First Steps in Reinforcement Learning

Policy Iteration, Monte Carlo Learning and SARSA

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with slides stolen from David Silver's lecture and Figs. from the book by Sutton and Barto

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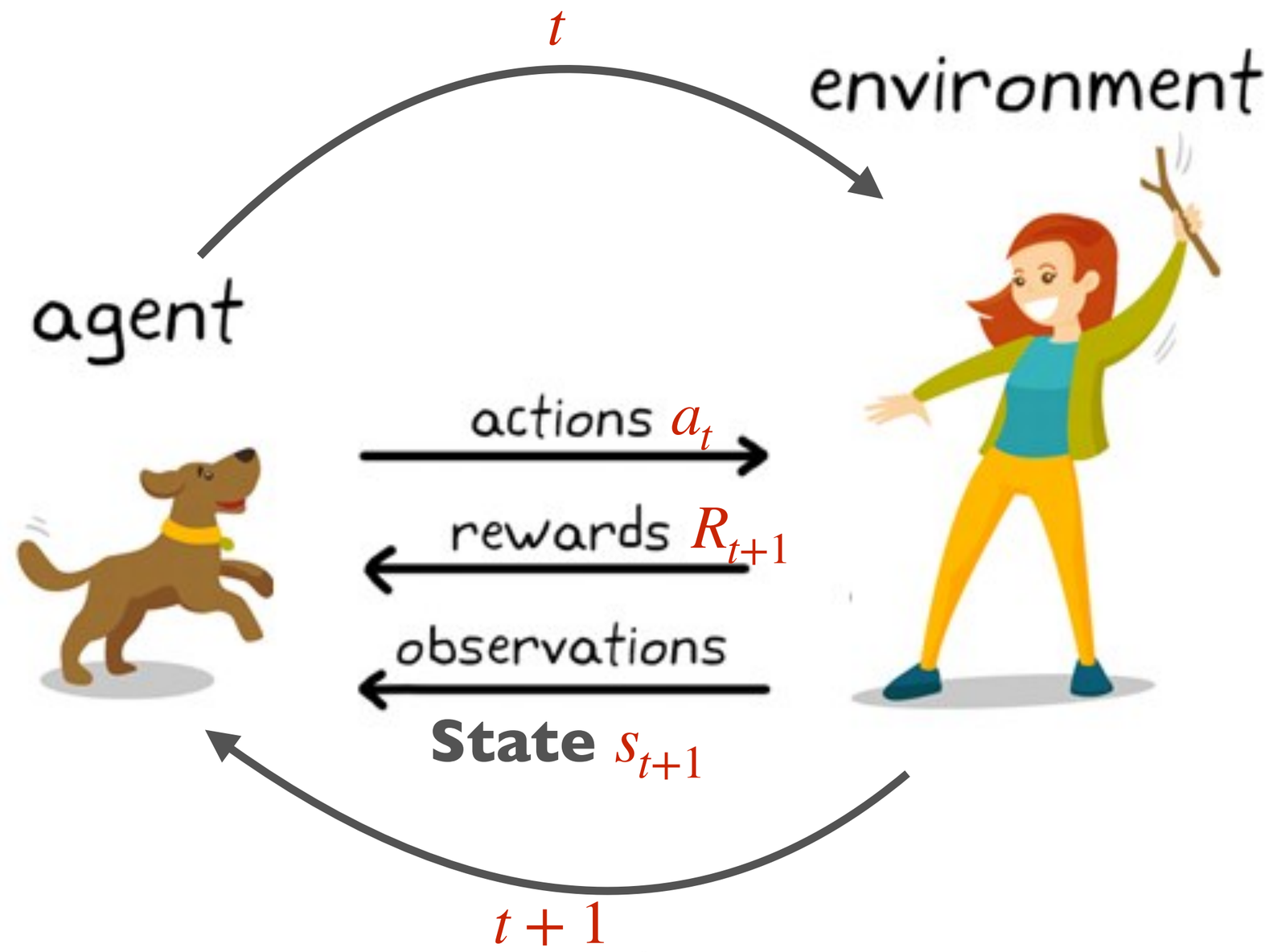
Revisiting
Markov Decision Processes


t
environment
agent
actions a_t
rewards R_{t+1}
observations
State s_{t+1}
t + 1

Markov Decision Process

- A Markov Decision Process is a Markov Reward Process with decisions on actions

Definition: A **Markov Decision Process** is a tupel $(\mathbf{S}, \mathbf{A}, \mathbf{P}, \mathbf{R}, \gamma)$

- $\mathbf{S}$ is a finite set of states
- $\mathbf{A}$ is a finite set of actions
- $\mathbf{P}$ is a state transition probability matrix $P_{ss'}^{a} = P\left(S_{t+1} = s' \,|\, S_t = s, A_t = a\right)$
- $\mathbf{R}$ is a reward function $R_s^a = E\left(R_{t+1} \,|\, S_t = s, A_t = a\right)$
- γ is a discount factor $\gamma \in [0,1]$

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Value Functions

Definition: The *state-value function* $v_\pi(s)$ of an MDP is the expected return starting from state s and following policy π

$$v_\pi(s) = E_\pi\left\{G_t \,|\, S_t = s\right\} = E_\pi\left\{R_{t+1} + \gamma v_\pi(S_{t+1}) \,|\, S_t = s\right\}$$

Definition: The *action-value function* $q_\pi(s, a)$ of an MDP is the expected return starting from state s by taking action a and following policy π

$$q_\pi(s, a) = E_\pi\left\{G_t \,|\, S_t = s, A_t = a\right\} = E_\pi\left\{R_{t+1} + \gamma q_\pi(S_{t+1}, A_{t+1}) \,|\, S_t = s, A_t = a\right\}$$

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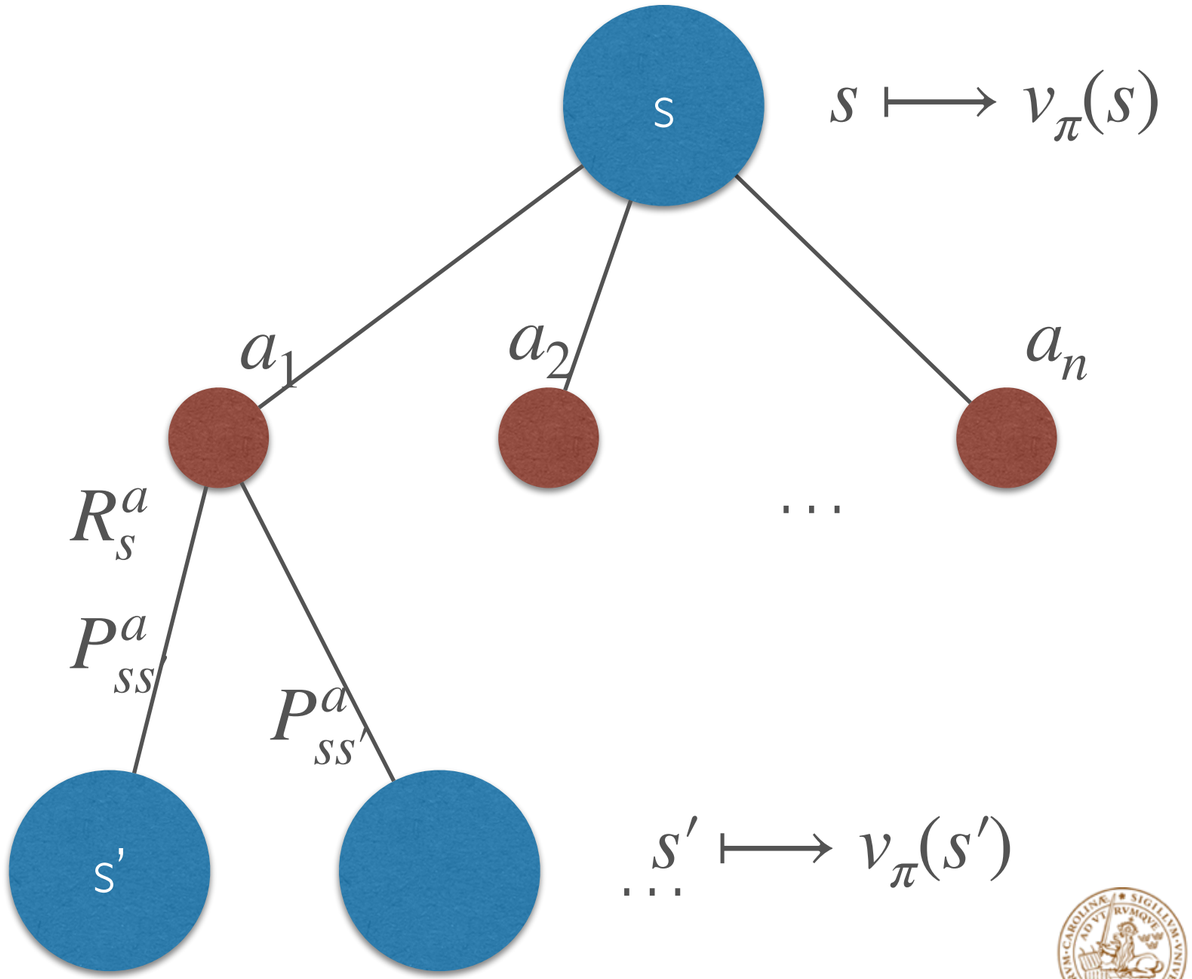
Bellman Expectation Equation

- From state s, we can take different actions $a_i \in \mathbf{A}$ chosen according to $\pi(a \mid s)$
- Based on s and a we get a reward R_s^a

$$v_\pi(s) = \sum_{a \in \mathbf{A}} \pi(a \mid s) \left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_\pi(s') \right)$$

