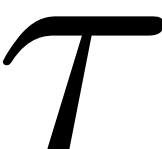
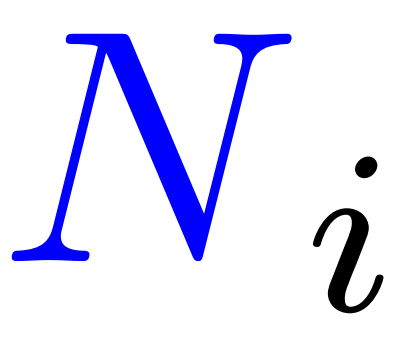
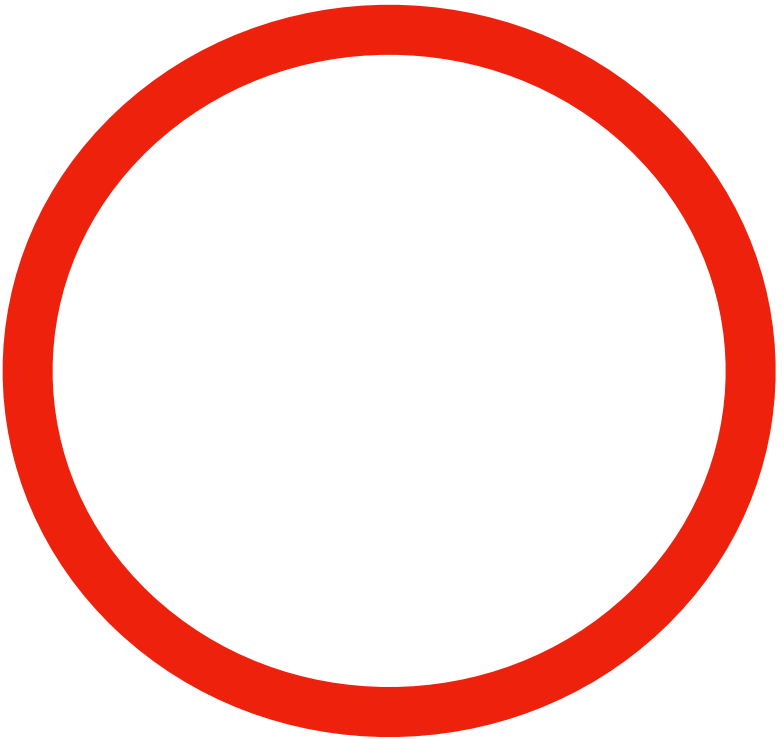

INSEA

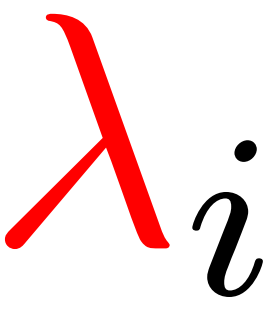
$$N_i | \lambda_i \sim \mathcal{P}(\lambda_i)$$

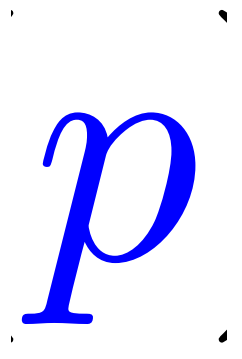


Données de l’individu





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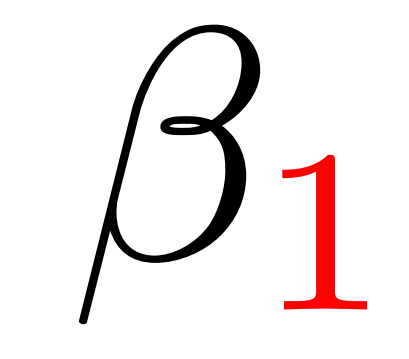
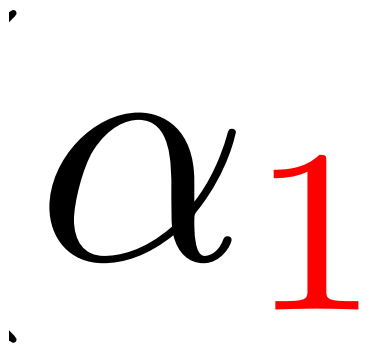
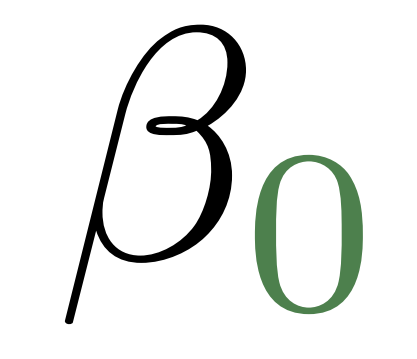
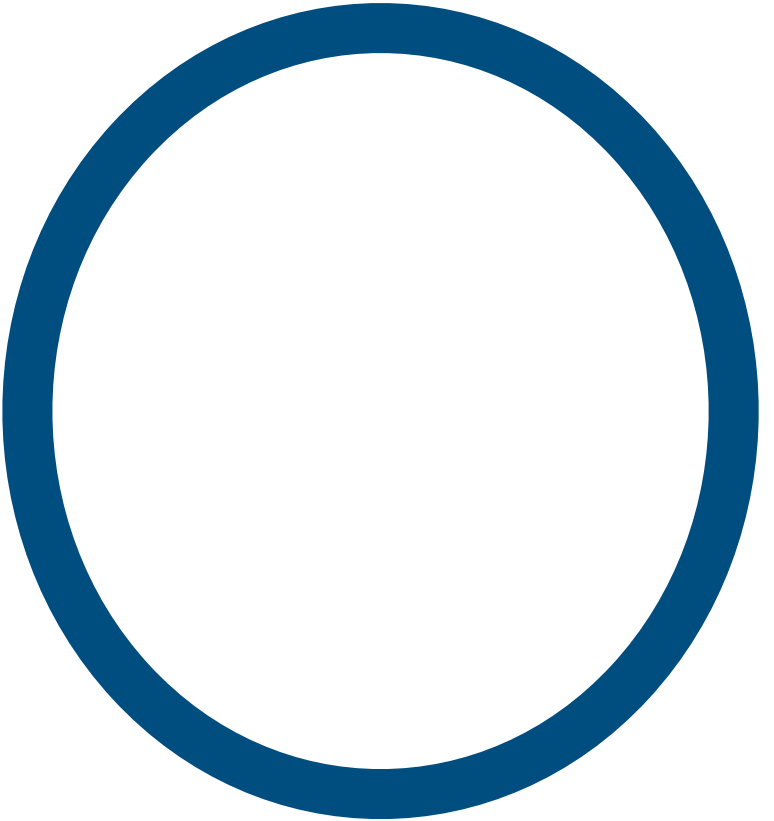
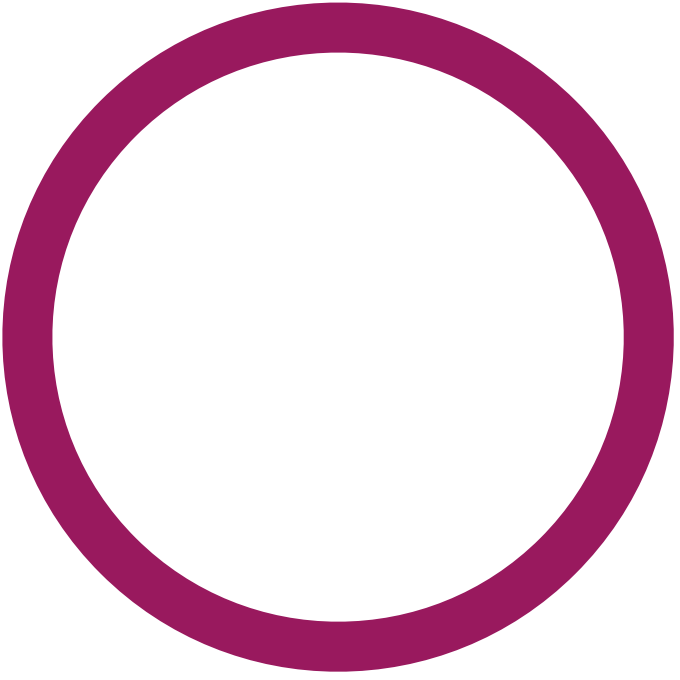

