

Image Analysis, Assignment 2

In this assignment you will study convolution, filtering, interpolation, classification and continue your work on your own Optical Character Recognition system. The data for the assignments is on the course homepage:

<http://www.ctr.maths.lu.se/course/newimagean/2018/>

The Rules

The deadline is on **Sunday 30 September** (23:59 CET). Each student should hand in his or her own individual solution and should, upon request, be able to present the details in all the steps of the used algorithm. You are, however, allowed to discuss the assignment-problem with others. You may also ask your teachers and the course assistants for advice, if needed.

The report. Present your work in a report of approximately four A4-pages written in English or Swedish. Don't worry if your report is longer. Make sure you answer **all questions** and provide complete solutions to the exercises.

You may submit your code along with the report, if you want to.¹ However, the teacher is going to judge your work based on the report alone. Usually the teacher will check the code only in very special cases, for instance, when the results seem strange or if very persistent problems remain with your implementation.

Submitting your work. Email the report as a **pdf-file** with the name format `assignment_2_yourname.pdf` to the address

`fman20@maths.lth.se`

Write **your name and the assignment number** in the subject line of your email.

¹Make sure that your matlab scripts are well commented and can be executed directly (that is, without loading any data, setting parameters etc. Such things should be done in the script).

1 Filtering

Shown below is an image and the results of convolving this image with six different filters. Combine each convolved image with the correct filter.

$$f_1 = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

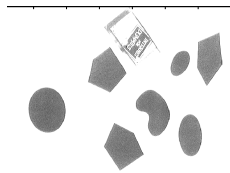
$$f_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$f_3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

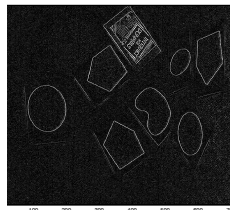
$$f_4 = \frac{1}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$f_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

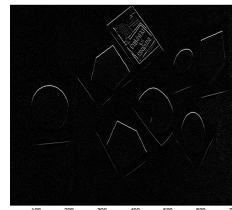
$$f_6 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



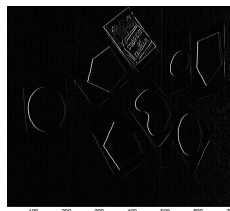
original



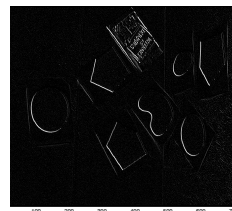
A



B



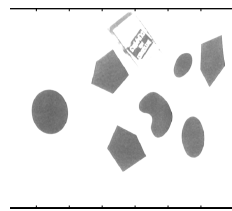
C



D



E



F

For the report: Write down your answers with a short motivation.

2 Interpolation

Assume a one-dimensional image (or signal)

$$f = [1 \quad 2 \quad 2 \quad 4 \quad 7 \quad 6 \quad 5] \quad .$$

a) Explain what linear interpolation means. Sketch what the function f looks like after linear interpolation.

b) Interpolation can often be expressed using the following equation

$$F_g(x) = \sum_i g(x-i)f(i) \quad , \quad (1)$$

where the function $g(x)$ is different for different types of interpolation. Which function $g(x)$ correspond to linear interpolation?

c) Assume that the position for the seven pixels are 0, 1, 2, 3, 4, 5, 6. Using Equation (1), what is the intensity and the derivative of the intensity between the pixels, i.e. at positions $x = 0.5, 1.5, 2.5, 3.5, 4.5, 5.5$? Calculate the derivative both by differentiating the equation (1) with respect to x and evaluating the result, and graphically using the figure obtained above. What is the connection between the derivative of F_g at points $x = 0.5 + k, k = 0 \dots 5$ and the discrete 'differentiation' $f * w$, where $w = [1 \quad -1]$?

d) Write out the expression of $\frac{\partial F_g}{\partial x}(k)$. Write the definition of convolution $(f * w)(k)$. Compare these two expressions and show that

$$\frac{\partial g}{\partial x}(j) = w(j), \quad \forall j \in \mathbb{Z}$$

e) Give one example of an interpolation function g such that

$$\frac{\partial F_g}{\partial x}(k) = (f * w_2)(k), \quad k = 1, \dots, 5 \quad ,$$

where $w_2 = [1/2 \quad 0 \quad -1/2]$.

