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MODULE 1

Exercise:01

1.LEAST SQUARE METHOD

Objective:

To fit the given data line/curve using least square method

Input:

No.of countries	1	2	3	4	5	6	7	8	9	10	11
Literacy rate	99	97	95	91	88	87	86	80	74	73	62

Procedure/Methodology:

x	y	u=x-6	v=y-87	u^2	u^3	u^4	u^2v	uv
1	99	-5	12	25	-125	625	300	-60
2	97	-4	10	16	-64	256	160	-40
3	95	-3	8	9	-27	81	72	-24
4	91	-2	4	4	-8	16	16	-8
5	88	-1	1	1	-1	1	1	-1
6	87	0	0	0	0	0	0	0
7	86	1	-1	1	1	1	-1	-1
8	80	2	-7	4	8	16	-28	-14
9	74	3	-13	9	27	81	-117	-39
10	73	4	-14	16	64	256	-224	-56
11	62	5	-25	25	125	625	-625	-125
		Σu=0	Σv=-25	Σu^2=110	Σu^3=0	Σu^4=1958	Σu^2v=-446	Σuv=-368

Line fitting:

$$\Sigma v = a\Sigma u + bn$$
$$\Sigma uv = a\Sigma u^2 + b\Sigma u$$

$$-25 = 0a + 11b \quad (1)$$
$$-368 = 110a + 0b \quad (2)$$

Solving (1) and (2)
(1)$\Longrightarrow$

-25=11b

b=$\frac{-25}{11}$

$$\boxed{\mathbf{b = -2.2727}}$$

(2)$\Longrightarrow$

-368=110a

a=$\frac{-368}{110}$

$$\boxed{\mathbf{a = -3.3454}}$$

v=au+b

y-87=-3.3454(x-6)+(-2.2727)

y-87=-3.3454x+20.0724-2.2727

y=-3.3454x+104.7997

$$\boxed{\mathbf{3.3454x + y = 104.7997}}$$

Curve fitting:

$$\Sigma v = a\Sigma u^2 + b\Sigma u + cn$$
$$\Sigma uv = a\Sigma u^3 + b\Sigma u^2 + c\Sigma u$$
$$\Sigma u^2 v = a\Sigma u^4 + b\Sigma u^3 + c\Sigma u^2$$

$$-25 = 110a + 0b + 11c \quad (3)$$
$$-368 = 0a + 110b + 0c \quad (4)$$
$$-446 = 1958a + 0b + 110c \quad (5)$$

Solving (3),(4) and (5)

From (3) and (5) $\Longrightarrow$

$$110a+11c=-25$$
$$1958a+110c=-446$$

(3)X10 $\Longrightarrow$

$$1100a+110c=-250$$
$$1958a+110c=-446$$

$$-858a=196$$
$$a=-\frac{196}{858}$$
$$\boxed{\mathbf{a=-0.2284}}$$

From (4) $\Longrightarrow$

$$110b=-368$$
$$b=-\frac{368}{110}$$
$$\boxed{\mathbf{b=-3.3455}}$$

Substitue the value of a and b in (3)

$$110a+0b+11c=-25$$
$$110a+11c=-25$$
$$110(-0.2284)+11c=-25$$
$$11c=-25+25.124$$
$$\boxed{\mathbf{c=0.0113}}$$

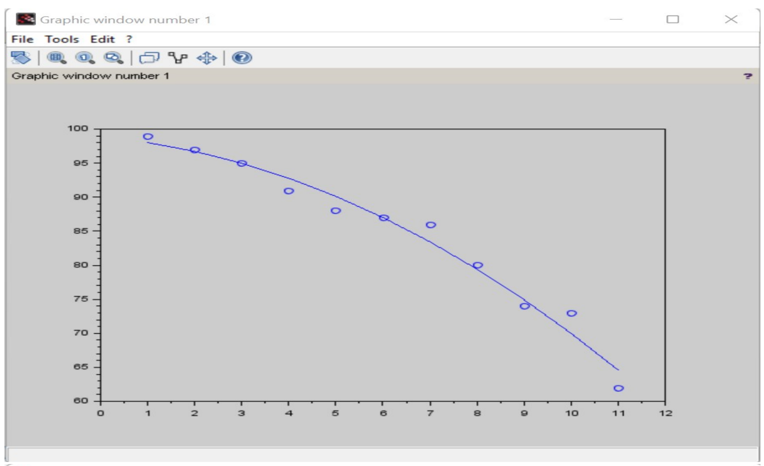
$$v=au^2+bu+c$$
$$y-87=-0.2284(x-6)^2+(-3.3455)(x-6)+0.0113$$
$$y-87=-0.2284(x^2+36-12x)-3.3455x+20.0730+0.0113$$
$$y-87=-0.2284x^2-8.2224+2.7408x-3.3455x+20.0843$$
$$\boxed{\mathbf{y=-0.2284x^2-0.6047x+98.8619}}$$

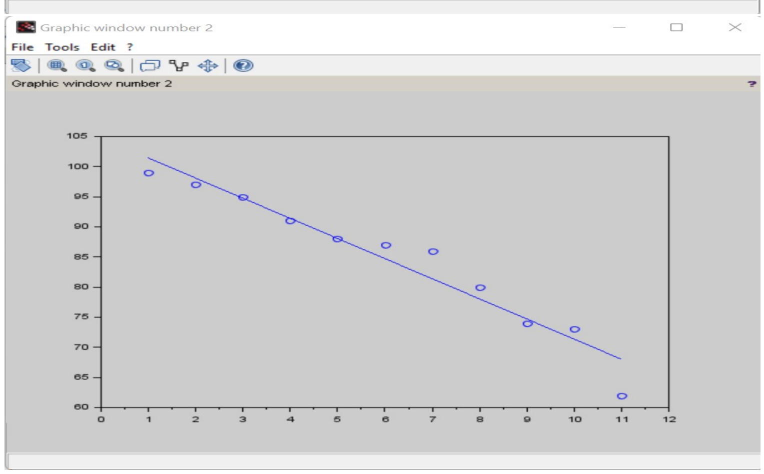
Coding/Programming:

```
clc; clear all;
x=[1:1:11];
y=[99,97,95,91,88,87,86,80,74,73,62];
n=length(x);

sx=sum(x);
sy=sum(y);
sx4=sum(x.^4);
sx3=sum(x.^3);
sx2=sum(x.^2);
sxy=sum(x.*y);
sx2y=sum((x.^2).*y);
//
disp("Curve Fitting")

//sy=a1*sx2+b1*sx+c1*n;
//sxy=a1*sx3+b1*sx2+c1*sx;
//sx2y=a1*sx4+b1*sx3+c1*sx2;
A=[sx2,sx,n;sx3,sx2,sx;sx4,sx3,sx2];
B=[sy,sxy,sx2y]';
//X=[a1,b1,c1]';
R1=inv(A);
X=R1*B;
disp(A,'A=')
disp(B,'B=')
disp(R1,'R1=')
a1=X(1); b1=X(2); c1=X(3);
disp(a1,'a1=')
disp(b1,'b1=')
disp(c1,'c1=')
printf("Equation : y = (%.4f).x^2 + (%.4f).x +(%.4f)",a1,b1,c1);
figure(1);
y1=a1*x^2+b1*x+c1
scatter(x,y)
plot(x,y1)
//
disp("Line Fitting")
//sy=a2*sx+b2*n;
//sxy=a2*sx2+b2*sx;
A1=[sx,n;sx2,sx];
B1=[sy,sxy]';
```

```
//X=[a2,b2]';
R2=inv(A1);
X1=R2*B1;
disp(A1,'A1=')
disp(B1,'B1=')
disp(R2,'R2=')
a2=X1(1); b2=X1(2);
disp(a2,'a2=')
disp(b2,'b2=')
printf("Equation : y = (%.4f).x + (%.4f)",a2,b2);
figure(2);
y2=a2*x+b2
scatter(x,y)
plot(x,y2)
```

Output:





Exercise:2

2(A)BISECTION METHOD

Objective:

To find roots of the given equation using Bisection method

Input:

x^3+x^2+$tanx$-e^x=0

f(0)=-1 (-ve)

f(1)=0.839 (+ve)

Procedure/Methodology:

Iteration 1:

Here f(0) $= -1 < 0$ and f(1) $= 0.8391 > 0$

$\therefore$Root lies between 0 and 1

$x_0 = \frac{0+1}{2}$

=0.5

$f(x_0)$= $(0.5)^3 + (0.5)^2 + tan(0.5) - e^{0.5}$

$$\boxed{\mathbf{f(x_0) = -0.7274 < 0}}$$

Iteration 2:

Here f(0.5)= $-0.7274 < 0$ and f(1)= $0.8391 > 0$

$\therefore$Root lies between 0.5 and 1

x_1=$\frac{0.5+1}{2}$

= 0.75

$f(x_1)$=$(0.75)^3 + (0.75)^2 + tan(0.75) - e^{0.75}$

$$\boxed{\mathbf{f(x_1) = -0.201 < 0}}$$

Iteration 3:

Here f(0.75)= $-0.201 < 0$ and f(1)= $0.8391 > 0$

$\therefore$Root lies between 0.75 and 1

$$x_2 = \frac{0.75+1}{2}$$

$$= 0.875$$

$$f(x_2) = (0.875)^3 + (0.875)^2 + tan(0.875) - e^{0.875}$$

$$\boxed{\mathbf{f(x_2) = -0.234 < 0}}$$

Iteration 4:

Here f(0.75)= $-0.201 < 0$ and f(0.875)= $0.234 > 0$

$\therefore$Root lies between 0.75 and 0.875

$$x_3 = \frac{0.75+0.875}{2}$$

$$= 0.812$$

$$f(x_3) = (0.812)^3 + (0.812)^2 + tan(0.812) - e^{0.812}$$

$$\boxed{\mathbf{f(x_3) = -0.001 < 0}}$$

Iteration 5:

Here f(0.812)= $-0.001 < 0$ and f(0.875)= $0.234 > 0$

$\therefore$Root lies between 0.812 and 0.875

$$x_4 = \frac{0.812+0.875}{2}$$

$$= 0.844$$

$$f(x_4) = (0.844)^3 + (0.844)^2 + tan(0.844) - e^{0.844}$$

$$\boxed{\mathbf{f(x_5) = 0.112 > 0}}$$

Iteration 6:

Here f(0.812)= $-0.001 < 0$ and f(0.844)= $0.112 > 0$

$\therefore$Root lies between 0.812 and 0.844

$$x_5 = \frac{0.812+0.844}{2}$$

$$= 0.828$$

$$f(x_5) = (0.828)^3 + (0.828)^2 + tan(0.828) - e^{0.828}$$

$$\boxed{\mathbf{f(x_5) = 0.054 > 0}}$$

Iteration 7:

Here f(0.812)= $-0.001 < 0$ and f(0.828)= $0.054 > 0$

$\therefore$Root lies between 0.812 and 0.828

$$x_6 = \frac{0.812+0.828}{2}$$

$$= 0.82$$

$$f(x_6) = (0.82)^3 + (0.82)^2 + tan(0.82) - e^{0.82}$$

$$\boxed{\mathbf{f(x_6) = 0.026 > 0}}$$

Iteration 8:

Here f(0.812)= $-0.001 < 0$ and f(0.82)= $0.026 > 0$

$\therefore$Root lies between 0.812 and 0.82

$$x_7 = \frac{0.812+0.82}{2}$$

$$= 0.816$$

$$f(x_7) = (0.816)^3 + (0.816)^2 + tan(0.816) - e^{0.816}$$

$$\boxed{\mathbf{f(x_7) = 0.012 > 0}}$$

Iteration 9:

Here f(0.812)= $-0.001 < 0$ and f(0.816)= $0.012 > 0$

$\therefore$Root lies between 0.812 and 0.816

$$x_8 = \frac{0.812+0.816}{2}$$

$$= 0.814$$

$$f(x_8) = (0.814)^3 + (0.814)^2 + tan(0.814) - e^{0.814}$$

$$\boxed{\mathbf{f(x_8) = 0.006 > 0}}$$

Iteration 10:

Here f(0.812)= $-0.001 < 0$ and f(0.814)= $0.006 > 0$

$\therefore$Root lies between 0.812 and 0.814

$$x_9 = 0.812 + 0.8142$$

$$= 0.813$$

$$f(x_9) = (0.813)^3 + (0.813)^2 + tan(0.813) - e^{0.813}$$

$$\boxed{\mathbf{f(x_9) = 0.002 > 0}}$$

Iteration 11:

$$\text{Here f(0.812)} = -0.001 < 0 \text{ and f(0.813)} = 0.006 > 0$$

$$\therefore \text{Root lies between 0.812 and 0.813}$$

$$x_{10} = \frac{0.812+0.813}{2}$$

$$= 0.813$$

$$f(x_{10}) = (0.813)^3 + (0.813)^2 + tan(0.813) - e^{0.813}$$

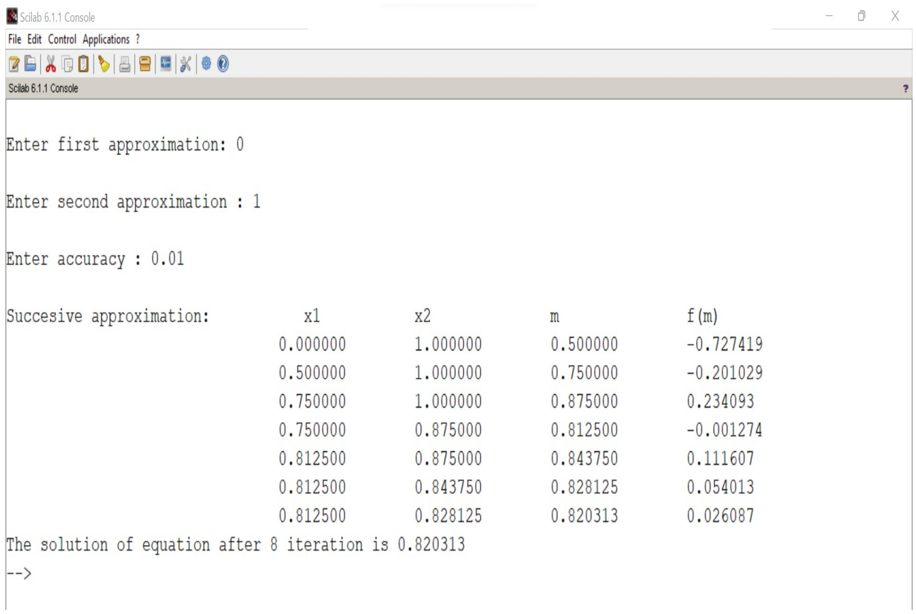
$$\boxed{\mathbf{f(x_{10}) = 0 > 0}}$$

Hence, approximate root of the equation is 0.813

Coding/Programming:

```
clc;clear all;
deff('y=f(x)','y=x^3+x^2+tan(x)-%e^x');
x1=input("Enter first approximation: ");
x2=input("Enter second approximation : ");
d=input("Enter accuracy : ");
c=1;
printf('Succesive approximation: \tx1\\tx2\t\tm\t\tf(m)\n');
while abs(x1-x2)>d
    m=(x1+x2)/2;
    printf('...\t%f\t%f\t%f\t%f\n',x1,x2,m,f(m));
    if f(m)*f(x1)>0
        x1=m;
    else
        x2=m;
    end
    c=c+1;
end
printf('The solution of equation after %i iteration is %g',c,m);
```

Output:



```
Scilab 6.1.1 Console
File  Edit  Control  Applications  ?
Scilab 6.1.1 Console

Enter first approximation: 0

Enter second approximation : 1

Enter accuracy : 0.01

Succesive approximation:          x1          x2          m           f(m)
                              0.000000    1.000000    0.500000    -0.727419
                              0.500000    1.000000    0.750000    -0.201029
                              0.750000    1.000000    0.875000    0.234093
                              0.750000    0.875000    0.812500    -0.001274
                              0.812500    0.875000    0.843750    0.111607
                              0.812500    0.843750    0.828125    0.054013
                              0.812500    0.828125    0.820313    0.026087
The solution of equation after 8 iteration is 0.820313
-->
```

Exercise:3

2(B)REGULAR FALSI METHOD

Objective:

To find roots of the given equation using Regular Falsi method

Input:

$$x^3+x^2+tanx\text{-}e^x=0$$

Procedure/Methodology:

$$\text{f(x)}=x^3+x^2+tanx\text{-}e^x$$

f(0)=-1 (-ve)

f(1)=0.839 (+ve)

Iteration1:

$$\text{Here } f(0) = -1 < 0 \text{ and } f(1) = 0.839 > 0$$

$$\therefore \text{Root lies between } x_0\text{=0 and } x_1\text{=1}$$

$$x_2 = \frac{x_0.f(x_1)-x_1.f(x_0)}{f(x_1)-f(x_0)}$$

$$=\frac{0(0.839)-1(-1)}{0.839-(-1)}$$

$$x_2 = 0.544$$

$$f(x_2) = \text{f } (0.544) = (0.544)^3 + (0.544)^2 + tan(0.544) + e^{0.544}$$

$$\boxed{\mathbf{f(x_2) = -0.662 < 0}}$$

Iteration2:

$$\text{Here } f(0.544) = -0.662 < 0andf(1) = 0.839 > 0$$

$$\therefore \text{Root lies between } x_0 = 0.544 \text{ and } x_1\text{=1}$$

$$x_3 = \frac{x_0.f(x_1)-x_1.f(x_0)}{f(x_1)-f(x_0)}$$

$$=\frac{0.544(0.839)-1(-0.662)}{0.839-(-0.662)}$$

$$x_3 = 0.745$$

$$f(x_3) = \text{f } (0.745) = (0.745)^3 + (0.745)^2 + tan(0.745) + e^{0.745}$$

$$\boxed{\mathbf{f(x_2) = -0.216 < 0}}$$

Iteration3:

$$\text{Here } f(0.745) = -0.216 < 0 and f(1) = 0.839 > 0$$

$$\therefore \text{Root lies between } x_0 = 0.745 \text{ and } x_1{=}1$$

$$x_4 = \frac{x_0.f(x_1)-x_1.f(x_0)}{f(x_1)-f(x_0)}$$

$$= \frac{0.745(0.839)-1(-0.216)}{0.839-(-0.216)}$$

$$x_4 = 0.797$$

$$f(x_4) = \text{f } (0.797) = (0.797)^3 + (0.797)^2 + tan(0.797) + e^{0.797}$$

$$\boxed{\mathbf{f(x_4) = -0.054 < 0}}$$

Iteration4:

$$\text{Here } f(0.797) = -0.054 < 0 and f(1) = 0.839 > 0$$

$$\therefore \text{Root lies between } x_0 = 0.797 \text{ and } x_1{=}1$$

$$x_5 = \frac{x_0.f(x_1)-x_1.f(x_0)}{f(x_1)-f(x_0)}$$

$$= \frac{0.797(0.839)-1(-0.054)}{0.839-(-0.054)}$$

$$x_5 = 0.809$$

$$f(x_5) = \text{f } (0.809) = (0.809)^3 + (0.809)^2 + tan(0.809) + e^{0.809}$$

$$\boxed{\mathbf{f(x_5) = -0.012 < 0}}$$

Iteration5:

$$\text{Here } f(0.809) = -0.012 < 0 and f(1) = 0.839 > 0$$

$$\therefore \text{Root lies between } x_0 = 0.809 \text{ and } x_1{=}1$$

$$x_6 = \frac{x_0.f(x_1)-x_1.f(x_0)}{f(x_1)-f(x_0)}$$

$$= \frac{0.809(0.839)-1(-0.012)}{0.839-(-0.012)}$$

$$x_6 = 0.812$$

$$f(x_6) = \text{f } (0.812) = (0.812)^3 + (0.812)^2 + tan(0.812) + e^{0.812}$$

$$\boxed{\mathbf{f(x_6) = -0.003 < 0}}$$

Iteration6:

$$\text{Here } f(0.812) = -0.003 < 0 and f(1) = 0.839 > 0$$

$$\therefore \text{Root lies between } x_0 = 0.812 \text{ and } x_1{=}1$$

$$x_7 = \frac{x_0.f(x_1) - x_1.f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0.812(0.839) - 1(-0.003)}{0.839 - (-0.003)}$$

$$x_7 = 0.813$$

$$f(x_7) = \text{f } (0.813) = (0.813)^3 + (0.813)^2 + tan(0.813) + e^{0.813}$$

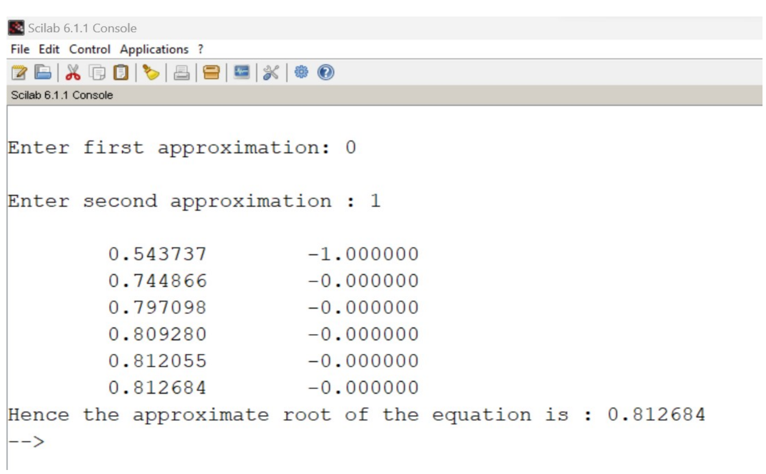
$$\boxed{\mathbf{f(x_7) = 0 < 0}}$$

Hence the approximate root of the equation is 0.813

Coding/Programming:

```
clc; clear all;
deff('y=f(x)','y=x^3+x^2+tan(x)-%e^x');
a=input("Enter first approximation: ");
b=input("Enter second approximation : ");
i=1;
while(i<=6)
    c=(a*f(b)-b*f(a))/(f(b)-f(a));
    if (f(a)*f(c)<0) then
        b=c;
    else
        a=c;
    end
    i=i+1;
    printf("\t%f\t%f\n",c,round(f(c)))
end
//disp(i)
printf('Hence the approximate root of the equation is : %g',c,i);
```

Output:



Exercise:4

2(C)NEWTON RAPHSON METHOD

Objective:

To find roots of the given equation using Newton Raphson method

Input:

$$x^3+x^2+tanx\text{-}e^x=0$$

Procedure/Methodology:

$$\text{f(x)}=x^3+x^2+tanx\text{-}e^x$$

$$f'(x)=3x^2+2x+sec^2x-e^x$$

$$\text{f(0)=-1 (-ve)}$$

$$\text{f(1)=0.839 (+ve)}$$

Here $f(0)=-1<0$ and $f(1)=0.839>0$

$\therefore$Root lies between 0 and 1

$$x_0=\frac{0+1}{2}$$

$$x_0=0.5$$

Iteration1:

$$f(x_0)=f(0.5)=(0.5)^3+(0.5)^2+tan(0.5)-e^{0.5}$$

$$f(x_0)=0.727$$

$$f'(x_0)=f'(0.5)=3(0.5)^2+2(0.5)+sec^2(0.5)-e^{0.5}$$

$$f'(x_0)=1.4$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$x_1=0.5-\frac{(-0.727)}{1.4}$$

$$\boxed{\mathbf{x_1=1.02}}$$

Iteration2:

$$f(x_1) = f(1.02) = (1.02)^3 + (1.02)^2 + tan(1.02) - e^{1.02}$$
$$f(x_1){=}0.955$$
$$f'(x_1) = f'(1.02) = 3(1.02)^2 + 2(1.02) + sec^2(1.02) - e^{1.02}$$
$$f'(x_1){=}6.033$$
$$x_2 = x_1 - \frac{f(x_1}{f'(x_1)}$$
$$x_2 = 1.02 - \frac{(0.955)}{6.033}$$
$$\boxed{\mathbf{x_2 = 0.861}}$$

