



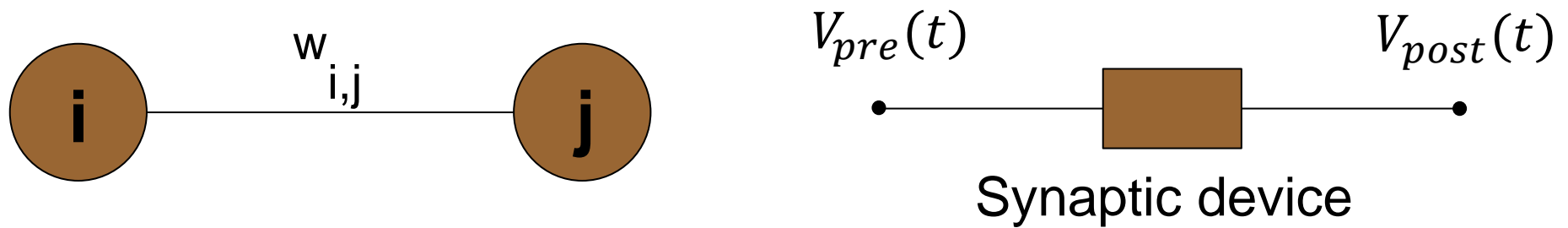
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# Lecture 7 – Spiking Neural Networks Part 2

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# Last lecture



But what about the neurons? Spike generation? Spike handling?

# Outline – Lecture 7

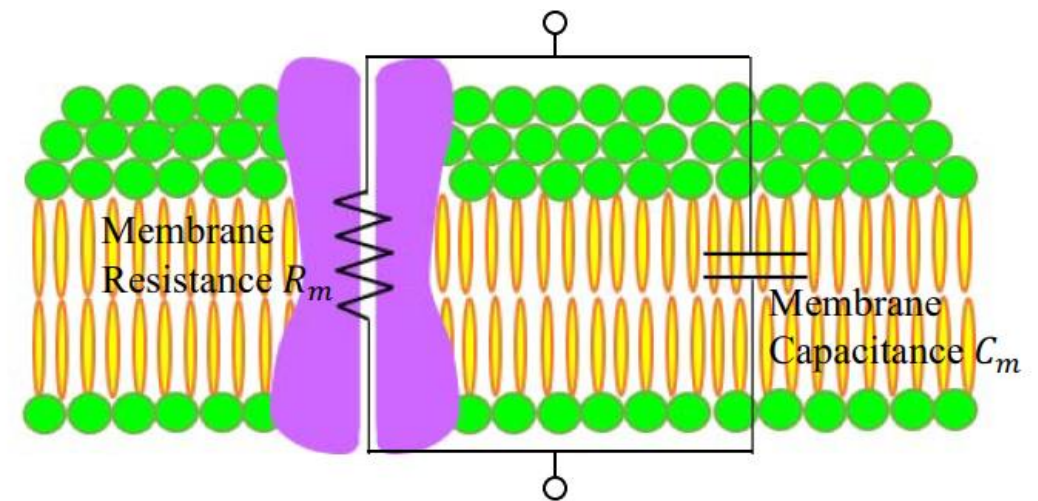
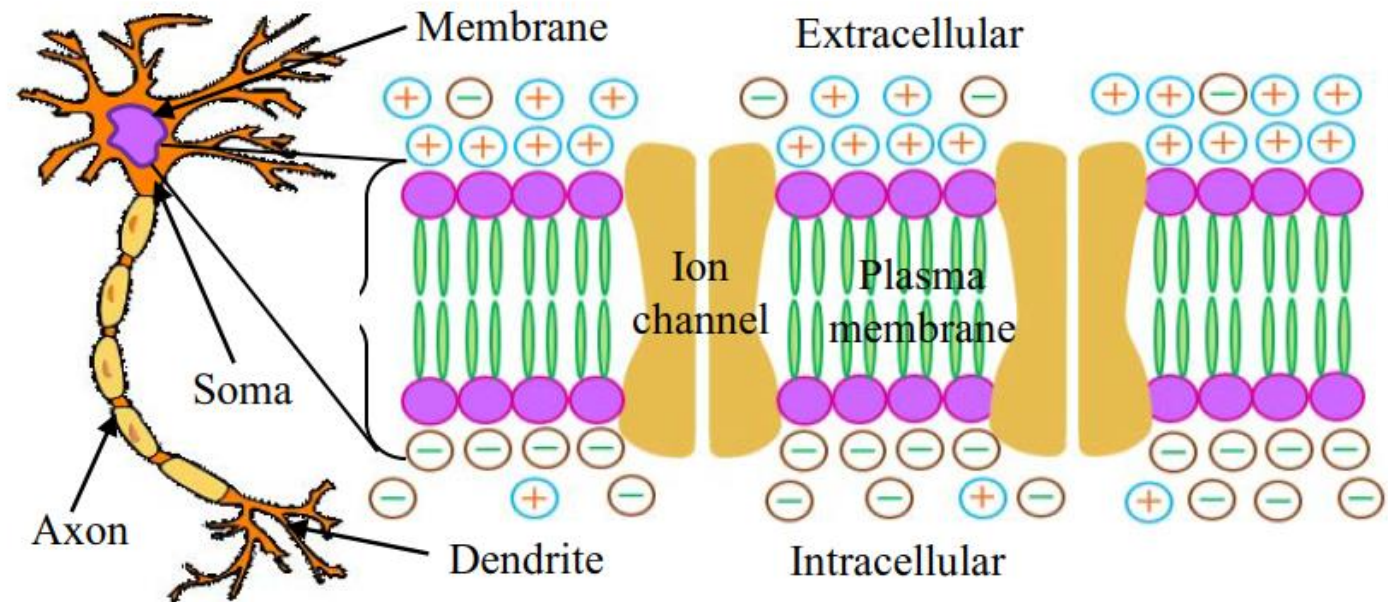
- The biological neuron and it's functionality
- Mathematical neuron models
- SNN features important for learning
- Examples of SNN implementations (software)
- Make your own quiz question!





# The biological neuron

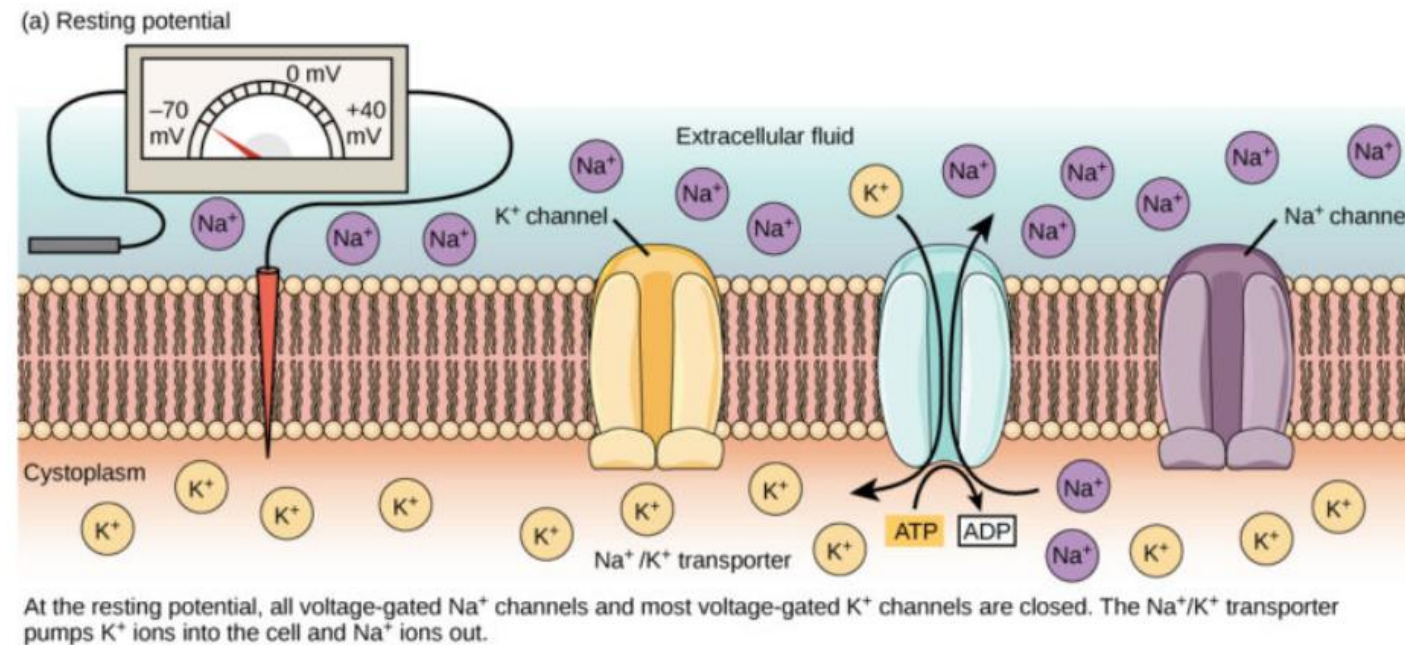
- **Electrochemical system**
  - A cell membrane separating two ionic liquids
  - $\text{Na}^+$ ,  $\text{K}^+$ ,  $\text{Ca}^{2+}$ , and others...
- **Neuronal membrane**
  - Lipid bilayer with protein ion channels
  - Ion concentration gradient  $\rightarrow$  Electric field across membrane  $\rightarrow$  membrane capacitance  $C_m$ .
  - Ionic channels “leak” ions through  $\rightarrow R_m$