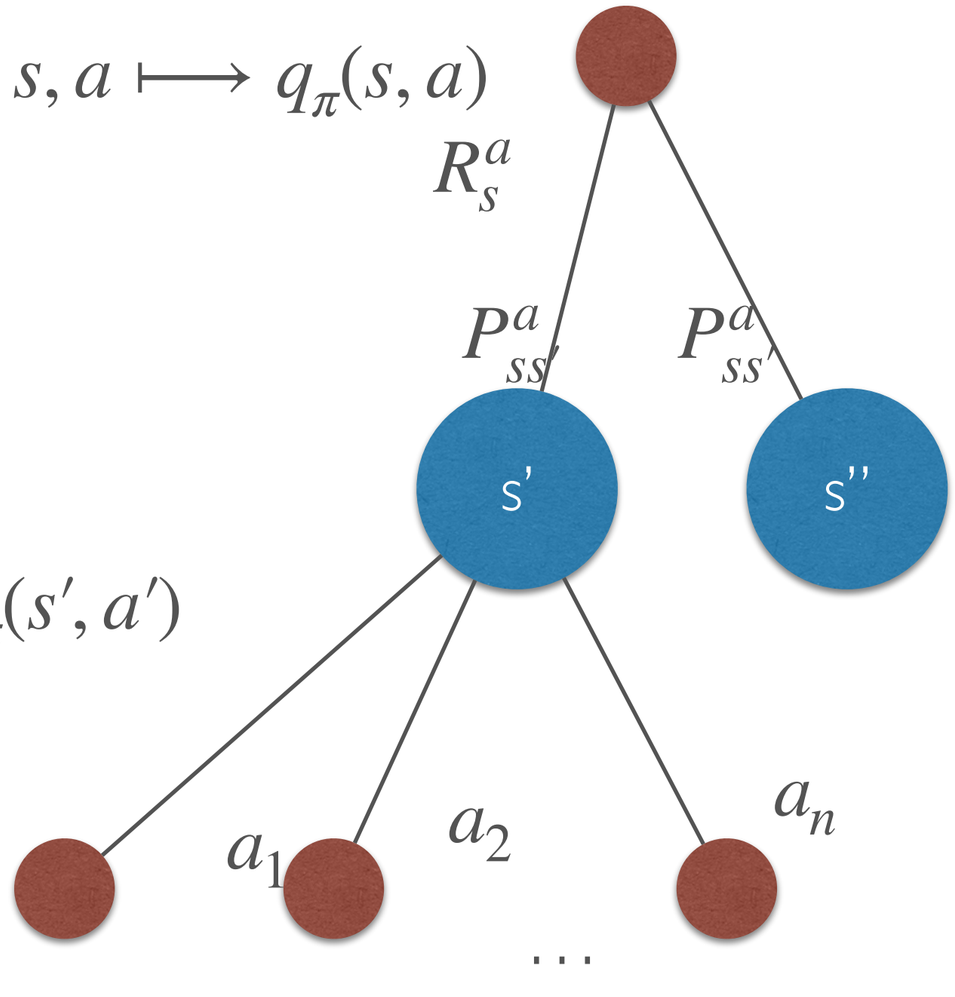


Bellman Expectation Equation

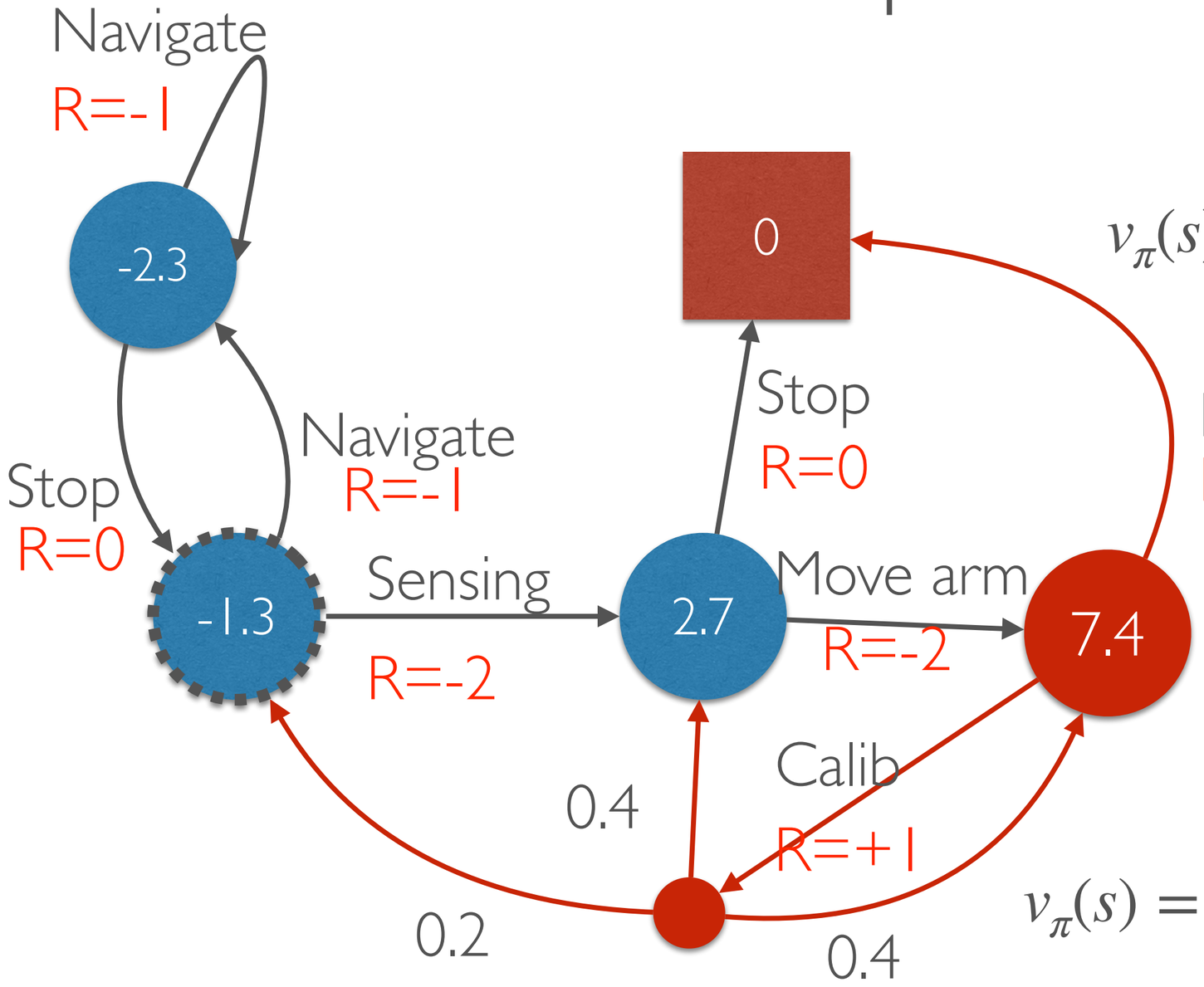
$$s, a \longmapsto q_\pi(s, a)$$

$$q_\pi(s, a) = R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a \sum_{a' \in \mathbf{A}} \pi(a' \mid s') q_\pi(s', a')$$

$$s', a_i \longmapsto q_\pi(s', a_i)$$



Bellman expectation Equation



$v_\pi(s)$ for $\pi(s,a) = 0.5,\ \gamma = 1$

$$v_\pi(s) = 0.5 \cdot \big(1 + 0.2 \cdot (-1.3) + 0.4 \cdot 2.7 + 0.4 \cdot v_\pi(s)\big) + 0.5 \cdot 10$$

$$v_\pi(s) = 7.4$$

$$v_\pi(s) = \sum_{a \in \mathbf{A}} \pi(a \mid s) \left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_\pi(s') \right)$$

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Plan for Today

- How can the state value and the action value functions be computed?
  - ***What do we need to know for doing this?***
- How can we find an optimal policy?

- Start with a first simple solution: **Iterative Policy Evaluation** and **Policy Iteration**
- Then: first steps in **Model-free Prediction and Control:**
  - Monte Carlo Learning
- Outlook: **Temporal Difference Learning** (TD-Learning) and SARSA
  - SARSA is similar to the famous Q-Learning appoach.

Prediction, Control

- We have given a fully defined MDP $(\mathbf{S}, \mathbf{A}, \mathbf{P}, \mathbf{R}, \gamma)$
- We also assume to be given some policy π
- We want to know the state values $v_\pi(s)$ of each state s for the policy.
  - This is called ***Prediction*** of how good the plan / policy is.
- Then, we want to know π^*, i.e., the optimal policy for the MDP
  - This is called ***Control***.

