Centre for Mathematical Sciences, Mathematics, LTH, September 2018

Image Analysis, Assignment 1

Introduction

In this assignment you will study basic histogram equalization, linear algebra for images and start your work on your own Optical Character Recognition system. The data for the assignments is on the course homepage:
`http://www.ctr.maths.lu.se/course/newimagean/2018/`

The Rules

The assignment is **published on the web page** at the lecture **Tuesday 4 September**. The deadline is on **Sunday 16 September** (23:59 CET). Each student should hand in his or her own individual solution and should, upon request, be able to present the details in all the steps of the used algorithm. You are, however, allowed to discuss the assignment-problem with others. You may also ask your teachers and the course assistants for advice, if needed.

The report. Present your work in a report of approximately four A4-pages written in English or Swedish. Make sure you answer **all questions in the grayed boxes** and provide complete solutions to the exercises.

You may submit your code along with the report, if you want to (see below.) However, the teacher is going to judge your work based on the report alone. Usually the teacher will check the code only in very special cases, for instance, when the results seem strange or if very persistent problems remain with your implementation.

Submitting your work. Email the report as a **pdf-file** with the name format `assignment_1_yourname.pdf` to the address

`fman20@maths.lth.se`

You may optionally attach your code to the message, for instance, by collecting all the m-files in a compressed folder[1]. Write **your name and the assignment number** in the subject line of your email.

[1]Make sure that your matlab scripts are well commented and can be executed directly (that is, without loading any data, setting parameters etc. Such things should be done in the script).

1 Image sampling

Consider a continuous monochrome image. The intensity (brightness) in a point (x, y) in the image is given by the function

$$f(x, y) = 4x(1 - x), \quad 0 \leq x \leq 1, 0 \leq y \leq 1.$$

Think about what the image looks like.

Sample the image evenly to a discrete image with 5×5 pixels. Let the lower left pixel be a sample from the point $(0, 0)$ in the continuous image and the upper right pixel a sample from $(1, 1)$. Quantify the discrete image with 16 different gray levels from 0 to 15. What is the result?

> For the report: Write out the resulting 5×5 image matrix. Explain how you did your calculations.

2 Histogram equalization

An image (in a continuous representation) has gray level histogram

$$p_r = \frac{e^r - 1}{e - 2}, \quad r \in [0, 1] \ .$$

What gray level transform $s = T(r)$ should be used so that the resulting histogram p_s is uniform, i.e.

$$p_s = 1, \quad s \in [0, 1] \ ?$$

> For the report: Specify the transformation $s = T(r)$ and show how you computed it.

3 Neighbourhood of pixels

Consider the following image

$$\begin{pmatrix}
3 & 3 & 2 & 2 & 2 & 3 & 3 & 3 & 0 & 2 & 2 & 2 \\
0 & 3 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 3 & 2 & 3 \\
1 & 2 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 2 & 3 \\
3 & 0 & 3 & 1 & 0 & 3 & 3 & 0 & 2 & 3 & 3 & 2 \\
3 & 2 & 1 & 0 & 1 & 3 & 1 & 3 & 2 & 0 & 3 & 2 \\
0 & 3 & 3 & 1 & 1 & 1 & 1 & 0 & 2 & 3 & 3 & 3 \\
2 & 0 & 2 & 1 & 1 & 0 & 1 & 0 & 2 & 2 & 0 & 2 \\
2 & 1 & 3 & 0 & 0 & 3 & 2 & 1 & 2 & 0 & 3 & 2 \\
2 & 0 & 3 & 1 & 0 & 3 & 2 & 3 & 2 & 2 & 0 & 3 \\
2 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & 1 \\
0 & 2 & 3 & 0 & 0 & 1 & 0 & 0 & 0 & 3 & 3 & 2 \\
0 & 2 & 3 & 0 & 3 & 3 & 2 & 1 & 2 & 1 & 0 & 2
\end{pmatrix} .$$

a) Mark each element that has intensity 0 or 1 with a circle.