# Dynamics of the membrane potential

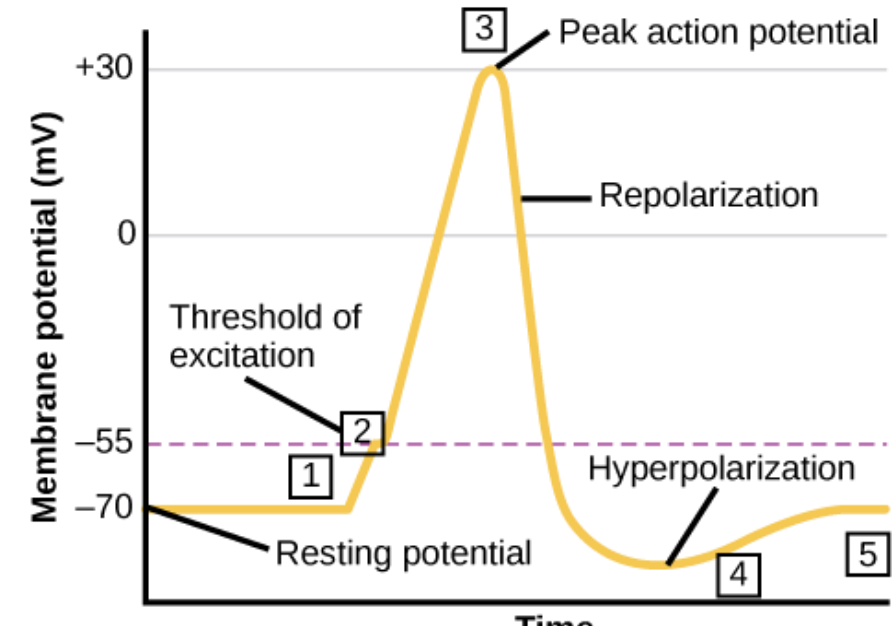
- Extracellular medium (outside the cell)
  - Rich on  $\text{Na}^+$ ,  $\text{Cl}^-$
- Intracellular medium (inside the cell)
  - Rich on  $\text{K}^+$ , negative proteins
- Ion transport through protein channels
  - K passive channels always open
  - Voltage controlled:  
Open/close depending on  $V_{mem}$
  - Na channel open for  $V_{mem} > -55\text{mV}$
  - K active channel open for  $V_{mem} > 40\text{mV}$
- Drift-Diffusion →  
Each ion have its own equilibrium state...
  - $\text{Na}^+ \rightarrow V_{\text{Na}} = 58\text{mV}$
  - $\text{K}^+ \rightarrow V_{\text{K}} = -93\text{mV}$
- Stable resting state at  $-70\text{mV}$  maintained by active K/Na pump using  $\text{ATP} \rightarrow \text{ADP}$ 
  - 3  $\text{Na}^+$  out, 2  $\text{K}^+$  in per ATP

$$J = -\frac{D\partial c_i}{\partial x} + qc_i\varepsilon \quad \text{Drift-Diffusion equation}$$

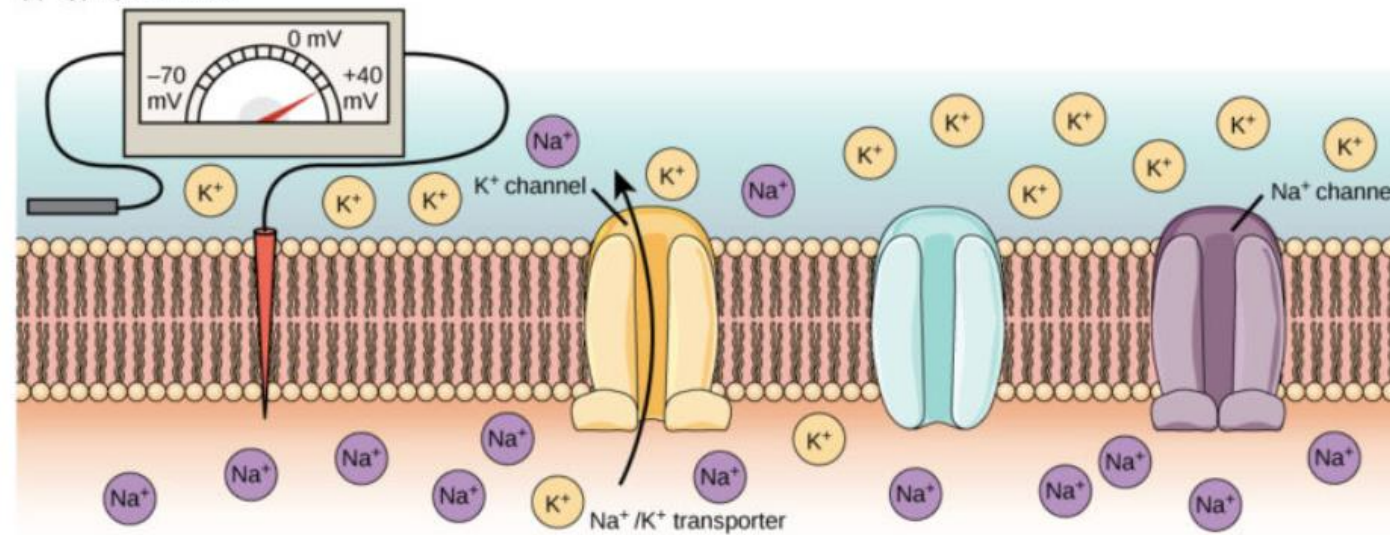


# Creating an action potential

1. Incoming signal
  - neurotransmitters binds to receptors
  - Some  $\text{Na}^+$  channels open
  - depolarization
2. When  $V_{mem} > -55\text{mV}$  :  
Majority  $\text{Na}$  channels open (THRESHOLD!)
  - $\text{Na}$  rushes into cell
  - $V_{mem}$  towards  $\text{Na}^+$  equil. (58 mV)
3. When  $V_{mem} > 30 - 40\text{ mV}$  :  
Majority  $\text{K}$  channels opens
  - $\text{K}^+$  rushes out!
  - $V_{mem}$  again towards  $\text{K}^+$  equil. (-93 mV)
4. Inverted concentrations restored by  $\text{Na}/\text{K}$  pump