Policy Evaluation To Find v_π

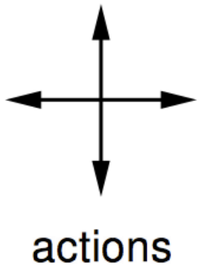
Iterative Policy Evaluation

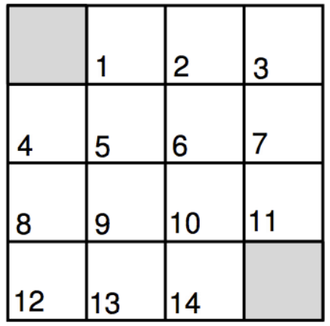
- Goal: evaluate a given policy π
- Solution: Iterative application of the Bellman expectation equation $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_\pi$

- Initialise all state value to 0: $v_1(s) = 0$ for all $s \in S$
- At each iteration $k + 1$
  - For all states $s \in S$
  - Update $v_{k+1}(s)$ from $v_k(s')$ where s' is a successor state of s

$$v_{k+1}(s) = \sum_{a \in \mathbf{A}} \pi(a \mid s) \left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_k(s') \right)$$

Example: Random Policy in the Small Gridworld





$r = -1$ for all transition

$\gamma = 1$

- Non-terminal states $1 \ldots 14$
- terminal state in grey
- Actions leading out of the grid leave state unchanged
- Reward is -1 until the terminal state is reached
- Agent follows uniform random policy

$$\pi(n, \cdot) = \pi(e, \cdot) = \pi(s, \cdot) = \pi(w, \cdot)$$

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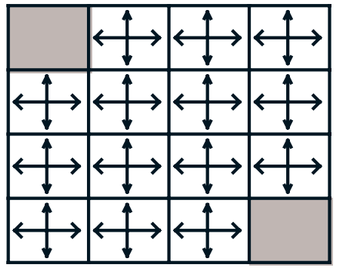
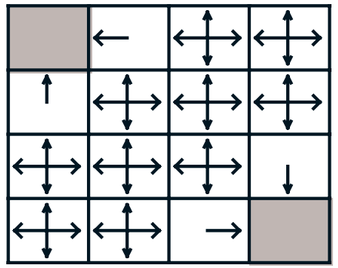
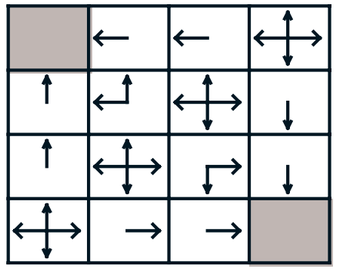
Example: Random Policy in the Small Gridworld

	v_k for the Random Policy	Greedy Policy w.r.t. v_k
$k = 0$	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	 ← random policy
$k = 1$	0.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 0.0	
$k = 2$	0.0 -1.7 -2.0 -2.0 -1.7 -2.0 -2.0 -2.0 -2.0 -2.0 -2.0 -1.7 -2.0 -2.0 -1.7 0.0	

$$R = -1, \gamma = 1$$

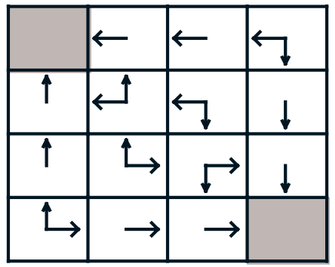
$$\pi(n, \cdot) = \pi(e, \cdot) = \pi(s, \cdot) = \pi(w, \cdot)$$

$$v_{k+1}(s) = \sum_{a \in \mathbf{A}} \pi(a \mid s) \left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_k(s') \right)$$

Example: Random Policy in the Small Gridworld

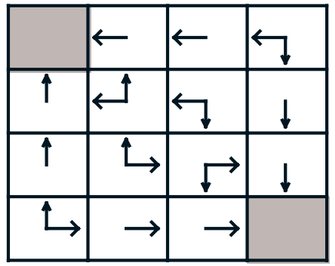
$k = 3$

0.0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0.0



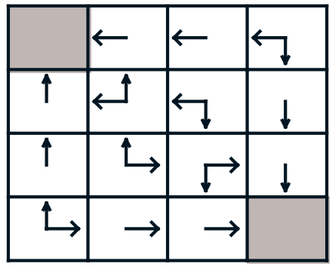
$k = 10$

0.0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0.0



$k = \infty$

0.0	-14.	-20.	-22.
-14.	-18.	-20.	-20.
-20.	-20.	-18.	-14.
-22.	-20.	-14.	0.0



optimal policy

Policy Iteration To Improve the Policy

How to improve the Policy

- Given a policy.
  - Evaluate the policy

$$v_\pi = E\left\{R_{t+1} + \gamma R_{t+2} + \ldots \mid S_t = s\right\} = \sum_{a \in \mathbf{A}} \pi(a \mid s)\left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_k(s')\right)$$

  - Improve the policy by acting greedily with respect to v_π

$$\pi' = \text{greedy}(v_\pi) \qquad \pi'(s) = \arg\max_{a \in A}\left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_k(s')\right)$$

- In the Gridworld the improved policy was optimal $\pi^{k \geq 3} = \pi^*$
- In general, more iterations are needed
- But this process of **policy iteration** always converges to π^*

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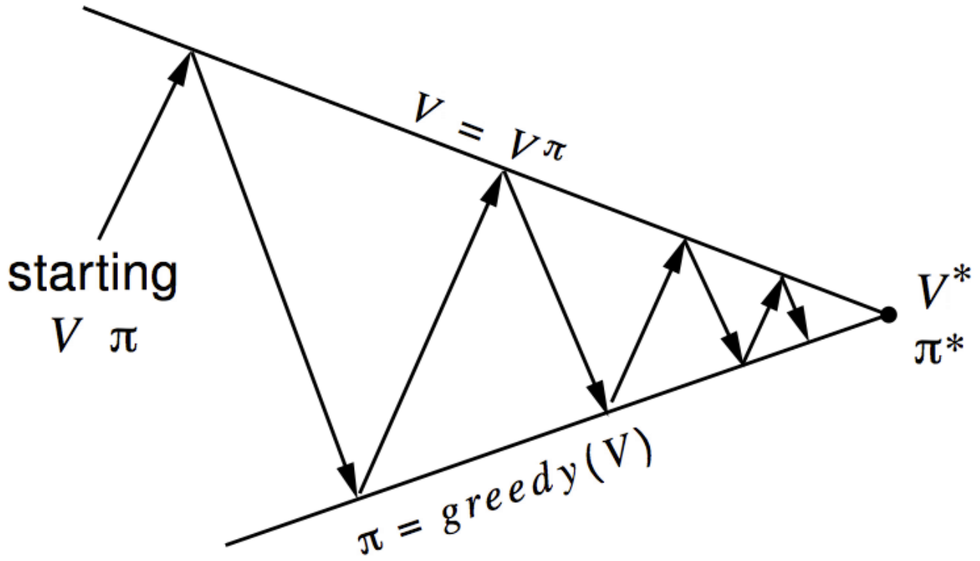
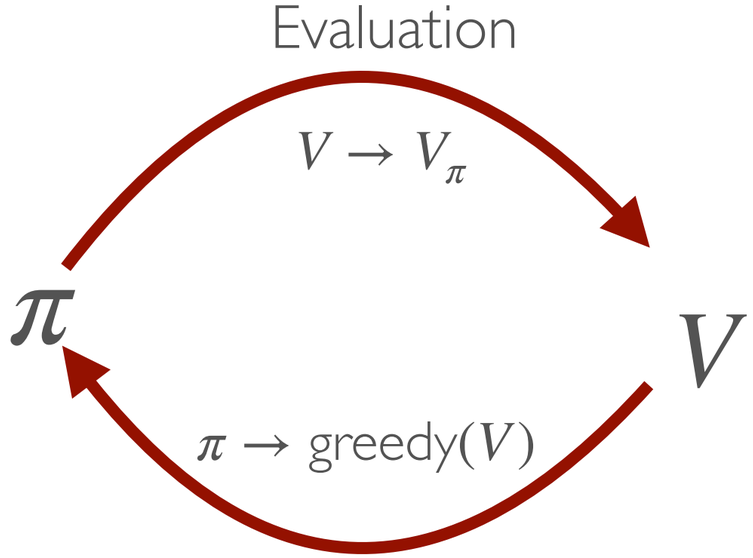
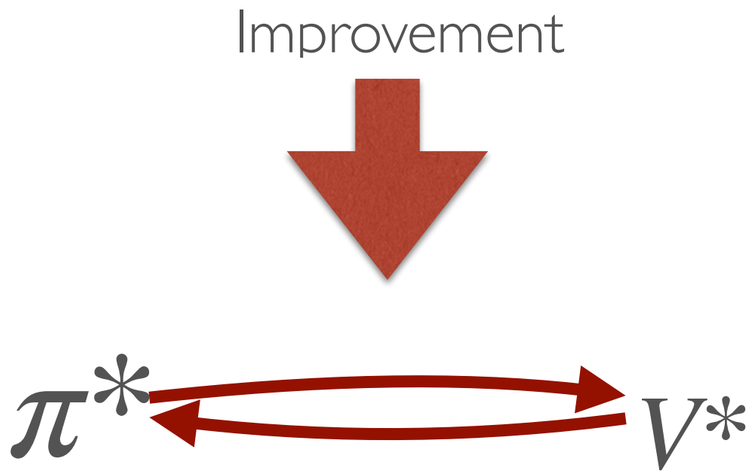
Policy Iteration





- **Policy Evaluation**: Estimate v_π
  - Iterative policy evaluation
- **Policy improvement**: Generate $\pi' \geq \pi$
  - Greedy policy improvement



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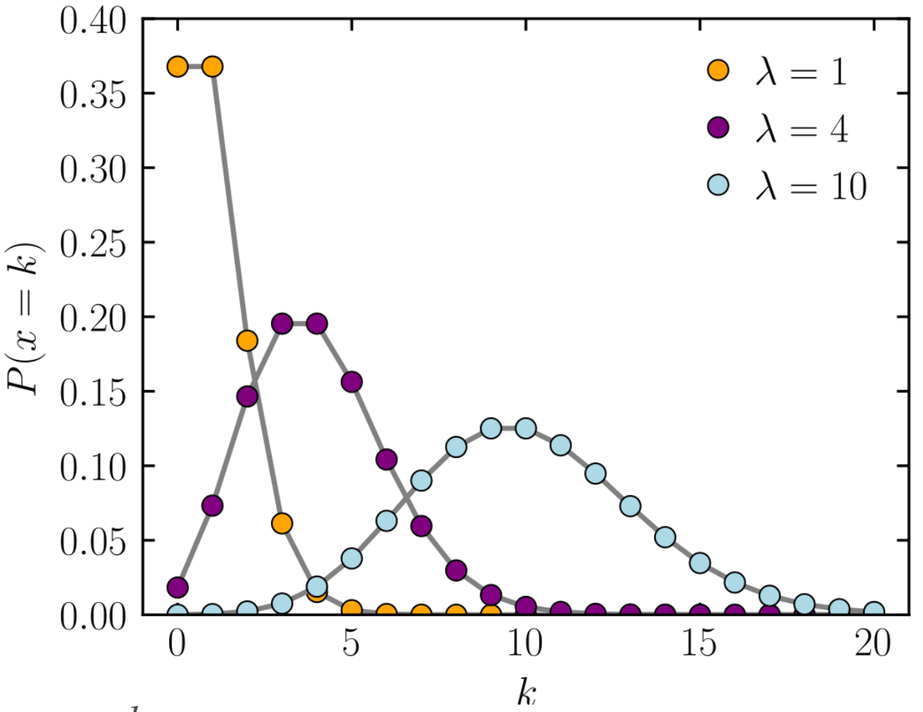
Christmas Task: Play with Jack's Car Rental



- **States**: Two locations, maximum 20 cars at each location
- **Actions**: Move up to 5 cars between locations overnight
- **Reward**: 10€ for each car rented if available, -2€ for moving a car
- **Transitions**: Cars returned and requested randomly
  - Poisson distribution, *k* returns/requests with probability $P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$
  - 1st location: average requests=3, average returns=3
  - 2nd location: average requests=4, average returns =2

Policy: how many cars to move?