For a pixel with coordinates (m, n) the 4-neighbours (as defined in lecture 1) are the four pixels with coordinates $(m \pm 1, n)$ and $(m, n \pm 1)$.

b) Fill the circles of those pixels that has intensity 0 or 1 and has **at least two** 4-neighbours that also have intensities 0 or 1.

For the report: Perhaps the easiest way to do this is by hand, e.g. by taking a photo of your solution and putting it in the report.

4 Segmentation part of OCR

During all of the four assignments you are gradually going to build and test a small system for ocr (optical character recognition). During this first assignment your task is to write one function S = im2segment(image) in matlab. And to test this function on a few images using a benchmark script `inl1_test_and_benchmark`.

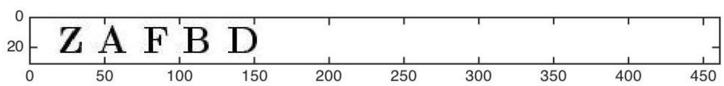
On the web page for the course in image analysis there is a zip-file,
`inl1_to_students.zip`
By downloading and unpacking the file you will obtain a folder `ocr_project` and in this folder there are two folders
`datasets`
and
`matlab`
In the datasets folder there is for now only one folder 'short1', which contains a few test examples and ground-truth both for segmentation and for recognition. Later on we will add more folders with additional (and more challenging images).

Study the script `inl1_stub.m`, that reads one of the images in the folder short1. Each image in the folder contains text (dark against a light background) Write a matlab program `im2segment` that takes such an image matrix I as input and returns a segmentation, i.e. a set of images $S = (S_1, \ldots, S_n)$ one for each letter in the image. Each such image matrix S_i should be a matrix with ones at the pixels for that letter and zeroes for all other pixels. In order to make things easy for all of us, you should all use the same convention to let the output be a so called cell array in matlab.

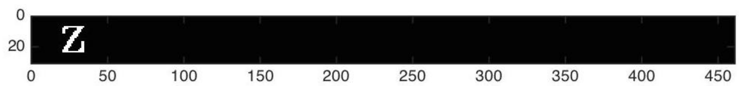
```
S = cell(1,n);
S{1} = bild1;
...
S{n} = bildn;
```

Also make sure that each image (e.g. `bild1`) has the same size as the original image.

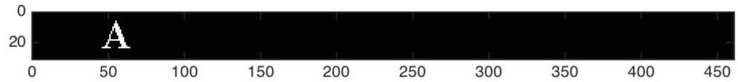
If all goes well when running your segmentation on the follwing image (im)



the resulting cell array will then contain 5 images, i.e. `S{1}` is



the second segment `S{2}` is



and so on. Again notice that each segmented image is of the same size as the original image.

Test your function `im2segment` using the benchmark script `inl1_test_and_benchmark.m` that loads each image in a folder and each ground truth segmentation and measures how well it works.

For the report: Print your code im2segment.m and the text results of running the benchmark script. You could also add a figure of one example of an input image and also the resulting segmentation images, similar to the ones above. Make sure that the code is commented so that it is easy to follow, or write a description of what it does.

5 Dimensionality

A **vector space** is a collection of objects and two operations **addition** and **multiplication by scalar**. Objects do not necessarily have to be geometric vectors. See `https://en.wikipedia.org/wiki/Vector_space` for more examples. For finite dimensional vector spaces one may choose a basis of elements $e_1, \ldots, e_k$ so that every example u can be written as

$$u = x_1 e_1 + \ldots, + x_k e_k$$

for some set of scalars $(x_1, \ldots, x_k)$. Here k is called the dimension of the vector space.
A: The set of gray-scale (monochrome) images with 2×2 pixels is a vector space. How many dimensions does this vector space have? Give an example of a basis $e_1, \ldots, e_k$.
B: The same question, but for the set of gray-scale (monochrome) images with 2000×3000 pixels.