Iteration3:

$$f(x_2) = f(0.861) = (0.861)^3 + (0.861)^2 + tan(0.861) - e^{0.861}$$
$$f(x_2){=}0.18$$
$$f'(x_2) = f'(0.861) = 3(0.861)^2 + 2(0.861) + sec^2(0.861) - e^{0.861}$$
$$f'(x_2){=}3.94$$
$$x_3 = x_2 - \frac{f(x_2}{f'(x_2)}$$
$$x_3 = 0.861 - \frac{(0.18)}{3.94}$$
$$\boxed{\mathbf{x_3 = 0.816}}$$

Iteration4:

$$f(x_3) = f(0.816) = (0.816)^3 + (0.816)^2 + tan(0.816) - e^{0.816}$$
$$f(x_3){=}0.01$$
$$f'(x_3) = f'(0.816) = 3(0.816)^2 + 2(0.816) + sec^2(0.816) - e^{0.816}$$
$$f'(x_3){=}3.497$$
$$x_4 = x_3 - \frac{f(x_3}{f'(x_3)}$$
$$x_4 = 0.816 - \frac{(0.01)}{3.497}$$
$$\boxed{\mathbf{x_4 = 0.813}}$$

Iteration5:

$$f(x_4) = f(0.813) = (0.813)^3 + (0.813)^2 + tan(0.813) - e^{0.813}$$
$$f(x_4){=}0$$

$$f'(x_4) = f'(0.813) = 3(0.813)^2 + 2(0.813) + sec^2(0.813) - e^{0.813}$$

$$f'(x_4)\text{=}3.47$$

$$x_5 = x_4 - \frac{f(x_4}{f'(x_4)}$$

$$x_5 = 0.813 - \frac{(0)}{3.47}$$

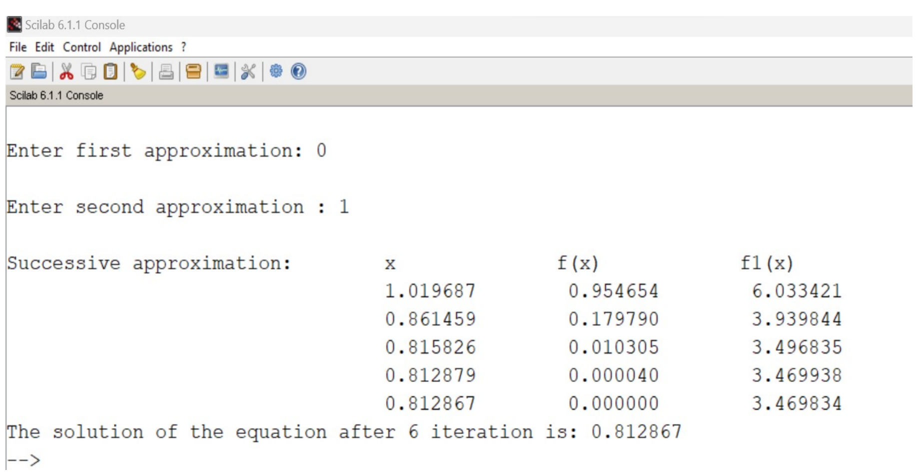
$$\boxed{\mathbf{x_5 = 0.813}}$$

Hence the approximate root of the equation is 0.813

Coding/Programming:

```
clc; clear all;
deff('y=f(x)','y=x^3+x^2+tan(x)-%e^(x)');
deff('z=f1(x)','z=3*x^2+2*x+(sec(x))^2-%e^(x)');
a=input("Enter first approximation: ");
b=input("Enter second approximation : ");
x=(a+b)/2;
i=1;
printf('Successive approximation:\t x\t \tf(x)\t \tf1(x)\n');
while(i<=5)
    x=x-(f(x)/f1(x))
    printf('                          \t %f\t %f\t %f\n',x,f(x),f1(x));
    i=i+1;
end
printf('The solution of the equation after %d iteration is: %f',i,x);
```

Output:


```
Scilab 6.1.1 Console
File  Edit  Control  Applications  ?
Scilab 6.1.1 Console

Enter first approximation: 0

Enter second approximation : 1

Successive approximation:        x              f(x)             f1(x)
                                 1.019687        0.954654         6.033421
                                 0.861459        0.179790         3.939844
                                 0.815826        0.010305         3.496835
                                 0.812879        0.000040         3.469938
                                 0.812867        0.000000         3.469834
The solution of the equation after 6 iteration is: 0.812867
-->
```

Exercise:5

3(A)GAUSS ELIMINATION METHOD

Objective:

To find solution for the system of linear equations using Gauss elimination method

Input:

$$\begin{aligned} 2x + 4y + 6z &= 22 \\ 3x + 8y + 5z &= 27 \\ -x + y + 2z &= 2 \end{aligned}$$

Procedure/Methodology:

$$\begin{aligned} 2x + 4y + 6z &= 22 \\ 3x + 8y + 5z &= 27 \\ -x + y + 2z &= 2 \end{aligned}$$

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 8 & 5 \\ -1 & 1 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 22 \\ 27 \\ 2 \end{bmatrix}$$

AX=B

$$\begin{bmatrix} 2 & 4 & 6 \\ 3 & 8 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 22 \\ 27 \\ 2 \end{bmatrix}$$

Augmented matrix:

$$(A, B) = \left[\begin{array}{ccc|c} 2 & 4 & 6 & 22 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array}\right] \begin{array}{c} R_1 \to \frac{R_1}{2} \\ R_2 \\ R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_1 + R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 2 & -4 & -6 \\ 0 & 3 & 5 & 13 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \to 2R_3 - 3R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 2 & -4 & -6 \\ 0 & 0 & 22 & 44 \end{array}\right]$$

$$x + 2y + 3z = 11 \tag{6}$$
$$y - 4z = -6 \tag{7}$$
$$22z = 44 \tag{8}$$

From (8) $\Longrightarrow$

$$\boxed{\mathbf{z = 2}}$$

Substitute the value of z in (7) $\Longrightarrow$

2y-4(2)=-6

$$\boxed{\mathbf{y = 1}}$$

Substitute the value of z and y in (6) $\Longrightarrow$

x+2(1)+3(2)=11

$$\boxed{\mathbf{x = 3}}$$

Coding/Programming:

```
clc;clear all;
disp('performing Gaussian elimination')
A=[2,4,6;3,8,5;-1,1,2];
B=[22,27,2]';
disp('the co-efficient matrix is:')
disp(A)
disp(B)
c=[A B]
disp('the augmented matrix is:')
disp(c)
disp('R1/2')
c(1,:)=c(1,:)/2
disp(c)
disp('R2=R2-3*R1')
disp('R3=R3+R1')
c(2,:)=c(2,:)-3*c(1,:)
c(3,:)=c(3,:)+c(1,:)
disp(c)
disp('R3=2*R3-3*R2')
c(3,:)=2*c(3,:)-3*c(2,:)
z=c(3,4)/c(3,3)
y=(c(2,4)+4*z)/c(2,2)
x=c(1,4)-(3*z)-2*y
//disp(z)
//disp(y)
//disp(x)
printf('\t The solution is:\n\tx=%d\ty=%d\n\tz=%d',x,y,z)
```

Output:

```
"performing Gaussian elimination"

"the co-efficient matrix is:"

 2.   4.   6.
 3.   8.   5.
-1.   1.   2.

 22.
 27.
 2.

"the augmented matrix is:"

 2.   4.   6.   22.
 3.   8.   5.   27.
-1.   1.   2.   2.

"R1/2"
```

```
 1.   2.   3.   11.
 3.   8.   5.   27.
-1.   1.   2.   2.

"R2=R2-3*R1"

"R3=R3+R1"

 1.   2.   3.   11.
 0.   2.  -4.  -6.
 0.   3.   5.   13.

"R3=2*R3-3*R2"

 2.

 1.

 3.
```

Exercise:6

3(B)GAUSS JORDAN METHOD

Objective:

To find the solution of system of linear equations using Gauss Jordan method

Input:

$$\begin{aligned} 2x + 4y + 6z &= 22 \\ 3x + 8y + 5z &= 27 \\ -x + y + 2z &= 2 \end{aligned}$$

Procedure/Methodology:

$$\begin{aligned} 2x + 4y + 6z &= 22 \\ 3x + 8y + 5z &= 27 \\ -x + y + 2z &= 2 \end{aligned}$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 3 & 8 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 22 \\ 27 \\ 2 \end{bmatrix}$$

AX=B

where

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 8 & 5 \\ -1 & 1 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 22 \\ 27 \\ 2 \end{bmatrix}$$

Augmented Matrix:

$$(A,B)=\left[\begin{array}{ccc|c} 2 & 4 & 6 & 22\\ 3 & 8 & 5 & 27\\ -1 & 1 & 2 & 2 \end{array}\right]\begin{array}{c} R_1\to\frac{R1}{2}\\ R_2\\ R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 11\\ 3 & 8 & 5 & 27\\ -1 & 1 & 2 & 2 \end{array}\right]\begin{array}{c} R_1\\ R_2\to R_2-3R_1\\ R_3\to R_3+R_1 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 11\\ 0 & 2 & -4 & -6\\ 0 & 3 & 5 & 13 \end{array}\right]\begin{array}{c} R_1\\ R_2\to\frac{R_2}{2}\\ R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 11\\ 0 & 1 & -2 & -3\\ 0 & 3 & 5 & 13 \end{array}\right]\begin{array}{c} R_1\\ R_2\\ R_3\to R_3-3R_2 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 11\\ 0 & 1 & -2 & -3\\ 0 & 0 & 11 & 22 \end{array}\right]\begin{array}{c} R_1\\ R_2\\ R_3\to\frac{R_3}{11} \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 2 & 3 & 11\\ 0 & 1 & -2 & -3\\ 0 & 0 & 1 & 2 \end{array}\right]\begin{array}{c} R_1\to R_1-2R_2\\ R_2\\ R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17\\ 0 & 1 & -2 & -3\\ 0 & 0 & 1 & 2 \end{array}\right]\begin{array}{c} R_1\to R_1-7R_3\\ R_2\\ R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3\\ 0 & 1 & -2 & -3\\ 0 & 0 & 1 & 2 \end{array}\right]\begin{array}{c} R_1\\ R_2\to R_2+2R_3\\ R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\Longrightarrow$

$$\boxed{\mathbf{x = 3}}$$

$$\boxed{\mathbf{y = 1}}$$

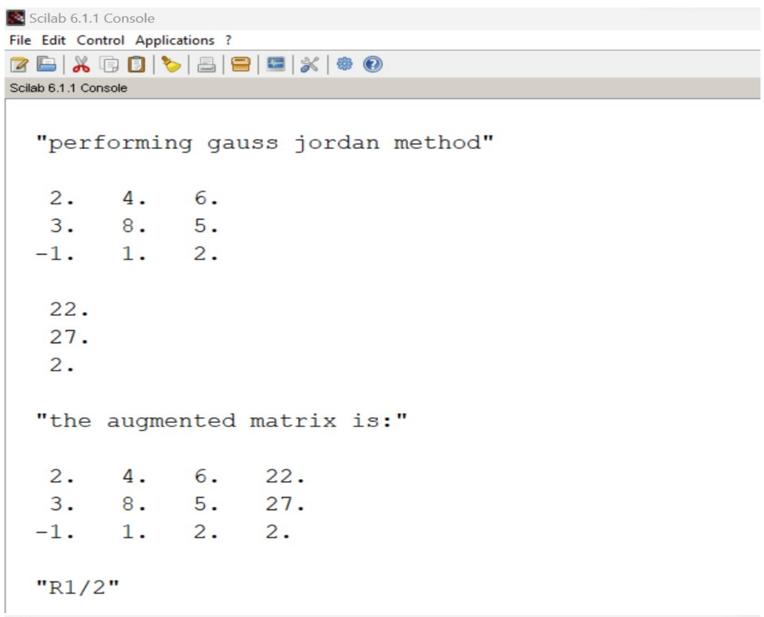
$$\boxed{\mathbf{z = 2}}$$

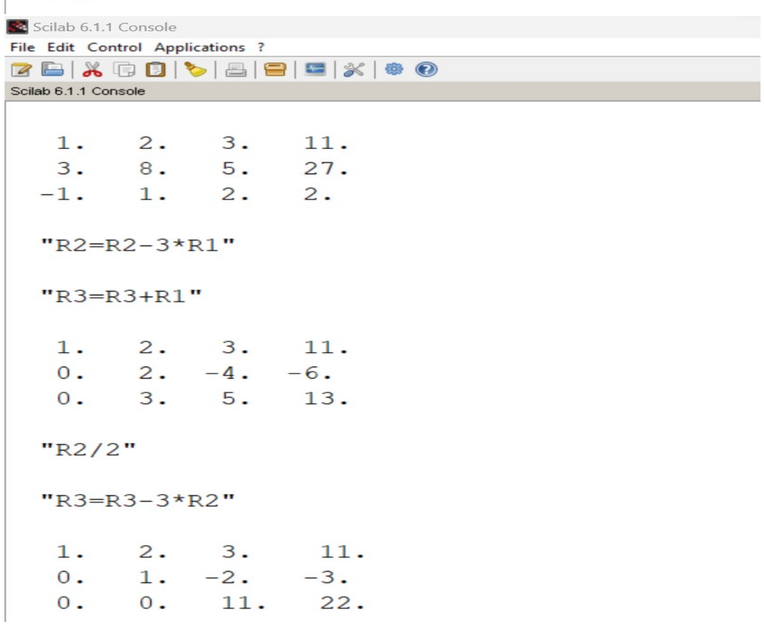
Coding/Programming:

```
clc;clear all;
disp('performing gauss jordan method')
a=[2,4,6;3,8,5;-1,1,2];
b=[22,27,2]';
disp(a)
disp(b)
c=[a b]
disp('the augmented matrix is:')
disp(c)
disp('R1/2')
c(1,:)=c(1,:)/2
disp(c)
disp('R2=R2-3*R1')
disp('R3=R3+R1')
c(2,:)=c(2,:)-3*c(1,:)
c(3,:)=c(3,:)+c(1,:)
disp(c)
disp('R2/2')
disp('R3=R3-3*R2')
c(2,:)=c(2,:)/2
c(3,:)=c(3,:)-3*c(2,:)
disp(c)
disp('R3=R3/11')
disp('R1=R1-2*R2')
c(3,:)=c(3,:)/11
c(1,:)=c(1,:)-2*c(2,:)
disp(c)
disp('R1=R1-7*R3')
disp('R2=R2+2*R3')
c(1,:)=c(1,:)-7*c(3,:)
c(2,:)=c(2,:)+2*c(3,:)
disp(c)
```

```
z=c(3,4)/c(3,3)
y=c(2,4)/c(2,2)
x=c(1,4)/c(1,1)
//disp(x)
//disp(y)
//disp(z)
printf('\t the solution is:\n\tx=%d\ty=%d\n\tz=%d',x,y,z)
```

Output:





```
Scilab 6.1.1 Console
File  Edit  Control  Applications  ?
Scilab 6.1.1 Console
   0.    0.    11.    22.

  "R3=R3/11"

  "R1=R1-2*R2"

   1.    0.    7.    17.
   0.    1.   -2.   -3.
   0.    0.    1.    2.

  "R1=R1-7*R3"

  "R2=R2+2*R3"

   1.    0.    0.    3.
   0.    1.    0.    1.
   0.    0.    1.    2.
         the solution is:
        x=3      y=1
        z=2
-->
```

Exercise:7

4.GAUSS SEIDEL METHOD

Objective:

To find solution for the system of equations using Gauss Seidel method

Input:

$$\begin{aligned} 12x + 3y - 5z &= 1 \\ x + 5y + 3z &= 28 \\ 3x + 7y + 13z &= 76 \end{aligned}$$

Procedure/Methodoly:

$$\begin{aligned} 12x + 3y - 5z &= 1 \\ x + 5y + 3z &= 28 \\ 3x + 7y + 13z &= 76 \end{aligned}$$

Diagonally dominant:

$$\begin{gathered} |12| \geq |3| + |-5| \\ |5| \geq |1| + |3| \\ |13| \geq |3| + |7| \end{gathered}$$

$\Longrightarrow$

$$x = \tfrac{1}{12}(1 - 3y + 5z)$$

$$y = \tfrac{1}{5}(28 - x - 3z)$$

$$z = \tfrac{1}{13}(76 - 3x - 7y)$$

Initial values:

(x,y,z)=(0,0,0)

Iteration1:

$$x_1 = \frac{1}{12}(1 - 3(0) + 5(0))$$
$$\boxed{\mathbf{x_1 = 0.0833}}$$
$$y_1 = \frac{1}{5}(28 - 0.0833 - 3(0))$$
$$\boxed{\mathbf{y_1 = 5.5833}}$$
$$z_1 = \frac{1}{13}(76 - 3(0.0833) - 7(5.5833))$$
$$\boxed{\mathbf{z_1 = 2.8205}}$$

Iteration2:

$$x_2 = \frac{1}{12}(1 - 3(5.5833) + 5(2.8205))$$
$$\boxed{\mathbf{x_2 = -0.1373}}$$
$$y_2 = \frac{1}{5}(28 + 0.1373 - 3(2.8205))$$
$$\boxed{\mathbf{y_2 = 3.9352}}$$
$$z_2 = \frac{1}{13}(76 - 3(00.1373) - 7(3.9352))$$
$$\boxed{\mathbf{z_2 = 3.7589}}$$

Iteration3:

$$x_3 = \frac{1}{12}(1 - 3(3.9352) + 5(3.7389))$$
$$\boxed{\mathbf{x_3 = 0.6657}}$$
$$y_3 = \frac{1}{5}(28 - 0.6657 - 3(3.7589))$$
$$\boxed{\mathbf{y_3 = 3.2115}}$$
$$z_3 = \frac{1}{13}(76 - 3(0.6657) - 7(3.2115))$$
$$\boxed{\mathbf{z_3 = 3.9633}}$$

Iteration4:

$$x_4 = \frac{1}{12}(1 - 3(3.2115) + 5(3.9633))$$
$$\boxed{\mathbf{x_4 = 0.9318}}$$
$$y_4 = \frac{1}{5}(28 - 0.9318 - 3(3.9633))$$
$$\boxed{\mathbf{y_4 = 2.8357}}$$
$$z_4 = \frac{1}{13}(76 - 3(0.9318) - 7(2.8357))$$
$$\boxed{\mathbf{z_4 = 4.1042}}$$

Iteration5:

$$x_5 = \frac{1}{12}(1 - 3(2.8357) + 5(4.1042))$$
$$\boxed{\mathbf{x_5 = 1.0845}}$$
$$y_5 = \frac{1}{5}(28 - 1.0845 - 3(4.1042))$$
$$\boxed{\mathbf{y_5 = 2.7685}}$$
$$z_5 = \frac{1}{13}(76 - 3(1.0845) - 7(2.7685))$$
$$\boxed{\mathbf{z_5 = 4.1052}}$$

Iteration6:

$$x_6 = \frac{1}{12}(1 - 3(2.7685) + 5(4.1052))$$
$$\boxed{\mathbf{x_6 = 1.1017}}$$
$$y_6 = \frac{1}{5}(28 - 1.1017 - 3(4.1052))$$
$$\boxed{\mathbf{y_6 = 2.9165}}$$
$$z_6 = \frac{1}{13}(76 - 3(1.1017) - 7(2.9165))$$
$$\boxed{\mathbf{z_6 = 4.0215}}$$

Iteration7:

$$x_7 = \frac{1}{12}(1 - 3(2.9165) + 5(4.0215))$$

$$\boxed{\mathbf{x_7 = 1.0298}}$$

$$y_7 = \frac{1}{5}(28 - 1.0298 - 3(4.0215))$$

$$\boxed{\mathbf{y_7 = 2.9811}}$$

$$z_7 = \frac{1}{13}(76 - 3(1.0298) - 7(2.9811))$$

$$\boxed{\mathbf{z_7 = 4.0033}}$$

Iteration8:

$$x_8 = \frac{1}{12}(1 - 3(3.2.9811) + 5(4.0033))$$

$$\boxed{\mathbf{x_8 = 1.0061}}$$

$$y_8 = \frac{1}{5}(28 - 1.0061 - 3(4.0033))$$

$$\boxed{\mathbf{y_8 = 2.9968}}$$

$$z_8 = \frac{1}{13}(76 - 3(1.0061) - 7(2.9968))$$

$$\boxed{\mathbf{z_8 = 4.0003}}$$

Iteration9:

$$x_9 = \frac{1}{12}(1 - 3(2.9968) + 5(4.0003))$$

$$\boxed{\mathbf{x_9 = 1.0009}}$$

$$y_9 = \frac{1}{5}(28 - 1.0009 - 3(4.0003))$$

$$\boxed{\mathbf{y_9 = 2.9996}}$$

$$z_9 = \frac{1}{13}(76 - 3(1.0009) - 7(2.9996))$$

$$\boxed{\mathbf{z_9 = 4.0000}}$$

Iteration10:

$$x_{10} = \frac{1}{12}(1 - 3(2.9996) + 5(4))$$

$$\boxed{\mathbf{x_{10} = 1.0001}}$$

$$y_{10} = \frac{1}{5}(28 - 1.0001 - 3(4))$$

$$\boxed{\mathbf{y_{10} = 2.9999}}$$

$$z_{10} = \frac{1}{13}(76 - 3(1.0001) - 7(2.9999))$$

$$\boxed{\mathbf{z_{10} = 4.0000}}$$

Iteration11:

$$x_{11} = \frac{1}{12}(1 - 3(2.9999) + 5(4))$$

$$\boxed{\mathbf{x_{11} = 1}}$$

$$y_{11} = \frac{1}{5}(28 - 1 - 3(4))$$

$$\boxed{\mathbf{y_{11} = 3}}$$

$$z_{11} = \frac{1}{13}(76 - 3(1) - 7(3))$$

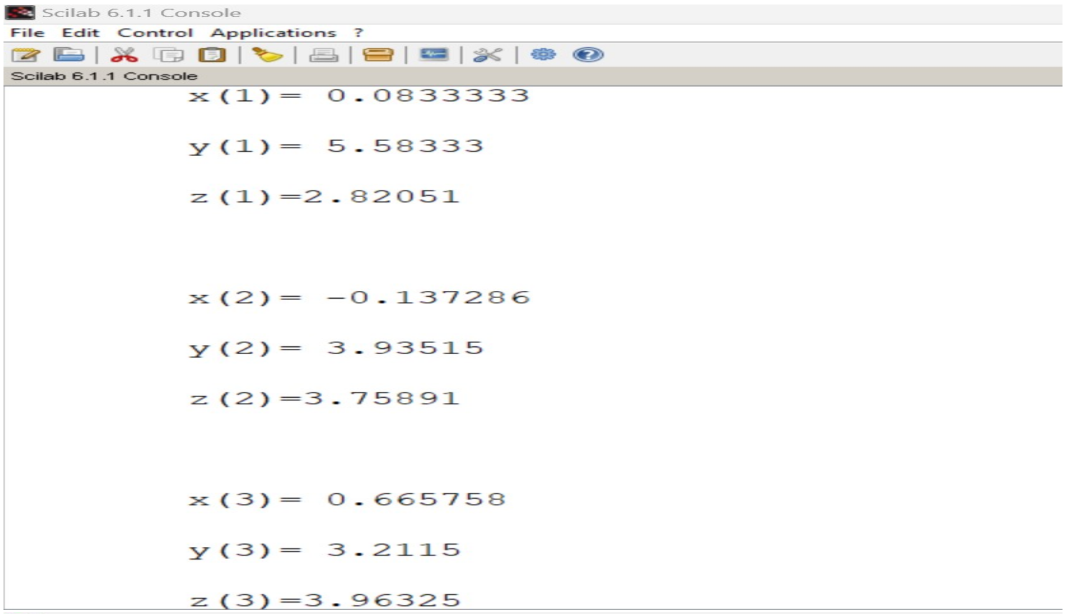
$$\boxed{\mathbf{z_{11} = 4}}$$

Hence,

$$x \simeq 1$$
$$y \simeq 3$$
$$z \simeq 4$$

Output:

```
clc; clear all;
x0=0;y0=0;z0=0;
deff('x=f1(y,z)','x=(1-3*y+5*z)/12')
deff('y=f2(x,z)','y=(28-x-3*z)/5')
deff('z=f3(x,y)','z=(76-3*x-7*y)/13')
for i=1:11
    x0=f1(y0,z0);
    y0=f2(x0,z0);
    z0=f3(x0,y0);
printf('\tx(%i)= %g\n\n\ty(%i)= %g\n\n\tz(%i)=%g\n\n\n\n',i,x0,i,y0,i,z0)
end
printf('the solution converges to %g,%g and %g',round(x0),round(y0),round(z0))
```