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Policy Improvement

Consider a deterministic policy $a = \pi(s)$

We can *improve* the policy by acting greedily $\pi'(s) = \arg\max_{a \in A} q_\pi(s, a)$

This improves the value from any state s over one step

Blackboard

- If improvement stops

$$v_{\pi'}(s) = q_{\pi}(s, \pi'(s)) = \max_{a \in A} q_{\pi}(s, a) = q_{\pi}(s, \pi(s)) = v_{\pi}(s)$$

- Bellman optimality equation has been satisfied

$$v_{\pi}(s) = \max_{a \in A} q_{\pi}(s, a)$$

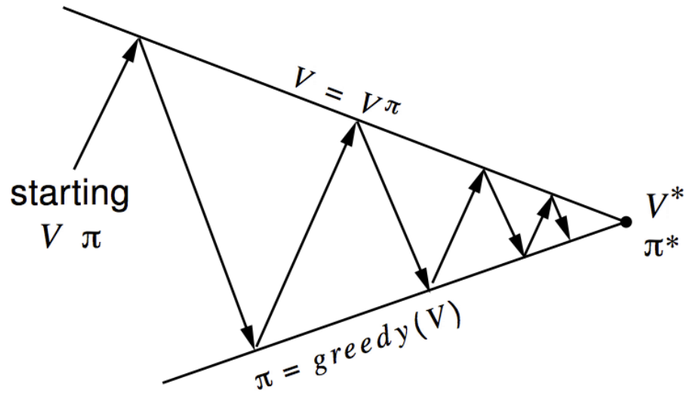
- Thus, $v_{\pi}(s) = v^*(s)$ for all $s \in S$
- and π is the optimal policy.

Some amazing facts

- **Policy evaluation** always converges to v_π
- **Policy Iteration** always converges to π^*
- *Synchronous* approach used above
- *Asynchronous* approach converges as well if all states are selected
- How do we know we have reached v_π ?
  - How many repetitions k ?
- How do we know we have reached π^* ?
- Simply stop after $k = 1$ and update after each iteration
  - This is equivalent to ***value iteration***

Generating a new improved π'

Improving the same π



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Value Iteration

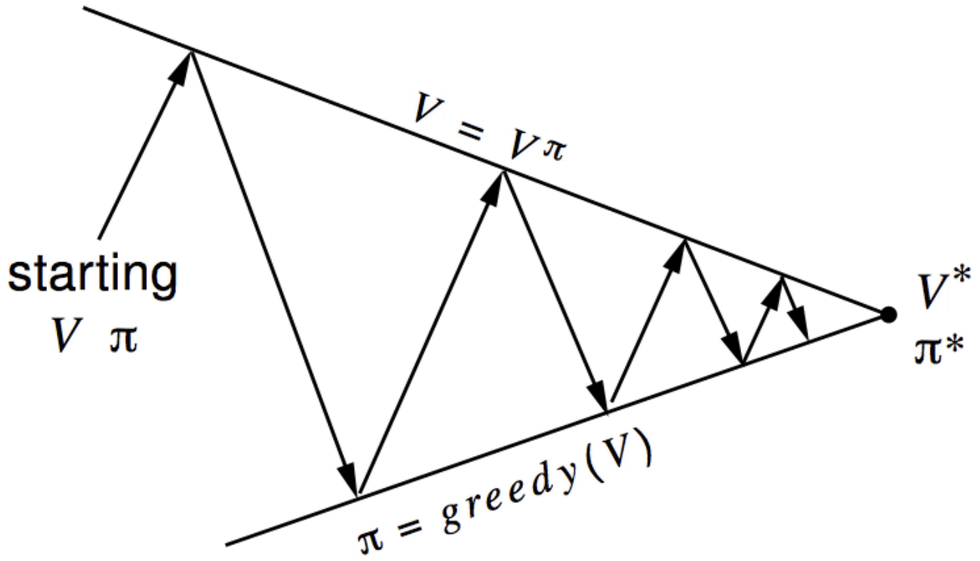
- Goal: find the optimal policy π
- Solution: Iterative application of the Bellman expectation equation $v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_*$
- Initialise all state value to 0: $v_1(s) = 0$ for all $s \in S$
- At each iteration $k + 1$
  - For all states $s \in S$
    - Update $v_{k+1}(s)$ from $v_k(s')$ where s' is a successor state of s

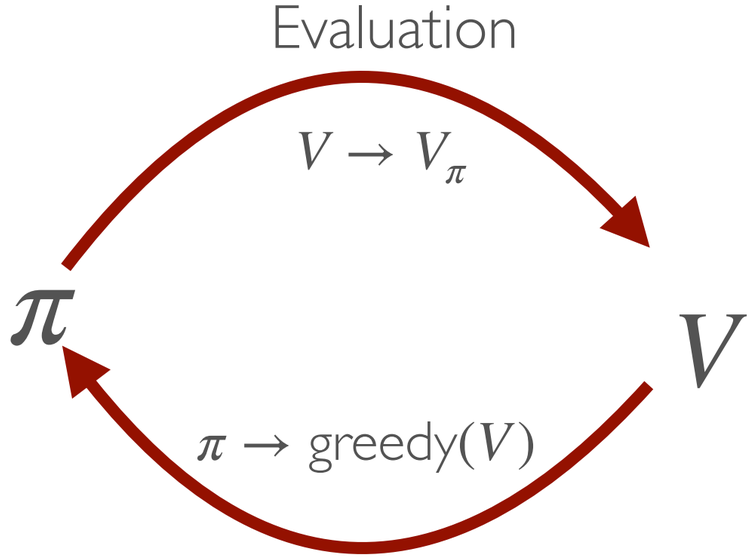
$$v_{k+1}(s) = \max_a \left(R_s^a + \gamma \sum_{s' \in \mathbf{S}} P_{ss'}^a v_k(s') \right)$$

- Note: there is no(!!) explicit policy here
  - we simply go to the state with the biggest reward
- Intermediate value functions might not correspond to any policy.

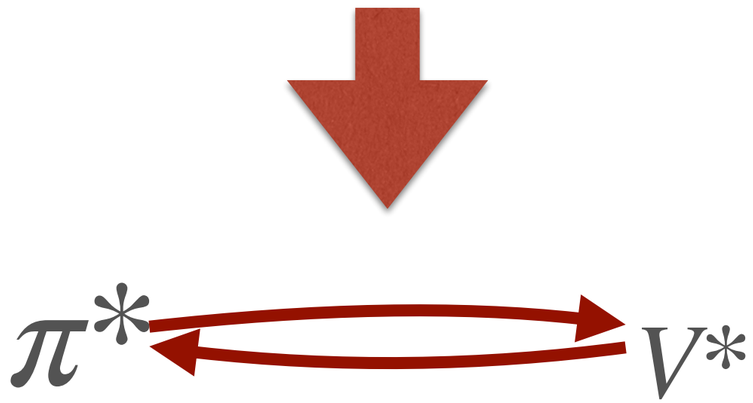
Done when there are no more updates

Generalized Policy Iteration





- **Policy Evaluation**: Estimate v_π
  - ***Any*** policy evaluation approach
- **Policy improvement**: Generate $\pi' \geq \pi$
  - ***Any*** policy improvement approach



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Summary so far

- Identified the optimal policy, state value function and action value function
- **Policy Iteration** and **Value Iteration** for finding the optimal policy.
  - Value Iteration: very simple approach, but still finds π^*
- The MDP needs to be known completely
  - ⛔ The transition probabilities $P^a_{s,s'}$ are usually not known
  - ⛔ It's not obvious how one can extend this to continuous actions and states
  - ➕ Everything is mathematically exact, expressible and analysable and best for small problems.

Summary so far

- Transition function $P^a_{s,s'}$? Rewards R^a_s ?
  - Two common cases
    1. The MDP is *unknown* but the experience is available: *Let's try things out and see what happens*…
    2. The MDP is known but too complicated to use.
- **Goal**:
  - ***Estimate*** the value function for an *unknown* MDP
  - ***Optimise*** the value function and the policy of an *unknown* MDP

Monte Carlo Learning for Model-Free Prediction and Control

Concrete Example: Blackjack

- **States** (200 of them):
  - Current sum (12-21)
  - Dealer's showing card (ace-10)
  - Do I have a "useable" ace? (yes-no)
- **Action stick**: Stop receiving cards (and terminate)
- **Action twist**: Take another card (no replacement)
- **Reward** for **stick**:
  - +1 if sum of cards > sum of dealer cards
  - 0 if sum of cards = sum of dealer cards
  - -1 if sum of cards < sum of dealer cards
- **Reward** for **twist**:
  - -1 if sum of cards > 21 (and terminate)
  - 0 otherwise
- **Transitions**: automatically twist if sum of cards < 12

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Learning from Episodes

- The goal: estimate the value function of an *unknown* MDP
- The trick: Use experiences!!

