Courbes_collision

November 29, 2022

Probabilité qu'il n'y ait pas de collision

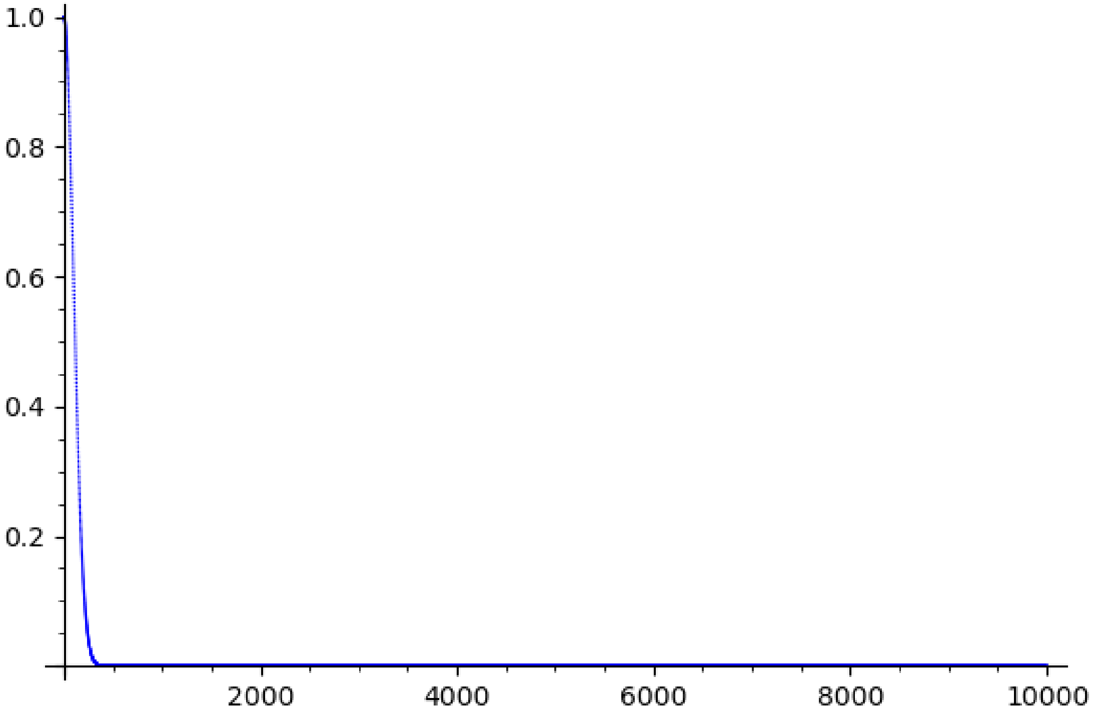
[1]:

```
m = 10**4
Pts = []
proba = 1
for n in range(m):
    proba = proba * ( (m-n) / m )
    Pts.append((n, proba))
```

[2]:

```
point(Pts, size=1)
```

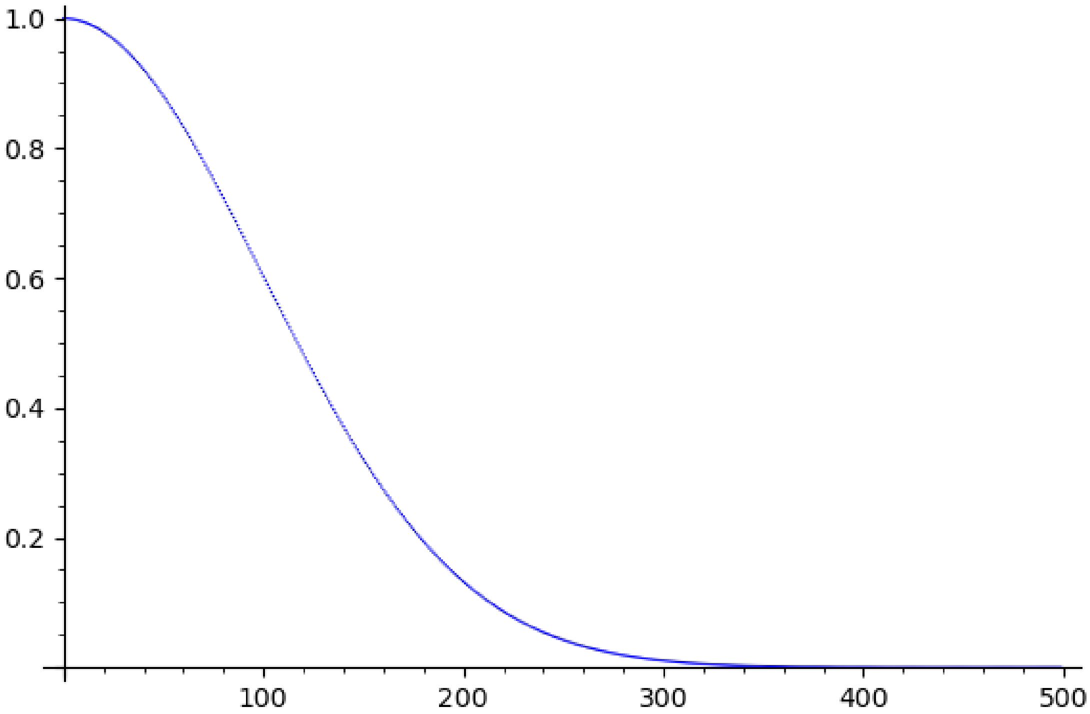
[2]:



[3]:

```
point(Pts[:500], size=1)
```

[3]:

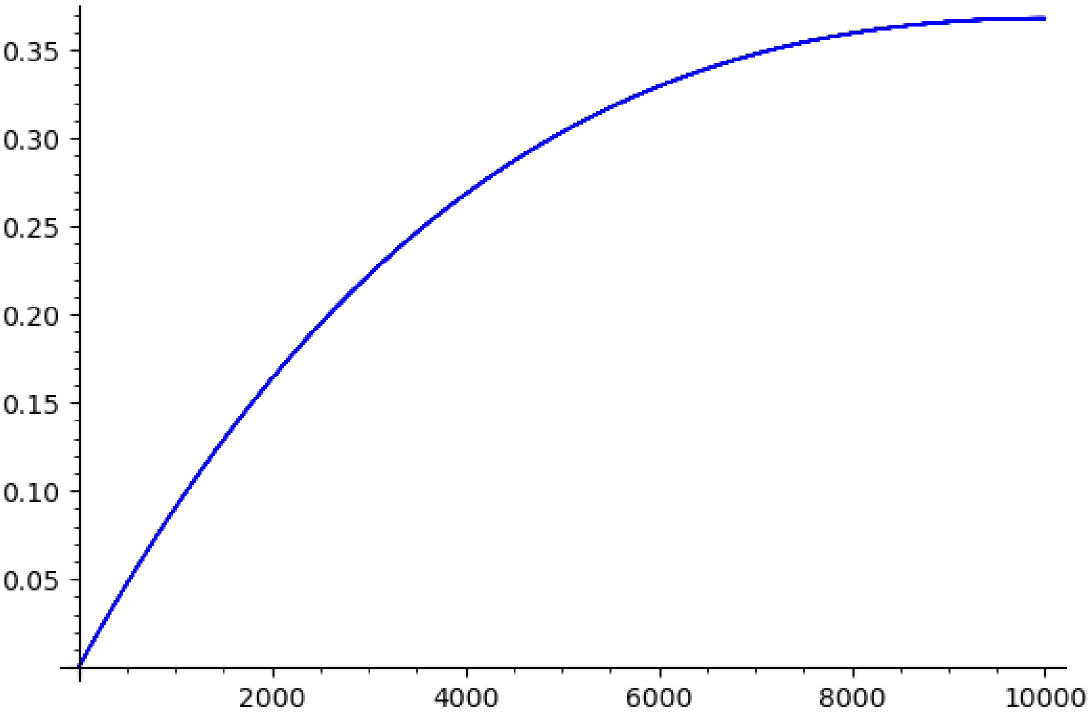


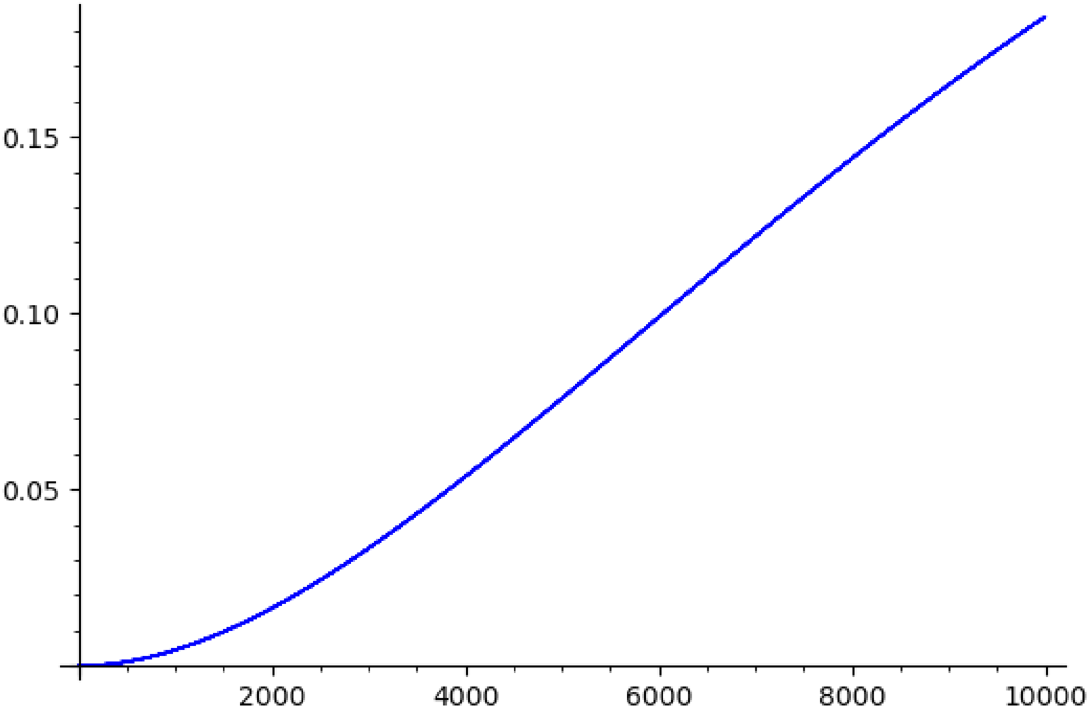
Probabilité que k clé aient la même valeur de hachage fixée

(en supposant un hachage uniforme)

[4]:

```
m = 10**4
for k in range(1, 5):
    Pts = []
    proba = 1 / m**k
    for n in range(k, m):
        Pts.append((n, proba))
        proba = proba * ( (n+1)/(n+1-k) * (m-1) / m )
    point(Pts, size=1).show()
```