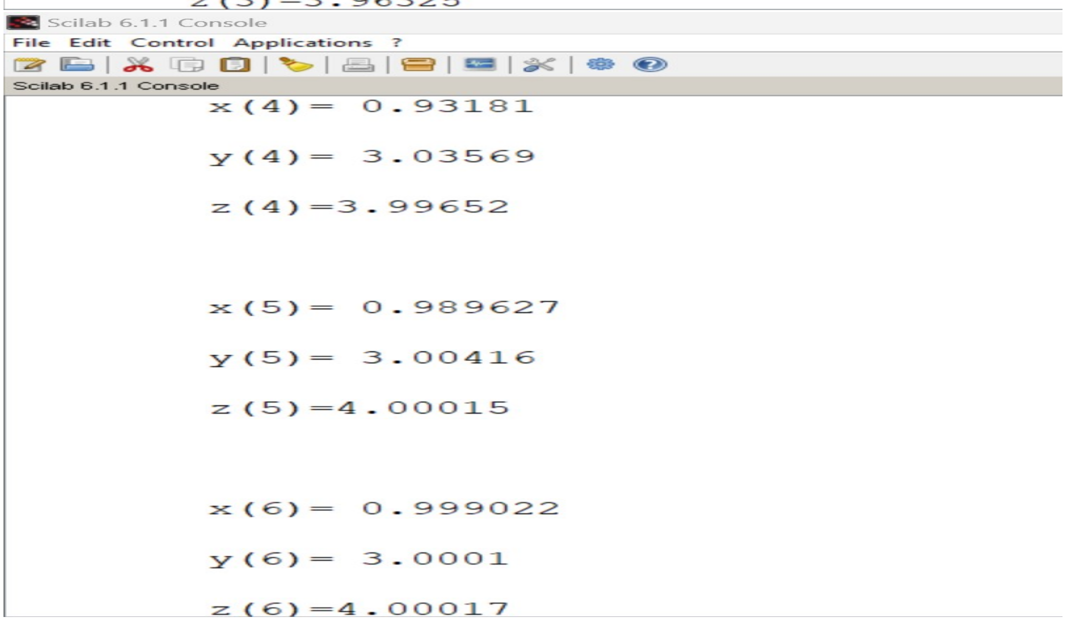


Scilab 6.1.1 Console

File Edit Control Applications ?

Scilab 6.1.1 Console

```
x(1)= 0.0833333

y(1)= 5.58333

z(1)=2.82051


x(2)= -0.137286

y(2)= 3.93515

z(2)=3.75891


x(3)= 0.665758

y(3)= 3.2115

z(3)=3.96325
```

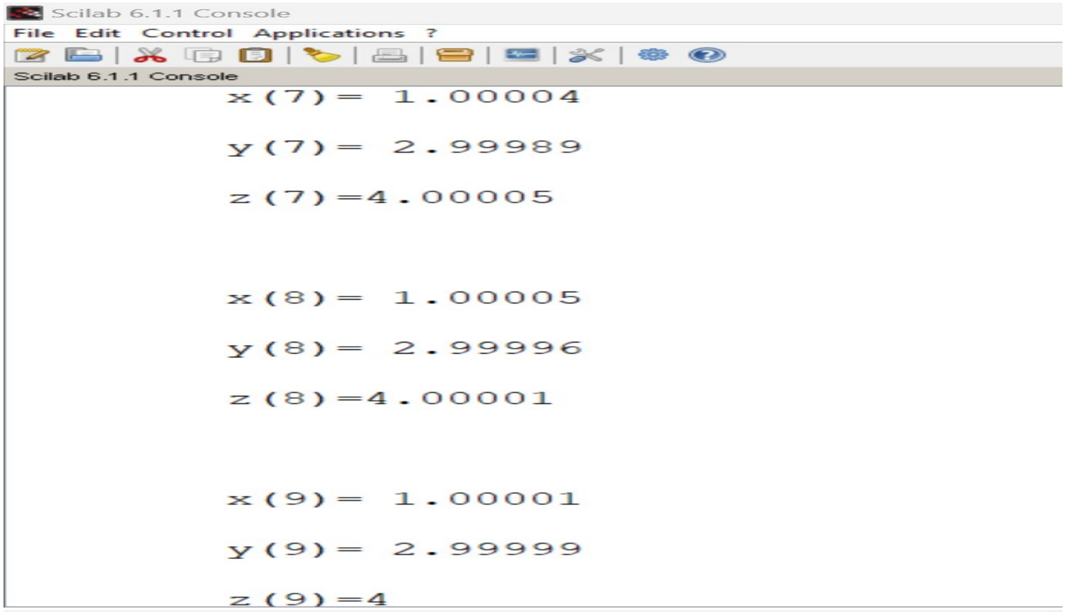


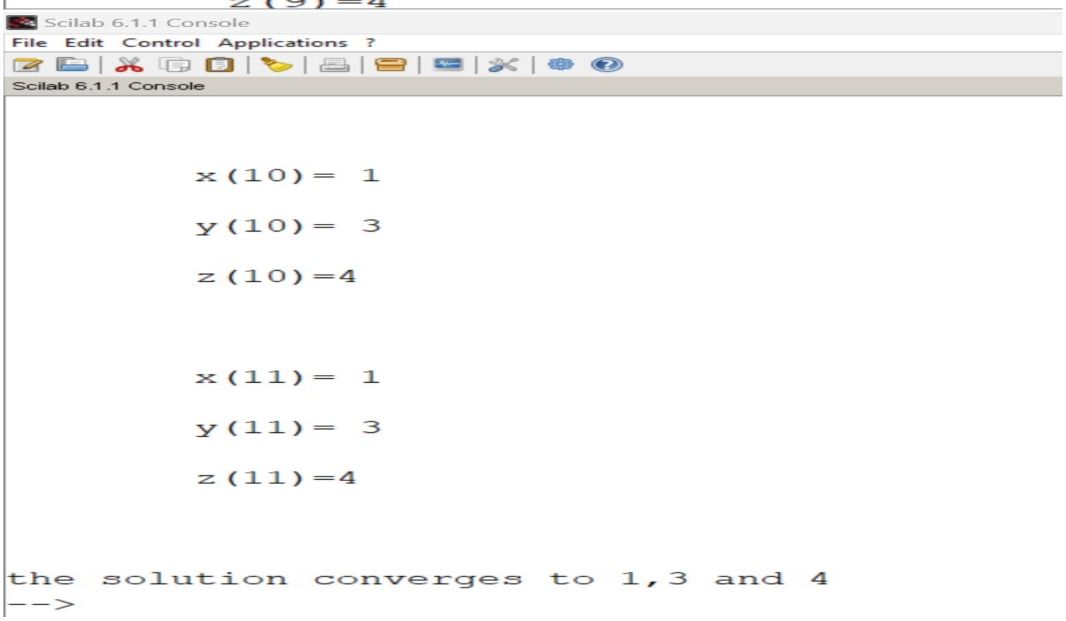
Scilab 6.1.1 Console

File Edit Control Applications ?

Scilab 6.1.1 Console

```
x(4)= 0.93181

y(4)= 3.03569

z(4)=3.99652


x(5)= 0.989627

y(5)= 3.00416

z(5)=4.00015


x(6)= 0.999022

y(6)= 3.0001

z(6)=4.00017
```


Scilab 6.1.1 Console
File Edit Control Applications ?
Scilab 6.1.1 Console
x(7)= 1.00004
y(7)= 2.99989
z(7)=4.00005
x(8)= 1.00005
y(8)= 2.99996
z(8)=4.00001
x(9)= 1.00001
y(9)= 2.99999
z(9)=4


Scilab 6.1.1 Console
File Edit Control Applications ?
Scilab 6.1.1 Console
x(10)= 1
y(10)= 3
z(10)=4
x(11)= 1
y(11)= 3
z(11)=4
the solution converges to 1,3 and 4
-->

MODULE 2

Exercise:8

5(A)NEWTON-GREGORY FORWARD INTERPOLATION

Objective:

To find derivative for the given data using Newton-Gregory forward interpolation method

Input:

x	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910
y	46	66	81	85	90	92	97	99	101	105

Procedure/Methodology:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$	$\Delta^8 y$	$\Delta^9 y$
1901	**46**									
		20								
1902	66		**-5**							
		15		**-6**						
1903	81		-11		**18**					
		4		12		**-34**				
1904	85		1		-16		**60**			
		5		-4		26		**-108**		
1905	90		-3		10		-48		**199**	
		2		6		-22		91		**-364**
1906	92		3		-12		43		-165	
		5		-6		21		-74		
1907	97		-3		9		-31			
		2		3		-10				
1908	99		0		-1					
		2		2						
1909	101		2							
		4								
1910	105									

The value of x at which we want to find is x=1902.5
Here,

$$h = x_1 - x_0$$

$$=1902-1901$$

$$\boxed{\mathbf{h = 1}}$$

$$p = \frac{x-x_0}{h}$$

$$=\frac{1902.5-1901}{1}$$

$$\boxed{\mathbf{p = 1.5}}$$

Newton's forward interpolation formula is:

$$y(x) = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + ...$$

$$\text{y(x)=y(1902.5)=46}+\frac{1.5(20)}{1!}+\frac{1.5(1.5-1)}{2!}(-5)+\frac{1.5(1.5-1)(1.5-2)}{3!}(-6)$$

$$+\frac{1.5(1.5-1)(1.5-2)(1.5-3)}{4!}(18)$$

$$+\frac{1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4)}{5!}(-34)$$

$$+\frac{1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)}{6!}(60)$$

$$+\frac{1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(1.5-6)}{7!}(-108)$$

$$+\frac{1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(1.5-6)(1.5-7)}{8!}(199)$$

$$+\frac{1.5(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(1.5-6)(1.5-7)(1.5-8)}{9!}(-364)$$

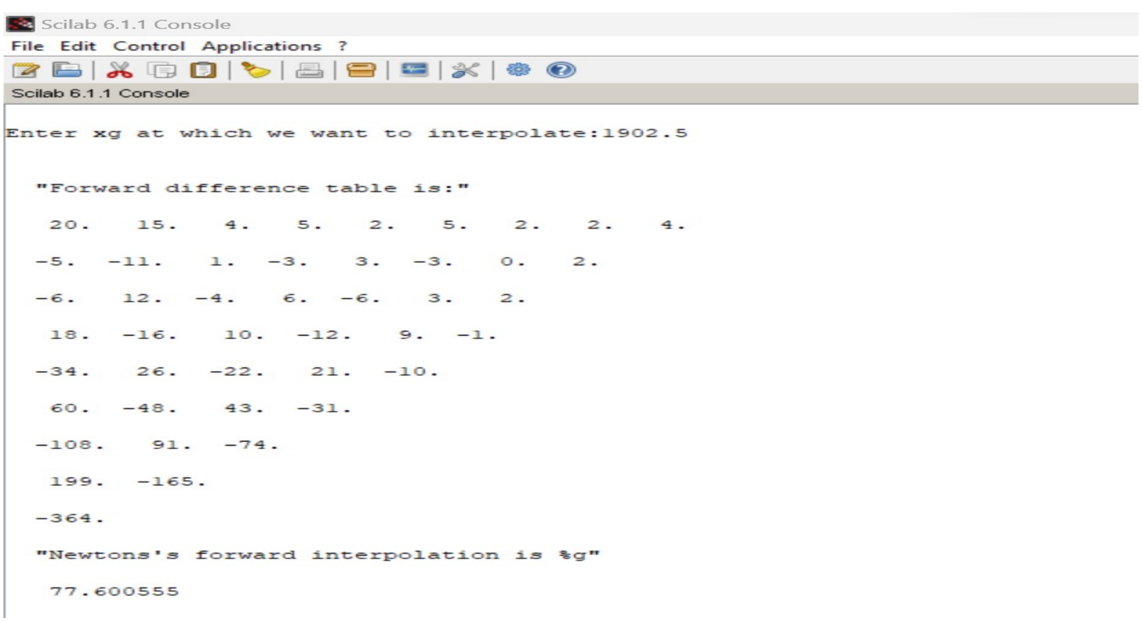
$$=46+30-1.875+0.375+0.4219+0.3984+0.4102+0.4746+0.6012+0.7943$$

$$\boxed{\mathbf{y(x) = y(1902.5) = 77.6006}}$$

Coding/Programming:

```
clc; clear all;
x=[1901:1:1910];
y=[46,66,81,85,90,92,97,99,101,105];
xg=input("Enter xg at which we want to interpolate:")
n=length(x)
h=x(2)-x(1)
u=(xg-x(1))/h
disp("Forward difference table is:")
for i=1:n-1
    disp(diff(y,i))
end
yg=y(1)
p=u
for i=1:n-1
    d=diff(y,i)
    yg=yg+p*d(1)
    p=p*((u-i)/(i+1))
end
disp("Newtons''s forward interpolation is %g",yg)
```

Ouput:



Exercise:9

5(B)NEWTON GREGORY BACKWARD INTERPOLATION

Objective:

To find derivative for the given data using Newton-Gregory backward interpolation method

Input:

x	1901	1902	1903	1904	1905	1906	1907	1908	1909	1910
y	46	66	81	85	90	92	97	99	101	105

Procedure/Methodology:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$	$\nabla^7 y$	$\nabla^8 y$	$\nabla^9 y$
1901	46									
		20								
1902	66		-5							
		15		-6						
1903	81		-11		18					
		4		12		-34				
1904	85		1		-16		60			
		5		-4		26		-108		
1905	90		-3		10		-48		199	
		2		6		-22		91		**-364**
1906	92		3		-12		43		**-165**	
		5		-6		21		**-74**		
1907	97		-3		9		**-31**			
		2		3		**-10**				
1908	99		0		**-1**					
		2		**2**						
1909	101		**2**							
		4								
1910	**105**									

The value of x at which we want to find is x=1908.5
Here,

$$h = x_1 - x_0$$

$$=1902\text{-}1901$$

$$\boxed{\mathbf{h = 1}}$$

$$p = \frac{x - x_n}{h}$$

$$= \frac{1908.5 - 1910}{1}$$

$$\boxed{\mathbf{p = -1.5}}$$

Newton's backward interpolation formula is:

$$y(x) = y_n + \frac{p}{1!}\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + ...$$

$$\text{y(x)=y(1908.5)=}\ 105 + \frac{(-1.5)}{1!}(4) + \frac{(-1.5)(-1.5+1)}{2!}(2) + \frac{(-1.5)(-1.5+1)(-1.5+1)(-1.5+2)}{3!}(2) +$$

$$\frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)}{4!}(-1) + \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)}{5!}(-10) +$$

$$\frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)(-1.5+5)}{6!}(-31) +$$

$$\frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)(-1.5+5)(-1.5+6)}{7!}(-74) +$$

$$\frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)(-1.5+5)(-1.5+6)(-1.5+7)}{8!}(-165) +$$

$$\frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)(-1.5+5)(-1.5+6)(-1.5+7)(-1.5+8)}{9!}(-364)$$

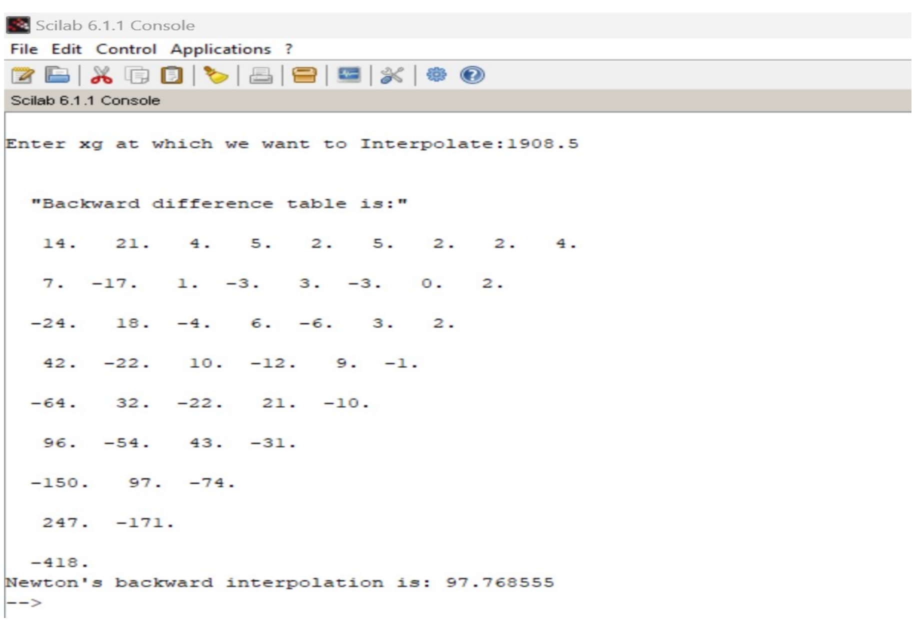
$$=105\text{-}6\text{+}0.75\text{+}0.125\text{-}0.0234\text{-}0.1172\text{-}0.2119\text{-}0.3252\text{-}0.4985\text{-}0.7943$$

$$\boxed{\mathbf{y(x) = y(1908.5) = 97.9045}}$$

Coding/Programming:

```
clc; clear all;
x=[1901:1:1910];
y=[46,60,81,85,90,92,97,99,101,105];
xg=input("Enter xg at which we want to Interpolate:")
n=length(x)
h=x(2)-x(1)
u=(xg-x(n))/h
disp("Backward difference table is:")
for i=1:n-1
    disp(diff(y,i))
end
yg=y(n)
p=u
for i=1:n-1
    d=diff(y,i)
    yg=yg+p*d(n-i)
    p=p*((u+i)/(i+1))
end
printf("Newton''s backward interpolation is: %f",yg)
```

Output:



Exercise:10

5(C)NEWTON'S DIVIDED DIFFERENCE METHOD

Objective:

To find derivative for the given data using Newton's divided difference method

Input:

x	1903	1907	1909	2000	2005	2009	2017	2020	2022	2023
y	46	66	81	85	90	92	97	99	101	105

Procedure/Methodology:

x	y	1^{st}order	2^{nd}order	3^{rd}order	4^{th}order	5^{th}order	6^{th}order
1903	**46**						
		5					
1907	66		**0.4167**				
		7.5		**-0.0051**			
1909	81		0.0802		**0**		
		0.044		0.0009		**0**	
2000	85		0.01		0		**0**
		1		-0.0007		0	
2005	90		-0.0556		0		0
		-0.5		0.0039		0	
2009	92		0.0104		-0.0002		0
		0.625		-0.0004		0	
2017	97		0.0038		0.0003		0
		0.6667		0.0048		0.0006	
2020	99		0.0667		0.0108		
		1		0.1556			
2022	101		1				
		4					
2023	105						

The value at which we want to find is x=1908.5

$$f(x) = y_0 + (x - x_0)f[x_0, x_1] + (x - x_0)(x - x_1)f[x_0, x_1, x_2] + ...$$

=46+(1908.5-1903)(5)+(1908.5-1903)(1908.5-1907)(0.4167)

+(1908.5-1903)(1908.5-1907)(1908.5-1909)(-0.0051)

+(1908.5-1903)(1908.5-1907)(1908.5-1909)(1908.5-2000)(0)

+(1908.5-1903)(1908.5-1907)(1908.5-1909)(1908.5-2000)(1908.5-2005)(0)

+(1908.5-1903)(1908.5-1907)(1908.5-1909)(1908.5-2000)(1908.5-2005)

(1908.5-2009)(0)

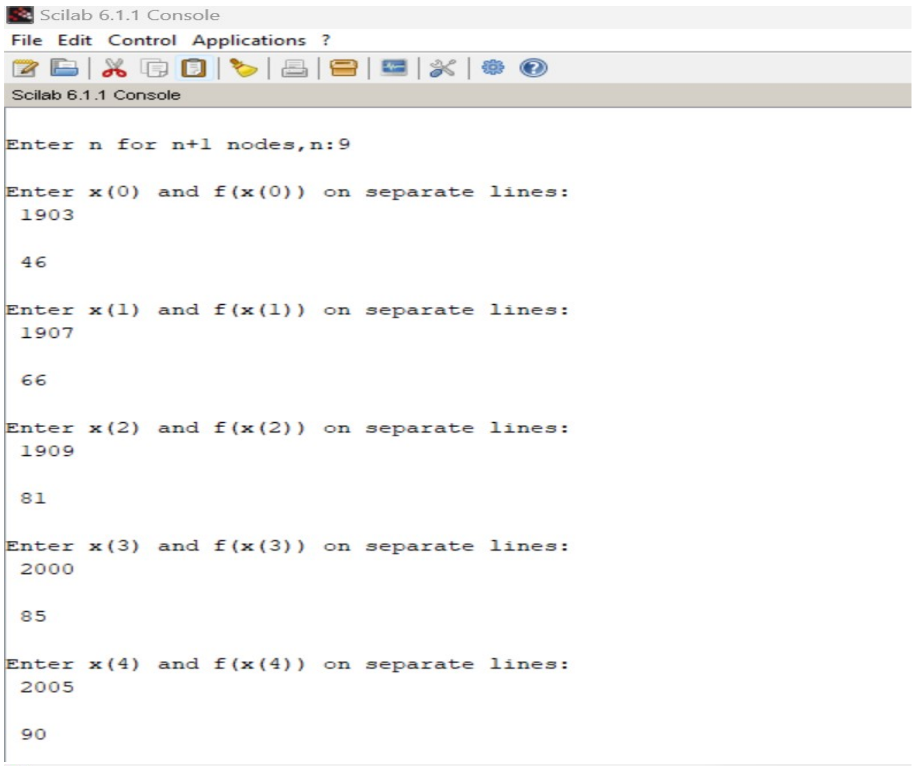
=46+27.5+3.4378+0.021+0+0+0

$$\boxed{\mathbf{f(x) = f(1908.5) = 76.9588}}$$

Coding/Programming:

```
clc; clear all;
n = input('Enter n for n+1 nodes,n:');
x = zeros(1,n+1);
y = zeros(n+1,n+1);
for i = 0:n
printf('Enter x(%d) and f(x(%d)) on separate lines: \n', i, i);
x(i+1) = input(' ');
y(i+1,1) = input(' ');
end
x0 = input('Now enter a point at which to evaluate the polynomial, x = ')
n = size(x,1);
if(n==1)
n = size(x,2);
end
for i = 1:n
D(i,1) = y(i);
end
for i = 2:n
for j = 2:i
D(i,j)=(D(i,j-1)-D(i-1,j-1))/(x(i)-x(i-j+1));
end
end
fx0 = D(n,n);
for i = n-1:-1:1
fx0 = fx0*(x0-x(i)) + D(i,i);
end
printf('Newtons iterated value: %.4f \n', fx0)
```

Output:

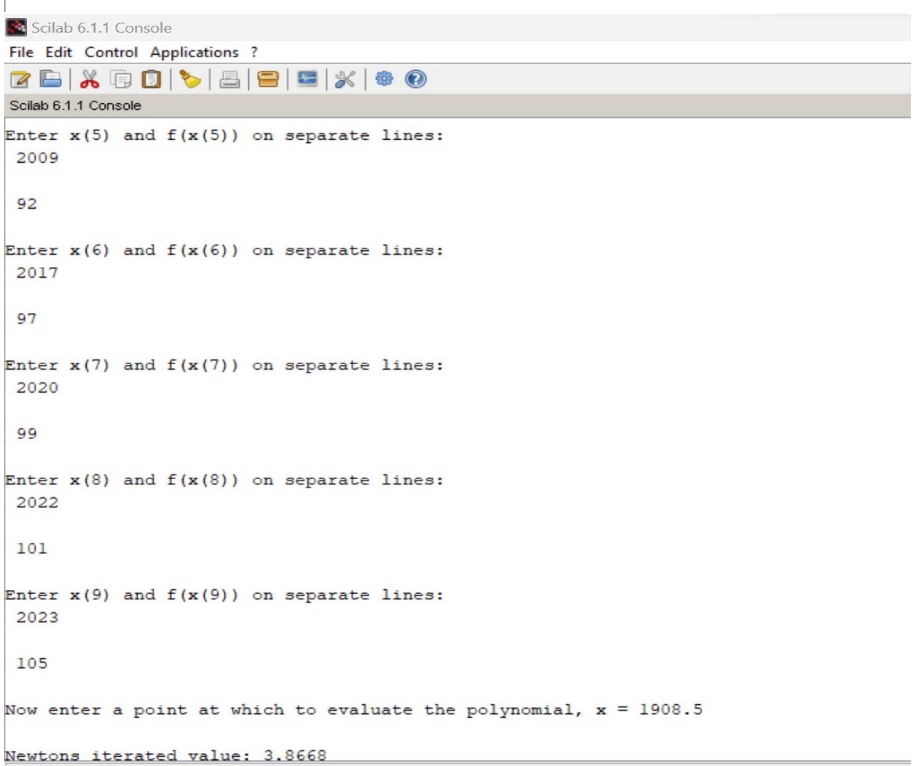


Scilab 6.1.1 Console

File Edit Control Applications ?

