Reinforcement learning

Applied Machine Learning (EDAN95)
Lectures13 and 14
2018-12-17 and 2018-12-19
Elin A. Topp

Material based on "Hands-on Machine Learning with SciKit-learn and TensorFlow" (course book, chapter 16),
and on lecture "Belöningsbaserad inlärning / Reinforcement learning"
by Örjan Ekeberg, CSC/Nada, KTH, autumn term 2006 (in Swedish)

Outline

- Reinforcement learning
  - Problem definition
    - Learning situation
    - Role of the reward
    - Simplified assumptions
    - Central concepts and terms
  - Known environment
    - Bellman’s equation
    - Approaches to solutions
  - Unknown environment
    - Temporal-Difference learning
    - Q-Learning
    - Sarsa-Learning
  - Improvements
    - The usefulness of making mistakes
    - Eligibility Trace

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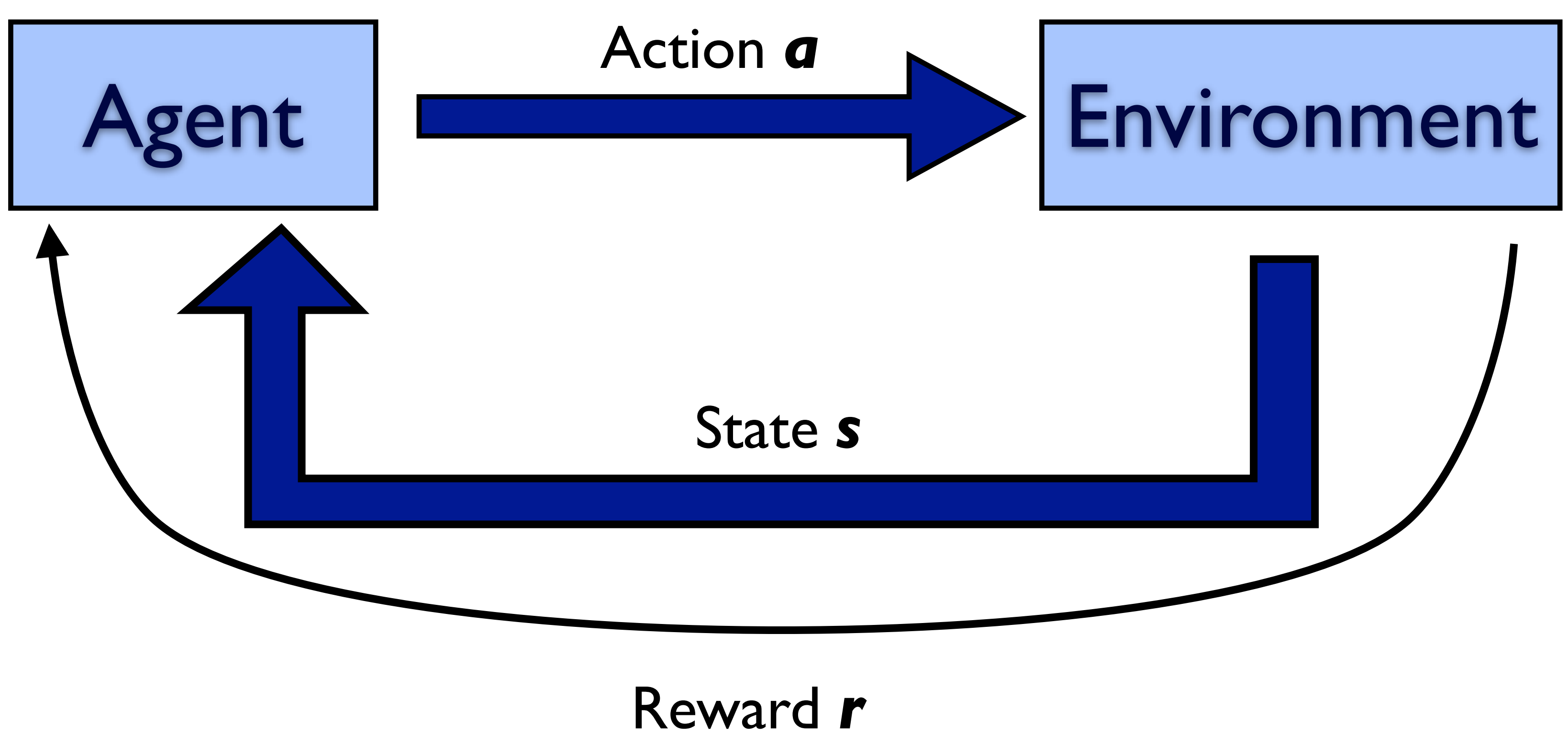
Learning situation: A model

An *agent* interacts with its *environment*

The agent performs *actions*

Actions have *influence* on the environment's *state*

The agent *observes* the environment's *state* and *receives* a *reward* from the environment



