Image Analysis - Lecture 4

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Lecture 4

Contents

- Features
- Scale Space
- Edge Detection
- Corner Detection

Features

Examples of features in images are

- Edges, where the intensity gradients are large.
- Ridges, the centre of dark or light stripes.
- Interesting points "Corners" that can be tracked reliably.

The focus of this lecture is **feature detection** and in particular on **edge detection**, but we are going to study **blob detection** and **ridge detection** and the theory behind **interpolation**, **scale space theory**, **differentiation**, and **noise**.

Edge detection

Properties of edges

- Edges usually contain important information.
- Compare with drawings. Usually we focus on the edges/contours of an object.
- At edges the intensity changes fast. Thus the derivative is large.
- Main idea: Edges are places where the gradient of the intensity is large.

Edge detection (ctd.)

Problems to consider

- What do we mean by differentiation in images?
- How do you calculate the derivative?
- How are the calculations affected by noise?
- This effect can be decreased by smoothing. Why?
- What are the connections between interpolation and differentiation?
- This motivates the study of differentiation, smoothing, noise and interpolation.

Scale Space Theory

Example: What is a cloud?

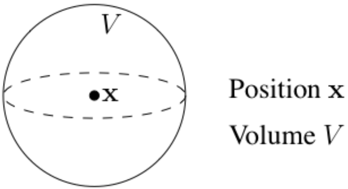
- something in the sky
- Regions in the atmosphere, where the density of condensed H_2O is above $0.4gm^{-3}$ at a resolution of about 1 m.

Scale Space Theory (ctd.)

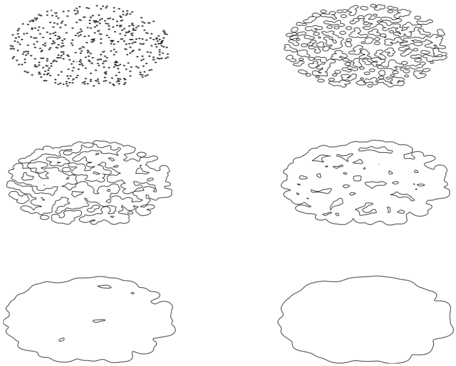
Scale selection:

- What does a cloud look like at a resolution of 1 μm?
- What does a cloud look like at a resolution of 10 km?

Let $M(x, V)$ denote the mass of condensed H_2O in a sphere with volume V centred at **x**.



Illustration



Properties of a scale space

Principal of causality

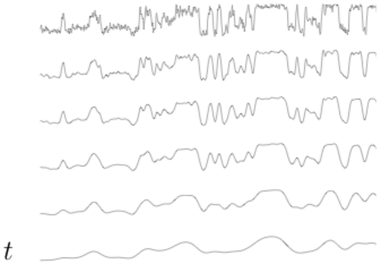
If $V_2 > V_1$ then $d(x, V_2)$ can be calculated from $d(x, V_1)$ but not vice versa.

We can go from a finer scale to a courser scale but not the other way!

Properties of a scale space (ctd.)

Atlas principle

If $V_1 < V_1^\star < \ldots < V_N^\star < V_2$ then the same result is obtained for every other partition $V_1 \to V_1^\star \to \ldots \to V_N^\star \to V_2$.



Axiomatic scale space theory

The idea behind scale space theory is to every function $f : \mathbb{R}^n \to \mathbb{R}$ associate a family $\{T_t f | t \geq 0\}$ of gradually smoothed functions

$$T_t f : \mathbb{R}^n \to \mathbb{R} \quad .$$

The original signal corresponds to scale $t = 0$. Increasing scale simplifies the signal but should not introduce new features (e.g. new local minima or maxima).

Gaussian scale space theory

Definition

The **Gaussian kernel** in two dimensions is defined as

$$G_b(x) = \frac{1}{2\pi b^2} e^{-|x|^2/2b^2}, \qquad x \in \mathbb{R}^2 .$$

Definition

The **Gaussian scale space** corresponding to the function $f : \mathbb{R}^2 \to \mathbb{R}$ is a family of functions $\{T_t f | t \geq 0\}$ parameterized by the variable t, where

$$T_t f = f * G_{\sqrt{t}} \quad .$$

Observe that the function $T_t f$ is obtained by solving the heat equation with initial values f until time $\sqrt{t}$.