Scilab 6.1.1 Console

```
Enter n for n+1 nodes,n:9

Enter x(0) and f(x(0)) on separate lines:
 1903

 46

Enter x(1) and f(x(1)) on separate lines:
 1907

 66

Enter x(2) and f(x(2)) on separate lines:
 1909

 81

Enter x(3) and f(x(3)) on separate lines:
 2000

 85

Enter x(4) and f(x(4)) on separate lines:
 2005

 90
```



Scilab 6.1.1 Console

File Edit Control Applications ?

Scilab 6.1.1 Console

```
Enter x(5) and f(x(5)) on separate lines:
 2009

 92

Enter x(6) and f(x(6)) on separate lines:
 2017

 97

Enter x(7) and f(x(7)) on separate lines:
 2020

 99

Enter x(8) and f(x(8)) on separate lines:
 2022

 101

Enter x(9) and f(x(9)) on separate lines:
 2023

 105

Now enter a point at which to evaluate the polynomial, x = 1908.5

Newtons iterated value: 3.8668
```

Exercise:11

5(D)LAGRANGE'S METHOD

Objective:

To find derivative for the given data using Lagrange's method.

Input:

x	1903	1907	1909	2000	2005	2009	2017	2020	2022	2023
y	46	66	81	85	90	92	97	99	101	105

Procedure/Methodology:

The value at which we want to fing x=1905

$$\begin{aligned} f(x) &= \frac{(x-x_1)(x-x_2)\cdots(x-x_9)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_9)}(y_0) \\ &+\frac{(x-x_0)(x-x_2)\cdots(x-x_9)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_9)}(y_1) \\ &+\cdots\cdots+\frac{(x-x_1)(x-x_2)\cdots(x-x_8)}{(x_9-x_1)(x_0-x_2)\cdots(x_9-x_8)}(y_9) \end{aligned}$$

$$\begin{aligned} y(1905) &= \frac{(1905-1907)(1905-1909)\cdots(1905-2023)}{(1903-1907)(1903-1909)\cdots(1903-2023)}(46) \\ &+\frac{(1905-1903)(1905-1909)\cdots(1905-2023)}{(1907-1903)(1907-1909)\cdots(1907-2023)}(66) \\ &+\cdots\cdots+\frac{(1905-1903)(1905-1907)\cdots(1905-2022)}{(2023-1903)(2023-1907)\cdots(2023-2022)}(105) \end{aligned}$$

=0.2932(46)+1.1397(66)+(-0.4341)(81)+4.6559(85)

+(-26.5913)(90)+43.3575(92)+(-126.1707)(97)+280.4072(99)

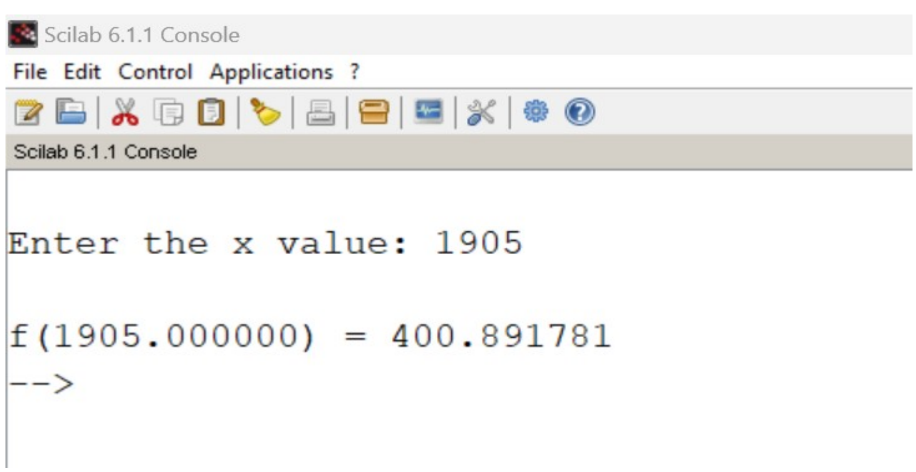
+(-319.5484)(101)+143.891(105)

$$\boxed{\mathbf{y(1905) = 400.8918}}$$

Coding/Programming:

```
clc; clear all;
X=[1903,1907,1909,2000,2005,2009,2017,2020,2022,2023];
Y=[46,66,81,85,90,92,97,99,101,105];
x=input("Enter the x value: ")
n=length(X);
L=0;
for i=1:n
    N=1;D=1;
    for j=1:n
        if(i==j)
            continue;
        else
            N=N*(x-X(j));
            D=D*(X(i)-X(j));
        end
    end
    L=L+(N/D)*Y(i);
end
printf("f(%f) = %f",x,L);
```

Output:



MODULE 3

Exercise:12

6(A)NEWTON'S FORWARD DIFFERENCE

Objective:

To find derivative for the given data using Newton's forward difference method

Input:

x	50	51	52	53	54	55	56
y	3.684	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Procedure/Methodology:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
50	**3.684**						
		0.0244					
51	3.7084		**-0.0003**				
		0.0241		**0**			
52	3.7325		-0.0003		**0**		
		0.0238		0		**0**	
53	3.7563		-0.0003		0		**0**
		0.0235		0		0	
54	3.7798		-0.0003		0		
		0.0232		0			
55	3.8030		-0.0003				
		0.0229					
56	3.8259						

The value of x at you want to find x=50

$$h = x_1 - x_0 = 51 - 50 = 1$$

$$u = \frac{x - x_0}{h}$$

$$= \frac{50. - 50}{1}$$

$$= 0$$

Newton's forward difference formula is:

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{3!}\Delta^3 y_0 + \frac{4u^3 - 18u^2 + 22u - 6}{4!}\Delta^4 y_0\right]$$

$$= 1\left[0.0244 - \frac{1}{2}(-0.0003)\right]$$

$$\boxed{\mathbf{\frac{dy}{dx} = 0.02455}}$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{12u^2 - 36u + 22}{24}\Delta^4 y_0\right]$$

$$= 1\left[-0.0003\right]$$

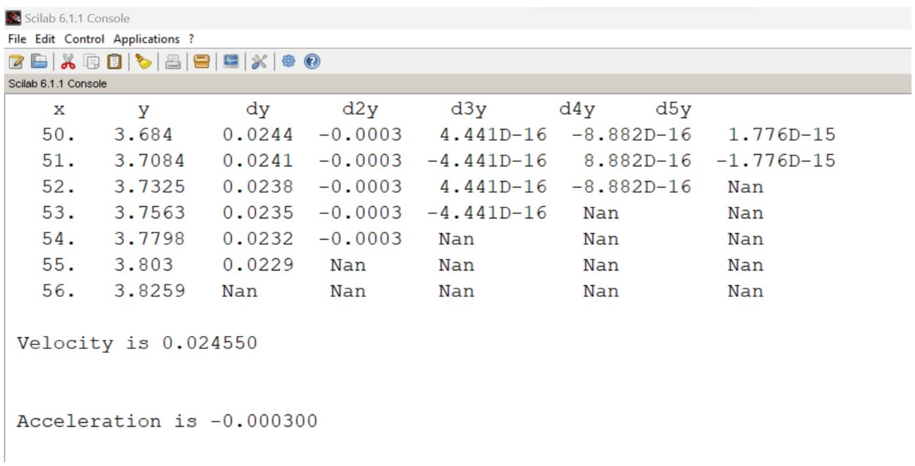
$$\boxed{\mathbf{\frac{d^2y}{dx^2} = -0.0003}}$$

Coding/Programming:

```
clc;clear all;
x=[50:56]
y=[3.684,3.7084,3.7325,3.7563,3.7798,3.8030,3.8259];
n=length(x);
xg=50;
h=x(2)-x(1);
p=(xg-x(1)/h);
d=%nan*ones(n,6);
d(:,1)=y';
for j=2:6
    for i=1:n-j+1
        d(i,j)=d(i+1,j-1)-d(i,j-1);
    end
```

```
end
mprintf("%5s %6s %9s %8s %8s %8s %7s",'x','y','dy','d2y',
                              'd3y','d4y','d5y')
disp([x' d])
dy=(1/h)*[(d(1,2)+((2*p-1)/2)*d(1,3)+((3*p^2-6*p+2)/6)*d(1,4)+
                             ((4*p^3-18*p^2+22*p-6)/24)*d(1,5))]
d2y=(1/h^2)*[(d(1,3)+(p-1)*d(1,4)+(12*p^2-36*p+22)/24*d(1,5))]
printf("\n Velocity is %4f \n\n",dy);
printf("\n Acceleration is %4f \n\n",d2y);
```

Output:



x	y	dy	d2y	d3y	d4y	d5y
50.	3.684	0.0244	-0.0003	4.441D-16	-8.882D-16	1.776D-15
51.	3.7084	0.0241	-0.0003	-4.441D-16	8.882D-16	-1.776D-15
52.	3.7325	0.0238	-0.0003	4.441D-16	-8.882D-16	Nan
53.	3.7563	0.0235	-0.0003	-4.441D-16	Nan	Nan
54.	3.7798	0.0232	-0.0003	Nan	Nan	Nan
55.	3.803	0.0229	Nan	Nan	Nan	Nan
56.	3.8259	Nan	Nan	Nan	Nan	Nan

Velocity is 0.024550

Acceleration is -0.000300

Exercise:13

6(B)NEWTON'S BACKWARD DIFFERENCE

Objective:

To find derivative for the given data using Newton's backward difference method

Input:

x	50	51	52	53	54	55	56
y	3.684	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Procedure/Methodology:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
50	3.684						
		0.0244					
51	3.7084		-0.0003				
		0.0241		0			
52	3.7325		-0.0003		0		
		0.0238		0		0	
53	3.7563		-0.0003		0		**0**
		0.0235		0		**0**	
54	3.7798		-0.0003		**0**		
		0.0232		**0**			
55	3.8030		**-0.0003**				
		0.0229					
56	**3.8259**						

The value of x at you want to find x=56

$$h = x_1 - x_0 = 51 - 50 = 1$$

$$v=\frac{x-x_n}{h}$$
$$=\frac{56-56}{1}$$
$$=0$$

Newton's backward difference formula is:

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{4v^3+18v^2+22v+6}{4!}\nabla^4 y_n\right]$$

$$= \frac{1}{1}\left[0.0229 + \frac{1}{2}(-0.0003)\right]$$

$$\boxed{\mathbf{\frac{dy}{dx} = 0.02275}}$$

$$\frac{d^2y}{dx^2 =} \frac{1}{h^2}\left[\nabla^2 y_n + \frac{(6v+6)}{3!}\nabla^3 y_n + \frac{(12v^2+36v+22)}{24}\nabla^4 y_n\right]$$

$$= \frac{1}{1}\left[-0.0003\right]$$

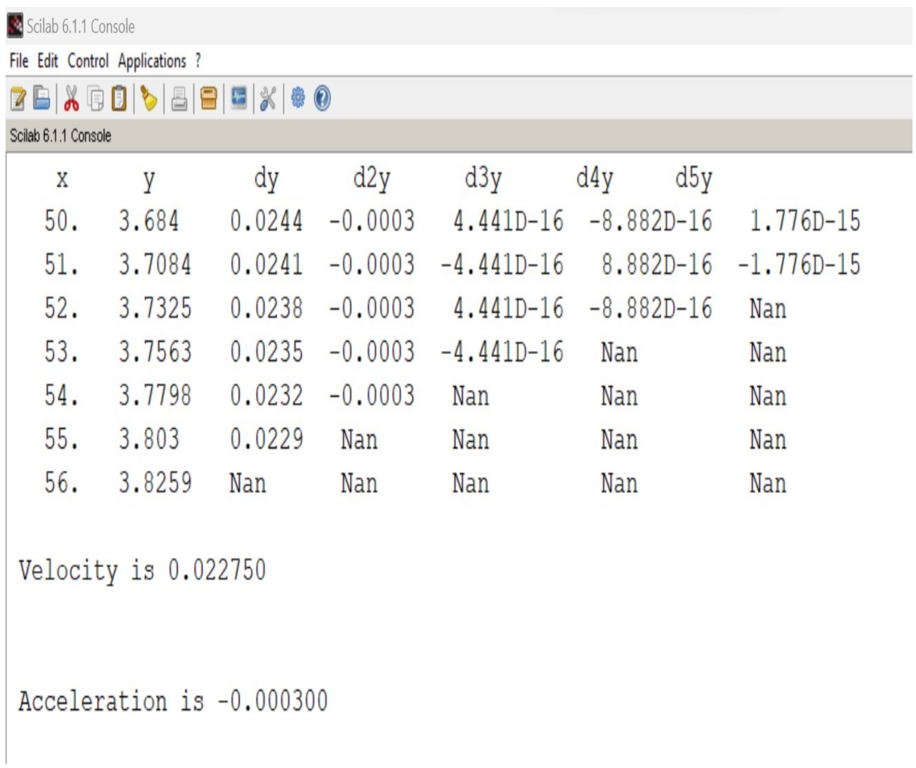
$$\boxed{\mathbf{\frac{d^2y}{dx^2} = -0.0003}}$$

Coding/Programming:

```
clc;clear all;
x=[50:56];
y=[3.684,3.7084,3.7325,3.7563,3.7798,3.8030,3.8259];
n=length(x);
xg=56;
h=x(2)-x(1);
p=(xg-x(n))/h;
d=%nan*ones(n,6);
d(:,1)=y';
for j=2:6
    for i=1:n-j+1
        d(i,j)=d(i+1,j-1)-d(i,j-1);
        end
```

```
end
mprintf("%5s %6s %9s %8s %8s %8s %7s",'x','y','dy','d2y','d3y','d4y','d5y');
disp([x' d])
dy=(1/h)*[(d(6,2)+((2*p+1)/2)*d(5,3)+((3*p^2+6*p+2)/6)*d(4,4)+
                           ((4*p^3+18*p^2+22*p+6)/24)*d(3,5))]
d2y=(1/h^2)*[(d(5,3)+((p+1)*d(4,4))+((12*p^2+36*p+22)/24)*d(3,5))]
printf("\n Velocity is %4f \n\n",dy)
printf("\n Acceleration is %4f \n\n",d2y)
```

Output:



Exercise:14

7(A)TRAPEZOIDAL RULE

Objective:

To find derivative for the given data using Trapezoidal rule

Input:

$$\int_4^{5.2} log_e x.dx$$

Procedure/Methodology:

Here a=4 ; b=5.2 ; n=6

$$h = \frac{Upperlimit-Lowerlimit}{No.of intervals}$$

$$= \frac{5.2-4}{6}$$

$$= \frac{1.2}{6}$$

$$=0.2$$

Hence the table is formulated as below:

x	4	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1}\right]$$

$$= \int_4^{5.2} log_e x.dx = \frac{0.2}{2}[(1.3863 + 1.6487) + 2(1.4351 + 1.4816$$

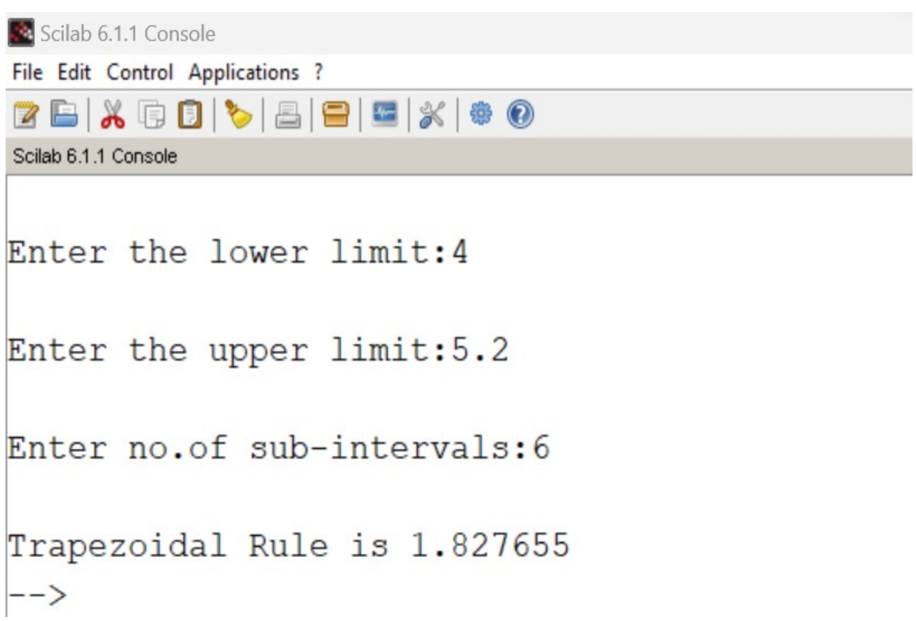
$$+1.5261+1.5686+1.6094)]$$

$$\boxed{= \int_4^{5.2} \mathbf{log_e x.dx = 1.82766}}$$

Coding/Programming:

```
clc;clear all;
deff("y=f(x)",'y=log(x)')
a=input("Enter the lower limit:");
b=input("Enter the upper limit:");
n=input("Enter no.of sub-intervals:");
h=(b-a)/n;
sum1=0;
for i=1:n-1
    x=a+i*h;
    sum1=sum1+f(x);
end
I=(h/2)*(f(a)+f(b)+2*sum1);
printf("Trapezoidal Rule is %f",I)
```

Output:



Exercise:15

7(B)SIMPSON'S $(1/3)^{\mathbf{rd}}$ RULE

Objective:

To find derivative for the given data using Simpson's $(1/3)^{rd}$ rule

Input:

$$\int_4^{5.2} log_e x.dx$$

Procedure/Methodology:

Here a=4 ; b=5.2 ; n=6

$$h = \frac{Upperlimit-Lowerlimit}{No.of intervals}$$
$$= \frac{5.2-4}{6}$$
$$= \frac{1.2}{6}$$
$$=0.2$$

Hence the table is formulated as below:

x	4	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Simpson's $(\mathbf{1/3})^{\mathbf{rd}}$rule:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}\left[(y_0 + y_n) + 2(y_2 + y_4 + \cdots) + 4(y_1 + y_3 + \cdots)\right]$$

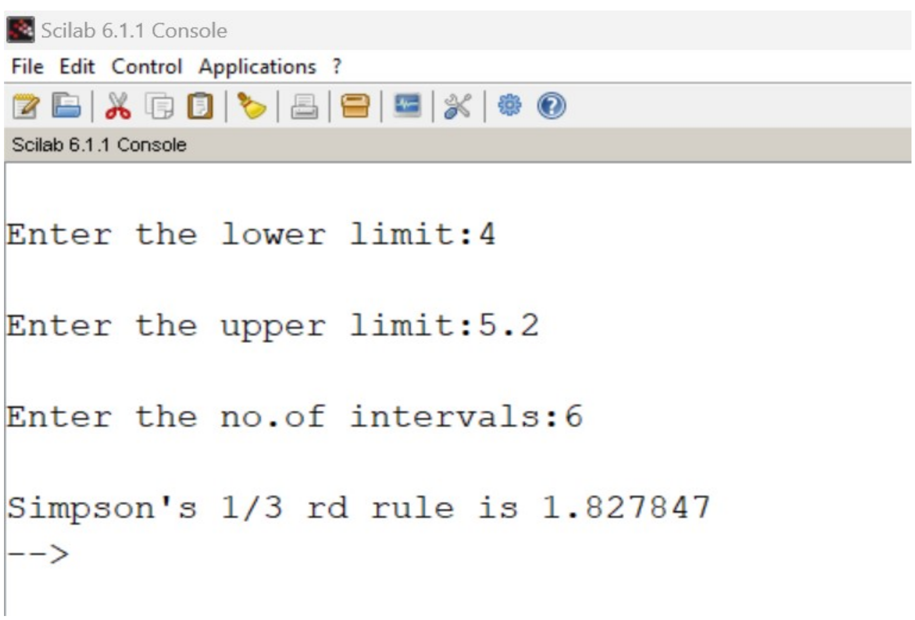
$$= \int_4^{5.2} log_e x.dx = \frac{0.2}{3}[(1.3863 + 1.6487) + 2(1.4816 + 1.5686)+$$
$$4(1.4351+1.5261+1.6094)]$$

$$\boxed{\int_4^{5.2} \mathbf{log_e x.dx = 1.8278}}$$

Coding/Programming:

```
clc;clear all;
deff("y=f(x)",'y=log(x)')
a=input("Enter the lower limit:");
b=input("Enter the upper limit:");
n=input("Enter the no.of intervals:");
h=(b-a)/n;
sum1=0;
for i=1:n-1
    x=a+i*h;
    if modulo (i,2)==0
        sum1=sum1+2*f(x);
    else
        sum1=sum1+4*f(x)
        end
end
I=(h/3)*(f(a)+f(b)+sum1);
printf("Simpson''s 1/3 rd rule is %f",I)
```

Output:



Exercise:16

7(C)SIMPSON'S $(3/8)^{\mathbf{th}}$ RULE

Objective:

To find derivative for the given data using Simpson's $(3/8)^{th}$ rule

Input:

$$\int_4^{5.2} log_e x.dx$$

Procedure/Methodology:

Here a=4 ; b=5.2 ; n=6

$$h = \frac{Upperlimit - Lowerlimit}{No.of intervals}$$
$$= \frac{5.2-4}{6}$$
$$= \frac{1.2}{6}$$
$$=0.2$$

Hence the table is formulated as below:

x	4	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Simpson's $(3/8)^{\mathbf{th}}$rule:

$$\int_{x_0}^{x_n} f(x)dx = \frac{3h}{8}\left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y + 5) + 2(y_3 + y_6 + \cdots)\right]$$

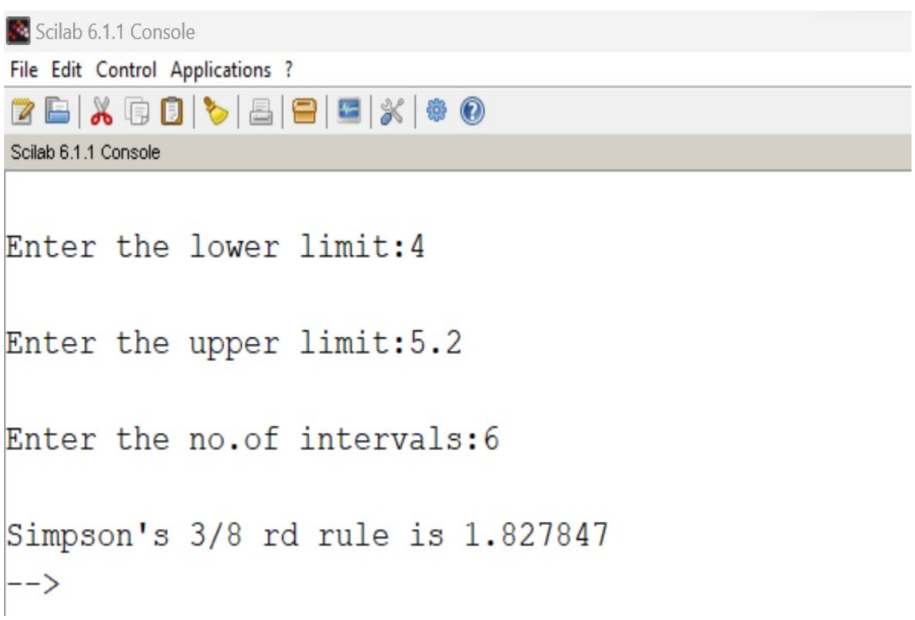
$$= \int_4^{5.2} log_e x.dx = \frac{3(0.2)}{8}[(1.3863 + 1.6487) + 3(1.4351 + 1.4816 + 1.5686+1.6094)+2(1.5261)]$$

$$\boxed{\int_4^{5.2} \mathbf{log_e x.dx = 1.8278}}$$

Coding/Programming:

```
clc;clear all;
deff("y=f(x)",'y=log(x)')
a=input("Enter the lower limit:");
b=input("Enter the upper limit:");
n=input("Enter the no.of intervals:");
h=(b-a)/n;
sum1=0;
for i=1:n-1
    x=a+i*h;
    if modulo (i,3)==0
        sum1=sum1+2*f(x);
    else
        sum1=sum1+3*f(x)
        end
end
I=(3*h/8)*(f(a)+f(b)+sum1);
printf("Simpson''s 3/8 rd rule is %f",I)
```

Output:



MODULE 4

Exercise:17

8(A)TAYLOR SERIES

Objective:

To find solution of given differential equation using Taylor's method

Input:

$\frac{dy}{dx}$=1-2xy with h=0.2 and y(0)=0

Procedure/Methodology:

h=0.2
Here f(x,y)=1-2xy
y(0)=0
$\implies x_0 = 0\ ,\ y_0 = 0$

Now,

$$x_1 = x_0 + h = 0 + 0.2 + 0.2$$
$$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$$

Taylor series:

$$y_{n+1} = y_n + \frac{h}{1!}y_n' + \frac{h^2}{2!}y_n'' + \frac{h^3}{3!}y_n''' + \cdots \tag{9}$$

Put n=0 in (1) $\implies y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \cdots$

Put n=1 in (1) $\implies y_2 = y_1 + \frac{h}{1!}y_1' + \frac{h^2}{2!}y_1'' + \cdots$

$$y' = 1 - 2xy$$
$$y''\text{=-}2[xy' + y.1]$$
$$\text{=-}2(xy' + y)$$
$$y'''\text{=-}2[xy'' + y'.1 + y']$$
$$\text{=-}2(xy'' + 2y')$$
$$y^{IV}\text{=-}2[xy'' + y''.1 + 2y'']$$
$$\text{=-}2(xy''' + 3y'')$$
$$y^{V}\text{=-}2[xy^{IV}+y'''.1 + 3y''']$$
$$\text{=-}2(xy^{IV} + 4y''')$$

To find $y_1 = y(x_1) = y(0.2)$

$$\text{At } x_0 = 0; y_0 = 0$$
$$y' = 1 - 2x_0y_0$$
$$=1\text{-}2(0)(0)$$
$$=1$$
$$y_0'' = -2[x_0y_0' + y_0]$$
$$\text{=-}2[(0)(1)+0]$$
$$=0$$
$$y_0''' = -2[x_0y_0'' + 2y_0']$$
$$\text{=-}2[(0)(0)+2(1)]$$
$$\text{=-}4$$
$$y_0^{IV} = -2[x_0y_0''' + 3y_0'']$$
$$\text{=-}2[(0)(\text{-}4)+3(0)]$$
$$=0$$
$$y_0^{V} = -2[x_0y_0IV + 4y_0''']$$
$$\text{=-}2[(0)(0)+4(\text{-}4)]$$
$$=32$$

Now,

$$y_1 = y_0 + \tfrac{h}{1!}y_0' + \tfrac{h^2}{2!}y_0'' + \cdots$$

$$y_1 = 0 + \tfrac{0.2}{1!}(1) + \tfrac{(0.2)^2}{2!}(0) + \tfrac{(0.2)^3}{3!}(-4) + \tfrac{(0.2)^4}{4!}(0) + \tfrac{(0.2)^5}{5!}(32) + \cdots$$

=0.2-0.00533+0.00008

$$\boxed{\mathbf{y_1 = y(x_1) = y(0.2) = 0.19475}}$$

To find $y_2 = y(x_2) = y(0.4)$

$$\text{At } x_1 = 0.2; y_1 = 0.19475$$

$$y' = 1 - 2x_1y_1$$

=1-2(0.2)(0.19475)

=0.9221

$$y_1{''} = -2[x_1y_1{'} + y_1]$$

=-2[(0.2)(0.9221)+0.19475]

=-0.75834

$$y_1{'''} = -2[x_1y_1'' + 2y_1']$$

=-2[(0.2)(-0.75834)+2(0.9221)]

=-3.385064

$$y_1^{IV} = -2[x_1y_1''' + 3y_1'']$$

=-2[(0.2)(-3.385064)+3(-0.75834)]

=5.9040656

Now,

$$y_2 = y_1 + \tfrac{h}{1!}y_1' + \tfrac{h^2}{2!}y_1'' + \cdots$$

$$y_2 = 0.19475 + \tfrac{0.2}{1!}(0.9221) + \tfrac{(0.2)^2}{2!}(-0.75834) + \tfrac{(0.2)^3}{3!}(-3.385064) + \tfrac{(0.2)^4}{4!}(5.9040656) + \cdots$$

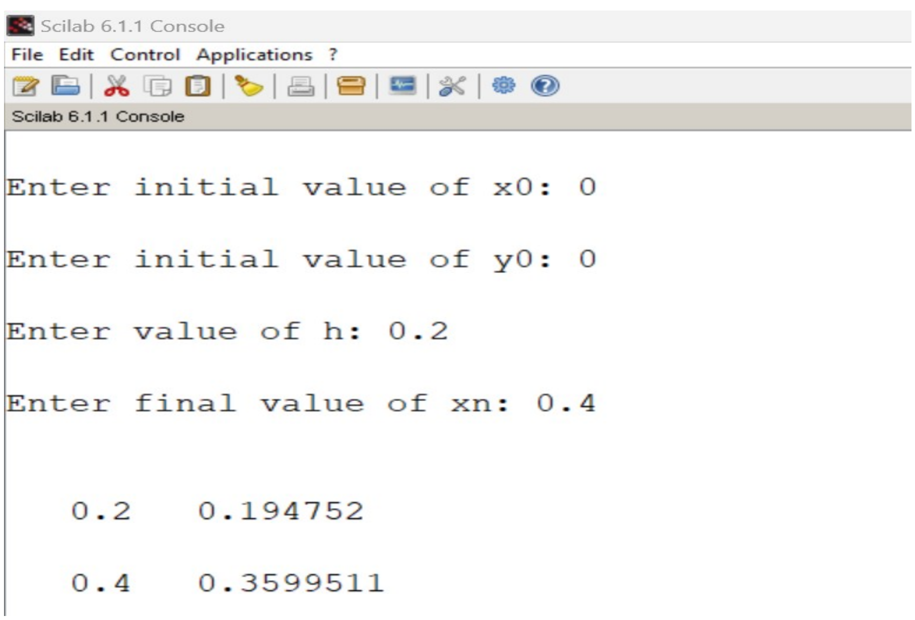
=0.19475+0.18442-0.0151668-0.00451342+0.000393604

$$\boxed{\mathbf{y_2 = y(x_2) = y(0.4) = 0.35988}}$$

Coding/Programming:

```
clc; clear all;
deff("z=f(x,y)","z=1-2*x*y")
deff("z1=f1(x,y)","z1=-2*(x*f(x,y)+y)")
deff("z2=f2(x,y)","z2=-2*(x*f1(x,y)+2*f(x,y))")
deff("z3=f3(x,y)","z3=-2*(x*f2(x,y)+3*f1(x,y))")
deff("z4=f4(x,y)","z4=-2*(x*f3(x,y)+4*f2(x,y))")
x0=input("Enter initial value of x0: ")
y0=input("Enter initial value of y0: ")
h=input("Enter value of h: ")
xn=input("Enter final value of xn: ")
N=(xn-x0)/h
for i=1:N
y1=y0+h*f(x0,y0)+((h^2)/2)*f1(x0,y0)+((h^3)/6)*f2(x0,y0)+
               ((h^4)/24)*f3(x0,y0)+((h^5)/120)*f4(x0,y0)
x0=x0+h
disp([x0 y1])
y0=y1
end
```

Output:



Exercise:18

8(B_1)EULER METHOD

Objective:

To find solution of given differential equation using Euler's method

Input:

y'=x+y ; y(0)=1 for x=0(0.2)(1.0)

Procedure/Methodology:

y'=x+y
Here f(x,y)=x+y
y(0)=1
$\implies x_0 = 0$, $y_0 = 1$
Here h=0.2

To find values of y when x=0.2,0.4,0.6,0.8,1.0
Now, $x_0 = 0$
Then,

$$x_1 = x_0 + h = 0.2$$
$$x_2 = x_1 + h = 0.4$$
$$x_3 = x_2 + h = 0.6$$
$$x_4 = x_3 + h = 0.8$$
$$x_5 = x_4 + h = 1.0$$

(i.e)

To find : $y_1 = y(x_1)$; $y_2 = y(x_2)$; $y_3 = y(x_3)$; $y_4 = y(x_4)$; $y_5 = y(x_5)$

Euler method formula is:

$$y_{n+1} = hf(x_n, y_n)$$

<u>Value of $y_1 = y(0.2)$</u>

$$y_1 = y_0 + hf(x_0, y_0) \text{ by putting n=0 in Euler's formula}$$
$$=1+(0.2)f(0,1)$$
$$=1+0.2[0+1]$$
$$=1.2$$

<u>Value of $y_2 = y(0.4)$</u>

$$y_2 = y_1 + hf(x_1, y_1) \text{ by putting n=1 in Euler's formula}$$
$$=1.2+(0.2)f(0.2,1.2)$$
$$=1.2+0.2[0.2+1.2]$$
$$=1.48$$

<u>Value of $y_3 = y(0.6)$</u>

$$y_3 = y_2 + hf(x_2, y_2) \text{ by putting n=2 in Euler's formula}$$
$$=1.48+(0.2)f(0.4,1.48)$$
$$=1.48+0.2[0.4+1.48]$$
$$=1.856$$

<u>Value of $y_4 = y(0.0.8)$</u>

$$y_4 = y_3 + hf(x_3, y_3) \text{ by putting n=3 in Euler's formula}$$
$$=1.856+(0.2)f(0.6,1.856)$$
$$=1.856+0.2[0.6+1.856]$$
$$=2.3472$$

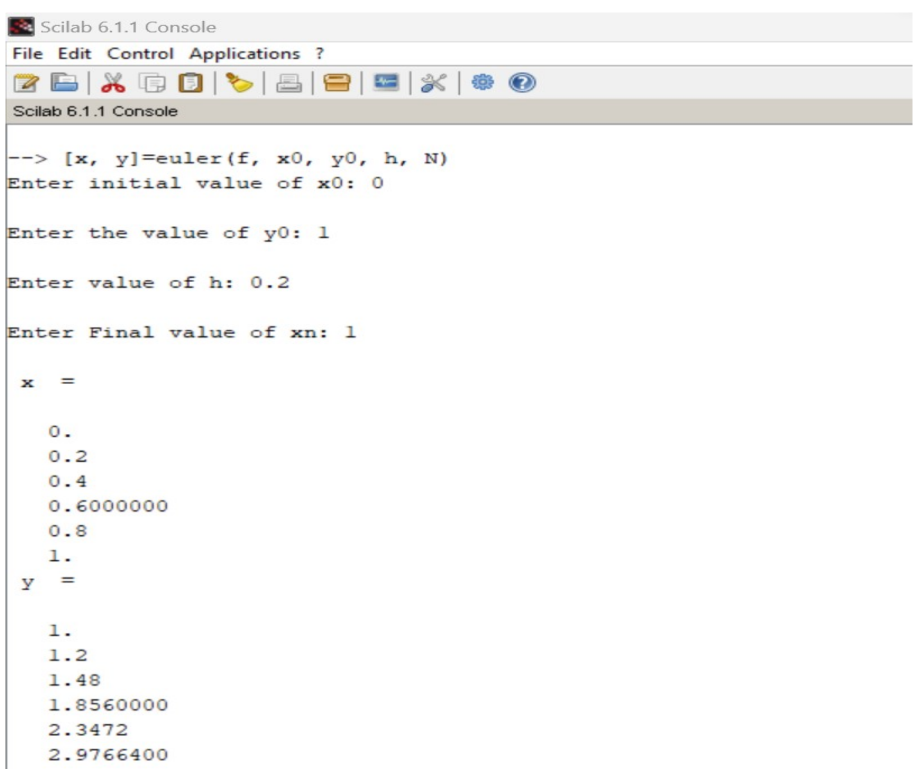
<u>Value of $y_5 = y(1.0)$</u>

$$y_5 = y_4 + hf(x_4, y_4) \text{ by putting n=4 in Euler's formula}$$
$$=2.3472+(0.2)f(0.8,2.3472)$$
$$=2.3472+0.2[0.8+2.3472]$$
$$=2.97664$$

Coding/Programming:

```
clc; clear all;
deff("z=f(x,y)","z=x+y")
function [x, y]=euler(f, x0, y0, h, N)
x0 = input("Enter initial value of x0: ")
y0 = input("Enter the value of y0: ")
h = input("Enter value of h: ")
xn = input("Enter Final value of xn: ")
N = (xn - x0)/h
x = zeros(N+1, 1)
y = zeros(N+1,1)
x(1) = x0
y(1) = y0
for j = 1:N
x(j+1) = x(j) + h
y(j + 1) = y(j) + h*f(x(j), y(j))
end
endfunction
```

Output:



Exercise:19

8(B_2)IMPROVED EULER METHOD

Objective:

To find solution of given differential equation using Improved Euler's method

Input:

y'=$y - \frac{2x}{y}$ at x=0.1,0.2 and y(0)=1

Procedure/Methodology:

Here f(x,y)=$y - \frac{2x}{y}$

y(0)=1$\Longrightarrow$ $x_0 = 0; y_0 = 1$

h=0.1+0

=0.1

$\Longrightarrow$

$x_1 = x_0 + h$

=0+0.1

=0.1

$x_2 = x_1 + h$

=0.1+0.1

=0.2

To find values of y when x=0.1 and 0.2
(i.e)

To find : y(0.1)=y(x_1)=y_1 ; y(0.2)=y(x_2)=y_2

Improved Euler method formula is:

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))\right]$$

To find y_1:

$$x_0 = 0; y_0 = 1$$

$$y_1 = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_0 + h, y_0 + hf(x_0, y_0))\right] putting \quad n = 0 \quad in$$
$$Improved \quad Euler's \quad formula$$

f(x_0, y_0)=f(0,1)

=1-$\frac{2(0)}{1}$

=1

x_0+h=0+0.1

=0.1

$\Longrightarrow$

$y_0 + hf(x_0, y_0)$=1+(0.1)(1)

=1.1

Now,

$f(x_0 + h, y_0 + hf(x_0, y_0)$=f(0.1,1.1)

=1.1-$\frac{2(0.1)}{1.1}$

=0.91818

$\therefore y_1$=1+$\frac{0.1}{2}$$[1 + 0.91818]$

$$\boxed{\mathbf{y_1 = y(x_1) = y(0.1) = 1.095901}}$$

To find y_2:

$$x_1 = 0.1; y_1 = 1.095909$$

$$y_2 = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_1 + h, y_1 + hf(x_1, y_1))\right] putting \quad n = 1 \quad in$$
$$Improved \quad Euler's \quad formula$$

f(x_1, y_1)=f(0.1,1.095909)

=1.095909-$\frac{2(0.1)}{1.095909}$

=0.913412

x_1+h=0.1+0.1

=0.2

$\Longrightarrow$

$$y_1 + hf(x_1, y_1) = 1.095909 + (0.1)(0.913412)$$
$$= 1.1872502$$

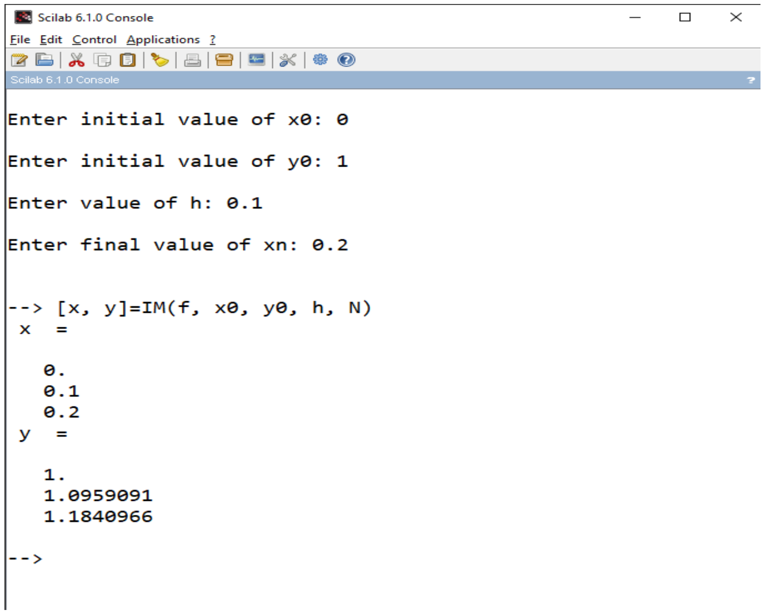
Now,

$$f(x_1 + h, y_1 + hf(x_1, y_1) = \text{f}(0.2, 1.1872502)$$
$$= 1.872502 - \frac{2(0.2)}{1.1872502}$$
$$= 0.8503372$$
$$\therefore y_2 = 1.095909 + \frac{0.1}{2}[0.913412 + 0.8503372]$$
$$\boxed{\mathbf{y_2 = y(x_2) = y(0.2) = 1.18409646}}$$

Coding/Programming:

```
clc; clear all;
function ydot=f(x, y)
    ydot=(y-((2*x)/y))
endfunction
x0 = input("Enter initial value of x0: ")
y0 = input("Enter initial value of y0: ")
h = input("Enter value of h: ")
xn = input("Enter final value of xn: ")
N = (xn - x0)/h
function [x, y]=IM(f, x0, y0, h, N)
    x = zeros(N+1,1)
    y = zeros(N+1,1)
    x(1) = x0
    y(1) = y0
    for j = 1:N
        x(j+1) = x(j)+h
        y(j+1) = y(j)+(h/2)*(f(x(j),y(j))+f(x(j)+h,y(j)+h*f(x(j),y(j))))
    end
endfunction
```

Output:


Scilab 6.1.0 Console
File Edit Control Applications ?
Scilab 6.1.0 Console
Enter initial value of x0: 0
Enter initial value of y0: 1
Enter value of h: 0.1
Enter final value of xn: 0.2
--> [x, y]=IM(f, x0, y0, h, N)
x =
0.
0.1
0.2
y =
1.
1.0959091
1.1840966
-->

Exercise:20

8(B_3)MODIFIED EULER METHOD

Objective:

To find solution of given differential equation using Modified Euler's method

Input:

$$y' = x^2 + y^2 \text{ ; h=0.1 and y(0)=1}$$

Procedure/Methodology:

$$\text{Here f(x,y)}=x^2 + y^2$$

$$\text{y(0)=1} \implies x_0 = 0; y_0 = 1$$

h=0.1 $\implies$

$$x_1 = x_0 + h\text{=0+0.1=0.1}$$
$$x_2 = x_1 + h\text{=0.1+0.1=0.2}$$

Modified Euler formula is:

$$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2}, y_n + \frac{h}{2}f(x_n, y_n)\right]$$

To find y_1:

$$x_0 = 0; y_0 = 1$$

$$y_1 = y_0 + hf\left[x_0 + \frac{h}{2}, y_0 + \frac{h}{2}f(x_0, y_0)\right] putting \quad n = 0$$

$$x_0 + \frac{h}{2} = 0 + \frac{0.1}{2}$$
$$=0.05$$
$$f(x_0, y_0) = f(0,1) = 0^2 + 1^2$$
$$=1$$
$$\frac{h}{2} f(x_0, y_0) = \frac{0.1}{2}(1)$$
$$=0.05$$
$$y_0 + \frac{h}{2} f(x_0, y_0) = 1 + 0.05$$
$$=1.05$$

Now,

$$y_1 = 1 + (0.1) f(0.05, 1.05)$$
$$= 1 + (0.1)[(0.05)^2 + (1.05)^2]$$
$$= 1 + (0.1)(1.105)$$
$$\boxed{\mathbf{y_1 = y(x_1) = y(0.1) = 1.1105}}$$

<u>To find y_2:</u>

$$x_1 = 0.1; y_1 = 1.1105$$

$$y_1 = y_1 + hf\left[x_1 + \frac{h}{2}, y_1 + \frac{h}{2} f(x_1, y_1)\right] putting \quad n = 1$$

$$x_1 + \frac{h}{2} = 0.1 + \frac{0.1}{2}$$
$$=0.15$$
$$f(x_1, y_1) = f(0.1, 1.1105) = (0.1)^2 + (1.1105)^2$$
$$=1.2432$$
$$\frac{h}{2} f(x_1, y_1) = \frac{0.1}{2}(1.2432)$$
$$=0.06216$$
$$y_1 + \frac{h}{2} f(x_1, y_1) = 1.1105 + 0.06216$$
$$=1.17266$$