We learn v_π from **episodes of experience** under the policy π.

$$S_1, A_1, R_2, \ldots, S_k \sim \pi$$

Monte Carlo Reinforcement Learning

- MC methods learn directly from episodes of experience / observations
- MC uses simplest possible idea: value = sample mean of the return

Reminder: Expected Return

- Remember:
  - the ***return*** $G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$
  - The ***value function*** is the expected return: $v_\pi(s) = E_\pi \left\{ G_t \,|\, S_t = s \right\}$

- The goal is to estimate the expected return from the observed episodes

$$E^1 : S_1^1, A_1^1, R_2^1, \ldots, S_k^1$$
$$E^2 : S_1^2, A_1^2, R_2^2, \ldots, S_k^2$$
$$\ldots$$
$$E^n : S_1^n, A_1^n, R_2^n, \ldots, S_k^n$$

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Monte Carlo Policy Evaluation

Counting and Averaging

- Episodes $E^1, E^2, \ldots, E^n$ are given as training data
- For every episode $E^i : S_1^i, A_1^i, R_2^i, \ldots, S_k^i$
  - **First** times step t where state s is visited in episode E_i
    - increment counter $N(s) \leftarrow N(s) + 1$
    - Increment total return $S(s) \leftarrow S(s) + G_t$
- The estimated mean return: $V(s) \leftarrow S(s)/N(s)$

$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- Law of large numbers: $V(s) \rightarrow V_\pi(s)$ as $N(s) \rightarrow \infty$

Monte Carlo Policy Evaluation

Counting and Averaging

- Episodes $E^1, E^2, \ldots, E^n$ are given as training data
- For every episode $E^i : S_1^i, A_1^i, R_2^i, \ldots, S_k^i$
  - **Every** times step t that state s is visited in episode E^i
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$$G_t = R_{t+1} + \gamma R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

- Law of large numbers: $V(s) \rightarrow V_\pi(s)$ as $N(s) \rightarrow \infty$

We have to have all episodes in memory!!

Recursive computation of mean

- The mean μ_k can be computed recursively from the sequence $x_1, x_2, \ldots$

$$
\begin{aligned}
\mu_k &= \frac{1}{k}\sum_{j=1}^{k} x_j \\
&= \frac{1}{k}(x_k + \sum_{j=1}^{k-1} x_j) \\
&= \frac{1}{k}(x_k + (k-1)\mu_{k-1}) \\
&= \mu_{k-1} + \frac{1}{k}\underbrace{(x_k - \mu_{k-1})}_{\text{error-term}}
\end{aligned}
$$

Incremental Monte-Carlo Updates

- Update each $V(s)$ after each episode E^i:
  - For every state S_t with return G_t
    - increment counter $N(s) \leftarrow N(s) + 1$
    - ~~Increment total return $S(s) \leftarrow S(s) + G_t$~~
- The estimated mean return: ~~$V(s) \leftarrow S(s)/N(s)$~~

Incremental Monte-Carlo Updates

- Update each $V(s)$ after each episode E^i:
  - For every state S_t with return G_t
    - increment counter $N(s) \leftarrow N(s) + 1$
    - ~~Increment total return $S(s) \leftarrow S(s) + G_t$~~
    - The estimated mean return: ~~$V(s) \leftarrow S(s)/N(s)$~~

$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)} \left(G_t - V(S_t)\right)$$

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t)\right)$$

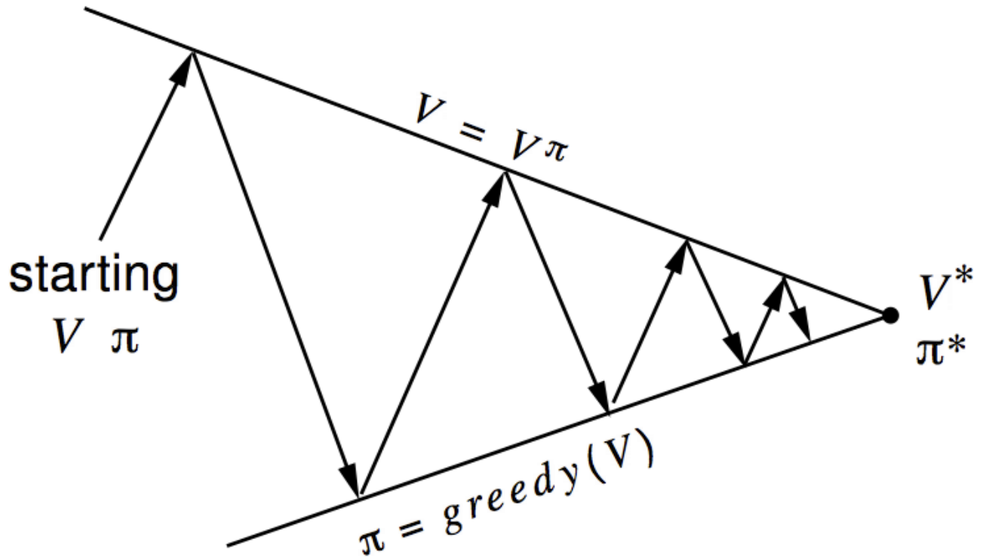
We can work incrementally!!
Sample one episode at a time

Optimizing the Policy

Monte Carlo Control

Policy Iteration

- **Policy Evaluation**: Estimate v_π
  - Iterative Policy Evaluation
  - Monte Carlo Policy Evaluation
- **Policy improvement**: Generate $\pi' \geq \pi$
  - Greedy policy improvement **???**



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Computing the Policy

- Greedy policy improvement over $V(s)$ requires knowledge of the MDP:

$$\pi'(s) = \arg\max_{a \in A} \left(R_s^a + \gamma \sum_{s'} P_{ss'}^a v_\pi(s') \right)$$

- Greedy improvement over $Q(s,a)$ is model-free.

$$\pi'(s) = \arg\max_{a \in A} Q(s,a)$$

This is the Trick!!!

Computed incrementally exactly like $V(S_t)$ with incremental monte carlo updates!!

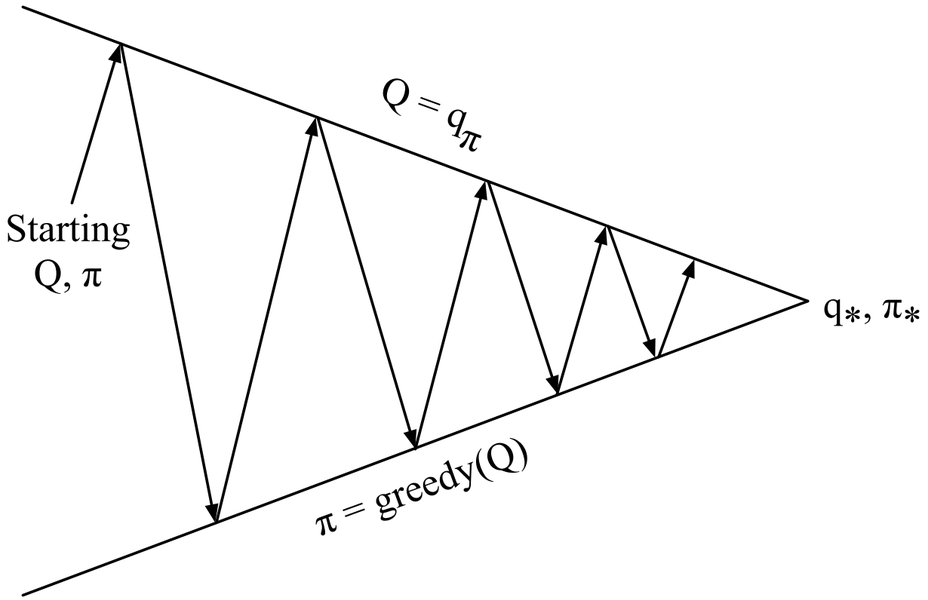
- What we need is a statistic of state transitions s given an action a

$$S_1, A_1, R_2, S_2, A_2, R_3, \ldots, S_k$$

Generalized Policy Iteration for Action Value Function

- Policy Evaluation:
  - Monte Carlos Policy Evaluation of $Q = q_\pi$
- Policy Improvement:
  - Greedy policy improvement **???**

$$\pi'(s) = \arg\max_{a \in A} Q(s, a) \; ??$$



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Example of a greedy action

- Two restaurants you can choose from
  - The top one was closed and we got no food.
    - $V(top) = 0$
  - You go to the bottom one and get a reward +1
    - $V(bottom) = +1$
  - You go to the bottom one and get a reward +2
    - $V(bottom) = +2$
  - You go to the bottom one and get a reward +1
    - $V(bottom) = +1$
  - …
  - Are you sure you chose the right restaurant?

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We learn on the job!!!
The policy we can find when using greedy improvement will depend on the policy we have now.
We very probably miss certain state-action pairs.