The theory of Gaussian scale space

Theorem

An operator T_t with the following properties

- *T_t is a* **linear** *and* **translation invariant** *operator for every t,*
- **Scale invariance**. *If a function is scaled with a factor λ, i.e. $g(x) = f(x/\lambda)$ then there exists a scale $t' = t'(t, \lambda)$ such that $T_t g(x) = (T_{t'} f)(x/\lambda)$,*
- **Semi group property***: $T_{t_1}(T_{t_2} f) = T_{t_1+t_2} f$,*
- **Positivity preserving***: $f > 0 \ \Rightarrow \ T_t f > 0$,*

is given by

$$T_t f = f * G_{\sqrt{t}} \ .$$

What do we mean by scale 0

What does

$$f_t = T_t f_0 = f_0 * G_{\sqrt{t}} \ ?$$

There is no image with infinite resolution, i.e. the image at scale 0, f_0.
The only information we have about the image is an observation at one scale t_0, i.e. f_{t_0}.
The equation above is only symbolic.
What is the infinite resolution of a cloud or a photo in a newspaper?
Most images do not exist in all scales.
What is the curvature of the earth, a cloud, a tree?
How long is the coastline of Sweden?

Two popular uses of scale space

- **The coarse to fine principle.** In many applications it is useful to first search through the image on a coarse scale and then refine the search on a finer scale in the most interesting regions.
- **Scale space analysis:** Many features (e.g. edges) can be defined on all scales. Using the whole scale space representation one can construct robust detectors. Often features are detected on a coarser scale and positioned more precisely on a finer scale.

Scale space pyramid

- Fast implementations can be made using scale space pyramids.
- After scale space smoothing one does not need to save all pixels and can subsample the image. Usually in steps of 2.
- Study the material and illustrations in Chapter 3.5 of Szeliski.

Noise

- A common noise model is stationary Gaussian noise. (Often assuming independent pixels).
- There are faults to this model, but it is often easy to use it.
- The course **stationary processes** contains the necessary theory for using such models.
- The Fourier transform of independent identically distributed noise is flat.
- The Fourier transform of images is usually large for low frequencies, but small for large frequencies.

Edge detection

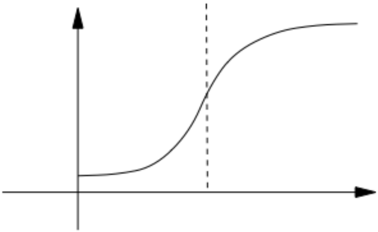
Edge detection is based on finding points in the image, where the first order derivatives are large.

Two main approaches

- Find points where the second derivative (in some sense) is zero (Laplacian methods).
- Find points where the first derivative is large (gradient methods).

Laplacian methods

Define the edge as the inflexion point. $\Leftrightarrow$ second derivative $= 0$



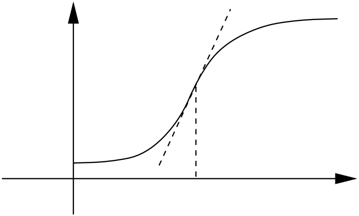
Laplacian methods (ctd.)

Laplacian methods have been used, but they have several disadvantages

- The Laplace filter is un-oriented
- The result is sometimes strange at sharp corners
- The result is strange where 3 or more intensities/colours intersect

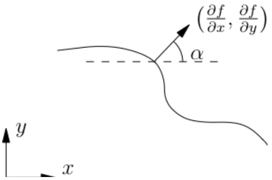
Gradient methods

One dimension:



Edges from thresholding

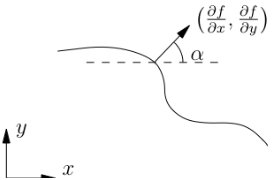
Rule: If $|\nabla f|^2$ > threshold, mark as edge point.
The gradient also contains directional information.



The normal gives α.