β_1

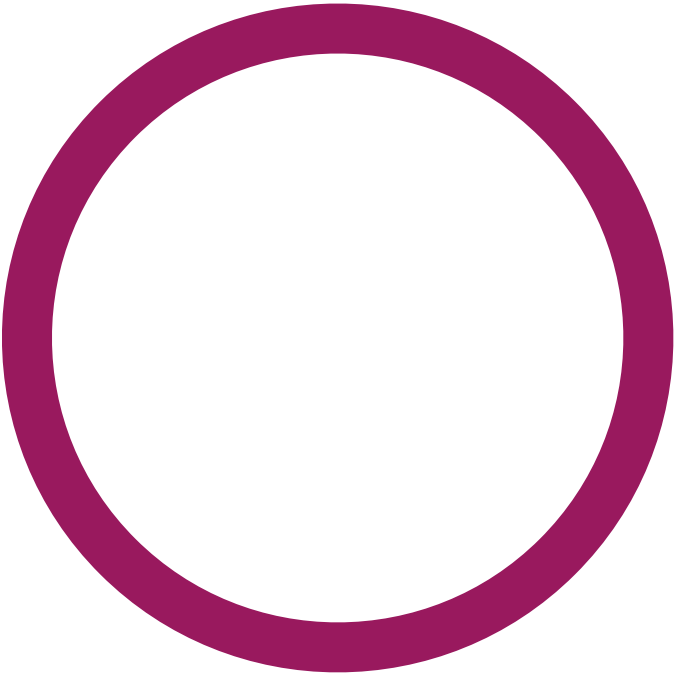

α_1

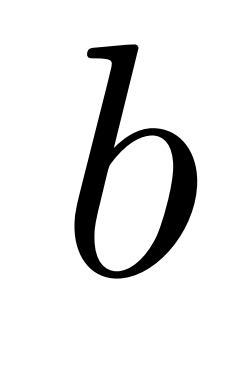

β_0

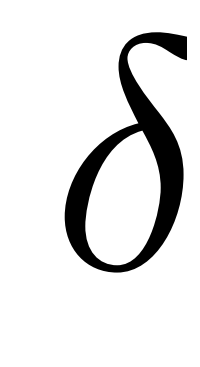

α_0

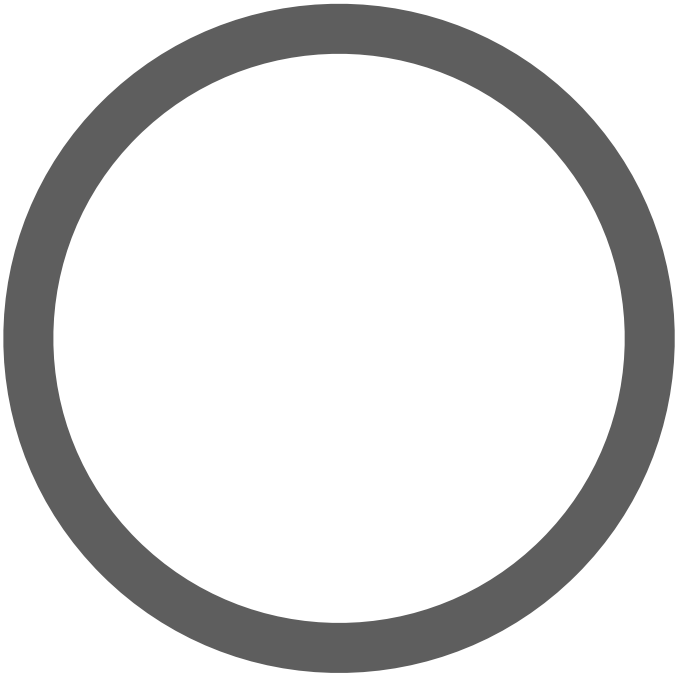


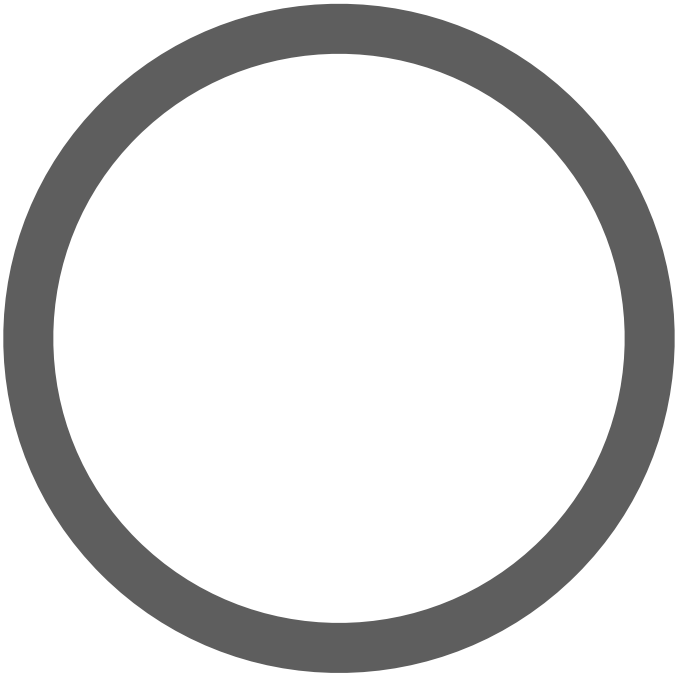


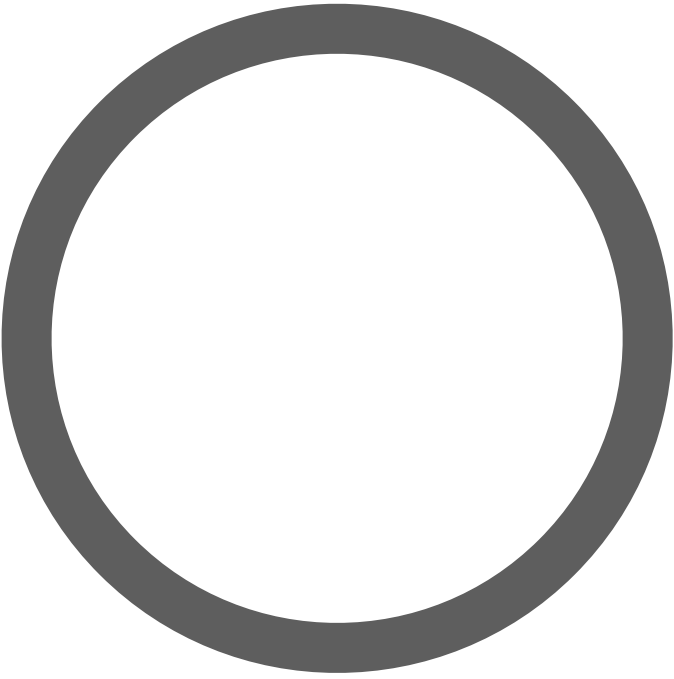


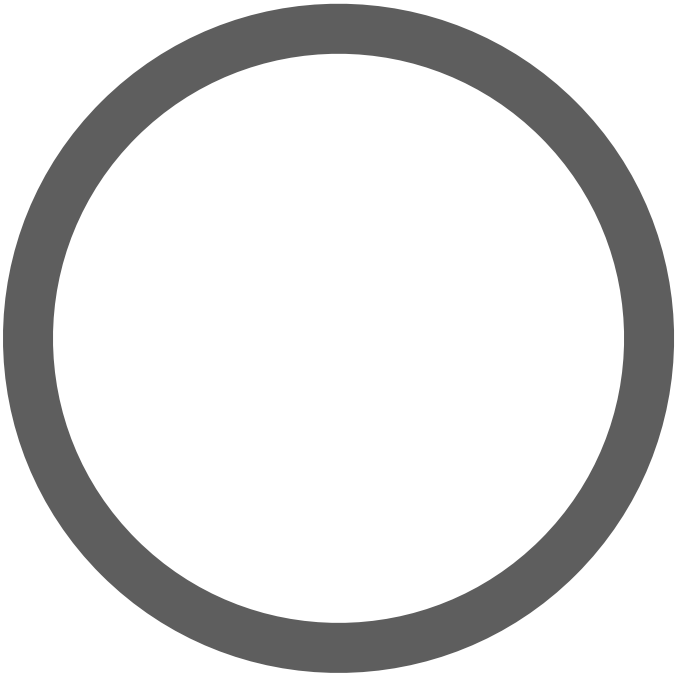


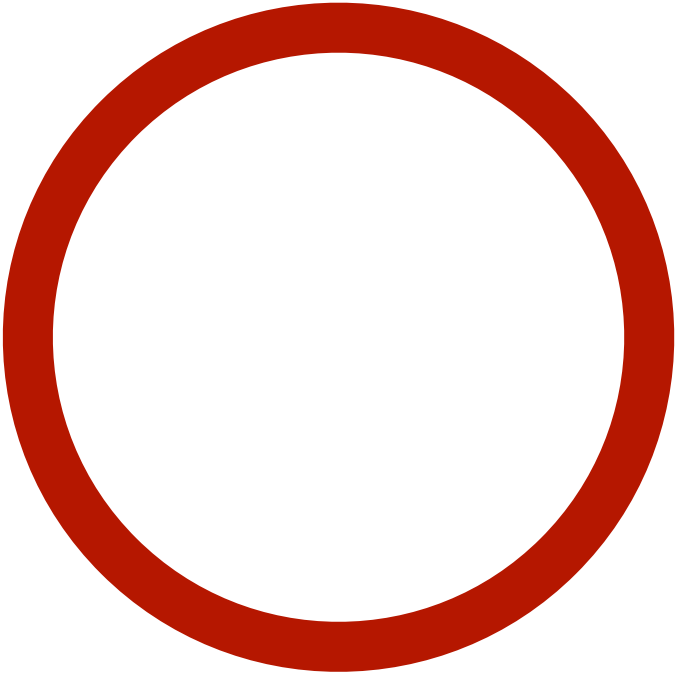


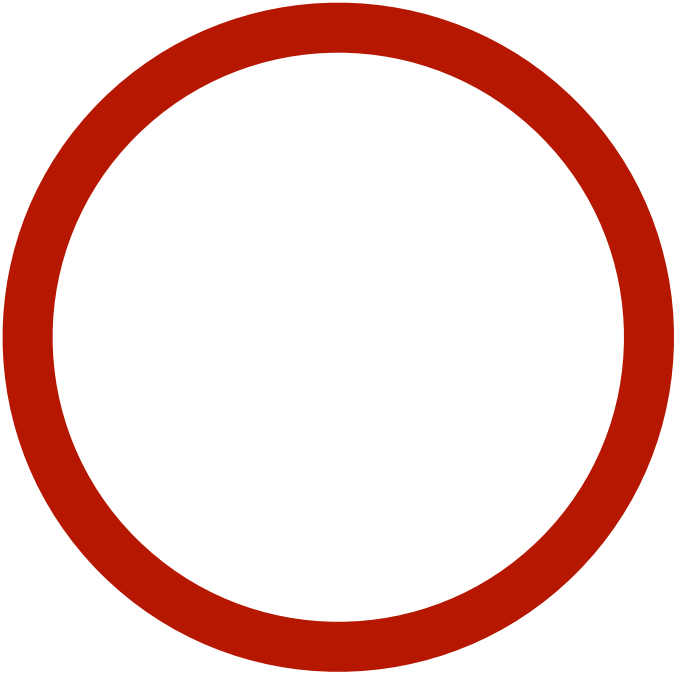