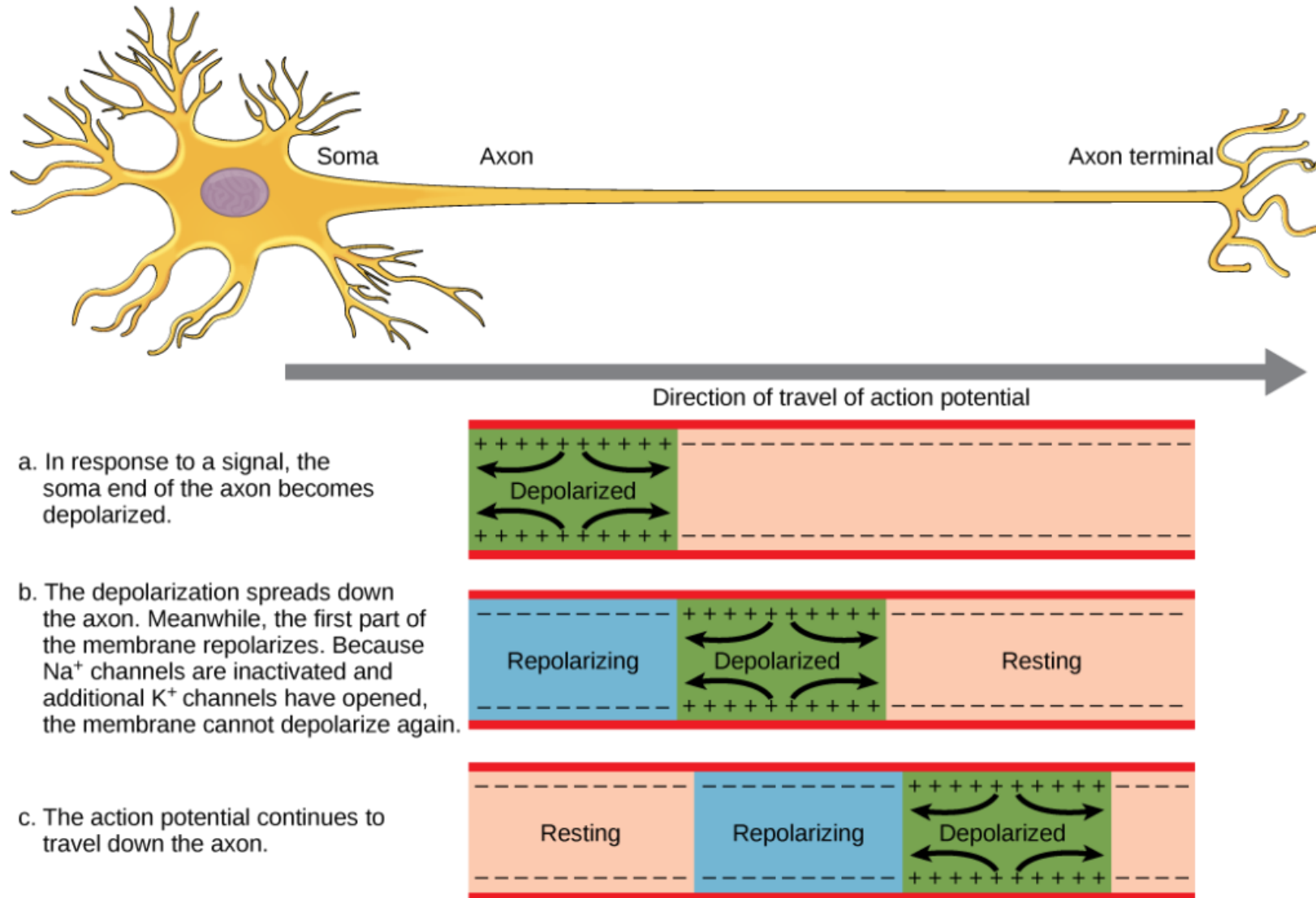
(c) Hyperpolarization



At the peak action potential,  $\text{Na}^+$  channels close while  $\text{K}^+$  channels open.  $\text{K}^+$  leaves the cell, and the membrane eventually becomes hyperpolarized.

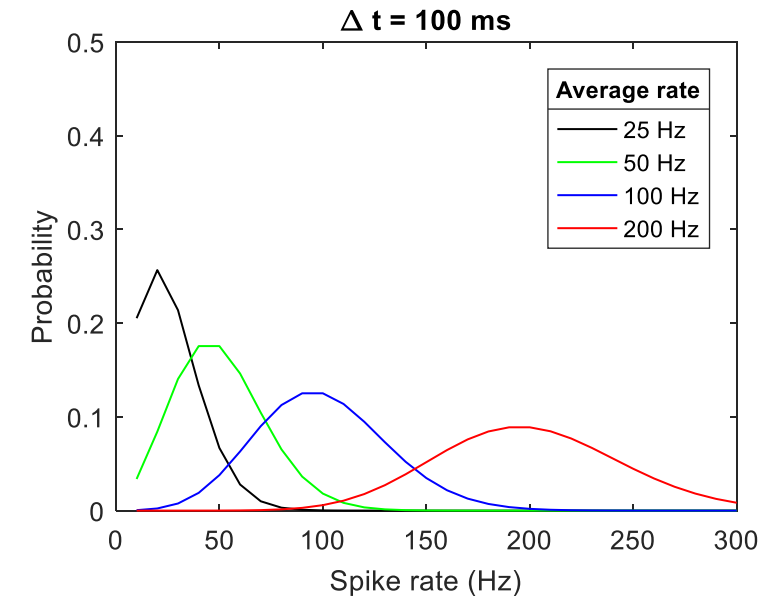
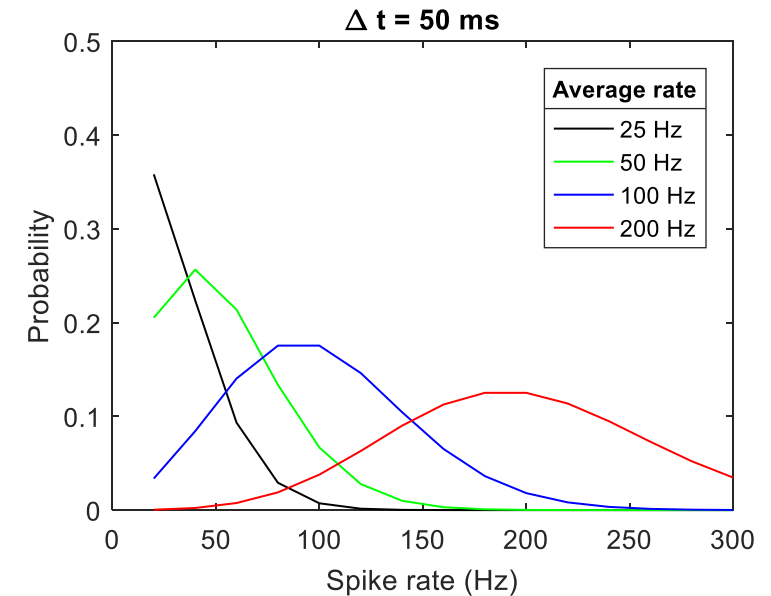


# Transmitting the action potential



# Neurons as Poisson process

- Neuron average activity can be known
  - Depends on input to neuron
- Exact spike timing stochiometric
  - Interactions → noise
- Neurons → Poisson spike sources
- Some randomness may help with learning
  - Avoids resonances, local minima, ...





# Key attributes of the biological neuron?

# Hodgkin-Huxley model

- A biologically plausible LIF model based on the squid axon:
- Three parallel current channels; Na, K and Leakage

$$\rightarrow C_n \frac{dv(t)}{dt} = I_{input}(t) - I_L - I_{Na} - I_K$$

Na activation  
function  $m \in f(v)$

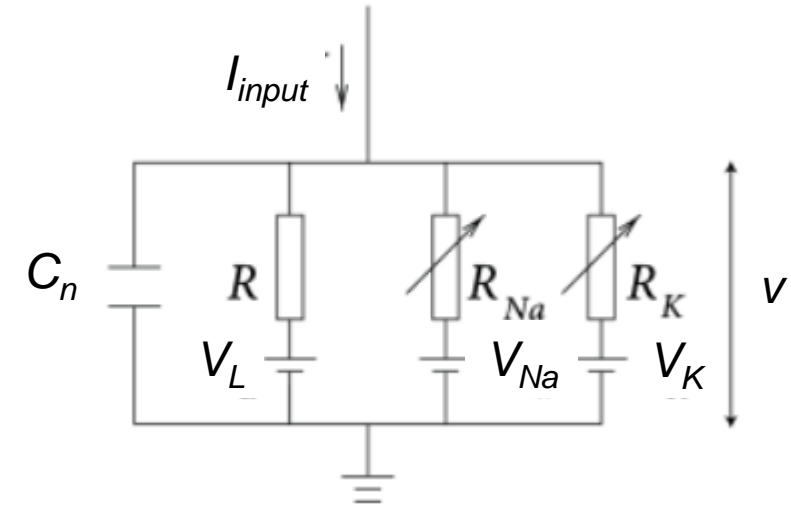
$$I_{Na} = g_{Na} m^3 h (v(t) - V_{Na})$$