Now,

$$y_2 = 1.1105 + (0.1) f(0.15, 1.17266)$$
$$= 1.1105 + (0.1)[(0.15)^2 + (1.17266)^2]$$
$$= 1.1105 + (0.1)(1.3976)$$
$$\boxed{\mathbf{y_2 = y(x_2) = y(0.2) = 1.25026}}$$

Coding/Programming:

```
clc;clear all;
deff("z=f(x,y)","z=((x^2)+(y^2))")
function [x,y]=modeuler(f,x0,y0,h,N)
    x0=input("Enter initial value of x0:")
    y0=input("Enter initial value of y0: ")
    h=input("Enter value of h: ")
    xn=input("Enter final value of xn: ")
    N=(xn-x0)/h
    x=zeros(N+1,1)
    y=zeros(N+1,1)
    x(1)=x0
    y(1)=y0
    for j=1:N
        x(j+1)=x(j)+h
        y(j+1)=y(j)+h*f(x(j)+(h/2),y(j)+(h/2)*f(x(j),y(j)))
    end
endfunction
```

Output:

```
--> [x,y]=modeuler(f,x0,y0,h,N)
Enter initial value of x0:0

Enter initial value of y0: 1

Enter value of h: 0.1

Enter final value of xn: 0.2

 x  =

   0.
   0.1
   0.2
 y  =

   1.
   1.1105
   1.2502633
```

Exercise:21

8(C_1)RUNGE-KUTTA 2nd ORDER METHOD

Objective:

To find solution of given differential equation using Rungee-Kutta 2^{nd} order method

Input:

$$y'=\frac{y^2-x^2}{y^2+x^2} \text{ at x=0.2,0.4 and y(0)=1}$$

Procedure/Methodology:

$$\text{Here f(x)}=\frac{y^2-x^2}{y^2+x^2}$$

$$\text{y(0)=1} \implies x_0\text{=0;}y_0\text{=1}$$

h=0.2 $\implies$

$$x_1 = x_0 + h\text{=0+0.2=0.2}$$

$$x_2 = x_1 + h\text{=0.2+0.2=0.4}$$

<u>To find y_1</u>

$$x_0 = 0; y_0 = 1$$

$$k_1 = hf(x_0, y_0)$$

$$=0.2f(0,1)$$

$$= 0.2\left[\frac{(1)^2-(0)^2}{(1)^2+(0)^2}\right]$$

$$\boxed{k_1 = 0.2}$$

$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1)$$

$$=0.2f(0+\tfrac{1}{2}(0.2), 1+\tfrac{1}{2}(0.2))$$

$$=0.2f(0.1, 1.1)$$

$$= 0.2\left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$$

$$\boxed{k_2 = 0.19672}$$

$$\Delta y = k_2 \implies y(x+h) = y + \Delta y$$

$$y(x+h)=y_0 + \Delta y$$

$$=1+0.19672$$

$$\boxed{\mathbf{y_1 = 1.19672}}$$

<u>To find y_2</u>

$$x_1 = 0.2; y_1 = 1.19672$$

$$k_1 = hf(x_1, y_1)$$

$$=0.2f(0.2, 1.19672)$$

$$= 0.2\left[\frac{(1.19672)^2 - (0.2)^2}{(1.19672)^2 + (0.2)^2}\right]$$

$$\boxed{k_1 = 0.18913}$$

$$k_2 = hf(x_1 + \tfrac{1}{2}h_1, y_1 + \tfrac{1}{2}k_1)$$

$$=0.2f(0.2+\tfrac{1}{2}(0.2), 1.19672+\tfrac{1}{2}(0.18913))$$

$$=0.2f(0.3, 1.291285)$$

$$= 0.2\left[\frac{(1.291285)^2 - (0.3)^2}{(1.291285)^2 + (0.3)^2}\right]$$

$$\boxed{k_2 = 0.17951}$$

$\Delta y = k_2 \implies y(x+h) = y + \Delta y$

$$y(x+h)=y_1 + \Delta y$$

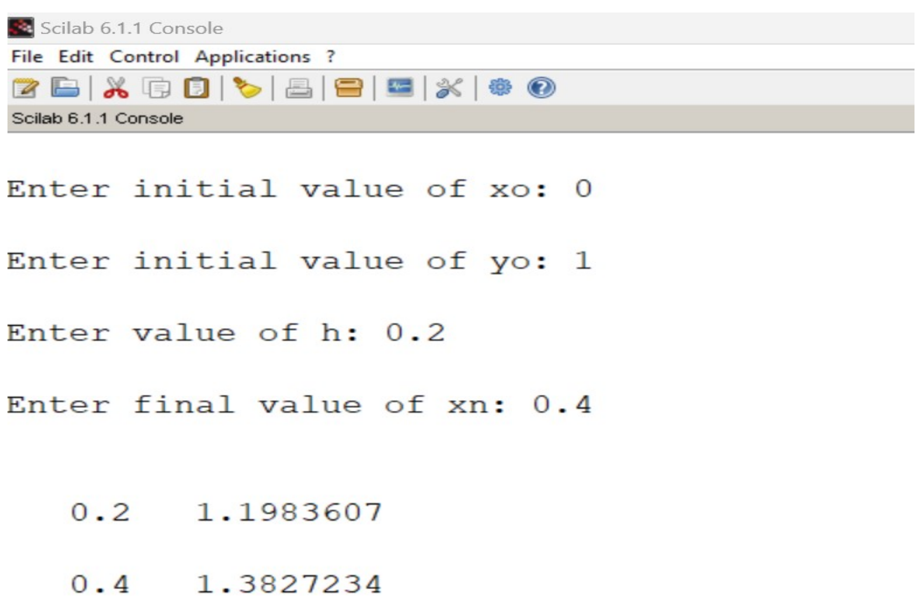
$$=1.19672+0.17951$$

$$\boxed{\mathbf{y_2 = 1.37623}}$$

Coding/Programming:

```
clc; clear all;
deff("z=f(x,y)","z=((y^2-x^2)/(y^2+x^2))")
x0=input("Enter initial value of x0: ")
y0=input("Enter initial value of y0: ")
h=input("Enter value of h: ")
xn=input("Enter final value of xn: ")
n=(xn-x0)/h
for i=1:n
k1=h*f(x0,y0)
k2=h*f(x0+(h/2),y0+(k1/2))
y1=y0+0.5*(k1+k2)
x0=x0+h
disp([x0 y1])
y0=y1
end
```

Output:



Exercise:22

8(C_2)RUNGE-KUTTA 3rd ORDER METHOD

Objective:

To find solution of given differential equation using Rungee-Kutta 3^{rd} order method

Input:

$$y'=\frac{y^2-x^2}{y^2+x^2} \text{ at x=0.2,0.4 and y(0)=1}$$

Procedure/Methodology:

$$\text{Here f(x)}=\frac{y^2-x^2}{y^2+x^2}$$

$$\text{y(0)=1} \implies x_0\text{=0;}y_0\text{=1}$$

h=0.2 $\implies$

$$x_1 = x_0 + h\text{=0+0.2=0.2}$$

$$x_2 = x_1 + h\text{=0.2+0.2=0.4}$$

To find y_1

$$x_0 = 0; y_0 = 1$$

$$k_1 = hf(x_0, y_0)$$

$$\text{=0.2f(0,1)}$$

$$= 0.2\left[\frac{(1)^2-(0)^2}{(1)^2+(0)^2}\right]$$

$$\boxed{k_1 = 0.2}$$

$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1)$$
$$=0.2f(0+\tfrac{1}{2}(0.2),1+\tfrac{1}{2}(0.2))$$
$$=0.2f(0.1,1.1)$$
$$= 0.2\left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$$
$$\boxed{k_2 = 0.19672}$$
$$k_3 = hf(x_0 + h, y_0 + 2k_2 - k_1)$$
$$=0.2f(0+0.2,1+2(0.196720-0.2)$$
$$=0.2f(0.2,1.19344)$$
$$= 0.2\left[\frac{(1.19344)^2 - (0.2)^2}{(1.19344)^2 + (0.2)^2}\right]$$
$$\boxed{k_3 = 0.18907}$$
$$\Delta y = \tfrac{1}{6}(k_1 + 4k_2 + k_3)$$
$$=\tfrac{1}{6}(0.2 + 4(0.19672) + 0.18907)$$
$$\boxed{\Delta y = 0.19599}$$

$\Longrightarrow$

$$y(x+h) = y_0 + \Delta y$$
$$=1+0.19599$$
$$\boxed{\mathbf{y_1 = 1.19599}}$$

<u>To find y_2</u>

$$x_1 = 0.2; y_0 = 1.19599$$
$$k_1 = hf(x_1, y_1)$$
$$=0.2f(0.2,1.19599)$$
$$= 0.2\left[\frac{(1.19599)^2 - (0.2)^2}{(1.19599)^2 + (0.2)^2}\right]$$
$$\boxed{k_1 = 0.18912}$$

$$k_2 = hf(x_1 + \tfrac{1}{2}h, y_1 + \tfrac{1}{2}k_1)$$

$$=0.2f(0.2+\tfrac{1}{2}(0.2),1.19599+\tfrac{1}{2}(0.18912))$$

$$=0.2f(0.3,1.29055)$$

$$= 0.2\left[\frac{(1.29055)^2 - (0.3)^2}{(1.29055)^2 + (0.3)^2}\right]$$

$$\boxed{k_2 = 0.17949}$$

$$k_3 = hf(x_1 + h, y_1 + 2k_2 - k_1)$$

$$=0.2f(0.2+0.2,1.19599+2(0.17949-0.18912)$$

$$=0.2f(0.4,1.36585)$$

$$= 0.2\left[\frac{(1.36585)^2 - (0.4)^2}{(1.36585)^2 + (0.4)^2}\right]$$

$$\boxed{k_3 = 0.16840}$$

$$\Delta y = \tfrac{1}{6}(k_1 + 4k_2 + k_3)$$

$$=\tfrac{1}{6}(0.18912 + 4(0.17949) + 0.16840)$$

$$\boxed{\Delta y = 0.17925}$$

$\Longrightarrow$

$$y(x+h) = y_0 + \Delta y)$$

$$=1.19599+0.17925$$

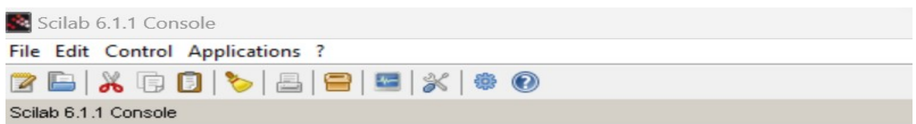
$$\boxed{\mathbf{y_2 = 1.37524}}$$

Coding/Programming:

```
clc; clear all;
deff("z=f(x,y)","z=((y^2-x^2)/(y^2+x^2))")
x0=input("Enter initial value of x0: ")
y0=input("Enter initial value of y0: ")
h=input("Enter value of h: ")
xn=input("Enter final value of xn: ")
n=(xn-x0)/h
for i=1:n
k1=h*f(x0,y0)
k2=h*f(x0+(h/2),y0+(k1/2))
```

```
k3=h*f(x0+h,y0+2*k2-k1)
y1=y0+(1/6)*(k1+4*k2+k3)
x0=x0+h
disp([x0 y1])
y0=y1
end
```

Output:




```
Enter initial value of x0: 0

Enter initial value of y0: 1

Enter value of h: 0.2

Enter final value of xn: 0.4


    0.2    1.1959931

    0.4    1.3752424
```

Exercise:23

RUNGE-KUTTA 4^{th} ORDER METHOD

Objective:

To find solution of given differential equation using Rungee-Kutta 4^{th} order method

Input:

$$y'=\frac{y^2-x^2}{y^2+x^2} \text{ at x=0.2,0.4 and y(0)=1}$$

Procedure/Methodology:

$$\text{Here f(x)=}\frac{y^2-x^2}{y^2+x^2}$$

$$\text{y(0)=1} \implies x_0\text{=0;}y_0\text{=1}$$

h=0.2 $\implies$

$$x_1 = x_0 + h\text{=0+0.2=0.2}$$

$$x_2 = x_1 + h\text{=0.2+0.2=0.4}$$

To find y_1

$$x_0 = 0; y_0 = 1$$

$$k_1 = hf(x_0, y_0)$$

$$\text{=0.2f(0,1)}$$

$$= 0.2\left[\frac{(1)^2-(0)^2}{(1)^2+(0)^2}\right]$$

$$\boxed{k_1 = 0.2}$$

$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1)$$

$$=0.2f(0+\tfrac{1}{2}(0.2),1+\tfrac{1}{2}(0.2))$$

$$=0.2f(0.1,1.1)$$

$$= 0.2\left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$$

$$\boxed{k_2 = 0.19672}$$

$$k_3 = hf(x_0 + \tfrac{h}{2}, y_0 + \tfrac{k_2}{2}$$

$$=0.2f(0+\tfrac{0.2}{2},1+\tfrac{0.19672}{2})$$

$$=0.2f(0.1,1.09836)$$

$$= 0.2\left[\frac{(1.09836)^2 - (0.1)^2}{(1.09836)^2 + (0.1)^2}\right]$$

$$\boxed{k_3 = 0.19671}$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$=0.2f(0+0.2,1+0.19671)$$

$$=0.2f(0.2,1.19671)$$

$$= 0.2\left[\frac{(1.19671)^2 - (0.2)^2}{(1.19671)^2 + (0.2)^2}\right]$$

$$\boxed{k_4 = 0.1891}$$

$$\Delta y = \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$=\tfrac{1}{6}[0.2+2(0.19672)+2(0.19671)+0.1891]$$

$$\boxed{\Delta y = 0.19599}$$

$$y(x+h)=y_0 + \Delta y$$

$$=1+0.19599$$

$$\boxed{\mathbf{y_1 = 1.19599}}$$

To find y_2

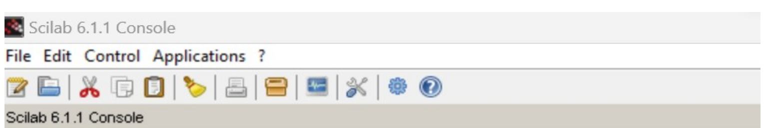
$$x_1 = 0.2; y_1 = 1.19599$$
$$k_1 = hf(x_1, y_1)$$
$$=0.2f(0.2,1.19599)$$
$$= 0.2\left[\frac{(1.19599)^2 - (0.2)^2}{(1.19599)^2 + (0.2)^2}\right]$$
$$\boxed{k_1 = 0.1891}$$
$$k_2 = hf(x_1 + \tfrac{h}{2}, y_1 + \tfrac{k_1}{2})$$
$$=0.2f(0.2+\tfrac{0.2}{2},1.19599+\tfrac{0.1891}{2})$$
$$=0.2f(0.3,1.29054)$$
$$= 0.2\left[\frac{(1.29054)^2 - (0.3)^2}{(1.29054)^2 + (0.3)^2}\right]$$
$$\boxed{k_2 = 0.17949}$$
$$k_3 = hf(x_1 + \tfrac{h}{2}, y_1 + \tfrac{k_2}{2}$$
$$=0.2f(0.2+\tfrac{0.2}{2},1.19599+\tfrac{0.17949}{2})$$
$$=0.2f(0.3,1.28573)$$
$$= 0.2\left[\frac{(1.28573)^2 - (0.3)^2}{(1.28573)^2 + (0.3)^2}\right]$$
$$\boxed{k_3 = 0.179347}$$
$$k_4 = hf(x_1 + h, y_1 + k_3)$$
$$=0.2f(0.2+0.2,1.19599+0.179347)$$
$$=0.2f(0.4,1.375337)$$
$$= 0.2\left[\frac{(1.375337)^2 - (0.4)^2}{(1.375337)^2 + (0.4)^2}\right]$$
$$\boxed{k_4 = 0.16880}$$
$$\Delta y = \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$=\tfrac{1}{6}[0.1891+2(0.17949)+2(0.179347)+0.1688]$$
$$\boxed{\Delta y = 0.1792}$$

$$y(x+h)=y_1+\Delta y$$
$$=1.19599+0.1792$$
$$\boxed{\mathbf{y_2 = 1.37519}}$$

Coding/Programming:

```
clc; clear all;
deff('z=f(x,y)','z=(y^2-x^2)/(y^2+x^2)')
x0=input("Enter initial value of x0: ")
y0=input("Enter the value of y0: ")
h=input("Enter value of h: ")
xn=input("Enter Final value of xn: ")
n=(xn-x0)/h
for i=1:n
    k1=h*f(x0,y0)
    k2=h*f(x0+(h/2),y0+(k1/2))
    k3=h*f(x0+(h/2),y0+(k2/2))
    k4=h*f(x0+h,y0+k3)
    y1=y0+(1/6)*(k1+2*k2+2*k3+k4)
    x0=x0+h
    disp([x0 y1])
    y0=y1
end
```

Output:




```
Enter initial value of x0: 0

Enter the value of y0: 1

Enter value of h: 0.2

Enter Final value of xn: 0.4


   0.2   1.1959995

   0.4   1.3752671
```

Exercise:24

8(D_1)MILNE'S PREDICTOR-CORRECTOR METHOD

Objective:

To find the solution of the given derivative using Milne's predictor-corrector method.

Input:

$$5xy' + y^2 - 2 = 0$$

Find y(4.4) with given

y(4)=1; y(4.1)=1.0049; y(4.2)=1.0097; y(4.3)=1.0143

Procedure/Methodology:

$$5xy' + y^2 - 2 = 0$$
$$y' = \frac{2-y^2}{5x}$$

$$y(4)=1 \implies x_0 = 4; y_0 = 1$$

Here h=0.1 $\implies$

$$x_1 = x_0 + h = 4+0.1 = 4.1$$
$$x_2 = x_1 + h = 4.1+0.1 = 4.2$$
$$x_3 = x_2 + h = 4.2+0.1 = 4.3$$
$$x_4 = x_3 + h = 4.3+0.1 = 4.4$$

(i.e)

$$x_0 = 4; y_0 = 1$$
$$x_1 = 4.1; y_1 = 1.0049$$
$$x_2 = 4.2; y_2 = 1.0097$$
$$x_3 = 4.3; y_3 = 1.0143$$

Milne's Predictor method formula is:

$$y_{n+1}, p = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

Put n=3

$$y_4, p = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3)$$
$$y'_1 = \frac{2-y_1^2}{5x_1}$$
$$=\frac{2-(1.0049)^2}{5(4.1)}$$
$$=0.04830$$
$$y'_2 = \frac{2-y_2^2}{5x_2}$$
$$=\frac{2-(1.0097)^2}{5(4.2)}$$
$$=0.0466$$
$$y'_3 = \frac{2-y_3^2}{5x_3}$$
$$=\frac{2-(1.0143)^2}{5(4.3)}$$
$$=0.04517$$

$\Longrightarrow$

$$y_4, p = 1 + \frac{4(0.1)}{3}[2(0.04830) - 0.0466 + 2(0.04517)]$$
$$\boxed{\mathbf{y_4, p = 1.018712}}$$

Milne's corrector method formula is:

$$y_{n+1}, c = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1})$$

Put n=3

$$y_4, c = y + 2 + \frac{h}{3}(y'_2 + 4y'_3 + y'_4)$$
$$y'_4 = \frac{2-y_4^2}{5x_4}$$
$$=\frac{2-(1.018712)^2}{5(4.4)}$$
$$=0.04373$$

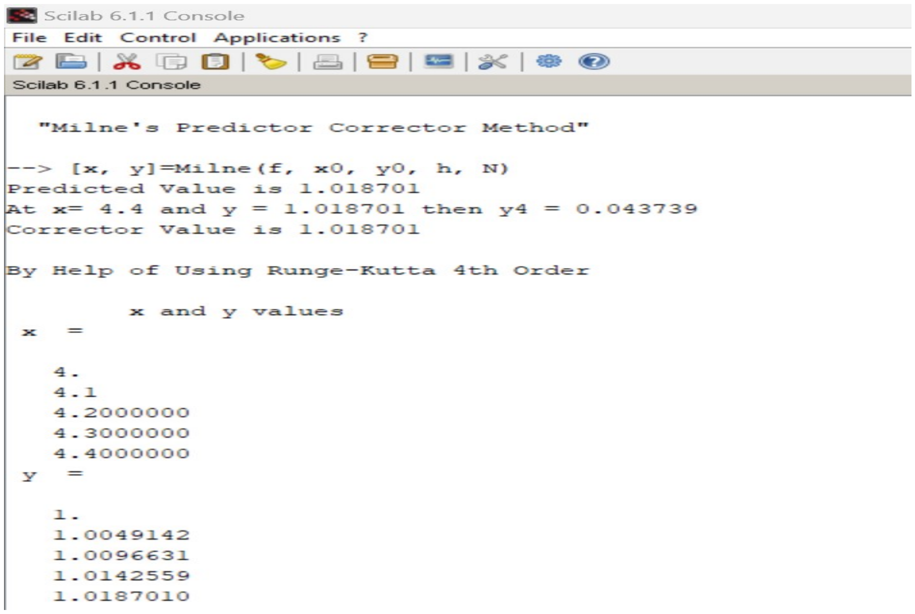
$\Longrightarrow$

$$y_4, c = 1.0097 + \frac{0.1}{3}[0.0466+4(0.04517)+0.04373]$$
$$\boxed{\mathbf{y_4, c = 1.01873}}$$

Coding/Programming:

```
clc; clear all;
disp("Milne''s Predictor Corrector Method")
deff("g=f(x,y)","g=(2-y^2)/(5*x)")
x0=4; y0=1; xn=4.4; xf=4.4;
h=0.1;
N=(xn-x0)/h
function [x, y]=Milne(f, x0, y0, h, N)
    x = zeros(N+1, 1)
    y = zeros(N+1,1)
    x(1) = x0
    y(1) = y0
    for i=1:N
        k1=h*f(x(i),y(i))
        k2=h*f(x(i)+(h/2),y(i)+(k1/2))
        k3=h*f(x(i)+(h/2),y(i)+(k2/2))
        k4=h*f(x(i)+h,y(i)+k3)
        y(i+1)=y(i)+(1/6)*(k1+2*k2+2*k3+k4)
        x(i+1) = x(i) + h
    end
    y1=f(x(2),y(2))
    y2=f(x(3),y(3))
    y3=f(x(4),y(4))
    p=y(1)+(4*h/3)*(2*y1-y2+2*y3)
    printf("Predicted Value is %f",p)
    y4=f(xf,p)
    printf("\nAt x= 4.4 and y = %f then y4 = %f",p,y4)
    c=y(3)+(h/3)*(y2+4*y3+y4)
    printf("\nCorrector Value is %f",c)
    printf("\n\nBy Help of Using Runge-Kutta 4th Order\n")
    printf("\n\tx and y values\n")
endfunction
```

Output:



Scilab 6.1.1 Console

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Scilab 6.1.1 Console

```
  "Milne's Predictor Corrector Method"

--> [x, y]=Milne(f, x0, y0, h, N)
Predicted Value is 1.018701
At x= 4.4 and y = 1.018701 then y4 = 0.043739
Corrector Value is 1.018701

By Help of Using Runge-Kutta 4th Order

        x and y values
 x  =

   4.
   4.1
   4.2000000
   4.3000000
   4.4000000
 y  =

   1.
   1.0049142
   1.0096631
   1.0142559
   1.0187010
```

Exercise:25

$8(D_2)$ADAM'S BASHFORTH METHOD

Objective:

To find the solution of the given derivative using Adam's Bashforth method.