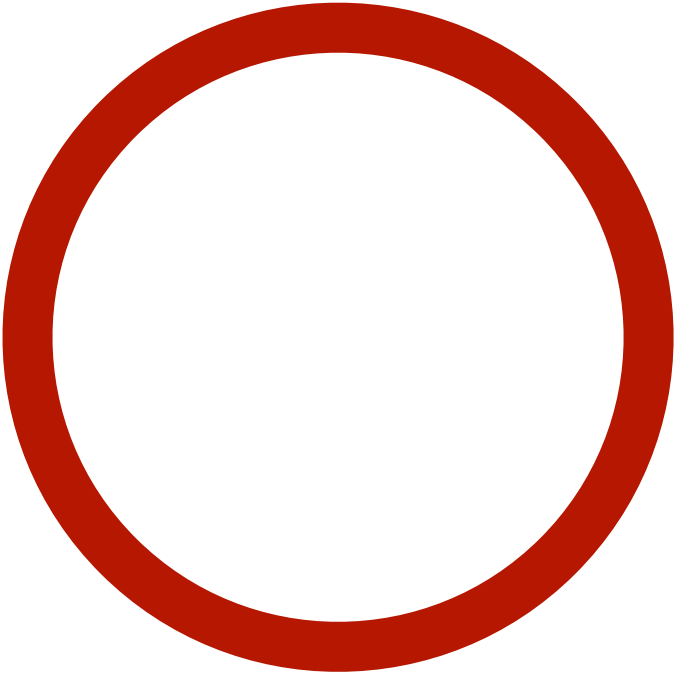


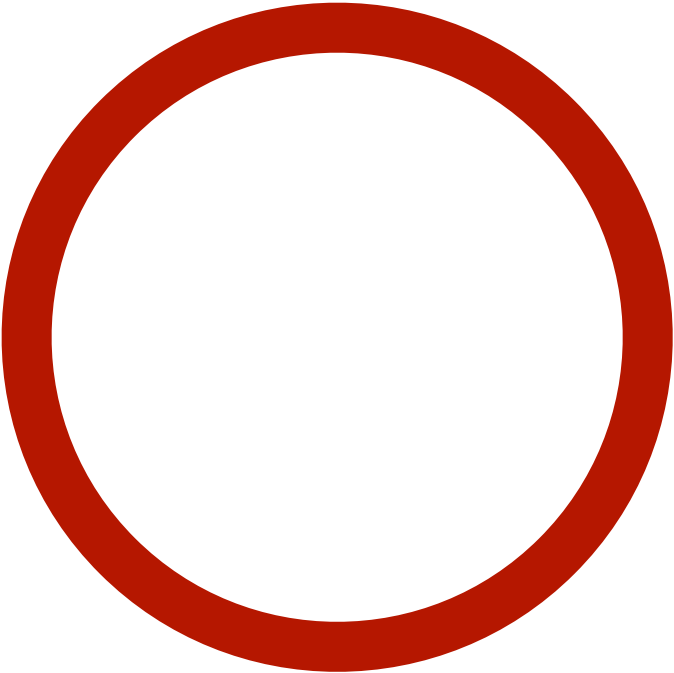


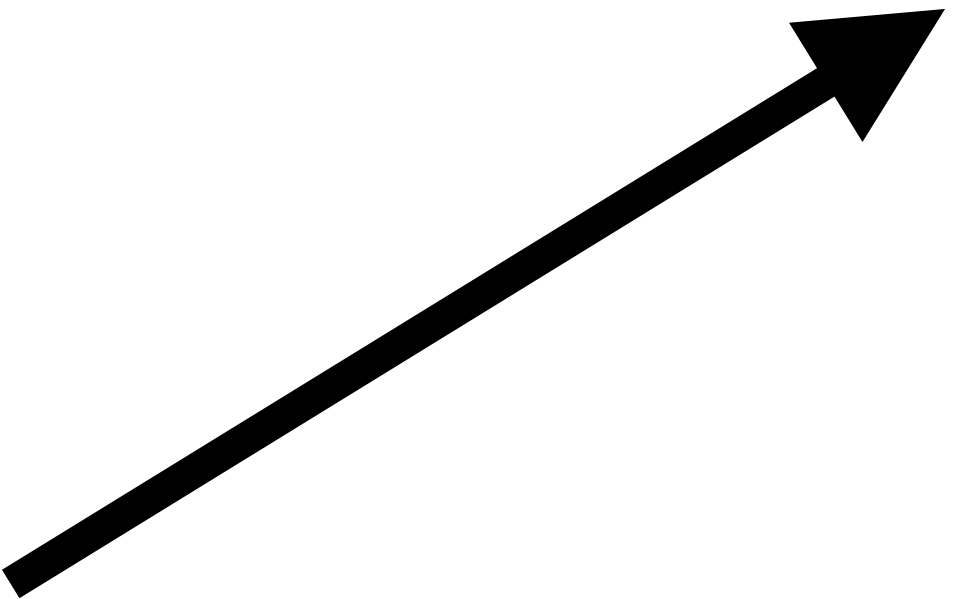


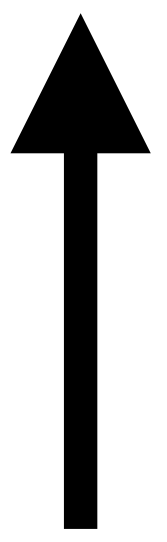


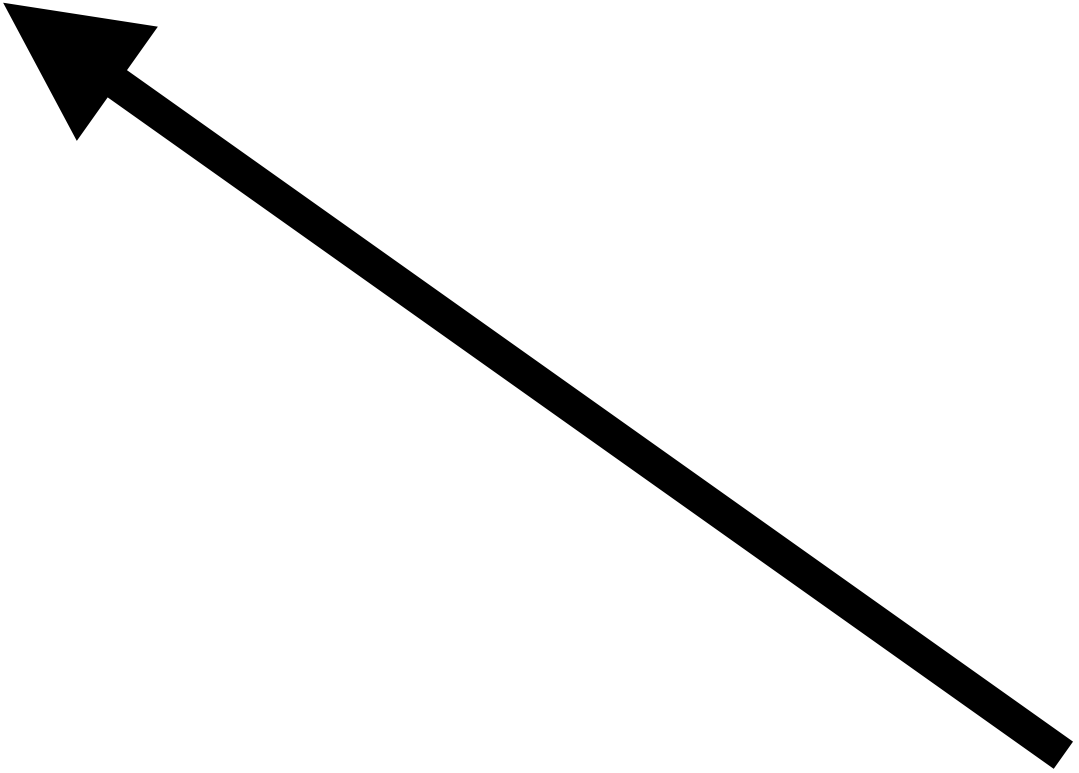


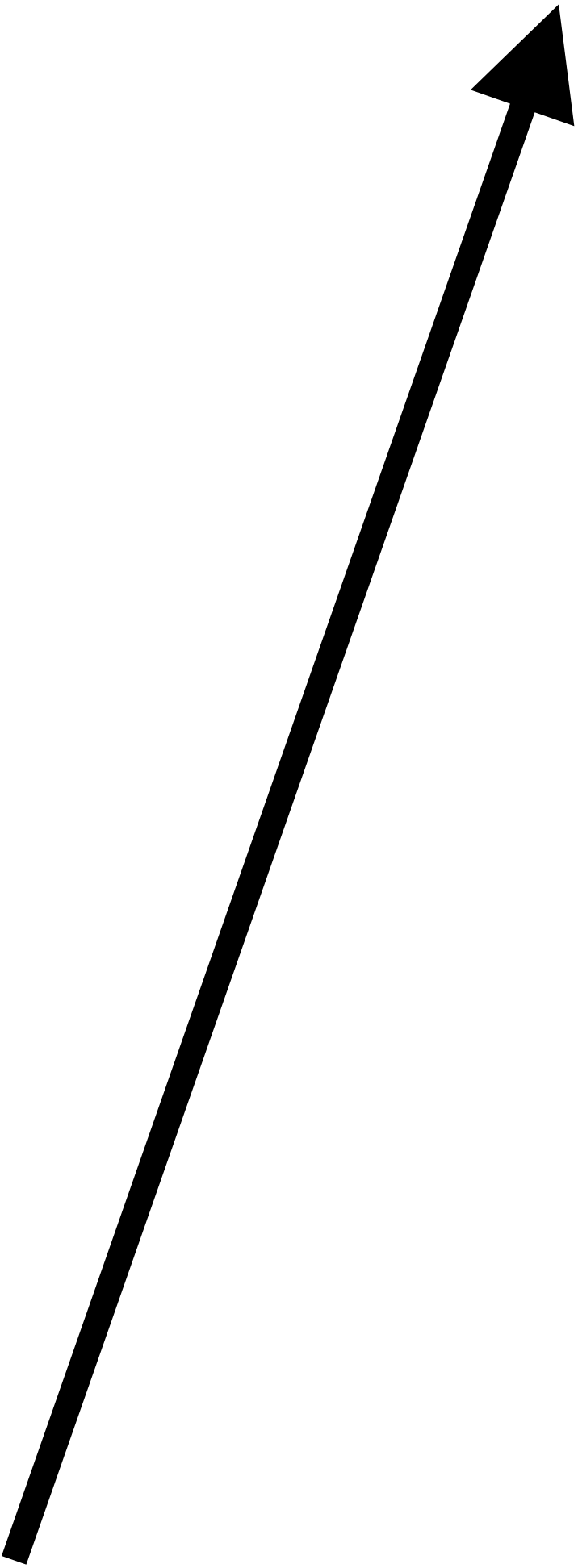


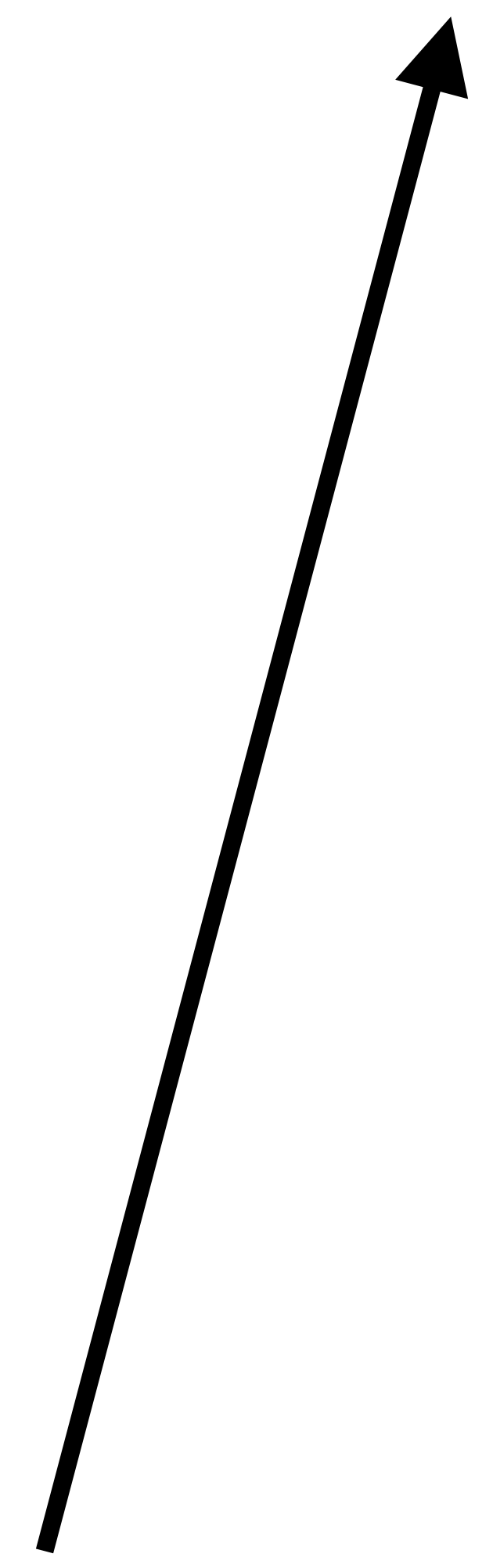


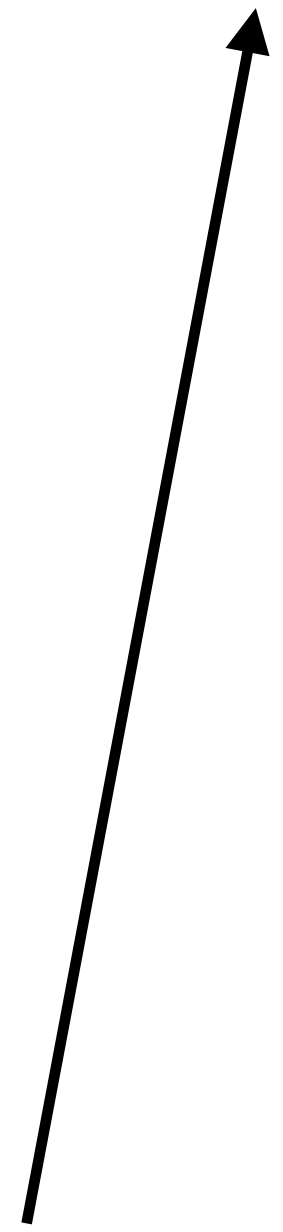


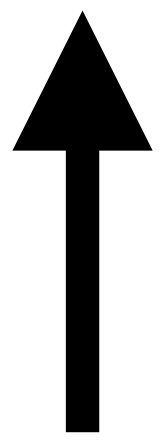