2. Edges from local maxima (Non-maximum suppression)

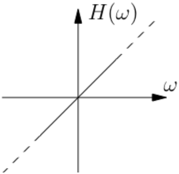
Rule: If a point is a local maxima in gradient in the direction of the gradient mark as edge point.



Analysis in frequency plane

One dimensional analysis.

$$f' \mapsto 2\pi i u F(u) \qquad \frac{1}{2\pi i}\frac{d}{dx} \mapsto u$$



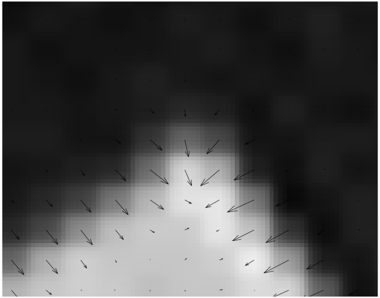
Orientation tensor

A common technique for finding so called ’corners’ (or interest points) is to study the **orientation tensor**.
With the tensor you can segment the images into regions that are

- flat
- flow-regions or edges (that have one dominant orientation)
- texture, corners, interest points (where there are several strong orientations)

Note: Read in the book (Chapter 4.1 in Szeliski). Both flow regions and texture regions can be extended, e.g. whole regions of the image with one dominant orientation.

Illustration of orientations



Illustrations of the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

The Orientation Tensor

Construct the matrix

$$M = \begin{bmatrix} W_{xx} & W_{xy} \\ W_{xy} & W_{yy} \end{bmatrix} = \begin{bmatrix} (\frac{\partial f}{\partial x})^2 * G_b & (\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}) * G_b \\ (\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}) * G_b & (\frac{\partial f}{\partial y})^2 * G_b \end{bmatrix} ,$$

where G_b denotes the Gaussian function with parameter b.
M - orientation tensor.
Note: We construct a matrix for every pixel.

Properties of the orientation tensor

The matrix M has the following properties:

- (Flat) Two small eigenvalues in a region - flat intensity.
- (Flow) One large and one small eigenvalue - edges and flow regions.
- (Texture) Two large eigenvalues - corners, interest points, texture regions.

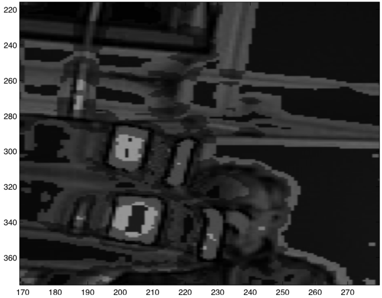
This can be used in algorithms for segmenting the image into (flat, flow, texture).

Calculation of the orientation tensor

Algorithm:

1. Calculate smoothed gradients: $L_x = f * \frac{\partial G_a}{\partial x}$, $L_y = f * \frac{\partial G_a}{\partial y}$.
2. Calculate products:
   $W_{xx} = (L_x.^2) * G_b, W_{xy} = (L_x. * L_y) * G_b, W_{yy} = (L_y.^2) * G_b,$.
3. Calculate eigenvalues:
   $W_{tr} = W_{xx} + W_{yy}, W_{det} = W_{xx} * W_{yy} - W_{xy} * W_{xy},$
   $W_\lambda = W_{tr}/2 \pm \sqrt{W_{tr}^2/4 - W_{det}}$.
4. Classify regions as texture if $W_\lambda > T$, flat if $W_\lambda < T$ and as flow if one is larger and one is smaller.

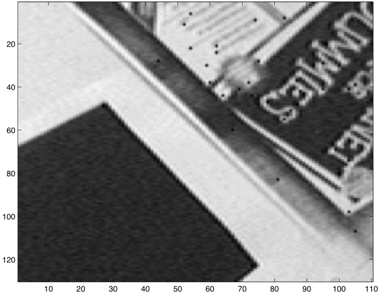
Illustration of the orientation tensor



Harris corner detector

Harris corner detector is based on local maxima to the function

$$f_{cr} = (k + \frac{1}{k})|\det(M)| - |\operatorname{trace}(M)^2 - 2\det(M)|$$



Corner/interest point detector

Edges are in **flow regions.** Search for maximum gradient magnitude perpendicular to the structure.