$$I_K = g_K n^4 (v(t) - V_K)$$

$$I_L = \frac{1}{R} (v(t) - V_L)$$

Na inactivation  
function  $h \in f(v)$

K activation  
function  $n \in f(v)$

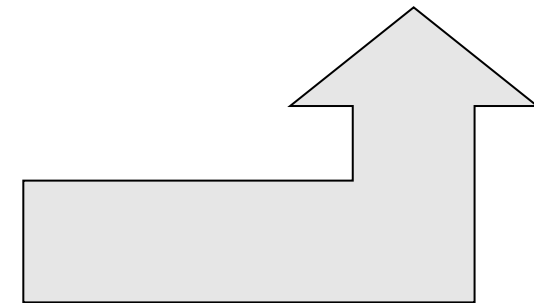
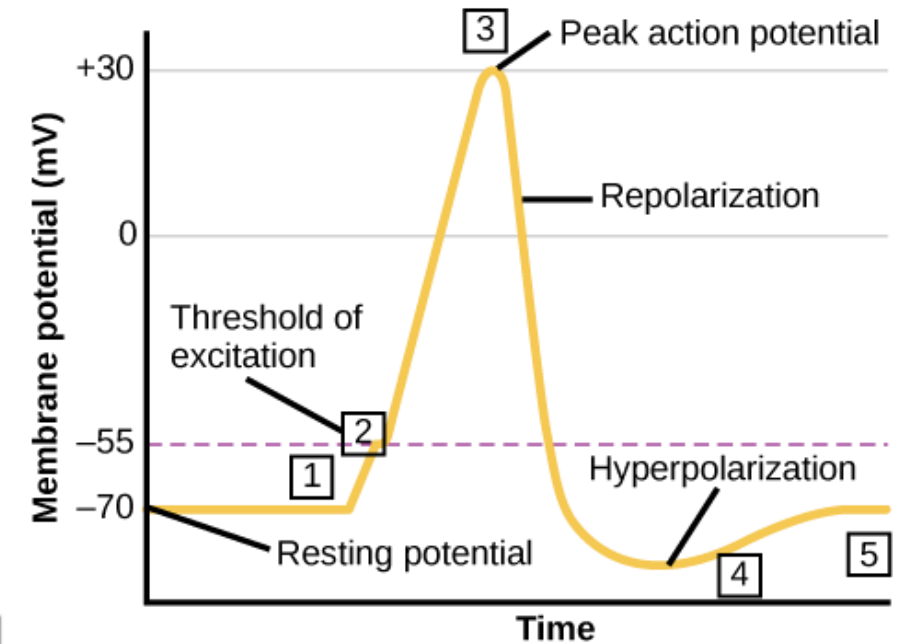
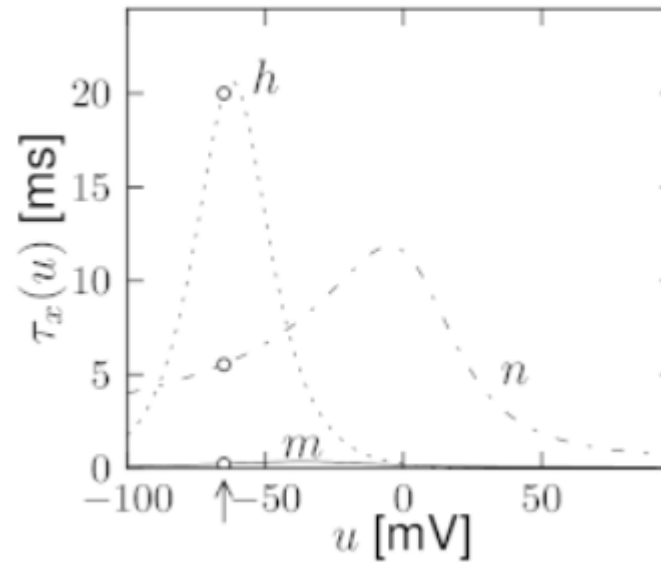
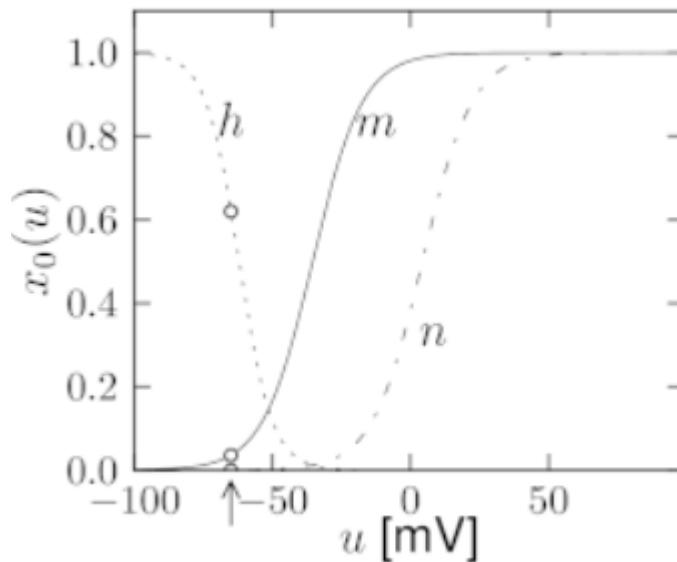


| $x$ | $V_K$ [mV] | $g_x$ [mS / cm <sup>2</sup> ] |
|-----|------------|-------------------------------|
| Na  | 55         | 40                            |
| K   | -77        | 35                            |
| L   | -65        | 0.3                           |

# Hodgkin-Huxley parameters

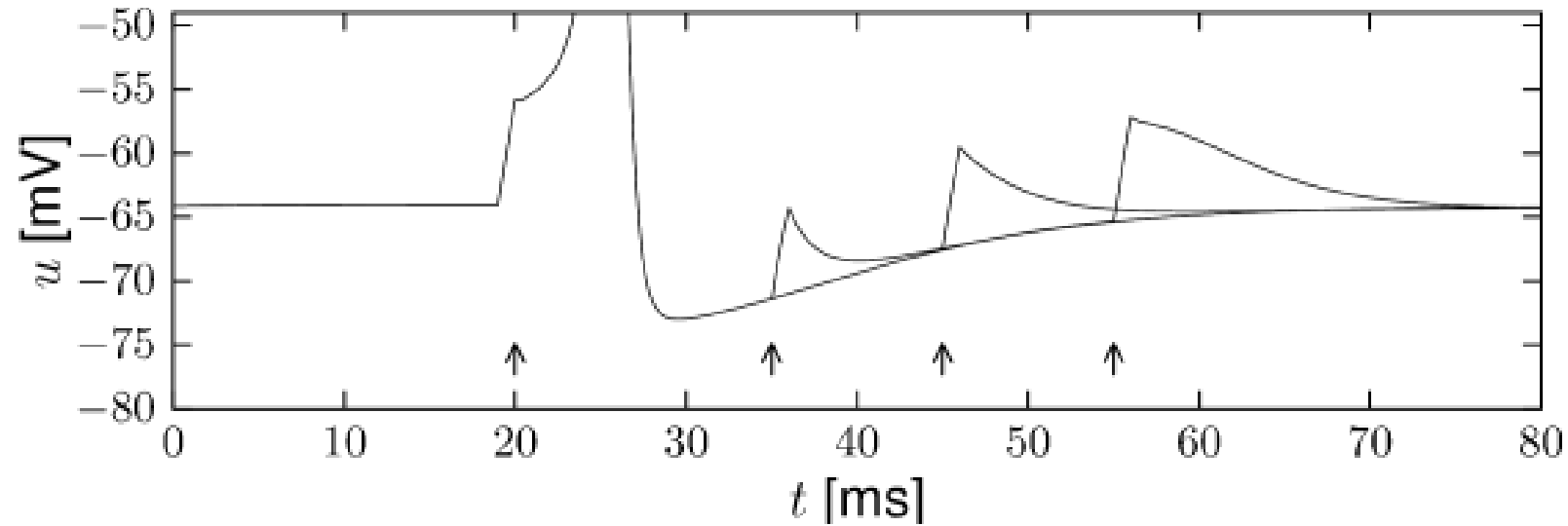
- $n, m$  and  $h \in f(v, t)$
- Parameters respond with time constant!
- → possibility for a potential spike