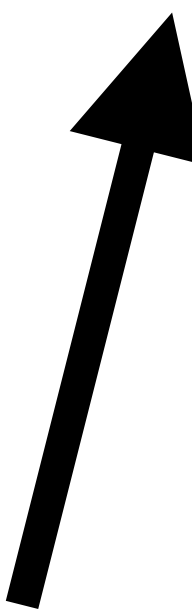


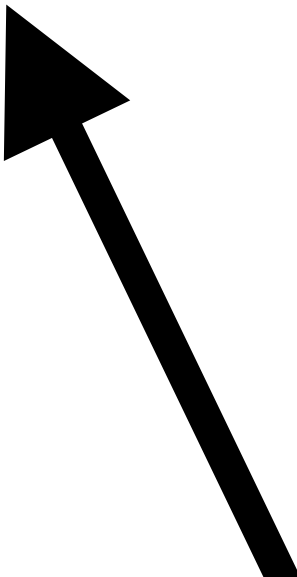


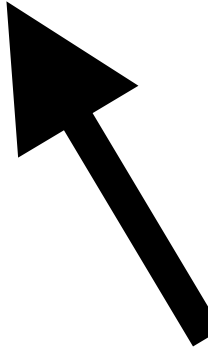


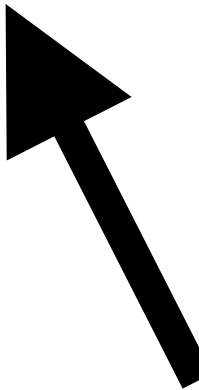


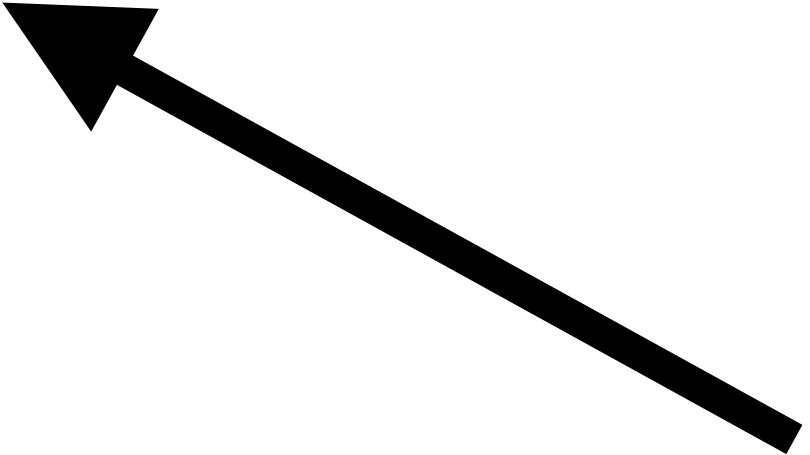


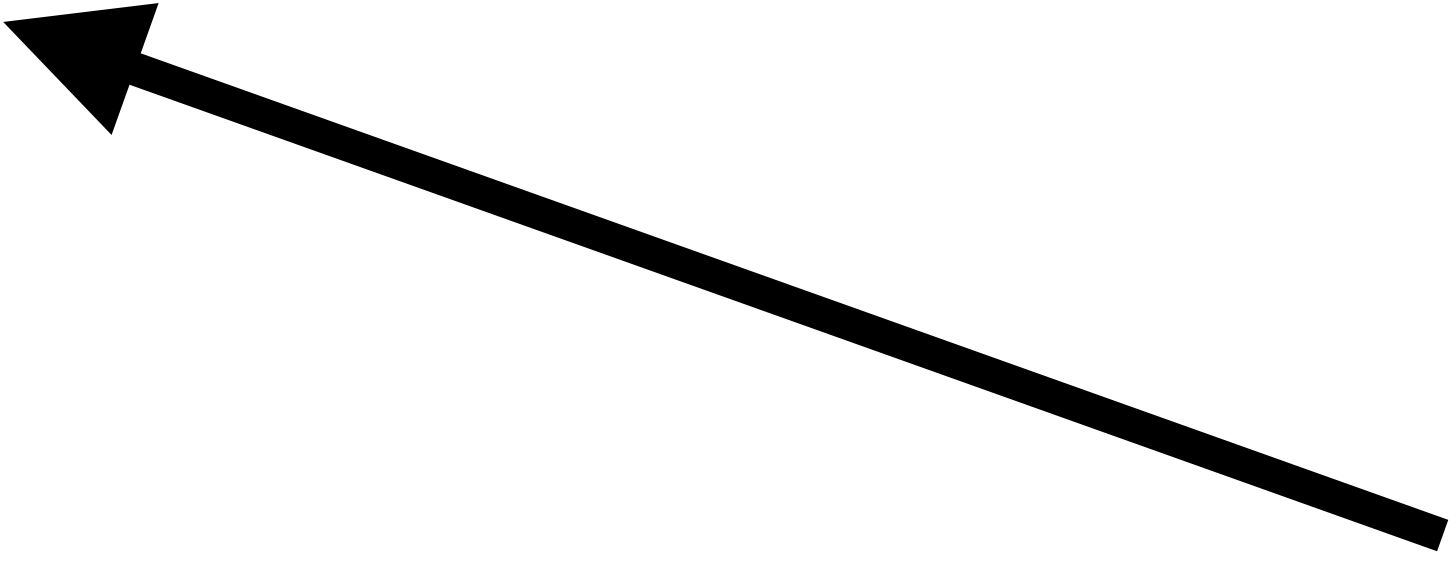


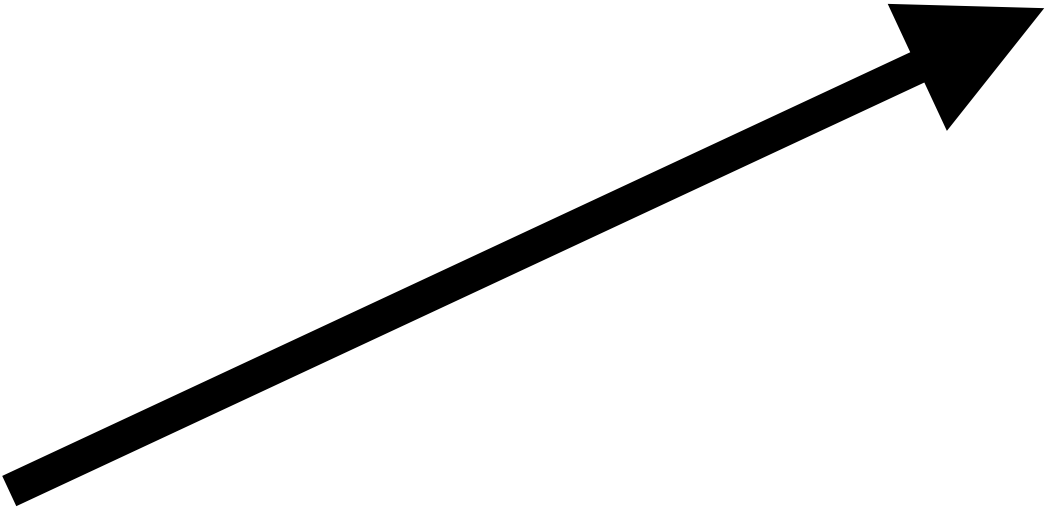


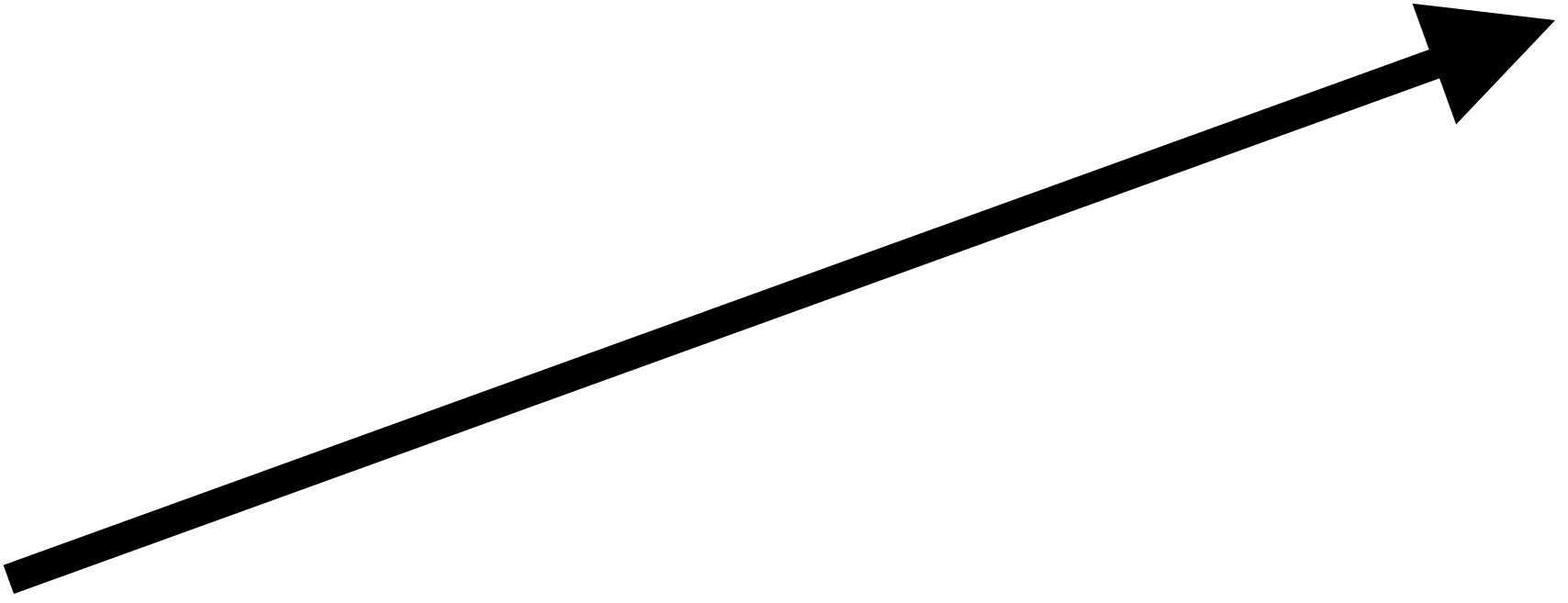












$$\lambda_i | z_i = 1 \sim \text{Gamma}(\alpha_1, \beta_1)$$