ϵ-greedy exploration

- *Try out something new for a change!!* **exploration vs exploitation**
- Simplest idea is to keep exploring: **ϵ-greedy**
  - *All* m actions are tried out with above zero probability:
  - With probability $\epsilon > 0$ we choose an action at random
  - With probability $1 - \epsilon$ we use the greedy action

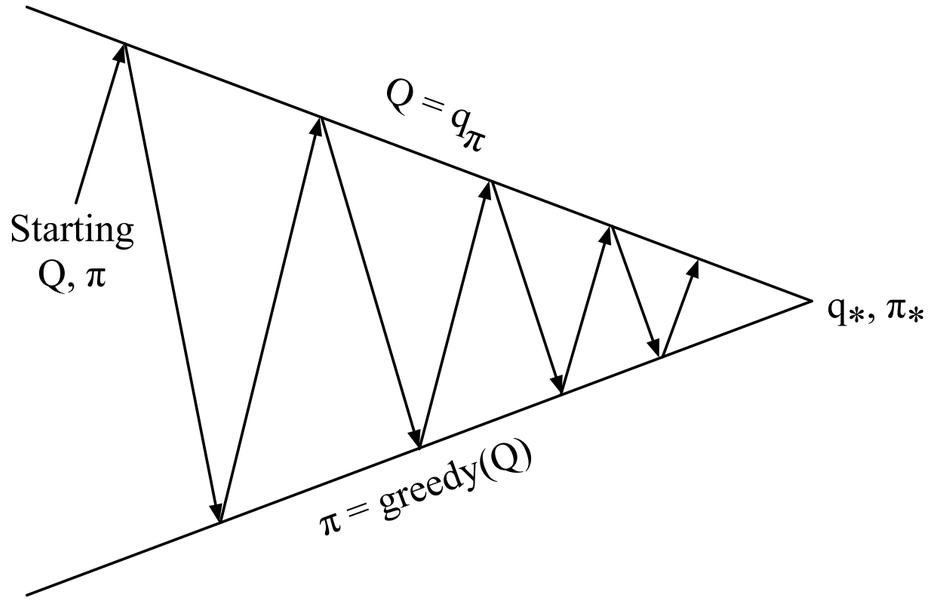
$$\pi'(a \mid s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a = \arg\max_{a' \in A} Q(s, a') \\ \epsilon/m & \text{otherwise} \end{cases}$$

ϵ-greedy Policy Improvement

Theorem: For any ϵ-greedy policy π, the ϵ-greedy policy π' with respect to q_π improves the value function: $v_{\pi'}(s) \geq v_\pi(s)$

Monte Carlo Control

- Policy Evaluation:
  - Monte Carlos Policy Evaluation of $Q = q_\pi$
- Policy Improvement:
  - ϵ-greedy policy improvement



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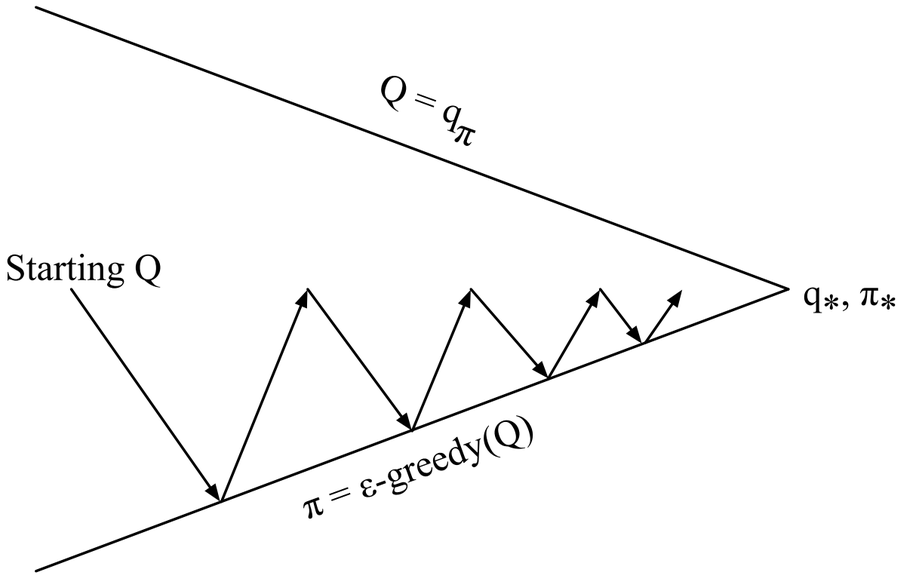
Monte Carlo Control more efficient

- For every episode:
  - Policy Evaluation:
    - Monte Carlos Policy Evaluation of $Q \approx q_\pi$
  - Policy Improvement:
    - Generate a new episode with ϵ-greedy policy



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GLIE Monte Carlo Control

- **Greedy in the Limit with Infinite Exploration** (GLIE)
- Two principles:
  1. keep exploring
  2. make sure that the policy eventually becomes greedy!
     - needed for the Bellman optimality equation.

- One possibility: ϵ-greedy policy with slow decay of ϵ: e.g., $\epsilon_k = \frac{1}{k}$

GLIE Monte Carlo Control

- Generate a new episode E^k using your policy $\pi : S_1, A_1, R_2, S_2, A_2, R_3, \ldots, R_T$
- For each state S_t and action A_t in the episode

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)} \left(G_t - Q(S_t, A_t)\right)$$

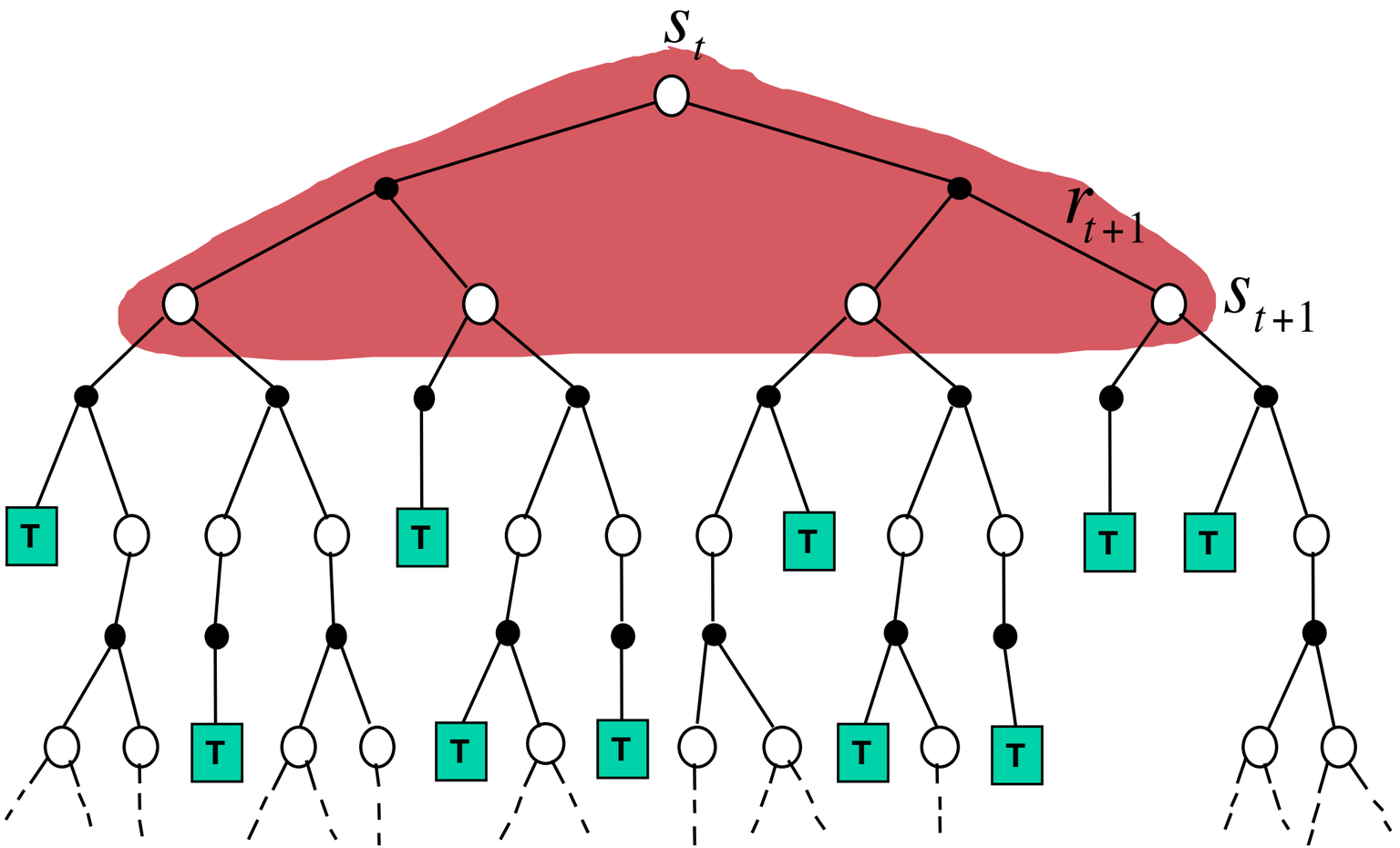
- Improve the policy based on the new $Q(S, A)$

$$\epsilon \leftarrow 1/k$$
$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