For the report: Provide a complete solution.

3 Classification using Nearest Neighbour and Bayes theorem

As output from an imaging system we get a measurement that depends on what we are seeing. For three different classes of objects we get the following measurements.

Class 1:

0.4003, 0.3985, 0.3998, 0.3997, 0.4015, 0.3995, 0.3991

Class 2:

0.2554, 0.3139, 0.2627, 0.3802, 0.3247, 0.3360, 0.2974

Class 3:

0.5632, 0.7687, 0.0524, 0.7586, 0.4443, 0.5505, 0.6469

3.1 Nearest Neighbours

Use nearest neighbour classification. Assume that the first four measurements in each class are used for training and the last three for testing. How many measurements will be correctly classified ?

3.2 Gaussian distributions

Assume that the measurements have normal distribution with known parameters (expected value m_i and variance σ^2). For class 1: $m_1 = 0.4, \sigma_1 = 0.01$. For class 2: $m_2 = 0.3, \sigma_2 = 0.05$. For class 3: $m_3 = 0.5, \sigma_3 = 0.2$. All classes are as likely to occur. What is the maximum a posteriori classification of a measured x ? How many measurements are correctly classified? In this case you can try to classify all seven samples in each class, since no measurements were used for training. Hint: Matlab has a function called 'normpdf'.

For the report: Provide a complete solution.

4 Classification

A scene is generated by some random mechanism. By studying a large number of noise free realizations of the scene it has been found that

$$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \quad \text{has prob} \quad \frac{1}{4}; \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \quad \text{has prob} \quad \frac{1}{2}; \quad \begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix} \quad \text{has prob} \quad \frac{1}{4}$$

(a priori probabilities). One of these images has been distorted by noise in the sense that the value of a pixel has been changed with the probability ϵ , i.e. $\text{Prob}(\text{observing } 0 \mid \text{the correct value is } 1) = P(\text{observing } 1 \mid \text{the correct value is } 0) = \epsilon$. Assume that noise in different pixels is independent. Now consider the image

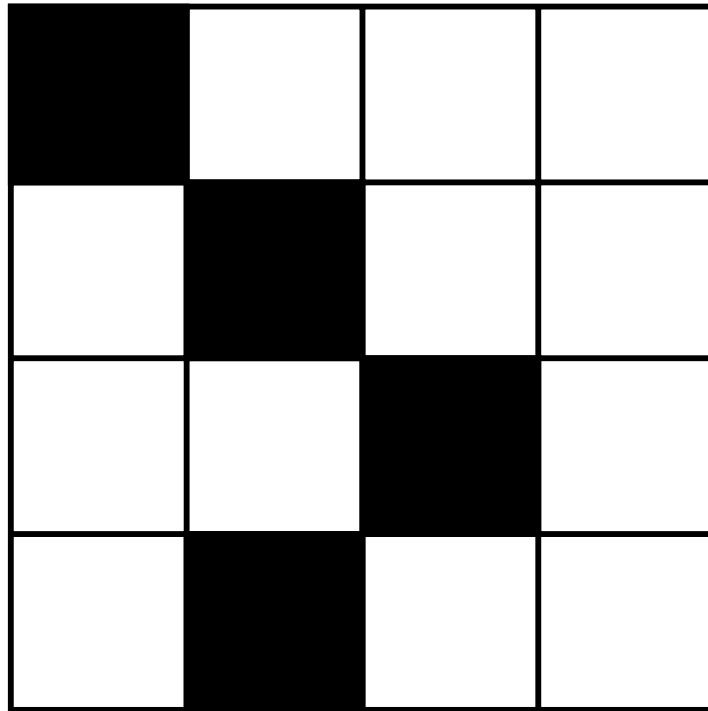
$$\begin{matrix} 0 & 1 \\ 1 & 1 \end{matrix}$$