Real life examples

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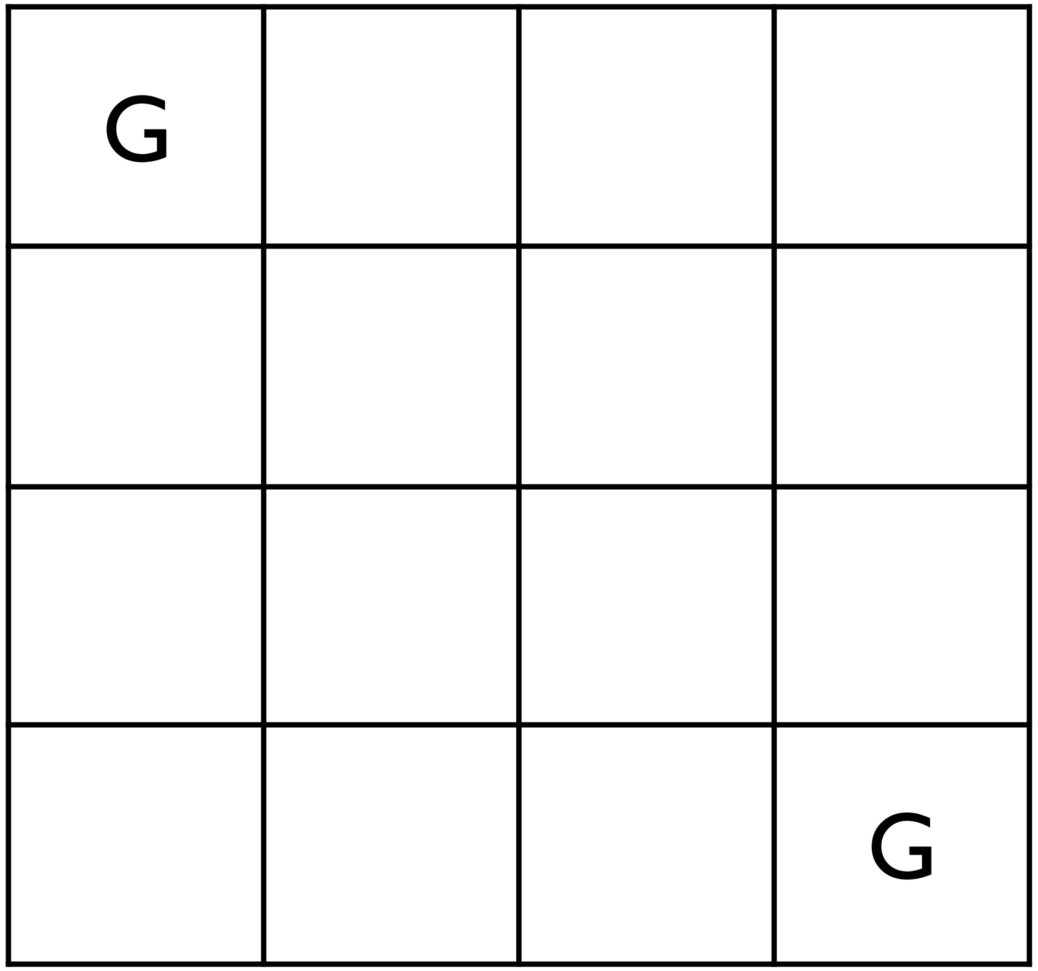
Riding a bicycle

Powder skiing

A classic example: Grid World

Simplified "Wumpus world" with just two gold pieces



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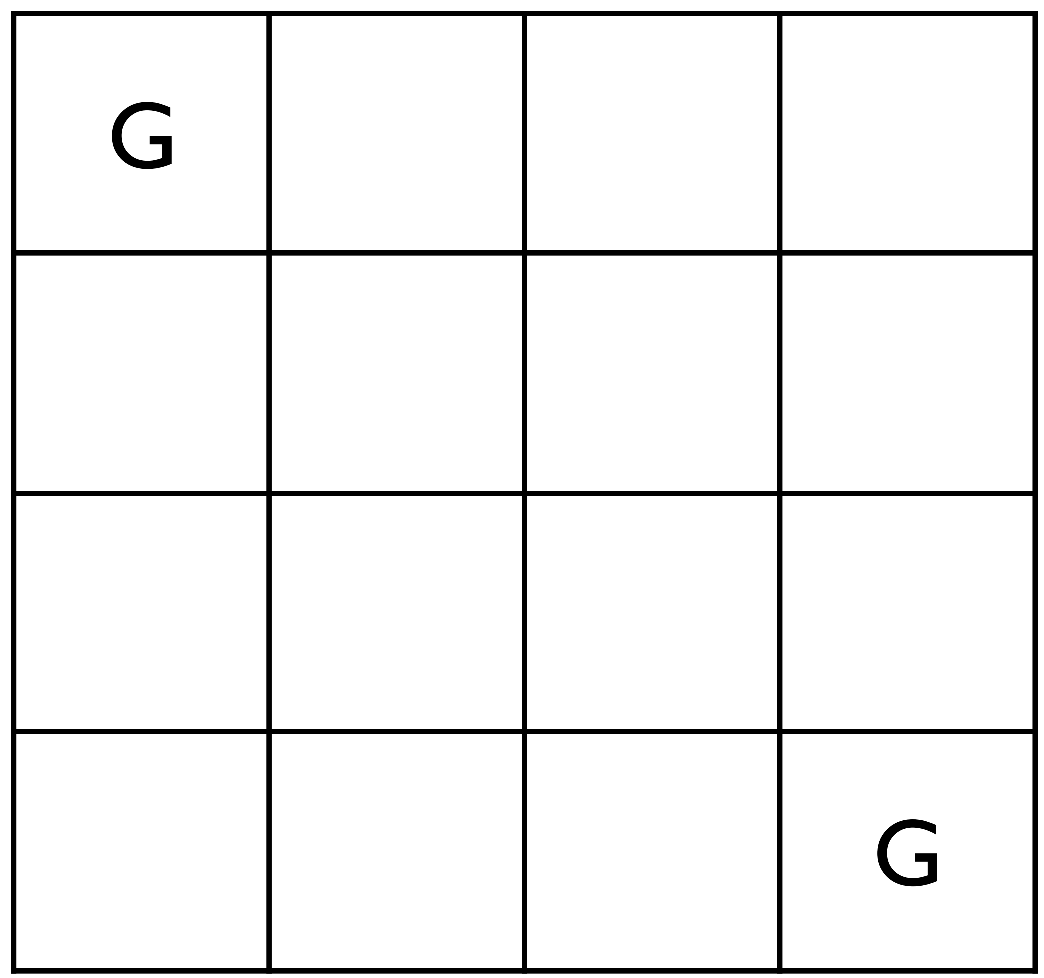

