Computer Laboratory Exercises in Optimization 2016
Lab 1. Line Search and Multidimensional Search.

The lab is to be done in groups (2-3 persons in a group). You can bring your own laptop with an installed MATLAB package of the version 2011 or later.

Preparation Exercises

The preparation is to be done **before** the actual lab session. Due to the lack of time, you are **strongly recommended** to download and run the m-file for the lab in advance to make sure that you understand the details of what is going on there.

- If you are not familiar with Matlab you should examine Matlab with the command `demo`. A quicker way may be to type `help demos`, choose a demo from the list, eg. intro and then type `intro`. You might find it usefull to look at the MATLAB links on the course homepage (see **Some Related Links** at the end).

  `http://www.maths.lth.se/matematiklth/personal/ghulchak/optimization/2016/`

- For each of the following methods give a short explanation of how and why the method works.

  Golden section

  Newton

  Armijo

  Steepest descent

  Modified Newton

- Run the lab m-file at home and make sure that you understand how it works and what's going on.

- Implement the Golden section, Dichotomous, Bisection and Newton's method in MATLAB to solve the following exercises: 2.1a, 2.3, 2.4ac, 2.6. Present your program and the solution at the lab.

At the computer

♡ Download the file lab1.m from the course homepage (see the link above).

♡ Type `lab1` in the Matlab window to start the program.

♡ A new window appears where you select the first method by clicking the name Golden section. As you can see the function to be investigated is called $fun1$ and it is defined on the interval $-0.25 \leq x \leq 0.5$. Select an initial interval by clicking two points in the window. For each next click, anywhere in the window, a new smaller interval of uncertainty will be shown. A new initial interval can be defined after you have clicked the restart button. When you have tried many different initial intervals, you can change the domain of definition to $-10 \leq x \leq 10$ by clicking the change button. Continue your experimentation. Investigate also the function $fun2$ on the interval $-10 \leq x \leq 10$.

Questions to be answered for Golden Section:

- Does the method always converge?
- What kind of points are found by this method? Global minimum, local minimum or something else?
- Would you say that it is a reliable method?

♡ Repeat your investigation for Newton's method by clicking the name. Then click once to choose an initial point and continue clicking to see the Newton steps. Change the domain and the function as before. Try different initial points all over the interval $0 \le x \le 2$ for $fun1$.

Questions to be answered for Newton's method:

- Does the method always converge?
- What kind of points are found by this method? Global minimum, local minimum or something else?
- Would you say that it is a reliable method?
- How far away from a minimum can you start without losing the convergence to the minimum?

♡ Investigate Armijo's method in the same way. Note that you don't have to use the restart button. A single click in the window selects an initial point, then some possible iteration points that *may occur* (depending on where you start) in the search are shown in red with the one chosen by Armijo's rule being blue.

♡ Try two Matlab methods for minimization of the first function $fun1$ by typing

1) `options=optimset('display','iter');`
`fminbnd(@lab1,-10,10,options)`

Here the interval $[-10, 10]$ is chosen (you can try another values).

2) `[x,fval,exitflag,output]=fminunc(@lab1,-1)`

Here the initial point -1 is chosen (you can try another one).

- Do you see any similarities with Golden Section or Newton's methods you tried above?
- Is the Matlab result satisfactory? (convergence? global minimum?)

♡ To continue with the multidimensional search click the multidim button.
In this new window you are going to minimize functions of two variables. The methods at your disposal are:

Steepest descent with Armijo's line search

Newton's method with the unit step

The Modified Newton's method with the unit step

All methods can be switched on at the same time but you are advised to begin with Steepest descent, click out an initial point and continue the clicking. After you have examined the behaviour for many different starting points, try Newton's method in the same way and finally the modified Newton for different values of ϵ (in the box to the right of EPSILON).

Questions to be answered for each of these three methods:

- Does the method always converge? Is it fast/slow? For what kind of functions (think of $f1$, $f2$, $f5$ and $f6$ as examples) does it work the best?
- What kind of points are found by the method? Global minimum, local minimum or something else?

Additional questions for (Modified) Newton's method:

- How does the size of ϵ affect the method?
- In what way could it be possible to improve the covergence of the method? By what price?

♡ There is also a method called Strategy which allows you to change the method as well as the value of ϵ after each step. This is useful for finding a mixed strategy that works well for all starting points. Try to figure out such a strategy and test it on the function $f2$.

For the functions $f3$ and $f4$ no contours are shown. Try to find the global minimum for these functions in as few steps as possible. Can you be sure that you have found the global minimum? Compare with Matlab's `fminsearch`. Type

```
[point,val]=fminsearch(@(x)lab1(x,'f3'),[5 5])
[point,val]=fminsearch(@(x)lab1(x,'f4'),[5 5])
```

Here [5 5] is the initial point (try several different).