SIR model with additional parameter m

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The model

$$\begin{cases} \dfrac{dS}{dt} = \mu(m-1)I - \dfrac{\beta}{N} IS \\ \\ \dfrac{dI}{dt} = -\mu(2m-1)I + \dfrac{\beta}{N} IS \\ \\ \dfrac{dR}{dt} = \mu m I \end{cases}$$

Approach of my choice (Determenistic)

The population is assumed to be fixed and sufficiently large ($N = 10^4$).Interactions between individuals are considered uniform, which justifies the use of the incidence term $\frac{\beta}{N}SI$.The available data consist of daily counts of infected individuals over a short observation period (61 days), which supports the use of a deterministic formulation.

Seeing that we have the data for the infected, we will attepmt to identify β and m from

$$\frac{dI}{dt} = -\mu(2m-1)I + \frac{\beta}{N}IS$$

Replacing S with $N - I - R$ and then $R(t) = \mu m \int_0^t I(s)ds$.

We have finally

$$I_{model} = \frac{dI}{dt} = I\Big(\frac{\beta}{N}(N - I - \mu m \int_0^t I(s)ds) - \mu(2m-1)\Big)$$

The goal is minimising the quantity :

$$J(t_i, \beta, m) = \sum_{i=0}^{T_{obs}-1} \Big(I_{obs}(t_i) - I_{model}(t_i)\Big)^2$$

where I_{obs} are the real values of the infected.

We can assume at the begining that R is very small, we thus have :

$$I_{model} = \frac{dI}{dt} = I\Big(\beta - \mu(2m-1)\Big)$$

and thus :

$$I(t) \approx I_0 exp((\beta - \mu(2m-1))t)$$

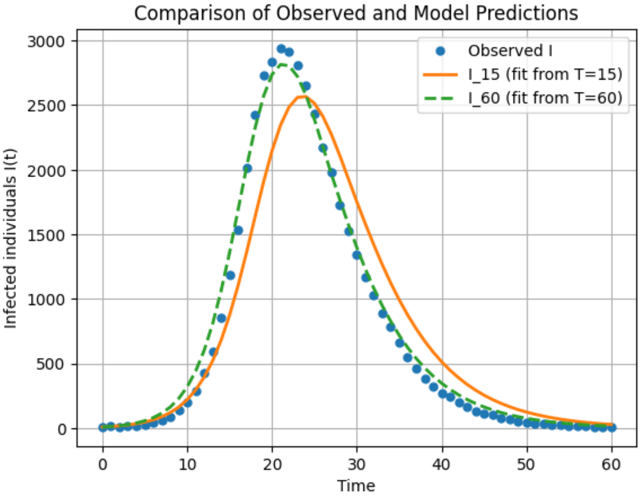
In this instance it is no longer possible to consider R very small, we can however consider $J(t) = \int_0^t I(s)ds$, we now have the system

$$\begin{cases} \dfrac{dI}{dt}(t) = I(t)\Big(\dfrac{\beta}{N}(N - I - \mu m J(t) - \mu(2m-1)\Big) \\ \\ J(t) = \displaystyle\int_0^t I(s)ds \end{cases}$$

Values

- For $T_{obs} = 15 : \beta = 0.5000, m = 0.9624$
- For $T_{obs} = 60 : \beta = 0.5421, m = 0.9699$

Plot


Comparison of Observed and Model Predictions
Observed I
I_15 (fit from T=15)
I_60 (fit from T=60)
Infected individuals I(t)
Time
3000
2500
2000
1500
1000
500
0
0
10
20
30
40
50
60

We use the sbi Python package for simulation-based inference.

Values

- $T_{obs} = 15 : beta = 0.4333$ and $m = 0.9498$.
- $T_{obs} = 60 : beta = 0.4001$ and $m = 0.9999$.

Histograms

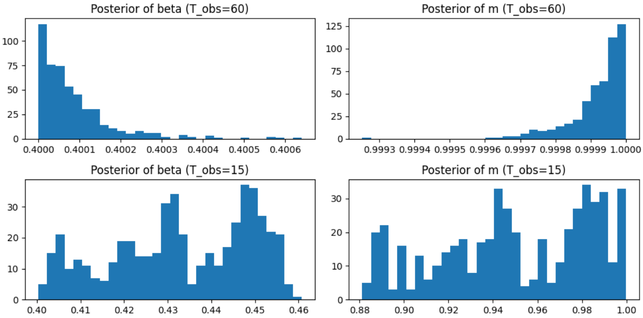


Figure – 500 samples of β and m .