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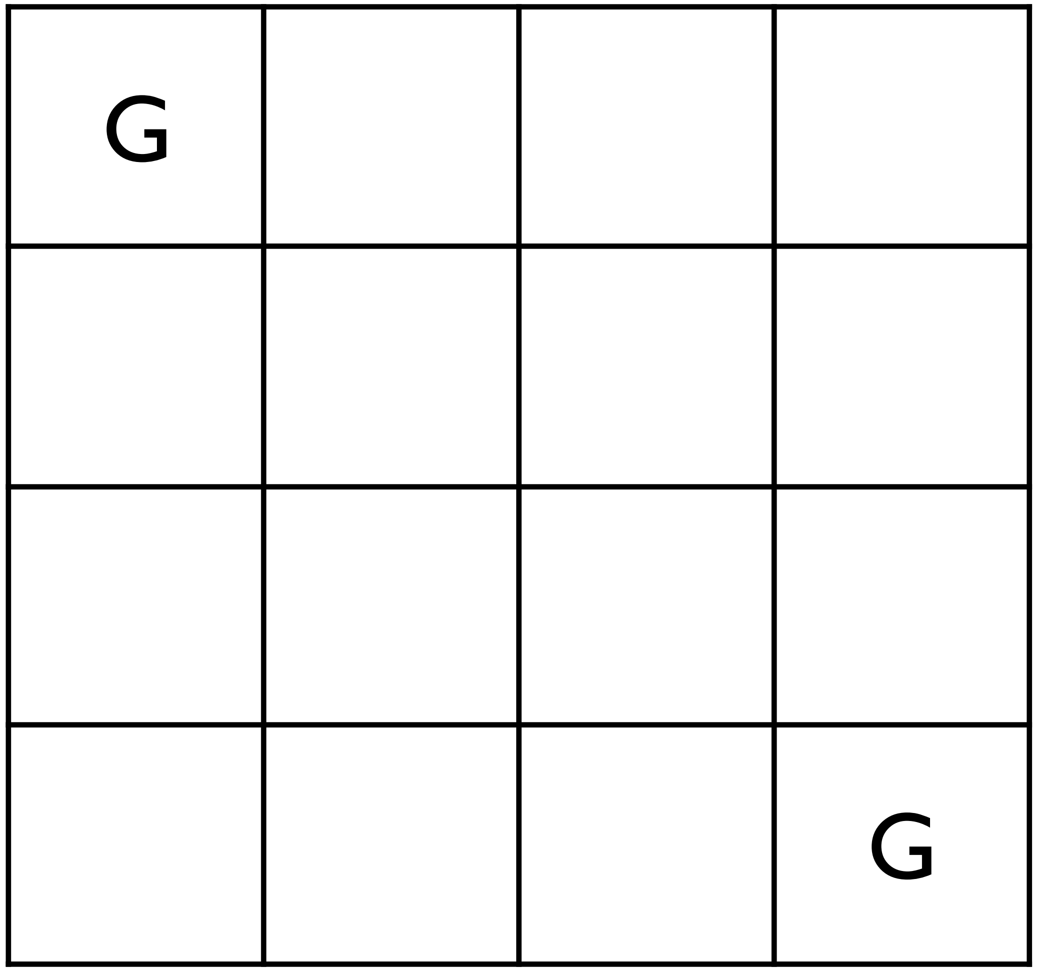