$$\frac{dx}{dt} = \frac{(x - x_0(v))}{\tau_x}$$



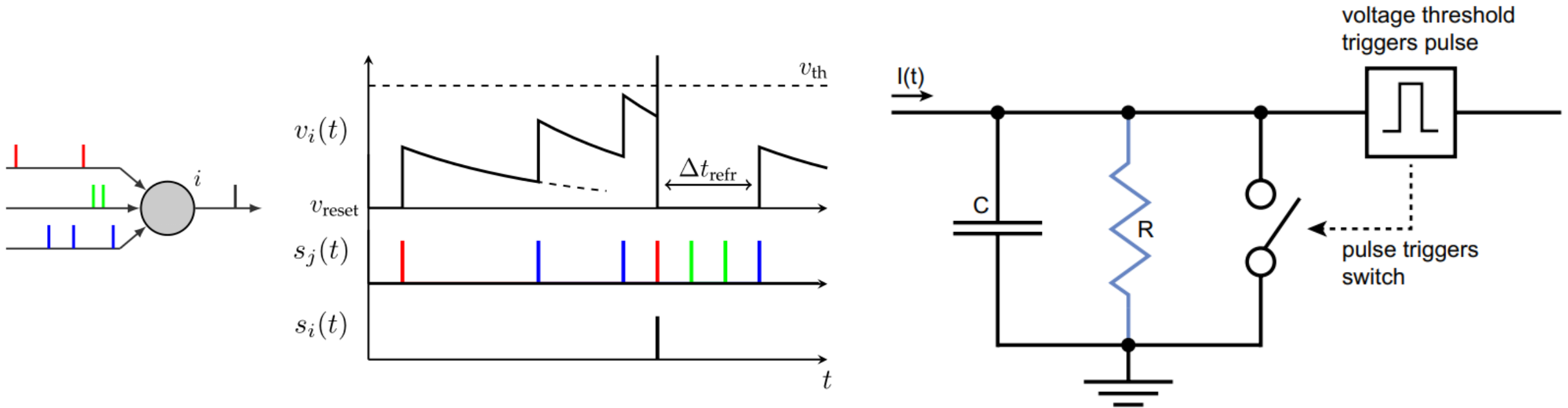
# Refractory behaviour with H-H model

- After spiking  $\rightarrow u < 0 \rightarrow$  Incoming spikes have harder time to initiate new spike  
 $\rightarrow$  Refractory behaviour, although not explicitly defined
- In practical implementations often defined explicitly instead





# The integrate-and-fire model



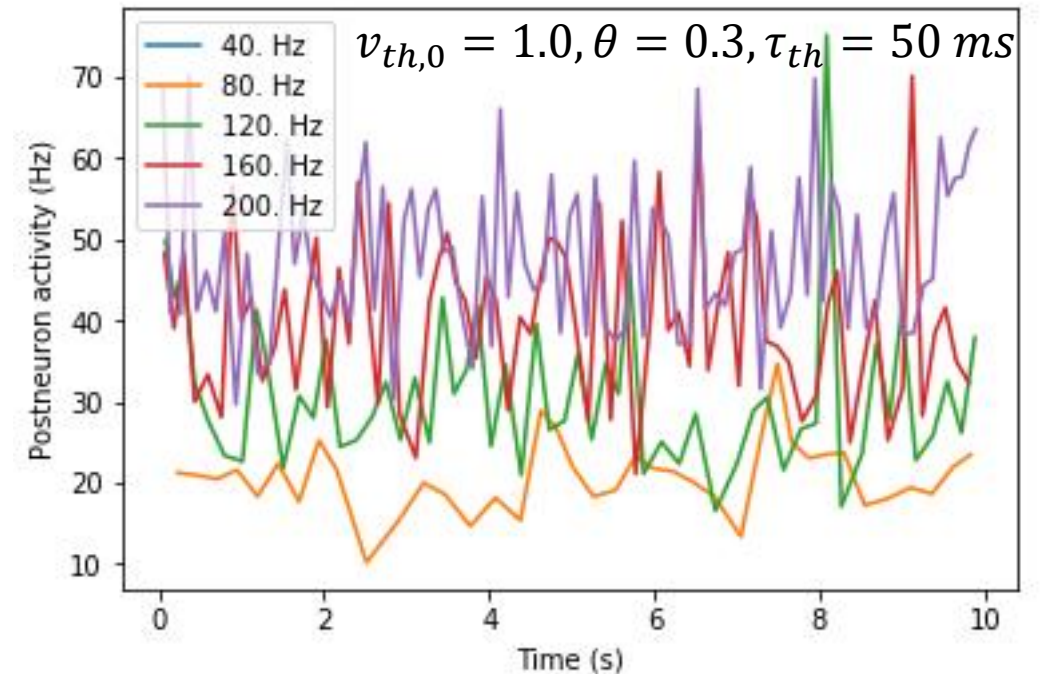
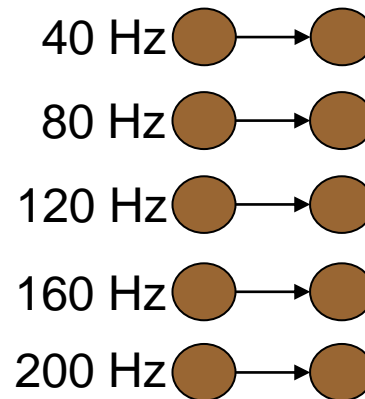
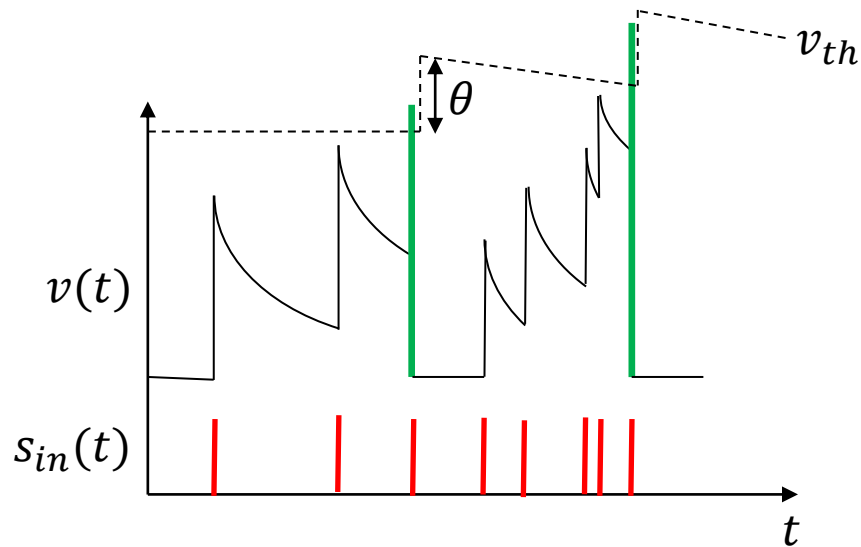
$$C_n \frac{dv(t)}{dt} = I_{\text{input}}(t) - \frac{1}{R_n} (v(t) - v_0) + I_{\text{spike}}^0 \delta(v(t) - v_t)$$

# Adaptive threshold

- For each spiking event  $\rightarrow$  increase threshold
- Threshold decays to “resting value”  
 $\rightarrow$  Hard to have high spiking rate
- Keeps overall spiking rate uniform