Input:

$$5xy' + y^2 - 2 = 0$$

Find y(4.4) with given

y(4)=1; y(4.1)=1.0049; y(4.2)=1.0097; y(4.3)=1.0143

Procedure/Methodology:

$$5xy' + y^2 - 2 = 0$$
$$y' = \frac{2-y^2}{5x}$$

$$\text{y(4)=1} \implies x_0 = 4; y_0 = 1$$

Here h=0.1 $\implies$

$$x_1 = x_0 + h\text{=4+0.1=4.1}$$
$$x_2 = x_1 + h\text{=4.1+0.1=4.2}$$
$$x_3 = x_2 + h\text{=4.2+0.1=4.3}$$
$$x_4 = x_3 + h\text{=4.3+0.1=4.4}$$

(i.e)

$$x_0 = 4; y_0 = 1$$
$$x_1 = 4.1; y_1 = 1.0049$$
$$x_2 = 4.2; y_2 = 1.0097$$
$$x_3 = 4.3; y_3 = 1.0143$$

Adam's Bashforth predictor method formula is:

$$y_{n+1}, p = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

Put n=3

$$y_4, p = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$
$$y'_0 = \frac{2-y_0^2}{5x_0}$$
$$=\frac{2-(1)^2}{5(4)}$$
$$=0.05$$
$$y'_1 = \frac{2-y_1^2}{5x_1}$$
$$=\frac{2-(1.0049)^2}{5(4.1)}$$
$$=0.04830$$
$$y'_2 = \frac{2-y_2^2}{5x_2}$$
$$=\frac{2-(1.0097)^2}{5(4.2)}$$
$$=0.04670$$
$$y'_3 = \frac{2-y_3^2}{5x_3}$$
$$=\frac{2-(1.0143)^2}{5(4.3)}$$
$$=0.04517$$

$\implies$

$$=1.0143+\frac{h}{24}(55(0.04517)\text{-}59(0.04670)+37(0.04830)\text{-}9(0.05))$$
$$\boxed{\mathbf{y_4, p = 1.01874}}$$
$$y'_4 = \frac{2-y_4'^2}{5x_4}$$
$$\frac{2-(1.01874)^2}{5(4.4)}$$
$$=0.04373$$

Adam's Bashforth corrector method formula is:

$$y_{n+1}, c = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})$$
$$y_4, c = y_3 + \frac{h}{24}(9y'_4 + 19y'_3 - 5y'_2 + y'_1)$$
$$=1.0143+\frac{0.1}{24}(9(0.04373)+19(0.04517)\text{-}5(0.04670)+0.04830)$$
$$\boxed{\mathbf{y_4, c = 1.01874}}$$

Coding/Programming:

```
clc; clear all;
disp("Adams Bashforth Predictor Corrector Method")
deff("g=f(x,y)","g=(2-y^2)/(5*x)")
x0=4; y0=1; xn=4.4; xf=4.4;
h=0.1;
N=(xn-x0)/h
function [x, y]=Adam(f, x0, y0, h, N)
    x = zeros(N+1, 1)
    y = zeros(N+1,1)
    x(1) = x0
    y(1) = y0
    for i=1:N
        k1=h*f(x(i),y(i))
        k2=h*f(x(i)+(h/2),y(i)+(k1/2))
        k3=h*f(x(i)+(h/2),y(i)+(k2/2))
        k4=h*f(x(i)+h,y(i)+k3)
        y(i+1)=y(i)+(1/6)*(k1+2*k2+2*k3+k4)
        x(i+1) = x(i) + h
    end
    yo=f(x(1),y(1))
    y1=f(x(2),y(2))
    y2=f(x(3),y(3))
    y3=f(x(4),y(4))
    p=y(4)+(h/24)*(55*y3-59*y2+37*y1-9*yo)
    printf("Predicted Value is %f",p)
    y4=f(xf,p)
    printf("\nAt x= 4.4 and y = %f then y4 = %f",p,y4)
    c=y(4)+(h/24)*(9*y4+19*y3-5*y2+y1)
    printf("\nCorrector Value is %f",c)
    printf("\n\nBy Help of Using Runge-Kutta 4th Order\n")
    printf("\n\tx and y values\n")
endfunction
```

Output:

Scilab 6.1.1 Console

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Scilab 6.1.1 Console

```
  "Adams Bashforth Predictor Corrector Method"

--> [x, y]=Adam(f, x0, y0, h, N)
Predicted Value is 1.018701
At x= 4.4 and y = 1.018701 then y4 = 0.043739
Corrector Value is 1.018701

By Help of Using Runge-Kutta 4th Order

        x and y values
 x  =

   4.
   4.1
   4.2000000
   4.3000000
   4.4000000
 y  =

   1.
   1.0049142
   1.0096631
   1.0142559
   1.0187010
```

MODULE 5

Exercise:26

9(A_1)LEIBMANN'S METHOD

Objective:

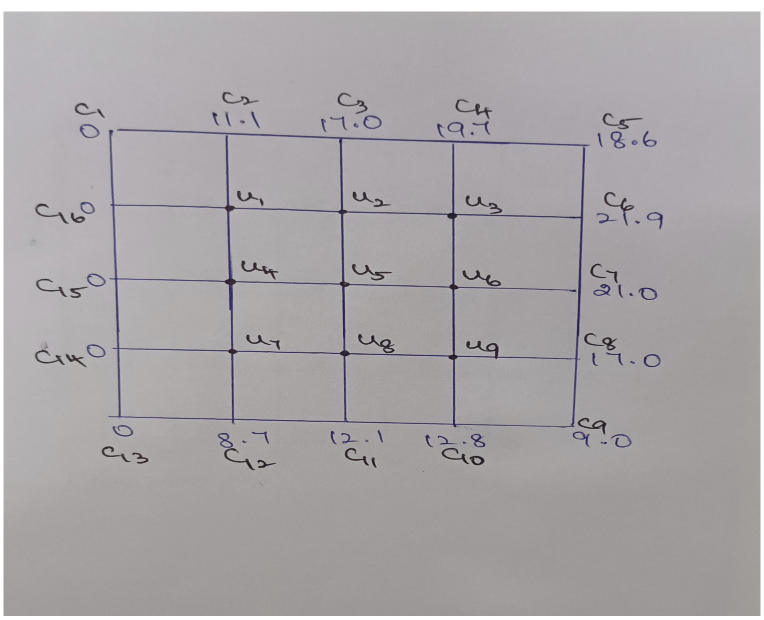
To find the Solution of Partial Differential Equation using Liebmann's Method.

Input:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Procedure/Methodology:

Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ at the nine mesh points of the squares given below. The values of u at the boundary are specified in the figure



Standard five point formula

$$u_2, u_4, u_5, u_6, u_8$$

Diagonal five point formula

$$u_1, u_3, u_7, u_9$$

$\Longrightarrow$

$$u_5^{(0)} = \frac{1}{4}(C_3 + C_7 + C_{11} + C_{15})$$
$$=\frac{1}{4}(17.5+21+12.1+0)$$
$$=12.5(\text{SFPF})$$

$$u_1^{(0)} = \frac{1}{4}(C_1 + u_5^{(0)} + C_3 + C_{15})$$
$$=\frac{1}{4}(0+12.5+17+0)$$
$$=7.4(\text{DFPF})$$

$$u_3^{(0)} = \frac{1}{4}(C_5 + u_5^{(0)} + C_3 + C_7)$$
$$=\frac{1}{4}(18.6+12.51+17+21)$$
$$=17.3(\text{DFPF})$$

$$u_7^{(0)} = \frac{1}{4}(C_{15} + C_{11} + u_5^{(0)} + C_{13})$$
$$=\frac{1}{4}(0+12.1+12.5+0)$$
$$=6.15(\text{DFPF})$$

$$u_9^{(0)} = \frac{1}{4}(C_9 + C_{11} + u_5^{(0)} + C_7)$$
$$=\frac{1}{4}(9+12.1+12.5+21)$$
$$=13.7(\text{DFPF})$$

$\Longrightarrow$

$$u_2^{(0)} = \frac{1}{4}(C_3 + u_1^{(0)} + u_3^{(0)} + u_5^{(0)}$$
$$=\frac{1}{4}(17+7.4+17.3+12.5)$$
$$=13.6(\text{SFPF})$$

$$u_4^{(0)} = \frac{1}{4}(C_{15} + u_1^{(0)} + u_7^{(0)} + u_5^{(0)}$$
$$=\frac{1}{4}(0+7.4+6.2+12.5)$$
$$=6.5(\text{SFPF})$$

$$u_6^{(0)} = \frac{1}{4}(u_3^{(0)} + u_5^{(0)} + C_7 + u_9^{(0)}$$
$$=\frac{1}{4}(17.6+12.5+21+13.7)$$
$$=16.1(\text{SFPF})$$

$$u_8^{(0)} = \frac{1}{4}(C_{11} + u_5^{(0)} + u_7^{(0)} + u_9^{(0)}$$
$$=\frac{1}{4}(12.1+12.5+6.2+13.7)$$
$$=11.1(\text{SFPF})$$

<u>Iteration 1:</u>

$$u_1^{(1)} = \frac{1}{4}(C_2 + C_{16} + u_2^{(0)} + u_4^{(0)}$$
$$=\frac{1}{4}(11.1+0+13.6+6.5)$$
$$=7.8$$

$$u_2^{(1)} = \frac{1}{4}(C_3 + u_5^{(0)} + u_1^{(0)} + u_3^{(0)}$$
$$=\frac{1}{4}(17+12.5+7.4+17.3)$$
$$=13.6$$

$$u_3^{(1)} = \frac{1}{4}(C_4 + u_2^{(0)} + C_6 + u_6^{(0)}$$
$$=\frac{1}{4}(19.7+13.6+21.9+16.1)$$
$$=17.8$$

$$u_4^{(1)} = \frac{1}{4}(C_{15} + u_7^{(0)} + u_1^{(0)} + u_5^{(0)}$$
$$=\frac{1}{4}(0+6.2+7.4+12.5)$$
$$=6.5$$

$$u_5^{(1)} = \frac{1}{4}(u_2^{(0)} + u_4^{(0)} + u_6^{(0)} + u_8^{(0)}$$
$$=\frac{1}{4}(13.6+6.5+16.1+11.1)$$
$$=11.8$$

$$u_6^{(1)} = \frac{1}{4}(C_7 + u_3^{(0)} + u_5^{(0)} + u_9^{(0)}$$
$$=\frac{1}{4}(21+17.3+12.5+13.7)$$
$$=16.1$$

$$u_7^{(1)} = \frac{1}{4}(C_{14} + u_4^{(0)} + u_8^{(0)} + C_{12}$$
$$=\frac{1}{4}(0+6.5+11.1+8.7)$$
$$=6.6$$

$$u_8^{(1)} = \frac{1}{4}(C_{11} + u_5^{(0)} + u_7^{(0)} + u_9^{(0)}$$
$$=\frac{1}{4}(12.1+12.5+6.2+13.7)$$
$$=11.1$$

$$u_9^{(1)} = \frac{1}{4}(C_{10} + u_6^{(0)} + u_8^{(0)} + C_8$$
$$=\frac{1}{4}(12.8+16.1+11.1+17.0)$$
$$=14.2$$

<u>Iteration 2:</u>

$$u_1^{(2)} = \frac{1}{4}(C_2 + C_{16} + u_2^{(1)} + u_4^{(1)}$$
$$=\frac{1}{4}(11.1+0+13.6+6.5)$$
$$=7.8$$

$$u_2^{(2)} = \frac{1}{4}(C_3 + u_5^{(1)} + u_1^{(1)} + u_3^{(1)}$$
$$=\frac{1}{4}(17+11.8+7.8+17.8)$$
$$=13.6$$

$$u_3^{(2)} = \frac{1}{4}(C_4 + u_2^{(1)} + C_6 + u_6^{(1)}$$
$$=\frac{1}{4}(19.7+13.6+21.9+16.1)$$
$$=17.8$$

$$u_4^{(2)} = \frac{1}{4}(C_{15} + u_7^{(1)} + u_1^{(1)} + u_5^{(1)}$$
$$=\frac{1}{4}(0+6.6+7.8+11.8)$$
$$=6.6$$

$$u_5^{(2)} = \frac{1}{4}(u_2^{(1)} + u_4^{(1)} + u_6^{(1)} + u_8^{(1)}$$
$$=\frac{1}{4}(13.6+6.5+16.1+11.1)$$
$$=11.8$$

$$u_6^{(2)} = \frac{1}{4}(C_7 + u_3^{(0)} + u_5^{(0)} + u_9^{(0)}$$
$$=\frac{1}{4}(21+17.8+11.8+14.2)$$
$$=16.2$$

$$u_7^{(2)} = \frac{1}{4}(C_{14} + u_4^{(1)} + u_8^{(1)} + C_{12}$$
$$=\frac{1}{4}(0+6.5+11.1+8.7)$$
$$=6.6$$

$$u_8^{(2)} = \frac{1}{4}(C_{11} + u_5^{(0)} + u_7^{(0)} + u_9^{(0)}$$
$$=\frac{1}{4}(12.1+11.8+6.6+14.2)$$
$$=11.1$$

$$u_9^{(2)} = \frac{1}{4}(C_{10} + u_6^{(0)} + u_8^{(0)} + C_8$$

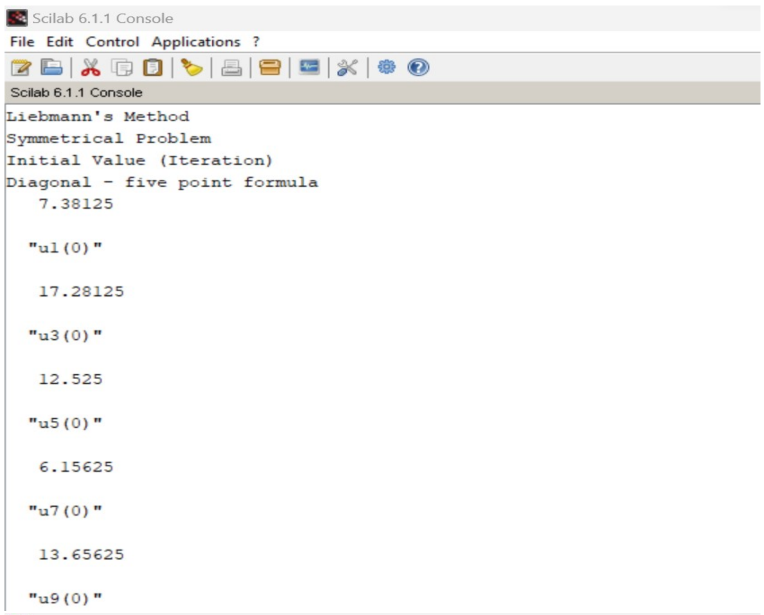
$$=\frac{1}{4}(12.8+16.1+11.1+17.0)$$
$$=14.2$$

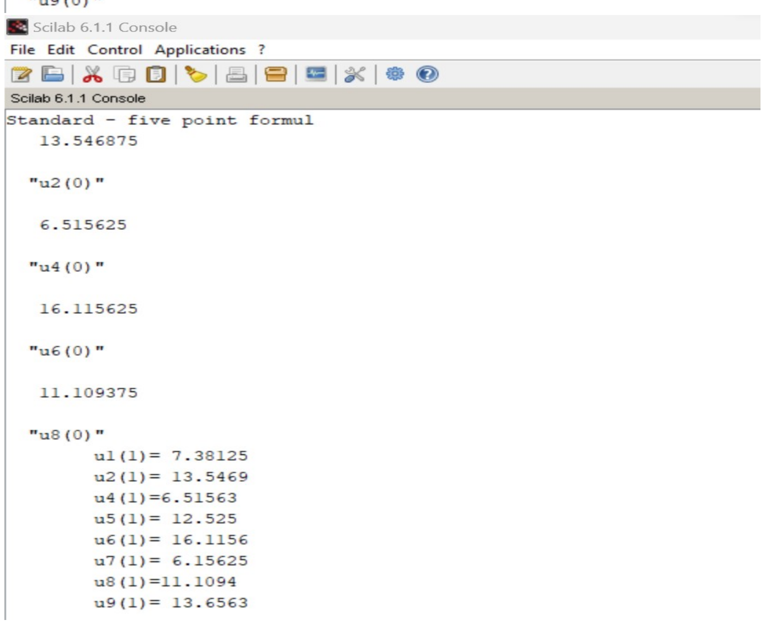
Coding/Programming:

```
clc; clear all;
b=[0,11.1,17,19.7,18.6,21.9,21.0,17,9,12.8,12.1,8.7,0,0,0,0];
a=1/4;
printf("\nSymmetrical Problem")
printf("\nInitial Value (Iteration)")
u5=a*(b(3)+b(11)+b(7)+b(15))
u1=a*(b(1)+u5+b(3)+b(15))
u3=a*(b(5)+u5+b(3)+b(7))
u7=a*(b(15)+b(11)+u5+b(13))
u9=a*(b(9)+b(11)+b(7)+u5)
u2=a*(b(3)+u1+u3+u5)
u4=a*(b(15)+u1+u5+u7)
u6=a*(b(7)+u3+u5+u9)
u8=a*(b(11)+u5+u7+u9)
printf("\nDiagonal - five point formula")
disp(u1,'u1(0)')
disp(u3,'u3(0)')
disp(u5,'u5(0)')
disp(u7,'u7(0)')
disp(u9,'u9(0)')
printf("Standard - five point formul")
disp(u2,'u2(0)')
disp(u4,'u4(0)')
disp(u6,'u6(0)')
disp(u8,'u8(0)')
for i=1:3
    u11=a*(b(2)+b(16)+u2+u4)
    u21=a*(b(3)+u5+u1+u3)
    u31=a*(b(4)+u2+b(6)+u6)
    u41=a*(b(15)+u1+u5+u7)
    u51=a*(u2+u4+u6+u8)
    u61=a*(u3+u5+u9+b(7))
    u71=a*(u4+u8+b(14)+b(12))
    u81=a*(b(11)+u5+u7+u9)
    u91=a*(b(8)+b(10)+u6+u8)
```

```
    printf('\tu1(%i)= %g\n\tu2(%i)= %g\n\tu4(%i)=%g\n\tu5(%i)= %g\n
                \tu6(%i)= %g\n\tu7(%i)= %g\n\tu8(%i)=%g\n\tu9(%i)=
                %g\n\t\n',i,u1,i,u2,i,u4,i,u5,i,u6,i,u7,i,u8,i,u9)
end
printf('Solution of converges Laplace''s Equation to \n\t%g, %g, %g  %g  %g,
                                %g, %g  %g  %g',u1,u2,u3,u4,u5,u6,u7,u8,u9)
```

Output:





```
        u1(2)= 7.38125
        u2(2)= 13.5469
        u4(2)=6.51563
        u5(2)= 12.525
        u6(2)= 16.1156
        u7(2)= 6.15625
        u8(2)=11.1094
        u9(2)= 13.6563

        u1(3)= 7.38125
        u2(3)= 13.5469
        u4(3)=6.51563
        u5(3)= 12.525
        u6(3)= 16.1156
        u7(3)= 6.15625
        u8(3)=11.1094
        u9(3)= 13.6563

Solution of converges Laplace's Equation to
        7.38125, 13.5469, 17.2813  6.51563  12.525, 16.1156, 6.15625  11.1094  13.6563
-->
```

Exercise:27

9(A_2)POISSON'S METHOD

Objective:

To find the Solution of Partial Differential Equation using Poisson's Method.

Input:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 8x^2y^2$$

Boundary Conditions:

$$u(x, y) = 0$$

Procedure/Methodology:

Solve the Elliptic equation $u_{xx} + u_{yy} = 8x^2y^2$ for a Square mesh with $u(x, y) = 0$ on the boundaries dividing the square into 16 sub-squares of length 1 unit (shown in figure below)

Solution:

$$\boxed{u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 8i^2j^2}$$

Here Boundary values are Symmetrical about x-axis and y-axis

$u_1 = u_3 = u_7 = u_9$ and

$u_2 = u_4 = u_6 = u_8$

Now, find the value u_1, u_2 and u_5

At $u_3(i = 1, j = 1)$

$$\begin{aligned} u_{0,1} + u_{2,1} + u_{1,2} + u_{1,0} - 4u_{1,1} &= 8 \\ u_2 + 0 + 0 + u_2 - 4u_3 &= 8 \\ 2u_2 - 4u_1 &= 8 \end{aligned} \tag{10}$$

At $u_5(i = 0, j = 0)$

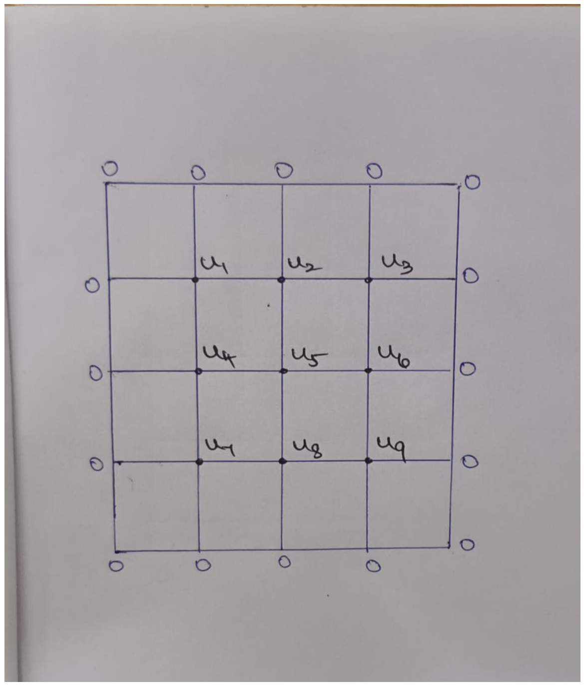


Figure 1: Boundary value with 16 square mesh

$$\begin{aligned} u_{-1,0}+u_{1,0}+u_{0,1}+u_{0,-1}-4u_{0,0} &= 0 \\ u_4+u_6+u_8+u_2-4u_5 &= 0 \\ u_2 &= u_5 \end{aligned} \tag{11}$$

At $u_2 (i=0, j=1)$

$$\begin{aligned} u_{-1,1}+u_{1,1}+u_{0,2}+u_{0,0}-4u_{0,1} &= 0 \\ u_1+u_1+0+u_5-4u_2 &= 0 \\ 2u_1+u_5 &= 4u_2 \end{aligned} \tag{12}$$

from (11) and (12)

$$2u_1 - 3u_2 = 0 \tag{13}$$

$$\begin{array}{lcrcl} (13)\times 2 & \Rightarrow & \cancel{4u_1}-6u_2 & = & 0 \\ (10)\times 1 & \Rightarrow & \cancel{-4u_1}+2u_2 & = & 8 \\ \hline & & -4u_2 & = & 8 \end{array}$$

$$
\begin{aligned}
u_2 &= \frac{8}{-4} \\
u_2 &= -2
\end{aligned}
$$

Substitution in (10)

$$
\begin{aligned}
-4 - 4u_1 &= 8 \\
-4u_1 &= 8 + 4 \\
u_1 &= \frac{12}{-4} \\
u_1 &= -3
\end{aligned}
$$

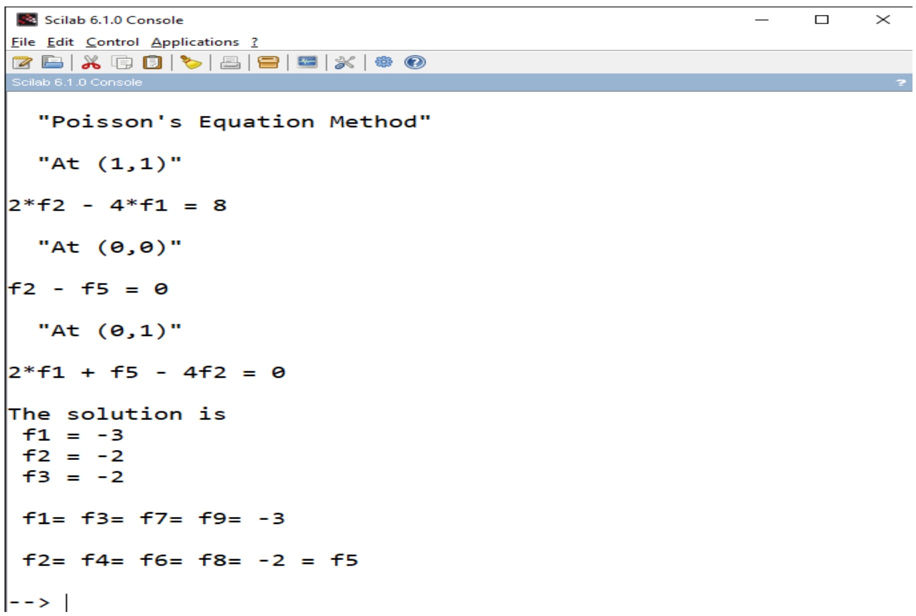
$$
u_1 = u_3 = u_7 = u_9 = -3
$$
$$
u_2 = u_4 = u_6 = u_8 = u_5 = -2
$$

Coding/Output:

```
clc; clear all;
disp("Poisson''s Equation Method")
deff("z=f(x,y)","z=8*x^2*y^2");
//D2f = 8*x^2 * y^2
// f = 0
// h = 1
b=[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
disp("At (1,1)")
//Point 1 : 0 + 0 + f2 + f2 - 4f1 = 8(1)^2*(1)^2
printf("\t\n2*f2 - 4*f1 = 8\n")
disp("At (0,0)")
//Point 2 : f8 + f6 + f2 + f4 - 4f5 = 0
printf("\t\nf2 - f5 = 0\n")
disp("At (0,1)")
//Point 3 : f1 + 0 + f3 + f5 - 4f2 = 0
printf("\t\n2*f1 + f5 - 4f2 = 0\n")
//Rearranging the equations
//           2*f2 - 4*f1 = 8
//           f2 - f5 = 0
//           2*f1 + f5 - 4f2 = 0
A = [-4,2,0;0,1,-1;2,-4,1]
B = [8,0,0]
C = inv(A)*B' ;
```

```
mprintf('\nThe solution is \n f1 = %d \n f2 = %.0f
        \n f3 = %.0f \n ', C(1),C(2),C(3))
printf('\t\n f1= f3= f7= f9= %d\n',C(1))
printf('\t\n f2= f4= f6= f8= %.0f = f5\n',C(3))
```

Output:



Exercise:28

9(B_1)BENDER SCHMIDT METHOD

Objective:

To find the Solution of Partial Differential Equation using Bender Schmidt Method.