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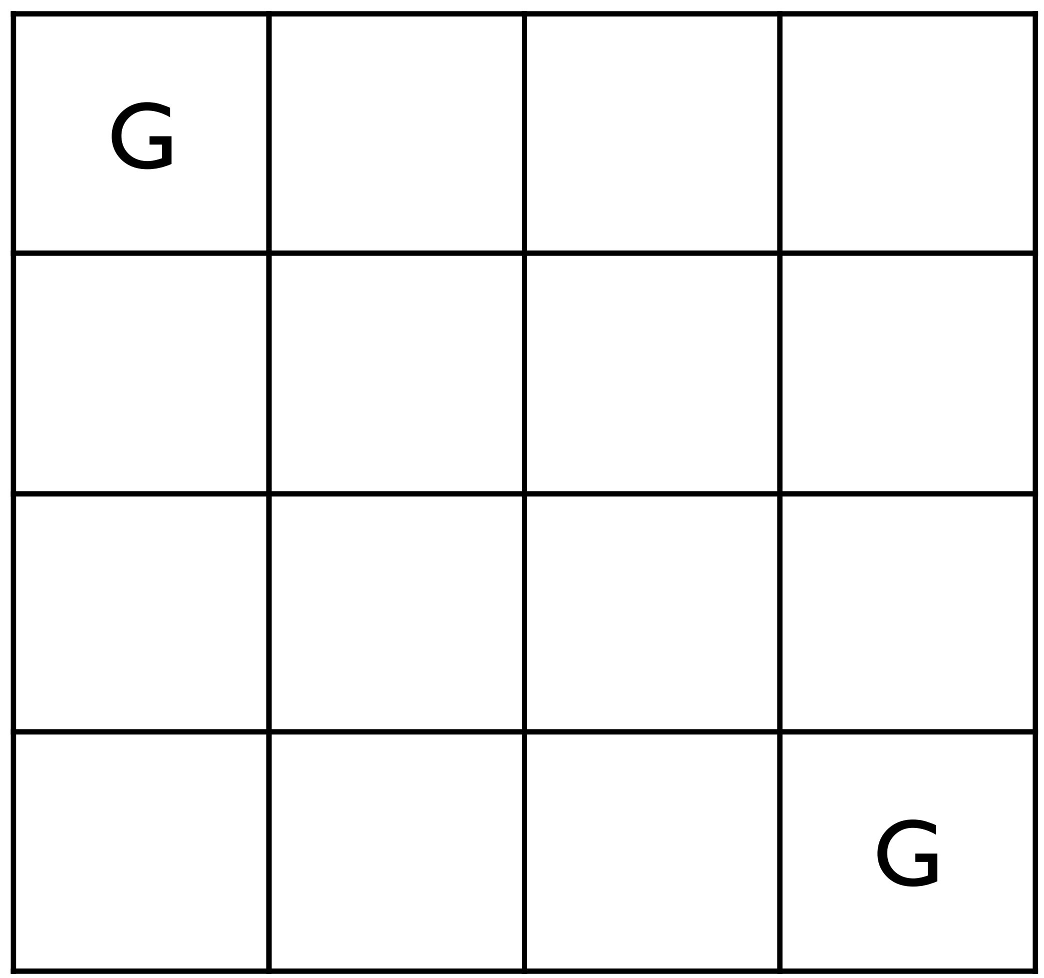
- Every state s_j is represented by a field in the grid
- Action *a* the agent can choose consists of moving one step to a neighbouring field
- *Reward*: -1 in every step until one of the goals (G) is reached.



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Note: The reward does not tell what exactly it was, that made the action "good" *(structural credit assignment)*

Learning situation: The agent's task

The task:

Find a behaviour (action sequence) that maximises the overall reward

How long into the future should we spy?

Finite time horizon:

$$max\ E\left[\ \sum_{t=0}^{h} r_t\right]$$

Infinite time horizon:

$$max\ E\left[\ \sum_{t=0}^{\infty} \gamma^t r_t\right]$$

with γ being a discount factor for future rewards ($0 < \gamma < 1$)

The reward function's role

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- Avoid mistakes and try to do something useful (Learning to walk towards a goal): Reward -10 when failing (falling) or -5 when moving backwards, +5 when an action leads to a forward movement
- Find the shortest / cheapest / fastest path to a goal: Reward -1 for each step that does not end in the goal

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- Environment is observable

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- An agent's *policy* π is the "rule" after which the agent chooses its action a in a given state s

$$\pi(s) \longmapsto a$$

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- An agent's *utility function* U describes the expected future reward given s, when following policy π

$$U^{\pi}(s) \longmapsto \mathbb{R}$$

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0	-1	-2	-3
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-2	-3	-2	-1
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U with optimal policy

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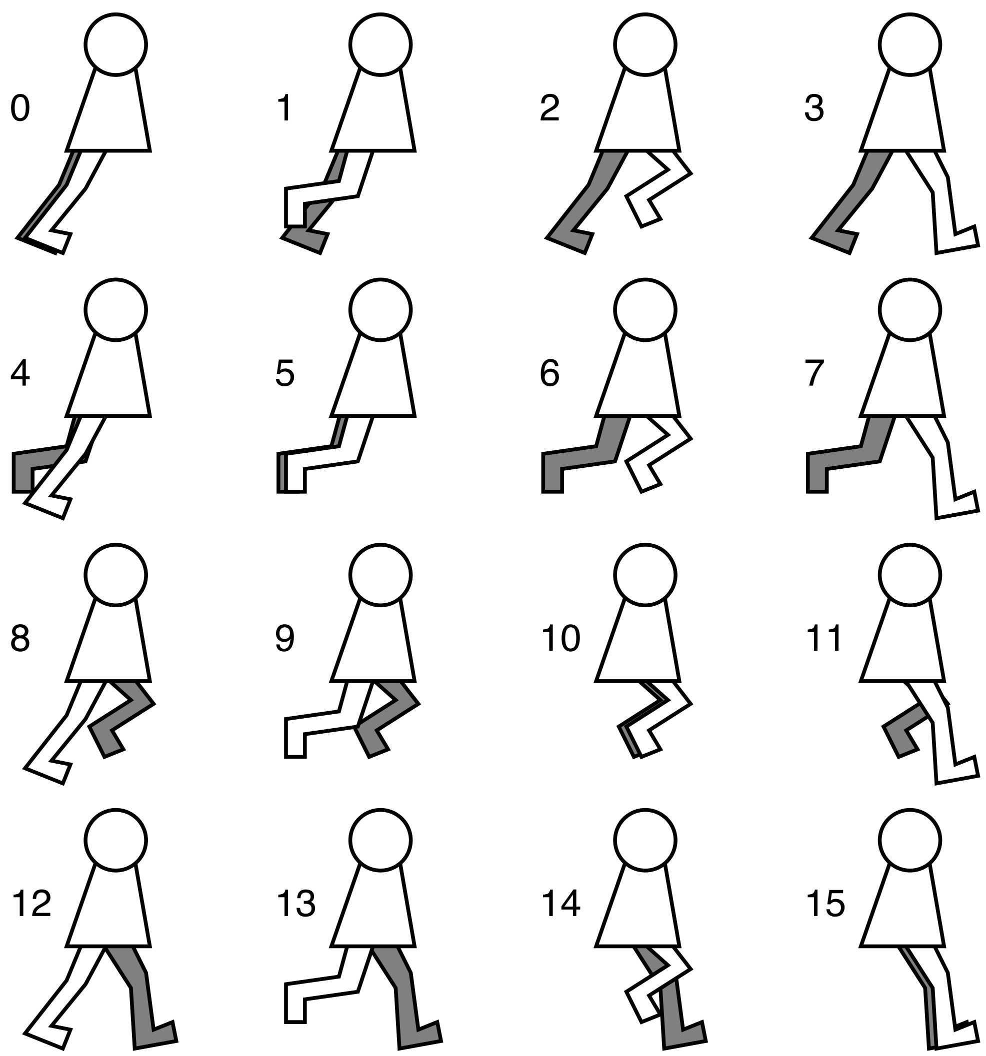
U with optimal policy