0.35
0.30
0.25
0.20
0.15
0.10
0.05
2000
4000
6000
8000
10000


0.15
0.10
0.05
2000
4000
6000
8000
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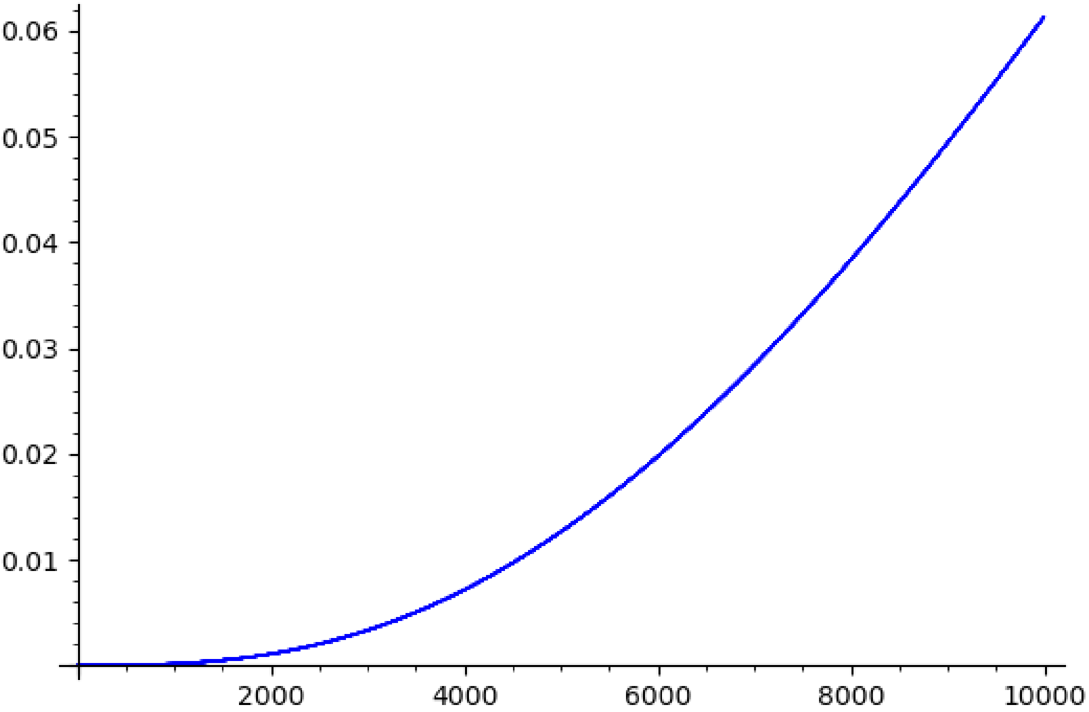
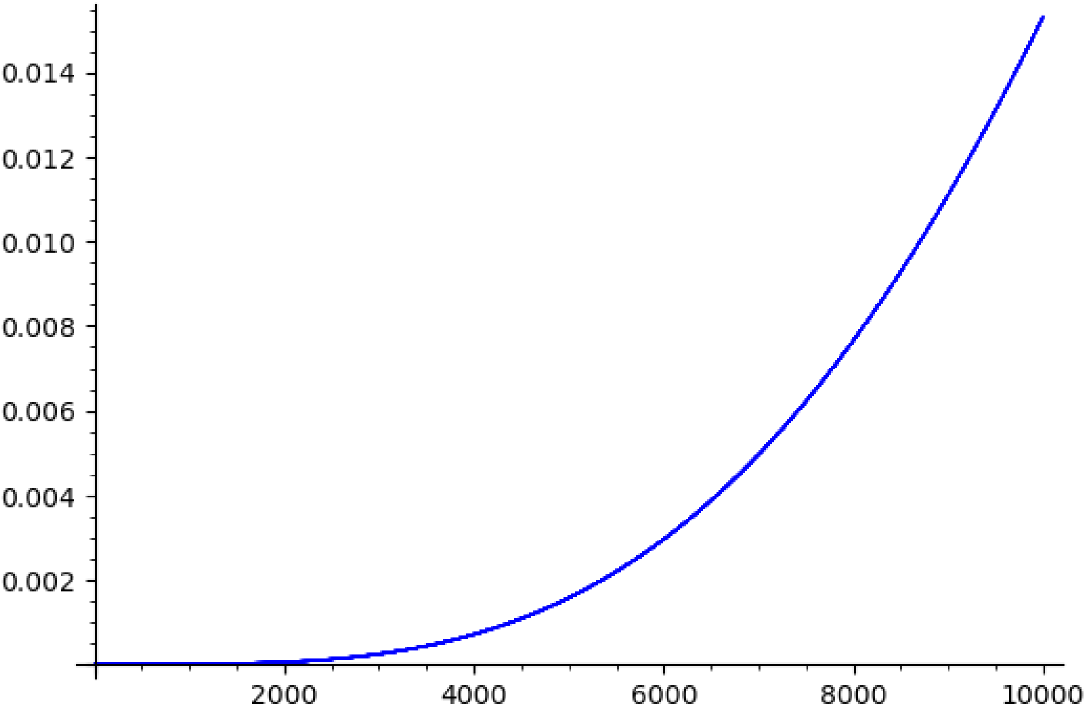
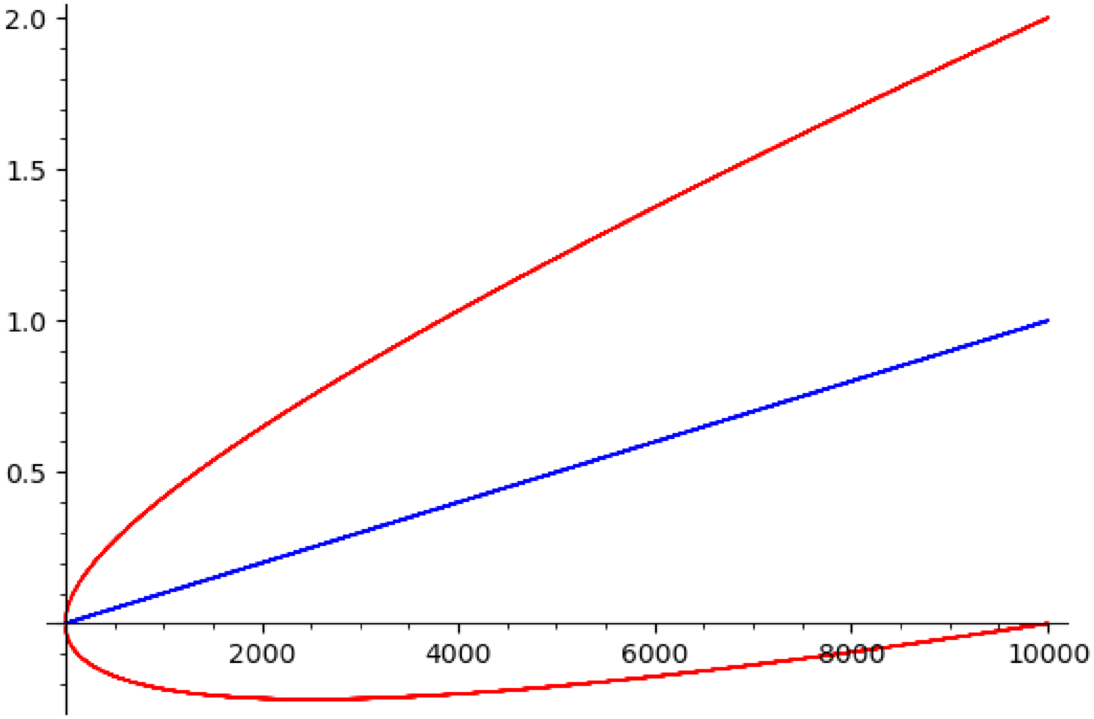