GLIE Monte Carlos converges to the optimal $q^*(s, a)$

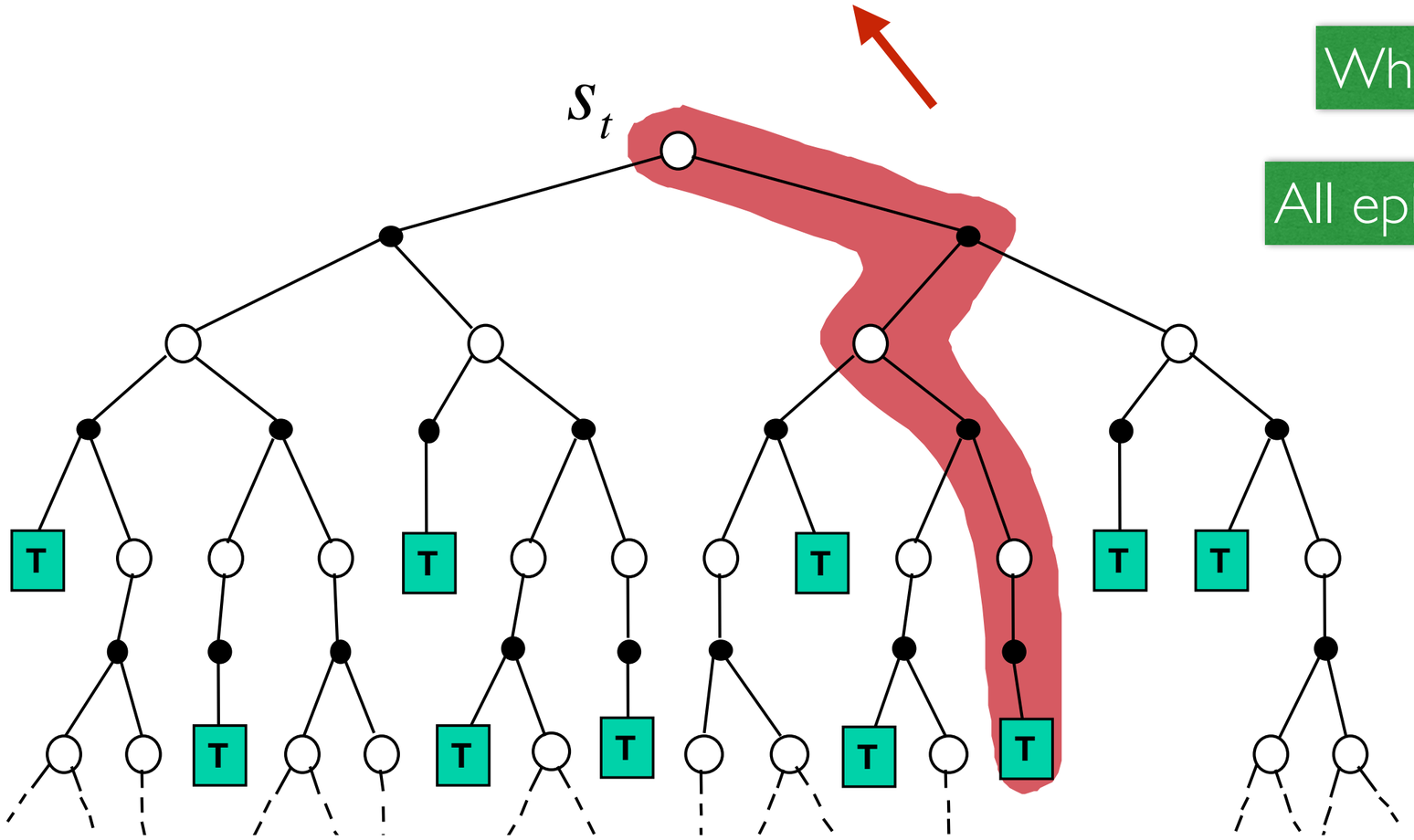
Dynamic programming backup

$$V(S_t) \leftarrow \mathbb{E}_\pi \left[R_{t+1} + \gamma V(S_{t+1}) \right]$$



Monte Carlo Backup

$$V(S_t) \leftarrow V(S_t) + \alpha\,(G_t - V(S_t))$$



What's the problem?

All episodes must end!!

What was the Bellman Eq again?

$$v_\pi(s) = E_\pi\left\{G_t \,|\, S_t = s\right\} = E_\pi\left\{R_{t+1} + \gamma v_\pi(S_{t+1}) \,|\, S_t = s\right\}$$

Temporal Difference Learning: TD-Learning

- Goal: learn v_π from online experiences under the policy π

- Simplest temporal difference learning algorithm: TD(0)
  - Update $V(S_t)$ towards the ***estimated return*** $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$

  - $R_{t+1} + \gamma V(S_{t+1})$ is called *TD target*
  - $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called *TD error*

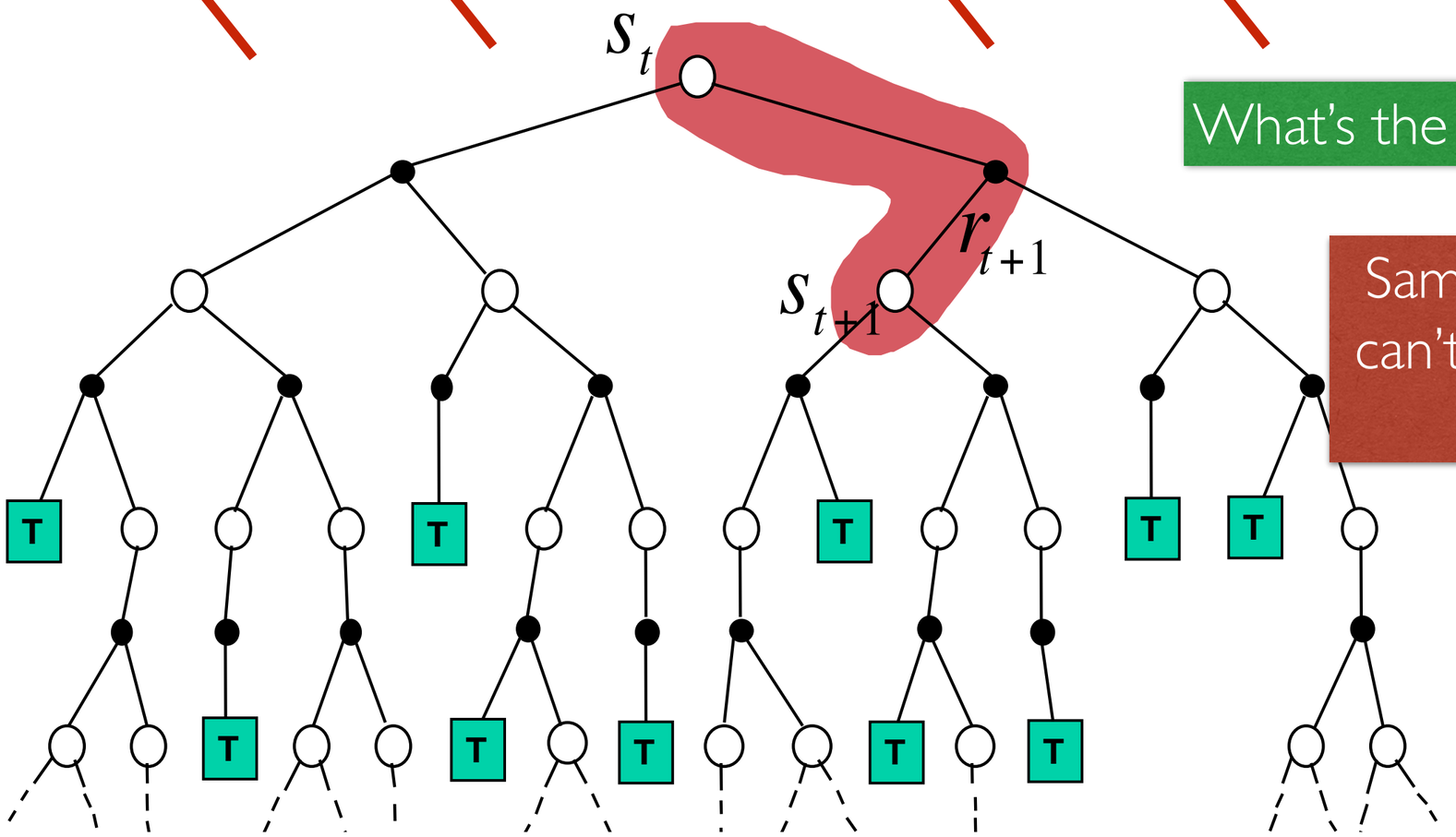
$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$

Comparison TD vs MC

- TD: updates ***online towards the best guess***, *before* the final outcome
- MC has to wait until the episode is complete and we know the final return G_t
- TD learns from incomplete episodes, e.g., without final outcome.
- MC requires complete episodes

Temporal Difference Backup

$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$



What's the problem this time?

Same as with MC: we can't optimise π based on $V(S)$

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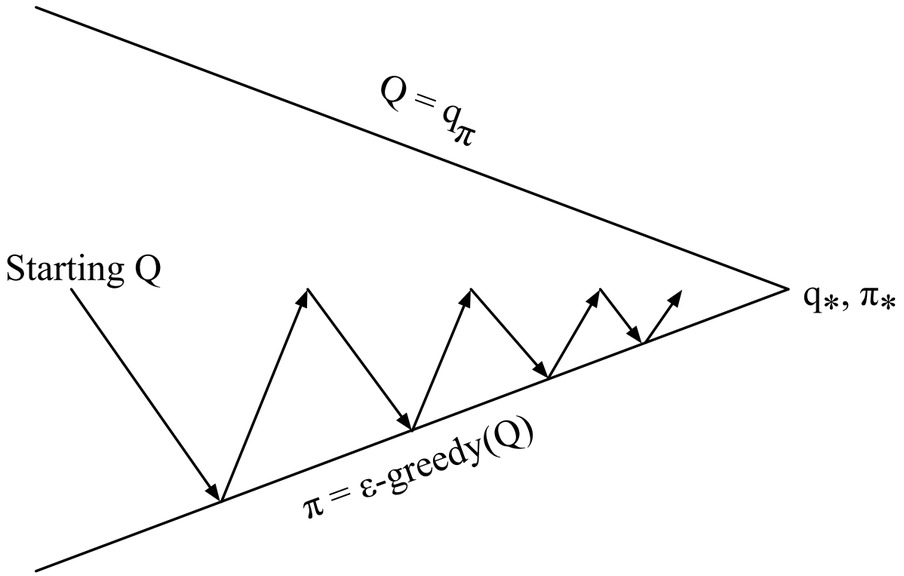
Policy Optimisation with TD

- For every episode:
  - Policy Evaluation:
    - TD-Learning idea to find $Q \approx q_\pi$
  - Policy Improvement:
    - ϵ-Greedy policy improvement



Results in the SARSA approach: Closely related to famous Q-Learning

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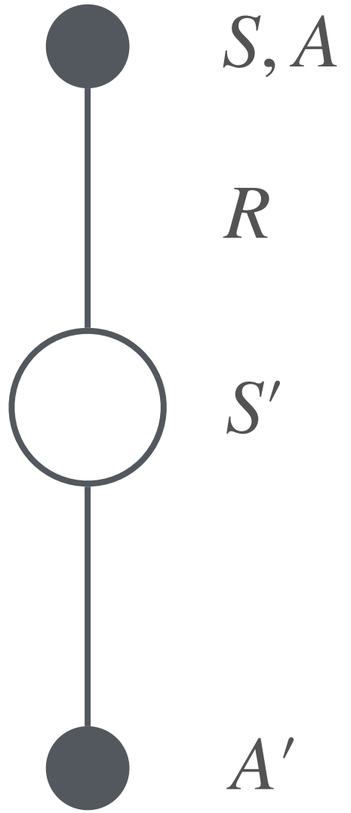
Action-Value Function with SARSA