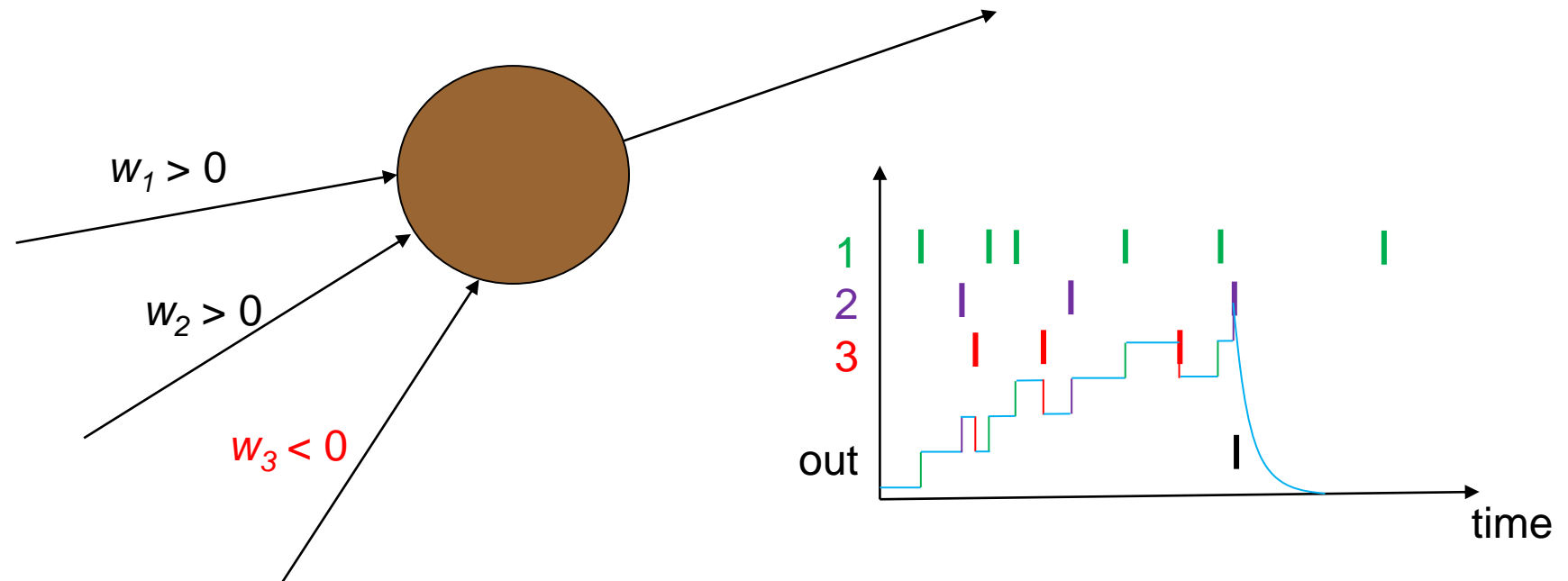
$$\frac{dv_{th}}{dt} = -\frac{(v_{th} - v_{th,0})}{\tau_{th}}$$

On spike:  $v_{th} = v_{th} + \theta$



# Excitatory / Inhibitory synapses

- Not all synapses shift the neuron potential towards threshold (excitation)
- Some reduce the neuron potential (inhibition)
  - Inhibition can be a crucial feature for learning in SNNs
- Inhibition can be modeled by synapse with negative weight



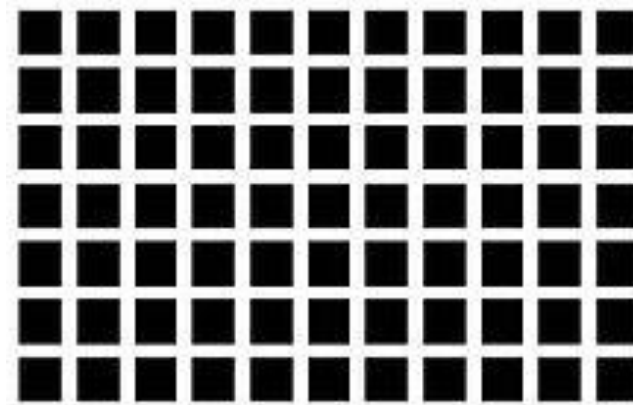
# Lateral inhibition - Example

- Upon firing (activation) a neuron inhibits potentiation of neighbouring neurons (in the same layer!)  
→ extra spatial contrast



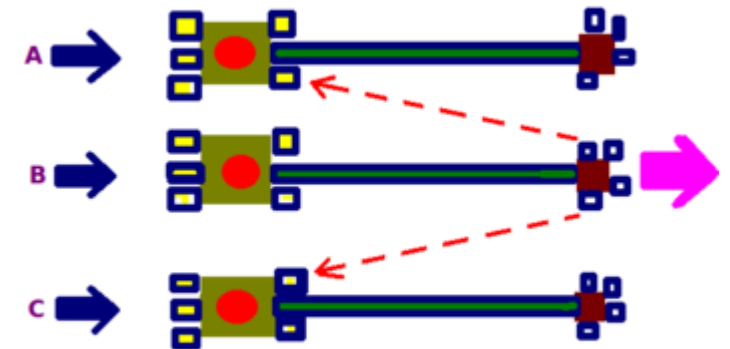
**Mach bands illusion**

Darker areas in contact with lighter areas appear darker than they are



**Herman grid illusion**

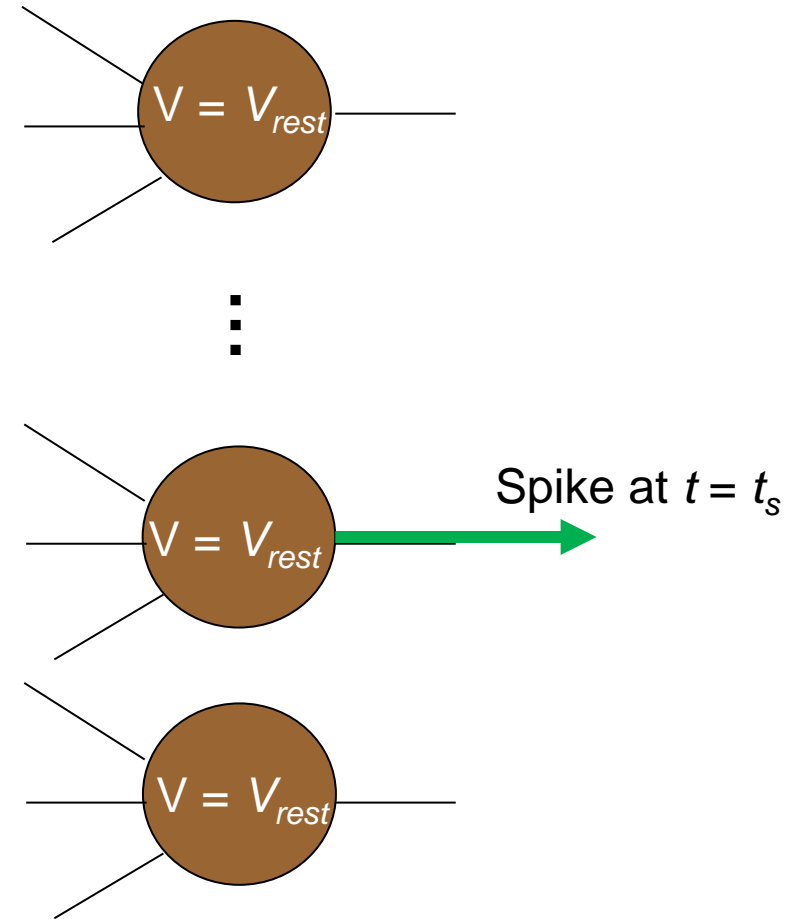
Darker blobs appear at intersections





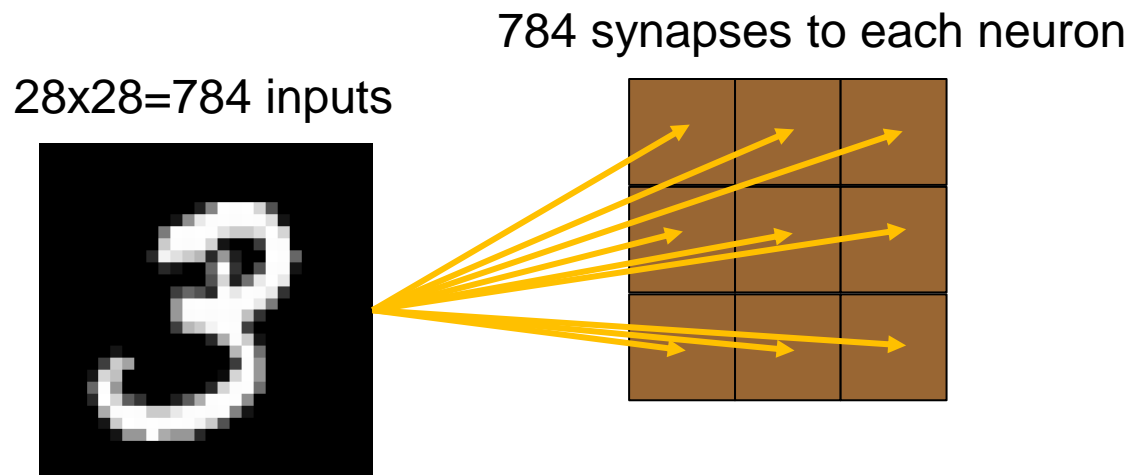
# “Winner-takes-all”