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = |\nabla f|^2 .$$

Corners or interest points can be found in **texture regions.** Search for maximum 'cornerness' or 'interest'

$$f_{cr} = (k + \frac{1}{k})|\det(M)| - |\operatorname{trace}(M)^2 - 2\det(M)|$$

Other detectors

Both edge detection and corner/interest point detection are extremely useful in numerous applications.
There is a general pattern to the technique:

- Calculated smoothed derivatives
- Search for points, curves or regions fulfilling certain criteria based on the derivatives.

Other detectors can be defined, e.g. for

- Finding the centre of valleys or ridges.
- Finding particular patterns such as 'text', licence plates, bricks, skin, cloud, faces. More on this in Lecture 5.

Ridge detection

Example from Masters thesis project in medical image analysis

Calculated smoothed second derivatives

$$\frac{\partial^2 G_a}{\partial x^2} * f$$

Ridge detection (ctd.)

Different scales (smoothing) is used to find ridges of different scales (widths)
The second derivatives in an arbitrary direction can be calculated from a combination of the three second order derivatives.
Compare with gradient.

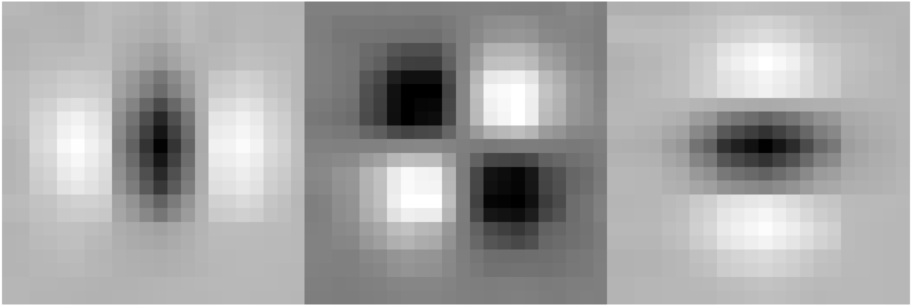
$$R_{xx} = \frac{\partial^2 G_a}{\partial x^2} * f$$

$$R_{xy} = \frac{\partial^2 G_a}{\partial x \partial y} * f$$

$$R_{yy} = \frac{\partial^2 G_a}{\partial y^2} * f$$

Ridge detection (ctd.)

The second order filters:



A filter in an arbitrary direction given by θ:

$$\begin{pmatrix}\cos(\theta) & \sin(\theta)\end{pmatrix}\begin{pmatrix}R_{xx} & R_{xy}\\ R_{xy} & R_{yy}\end{pmatrix}\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}$$

Invariant detectors

'Harris/Föstner' corner detectors have the property that it is invariant (independent) of translation and rotation.
Recently researchers have developed interest point detectors that are invariant also for

- changes in scale
- affine transformations

Today's masters thesis suggestion: Features detection and tracking in sonograms

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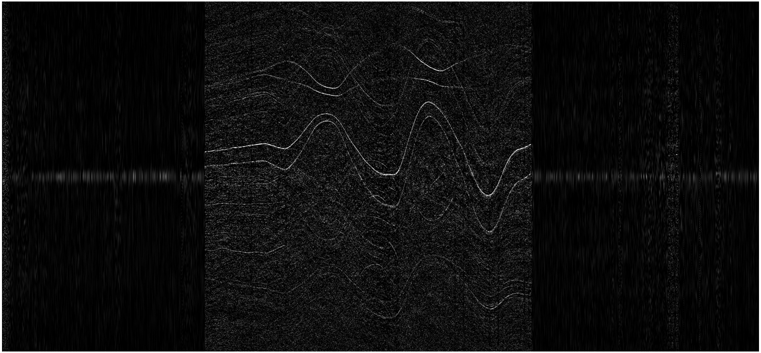

Review - Lecture 4

- Technique:
  - Scale space (smoothing)
  - Interpolation
  - Differentiation
  - Noise
  - Scale space pyramid
- Edge detection
- Orientation tensor
- Corner/interest point detection
- Other detectors: Ridges, blobs, sift (in ppt material)