Input:

$$\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$$

Boundary Conditions :

$$\begin{aligned} u(0,t) &= u(4,t) = 0 \\ u(x,0) &= x(4-x) \end{aligned}$$

PROCEDURE/METHODOLOGY :

Solve $u_{xx} = 2u_t$ under conditions $u(0,t) = u(4,t) = 0$ and $u(x,0) = x(4-x)$ taking $h = 1$ find the value up to $t = 5$

Solution:

$$u_{xx} = 2u_t$$

$$\alpha = 2, h = 1$$

$$k = \frac{\alpha h^2}{2} = 1$$

$$\lambda = \frac{k}{\alpha h^2} = \frac{1}{2(1)^2} = \frac{1}{2}$$

$$\boxed{\lambda = \frac{1}{2}}$$

General Formula

$$\boxed{u_{i,j+1} = \frac{1}{2}[u_{i+1,j} + u_{i-1,j}]}$$

$$\begin{aligned} u(x,0) &= x(4-x) \\ u(1,0) &= 1(4-1) = 3 \\ u(2,0) &= 2(4-2) = 4 \\ u(3,0) &= 3(4-3) = 3 \end{aligned}$$

for $j = 0$

$$\begin{aligned} u_{i,1} &= \frac{1}{2}[u_{i+1,0} + u_{i-1,0}] \\ u_{1,1} &= \frac{1}{2}[u_{2,0} + u_{0,0}] = 2 \\ u_{2,1} &= \frac{1}{2}[u_{3,0} + u_{1,0}] = 3 \\ u_{3,1} &= \frac{1}{2}[u_{4,0} + u_{2,0}] = 2 \end{aligned}$$

for $j = 1$

$$\begin{aligned} u_{i,2} &= \frac{1}{2}[u_{i+1,1} + u_{i-1,1}] \\ u_{1,2} &= \frac{1}{2}[u_{2,1} + u_{0,1}] = 1.5 \\ u_{2,2} &= \frac{1}{2}[u_{3,1} + u_{1,1}] = 2 \\ u_{3,2} &= \frac{1}{2}[u_{4,1} + u_{2,1}] = 1.5 \end{aligned}$$

for $j = 2$

$$\begin{aligned} u_{i,3} &= \frac{1}{2}[u_{i+1,2} + u_{i-1,2}] \\ u_{1,3} &= \frac{1}{2}[u_{2,2} + u_{0,2}] = 1 \\ u_{2,3} &= \frac{1}{2}[u_{3,2} + u_{1,2}] = 1.5 \\ u_{3,3} &= \frac{1}{2}[u_{4,2} + u_{2,2}] = 1 \end{aligned}$$

for $j = 3$

$$\begin{aligned}
u_{i,4} &= \frac{1}{2}[u_{i+1,4} + u_{i-1,4}] \\
u_{1,4} &= \frac{1}{2}[u_{2,4} + u_{0,4}] = 0.75 \\
u_{2,4} &= \frac{1}{2}[u_{3,4} + u_{1,4}] = 1 \\
u_{3,4} &= \frac{1}{2}[u_{4,4} + u_{2,4}] = 0.75
\end{aligned}$$

for $j = 4$

$$\begin{aligned}
u_{i,5} &= \frac{1}{2}[u_{i+1,} + u_{i-1,}] \\
u_{1,5} &= \frac{1}{2}[u_{2,5} + u_{0,5}] = 0.5 \\
u_{2,5} &= \frac{1}{2}[u_{3,5} + u_{1,5}] = 0.75 \\
u_{3,5} &= \frac{1}{2}[u_{4,5} + u_{2,5}] = 0.5
\end{aligned}$$

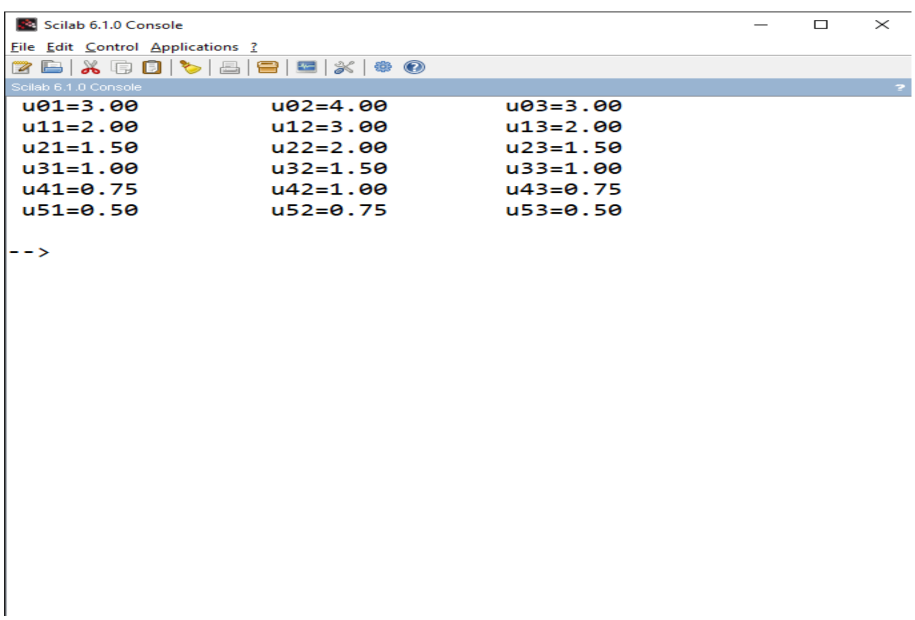
j=t\ i=x	0	1	2	3	4
0	0	3	4	3	0
1	0	2	3	2	0
2	0	1.5	2	1.5	0
3	0	1	1.5	1	0
4	0	0.75	1	0.75	0
5	0	0.5	0.75	0.5	0

Coding/Output:

```
clc; clear all;
deff('y=f(x)','y=4*x-x^2');
u=[f(0) f(1) f(2) f(3) f(4)];
printf(' u01=%0.2f\t  u02=%0.2f\t  u03=%0.2f\t \n',f(1),f(2),f(3))
u11=(u(1)+u(3))/2;
u12=(u(2)+u(4))/2;
u13=(u(3)+u(5))/2;
printf(' u11=%0.2f\t  u12=%0.2f\t  u13=%0.2f\t \n',u11,u12,u13)
```

```
u21=(u(1)+u12)/2;
u22=(u11+u13)/2;
u23=(u12+0)/2;
printf(' u21=%0.2f\t  u22=%0.2f\t  u23=%0.2f\t \n',u21,u22,u23)
u31=(u(1)+u22)/2;
u32=(u21+u23)/2;
u33=(u22+u(1))/2;
printf(' u31=%0.2f\t  u32=%0.2f\t  u33=%0.2f\t \n',u31,u32,u33)
u41=(u(1)+u32)/2;
u42=(u31+u33)/2;
u43=(u32+u(1))/2;
printf(' u41=%0.2f\t  u42=%0.2f\t  u43=%0.2f\t \n',u41,u42,u43)
u51=(u(1)+u42)/2;
u52=(u41+u43)/2;
u53=(u42+u(1))/2;
printf(' u51=%0.2f\t  u52=%0.2f\t  u53=%0.2f\t \n',u51,u52,u53)
```

Output:



Exercise:29

9(B_2)CRANK-NICHOLSON METHOD

Objective:

To find the Solution of Partial Differential Equation using Crank Nicholson Method.

Input:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

Boundary Conditions :

$$\begin{aligned} u(0,t) &= u(1,t) = 0 \\ u(x,0) &= 100(x - x^2) \end{aligned}$$

PROCEDURE/METHODOLOGY:

Solve using Crank - Nicholson simplified formula $u_t = u_{xx}$ with $u(0,t) = u(1,t) = 0$ & $u(x,0) = 100(x - x^2)$ with $h = 0.25$ for one time step.

Solution:

$$a = 1, h = 0.25$$

$$k = ah^2 = (1)(0.25)^2 = 0.0625$$

$$\begin{aligned} u(x,0) &= 100(x - x^2) \\ u(0.25,0) &= 100(0.25 - (0.25)^2) = 18.75 \\ u(0.5,0) &= 100(0.5 - (0.5)^2) = 25 \\ u(0.75,0) &= 100(0.75 - (0.75)^2) = 18.75 \end{aligned}$$

General Formula:

$$u_{i,j+1} = \frac{1}{4}[u_{i+1,j+1} + u_{i-1,j+1} + u_{i-1,j} + u_{i+1,j}]$$

$$\text{for } (i = 1, j = 0)$$

$$\begin{aligned} u_{1,0} &= \frac{1}{4}[u_{2,1} + u_{0,1} + u_{0,0} + u_{2,0}] \\ u_1 &= \frac{1}{4}[u_2 + 0 + 0 + 25] \\ \Rightarrow 4u_1 - u_2 &= 25 \end{aligned} \tag{14}$$

$$\text{for } (i = 2, j = 0)$$

$$\begin{aligned} u_{2,1} &= \frac{1}{4}[u_{3,1} + u_{1,1} + u_{1,0} + u_{3,0}] \\ u_2 &= \frac{1}{4}[18.75 + 18.75 + u_1 + u_3] \\ \Rightarrow 4u_2 - u_1 - u_3 &= 37.5 \end{aligned} \tag{15}$$

$$\text{for } (i = 3, j = 0)$$

$$\begin{aligned} u_{3,1} &= \frac{1}{4}[u_{4,1} + u_{2,1} + u_{2,0} + u_{4,0}] \\ u_3 &= \frac{1}{4}[u_2 + 0 + 0 + 25] \\ \Rightarrow 4u_3 - u_2 &= 25 \end{aligned} \tag{16}$$

$$\text{solving (14), (15), \& (16)}$$

$$A = \begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}; X = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}; B = \begin{pmatrix} 25 \\ 37.5 \\ 25 \end{pmatrix}$$

$$A^{-1} = \frac{1}{56}\begin{pmatrix} 15 & 4 & 1 \\ 4 & 16 & 4 \\ 1 & 4 & 15 \end{pmatrix}$$

$X = A^{-1}B$
We get

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \frac{1}{56}\begin{pmatrix} 15 & 4 & 1 \\ 4 & 16 & 4 \\ 1 & 4 & 15 \end{pmatrix} \times \begin{bmatrix} 25 \\ 37.5 \\ 25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 9.8214 \\ 14.2857 \\ 9.8214 \end{bmatrix}$$

$$\boxed{u_1 = 9.8214, u_2 = 14.2814, u_3 = 9.8214}$$

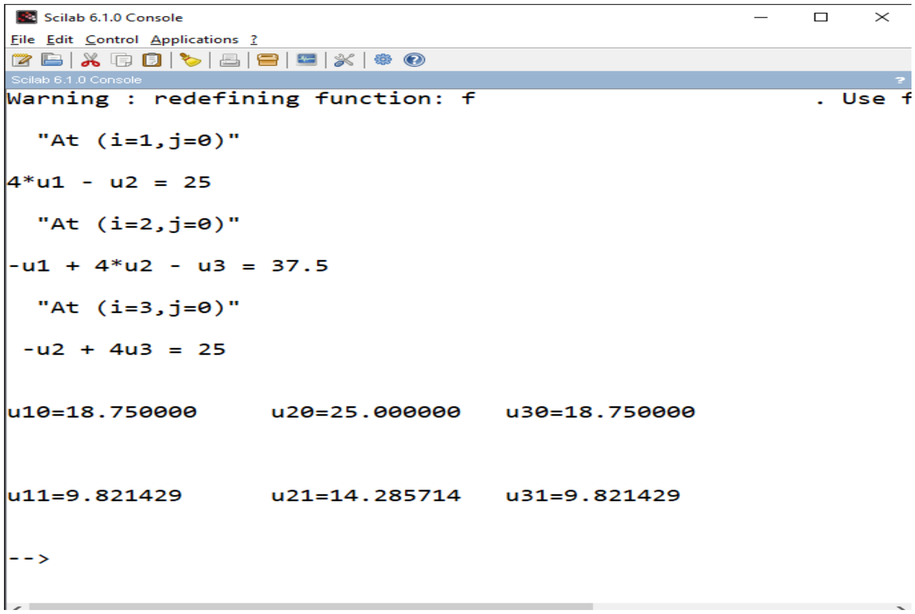
	i	0	1	2	3	4
j	t\\x	0	0.25	0.50	0.75	1
0	0	0	18.75	25	18.75	0
1	1/16	0	9.8214	14.2857	9.8214	0

Coding/Output:

```
clc ; clear all;
deff('y=f(x,t)','y=100*(x-x^2)');
u=[f(0,0) f(0.25,0) f(0.5,0) f(0.75,0) f(1,0)];
//by using crank-nicolson formula
//by putting i=1,2,3 we obtain four equation
disp("At (i=1,j=0)")
//Point 1 : 0 + 0 + u2 + 25 = 4u1
printf("\t\n4*u1 - u2 = 25\n")
disp("At (i=2,j=0)")
//Point 2 : 18.75 + 18.75 + u1 + u3 = 4u2
printf("\t\n-u1 + 4*u2 - u3 = 37.5\n")
disp("At (i=3,j=0)")
//Point 3 : u2 + 0 + 0 + 25 = 4u3
printf("\t\n -u2 + 4u3 = 25\n")
//Rearranging the equations
//            4*u1 - u2 = 25
//            -u1 + 4*u2 - u3 = 37.5
//            -u2 + 4u3 = 25
A=[4 -1 0 ;-1 4 -1 ;0 -1 4];
C=[25;37.5;25];
X=A^-1*C;
printf( ' \n\nu10=%f\t  u20=%f\t  u30=%f\t\n\n',u(2),u(3),u(4))

printf( ' \n\nu11=%f\t  u21=%f\t  u31=%f\t\n\n',X(1,1),X(2,1),X(3,1))
```

Output:


Scilab 6.1.0 Console
File Edit Control Applications ?
Scilab 6.1.0 Console
Warning : redefining function: f . Use f
"At (i=1,j=0)"
4*u1 - u2 = 25
"At (i=2,j=0)"
-u1 + 4*u2 - u3 = 37.5
"At (i=3,j=0)"
-u2 + 4u3 = 25
u10=18.750000 u20=25.000000 u30=18.750000
u11=9.821429 u21=14.285714 u31=9.821429
-->

Exercise:30

9(C)EXPLICIT SCHEME METHOD

Objective:

To find the Solution of Partial Differential Equation using Explicit Scheme Method.

Input:

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

Boundary Conditions:

$$\begin{aligned} u(0,t) &= u(4,t) = 0 \\ u(x,0) &= x(4-x) \end{aligned}$$

PROCEDURE/METHODOLOGY :

Solve the wave equation $u_{tt} = 4u_{xx}$ subject to $u(0,t) = u(4,t) = 0$ & $u(x,0) = x(4-x)$ by taking step length in x, $h = 1$

$$\text{Standard form of Wave Equation is } u_{tt} = c^2 u_{xx}$$

$$c^2 = 4 \Rightarrow c = \pm 2$$

$$\text{step length in } x,\ h = 1$$

$$\text{step length in } t,\ k = \frac{h}{c} = \frac{1}{2} = 0.5$$

By Boundary Condition

$$\text{At } x = 0, 1, 2, 3, 4 = i$$
$$\text{At } t = 0, 0.5, 1, 1.5, 2 = j$$
$$u(0, t) = u(4, t) = 0$$

$$\begin{aligned}
u(x, 0) &= x(4 - x) \\
u(1, 0) &= 1(4 - 1) = 3 \\
u(2, 0) &= 2(4 - 2) = 4 \\
u(3, 0) &= 3(4 - 3) = 3
\end{aligned}$$

for $j = 0$

$$\begin{aligned}
u_{i,1} &= \frac{1}{2}[u_{i+1,0} + u_{i-1,0}]r \\
u_{1,1} &= \frac{1}{2}[u_{2,0} + u_{0,0}] = 2 \\
u_{2,1} &= \frac{1}{2}[u_{3,0} + u_{1,0}] = 3 \\
u_{3,1} &= \frac{1}{2}[u_{4,0} + u_{2,0}] = 2
\end{aligned}$$

General Formula:

$$\boxed{u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}}$$

for $j = 1$

$$\begin{aligned}
u_{i,2} &= u_{i+1,1} + u_{i-1,1} - u_{i,0} \\
u_{1,2} &= u_{2,1} + u_{0,1} - u_{1,0} = 0 \\
u_{2,2} &= u_{3,1} + u_{1,1} - u_{2,0} = 0 \\
u_{3,2} &= u_{4,1} + u_{2,1} - u_{3,0} = 0
\end{aligned}$$

for $j = 2$

$$\begin{aligned}
u_{i,3} &= u_{i+1,2} + u_{i-1,2} - u_{i,1} \\
u_{1,3} &= u_{2,2} + u_{0,2} - u_{1,1} = -2 \\
u_{2,3} &= u_{3,2} + u_{1,2} - u_{2,1} = -3 \\
u_{3,3} &= u_{4,2} + u_{2,2} - u_{3,1} = -2
\end{aligned}$$

for $j = 3$

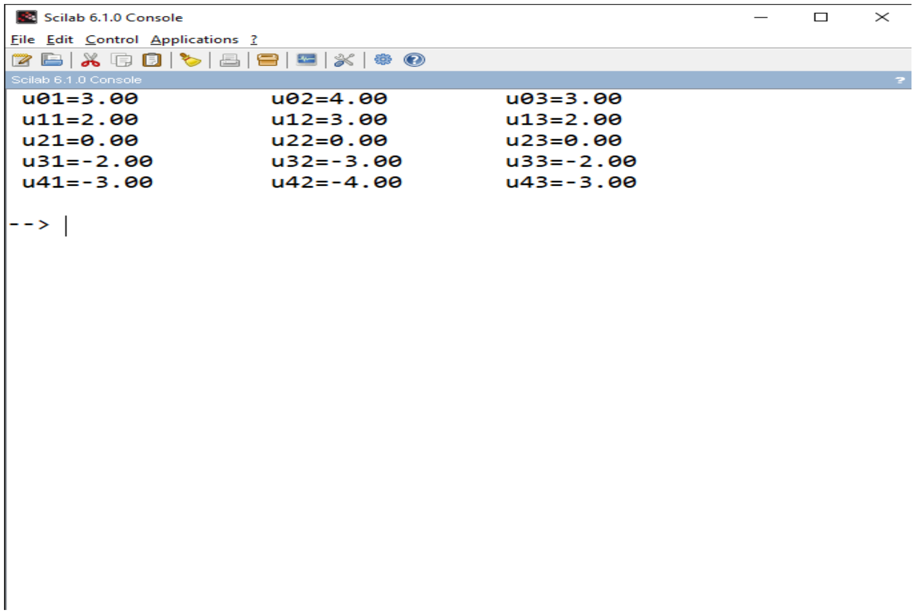
$$
\begin{aligned}
u_{i,4} &= u_{i+1,3} + u_{i-1,3} - u_{i,2} \\
u_{1,4} &= u_{2,3} + u_{0,3} - u_{1,2} = 0 \\
u_{2,4} &= u_{3,3} + u_{1,3} - u_{2,2} = 0 \\
u_{3,4} &= u_{4,3} + u_{2,3} - u_{3,2} = 0
\end{aligned}
$$

j	t\\$x = i$	0	1	2	3	4
0	0	0	3	4	3	0
1	0.5	0	2	3	2	0
2	1	0	0	0	0	0
3	1.5	0	-2	-3	-2	0
4	2	0	-3	-4	-3	0

Coding/Output:

```
clc; clear all;
deff('y=f(x)','y=4*x-x^2');
u=[f(0) f(1) f(2) f(3) f(4)];
b=0;
printf(' u01=%0.2f\t  u02=%0.2f\t  u03=%0.2f\t \n',f(1),f(2),f(3))
u11=(u(1)+u(3))/2;
u12=(u(2)+u(4))/2;
u13=(u(3)+u(5))/2;
printf(' u11=%0.2f\t  u12=%0.2f\t  u13=%0.2f\t \n',u11,u12,u13)
u21=b+u12-u(2);
u22=u11+u13-u(3);
u23=u12+b-u(4);
printf(' u21=%0.2f\t  u22=%0.2f\t  u23=%0.2f\t \n',u21,u22,u23)
u31=b+u22-u11;
u32=u21+u21-u12;
u33=b+u22-u13;
printf(' u31=%0.2f\t  u32=%0.2f\t  u33=%0.2f\t \n',u31,u32,u33)
u41=b+u32-u21;
u42=u33+u31-u22;
u43=b+u32-u23;
printf(' u41=%0.2f\t  u42=%0.2f\t  u43=%0.2f\t \n',u41,u42,u43)
```

Output:


Scilab 6.1.0 Console
File Edit Control Applications ?
Scilab 6.1.0 Console
u01=3.00 u02=4.00 u03=3.00
u11=2.00 u12=3.00 u13=2.00
u21=0.00 u22=0.00 u23=0.00
u31=-2.00 u32=-3.00 u33=-2.00
u41=-3.00 u42=-4.00 u43=-3.00
-->