- If all neurons in a layer can completely inhibit all others in the layer
  - i.e. the first to fire will be the only one to fire...
- Typically implemented as inhibition for a certain time, matching refractive period
  - $V = V_{rest}$  on all until  $t = t_s + t_{refr}$

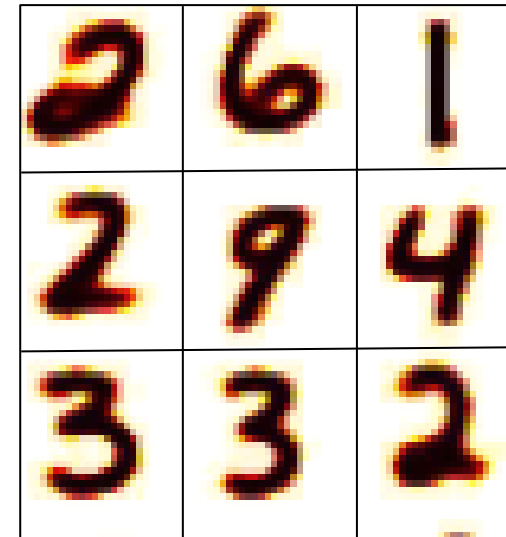


# Learning prototypes

- Lateral inhibition promotes neuronal learning of specific prototypes in the data, since only a few neurons can spike and adjust their receptive field towards a specific prototype
- Lateral inhibition also prevents overfitting as the competition between neurons forces them to learn different features/prototypes



Receptive field graph

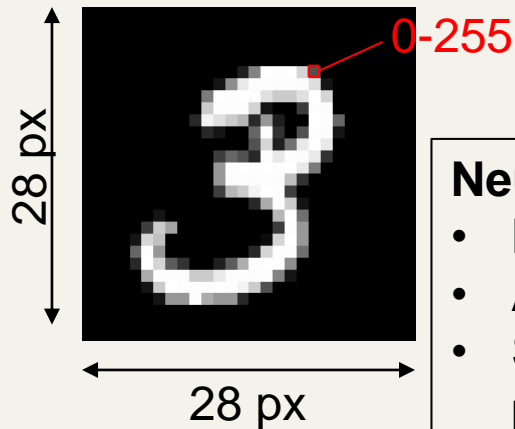


# Example of SNN in action

- Classification of MNIST data set
- Rate encoding + conductance synapses

## Input encoding

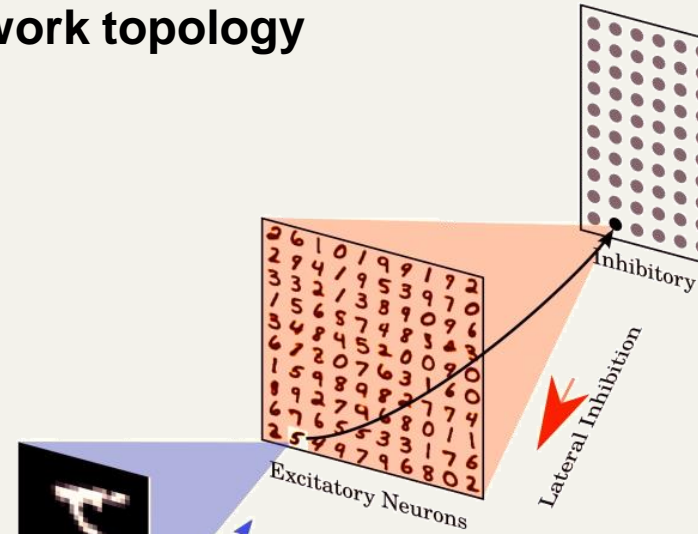
$$f = \frac{I_{ij}}{4} < 63.75 \text{ Hz}$$



## Neuron features

- LIF neurons
- Adaptive threshold
- Short refractive period
- “Soft” lateral inhibition via inhibitory neurons

## Network topology



## Network size

- 100, 400, 1600 and 6400 excitatory neurons
- Trained on 3-15x training set

## Training

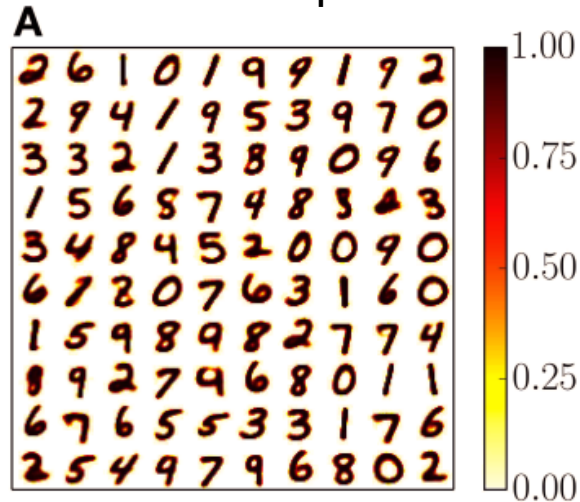
- Image presented for 350 ms
- 150 ms pause between images
- Weights updated by STDP (4 variants)

## Testing

- Set learning rate = 0
- Fix neuron thresholds
- Present testing set
- Each neuron is assigned a class depending on highest response over one set in training

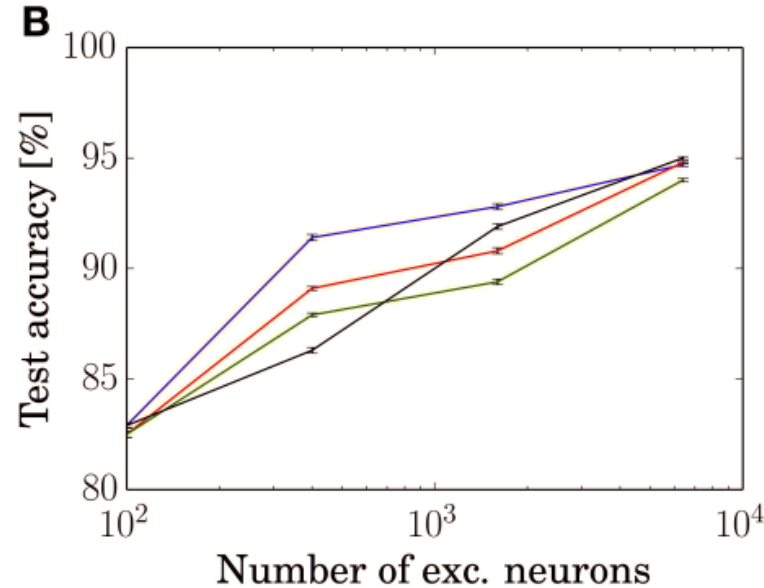
# Training results of SNN

Inhibition → specialization of the neuron receptive fields

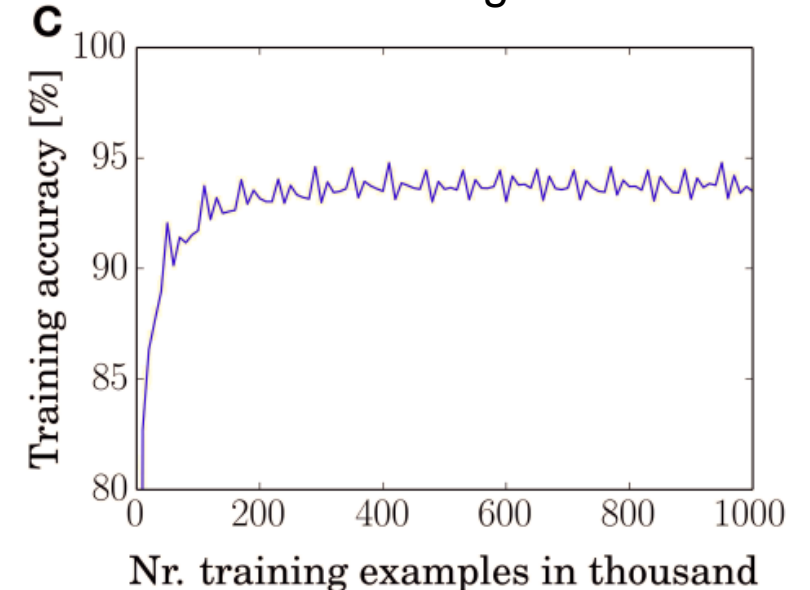


2D receptive fields

Robust with respect to the STDP variant.