0.06
0.05
0.04
0.03
0.02
0.01
2000
4000
6000
8000
10000


0.014
0.012
0.010
0.008
0.006
0.004
0.002
2000
4000
6000
8000
10000

Moyenne et écart-type du nombre de clés par case

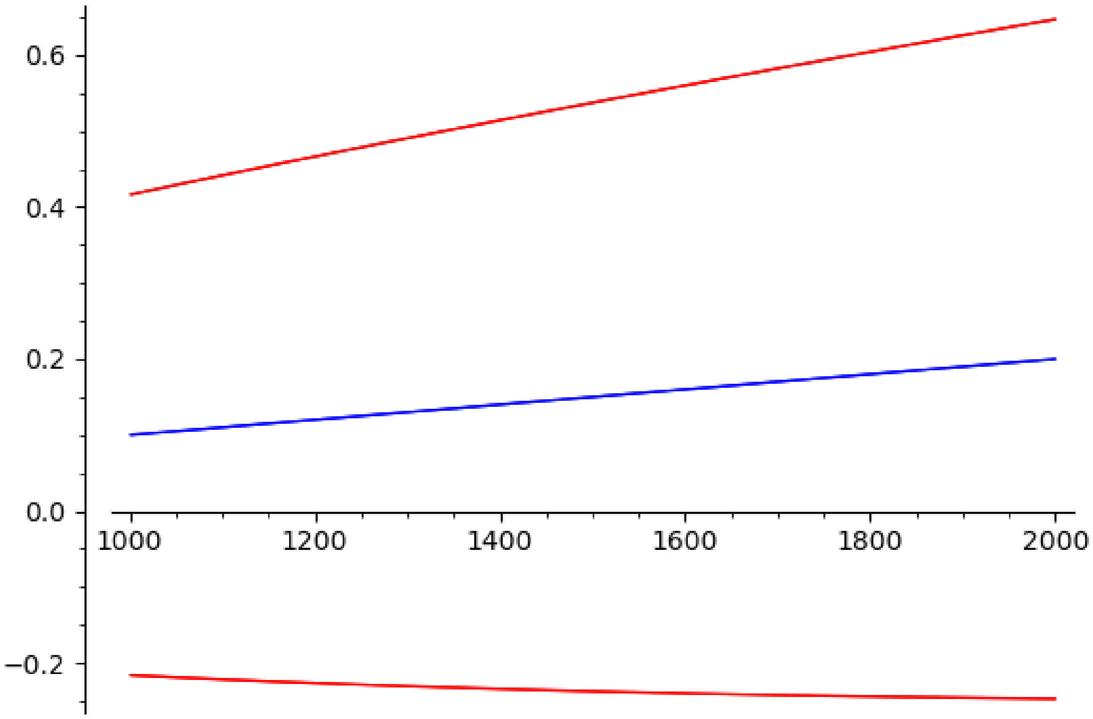
[9]:

```
m = 10**4
Pts = []
Pts2 = []
Pts3 = []
for n in range(1,m):
    proba = n/m
    ecart = sqrt(proba * ((m-1)/m))
    Pts.append((n, proba))
    Pts2.append((n, proba+ecart))
    Pts3.append((n, proba-ecart))
    p1 = point(Pts, size=1)
p2 = point(Pts2, size=1, color='red')
p3 = point(Pts3, size=1, color='red')
show(p1+p2+p3)
```



[10]:

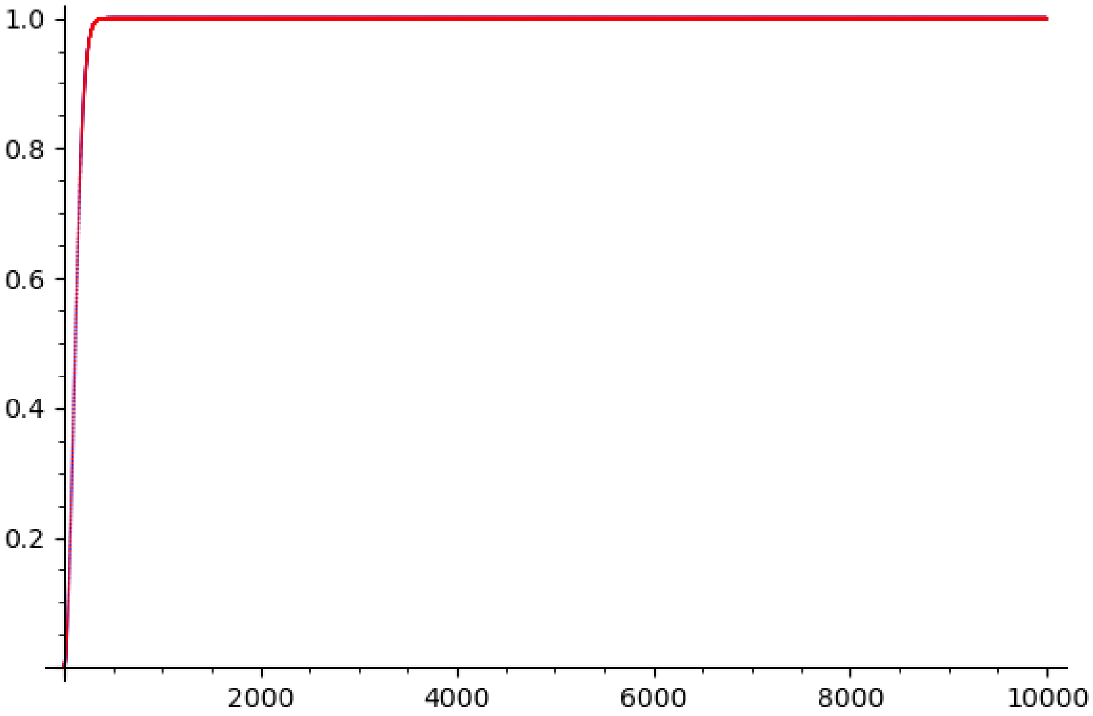
```
p1 = point(Pts[1000:2000], size=1)
p2 = point(Pts2[1000:2000], size=1, color='red')
p3 = point(Pts3[1000:2000], size=1, color='red')
show(p1+p2+p3)
```



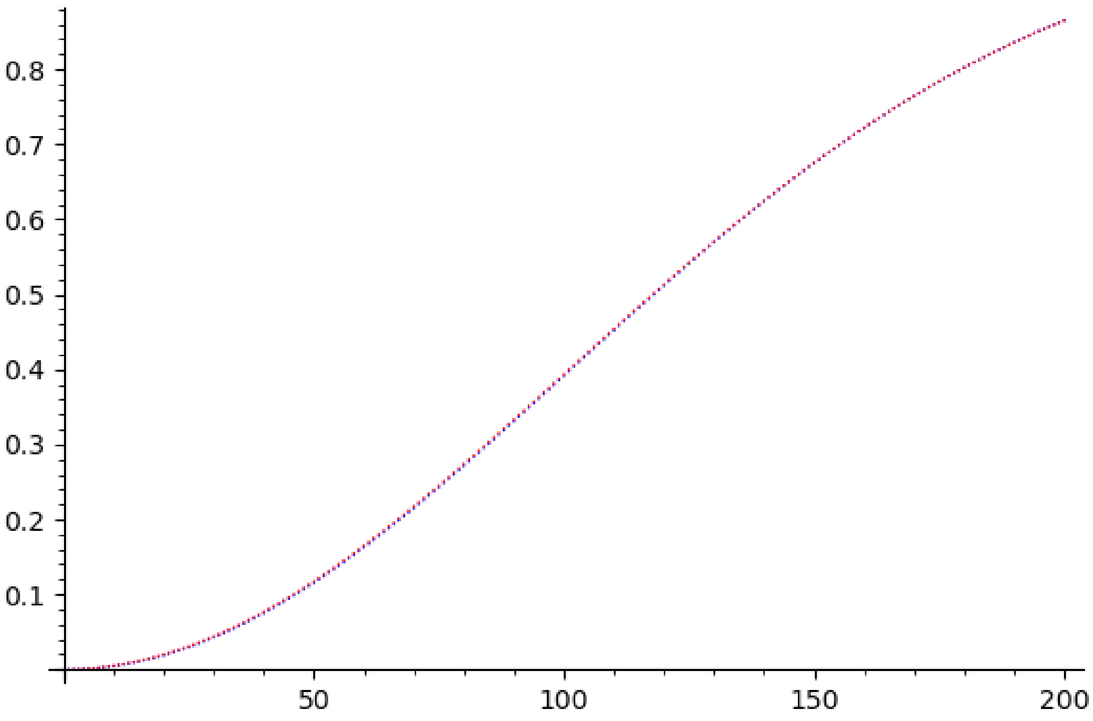
Comparaison Probabilité exacte et approximation qu'il y ait au moins une collision

(paradoxe des anniversaire)

[11]:
```
m = 10**4
Pts = []
App = []
p = 1
for n in range(1,m):
    proba = 1-p
    p = p * (1-n/m)
    Pts.append((n, proba))
    approx = 1-exp(-n**2/(2*m))
    App.append((n, approx))
p1 = point(Pts, size=1)
p2 = point(App, size=1, color='red')
show(p1+p2)
```