For the report: Answer the questions A and B above and motivate your answer.

6 Scalar products and norm on images

How is the scalar product defined for images? How is the norm of an image defined? Given three images

$$u = \begin{bmatrix} 1 & -3 \\ 4 & -1 \end{bmatrix},$$

$$v = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix},$$

$$w = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \quad .$$

Calculate $||u||$, $||v||$, $||w||$, $u \cdot v$, $u \cdot w$, $v \cdot w$. Are the matrices $\{v, w\}$ orthonormal? What is the orthogonal projection of u on the subspace spanned by $\{v, w\}$?

(Notice that there are many possible matrix norms! There is a potential pitfall here if you use a computer program, e.g. matlab. You might accidentally be using the wrong norm!)

For the report: Do the calculations and answer all the questions.

7 Image compression

A small camera delivers low resolution images with 3×4 pixels. Before transmitting the image to a computer, one would like to compress the images consisting of 12 intensities to 4 numbers. After studying numerous images and using principal component analysis one has determined that the following four images represent typical images well,

$$\phi_1 = \frac{1}{3}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \phi_2 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \phi_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \phi_4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{pmatrix}.$$

Show that these four images are orthonormal in the scalar product

$$(f, g) = \sum_i \sum_j f(i,j)g(i,j) \ .$$

How should we determine the four numbers (coordinates) x_1, x_2, x_3, x_4 such that the approximate image

$$f_a = x_1\phi_1 + x_2\phi_2 + x_3\phi_3 + x_4\phi_4$$

is as close to f as possible, i.e. such that $|f - f_a|^2 = (f - f_a, f - f_a)$ is as small as possible?

Then calculate x_1, x_2, x_3, x_4 for the image

$$f = \begin{pmatrix} -2 & 6 & 3 \\ 13 & 7 & 5 \\ 7 & 1 & 8 \\ -3 & 4 & 4 \end{pmatrix} \ .$$

8 Image bases

On the web page for the course there is a file, zip-file, `assignment1bases.mat` . with two variables `bases` and `stacks`.

The variable `stacks` is a cell array. It contains two stacks of images

- 400 test images of faces of size 19×19. These are stored in a variable `stacks{1}`, which is a three-dimensional data structure of size $19 \times 19 \times 400$.
- 400 other test images of size 19×19. These are stored in a variable `stacks{2}`, which is a three-dimensional data structure of size $19 \times 19 \times 400$.

In the previous exercise we saw how to project an image onto a low-dimensional subspace defined by a set of basis images. Some interesting questions for discussion are:
D1: Is there a difference between different bases?
D2; What is the best basis?
D3: How can one calculate a good basis?
We do not expect a full answer to these three questions, but we encourage you to think about them and to discuss them with your fellow students.

The variable `bases` is also a cell array. It contains three sets of bases for 3 different subspaces of dimension 4. The first basis is stored in a variable `bases{1}`, which is a tensor of size $19 \times 19 \times 4$. Thus the four basis images are `bases{1}(:,:,1)`, `bases{1}(:,:,2)`, `bases{1}(:,:,3)`, `bases{1}(:,:,4)`.

TO DO Write a matlab function that projects an image u onto a basis (e_1, e_2, e_3, e_4) and returns the projection u_p and error norm r, i.e. the norm of the difference $r = |u - u_p|$.

TO DO Then write a script that tests all of the 400 test images in a test set and returns the mean of the error norms. Calculate this mean for each of the 2 test sets on each of the three bases.

Discussion? How should one calculate the best basis for a stack of images?

For the report: Include printouts of the matlab function for projection. In the report plot a few images in each of the two test sets. In the report describe in your own words what the images look like in the two test sets. Also in the report plot the four images in each of the three bases and describe what the three different choices of basis look like. Then print the mean of the error norms for the six combinations (2 test sets against the three bases). Which basis works best for test set 1? Why? Discuss. Which basis works best for test set 2? Why? Discuss.