Quick to converge. Maximum 95.0% for the largest network.



Possible improvements: Additional layers, localised receptive fields,...

Limitations: Few spikes + Rate-encoding limits minimum presentation time



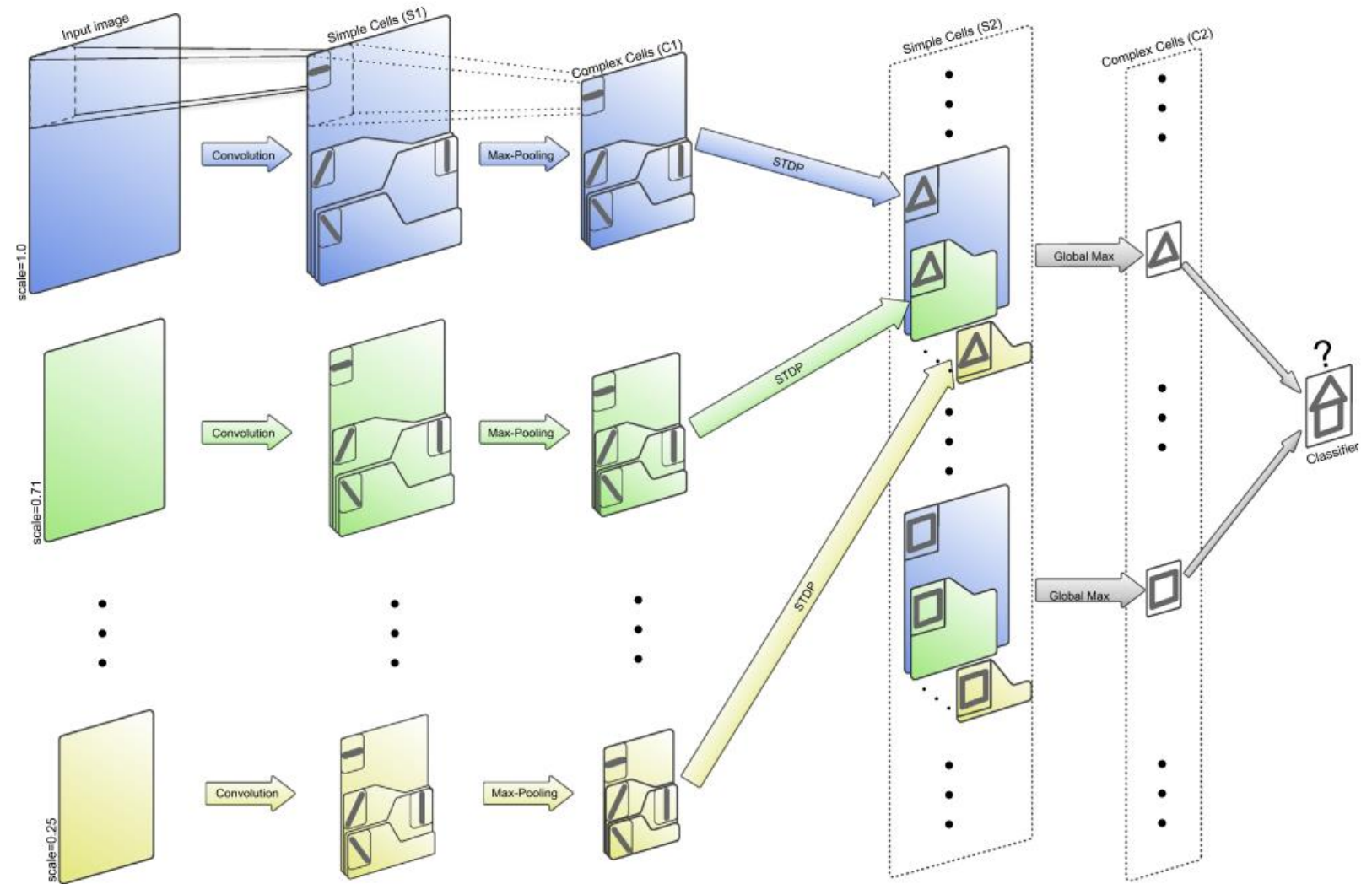
# Energy usage in SNN

- Variable threshold → keeps # pulses low
- Very sparse spiking: 17 spikes for one example (16 spikes correct class + 1 spike incorrect)
- Typical energy / spike in hardware? 1 spike ~1 pJ, synapse ~1 pW
- Authors estimate ~ 1 mW consumption! (smart phone ~ 1 W)

Compare to NVIDIA V100 GPU = 250W (at least)  
→ 250 000 times better

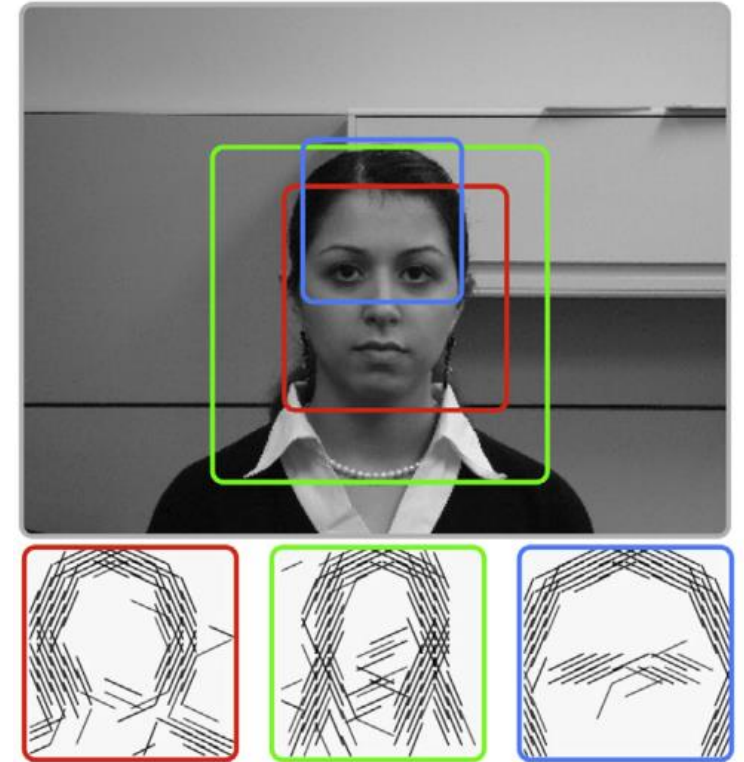
# More advanced SNN

- Time-encoding
  - Darker pixel  
→ Larger input spike delay
- First layer: Convolution
  - 5x5 kernels with four predefined features
  - No learning here!
- Second layer: Data pooling
  - Max pool implemented as first pulse is propagated
- Data scaling
  - 0th-2nd layers implemented at 5 scales
- STDP learning in third layer
  - Unsupervised feature learning
  - Input from all scales!



# Performance benchmark

- Beats DeepConvNet on the **3D-object** data set:
  - 10 classes of objects at 72 different conditions (various angles, scales, tilt, ...)
  - The SNN got 96% vs 85.8% for DeepConvNet
- **Much fewer parameters**: thousands vs 60 million (DeepConvNet)
  - Allows for smaller data sets without overfitting
  - Only 3500 images used here (humans can generalize from a few images only).



*Ex. Feature learning at three scales*



# Make your own quiz question!

- Come up with a good quiz question based on the topics of L4-L7
- You have until the end of the lecture/day → Send it to [mattias.borg@eit.lth.se](mailto:mattias.borg@eit.lth.se)
- Quiz will be posted on Canvas for you to practise on..

**What is the chance that you win on the lottery?**

1. Chance? I always win
2. 1 in 100 000
3. As good as dying in a plane crash
4. I will win when pigs can fly

Lecture 4 – Machine Learning Topologies  
*Convolutional Neural Network*  
*Recurrent Neural Network*  
*Locally Connected Network*

Lecture 5 – Hardware for Big Data  
*GPU*  
*TPU*  
*In-memory computing*

Lecture 6 – Spiking Neural Networks 1  
*Plasticity*  
*Hebbian learning*  
*STDP*

Lecture 7 – Spiking Neural Networks 2  
*Biological neuron*  
*Hodgkin-Huxley*  
*Integrate-and-fire*