$$\lambda_i | z_i = 0 \sim \text{Gamma}(\alpha_0, \beta_0)$$

$$z_i | p \sim \text{Bernoulli}(p)$$

$$p \sim \beta(a, b)$$

$$\alpha_0, \alpha_1, \beta_0, \beta_1 \sim \text{Gamma}(\tau, \delta)$$

Paramètres individuels

Variables latentes (non-observées)

Hyper-paramètres (non aléatoires)

Paramètres collectifs

Modèle bayésien hiérarchique comme un PGM (*Probabilistic graphical model*)

On représente les dépendances entre variables aléatoires avec une flèche:

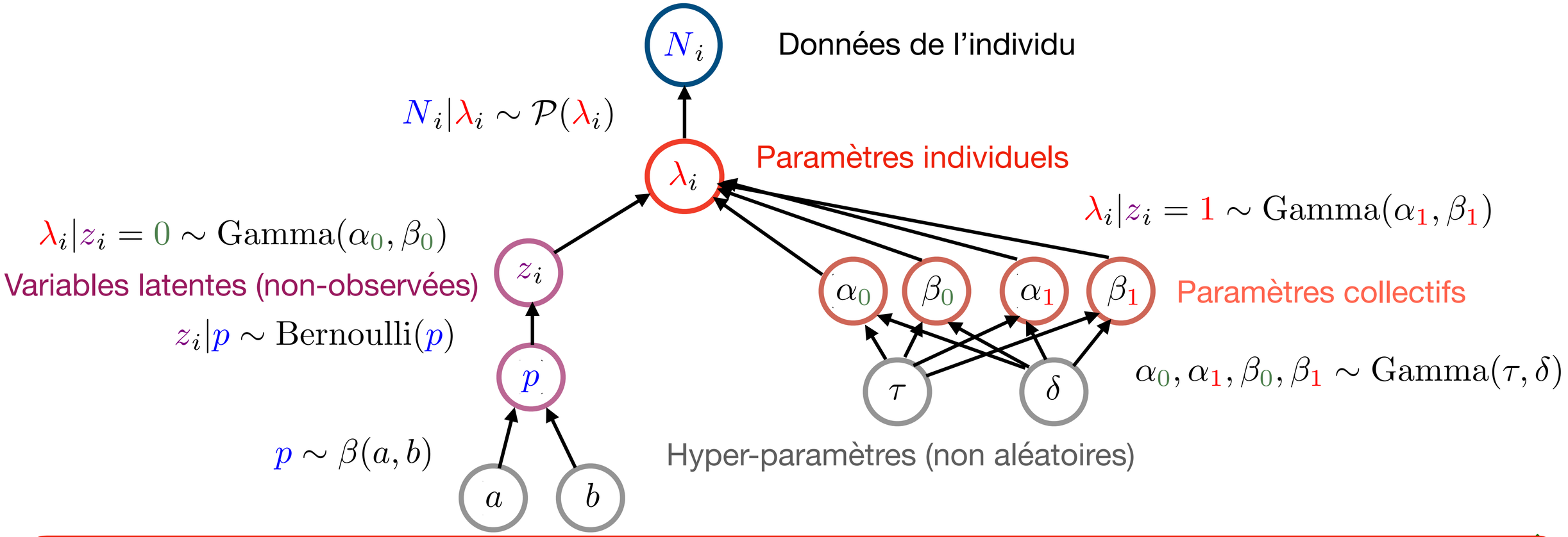
Question: trouver la loi a posteriori jointe de tous les paramètres $(\{\lambda_i\}_{i=1}^n, \{z_i\}_{i=1}^n, p, \alpha_0, \alpha_1, \beta_0, \beta_1) \,|\{N_i\}_{i=1}^n$.

Pourquoi Monte-Carlo ?

Modèle bayésien hiérarchique

Modèle bayésien hiérarchique comme un PGM (*Probabilistic graphical model*)

On représente les dépendances entre variables aléatoires avec une flèche:



Question: trouver la loi a posteriori jointe de tous les paramètres $(\{\lambda_i\}_{i=1}^n, \{z_i\}_{i=1}^n, p, \alpha_0, \alpha_1, \beta_0, \beta_1) \,|\{N_i\}_{i=1}^n$.

1. Pourquoi Monte-Carlo ? (Exemple de modèle hiérarchique)
2. Introduction à la méthode Monte-Carlo (historique, PRNG)
3. Algorithmes de simulation i.i.d (PRNG, transformation, rejet)
4. Méthodes MCMC (Gibbs, Metropolis)
5. Diagonstics de convergence MCMC
6. Méthodes MCMC avancées (Langevin, HMC, NUTS)

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La loi jointe $(\{\lambda_i\}_{i=1}^n, \{z_i\}_{i=1}^n, p, \alpha_0, \alpha_1, \beta_0, \beta_1) \,|\{N_i\}_{i=1}^n$ est