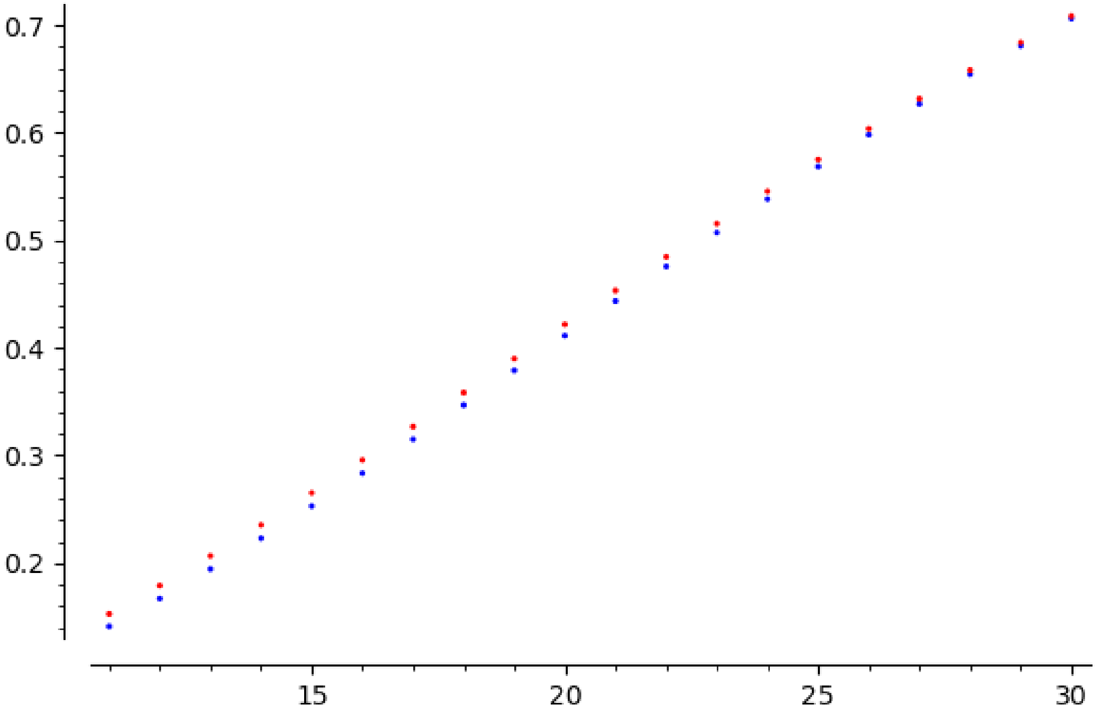
```
[12]: p1 = point(Pts[:200], size=1)
      p2 = point(App[:200], size=1, color='red')
      show(p1+p2)
```



[ ]:

[13]:

```
m = 365
Pts = []
App = []
p = 1
for n in range(1,m):
    proba = 1-p
    p = p * (1-n/m)
    Pts.append((n, proba))
    approx = 1-exp(-n**2/(2*m))
    App.append((n, approx))
p1 = point(Pts[10:30], size=5)
p2 = point(App[10:30], size=5, color='red')
show(p1+p2)
```

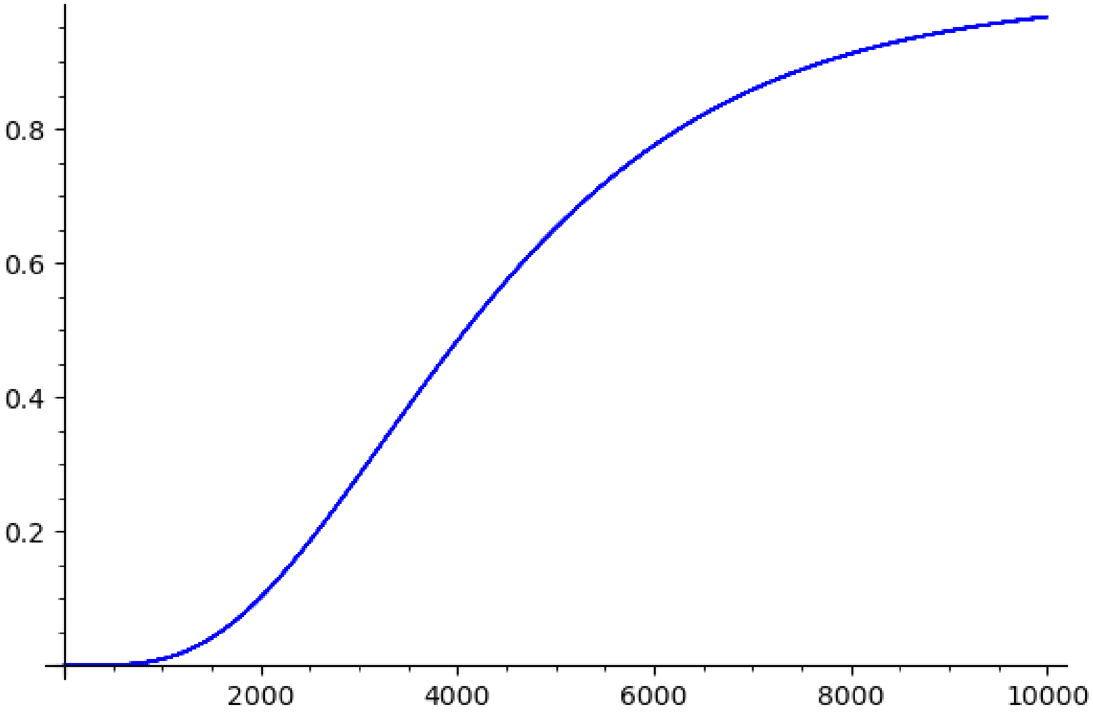


```
[14]: (sqrt(2*ln(2))).n()
```

```
[14]: 1.17741002251547
```