For the report: Using Bayes theorem calculate a *MAP* (maximum a posteriori) estimation of the scene if (a) $\epsilon = 5\%$ and (b) if $\epsilon = 50\%$. Show the answers in percentages or percentage decimals

5 Classification

Consider a binary 4×4 image of a scene with a vertical line. In the *correct* image all pixels would be white except one vertical row with black pixels. Unfortunately the camera used is far from perfect. Errors in different pixels are independent with $p(\text{white} \mid \text{line}) = p(\text{black} \mid \text{not line}) = \epsilon$ and consequently, $\text{Prob}(\text{black} \mid \text{line}) = \text{Prob}(\text{white} \mid \text{no line}) = 1 - \epsilon$. Assume the a priori probability for the line to be located in column 1 or 4 is 0.3 (each) and the the

a priori probability that the line is in column 2 or 3 is 0.2 (each). Calculate the maximum a posteriori estimation of the following image when $\epsilon = 0.2$.



For the report provide the a priori probabilities of the four images. Calculate the a posteriori probabilities for the four images. Which is the most probable image. Show the answers in percentages or percentage decimals

6 Classification

An image (with 5×3 pixels) has probability 0.25 of being a 'B' (image ω_1), 0.4 of being an '0' (image ω_2) and 0.35 of being an '8' (image ω_3). In the final image there is also a measurement noise The probability of error is bigger for white pixels. An originally white pixel has a probability 0.35 of being black in x . An originally black pixel has a probability 0.25 of being white in x . How should the image x below be classified using maximum a posteriori probabilities (MAP) in the classification criterion.

For the report provide the a priori probabilities of the three images. Calculate the a posteriori probabilities for the three images. Which is the most probable image. Show the answers in percentages or percentage decimals

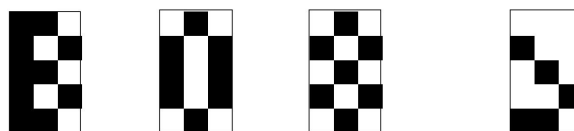


Figure 1: Mean images for three different classes, and an observed image x .

7 The OCR system - part 2 - Feature extraction

During all of the four assignments you are gradually going to build and test a small system for ocr (optical character recognition). During this second assignment your task is to write one function $\mathbf{x} = \text{segment2features}(B)$ in matlab. And to test this function on a few images using a benchmark script `inl2_test_and_benchmark`.

On the web page for the course in image analysis there is a zip-file, `inl2_to_students.zip`. By downloading and unpacking the file you will obtain a folder `ocr_project` and in this folder there are two folders `datasets` `matlab`. In the `datasets` folder there is for now only one folder 'short1', which contains a few test examples and ground-truth both for segmentation and for recognition. Later on we will add more folders with additional (and more challenging images).

Given a binary image B (thus only with zeros and ones), study the region of pixels that are equal to one. Use your imagination to define at least 6 different features (numbers), that can be used to classify which letter the region corresponds to. It is good if the feature (the value) does not depend on which position the region has. In other words, if the regions is shifted in the x or y direction, then the same features will be obtained.

Write a function in matlab that, given such and image matrix B as input, returns a feature vector

$$\mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}.$$

One suitable function name is

```
function x = segment2features(B);
```

Suggestion: One could try using moments, histograms, the sum of the pixel values along a row or in a subregion. To remove the influence of the position one could first translate the image so that the centre of mass is in the origin.

On the course home page there is a zip-file `inl2_to_students.zip`. Here you can find both images and an unfinished routine `inl2_stub.m` that can be used to try things out. Try both `im2segment` (from Handin 1) and then `segment2features` on such images. Do you get similar feature vectors for the same character? Do you get different feature vectors for different characters?

When you are finished you can also try running `inl2_test_and_benchmark`. In the script there is a simple visualization part that might be used to get a better feeling for the features.

For the report describe and motivate your features, supply both code (e.g. matlab code) and a printout the results of using your algorithm, i.e. supply examples of input data (e.g. as images) and the corresponding feature vectors, both for different characters and for different images of the same character.