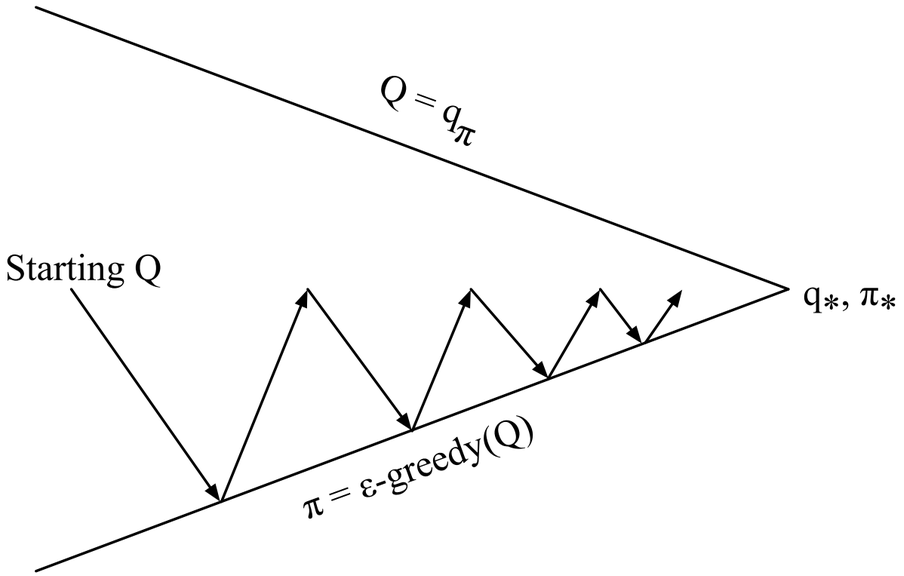
- At **every time step** we improve our policy
  - S, A: In state S choose A according to ϵ-greedy policy π
  - R, S' sampled from the environment
  - A' chosen ϵ-greedy: according to ϵ-greedy policy π

$$Q(S, A) \leftarrow Q(S, A) + \alpha \left(R + \gamma Q(S', A') - Q(S, A)\right)$$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)} \left(G_t - Q(S_t, A_t)\right)$$

On Policy Control with SARSA

- For **every single time step**:
  - Policy Evaluation:
    - **SARSA** update of $Q \approx q_\pi$
  - Policy Improvement:
    - ϵ-greedy policy improvement
    - no explicit policy necessary



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SARSA Algorithm for on-policy control

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Repeat (for each step of episode):
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma Q(S',A') - Q(S,A)\big]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

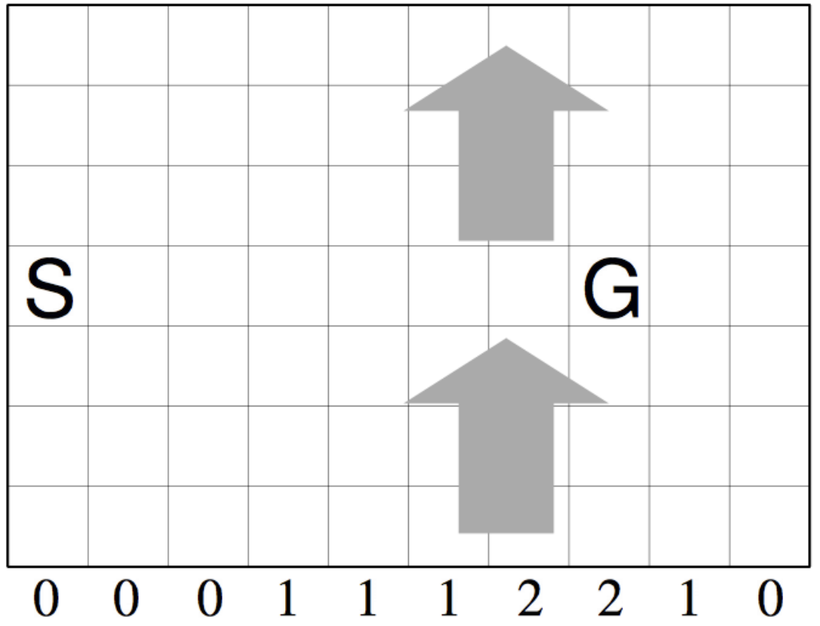
Convergence of SARSA

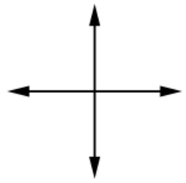
- **Theorem**: *SARSA converges to the optimal action-value function* $Q(S, A) \rightarrow q^*(s, a)$, *under the following conditions:*
  - *GLIE sequence of policies* $\pi_t(a, s)$
  - *Robbins-Monro sequence of step-sizes* α_t

$$\sum_{t=1}^{\infty} \alpha_t = \infty \text{ and } \sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

  - $\alpha_k = \frac{1}{k}$ *works*
  - SARSA almost always works, no matter GLIE and α_t 😀

Windy Gridworld example



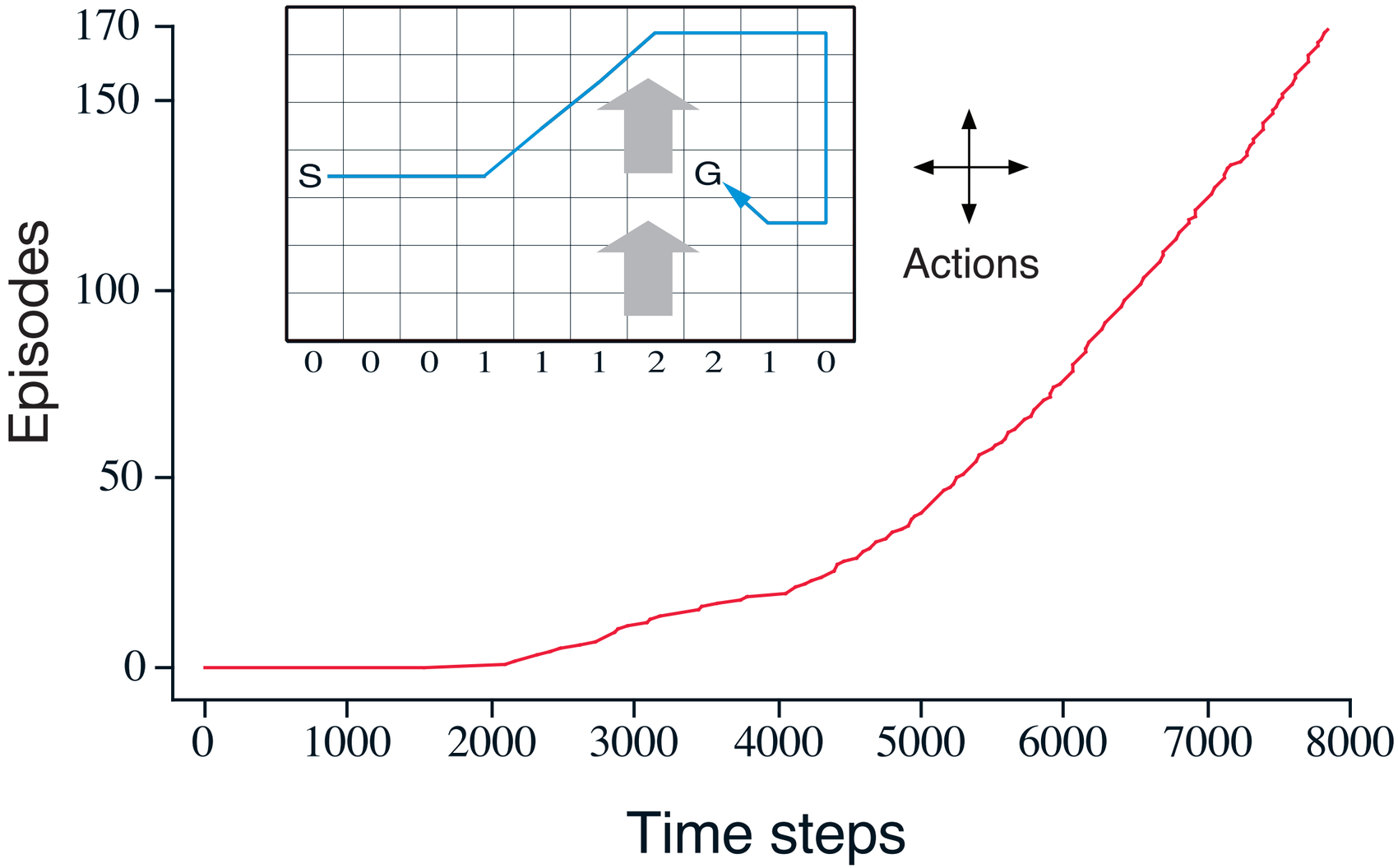


standard moves

wind pushing upward n gridpoints

- Reward $= -1$ per time step until goal is reached
- undiscounted: $\gamma = 1$

SARSA on the Windy Gridworld



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What we did today

- First real RL approach for finding an optimised policy
- MC is *model-free*: no knowledge of the MDP transitions / rewards are needed
- Can apply MC only to *episodic* MDPs: all episodes must end.
- based on observations only
- policy optimisation done via the state action value Q
- ϵ-greedy approach for finding the best policy while exploring at the same time.
- GLIE Monte Carlo Control allows to slowly reduce ϵ to 0 while assuring convergence (for proof, see Sect 5 of the book)
- Exploit the Bellman Equation for TD-Learning and SARSA.