Probabilité d'erreur filtre de Bloom

```
[22]: m = 10**4
      for k in range(5, 55, 10):
          Pts = []
          for n in range(k, m):
              proba = (1 - exp((-k*n)/m))**k
              Pts.append((n, proba))
          point(Pts, size=1).show()
```


0.8
0.6
0.4
0.2
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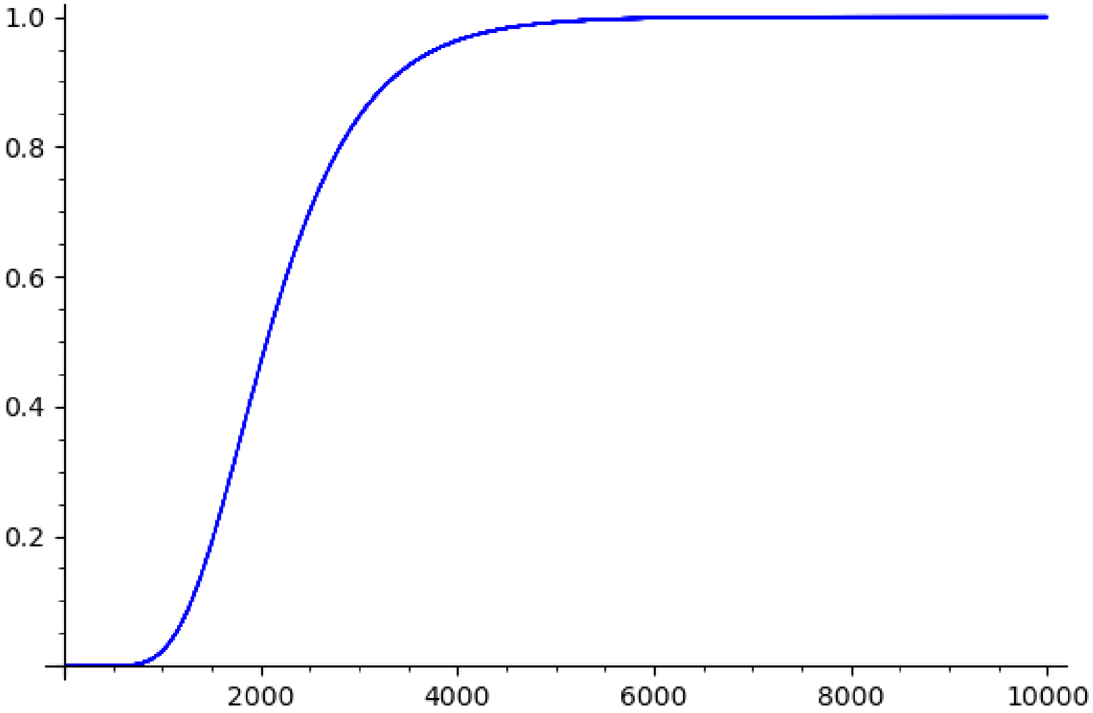
1.0
0.8
0.6
0.4
0.2
