Computer Vision: Lecture 8

Carl Olsson

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Today's Lecture

Outline

- Outliers and RANSAC
- Minimal Solvers

The Outlier Problem

What is an outlier?

An outlier is a measurement that does not fulfill the noise assumption. Arises from for example mismatches.

Demonstration ...

RANdom SAmpling Consensus - RANSAC

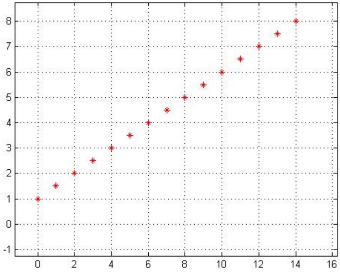
Idea

If the number of outliers is small, pick a random subset of measurements. With high probability this set will be outlier free.

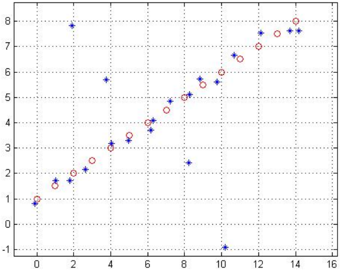
Algorithm

1. Randomly select a small number of the measurements, and fit the model to these.
2. Evaluate the error with respect to the estimated model fore the rest of the measurements. The Consensus set is the set of measurements with error less than some predefined threshold.
3. Repeat a number of times and select the model fit that gives the largest consensus set.

Line Fitting Example



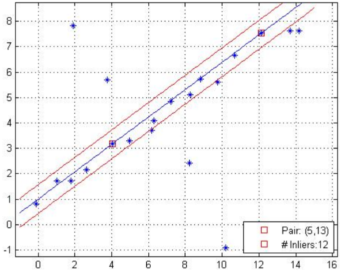
Real data



Noisy measurements and real data

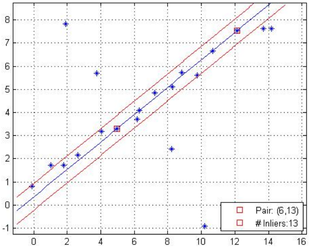
Line Fitting Example

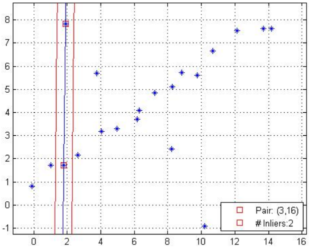
Select 2 points and fit a line.
First iteration:

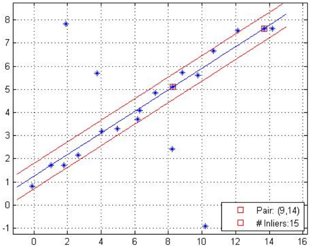


Line Fitting Example

More iterations:

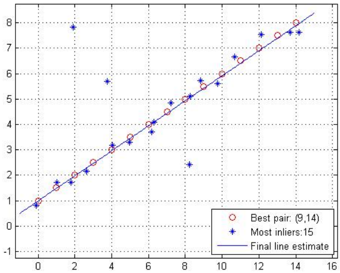






Line Fitting Example

Final result:



RANSAC

Relative Orientation demo...

RANSAC

How many iterations do we need?
See lecture notes...

Minimal Solvers

See lecture notes...

Minimal Solvers

Algorithm

- Select a monomial basis.
- Apply the mapping T_x to the monomial basis and reduce the expressions until the result consists of monomials from the basis.
- Construct the action matrix M_x^T.
- Compute eigenvalues and eigenvectors of M_x^T.
- Extract the solutions from the eigenvectors.

Note: The theory guarentees that the solutions will be among the eigenvectors, but not all eigenvectors are solutions. May need to test the result.

Minimal Solvers

What degree of monomials should we choose? Monomials of order 2:

$$
\begin{align}
1 &\mapsto x_0 \tag{1}\\
x_0 &\mapsto x_0^2 \tag{2}\\
y_0 &\mapsto x_0 y_0 \tag{3}\\
x_0^2 &\mapsto x_0^3 = x_0(y_0+3) = x_0 y_0 + 3x_0 \tag{4}\\
x_0 y_0 &\mapsto x_0^2 y_0 = x_0^2 \tag{5}\\
y_0^2 &\mapsto x_0 y_0^2 = x_0 y_0. \tag{6}
\end{align}
$$

Minimal Solvers

What degree of monomials should we choose? Action matrix:

$$M_x = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

M_x^T has eigenvalues $\lambda = -2, 0, 2$.
(The eigenvalue 0 has multiplicity 4.)