0	-14	-20	-22
-14	-18	-22	-20
-20	-22	-18	-14
-22	-20	-14	0

U with random policy

Cartoon Walker

16 discrete states, some really bad, 4 discrete actions, only some making the walker walk



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We are not going into details here!

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The utility values of different states obey *Bellman's equation,* given a fixed policy π:

$$U^{\pi}(s) = r(s, \pi(s)) + \gamma \cdot U^{\pi}(\delta(s, \pi(s)))$$

Solving the equation

There are two ways of solving (this "optimal" version of) *Bellman's equation*

$$U^{\pi}(s) = r(s, \pi(s)) + \gamma \cdot U^{\pi}(\delta(s, \pi(s)))$$

- Directly: $U^{\pi}(s) = r(s, \pi(s)) + \gamma \cdot \sum_{s'} P(s' \mid s, \pi(s))\, U^{\pi}(s')$

- Iteratively (*Value / utility iteration*), stop when equilibrium is reached, i.e., "nothing happens"

$$U^{\pi}_{k+1}(s) \longleftarrow r(s, \pi(s)) + \gamma \cdot U^{\pi}_{k}(\delta(s, \pi(s)))$$

Finding optimal policy and value function

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Tricky to solve ... but possible:

Combine policy and value iteration by switching in each iteration step

Policy iteration

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Policy iteration provides exactly this switch.

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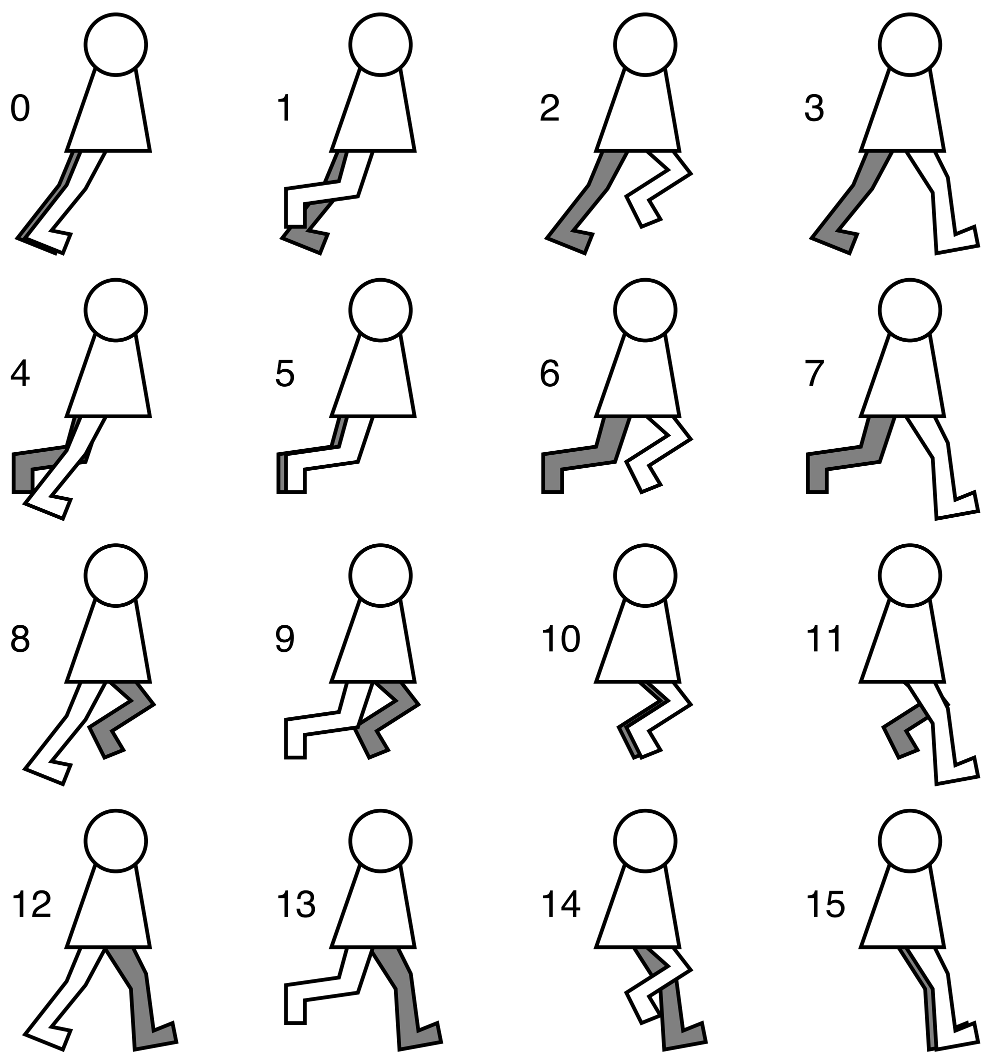
For each iteration step k:

$$\pi_k(s) = \underset{a}{\mathrm{argmax}}\left(r(s, a) + \gamma \cdot U_k(\delta(s, a))\right)$$

$$U_{k+1}(s) = r(s, \pi_k(s)) + \gamma \cdot U_k(\delta(s, \pi_k(s)))$$

Policy Iteration for Cartoon Walker

We cheat a bit, and use entirely known reward and transition functions…



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```
for s in range(len(policy)):
    policy[s] = argmax(
        lambda a: rew[s][a] + gamma * value[trans[s][a]],
        range(len(trans[s])))
```

```
for s in range(len(value)):
    a = policy[s]
    value[s] = rew[s][a] + gamma * value[trans[s][a]]
```

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Still, we can estimate U^* from *experience*, as a *Monte Carlo approach* will do:

- Start with a randomly chosen *s*
- Follow a policy π, store rewards and s_t for the step at time *t*
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- Start over with a randomly chosen *s* ...

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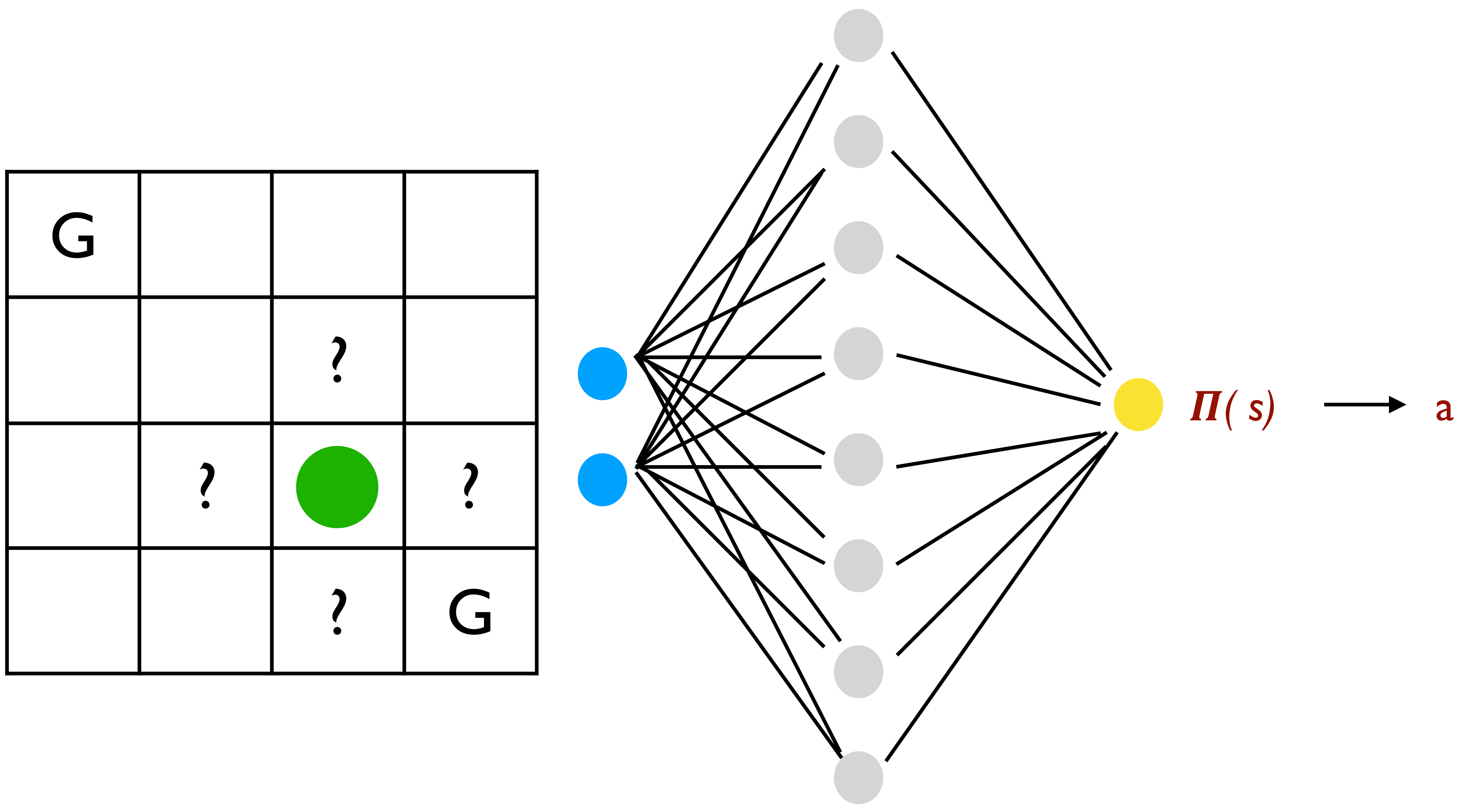
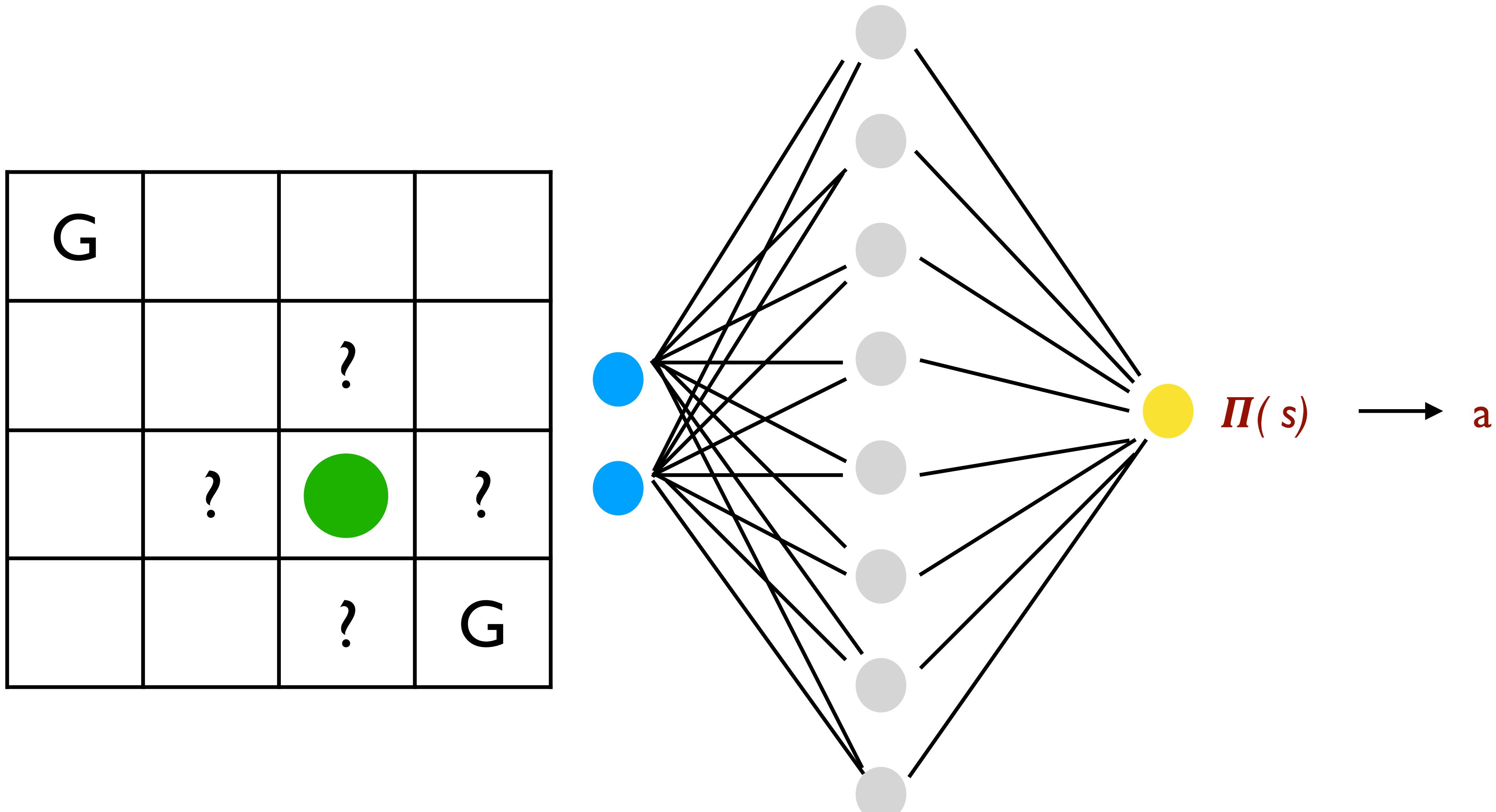
Converges slowly...

Policy gradients



Policy gradients

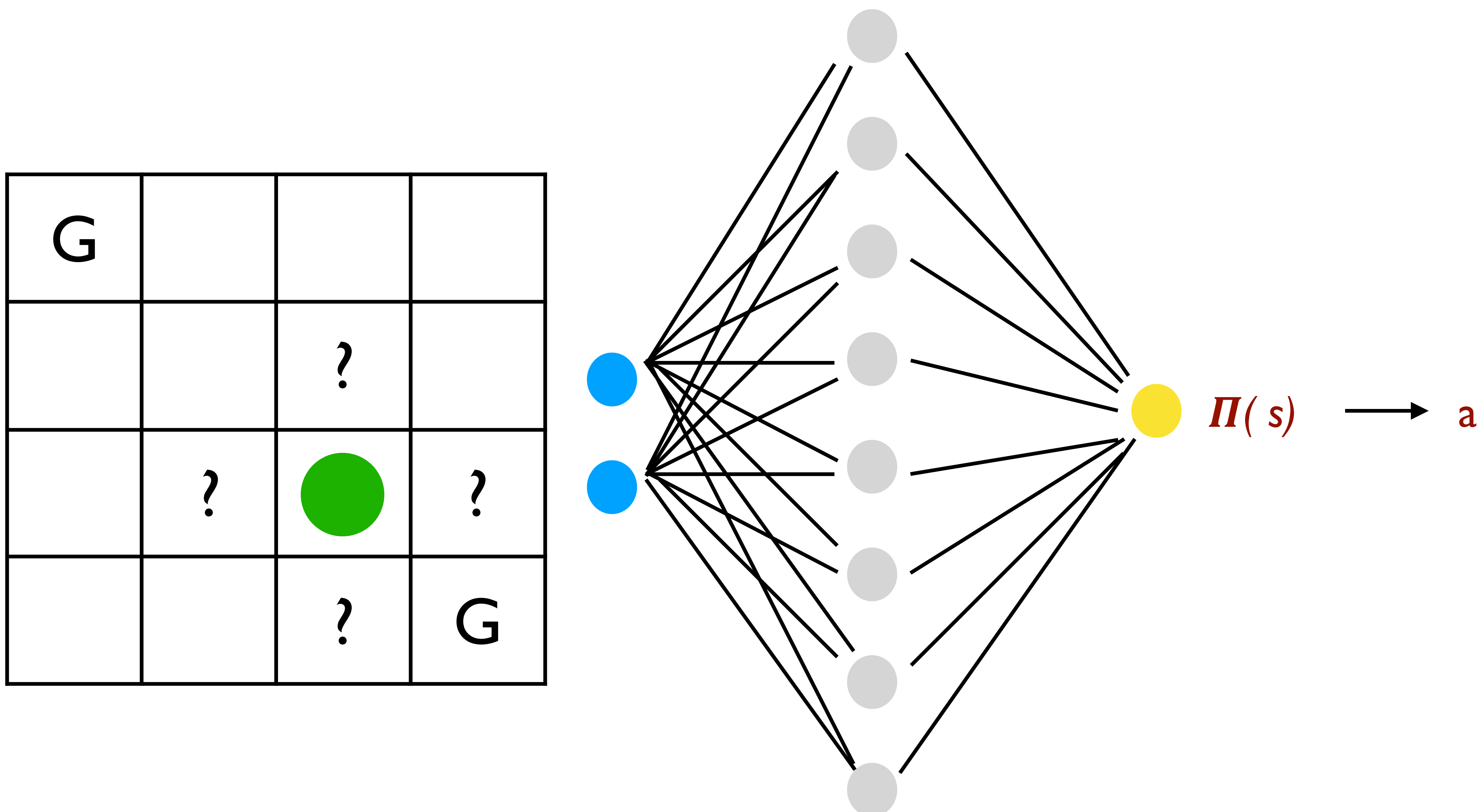
What if…



Policy gradients

What if…

… we take help of an ANN to learn a good policy?



Training the network

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If we had a "label" saying after a forward run that DOWN is the optimal thing to do for this state…

… we would compute the loss as:

- log P(y=DOWN | x)

… but we do not have this label, so we use the reward R we get from using our

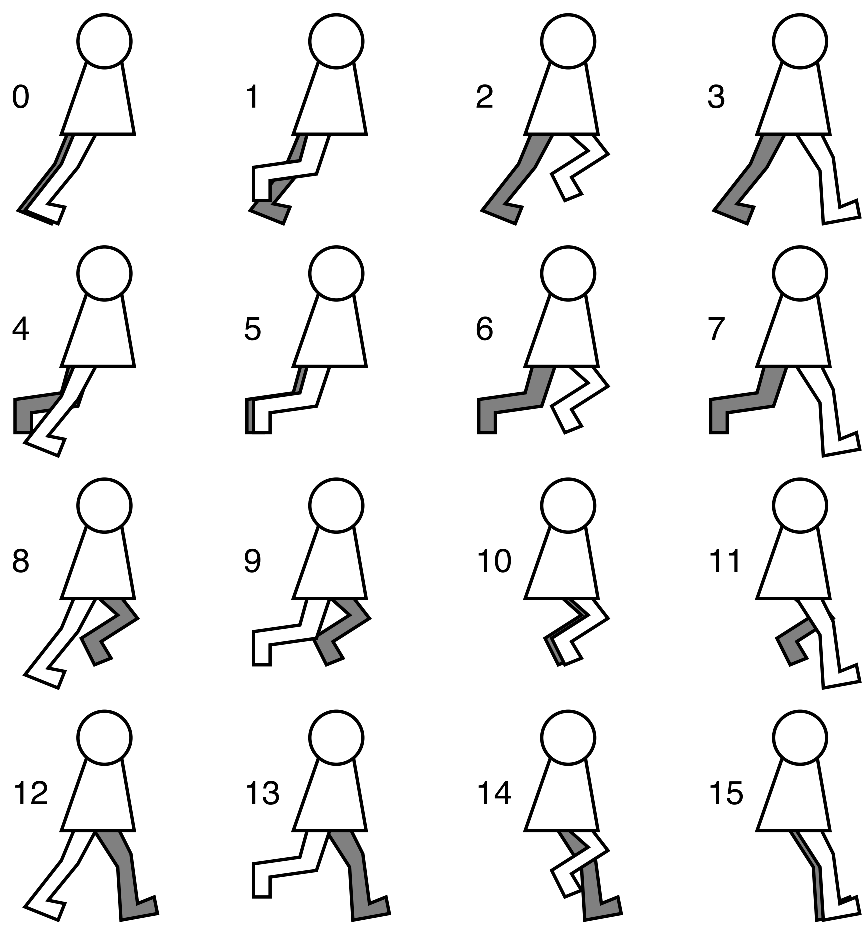
policy (the sampled action) to compute the loss:

Loss = - *R* log P(a) with R being r(s, a)

but that means that we have to save the gradients along our path through the state-action space

Policy Gradients for Cartoon Walker

Represent the walker's policy in a network with
- a single valued array (one input value) for the state
- one of four possible output "classes" (sampled from probability distribution)
- softmax activation
- and not too many hidden neurons ;-)



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Will need a lot more time and tweaking than the policy iteration!

Lab assignment 7

- The lab assignment is given as a package with instructions, code skeleton and some useful links also to hands-on material at https://github.com/ErikGartner/edan95-rlagent-handout

- Some hands-on experimenting material can be found at https://github.com/ageron/handson-ml