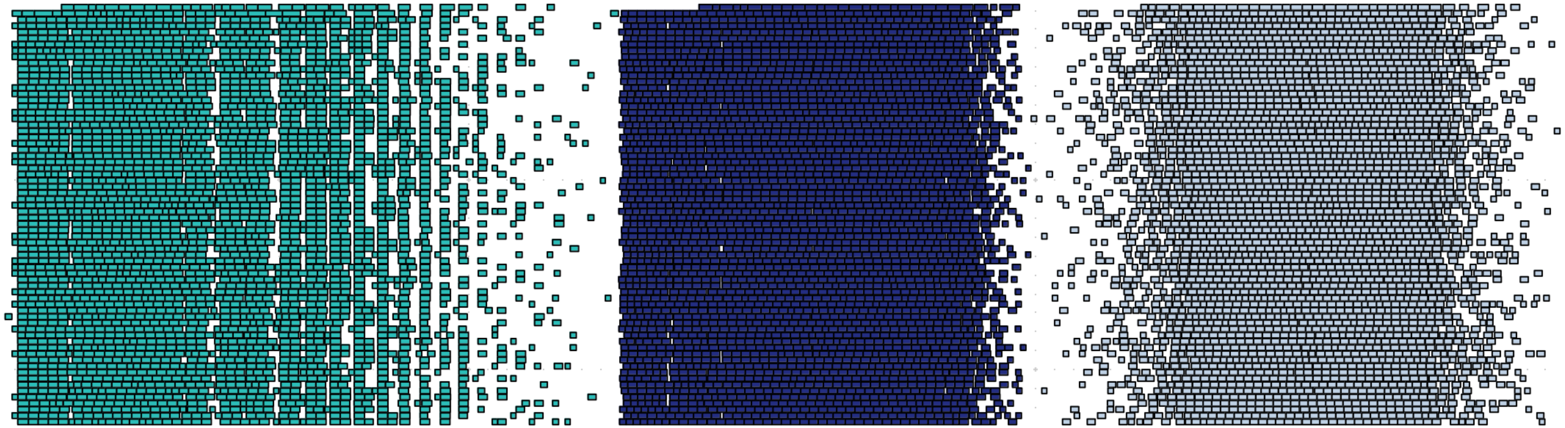
PLASMA Cholesky inversion using OpenMP
Intel Xeon E5-2650 v3 (Haswell) 2.3GHz 20 cores
tiles of size 224 x 224, matrix of size 13 x 13 tiles (2912 x 2912)



PLASMA OpenMP Cholesky Factorization Performance



PLASMA Traces When Routines Are Merged on KNL



Intel Xeon Phi, Knights Landing, 68 cores, 1.4 GHz

N = 7200
NB = 288
(25x25 tiles)

sync:
0.485 sec
770 Gflop/s

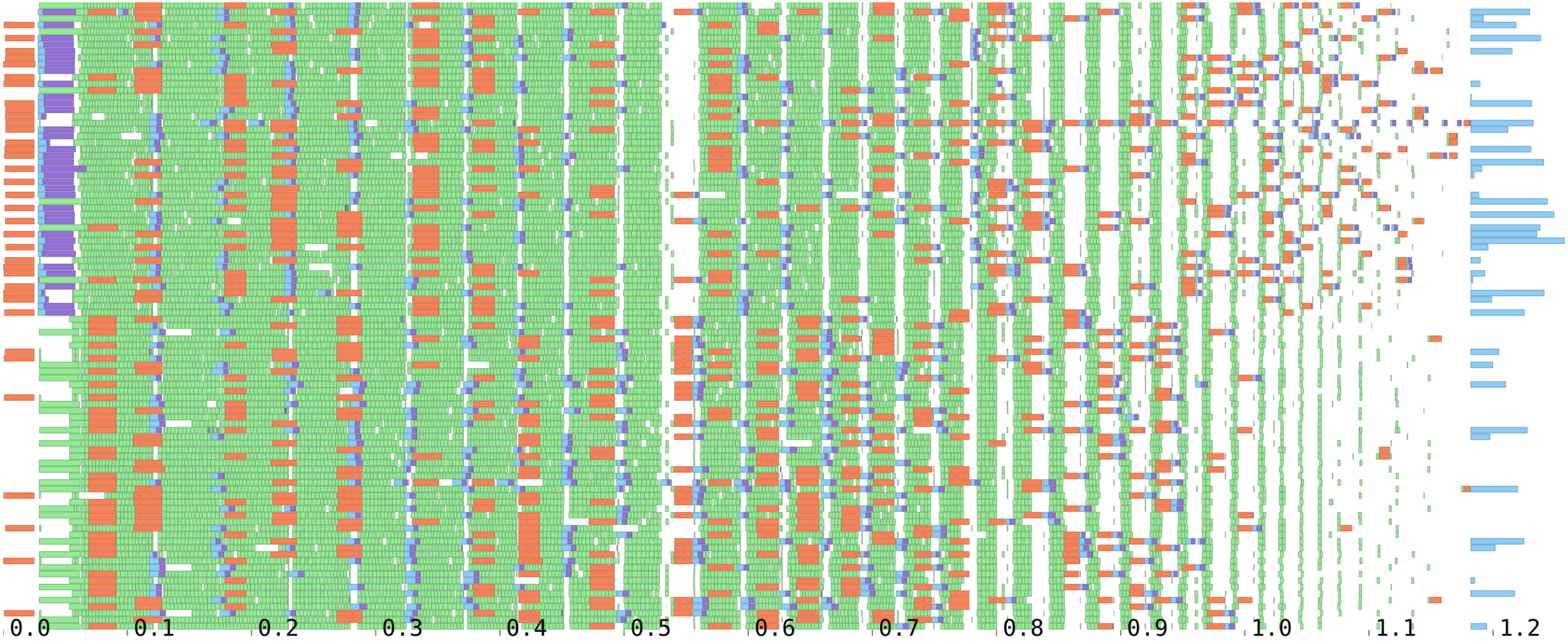
async:
0.373 sec
1001 Gflop/s

PLASMA 17 Documents

- LAPACK Working Notes
  - http://www.netlib.org/lapack/lawns/downloads/
- LAWN 293: PLASMA 17.1 Functionality Report
  - http://www.netlib.org/lapack/lawnspdf/lawn293.pdf
- LAWN 292: PLASMA 17 Performance Report
  - http://www.netlib.org/lapack/lawnspdf/lawn292.pdf

PLASMA on 96 ARM Cores: LU

LU factorization on ARMv8 (Cavium ThunderX) – 5K × 5K matrix, tile size of 128



PLASMA on 96 ARM Cores: QR

QR factorization on ARMv8 (Cavium ThunderX) – 5K × 5K matrix, tile size of 128