The degree needs to be large enough for give monomials of the same degree.

Minimal Solvers

Other mappings than T_x also work:
E.g. T_y :

$$\begin{aligned}
1 &\mapsto y_0 && (8)\\
x_0 &\mapsto x_0 y_0 && (9)\\
y_0 &\mapsto y_0^2 && (10)\\
x_0^2 &\mapsto x_0^2 y_0 = x_0^2 && (11)\\
x_0 y_0 &\mapsto x_0 y_0^2 = x_0 y_0 && (12)\\
y_0^2 &\mapsto y_0^3 = y_0^2(x_0^2 - 3) = x_0^2 - 3y_0^2. && (13)
\end{aligned}$$

Minimal Solvers

$$M_y = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -3 \end{pmatrix}. \tag{14}$$

M_y^T has eigenvalues $-3, 1, 0$.
Since two solutions have $y_0 = -3$ the multiplicity of -3 will always be at least 2.

The 5-point Solver

Finding an essential matrix can be done with five correspondences by solving

$$\bar{\mathbf{x}}_i^T E \mathbf{x}_i = 0, \qquad i = 1, ..., 5. \tag{15}$$

$$\det(E) = 0, \tag{16}$$

$$2EE^T E - \text{trace}(EE^T)E = 0. \tag{17}$$

Form the M-matrix with the five correspondences. M has a 4 dimmensional null-space:

$$M(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \alpha_4 v_4) = 0. \tag{18}$$

Reshaping into matrices

$$\bar{\mathbf{x}}_i^T (\alpha_1 E_1 + \alpha_2 E_2 + \alpha_3 E_3 + \alpha_4 E_4)\mathbf{x}_i = 0, \quad i = 1, ..., 5. \tag{19}$$

The 5-point solver

Use the remaining equations to determine $\alpha_1, \alpha_2, \alpha_3, \alpha_4$.
Nine equations:

$$2EE^TE - \text{trace}(EE^T)E = \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4} \alpha_i\alpha_j\alpha_k \left(2E_iE_j^TE_k - \text{trace}(E_iE_j^T)E_k\right). \tag{20}$$

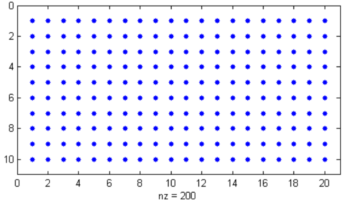
One equation:

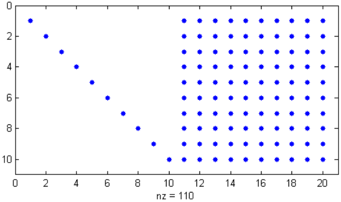
$$\det(E) = \sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{4} \alpha_i\alpha_j\alpha_k \big(e_{11}^i e_{22}^j e_{33}^k + e_{12}^i e_{23}^j e_{31}^k + e_{13}^i e_{21}^j e_{32}^k - e_{11}^i e_{23}^j e_{32}^k - e_{12}^i e_{21}^j e_{33}^k - e_{13}^i e_{22}^j e_{31}^k\big), \tag{21}$$

The Five Point Solver

α_1 can be assumed to be one (scale ambiguity).
Monomial order:

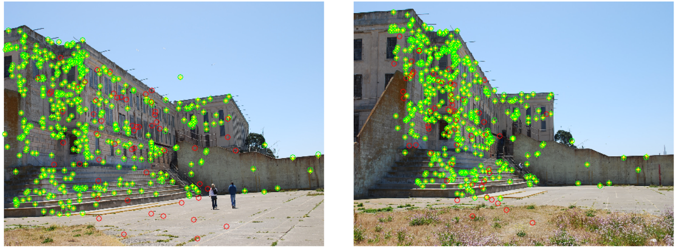
$$\{\alpha_4^3, \alpha_3\alpha_4^2, \alpha_3^2\alpha_4, \alpha_3^3, \alpha_2\alpha_4^2, \alpha_2\alpha_3\alpha_4, \alpha_2\alpha_3^2, \alpha_2^2\alpha_4, \alpha_2^2\alpha_3, \alpha_2^3, \\ \alpha_4^2, \alpha_3\alpha_4, \alpha_3^2, \alpha_2\alpha_4, \alpha_2\alpha_3, \alpha_2^2, \alpha_4, \alpha_3, \alpha_2, 1\}.$$





Gaussian elimination gives reductions for all third order terms.

The 5-point solver



Histogram over the size of the consensus set in each iteration of RANSAC (1000 iterations), using 5 points, 8 points and 10 points respectively.