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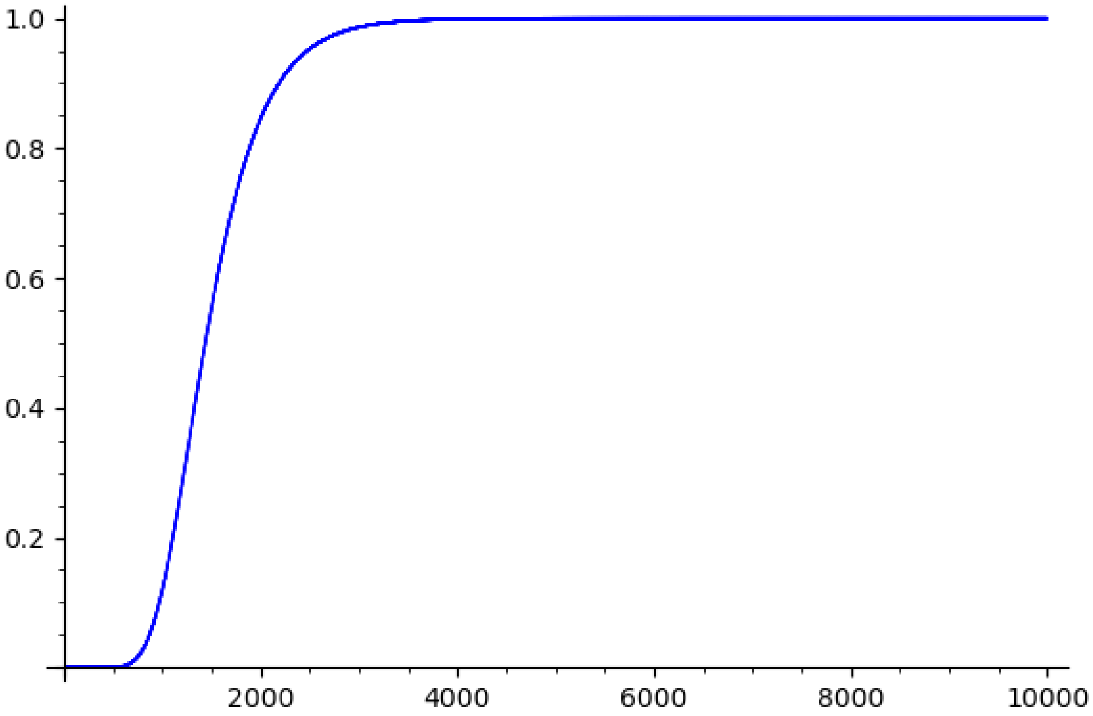
1.0
0.8
0.6
0.4
0.2
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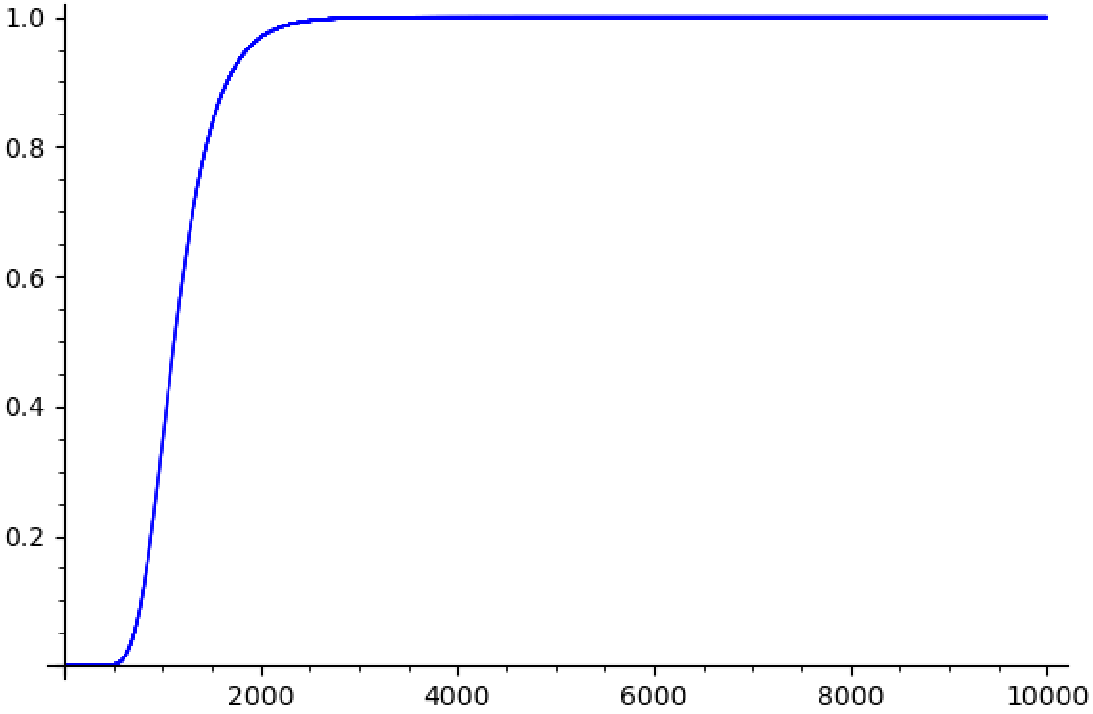
1.0
0.8
0.6
0.4
0.2
2000
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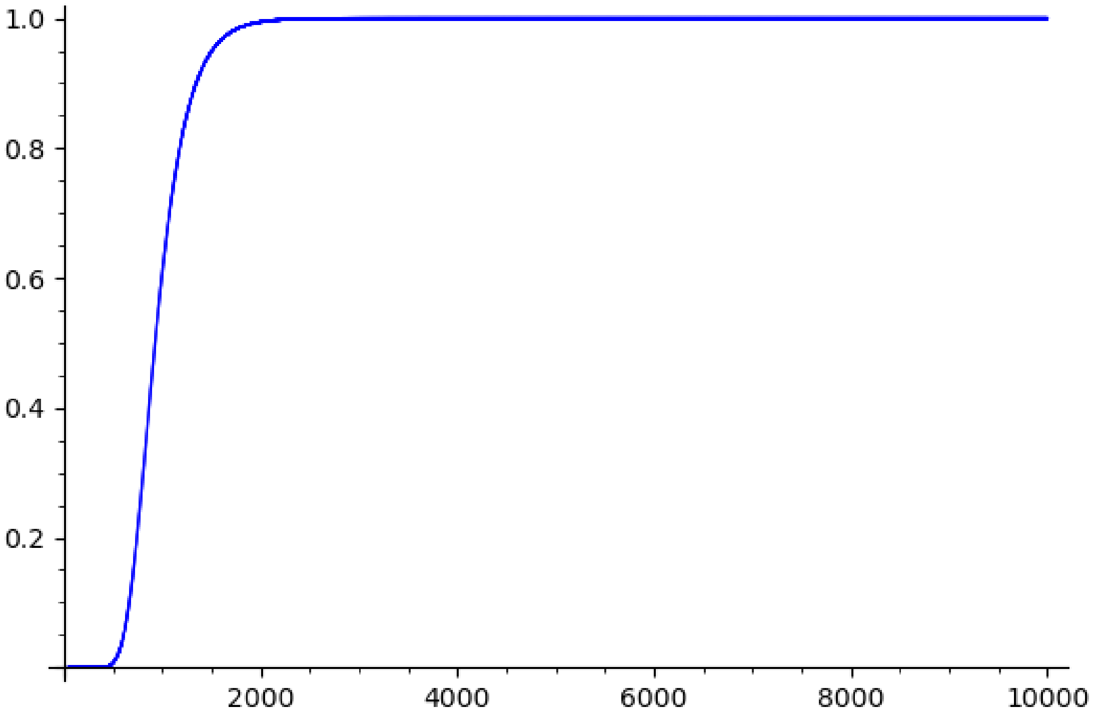
1.0
0.8
0.6
0.4
0.2
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[ ]: